Childcare Support and Economic Growth in an Ultra-Declining Birthrate Society

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Abstract This paper analyzes weather public capital investment or childcare support maximize growth rate in an ultra-declining birthrate society using a labor augmented model with the public capital. We clarify the global stability of the private capital-public capital ratio to the steady state. In addition, we analyze the effect of increasing expenditure share of a tax revenue on the economic growth. The result of this analysis shows that increased share on the public capital investment brings the higher economic growth. This means that if all tax revenue is allocated to the public capital investment, the growth rate will be maximized.

Keywords: Public capital investment・Childcare support・Income tax・Economic growth

JEL classification: D91・E62・O41

1. Introduction

The number of children continues to decrease. The total fertility rate was 1.36[2] in 2019-the lowest level to date-as indicated by the Ministry of Health, Labor and Welfare (MHLW) in Japan. According to the Cabinet Office continues to insist that Japan is already in the state the ultra-birthrate declining society in a long time. The society will become that one out of 2.5 people is among the elderly (aged 65 or older) by 2050.[3] Viewing life in the long term, workers should determine their spending based on their estimated lifetime income. According to the overlapping generation model advocated by Diamond (1965), a person's lifetime income consists of earnings in the two periods: during their worker's period and in later life. Individuals make decisions today from a lifetime perspective within a budget constraint. The number of children in the developed country will decline is shown by Becker (1981) and Becker and Lewis(1973). At first glance this to be a contradiction when considering the children as a good. This is because the price of a childcare cost is proportional to scale of the quantity multiplied a quality. On this study, models are established based on a neoclassical theory that the

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[2] "Current population survey," MHLW website (https://www.mhlw.go.jp/toukei/list/81-1a.html) (accessed on September 20, 2020)

[3] "Situations of Aging" (Japanese), Cabinet Office website (https://www8.cao.go.jp/kourei/whitepaper/w-2012/zenbun/pdf/1s1s_1.pdf) (accessed on June 15, 2020)

growth of the capital will drive up the gross domestic product (GDP) and leads to a greater growth rate of the whole nation. This article's main portion utilizes Romer's endogenous growth model (1986) to introduce the public capital which is proposed by Barro (1990), Barro and Sala-i-Martin (1992), Futagami et al. (1993), Turnovsky (1997), and Yakita (2008) and Maebayashi (2013). They indicate that the public capital stock boosts the labor productivity. The financial resource of the public capital investment is an income tax (labor income and capital income). Yakita (2008) considers two public expenditures both the public capital investment and public capital maintenance using the birth rate internalization model. Maebayashi (2013) shows the dynamics of the private-public capital ratio and certification in the existence of the steady state and the global stability to there. Furthermore, he analyzes the optimal allocation of the tax revenue between the expenditure on the public capital investment and the public pension subsidy in pay-as-you-go pension system. His insistency is clear that the best policy for the growth is to allocate all financial resources to the public capital investment. However, from the perspective of the social welfare, he shows that the optimal allocation rate in the tax revenue can be derived depending on the scale of the social discount rate.

In this paper, we analyze priority policy between the public capital investment and childcare support in the growth rate under the government budget constraints which are the income tax revenues only.

First, we proof the existence of a steady state and the economy will converge to the steady state globally and stably. And we show that all variables the public capital, private capital, and GDP grow at the same rate on the balanced growth path (BGP). Second, we analyze the effect of increasing the share of the public capital investment on growth under constant tax revenue. Using a numerical example, the growth will be positive. If we interpret it intuitively without using it, keywords will be the elasticity of the increasing share of public capital investment for the relative ratio scale on the private-public capital and the share of labor on GDP. That is, the absolute value of elasticity is less than 1 for the marginal increase of the public capital investment. This means that the effect of rising the wage rate due to the increase in the public capital has not contributed so much to the increase in savings. In this case, a considerably large labor share in GDP is required for positive growth. This reason for this is very clear. First, this depends on the shape of the utility function shown in the linear logarithm. The use of this function means that savings depend only on income in the first period, not on interest rate. In other words, the substitution effect and the income effect cancel each other out, and the effect on savings against changes in the interest rate becomes zero. Second, the childcare support as a government child rearing support measure does not contribute to the increase in the labor force. In this analysis, the public pension system and long-term care insurance system in the social security system are not considered, so there is no externality to the parent generation. Therefore, the incentive for parents to have children is considered to be consumer goods, not capital goods, form economic point of view. That's why the size of the child's preference rate plays a very important role. In fact, this model shows that the number of children matches the parent's preference for children, regardless of the government's childcare support. This argues that no matter how much the government implements a child-rearing support policy, it will not lead to an increase in the number of children and, by extension, the labor force in Japan as a whole in the future unless the parents' love for their children is greatly poured. In other words, even if the allocation of the childcare support as a countermeasure against the declining birthrate is increased under certain financial resources, it does not lead to an increase in the labor force in the future.

