How Does Automation Affect Economic Growth and Income Distribution in a Two-Class Economy?

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Hiroaki Sasaki* Takefumi Hagiwara Pham Thu Giang Huong
Noriki Fukatani Shogo Ogawa Naoto Okahara

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Abstract

This study uses a growth model with automation technology to consider two classes—workers and capitalists—and investigates how advances in automation technology affect economic growth and income distribution. In addition to the two production factors labor and traditional capital, we consider automation capital as the third production factor. We also introduce Pasinetti-type saving functions into the model to investigate how the difference between the capitalists' and workers' saving rates affect economic growth and income distribution. When the capitalists' saving rate is higher than a threshold level, per capita output exhibits endogenous growth irrespective of the workers' savings rate. In this case, the income gap between workers and capitalists widens over time. When the capitalists' saving rate is less than the threshold level, two different long-run states occur depending on the workers' saving rate: the capitalists' own automation capital share approaches a constant, and it approaches zero. In both cases, the per capita output growth is zero and the income gap between the two classes becomes constant over time.

Keywords: automation technology; endogenous growth; income distribution

JEL Classification: E25; O11; O33; O41

1 Introduction

Automation technology has recently seen several advances such as artificial intelligence (AI) and robots. Technologies such as autonomous cars, face recognition systems, and high-

*Corresponding author. Graduate School of Economics, Kyoto University. Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: sasaki@econ.kyoto-u.ac.jp

frequency trading can operate without human support and substitute for human labor. How do advances in automation technology affect an economy? This study uses a theoretical model to investigate how the introduction of AI and robots affects economic growth and income distribution by incorporating two classes—workers and capitalists.

Frey and Osborne (2013, 2017) point out the possibility of AI and robots substituting human labor, and investigate how automation technology can affect the US labor market. They estimate that around 47% of labor can be substituted in the future. The McKinsey Global Institute (2017) considered more than 800 jobs and 2000 activities in the United States, to find that around 45% of labor activities can be substituted by AI. The Boston Consulting Group (2015) has predicted that 40–50% of jobs in the United States, the United Kingdom, Canada, and Japan will be substituted by AI and robots by 2025.

More attempts have been made to investigate the effect of automation technology on an economy from a macroeconomics perspective.[1] These works can be classified roughly under two approaches, task-based approach, represented by Daron Acemoglu, and automation capital-based approach, represented by Klaus Prettner.[2]

Acemoglu and Restrepo (2018, 2020) explain the effects of AI and robots on the macroeconomy as follows. AI and robots substitute human labor and decrease labor demand and the wage rate. Correspondingly, they generate new employment opportunities in the labor market, and thus increase the labor demand and wage rate. Thus, AI and robots have a counterbalancing effect. Under specific assumptions, the positive effects of AI and robots dominate their negative effects. Acemoglu and Restrepo (2018, 2020) thus conclude that the introduction of AI and robots does not lead to severe unemployment and wage declines.[3]

Prettner (2019) introduces a new production factor that perfectly substitutes labor, "automation capital" (e.g., AI and robots), differentiating it from "traditional capital" (e.g., machines and factories). To this end, he proposes an augmented Solow growth model with automation capital in the Cobb–Douglas production function. Specifically, he assumes that a representative household saves a constant fraction of income—a portion saved as accumulation of automation capital, and the rest saved as accumulation of traditional capital. The results show that the accumulation of automation capital reduces the wage rate and labor share of the national income and leads to the endogenous per capita output growth, but with no exogenous technological progress.[4]

[1] For the study of mechanization under a growth model, see Zeira (1998).

[2] For growth models with AI and robots, see Aghion et al. (2019).

[3] Some empirical studies report how the introduction of labor-substitutable technology affected employment and the wage rates (Graetz and Michaels 2018; Cords and Prettner 2019; Acemoglu and Restrepo 2020).

[4] Gasteiger and Prettner (2020) build an overlapping generations model with automation capital. They show that endogenous growth cannot be obtained, and that the economy will become stagnant over time. In the overlapping generations model, the working generations earns wage income, but this decreases owing to

This study aims to examine how advances in automation technology that substitute human labor can affect economic growth and income distribution. To this end, we adopt Prettner's approach. The substitution of human labor with automation technology has been shown in an empirical study. DeCanio (2016) empirically analyzes the US cross-section data to estimate the elasticity of substitution between robots and human labor and investigates the effect of automation technology on wage rates. He uses Houthakker's method to avoid problems that could arise from the use of aggregate capital. The study shows that the elasticity of substitution is around 1.9, indicating that the introduction of robots decreases the wage rates.

Studies in the literature have considered whether advances in automation technology affected economic growth. However, no study has examined the effect on income distribution and the income gap between workers and capitalists. Thus, we extend the literature by introducing two classes—workers and capitalists—and examine how advances in automation technology affect the income gap between the two classes and economic growth. As Piketty (2014) points out, wealth is largely concentrated in the hands of a few. Several recent studies focus on issues of disparity and gaps between people in the economy. The introduction of automation capital has affected people differently, leading to income disparities between people.

To incorporate workers' saving in the model, we follow the approach proposed by Pasinetti (1962).[5] Pasinetti (1962) argues that when workers save, they receive interest income by holding capital stock through savings. Thus, the total capital stock of the whole economy consists of the workers' and capitalists' own capital stock. He also shows that at the long-run equilibrium, where workers and capitalists coexist, the profit rate is the natural growth rate divided by the capitalists' saving rate. This is called the Pasinetti theorem, by which the long-run profit rate is independent of the workers' propensity to save.

Furthermore, Samuelson and Modigliani (1966) point out that the Pasinetti theorem assumes that the capitalists' propensity to save is considerably higher than the workers' propensity to save. They also show that unless this implicit assumption is satisfied, the

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the accumulation of automation capital. This decrease in wage income in turn reduces the saving of households, which then decreases the accumulation of traditional capital, to lead to economic stagnation. Moreover, mechanization can decrease the wage rate, stagnate the economy, and thus decrease the social welfare of the future generations. For a detailed discussion on this issue, see Benzell et al. (2015) and Sachs et al. (2015).

[5]Böhm and Kaas (2000) build a discrete time growth model with a Kaldorian saving function, and show chaotic dynamics when the production function takes the Leontief form. Dalgaard and Hansen (2005) extend the Solow growth model using two different saving rates, and show that multiple equilibria occur when the propensity to save from wage is higher than the propensity to save from profit. Saez and Zucman (2016) show that the saving rate inequality in the US economy has increased recently, with the top 1% saving more as a fraction of their income than the top 10–1% and bottom 90%. Stiglitz (1967) presents a two-sector (capital goods and consumption goods), two-class growth model, and investigates the existence and stability of steady-state equilibria.

long-run profit rate will be the natural growth rate times the output capital elasticity of the production function divided by the workers' propensity to save. This is called the "dual theorem."

