Foreign Reserve Management in an Oil Economy: Macroeconomic Risk as a Real Option

Scandizzo, Pasquale and Pagliacci, Carolina

UniversityTor Vergata, Central Bank of Venezuela

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Foreign Reserve Management in an Oil Economy: Macroeconomic Risk as a Real Option*

Pasquale Scandizzo[&]
Carolina Pagliacci‡

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Abstract

This paper assesses reserve management for determining optimal or minimal reserves for an oil producing economy under dynamic uncertainty. Reserve benchmarks are formulated taking into consideration the amount of contingent liabilities in foreign exchange that arises during currency crises. These contingent liabilities are derived based on the analogy between holding domestic money and possessing a financial option whose payoff depends on the expected behavior of oil proceeds. When reserve accumulation has an opportunity cost in terms of capital goods, an optimum level of reserves can be established, given the capability of reserves to delay and mitigate currency crises. Alternatively, when reserves constitute the best means to accumulate country wealth, an appropriate minimal reserve level may be calculated. In this case, reserves act as an instrument of self-insurance that guarantees honoring a selected amount of foreign exchange claims at the time of a crisis. Econometric estimates for Venezuela show reasonable numerical values for counterfactual optimal and minimal reserves.

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[&] Professor of the UniversityTor Vergata, email:scandizzoniroma2.it .

‡Senior Researcher of the Economic Research Office, Central Bank of Venezuela, e-mail: cpagliac@bcv.org.ve.

Manejo de Reservas Internacionales en una Economía Petrolera: El riesgo Macroeconómico como una Opción Real

Pasquale Scandizzo
Carolina Pagliacci

Este trabajo analiza el manejo de reservas para determinar reservas óptimas o mínimas para una economía petrolera en un contexto con incertidumbre dinámica. Estos niveles referenciales de reservas son formulados tomando en consideración los pasivos contingentes en moneda extranjera que emergen durante las crisis cambiarias. Estos pasivos contingentes son derivados basados en la analogía entre mantener moneda doméstica y poseer una opción financiera cuyo *payoff* depende del comportamiento esperado de los ingresos petroleros. Cuando la acumulación de reservas tiene un costo de oportunidad en términos de bienes de capital, un nivel de reservas óptimo puede der determinado, dada la capacidad de las reservas para retardar y mitigar las crisis cambiarias. Alternativamente, cuando las reservas son el mejor medio para acumular la riqueza de un país, un nivel mínimo de reservas puede ser calculado. En este caso, las reservas actúan como un instrumento de auto-aseguramiento que garantiza honrar un monto seleccionado de obligaciones en moneda extranjera al momento de una crisis. Las estimaciones para Venezuela muestran valores numéricos razonables para reservas óptimas y mínimas contrafactuales.

1. Introduction

With their spectacular rise to more than 3 trillion dollars in the past 20 years, foreign reserves have become a worldwide emblem of global imbalances in trade and finance flows. On one hand, they may be considered the mirror image of the record expansion of U.S. foreign deficit and its corresponding debt. On the other hand, they may be taken to reflect the growing concern of governments and central banks on currency instability and currency crises. So far, as most of the literature on the subject notices, foreign reserve accumulation has been mostly the result of passive policies, i.e. prudent, but residual accumulation of foreign exchange surpluses to protect the country from a "rainy day". Neither from the point of view of financial returns, nor from that of risk management, the choice of foreign currency accumulation or its allocation to financial assets appears to have followed a rational policy model.

The present paper addresses the policy question of foreign reserve accumulation in an oil economy as a case study of development planning under macroeconomic risk. Risk, in this paper, relates to the threat of currency crises, which are driven by the conditions of dynamic uncertainty that characterize foreign exchange markets. The approach proposed is based on the idea that an adequate level of foreign reserves can be defined either as an optimal level for mitigating currency crisis or as a minimal self-insurance level. Optimality arises because foreign reserves can be seen as an asset that competes with the allocation of the country's income flow in capital goods, but also as an asset that has the property to counterbalance latent liabilities that result from agents' financial valuation of two alternative assets -domestic and foreign currency-. In this case, the essential role assigned to reserves is that of discouraging potential foreign exchange claims against the Central Bank. For the case of a minimal level, reserves are exclusively used to reduce the exposure to risk according to the explicit preferences of policymakers.

The determination of these foreign exchange contingent liabilities at an aggregate level builds the connection of this paper with real option theory, generating also further implications for macroeconomic planning. According to Dixit and Pyndick (1991), the option value of any commodity determines the speculative component that contributes to explaining its contingent value. In the case of domestic money, this option value reflects the difference between the opportunity of buying foreign exchange today and the decision of keeping domestic currency. That is, holding liquid balances denominated in domestic currency embeds a right to convert them into foreign currency or other assets denominated in foreign money, creating the analogy with possessing a financial option. In this context, contingent liabilities against the Central Bank arise from considering the intertemporal valuation of the option to convert domestic into foreign assets.

Because domestic and foreign money are related through the concept of a financial option, a crucial role is played by depreciation expectations on the domestic currency. At the same time, these depreciation expectations will depend on the soundness of the fundamentals of an oil economy, which can be ultimately described by the expectations formed around the behavior of the country's foreign earnings (oil revenues). These two elements lead to assert that the expected depreciation of the domestic currency and contingent foreign liabilities rely on the uncertainty about the future performance of these earnings. However, the chances of experiencing currency crises, and their effect on the country wealth can be mitigated and delayed by the accumulation of international reserves, which act as an instrument of deterrence for speculators' attacks by reducing any depreciation expectations held by private agents.

In the literature, Heller (1966) was the first to cast the analysis of reserve demand in the context of precautionary behavior. Frenkel and Jovanovic (1981) also derived optimal reserves under the notion that reserves serve as a buffer that smoothes out the stochastic fluctuations of external transactions. The idea that reserves act as an instrument of self-insurance was mainly developed by the literature that followed the financial crises of the 1990's, with papers such as, Feldstein (1999), and Kletzer and Mody (2000). More recently, Aizenman and Lee (2005) and Jeanne and Rancière (2006) focus on the idea that reserves act as self-insurance because they minimize the costs of adjustments related to the occurrence of sudden stops. García and Soto (2004), instead, center the discussion on exploiting the potential of reserves to reduce the likelihood of a sudden stop, and not to diminish the cost of adjustment itself. Lee (2004) takes a different approach and determines the optimal coverage for self-insurance by computing the insurance value of reserves using real option theory.

We center our attention on the idea that reserves can delay the occurrence of currency crises as in García and Soto (2004), but we use the real option theory to derive the link between crises and the uncertainty exhibited by foreign exchange proceeds of the country. Implicitly, our model can also be seen as a way to analyze the sustainability of monetary policies in a condition where no fiscal base is available except for a primary sector (Ghatak and Sánchez-Fung, 2007).