Because we assume the log-linear type utility function, the savings does not depend on the interest rate. That is, it only depends on the income in the first period. Therefore, the effect of the increase in the rising the public capital on the private capital is not reflected. I construct the model using the Diamond model (1965), which has a two-period overlapping generations model. We introduce the public capital stock to construct the model which has labor augmented production technology.

The remainder of this paper is organized as follows. The next section presents the model indicating the dynamic system of capital-private and public capital. I also clarify the globally stability of the dynamics to the steady state. I attempt to show the effect that occurs when the government increases income tax and public capital investment shares in the steady state. The last section concludes the paper.

2. Model

2.1 Individuals

I consider the model, which is two-period OLG model presented by Diamond (1965). All markets are fully competitive. I assume a homogeneous consumer who obtains utility from consumption in the working and old periods and the number of children. We consider the child as consumer goods not capital goods and there is no public pension. Therefore, there is no self-denial. He supplies labor inelastic in only the first period, where I assume that every consumer has one unit of labor and supplies to the labor market. He allocates income for consumption, saving, and childcare costs in the first period; he also consumes all in the first period, and consumes all income, such as saving and interest with no bequests in the second period. I assume a logarithmic linear utility function and lifetime budget constraint as follows: time preference, child preference, childcare cost, childcare support, and income tax are indicated by $\rho \in (0,1)$, $\varepsilon > 0$, $z \in (0,1)$, $h_t \in (0,1)$, $z \geq h_t$, and $n_t \geq 1$ which must hold in order for the economy to be sustainable in the long term.

$$max. \quad u_t = logc_t + \rho logd_{t+1} + \varepsilon logn_t \tag{1}$$

$$s.t \quad w_t\,(1-\tau)[1 - n_t(z - h_t)] = c_t + \frac{d_{t+1}}{r_{t+1}(1-\tau)} \tag{2}$$

$$c_t^* = \frac{(1-\tau)w_t}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]} \tag{3}$$

$$n_t^* = \frac{\varepsilon}{[1+\varepsilon+\rho r_{t+1}(1-\tau)](z-h_t)} \tag{4}$$

$$d_{t+1}^* = \frac{\rho r_{t+1} w_t (1-\tau)^2}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]} \tag{5}$$

$$s_t^* = \left\{1 - \frac{(1+\varepsilon)}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]}\right\}(1-\tau)w_t \tag{6}$$

2.2 Production

I consider Cobb-Douglas production technology, where labor increases with a public capital investment, as in Romer (1986). I assume that there are many firms in the good market, and these firms have the same technology. Inputs are private capital stock and labor. The production function of firm i is indicated as follows:

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha} \tag{7}$$

$$A_t = \frac{G_t}{L_t} \tag{8}$$

$$Y_t = K_t^{\alpha} {G_t}^{1-\alpha} = \left(\frac{K_t}{G_t}\right)^{\alpha} G_t = x_t^{\alpha} G_t \tag{9}$$

The labor force in the t period is indicated by follows.

$$L_t = N_t[1 - n_t(z-h_t)] \tag{10}$$

Here, we consider about the population. The number of populations in the t+1 period is indicated as follows.

$$N(t+1) = N_t + N_{t+1} = n_t N_0 + n_{t+1} N_t \tag{11}$$

Where N_t indicates the number of households in the t period. Here, the generations born in the t period don't work in the t+1 period. Therefore, the labor force in the t period, L_t is indicated as follows.

$$L_t = N_t\left[1 - \frac{\varepsilon}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]}\right] \tag{12}$$

The growth of the labor force at t period can be derived using the equation (10) and (11). And it is indicated as follows.