Thus, in two-class models, the difference in saving rates between two classes would lead to different long-run situations or multiple equilibria.[6]

We extend Prettner's (2019) augmented Solow model and incorporate workers and capitalists with different asset holdings and saving behavior into our model. We divide households into two groups—workers and capitalists. While workers own labor and traditional capital, capitalists own traditional capital and automation capital. The saving rates of the two classes are assumed different, but constant: the saving rate of capitalists is higher than that of workers. The dynamics of our model can be summarized as a set of differential equations of workers' own capital per worker and capitalists' own capital per capitalist. From the saving rates of the two classes, we obtain various long-run situations, summarized as follows.

First, when the capitalists' saving rate is more than a threshold level, per capita output exhibits endogenous growth irrespective of the workers' saving rate. In this case, the income gap between workers and capitalists widens over time.

Second, when the capitalists' saving rate is less than the threshold level, two different long-run states occur depending on the workers' saving rate. In one case, the capitalists' own automation capital share approaches a constant value, and in the other, it approaches zero. In both cases, the per capita output growth is zero, with the income gap between the two classes becoming constant over time.

The remainder of this paper is organized as follows. Section 2 presents our model. Section 3 investigates the dynamics of two-class capital. Section 4 investigates the dynamics of income distribution and growth. Section 5 conducts numerical simulations. Section 6 concludes the paper.

2 Model

Assume an economy producing a final good with labor, traditional capital, and automation capital. The good is used for consumption, investment in traditional capital, and investment in automation capital. This assumption implies that the good can be converted into traditional capital and automation capital at no additional cost. The goods and factor markets are competitive. This economy has two classes, workers and capitalist. Workers own labor and

[6]For two-class models with dynamic optimization of workers and capitalists, see Michl and Foley (2004), Commendatore and Palmisani (2009), Kurose (2021), and Sasaki (2021).

traditional capital, and receive wage income as well as capital income accruing from traditional capital. Capitalists own traditional capital and automation capital, and receive two types of capital income, one accruing from traditional capital, and the other accruing from automation capital. Assume also that L and N_c denote the number of workers and capitalists, respectively. The total population is, therefore, $N = L + N_c$. Suppose the total population increases at the constant rate $n > 0$. Suppose also that the composition of workers and capitalist, L/N and N_c/N, stays constant over time. Then, both L and N_c increase at the rate $n > 0$. Finally, we assume that labor, traditional capital, and automation capital are fully utilized.

2.1 Firms and production

The production function takes the following modified Cobb–Douglas form:

$$Y = F(K, L, P) = K^{\alpha}(L + P)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where K denotes traditional capital, L denotes labor, and P denotes automation capital.

Traditional capital is owned by workers and capitalists.

$$K = K_w + K_c, \tag{2}$$

where K_w and K_c denote the workers' and capitalists' own capital, respectively.

Let w, R^k, and R^p denote the wage rate, rental price of capital, and rental price of automation capital, respectively. Then, the workers' and capitalists' income will be, respectively,

$$\text{Workers' income} = wL + R^k K_w, \tag{3}$$

$$\text{Capitalists' income} = R^p P + R^k K_c. \tag{4}$$

From the profit maximization of firms, their factor prices would be equal to their marginal productivities.

$$w = (1-\alpha)\frac{Y}{L+P} = (1-\alpha)\left(\frac{K}{L+P}\right)^{\alpha}, \tag{5}$$

$$R^k = \alpha\frac{Y}{K} = \alpha\left(\frac{K}{L+P}\right)^{-(1-\alpha)}, \tag{6}$$

$$R^p = (1-\alpha)\frac{Y}{L+P} = (1-\alpha)\left(\frac{K}{L+P}\right)^{\alpha}. \tag{7}$$

From equations (5) and (7), w and R^p are increasing in K but decreasing in P, whereas from equation (6), R^k is decreasing in K but increasing in P.

The production function does not satisfy the Inada conditions, because we have

$$\lim_{P\to 0} R^p = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}, \tag{8}$$

$$\lim_{K\to 0} R^k = \infty. \tag{9}$$

As stated above, R^p is increasing in K, whereas R^k is decreasing in K. Thus, only after traditional capital K becomes sufficiently accumulated will automation capital P begin to accumulate.

Gasteiger and Prettner (2020) impose a no-arbitrage condition between two assets K and P such that $R^k = R^p$. From this, we obtain [7]

$$P = \left(\frac{1-\alpha}{\alpha}\right)K - L \Rightarrow P = \max\left\{0, \left(\frac{1-\alpha}{\alpha}\right)K - L\right\}. \tag{10}$$

When $K > [\alpha/(1-\alpha)]L \equiv \bar{K}$, P will start to accumulate. Therefore, when $0 < K < \bar{K}$, we have $P = 0$, and when $\bar{K} < K$, we have $P > 0$.

From our assumption that the final good is used for consumption, and investment in traditional and automation capital, we obtain $R^k = R^p$. This no-arbitrage condition is similar to the no-arbitrage condition between physical capital and human capital used in the one-sector human capital endogenous growth model presented in Barro and Sala-i-Martin (2003), and leads to the AK growth model.

In sum, the production function leads to

$$Y = \begin{cases} K^{\alpha}L^{1-\alpha} & \text{if } 0 < K < \bar{K} \\ BK & \text{if } \bar{K} \le K,\ B \equiv \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}. \end{cases} \tag{11}$$

The second production function is given by incorporating $L + P = [(1-\alpha)/\alpha]K$ from the no-arbitrage condition into equation (1). Accordingly, if the traditional capital exceeds its threshold level, perpetual output growth can be achieved even without technological progress as long as traditional capital is accumulated.

[7] Heer and Irmen (2019) criticize Prettner (2019) for not using the no-arbitrage condition, and show that using the no-arbitrage condition will endogenize the division of investment between traditional capital and automation capital, which will not be given exogenously.

We rewrite the production functions in per capita terms as

$$y = \begin{cases} k^{\alpha} & \text{if } 0 < k < \bar{k} \\ Bk & \text{if } \bar{k} \leq k, \end{cases} \tag{12}$$

where $y = Y/L = (N/L)(Y/N)$ and $k = K/L = (N/L)(K/N)$ denote the per capita output and traditional capital, respectively, as N/L is constant by assumption.

At equilibrium, by substituting $K/(L+P) = \alpha/(1-\alpha)$ into equations (5)–(7), we have

$$w = R^k = R^p = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \equiv R. \tag{13}$$

The rate of return R takes the minimum value $1/2$ when $\alpha = 1/2$, whereas it takes the maximum value 1 when $\alpha = 0, 1$.