The structure of the paper is the following. First, in section 2, we explain the analogy of holding domestic currency with possessing a financial option, and include, as a particular case, the value of an option in the presence of a dual exchange rate. Then we derive the amount of contingent liabilities for the general case, and in section 4 we provide the definition of wealth for an oil economy. In section 5, we address the problem of reserve management by deriving the expressions for computing optimal and minimal reserves. Finally, in section 6, we give parameters estimates and show the numerical calculations for the Venezuelan economy.

2. Domestic money as a financial option

2.1. The general case

Consider the case of a country whose domestic currency is subject to a loss of external purchasing power. From the point of view of individuals, holding liquid balances denominated in domestic currency embeds the right, but not the obligation, to convert them into foreign currency or other assets denominated in foreign money. More precisely, at any given time, individuals have the possibility of switching their holdings of domestic money into foreign currency at the current exchange rate or waiting up to a future time, let the purchasing power of domestic money deteriorate, and make the conversion at a different (probably higher) exchange rate[1]. Because agents are constantly faced with such choice, we can say that holding domestic money is equivalent to possessing a call option on foreign assets, or analogously, to possessing a put option on domestic currency, i.e. an option to exit from domestic currency by acquiring foreign assets at the ongoing exchange rate. Therefore, the decision to buy foreign currency is equivalent to deciding when to exercise such an option.

Define F_i as the current value of the call (put) option, i.e. the value assigned by a single individual to maintaining open the possibility of buying foreign currency in the future (or selling long positions of domestic currency)[2]. If we rule out arbitrage opportunities, the maximum value for F_i is the gain that the individual would obtain by selling the domestic currency at the current exchange rate with respect to the (lower) value that he would obtain by selling the same amount at a rate fully reflecting the depreciation of the currency. Like with any financial option, as F_i gets larger, the incentives for individuals to exercise their option are greater. Ideally, the individual would want to exercise this option when the difference between the value of foreign assets at the current exchange rate and the one at the expected exchange rate is so large that it would not be justifiable to wait for a reversal[3].

Consider that each individual (the i^{th} operator) holds a fraction w_i of the stock of high powered money in Bs (M_0) that can eventually be converted into foreign currency. The current value assigned by the operator to the implicit option is given by:

$$(1)\quad F_i = \max\left\{ \frac{w_i\, M_0}{\varepsilon} - \frac{w_i\, M_0}{\varepsilon\, e^{D^e t}} \;,\; 0 \right\}$$

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[1] The exchange rate is defined as the price of the foreign currency in terms of domestic currency, e.g. Bs/$.

[2] Because conversion can happen at any time without any given expiration date, this real option would be equivalent to an American perpetual option.

[3] Nevertheless, on the basis of the "good news principle" of Bernanke (1983), the put option can be expected to be exercised when the operators believe that the best possible outcome has already occurred and no improvement of the situation can be expected.

where ε represents the current exchange rate (units of domestic currency per unit of foreign currency), and D^e is the expected annual rate of variation of the exchange rate in a year, i.e. the expected rate of depreciation of the domestic currency. In this expression, $\frac{w_i M_0}{\varepsilon}$ represents the current value of assets in foreign currency that could be bought at the current exchange rate with an amount $w_i M_0$ of domestic money. The second term indicates the value of the assets in foreign exchange that would be obtained, by converting an amount $w_i M_0$ of domestic money using the expected exchange rate ($\varepsilon e^{D^e t}$) at the exercise time t. This difference represents the gain that the individual would obtain by converting her domestic currency long positions at the current rate, rather than at the depreciated rate $\varepsilon e^{D^e t}$.

Given that all individuals in the economy hold domestic money, the aggregate payoff of the option is simply $F = \sum_i F_i$, provided that $\sum_i w_i = 1$. A linearization of equation (1) can be stated as:

$$(2)\ F\left(D^e\right) = \begin{cases} M_0^{\$}\left(D^e\right)t & \text{for } D^e > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $M_0^{\$} = {M_0}/{\varepsilon}$ is the high powered money of the economy in foreign currency (dollars) at the current exchange rate. Expression (2) indicates that the current value of the implicit option is equal to the expected loss of purchasing power (in terms of foreign exchange) that holders of domestic money face. Therefore, an increase in the expected depreciation of the domestic currency will increase the value of the option, and the incentives to exercise it. Also notice that, the intrinsic value of this option is different than zero, only for expectations of depreciation, since appreciation expectations induce agents to keep their holdings of domestic money and drive the value of the option to zero. For a higher level of the high powered money, the value of the option increases as well due to the greater base over which the expected loss is computed.

In any economy, the expected depreciation rate of the domestic currency typically depends on fundamentals, which can be thought as fiscal or financial indicators that signal the degree of soundness of the economy. In an economy whose foreign receipts depend essentially on the state oil exports, fundamentals ultimately rely on the expected oil revenues, which affect the performance of the fiscal variables and also determine the state of real economy and the financial system. This description is especially accurate, if the economy does not utilize any saving mechanism to buffer oil shocks, as it is the case for Venezuela.

On the other hand, an expected accumulation of reserves can play an important role in this type of economy by signaling lower vulnerability to currency crises. Models such as Sachs, Tornell and Velasco (1996) and Sims (2001) provide theoretical background to explain the negative relationship between the occurrence of external crises and the accumulation of international reserves. For instance, in Sachs, Tornell and Velasco (1996) agents observe the level of international reserves to determine whether capital outflows can occur without causing a balance of payment crisis. The story from this model is that when a country faces weak fundamentals, the probability of occurrence of a crisis due to self-fulfilling prophecies is higher if reserves are low. In the same line of reasoning, Sims (2001) in a stylized model of a small economy, shows that explosives paths of prices (i.e., the exchange rate) can be ruled out if the Central Bank commits to maintain enough reserves to back up the quantity of money in the economy.

Based on these elements, and acknowledging the role of time in forming expectations, the general form of the expected depreciation rate of the domestic currency for an oil economy can be written as:

$$(3)\ D_t^e = D_0 - g\,\Delta^e y_t - h\,\Delta^e r_t$$

where D_t^e is the expected depreciation rate of the domestic currency (not reflected in the current exchange rate) formed with the information set available at time $t\text{-}1$, D_0 is an autonomus annual rate of depreciation, $\Delta^e y_t \equiv E(y_t - y_{t-1})$ is the expected change in oil revenues for the year and $\Delta^e r_t$ is the expected accumulation of reserves for the same period. For g, h>0, this function indicates that as expected oil earnings and reserves increase, the expected depreciation diminishes because agents perceive a more robust economy. It also implies that the effect of negative oil shocks on the exchange rate can be counterbalanced by hoarding larger reserves. Alternatively, one could argue that oil booms increase government profligacy to over-expand its expenditure causing a greater demand of foreign assets and a larger depreciation of the currency. In this case g<0, but an expected increase in international reserves could still be stabilizing for the economy by reducing depreciation expectations (h>0)[4].