$$g_L = \frac{L_{t+1}}{L_t} = \frac{\left[1 - \frac{\varepsilon}{[1+\varepsilon+\rho r_{t+2}(1-\tau)]}\right]}{\left[1 - \frac{\varepsilon}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]}\right]} \frac{N_{t+1}}{N_t} \tag{13}$$

We assume a perfect competitive market and solve the problem of profit maximization as follows:

$$(1-\alpha)\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} A_t^{1-\alpha} = w_t \tag{14}$$

$$\alpha\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} A_t^{1-\alpha} = r_t \tag{15}$$

From (14) and (15), the private capital-labor ratio will become the same value as in $K_{ti}/L_{ti} = K_t/L_t$. It derives $\sum_{i=1}^{\infty} L_{it} = L_{t,}$ $\sum_{i=1}^{\infty} K_{it} = K_t$. L_t, K_t assigning the total labor supply and total private capital. Defining a new variable as $x = \frac{K}{G}$, the ratio of private and public capital. Then, (14) and (15) are rewritten by the following equations.

$$(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\alpha} \frac{G_t}{L_t} = (1-\alpha)x_t^{\alpha}\frac{G_t}{L_t} = w_t \tag{16}$$

$$\alpha\left(\frac{K_t}{G_t}\right)^{\alpha-1} = \alpha x_t^{\alpha-1} = r_t \tag{17}$$

2.3 Government

The government taxes on the income and divide tax revenues for the public capital investment and the childcare support, $E > 0$, $H > 0$. The share of spending on the public capital investment and the income tax rate are shown as $\varphi \in [0,1]$, $\tau \in [0,1]$. And the depreciation rate of the public and private capital is 1. Government budget constraint is shown as the following equations.

$$E_t + H_t = \tau Y_t = \tau x_t^{\alpha} G_t \tag{18}$$

$$E_t = G_{t+1} - G_t = \varphi\tau Y_t = \varphi\tau x_t^{\alpha} G_t \tag{19}$$

$$w_t h_t n_t N = (1-\varphi)\tau Y_t = (1-\varphi)\tau x_t^{\alpha} G_t \tag{20}$$

The per capita childcare support is indicated by (21) using (20) and it is indicated by the next equation and the value will be constant.

$$h_t = \frac{(1-\varphi)(1-\varepsilon z)\tau}{\varepsilon[1-(1-\varphi)\tau]} \tag{21}$$

Here, using the equation (20), rewrite the equation (11), which indicates the labor force in the period t, as follows.

$$L_t = N_t\left[1 - \frac{\varepsilon}{[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]}\right] \tag{22}$$

This equation implies that the labor force in the t period does not depend on the childcare support. Furthermore, not only it but also the share of childcare support expenditure on the governmental tax revenue. This clearly means that the government can only intervene the childcare support through the public capital investment. And it suggests that the labor force will continue to decline in the future, even if the late marriage is resolved and the preference for having children increases.
Next, let's use the equation (22) to see the labor growth.

$$g_L = \frac{L_{t+1}}{L_t} = \frac{[1+\rho\alpha x_{t+2}^{\alpha-1}(1-\tau)][1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]}{[1+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)][1+\varepsilon+\rho\alpha x_{t+2}^{\alpha-1}(1-\tau)]}\frac{N_{t+1}}{N_t} \tag{23}$$

Where the number of household in t+1 period is shown as $N_{t+1} = N_t n_t$ and the number of children is constant in the steady state. Therefore, the equation (23) can be written by the next equation which indicated the growth of the labor force as the number of children in the steady state.

$$g_L = \frac{L_{t+1}}{L_t} = \frac{n_t N_t}{N_t} = n_t. \tag{24}$$

2.4. Equilibrium

There are three market and we consider only capital market by Walras' Law. The equilibrium condition is as follows.

$$s_t L_t = K_{t+1} \tag{25}$$

We substitute the optimal savings (6) for the equilibrium condition (25) and substitute there for the wage rate (16) and the interest rate (17). Then, these allow us to rewrite the condition (25) to as the next equation.

$$K_{t+1} = \left\{1 - \frac{(1+\varepsilon)}{[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]}\right\}(1-\tau)(1-\alpha)x_t^{\alpha}G_t \tag{26}$$

And we can get the equation (27) by dividing both sides of the equation (26) by K_t.

$$g_K = \frac{K_{t+1}}{K_t} = \left\{\frac{[\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)^2](1-\alpha)x_t^{\alpha-1}}{[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]}\right\} \tag{27}$$

We can obtain the dynamics of the private capital.