2.2 Households and asset holdings

The asset holdings of workers and capitalists are, respectively,

$$A_w = K_w, \tag{14}$$

$$A_c = K_c + P, \tag{15}$$

where A_W and A_c denote the workers' and capitalists' asset holdings, respectively.

Let s_w and s_c denote the saving rates of workers and capitalists, respectively. Now, the saving of workers, S_w, and capitalists, S_c, are, respectively,

$$S_w = s_w(wL + R^k K_w), \tag{16}$$

$$S_c = s_c(R^p P + R^k K_c). \tag{17}$$

The dynamics of the two kinds of assets are specified as

$$\dot{A}_w = S_w - \delta A_w = s_w(wL + RA_w) - \delta A_w, \tag{18}$$

$$\dot{A}_c = S_c - \delta A_c = s_c R A_c - \delta A_c, \tag{19}$$

where $\delta \in [0, 1]$ denotes the capital depreciation rate. We assume a common capital depreciation rate for traditional and automation capital.

Let $a_w = A_w/L$ and $a_c = A_c/L$. Now, the differential equations of a_w and a_c are, respectively,

$$\dot{a}_w = s_w R(1 + a_w) - (n + \delta)a_w, \tag{20}$$

$$\dot{a}_c = [s_c R - (n + \delta)]a_c. \tag{21}$$

Note that these differential equations are independent of each other. Furthermore, as $a_c = A_c/L = (N_c/L)(A_c/N_c)$ and N_c/L is constant by assumption, a_c is the own asset per capitalist and a_w is the own asset per worker.

3 Dynamics of two-class capital

To further investigate the model dynamics, we rewrite our model's dynamics in terms of $k_c = K_c/L$ and $k_w = K_w/L$. We have

$$a_c = k_c + p, \tag{22}$$

$$a_w = k_w, \tag{23}$$

$$\dot{a}_c = \dot{k}_c + \dot{p} = [s_c R - (n + \delta)](k_c + p), \tag{24}$$

$$\dot{a}_w = \dot{k}_w = s_w R(1 + a_w) - (n + \delta)a_w, \tag{25}$$

$$p = \left(\frac{1 - \alpha}{\alpha}\right)(k_c + k_w) - 1, \tag{26}$$

$$\dot{p} = \left(\frac{1 - \alpha}{\alpha}\right)(\dot{k}_c + \dot{k}_w). \tag{27}$$

Note that because $k_c = K_c/L = (N_c/L)(K_c/N_c)$, $p = P/L = (N_c/L)(P/N_c)$, and N_c/L is constant by assumption, k_c and p are the own traditional and own automation capital per capitalist, respectively. Equation (27) is the dynamic version of the no-arbitrage condition, which is given by differentiating the no-arbitrage condition with respect to time.

From the above equations, we obtain the system of differential equations of k_c and k_w as follows:

$$\dot{k}_c(t) = [s_c R - (n + \delta)]k_c(t) + (1 - \alpha)(s_c - s_w)Rk_w(t) - [\alpha s_c + (1 - \alpha)s_w]R + \alpha(n + \delta), \tag{28}$$

$$\dot{k}_w(t) = s_w R[1 + k_w(t)] - (n + \delta)k_w(t), \tag{29}$$

$$k_c(t) + k_w(t) > \frac{\alpha}{1 - \alpha}. \tag{30}$$

The last inequality comes from the no-arbitrage condition: when $P > 0$, we have $K > [\alpha/(1 - \alpha)]L$, and hence $k = k_c + k_w > \alpha/(1 - \alpha)$.

From the dynamic system, we find that the dynamics of k_w are independent of those of k_s, whereas the dynamics of k_c depend on the dynamics of k_w. Note that depending on the conditions, both k_c and k_w continue to increase over time with no steady state such that $\dot{k}_c = \dot{k}_w = 0$. Thus, we cannot use the Routh–Hurwitz conditions to examine the stability of our dynamic system. Hence, we investigate the dynamics of k_w and k_c separately, assuming that k_w has already reached its steady-state value k_w in the stable case, or that k_w has been increasing exponentially at the rate $s_w R - (n+\delta) > 0$ in the unstable case, as explained below.

As regards the dynamics of k_w, we consider two cases, $s_w < (n+\delta)/R$ and $s_w > (n+\delta)/R$. In the former case, k_w converges to the steady-state value k_w^*. However, in the latter case, k_w continues to increase exponentially at the constant rate $\lim_{k_w \to +\infty} \dot{k}_w / k_w = s_w R - (n + \delta) > 0$ over time. See Figures 1 and 2.

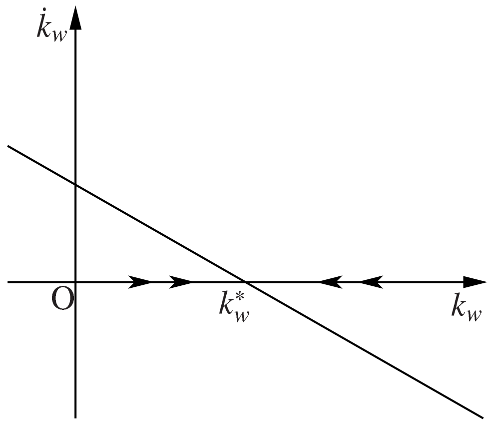

Figure 1: Dynamics of k_w when $s_w < (n + \delta)/R$

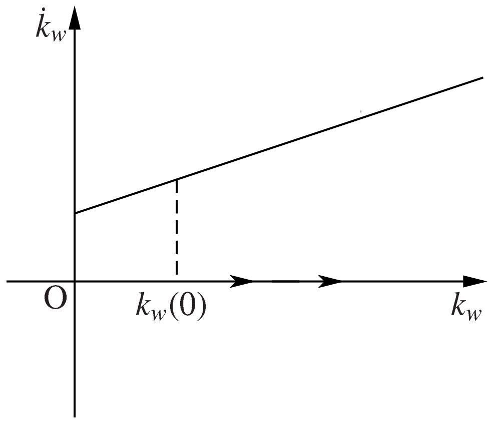

Figure 2: Dynamics of k_w when $s_w > (n + \delta)/R$

From the size of s_w and s_c, we can consider the following five cases:

Case 1-1-1 : $s_w < \dfrac{\alpha(n+\delta)}{R}$ and $\dfrac{n+\delta}{R} < s_c$.

Case 1-1-2 : $\dfrac{\alpha(n+\delta)}{R} < s_w < \dfrac{n+\delta}{R} < s_c$.