Incorporating the particular form of the exchange rate depreciation in (3), the aggregate current value of an option varies in time according to:

$$(4)\ F(\Delta^e y_t, \Delta^e r_t) = M_0^{\$}(D_0 - g\,\Delta^e y_t - h\,\Delta^e r_t)\ t$$

Assuming that g, h>0, this expression shows that for an oil economy, an increase in expected oil receipts and reserve accumulation will reduce the incentives for individuals to exercise the existing call option on foreign assets, by reducing its current payoff. It also shows that accumulating reserves can be seen as a way to deter foreign exchange claims that come from the conversion of domestic money liabilities when a huge depreciation of the domestic currency is expected.

[4] If reserve accumulation were also destabilizing (h<0), then the accumulation of reserves could not be understood as a mean to protect the economy from external shocks, making any consideration regarding reserve management useless.

2.1. The dual exchange rate case

Oil economies that have not implemented saving mechanisms to buffer oil shocks are typically subject to cycles of accumulation/desaccumulation of reserves that reflect the rollercoaster behavior of oil prices in combination with policies of fixing the nominal exchange rate. It can also be common that, in the downward phase of reserves, exchange rate controls are established as a way to evade an explicit depreciation of the domestic currency that would re-balance the ongoing current account deficit and that would stop capital flights. In other words, one can interpret exchange rate controls as an attempt at insulating reserves from a transitory negative shock in the supply of foreign exchange that does not want to be resolved with a temporary depreciation. In this section, we derive the particular form of the above option assuming that financial transactions are actively restricted by exchange rate controls.

These types of controls typically imply a form of a dual exchange rate system in which part of commercial transactions (imports) are priced at the official exchange rate (ε), while the rest of transactions (mostly financial) take place in a parallel market at the exchange rate (η)[5]. Provided always that $\eta > \varepsilon$, an exchange rate premium ($p_\eta = \eta/\varepsilon - 1$) arises to reflect the fact that the state centralizes the allocation of foreign currency and there is rationing in the official supply of foreign exchange for importers through licensing.

The official exchange rate is fixed by the authorities and we assume that does not change while the dual system is in place. The current level of the parallel exchange rate is basically determined by the supply and demand of foreign exchange for financial transactions. In particular, the expected depreciation rate in this market can have the same determinants as in the case of a floating or semi-fixed exchange rate described in equation (3).

One could argue that in the official exchange rate market, by over invoicing imports, a fraction of oil proceeds devoted to imports will be diverted to the parallel market, especially as the premium between the official and the parallel market grows. As in Guidotti (1988), this diversion of resources implies that there is an imperfect separation between the official and the parallel market that allows a certain degree of "leakages" in the system.

Taking into consideration all the above elements, the current value of the option on foreign assets can be written as:

$$(5)\quad F_i = \max\left\{ \left[q\,\frac{w_i\,M_0}{\varepsilon} + (1-q)\frac{w_i\,M_0}{\eta} \right] - \frac{w_i\,M_0}{v}\left[1 - D^e\,t\right] \;,\; 0 \right\}$$

[5] All exchange rates are expressed as quantities of domestic currency per unit of foreign currency, for instance Bs/$.

In this expression, $q\frac{w_i M_0}{\varepsilon}+(1-q)\frac{w_i M_0}{\eta}$ represents the average current value of assets in foreign currency that could be bought with an amount $w_i M_0$ of domestic money, considering the existence of a dual system. Since in this system there is an imperfect separation between the official and the parallel market, *q* represents the probability of obtaining a dollar at the official exchange rate (ε) if agents decided to liquidate their Bs long positions. Symmetrically, *1-q* represents the probability of obtaining a dollar at the parallel exchange rate (η). In other words, *q* can be described as the relative amount of financial transactions whose supply of foreign exchange is diverted from the official market. Intuitively, as the degree of financial leakages in the system increases, this fraction *q* should tend to rise. It will also rise, for instance, if the government intervenes in the parallel market trying to avoid an excessive gap between the official and the parallel rate by selling dollars at a price extremely close to the official rate.

The term $\frac{w_i M_0}{v}\left(1-D^e t\right)$ of expression (5) indicates the value of assets in foreign exchange that would be obtained, by converting an amount $w_i M_0$ of domestic money using the expected exchange rate at the exercise time. This expected exchange rate would be the one resulting from increasing the current average exchange rate prevailing in the economy (v) by the expected depreciation rate (D^e). This expected exchange rate is also the unique exchange rate that would arise from the abandonment of the dual system, which collapses as all the operators (or, in practice, a sufficiently large number of them) exercise their right and try to convert the stock of domestic assets into foreign exchange.

Since *v* is defined as the current average exchange rate prevailing in the economy, we could assume that $v=q\,\varepsilon+(1-q)\,\eta$, or it could also be expressed as a proportion of the current official exchange rate ($v\equiv f\,\varepsilon$), where $f=q+(1-q)(1+p_\eta)$ is a scalar necessarily greater than 1. The aggregate payoff of the option, provided that $\sum_i w_i=1$, can be written as:

$$(6)\quad F(D^e)=M_0^{\$}\left[D^e t+\mathrm{P}\right] \qquad \text{for } D^e>0$$

where $M_0^{\$}={M_0}/{v}$ is the high powered money of the economy in dollars at the current average exchange rate, and $P=f\left(q+\frac{1-q}{1+p_\eta}\right)-1$ is a term that can be interpreted as an indicator of the state of the dual system at the crisis point, corrected by the exchange rate premium.

In expression (6), note that if *q*=1 and *f*=1, or equivalently $\rho_\eta=0$ and *f*=1, this would correspond in practice to the elimination of the dual system, or a dual system with almost no restrictions on financial transactions. In this case, all transactions are undertaken at the official exchange rate, *P* is equal to zero, and the formula reduces to the original case shown in equation (2). Alternatively, for the case of a dual system with no leakages, *q*=0 and $f = \eta / \varepsilon$, the value of *P* becomes also zero, but the quantity of high powered money is expressed in terms of dollars converted at the parallel exchange rate (η). In this last instance, the current value of the option can also be written as:

$$(7)\quad F\left(D^e\right) = \frac{M_0}{\varepsilon}\left(\frac{1}{1+p_\eta}\right)\left(D^e\right)t \qquad \text{for } D^e>0$$

which indicates that higher current values of the parallel exchange rate (η), and therefore of the exchange rate premium (p_η), dissuade individuals to exercise their option of buying foreign assets today. This is the case because, part of the devaluation expectations for the official exchange rate have already materialized in the parallel market, and the actual devaluation rate is the difference between the current parallel exchange rate and the unique expected exchange rate when the dual system will be abandoned.