2.5. Dynamics

The dynamics of the public capital is indicated by the equation (28).

$$g_G = \frac{G_{t+1}}{G_t} = \varphi\alpha\tau x_t^{\alpha} + 1 \tag{28}$$

The growth of x is indicated by the next equation which is made by using the capital dynamic equations (27) and (28).

$$g_x = \frac{x_{t+1}}{x_t} = \frac{\frac{K_{t+1}}{K_t}}{\frac{G_{t+1}}{G_t}} = \frac{[\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)^2](1-\alpha)x_t^{\alpha-1}}{(\varphi\alpha\tau x_t^{\alpha}+1)[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]} \tag{29}$$

Let's put $[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)]$ as $\emptyset > 0$. Then the equation (29) can be written to follows.

$$g_x = \frac{x_{t+1}}{x_t} = \frac{\frac{K_{t+1}}{K_t}}{\frac{G_{t+1}}{G_t}} = \frac{[\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)^2](1-\alpha)x_t^{\alpha-1}}{(\varphi\alpha\tau x_t^{\alpha}+1)\emptyset} \tag{30}$$

$$\frac{\partial x_{t+1}}{\partial x_t} = \frac{A(x_t, x_{t+1})}{B(x_t, x_{t+1})} = f(x_t, x_{t+1}) = +0.081 > 0 \tag{31}$$

$$A(x_t, x_{t+1}) = \rho\alpha^2 x_{t+1}^{\alpha-1} x_t^{\alpha-1}(1-\tau)^2(1-\alpha) - \emptyset x_{t+1}\varphi\alpha^2\tau x_t^{\alpha-1} = -0.12 < 0 \tag{32}$$

$$B(x_t, x_{t+1}) = \rho\alpha(1-\alpha)^2(1-\tau)^2 x_{t+1}^{\alpha-2} x_t^{\alpha} + \emptyset(\varphi\alpha\tau x_t^{\alpha}+1) - \rho\alpha(\varphi\alpha\tau x_t^{\alpha}+1)(1-\tau)(1-\alpha)x_{t+1}^{\alpha-1} = -1.49 < 0 \tag{33}$$

Here, in order for the signs of "A" and "B" in the equation (31) to be positive, we will quantify the parameter in the equations (31) and (32) concretely as $(\alpha, \varepsilon, \rho, \tau, x, z, \varphi) = (0.5, 0.05, 0.95, 0.3, 3.2, 0.06, 0.83)$. Here, we put the extreme numerical examples such as $\varepsilon = 0.05, \rho = 0.95$ and so on. This is due to the following reasons. The engine of growth clearly the public capital investment in this model. Therefore, in order to connect the wage rate and interest rate pushed up by the public capital investment to higher growth, it is necessary to supply more labor time, that is, lower the opportunity cost for childcare, or higher the preference rate for future consumption. Next, we derive the second derivative of the equation (30). When x_t approaches 0, the growth of x will be 0 in the equation (30).

$\left(\lim_{x_t \to 0} \frac{x_{t+1}}{x_t} = 0\right)$. In other words, this shows that the curve in the graph passes through the origin.

$$\frac{\partial^2 x_{t+1}}{(\partial x_t)^2} = \frac{\partial f(x_t, x_{t+1})}{\partial x_t} = \frac{A'B - AB'}{B^2} = -0.019 < 0 \tag{34}$$

$$A' = \frac{\partial A(x_t, x_{t+1})}{\partial x_t} = -\rho\alpha^2 x_t^{\alpha-2} x_{t+1}^{\alpha-1}(1-\tau)^2(1-\alpha) + \emptyset(1-\alpha)x_t^{\alpha-2}x_{t+1}\varphi\alpha^2\tau = 0.016 > 0 \tag{35}$$

$$B' = \frac{\partial B(x_t, x_{t+1})}{\partial x_t} = \rho\alpha^2(1-\alpha)^2(1-\tau)^2 x_{t+1}^{\alpha-2} x_t^{\alpha-1} + \emptyset\varphi\alpha^2\tau x_t^{\alpha-1} - \rho\alpha^3\varphi\tau x_t^{\alpha-1}(1-\alpha)(1-\tau)x_{t+1}^{\alpha-1} = 0.045 > 0$$

$$\lim_{x_t \to 0} \frac{dx_{t+1}}{dx_t} = \infty \qquad \lim_{x_t \to \infty} \frac{dx_{t+1}}{dx_t} = 0 \tag{36}$$