Case 1-2-1 : $s_w < \dfrac{\alpha(n+\delta)}{R}$ and $s_c < \dfrac{n+\delta}{R}$.

Case 1-2-2 : $\dfrac{\alpha(n+\delta)}{R} < s_w < s_c < \dfrac{n+\delta}{R}$.

Case 2 : $\dfrac{n+\delta}{R} < s_w < s_c$.

Figure 3 shows these five cases on the (s_c, s_w) plane.

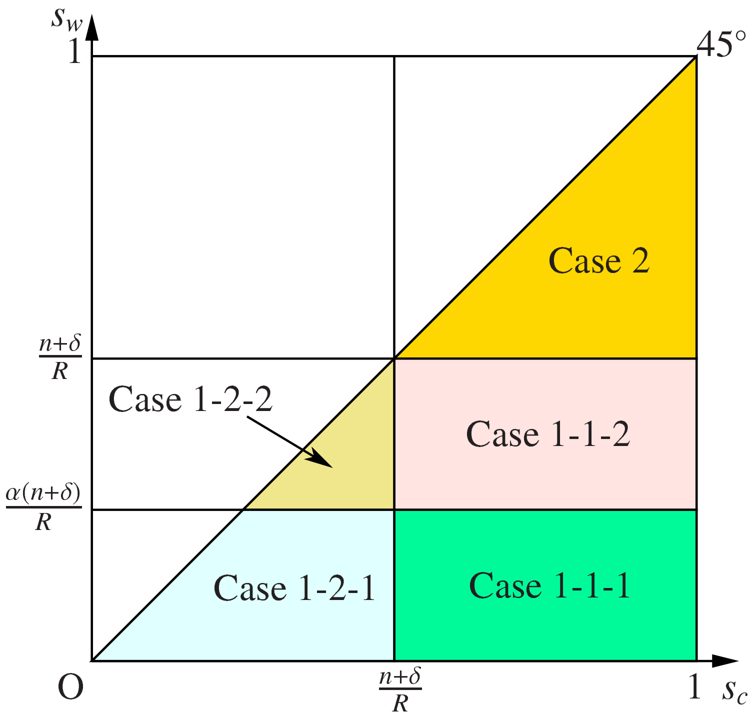

Figure 3: Classifications of five cases according to the size of s_c and s_w

3.1 Case of relatively small worker saving rate

When $s_w < (n+\delta)/R$, k_w converges to the steady-state value given by

$$k_w^* = \frac{s_w R}{n + \delta - s_w R} > 0. \tag{31}$$

When k_w converges to k_w^*, by substituting equation (31) into equation (29), we can rewrite the equation of motion of k_c as

$$\dot{k}_c(t) = [s_c R - (n+\delta)]\left[k_c(t) + \frac{s_w R - \alpha(n+\delta)}{n+\delta - s_w R}\right]. \tag{32}$$

From the no-arbitrage condition, we have

$$k_c + k_w^* > \frac{\alpha}{1-\alpha} \Longrightarrow k_c > k_w^* - \frac{\alpha}{1-\alpha} = \frac{\alpha(n+\delta) - s_w R}{(1-\alpha)(n+\delta - s_w R)} \equiv \hat{k}_c. \tag{33}$$

Therefore, to satisfy the non-arbitrage condition, k_c must be larger than $\hat{k}_c$.

The value of k_c, assuming that $\dot{k}_c = 0$, is given by

$$\bar{k}_c = \frac{\alpha(n+\delta) - s_w R}{n + \delta - s_w R}. \tag{34}$$

From the above discussion, when $s_w < (n+\delta)/R$, we have four cases, Cases 1-1-1, 1-1-2, 1-2-1, and 1-2-2, as shown in Figures 4–7, respectively.

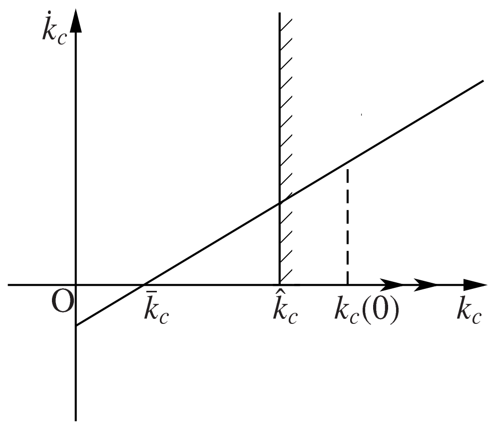

Figure 4: Dynamics of k_c in Case 1-1-1

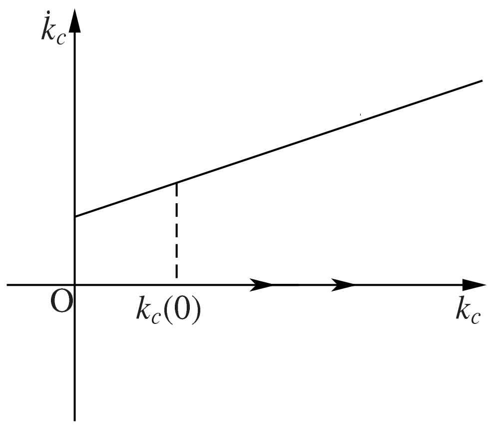

Figure 5: Dynamics of k_c in Case 1-1-2

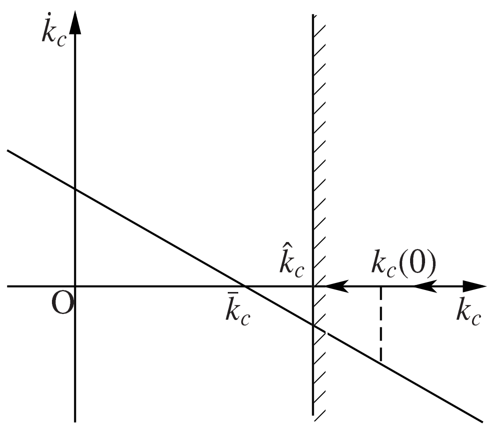

Figure 6: Dynamics of k_c in Case 1-2-1

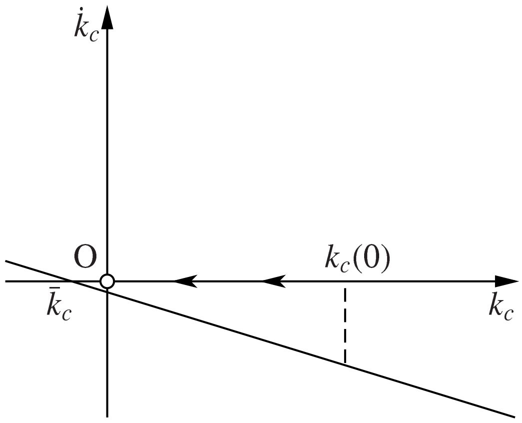

Figure 7: Dynamics of k_c in Case 1-2-2

In Case 1-1-1, k_c continues to increase over time as long as the initial value satisfies $k_c(0) > \hat{k}_c$. In Case 1-1-2, k_c continues to increase over time irrespective of the size of $k_c(0)$.