Notice that in both extreme cases, an elimination of the dual system (*q*=1) or a dual system with no leakages (*q*=0), the term *P* drops to zero, but for $0 < q < 1$, *P* is positive and maximizes the incentives to exercise the implicit option as $q \to 1/2$. This non-linearity of *P* on *q* seems to show that the mere existence of a dual system with two simultaneous meaningful prices would exacerbate depreciation expectations[6]. On the other hand, the effect of the exchange rate premium (p_η) on *P* is strictly positive ($\partial P / \partial p_\eta = q(1-q)p_\eta\left(2+p_\eta\right)/(1+p_\eta)^2 > 0$). This suggests that a greater premium will induce a larger instantaneous gain that will push individuals to exercise their option, especially as $q \to 1/2$.

With respect to government interventions in the parallel market, which can be thought as increasing the value of *q* by selling dollars at the official exchange rate, these interventions might be counterproductive, if the initial value of $q < 1/2$, that is, if most transactions are already priced at the parallel exchange rate. Nonetheless, because the sale of foreign exchange by the government can also be understood as an increase in the supply of dollars, an appreciation in the parallel exchange rate should occur as well, for instance as $\partial D_0 / \partial q < 0$. Therefore, we find that a government intervention in the parallel market might be successful, even for an initial $q < 1/2$, if the appreciation of parallel exchange rate is greater than the increase in *P* due to the increase in *q*, or equivalently:

$$(8)\quad \frac{\partial D_0}{\partial q} + \frac{\partial P}{\partial q} < 0$$

[6] P has a maximum in at *q*=0.5, since $\partial P / \partial q > 0$ for q<0.5 and $\partial P / \partial q < 0$ for q>0.5.

3. Contingent liabilities on reserves

So far, we have compared holding domestic currency with acquiring a financial option, and we have provided an explicit expression for the current payoff of such option. Since these payoffs are tied to the future performance of oil receipts, in the context of stochastic uncertainty for oil proceeds, agents' decision to exercise their option will depend on the inter-temporal valuation of the payoffs that could be generated upon. This means that we can find an expression that computes the price of the option, and determines its exit value, in terms of oil proceeds.

We start by assuming that development of the oil activity occurs under dynamic uncertainty. This implies that, as a consequence of a plurality of concurring factors, both the average value and the variance of oil revenue tends to increase over time. Adopting the convenient assumption of a geometric Brownian process, we thus assume that oil production yields a net cash flow formed by a systematic part, which is normalized to unity, and a stochastic part, denoted by *y*, observable in every period, evolving according to the expression:

$$(9)\quad dy_t = \alpha\, y_t\, dt + \sigma\, y_t\, dz$$

where α represents the drift (constant trend component) of oil earnings, σ^2 their (constant instantaneous) variance, and dz is a normally distributed random variable that satisfies $E(dz)=0$ and $E(dz^2) = dt$. We also define $\delta = \rho - \alpha$, and assume that $\rho > \alpha$, being ρ the discount rate in equation (10).

From the point of view of its holders, the price of the option is the one that solves the dynamic valuation of the payoff of buying foreign currency (or exiting domestic currency):

$$(10)\quad V = \sup\left\{ E\left[e^{-\rho\,\tau} \int_{\tau}^{\infty} e^{-\rho(s-\tau)} F\left(\Delta^e y_s, s\right) ds \Big/ y_0 = y \right]\right\}$$

where $E[\circ/ y_0 = y]$ is the operator that takes expectations conditional on the available information at time zero regarding the oil income, and *V* is the price of the option represented by the maximum expected present worth of the option value held by all individuals at any time. At the time the option is exercised, *V* represents the aggregate value of foreign assets claimed against the foreign exchange (reserves) held by the Central Bank. This is the case because, when individuals exercise their option, the Central Bank provides the foreign currency in exchange for domestic money. At any other time, *V* is interpreted as the aggregate contingent foreign exchange liabilities that could materialize against international reserves.

To solve this general class of valuations, the following standard dynamic programming conditions need to be satisfied (Dixit and Pyndick, 1994):

(11) $\rho V^*(y)\,dt = E\left[e^{-\rho\tau} \int_{\tau}^{\infty} e^{-\rho(s-\tau)} dF\left(\Delta^e y_s, s\right) ds \Big/ y_0 = y \right]$

(12) $V^*(y^*) = \tilde{F}(y^*)$

(13) $\dfrac{dV^*(y^*)}{dy} = \dfrac{d\tilde{F}(y^*)}{dy}$

where $V^*(y)$ is the solution to the optimization problem, y^* is the level of foreign exchange earnings at which operators abandon the domestic currency, $\tilde{F}(y)$ is the expected present value of the option, and (11) and (12) represent the value matching and the smooth pasting condition respectively. If arbitrage possibilities are exhausted, all operators are identical, except for the fact that they hold different shares of domestic money, so that they all exercise their option at y^*. When this happens, a currency crisis occurs.

Using the aggregate current value of the option derived in expression (4), we have that:

(14) $\tilde{F}(y) = E\left[e^{-\rho\tau} \int_{\tau}^{\infty} e^{-\rho(s-\tau)} F\left(\Delta^e y_s, s\right) ds \Big/ y_0 = y \right] = \tilde{M}_0^{\$}\left[\tilde{D}_0 - G\dfrac{y}{\delta} - h\,\tilde{r} \right]$

where $\tilde{M}_0^{\$} = \dfrac{M_0^{\$}}{\rho}$ represents the current stock of high powered money divided by ρ, and $\tilde{D}_0 = \dfrac{D_0}{\rho}$ stands for the present value of the autonomous annual depreciation, and $\tilde{r} = \dfrac{\Delta r}{\rho}$ represents the present value of the annual expected (constant) accumulation of reserves, or equivalently, a once and for all change in the current stock of international reserves. Since $E[\Delta y_t / y_0 = y] = y_0 e^{\alpha t} - y_0 e^{\alpha(t-1)} = \left(1 - e^{-\alpha}\right) y_0\, e^{\alpha t}$, then we can define $G = g\left(1 - e^{-\alpha}\right)(\rho/\delta)$ and let y/δ refer to the present value of a constant annual oil income.

Condition (11) is satisfied by the general class of solutions $V^*(y) = A\,y^{\beta}$. Assuming $V^*(y) = A\,y^{\beta}$ and manipulating terminal conditions (12) and (13), the value y^* is given by:

(15) $\dfrac{y^*}{\delta} = \dfrac{\beta}{G(\beta - 1)}\left[\tilde{D}_0 - h\,\tilde{r}\right]$

Implicitly, the value of y^* determines the probability of occurrence of a crisis, i.e. the probability that y falls under y^*, and therefore, the average waiting time for individuals to exercise their option. For any $\beta<0$ or $\beta>1$, if $G>0$, this expression suggests that a larger autonomous expected depreciation will increase the exit value y^*, and therefore, on average, rise the probability of a crisis and decrease the waiting time for individuals to exercise their option. Also, since $h>0$, an accumulation of foreign reserves will decrease the exit value, and imply on average, a smaller probability of a currency crisis[7]. Therefore, because reserve accumulation simultaneously increases the waiting time for individuals to exercise their option and reduces the probability of a currency crisis, it can be understood as a mechanism to delay currency crises by discouraging speculators' attacks.