The ratio of private-public capital will increase and the steady state of x is shown as x^*. If the equation (36) is satisfied with x^* the growth rate of GDP, the private capital and public capital will be the same.

$$[\rho\alpha(x^*)^{\alpha-1}(1-\tau)^2](1-\alpha)(x^*)^{\alpha-1} = (\varphi\alpha\tau(x^*)^\alpha + 1)[1 + \varepsilon + \rho\alpha(x^*)^{\alpha-1}(1-\tau)] \tag{37}$$

Proposition 1. There is a unique value which shows the ratio of the public-private capital in the steady state. If the equation (36) is satisfied, the public capital, private capital and GDP will grow at the same rate. That is we can gain the balanced growth path and it is globally stable.

$$\frac{\partial g}{\partial \varphi} = \tau\alpha^2(x^*)^\alpha \left[\frac{1}{\alpha} + \frac{\varphi}{x^*}\frac{dx^*}{d\varphi}\right] \tag{38}$$

$$\frac{dx^*}{d\varphi} = \frac{A}{B} > 0 \tag{39}$$

$$A = \varphi\alpha\tau(x^*)^\alpha\emptyset > 0 \tag{40}$$

$$B = -2(1-\alpha)^2(1-\tau)^2(x^*)^{2\alpha-3}\rho\alpha + [\varphi\alpha\tau(x^*)^\alpha + 1](1-\alpha)(x^*)^{\alpha-2}(1-\tau)\rho\alpha + \varphi\alpha^2\tau(x^*)^{\alpha-1}\emptyset > 0 \tag{41}$$

Where the second item in brackets means the elasticity of the share for the relative capital value and the sign will be positive. That is, the increment of the share on public capital investment raises the private-public capital scale. These indicate that the economy will grow regardless of the share of the private capital on GDP or the size of elasticity of the share.

Proposition 2. The economy will grows independently of the share for private capital on GDP or the elasticity of the allocation rate to private and public capital.

$$\frac{\partial g}{\partial \varphi}=\tau\alpha^2\left[\frac{2(1-\alpha)^2(1-\tau)^2(x^*)^{2\alpha-2}\rho+(\varphi\alpha\tau(x^*)^\alpha+1)(1-\alpha)(x^*)^{\alpha-2}(1-\tau)\rho+[\tau(x^*)^{-1}-\varphi(x^*)^\alpha]\varphi\alpha\emptyset}{2\alpha(1-\alpha)^2(1-\tau)^2(x^*)^{\alpha-2}\rho+(\varphi\alpha\tau(x^*)^\alpha+1)(1-\alpha)(x^*)^{-2}(1-\tau)\rho+\varphi\alpha\tau(x^*)^{-1}\emptyset}\right]>0 \tag{42}$$

The increase on the share of the public capital investment generates an ascending public capital. At the same time, it increases both the wage rate and interest rate. These lead to an upsurge in the private capital through the two effects on the income and the price. The policy of spending all tax revenue on the public capital investment will bring the first best results for growth.

Proposition 3. The policy of spending all tax revenue on public capital investment will be the first best policy for growth.

7. Concluding remarks

This study focuses on the relative value of the private-public capital with a policy of the childcare support. First, we clarify the global stability on the economic growth and there is the unique steady state which the economy converges. In the steady state the economy is on the balanced growth path, where the private capital, public capital and GDP grow at the same rate. Second, we analyze the effect of increasing the share on the public capital investment for the growth rate in the steady state. It does not depend on both the absolute value of the elasticity of increasing share on the public capital investment for the relative capital value and the share of the labor on GDP. This is because the expansion of both the interest rate and the wage with the rise of the public capital pushes up the relative capital value. That is, it is clear that the economy will grow as the public capital increases.

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