In Case 1-2-1, k_c continues to decrease over time and approaches a constant $\hat{k}_c$. When k_c reaches $\hat{k}_c$, the no-arbitrage condition is violated, and the system switches to a new system that does not include p. The long-run values are as follows:

$$k_w = k_w^* = \frac{s_w R}{n + \delta - s_w R}, \tag{35}$$

$$k_c = \hat{k}_c = \frac{\alpha(n + \delta) - s_w R}{(1 - \alpha)(n + \delta - s_w R)}, \tag{36}$$

$$p = 0. \tag{37}$$

Note that these values do not depend on the capitalists' saving rate s_c.

In Case 1-2-2, k_c continues to decrease over time, approaching $k_c = 0$. When $k_c = 0$, we have $a_c = p = [(1-\alpha)/\alpha]k_w^* - 1 > 0$. The long-run values are as follows:

$$k_w = k_w^* = \frac{s_w R}{n + \delta - s_w R}, \tag{38}$$

$$k_c = 0, \tag{39}$$

$$p = \frac{s_w R - \alpha(n+\delta)}{\alpha(n + \delta - s_w R)}. \tag{40}$$

Note that these values too do not depend on the capitalists' saving rate s_c.

Again, consider Case 1-2-1. As stated above, a regime switch occurs, and the system becomes a Pasinetti vs. Samuelson–Modigliani system. The Case 1-2-1 condition can be rewritten as

$$R < \frac{n+\delta}{s_c} \equiv R_1, \tag{41}$$

$$R < \frac{\alpha(n+\delta)}{s_w} \equiv R_2. \tag{42}$$

Now, we have either $R_1 < R_2$ or $R_1 > R_2$, from which

$$R_1 < R_2 \iff s_w < \alpha s_c, \tag{43}$$

$$R_1 > R_2 \iff s_w > \alpha s_c. \tag{44}$$

The first condition is identical to that for the Pasinetti equilibrium, whereas the second condition is identical to that for the dual equilibrium (Samuelson and Modigliani, 1966; Furuno, 1970). From Samuelson and Modigliani (1966) and Furuno (1970), the steady-state values of k_w and k_c can be given as [8]

$$k_w^P = \frac{(1-\alpha)s_w}{\alpha(s_c - s_w)}\left(\frac{\alpha s_c}{n+\delta}\right)^{1-\alpha}, \quad k_c^P = \frac{\alpha s_c - s_w}{\alpha(s_c - s_w)}\left(\frac{\alpha s_c}{n+\delta}\right)^{1-\alpha} \qquad \text{if } s_w < \alpha s_c, \tag{47}$$

$$k_w^D = \left(\frac{\alpha s_c}{n+\delta}\right)^{\frac{1}{1-\alpha}}, \quad k_c^D = 0 \qquad \text{if } s_w > \alpha s_c, \tag{48}$$

[8] These steady-state values are obtained by analyzing the following set of differential equations:

$$\dot{k}_c = [\alpha s_c (k_c + k_w)^{\alpha - 1} - (n+\delta)]k_c, \tag{45}$$

$$\dot{k}_w = \left[s_w (k_c + k_w)^{\alpha}\left(\frac{1-\alpha}{k_w} + \frac{\alpha}{k_c + k_w}\right) - (n+\delta)\right]k_w. \tag{46}$$

where P and D denote the "Pasinetti equilibrium" and "dual equilibrium," respectively.

3.2 Case of relatively large worker saving rate

When $s_w > (n+\delta)/R$, k_w continues to increase over time. Here, $s_c > (n+\delta)/R$ also holds because $s_c > s_w$ by assumption. Furthermore, the dynamic system leads to

$$\dot{k}_c(t) = [s_c R - (n+\delta)]k_c(t) + (1-\alpha)(s_c - s_w)Rk_w(t) - [\alpha s_c + (1-\alpha)s_w]R + \alpha(n+\delta), \tag{49}$$

$$k_w(t) = k_w(\bar{t}) \cdot \exp\{[s_w R - (n+\delta)](t-\bar{t})\}, \tag{50}$$

where $\bar{t}$ denotes the time when k_w begins to increase exponentially.

Assume that at $t = \bar{t}$, we have $k_w(\bar{t}) = \alpha/(1-\alpha)$. Then, at $t = \bar{t}$, the intercept of $\dot{k}_c = 0$ becomes $-s_w R + \alpha(n+\delta) < 0$, which is assumed to be negative. The locus $\dot{k}_c = 0$ continues to shift upward over time, and k_c continues to increase. This case is shown in Figure 8.

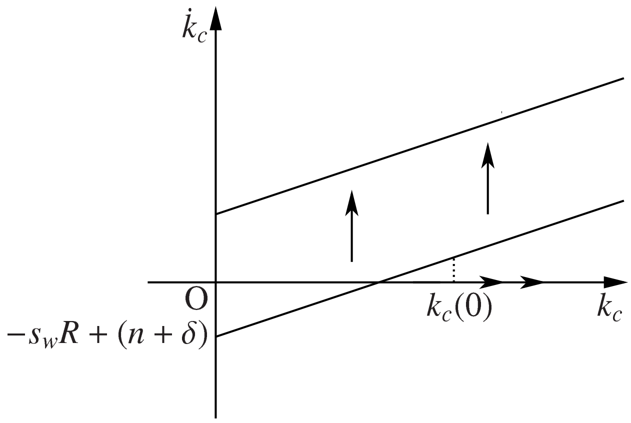

Figure 8: Dynamics of k_c in Case 2

The discussions in Sections 3.1 and 3.2 lead to the following two propositions.