Substituting y^* back into the smooth pasting condition, we find that the solution for the total value of contingent claims is:

$$(16)\quad V=\frac{\tilde{M}_0^{\$}}{(1-\beta)}\left[\frac{G\,(\beta-1)}{\beta}\right]^{\beta}\left[\tilde{D}_0-h\,\tilde{r}\right]^{1-\beta}\left(\frac{y}{\delta}\right)^{\beta}\qquad \text{for } y>y^*$$

$$(17)\quad V=\tilde{M}_0^{\$}\left[\tilde{D}_0-h\,\tilde{r}-G\,\frac{y}{\delta}\right]\qquad \text{for } y\leq y^*$$

where β has two roots: $\beta_1>1$ and $\beta_2<0$ that satisfy the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+(\rho-\delta)\beta-\rho=0$. Notice that $V>0$ only for $\beta<0$, which constrains our solution to using $\beta=\beta_2$. For this case, and h, $G>0$, the value of total contingent claims against the Central Bank is a decreasing function of the present value of oil proceeds and reserve accumulation, and an increasing function of the autonomous expected depreciation $\tilde{D}_0$. This shows that reserve accumulation also acts as an instrument to mitigate the effect of currency crisis by reducing the amount of foreign exchange liabilities that can materialize.

To understand solutions (16)-(17), consider the case in which the accumulation of reserves have already been decided by the authorities to an arbitrary level. Given these reserves and the other conditions of the economy ($\tilde{D}_0$), we can know the level of income y^* that triggers a currency crisis (when operators decide to convert domestic currency into foreign exchange). At this point, only the behavior of income determines the occurrence of such crisis. If income falls under y^*, the crisis instantly takes place with the amount of liabilities determined by (17). If income is higher than y^*, operators do not exercise their option, but there is a threat over reserves identical to the amount of contingent liabilities pointed out by (16).

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[7] If $G<0$, expression (15) will be greater than zero, only if, for $h>0$, $h\,\tilde{r}>\tilde{D}_0$, i.e. only if the beneficial effect of accumulating reserves is sufficiently large with respect to the autonomous depreciation expectations. In other words, since oil proceeds have a destabilizing effect, the accumulation of reserves needs to be sufficiently high to cause appreciation expectations. However, if these appreciation expectations are too strong, paradoxically the probability of a crisis increases.

The explicit expression for the negative root and its derivative with respect to the variance of oil proceeds are correspondingly:

$$(18)\ \beta_2 = \left[\frac{1}{2} - \frac{(\rho-\delta)}{\sigma^2}\right] - \sqrt{\left[\frac{(\rho-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}}$$

$$(19)\ \frac{d\beta_2}{d\sigma^2} = \frac{(\rho-\delta)}{\sigma^4} + \left[\left(\frac{\rho-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}\right]^{-\frac{1}{2}} \left[\left(\frac{\rho-\delta}{\sigma^2} - \frac{1}{2}\right)\left(\frac{\rho-\delta}{\sigma^4}\right) + \frac{\rho}{\sigma^4}\right] > 0$$

Expression (19) is positive for $\rho > \delta$, a condition that always holds for $\alpha \geq 0$, since by definition $\delta = \rho - \alpha$. Therefore, as volatility increases, the absolute value of the root gets smaller and the amount of liabilities increases. Likewise, a smaller absolute value of the root reduces the exit value y^*, indicating a smaller probability of a currency crisis. Statistically this happens because, as uncertainty increases, the probability of getting an instantaneous positive jump (drift) in income also increases, which is equivalent to saying that the probability of a crisis decreases. Nonetheless, if a crisis takes place, the amount of foreign exchange claims would be higher.

4. Wealth in an oil economy

In this section, we model the wealth of an economy that obtains most of its foreign exchange earnings from the extraction of oil. The resource is developed by the state, which can decide to keep international reserves or to import capital goods in order to accumulate productive assets. Individuals can also claim part of these oil earnings as savings in foreign exchange or as consumption of foreign goods.

Since there are only two assets in the economy, the intertemporal country wealth depends on the future accumulation and returns of these assets:

$$(20)\ W_t^e = W_0 + E\left\{\int_0^\infty e^{-\rho t}\left[(1+\varphi)\, I_t \quad + \quad (1+\theta)\,\Delta r_t\right] dt\right\}$$

where W_0 is the initial wealth, φ is the return of the domestic investment in capital goods per year, and θ is the annual rate of return of reserves. That is, the expected country wealth is defined as the expected present value of the asset accumulation in the economy.

Because capital goods are not produced domestically and oil proceeds are the only source of foreign exchange, the inter-temporal budget constraint of the economy is given by the present value of the balance of payment:

$$(21)\quad E\left[\int_0^\infty e^{-\rho t} y_t (1-\omega) dt \Big/ y_0 = y\right] = E\left[\int_0^\infty e^{-\rho t}\left[C_t + I_t + O_t + \Delta r_t\right] dt\right]$$

$$(22)\quad \Rightarrow\ \frac{y}{\delta}(1-\omega) = \tilde{C} + \tilde{I} + \tilde{O} + \tilde{r}$$

where ωy is the opportunity cost of depleting the natural resource[8], $\tilde{O} = O/\rho$, $\tilde{C} = C/\rho$ and $\tilde{I} = I/\rho$ are the present value of the expected (constant) annual financial outflows and imports of consumption and capital goods respectively. This expression points out that the present value of uses of foreign exchange (imports of consumption and capital goods, capital outflows and reserve accumulation) cannot exceed the present value of the expected oil proceeds.

Contingent liabilities derived in the preceding section represent, during currency crises, capital flights defined as the massive exchange of domestic money claims for foreign currency. The rest of the time, contingent liabilities characterize a threat over reserves that can be partially materialized depending on the exchange rate policy followed by the authorities. In this sense, we can define the present value of expected capital outflows ($\tilde{O}$) as a fraction *k* of the contingent liabilities. Also, the term *kV* can be thought as a measure of the disruptive effects that a crisis can engender on the real economy.