Proposition 1. *When the capitalists' saving rate is larger than the threshold level, their own traditional capital and automation capital per capitalist continue to increase over time. The workers' own traditional capital either converges to a constant value or continues to increase through time depending on whether the workers' saving rate is lower or higher than the threshold level.*

Proposition 2. *When the capitalists' saving rate is less than the threshold level, both their own and the workers' own traditional capital per worker approach a constant value, and two different long-run situations emerge. In one, the capitalists' own per capita automation capital approaches zero, and the economy becomes free of automation capital and converges* *to either the Pasinetti equilibrium or dual equilibrium. In the other, the capitalists' own per capita traditional capital approaches zero and automation capital approaches a positive value.*

4 Income distribution and growth

In this section, we analyze the income distribution and growth. For the analysis, we express the input factor ratios in terms of k_c and k_w as follows:

$$\frac{P}{K_c} = \frac{1-\alpha}{\alpha} \cdot \frac{k_c + k_w - \alpha}{k_c}, \tag{51}$$

$$\frac{P}{L} = \frac{1-\alpha}{\alpha}(k_c + k_w) - 1, \tag{52}$$

$$\frac{P}{K} = \frac{1-\alpha}{\alpha} - \frac{1}{k_c + k_w}, \tag{53}$$

$$\frac{K}{L} = k_c + k_w. \tag{54}$$

When k_c converges to constant values, these ratios also become constant, and, interestingly, independent of s_c, as k_w^* and the long-run values of k_c are independent of s_c.

However, when k_c continues to increase, these approach the following values.

$$\frac{P}{K_c} \to \frac{1-\alpha}{\alpha}, \tag{55}$$

$$\frac{P}{L} \to +\infty, \tag{56}$$

$$\frac{P}{K} \to \frac{1-\alpha}{\alpha}, \tag{57}$$

$$\frac{K}{L} \to +\infty. \tag{58}$$

From equation (55), capitalists allocate their saving between traditional and automation capital in the proportion of α to $1-\alpha$ over time.

The variables related to income distribution are calculated as

$$\frac{wL}{Y} = \frac{\alpha}{k_c + k_w}, \tag{59}$$

$$\frac{R^k K}{Y} = \alpha, \tag{60}$$

$$\frac{R^p P}{Y} = 1 - \alpha - \frac{\alpha}{k_c + k_w}, \tag{61}$$

$$\frac{R^k K_c + R^p P}{Y} = \frac{k_c + (1-\alpha)k_w - \alpha}{k_c + k_w}, \tag{62}$$

$$\frac{wL + R^k K_w}{Y} = \frac{\alpha(1 + k_w)}{k_c + k_w}. \tag{63}$$

When k_c converges to constant values, these ratios also become constant, and, interestingly, independent of s_c, as k_w^* and the long-run values of k_c are independent of s_c.

However, when k_c continues to increase, these approach the following values.

$$\frac{wL}{Y} \to 0, \tag{64}$$

$$\frac{R^k K}{Y} = \alpha, \tag{65}$$

$$\frac{R^p P}{Y} \to 1 - \alpha, \tag{66}$$

$$\frac{R^k K_c + R^p P}{Y} \to 1, \tag{67}$$

$$\frac{wL + R^k K_w}{Y} \to 0. \tag{68}$$

From equations (67) and (68), the income gap between workers and capitalists approaches the proportion of 0 to 1.

Summarizing the above discussions, we obtain the following proposition.

Proposition 3. *When the capitalists' saving rate is larger than the threshold level, the income gap between workers and capitalists becomes polarized over time. However, when the capitalists' saving rate is smaller than the threshold level, the income gap between workers and capitalists becomes constant over time.*

The per capita output level can be given by

$$y \equiv \frac{Y}{L} = Bk = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(k_c + k_w). \tag{69}$$

Thus, when k_c becomes constant over time, the per capita output stays constant. However, if k_c continues to increase, the per capita output growth rate will be as follows:

$$\begin{aligned} g_y \equiv \frac{\dot{y}}{y} &= \frac{\dot{k}_c + \dot{k}_w}{k_c + k_w} \\ &= \frac{k_c}{k_c + k_w}\left\{[s_c R - (n+\delta)] + (1-\alpha)(s_c - s_w)R\frac{k_w}{k_c} - \frac{[\alpha s_c + (1-\alpha)s_w]R}{k_c} + \frac{\alpha(n+\delta)}{k_c}\right\} \\ &\quad + \frac{k_w}{k_c + k_w}\left\{[s_w R - (n+\delta)] + \frac{s_w R}{k_w}\right\}. \end{aligned} \tag{70}$$

$$\Longrightarrow \lim_{k_c \to +\infty} g_y = \frac{\dot{a}_c}{a_c} = s_c R - (n+\delta) = s_c \alpha^{\alpha}(1-\alpha)^{1-\alpha} - (n+\delta) \geq 0. \tag{71}$$

The long-run growth rate g_y is increasing in s_c, decreasing in n, and decreasing in δ. Note that g_y does not depend on the workers' saving rate s_w. Also, g_y takes the minimum value when $\alpha = 1/2$ and the maximum value when $\alpha = 0, 1$.

From the above discussions, we obtain the following proposition.

Proposition 4. *When the capitalists' saving rate is smaller than the threshold level, the output per capita becomes constant. However, when the capitalists' saving rate is larger than the threshold level, the output per capita continues to increase at a constant rate over time even without exogenous technological progress.*

5 Numerical simulations

This section investigates the behavior of the main variables through numerical simulations, setting reasonable parameters. As AI and robots are production factors requiring advanced technology for production, we focus on the developed economies, with the parameters based on previous studies and public data obtained from international organizations.

For the capitalists' saving rate, we follow Saez and Zucman (2016), who estimate the average saving rate of the top 1% of income since 1913 in the United States. They estimate the saving rate as 20–25%, from which we set $s_c = 0.2$.

For the workers' saving rate, we follow Storm and Naastepad (2012), who estimate the average propensity to save from wage income in 12 OECD economies at around 0.098 for about 40 years. Thus, we set $s_w = 0.1$. According to Lieberknecht and Vermeulen (2018), the US saving rate of the top 1% of income is two times the average saving rate, which is consistent with our setting.

For population growth rate, we employ international data obtained from the World Bank 2020. The average population growth rate of 37 OECD economies during the period 2000–2019 is around 0.8%. Hence, we set $n = 0.01$. In addition, we normalize the initial-period labor input as $L(0) = 1$.

For capital depreciation rate, we follow Prettner (2019). From the Bureau of Economic Analysis (2004) data, we set $\delta = 0.07$.

Finally, we set the capital share of income. The Databook of International Labour Statistics published by The Japan Institute for Labour Policy and Training states that the labor share of income in terms of factor prices in major developed countries [9] during the period is

[9] The major developed countries are the United States, the United Kingdom, Germany, France, Italy, Sweden, Canada, and Japan.

about 68.2%. Therefore, we set the capital share of income to $\alpha = 0.3$.

For the initial workers' own per worker and capitalists' own per capitalist traditional capital values, we set $k_w(0) = 0.1$ and $k_c(0) = 0.5$, respectively. In this case, we have $k(0) = 0.6 > 0.428571 = \alpha/(1-\alpha)$, meaning that the no-arbitrage condition holds at the initial period.