Substituting out the present value of capital goods in expression (21), the expected wealth of the economy is:

$$(23)\quad W_t^e = W_0 + (1+\varphi)\left[\frac{y}{\delta}(1-\omega) - \tilde{C}\right] + (\theta - \varphi)\tilde{r} - (1+\varphi)\, k\, V\!\left(\frac{y}{\delta}, \tilde{r}\right)$$

The first addendum to the initial wealth W_0 represents the gains on wealth obtained by the transformation of oils proceeds into the stock of capital goods of the economy. This gain is positively related to the stock of natural resources and negatively related to the accumulated consumption of imported goods. The second addendum indicates the direct net expected contribution of reserves of foreign exchange to the country wealth. This contribution is proportional to the difference between the financial return earned by reserves and the rate of return that can be obtained by importing investment goods or by investing abroad. The third term on the right hand side represents the potential depletion of wealth (i.e. the contingent liability) that may come from the financial decisions of private agents, should they choose to convert their claims of domestic currency onto foreign exchange. This contingent reduction in wealth also accounts for the opportunity cost in terms of capital investments.

Notice that reserve accumulation will have an overall positive impact on wealth if the derivative $dW_t^e / d\tilde{r} > 0$, that is if:

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[8] The term ω can be thought as the rate that captures the implicit wedge between its present price and its superior long term price.

$$(24) \quad \tilde{M}_0^{\$}\,(1+\varphi)\,k\,h\left[\frac{\beta_2\delta}{G(\beta_2-1)y}\left(\tilde{D}_0-h\,\tilde{r}\right)\right]^{-\beta_2} > \quad \varphi-\theta$$

As a matter of fact, considering the order of magnitude of $\tilde{M}_0^{\$}$, this expression can be satisfied independently of $\varphi \neq \theta$, if $h\,\tilde{r} < \tilde{D}_0$, given G, h>0. This condition is the same that ensures that y*>0 and that V lies in the line of real numbers. Alternatively, if h>0, but $G<0$, then it is still sufficient that $h\,\tilde{r} > \tilde{D}_0$, which implies that, for compensating the destabilizing effect of oil income, the accumulation of reserves needs to be sufficiently large to cause appreciation expectations greater than the expected autonomous depreciation.

5. Reserve management

According to the preceding sections, foreign reserves have a beneficial effect on contingent liabilities by acting as a mechanism to delay and mitigate currency crises. In this context, risk is characterized by the probability assigned to a currency crisis, which occurs if oil revenues fall below the threshold at which asset holders will massively exercise their exit option from the domestic currency.

However, foreign reserves might cause a partial detriment on the expected wealth, if the return earned in international markets falls below the rate of reproduction of capital goods in the economy. In this case, opportunity costs are given by the investment opportunities foregone when holding financial resources in the form of a contingent asset rather than committing them to specific investments and/or development policies. Therefore, depending on net return earned by reserves, the strategy of reserve accumulation should be designed according to one of these two values: optimal or minimal reserves.

5.1. Optimal reserves

Assume also that country wealth is increasing in the overall value of reserves accumulated, but $\theta < \varphi$. If the return of capital investments is higher than the return of reserves, the accumulation of reserves has embedded an important opportunity cost. On the other hand, accumulating reserves still reduces the amount of potential foreign liabilities and reduces the probability of a crisis, all of which encourages its hoarding. This tradeoff allows arguing that the stock of reserves can be determined optimally by setting:

$$(25)\ \tilde{r}^* = \quad \arg\max \quad W^e(\tilde{r})$$

The FOC of this maximization problem is given by:

$$(26)\ \frac{\theta-\varphi}{k(1+\varphi)}=\frac{dV(\tilde{r})}{d\tilde{r}}$$

where $\frac{dV(\tilde{r})}{d\tilde{r}}=\tilde{M}_0^{\$}\left[\frac{G\,(\beta_2-1)}{\beta_2}\right]^{\beta_2}\left[\tilde{D}_0-h\,\tilde{r}\right]^{-\beta_2}\left(\frac{y}{\delta}\right)^{\beta_2}(-h)<0$. This condition, as most maximization conditions, equates the marginal cost of accumulating reserves ($\varphi-\theta$) to the marginal benefit (the absolute value of the right hand side term) in terms of smaller foreign exchange liabilities. Solving the FOC for the stock of reserves:

$$(27)\ \tilde{r}^* = \frac{1}{h}\tilde{D}_0 - \frac{G(\beta_2-1)}{\beta_2\,h}\left[\frac{\varphi-\theta}{h\ (1+\varphi)k\,\tilde{M}_0^{\$}}\right]^{-1/\beta_2}\left(y/\delta\right)$$

This expression contains two balancing terms. The first one shows the amount of reserve accumulation related to the existence of autonomous depreciation expectations. This whole term does not depend of the uncertainty of the oil proceeds, and the higher these expectations, the greater the need to accumulate foreign reserves as an asset to counterbalance the corresponding liability. The second term contains the variables that induce a lower accumulation of reserves, provided that $\varphi-\theta>0$: a greater expected oil income and smaller outstanding liabilities in the form of high powered money reduce the need to accumulate reserves, and so it does a reduction in volatility of oil income. This second term renders reserves dependent on uncertainty and transforms, therefore, the maximum amount of assets that would be needed to offset a sure liability into a form of contingent asset.

To evaluate the impact of income volatility on optimal reserves, recall that as volatility increases, the absolute value of the root (β_2) gets smaller. Since optimal reserves are higher when the β_2 approaches to zero, this implies that increasing volatility will increase the amount of optimal reserves[9]. This occurs because, as uncertainty increases, the probability of a crisis decreases, but the amount of contingent liabilities increases.

The SOC of the maximization problem requires that $\frac{d^2W^e(\tilde{r})}{d\tilde{r}^2}=-(1+\varphi)k\,\frac{d^2V(\tilde{r})}{d\tilde{r}^2}<0$. Since:

[9] As a matter of fact, $d\tilde{r}^*/d\beta_2>0$, if $\ln\left(\frac{\varphi-\theta}{h\ (1+\varphi)k\,\tilde{M}_0^{\$}}\right)<-1$, which is automatically satisfied for a sufficiently large magnitude of the present value of the high powered money in the economy.

$$(28)\frac{d^2V(\tilde{r})}{d\tilde{r}^2} = \tilde{M}_0^{\$}\left[\frac{G(\beta_2-1)}{\beta_2}\right]^{\beta_2}\left(\frac{y}{\delta}\right)^{\beta_2}(h)^2\left[\tilde{D}_0 - h\tilde{r}\right]^{-\beta_2-1}(-\beta_2) > 0$$

given $\beta_2 < 0$, then the SOC of the maximization problem is always satisfied.

5.2. Minimal reserves as a self insurance

When the difference between the return of reserves and the return of capital investment is positive, that is $\theta - \varphi > 0$, accumulating reserves only has beneficial effects. On one hand, there is an accumulation of oil proceeds in the form of the most productive asset available that increases wealth, and on the other hand, there is a gain associated to the reduction of foreign liabilities and the probability of a crisis, all of which encourages the hoarding of reserves. In this case, the tradeoff that called for an optimization procedure disappears, and the amount of reserves accumulated can be arbitrarily large.