Summarizing the above discussions, we obtain

$$s_w = 0.1,\ s_c = 0.2,\ \alpha = 0.3,\ n = 0.01,\ \delta = 0.07,\ k_w(0) = 0.1,\ k_c(0) = 0.5. \qquad (72)$$

The numerical simulation results are as follows.[10]

Figures 9 and 10 show that the workers' (capitalists') asset share increases (decreases) initially. This occurs because a_w rises faster than a_c. While a_w converges to a constant value, a_c increases at a constant rate. Therefore, the workers' (capitalists') asset share begins to decrease (increase) at some point in time, and then converges to zero (unity).

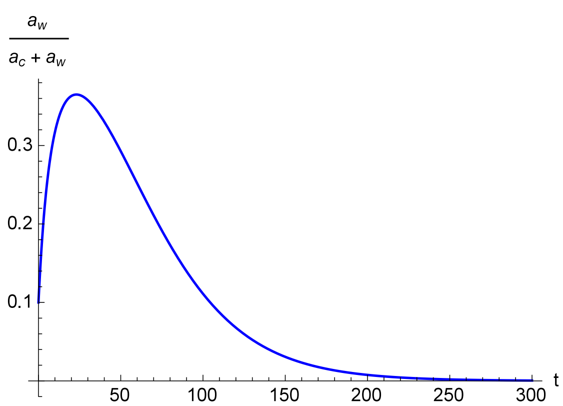

Figure 9: Behavior of workers' asset share

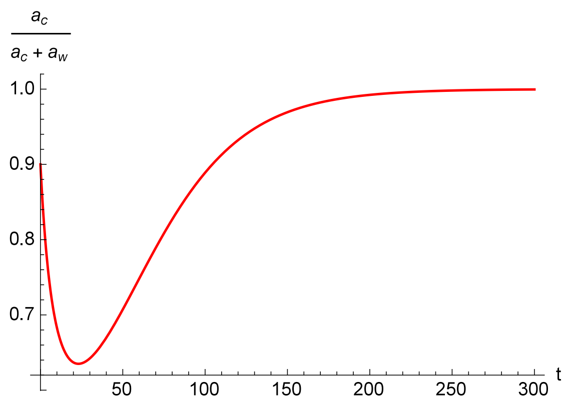

Figure 10: Behavior of capitalists' asset share

Figure 11 shows that the ratio of capitalists' own traditional capital to own total capital decreases at first, then increases, and finally converges to $\alpha = 0.3$. This means that capitalists divide their savings between traditional capital and automation capital investment in the proportion $\alpha = 0.3$ and $1 - \alpha = 0.7$ over time. This division depends on the transitional dynamics.

Figure 12 shows that the share of traditional capital in the whole capital decreases and converges to $\alpha = 0.3$, implying that the share of automation capital in the whole capital increases and converges to $1 - \alpha = 0.7$.

Figure 13 shows that the labor to sum of labor and automation capital ratio decreases

[10]Numerical simulations are conducted with the Wolfram Mathematica 10 software.

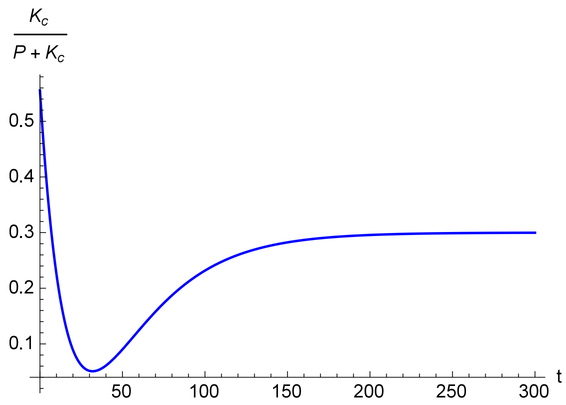

Figure 11: Behavior of ratio of capitalists' traditional capital to assets

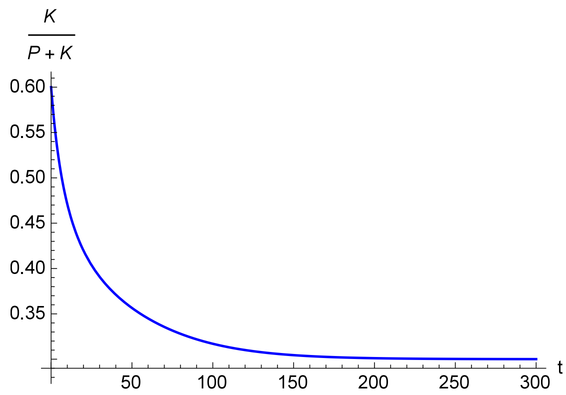

Figure 12: Behavior of ratio of traditional capital to the sum of traditional and automation capital

over time and converges to zero. This implies that labor becomes unnecessary relative to automation capital.

Figure 14 shows that the per capita output growth rate decreases for a while, then begins to increase at some point, and finally converges to a constant value over time. This behavior depends on the initial conditions and parameter values. Nevertheless, it implies that although advances in automation technology lead to sustainable economic growth over time, they lead to a decline in economic growth along with the transitional dynamics.

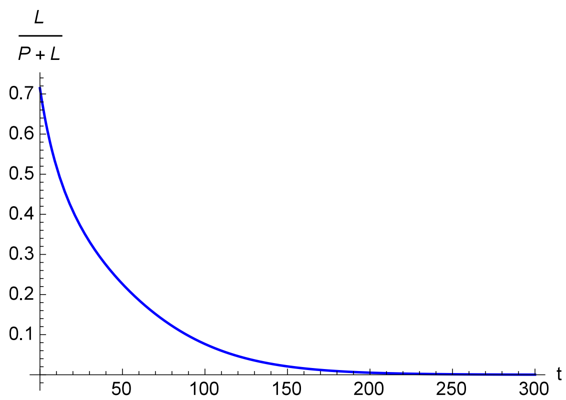

Figure 13: Behavior of ratio of labor to sum of labor and automation capital

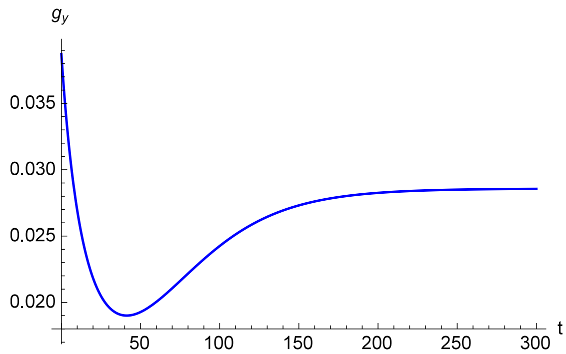

Figure 14: Behavior of per capita output growth

Figures 15 and 16 show the labor share of income and the rental share of automation capital, respectively. The labor share of income decreases over time and converges to zero, whereas the rental share of automation capital increases and converges to $1 - \alpha = 0.7$. The remaining $\alpha = 0.3$ is the capital share of income.