In this case, since there is not an upper bound to the accumulation of reserves, it would seem desirable to point out a lower bound or a minimal amount of reserves. This minimal amount could be established in terms of answering what level of reserves covers the expected foreign exchange liabilities that would arise at the time of a currency crisis when private agents exercise their claims. More generally, consider a rate of coverage (ϕ) applied on the expected amount of liabilities, the present value of this minimum amount of reserves should exactly match the selected value of shielded liabilities, such that:

$$(29)\quad \tilde{r}^m = \phi\ V\left[y^*\left(\tilde{r}^m\right), \tilde{r}^m\right]$$

$$(30) \Rightarrow \tilde{r}^m = \frac{\phi\,\tilde{M}_0^{\$}}{1-\beta_2 + h\,\phi\,\tilde{M}_0^{\$}}\,\tilde{D}_0$$

According to this expression, the minimal reserves should be directly proportional to the size of autonomous depreciation expectations, the stock of outstanding liabilities of domestic currency and the rate of coverage (ϕ) decided by authorities. Nonetheless, notice that changing the amount of minimal reserves, for instance by varying the desired size of the coverage, modifies both, the size of expected foreign liabilities during a crisis and the implicit probability of a crisis. In particular, as the size of coverage gets larger, contingent claims are reduced, and the probability of a crisis gets smaller as y^* diminishes[10].

[10] In fact, the closed form solution for the critical income level when accumulating minimal reserves is $\frac{y^*}{\delta} = \frac{\beta}{G\left(\beta - 1 - \phi\, h\, \tilde{M}_0^{\$}\right)}\tilde{D}_0$, which is inversely proportional to ϕ.

Therefore, selecting the size of the coverage against expected foreign claims during a crisis is identical to choosing the probability of a crisis that authorities will face and the amount of claims that effectively could be honored by the Central Bank. Because this amount of minimal reserves limits the exposure to risk according to the preferences revealed by authorities, they represent a form of self insurance mechanism against a currency crisis. This form of self insurance is in line with the approach followed by García y Soto (2004).

Differently than the approach presented by Lee (2005), we do not address the possibility that part of the insurance against crises can be obtained through market-based mechanisms (such as a put option) and neither can we refer to the optimal size of this insurance. Nevertheless, in our approach, authorities can discretionally decide the quality of the self insurance by evaluating its coverage in terms of the expected exposure to currency crisis.

6. Estimation of reserves for Venezuela

6.1. Parameters estimation.

Optimal or minimal reserves can be computed through this model, if parameters for the expected depreciation (g and h in equation 3) are estimated. However, to obtain estimates of these parameters for Venezuela, we need to consider that the dynamic of the nominal exchange rate has varied over time due to several modifications applied to the exchange rate regime. To overcome this difficulty, instead of modeling the nominal exchange rate, we model the *exchange rate market pressure* (*EMP*), which allows using a sufficiently long data set on the market, independently of the type of exchange rate regime in place. Empirical grounds for the estimation of the exchange market pressure are found in the literature of currency crises, in particular, in the works of Sachs, Tornell and Velasco (1996), García and Soto (2004) and Edwards (2004).

Inspired in Girton and Roper (1977), the exchange rate market pressure is a measure that linearly combines the growth rate of the nominal exchange rate D and the ratio of Central Bank net foreign exchange sales to international reserves (*Sl/R*), such that:

$$(31)\quad EMP \equiv f\left(D\,,\frac{Sl}{R}\right)$$

where *f*(.) denotes a linear function, whose coefficients are the inverse of the standard deviation of the variable. As it is defined, either a depreciation of the domestic currency or in an increase of the amount of dollars sold by the Central Bank increases the market pressure, capturing any change in the excess demand of foreign currency. During periods of exchange rate controls (dual systems), D refers to the growth rate of the parallel exchange rate, while *Sl* represents the amount of dollars supplied by authorities for commercial transactions. In periods of fixed exchange rate, the market pressure is completely captured by the foreign exchange sales of the Central Bank.

The estimation of the exchange rate market pressure is carried out with a GARCH model to control for time heteroskedasticity. The empirical explanatory variables for the market pressure are the annual change in the value of yearly oil exports (Δy), the annual accumulation of reserves (Δr). Estimation results are shown in the appendix. We retrieve the coefficients of equation (3) by a two step procedure: first, by computing the annual (instead of the quarterly) change in the market pressure per unit of change in oil exports and reserve accumulation using the dynamic structure of the equation, and second, by transforming these market pressure coefficients in units of nominal exchange rate depreciation (using the standard deviation of the exchange rate depreciation).

Finally, to retrieve the drift parameter (α) for equation (9), we use the fact that the mean of the change of the logarithm of the annual oil exports (μ) can be expressed as function of the standard deviation and the drift parameter ($\mu = \alpha - 0.5\,\sigma^2$). Since the sample available for the oil exports includes the oil boom registered from 2005 to 2008, the historical value of μ is equal to 0.1, which seems a very large value to use as a forecast for the expected behavior of the oil revenues. We set μ =0.05, σ takes its historical value of 0.25, and compute the roots of the characteristic equation according to the formulas provided. The discount rate ρ is set at 0.15, to satisfy that $\rho > \alpha$. Estimated coefficients are summarized in the following table:

Table 1. Estimated coefficients

Parameter	Estimate
g^*	0.093
h^*	0.285
G^*	0.016
D_0	0.169
σ	0.250
α	0.081
β_1	1.532
β_2	-3.132

*Coefficients are expressed in p.p. of currency depreciation per ten billions of US dollars

6.2. Numerical calculations

In this section, we provide numerical values for optimal and minimal reserves to assess the adequacy of the model for the case of Venezuela.

The closed form expressions that we use incorporate the existence of an exchange rate control implemented in Venezuela since 2003. For this reason, we need to estimate the probability of obtaining a dollar at the official exchange rate when undertaking financial transactions (the parameter q), which is pinned down to 0.25[11]. This parameter affects the average exchange rate prevailing in the economy (v), the amount of the high powered money in dollars valued at v, and the term P, interpreted as an indicator of the state of the exchange rate control. The exchange rate premium is computed as the wedge between the official exchange rate (Bs/$ 2.15) and the value of the parallel exchange rate set at Bs/$ 5.5 (an average value observed during 2009 in the parallel market).

In general, optimal reserves are highly dependent on the assumptions made regarding the current level of oil revenues (y) and the returns of capital goods and reserves. In this exercise, we consider several values for the annual oil income, and compute the expression y/δ, which is interpreted as the expected present value of receiving a constant annual oil income. However, providing a good estimate of the yield of capital investments goes beyond the scope of this paper. Instead, we provide possible values for the opportunity cost of reserves ($\varphi - \theta$), considering different levels of the capital investment return. Results are shown in Table 2.

Table 2. Optimal reserves*

Optimal Reserves				
Opportunity Costs		***y***		
φ	***θ***	**40,000**	**65,000**	**90,000**
	0.000	35,091	28,443	21,794
0.05	0.025	37,203	31,874	26,545
	0.049	42,678	40,771	38,864
	0.000	32,652	24,479	16,306
0.10	0.050	35,248	28,697	22,147
	0.099	42,723	40,844	38,965
	0.000	31,055	21,883	12,712
0.15	0.075	33,968	26,617	19,266
	0.149	42,765	40,913	39,060
Mean		37,043	31,613	26,184
Standard deviation		4,588	7,456	10,323

*Monetary figures are in MM of US $

According to Table 2, and to the formulas provided, the optimal reserves will decrease as the value of the expected oil income gets larger. Notice that, for any level of the return of investments, as $\varphi - \theta \to 0$, the optimal stocks of reserves tend to modestly vary around U.S.$ 40,800 MM, even for large variations in the expected oil income. In this case, the optimal level of reserves mostly reflects the state of the autonomous depreciation expectations (D_0), which are parameterized around 17% per year. When opportunity costs are different from zero, optimal reserves, on average,

[11] Estimates of this parameter can be computed marginally, given the approximate size of transactions carried out in the parallel market and the amount of over-invoiced imports. Recent estimates of q range between 0.2 and 0.3, but they could be subject to important changes.

decrease as the return of investments in capital goods is higher. Also, a greater gap between φ and θ will reduce the need for reserves, as capital goods become a better option of investment than reserves.

To illustrate the behavior of minimal reserves, we give several examples for different levels of the rate of coverage ϕ and the annual autonomous depreciation expectations D_0. Estimates are provided in table 3.

Table 3. Minimal reserves*

Minimal Reserves / Critical Oil Income				
ϕ	**variable**	**D_0=0.35**	**D_0=0.17**	**D_0=0.085**
0.5	r^m	27,085	14,143	8,031
	y^*	57,366	29,954	17,009
1.0	r^m	41,437	21,636	12,286
	y^*	43,881	22,913	13,011
1.5	r^m	50,325	26,278	14,922
	y^*	35,529	18,552	10,535

*Monetary figures are in MM of US $

Table 3 shows that as the level of coverage increases, not only the amount of contingent liabilities that could be honored increases, but the risk faced in terms of the probability of a currency crisis diminishes. On the other hand, larger autonomous depreciation expectations increase the amount of minimal reserves needed to satisfy a given rate of coverage.

It is important to point out that these values of optimal and minimal reserves represent average values, given a stylized expected behavior of the economy. In this sense, this exercise neither captures the changing behavior of the economy nor does it consider the impact that stochastic shocks could impinge on the different variables involved in calculations.

From the point of view of policymakers, these values should be interpreted as merely referential and cannot be understood as the exact amount of reserves that the Central Bank should hold at all times. Indeed, many oil economies maintain part of their foreign exchange proceeds in the form of saving or investment funds, which tend to gain higher returns than the reserves managed by the Central Bank[12]. Because these alternative forms of savings also constitute a stock of foreign assets that can deter speculators from triggering a currency crisis, one could interpret these amounts of optimal or minimal reserves simply as the sum of foreign assets that should be maintained altogether at the Central Bank and at the investment fund. The allocation of foreign assets between these two forms of accumulation can be ultimately a decision of policymakers based on the desirable degree of liquidity to be achieved.

[12] In comparison to the management of reserves at the Central Bank, the greater return of these investment funds is generally attributed to the less conservative portfolio allocation pursued. Such financial strategy typically entails a greater financial diversification at the expense of a lower degree of immediate availability of resources (liquidity).

7. Conclusions

In this paper we have looked at the problem of reserves from the point of view of their function as an instrument to control the contingent liability constituted by the accumulation of individual claims against the Central bank. These claims correspond to the issuance of high powered money denominated in domestic currency and to the consequent risk of massive attempt at conversion on the part of the claimholders. In this paper, reserves are one instrument to counteract the contingent liability with a correspondent contingent asset because they reduce the expectations of depreciation of the domestic currency and discourage contingent capital flight. In an oil dependent economy, these contingent values depend on the dynamic uncertainty of oil revenue, whose mean value and variance both tend to vary over time as a consequence of the myriad of factors affecting international demand and domestic supply. Under these conditions, reserve accumulation can be rationally planned according to two possible patterns. If the return on reserves is lower than the return obtainable on the alternative use for the same funds, a trade off emerges between the use of foreign exchange to obtain maximum returns and its use to deter holders of domestic balances to convert their claims. In this case, "optimal reserves" can be set at a level reflecting two separate components: a positive one that perfectly counterbalances the expected (non contingent) devaluation for the outstanding claims, and a negative one, reflecting the contingent nature of oil revenue and of its alternative uses. As the common wisdom would indicate, a greater volatility of oil proceeds should be compensated with a higher accumulation of reserves, in order to adequately face the materialization of larger contingent claims.

When the return on reserves is sufficiently high with respect to its alternative use, optimal levels cannot be specified, and reserves can be accumulated in arbitrarily huge amounts. In this case, the accumulation of reserves can still be used as an insurance to limit the exposure to the risk of a currency crisis. In particular, "minimal" reserves can be set to guarantee honoring a selected amount of foreign exchange claims at the time of a crisis.

The estimates developed for Venezuela show that the net effect of reserves on reducing expected depreciations is both significant and substantial. As a consequence, combined with the estimates of the other parameters of the model, the computed levels of optimal and minimal reserves appear to be reasonable with respect to the size and conditions of the economy.

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Appendix 1. Estimation of Market Pressure

Dependent Variable: EMP
Method: ML - ARCH
Sample (adjusted): 1991Q1 2008Q4
Included observations: 72 after adjustments
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.259536	0.063355	4.096511	0.0000
EMP(-1)	1.221471	0.102548	11.91120	0.0000
EMP(-2)	-0.394840	0.112410	-3.512481	0.0004
Δ y	-1.43E-05	8.48E-06	-1.688596	0.0913
Δ r (-1)	-4.39E-05	1.68E-05	-2.613750	0.0090
Variance Equation				
C	0.028590	0.026688	1.071273	0.2840
RESID(-1)^2	0.327440	0.246430	1.328734	0.1839
GARCH(-1)	0.494078	0.223569	2.209957	0.0271

R-squared	0.925534	Mean dependent var	0.774564
Adjusted R-squared	0.917390	S.D. dependent var	1.409578
S.E. of regression	0.405141	Akaike info criterion	1.035834
Sum squared resid	10.50492	Schwarz criterion	1.288796
Log likelihood	-29.29001	Hannan-Quinn criter.	1.136539
F-statistic	113.6367	Durbin-Watson stat	1.727430
Prob(F-statistic)	0.000000		

EMP: market pressure (in standardized units)
Δy: the annual change of the value of yearly oil exports (in millions of US $)
Δr: the annual accumulation of international reserves (in millions of US $)