Figure 17 shows that the workers' income share increases at first, then starts to decline,

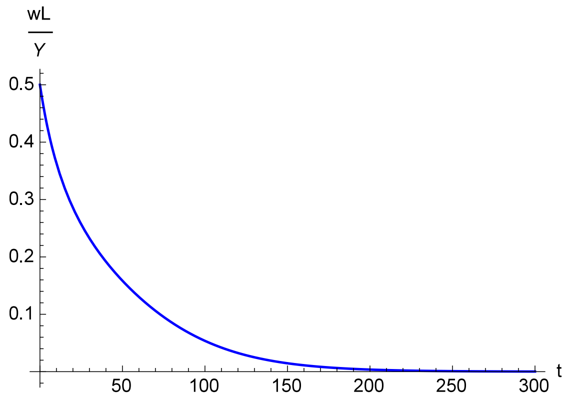

Figure 15: Behavior of labor share of income

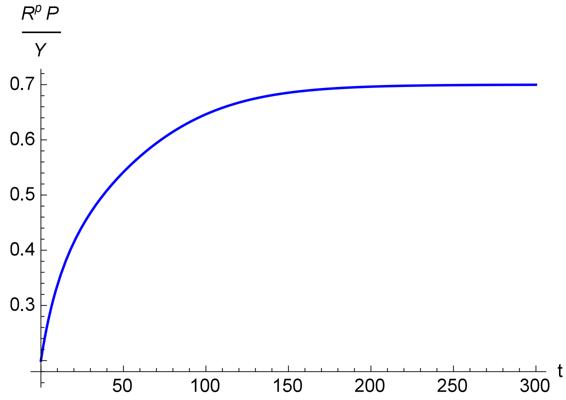

Figure 16: Behavior of rental share of automation capital

and finally converges to zero. The first-phase increase arises because the workers' asset share increases faster than capitalists' asset share. However, as time passes, the workers' asset share converges to a constant value, but the capitalists' asset share continues to increase, and so the workers' income share decreases. This means that advances in automation technology increase the income disparity between workers and capitalists over time.

Figure 18 shows the growth rate of automation capital. This growth rate is closely related to the per capita output growth rate. As $g_y = \dot{a}_c/a_c$ in the long run, the growth rate of y is equal to the growth rate of p. This is because we have $a_c = k_c + p$, and further, the growth rates of k_c and p are equalized. Thus, $g_y = g_p$ in the long run.

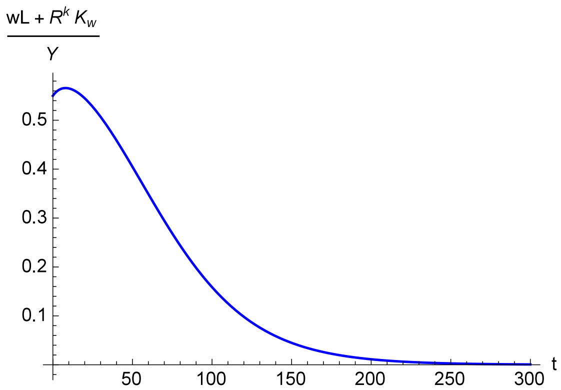

Figure 17: Behavior of workers' income share

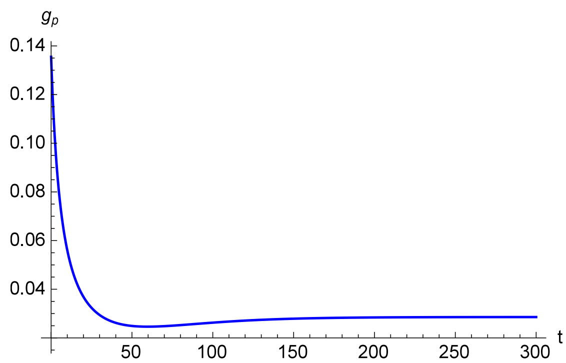

Figure 18: Behavior of growth of automation capital

6 Conclusions

This study built a growth model with automation capital as a perfect substitute for human labor, and investigated how advances in automation affect economic growth and income distribution. For this, we first introduced two classes—workers and capitalists.

The study mainly showed that several long-run situations occur depending on the saving rates of workers and capitalists. These can be summarized as follows.

When the capitalists' saving rate is more than the threshold level, the per capita output continues to increase over time irrespective of the workers' saving rate. In this case, the income disparity between the two classes continues to increase. These results suggest that advances in automation technology lead to economic growth and affect the two groups differently.

When the capitalists' saving rate is less than the threshold level, two different long-run situations occur depending on the workers' saving rate. First, when the workers' saving rate is more than the threshold level, the capitalists' traditional capital share becomes zero over time, but their automation capital share becomes positive. In this case, the per capita output growth rate is zero, and the income disparity between the two classes becomes constant. Second, when the workers' saving rate is less than the threshold level, the capitalists' automation capital share becomes zero, and their traditional capital share approaches a positive constant value in finite time. In this case, the per capita output growth rate is zero, while the income disparity between the two classes becomes constant. As time passes, a regime switch occurs, the economy turns into an economy where the final good is produced with labor and traditional capital, and then, the economy converges to either the Pasinetti equilibrium or the dual equilibrium according to the sizes of the saving rates of workers and capitalists.

To find the long-run situation that occurs, we carried out numerical simulations using data from developed countries. From the results, the capitalists' saving rate is higher than the threshold level, and the above-mentioned result (1) holds; that is, automation capital accumulates over time, the per capita output growth rate converges to a constant value, and the income disparity between workers and capitalists continues to expand.

Note that capitalists own automation capital, whereas workers do not. This asset holdings asymmetry in itself does not lead to an income gap between the two classes. Even if the workers own automation capital in addition to traditional capital, we would obtain similar conclusions. Our conclusions depend decisively on two assumptions, that is, the wage income of workers decreases as the automation technology advances, and the saving rate of workers is less than that of capitalists. These two assumptions result in the income gap between workers and capitalists.

In order to reduce the income disparity, we might tax the automation capital owned by capitalists and re-distribute the tax revenue to workers. This policy is essentially the same as workers owning automation capital, and as stated above, this might reduce the increase in income disparity, but not the income disparity. However, the income disparity can be reduced by increasing the workers' saving rate and accumulating the human capital that cannot be substituted by automation capital.

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