MPRA
Munich Personal RePEc Archive

A Markov-Switching Model of Inflation: Looking at the future during uncertain times

Barraez, Daniel and Pagliacci, Carolina

Central Bank of Venezuela, Central Bank of Venezuela

August 2009

Online at https://mpra.ub.uni-muenchen.de/106550/
MPRA Paper No. 106550, posted 22 Mar 2021 09:52 UTC

A Markov-Switching Model of Inflation: Looking at the future during uncertain times *

Carolina Pagliacci†

Daniel Barráez§

September 2009

Abstract

In this paper, we analyze the dynamic of inflation in Venezuela, in the last eighteen years, through a Markov-switching estimation of a New Keynesian Phillips curve. Estimation is carried out using the EM algorithm. The model's estimates distinguish between a "normal or backward looking" regime and a "rational expectation" regime consistent with episodes of high uncertainty regarding the performance of the economy. This characterization of regimes is based on two elements: the description of the process of formation of inflationary expectations and the main economic events occurred during each regime.

Key words: regime switching, Phillips curve, inflationary expectations.

JEL classification: C29, E31, and D84.

* Opinions on this paper are full responsibility of the authors and do not compromise those of the Banco Central de Venezuela. We are grateful for the research assistance of Jeison Pérez, and comments of Harold Zavarce and León Fernández. All errors and omissions are our own.

† Senior Researcher at the Banco Central de Venezuela.

§ Senior Researcher at the Banco Central de Venezuela and Professor at the Universidad Central de Venezuela.

I. Introduction

Since the seminal work of Hamilton (1989) many researchers have devoted to studying the economic growth from the perspective of regime changes, identifying two phases in the economic cycle: expansions and contractions. Through time, Markov-switching models have experienced refinements in the applied econometric techniques, but their estimation has mostly kept the original spirit of Hamilton's work: distinguishing between regimes of recessions and expansions. For instance, papers such as Kim and Murray (2002) and Kahn and Rich (2007) have incorporated the occurrence of regime switching in the non-observable components of growth (as in state-space models). Diebold and Rudebush (1996) study the business cycle assuming that the transition matrix that governs the process of switching is variable instead of being constant.

A less popular, but largely important use of the Markov-switching models has been the study of non-linearities in inflation. In an early work, Evans and Wachtel (1993) focus on analyzing the sources of uncertainty that affect the dynamics of inflation and agent's inflationary expectations collected in surveys. Assuming that inflation can either follow a random walk process or an autoregressive process, these authors established that the switch between these two regimes explains the presence of discrete jumps in the United States inflation during the postwar period. Also, the uncertainty attached to the changes of regimes is identified as the source of the recurrent differences between the forecasts of inflation collected in surveys and the actual rates of inflation. Other papers, like Simon (1996) and Blix (1999), emphasize the use of Markov-switching models to explain visible changes in the inflation dynamics and to improve inflation forecasts. Simon (1996) models inflation in Australia incorporating information of the output gap. More recently, Demers (2003) describes the non-linearities in Canadian inflation through the estimation of a Markov-switching backward looking Phillips curve.

In Venezuela, inflation dynamics has also been subject to important changes probably due to the continuous modifications impinged to the exchange rate regime at times of external crisis. These changes, or presumably structural breaks in the inflation dynamics, make linear models inappropriate tools for analyzing inflation through time. In order to fully capture these non-linearities, the objective of this paper is to model Venezuelan inflation through the estimation of a Markov-switching New Keynesian Phillips curve. The advantage of this type of non-linear models is that they allow combining the existence of different stochastic processes for inflation without imposing too many restrictions to the data generating process. On the other hand, the estimation of a New Keynesian Phillips curve, in a similar fashion as in Demers (2003), provides a basic understanding of the behavior of inflation from an economic point, using a theoretical structure that admits incorporating variables that traditionally have had a predominant figure in explaining inflation in Venezuela, such as the output gap. In fact, several works in Venezuela, such as Dorta et al. (2002), Arreaza et al. (2003) and Dorta (2006), estimated the impact of the output gap on inflation for different time periods, but using exclusively linear models. In our specification, we additionally allow the process of money creation by the public sector to influence the behavior of inflation. We also incorporate the rate of growth of the nominal exchange rate as an explanatory variable, to capture possible changes in its pass-through on inflation[1].

One of the challenges that arises within the evaluation of Markov-switching models with exogenous explanatory variables is that the characterization of regimes can not be done prior to estimation anymore. It is no longer clear that the regimes captured by this type of models (even in a two-regime setting) refer to high and low inflation regimes, analogously as it is done in the literature when considering contractions and expansions of the economy. On the contrary, after selecting the appropriate number of regimes, we need to make use of the estimation results and the nature of the relationship established between inflation and its explanatory variables, to understand and characterize the types of regimes found. To complete the categorization of regimes, we also observe the classification of periods provided by the probabilistic estimates of the most likely regime prevailing at each point in history along with information about the main economic historical events. This task, although more complicated, reveals a richer approach to understanding the dynamic of inflation.

[1] Mendoza (2006) studies exclusively the phenomenon of the pass-through of the nominal exchange rate in Venezuela in the

Another important feature that comes with the estimation of a New Keynesian curve Phillips with inflationary inertia is that endows the model with a sufficiently rich dynamical structure that can be employed to describe the process of formation of inflationary expectations. In the spirit of Sargent (1987), and differently than the approach of Evans and Wachtel (1993), in this paper, inflationary expectations are assumed to be the solution of the dynamic model estimated in each regime. This interpretation of how inflationary expectations are formed allows linking the behavior of expected inflation to the time trajectory of the explanatory variables, and offers an additional intuition of what factors may drive the changes in the inflation dynamics.

In order to estimate the type of Markov-switching model we are proposing, we need to adapt the EM algorithm explained in Hamilton (1990), which is mainly applied to autoregressive processes with a constant mean per regime. We chose using the EM algorithm in order to exploit its robustness and fast convergence property, as it is done in most of the non-economic literature, like for instance, the literature of speech pattern recognition. In most of the economic literature, although the advantages of the EM algorithm are acknowledged, the estimation of switching models is usually carried out with numerical maximization techniques.

Close to the results described in Evans and Wachtel (1993) and Simon (1996), we find that inflation can either follow an explosive stochastic process or a stationary autoregressive process. The estimated model also shows that the process of forming inflationary expectations switches from periods in which agents use the past values of variables to periods in which agents look rationally at forward information on variables. In many of these periods of "rationally" formed expectations, we also observe the occurrence of speculative attacks to the domestic currency and the implementation of reforms to the existing exchange rate system. In other periods, although the "rational expectation" regime is identified, we can only suggest the existence of conditions of overall uncertainty generated by a greater exposure of the economy to external shocks.

Under either type of circumstances, we could state that in the "rational expectation" regime

context of a non-linear VAR estimated with smooth transition techniques.

agents stop looking at the past behavior of economic variables and revert to using subjective information on such variables, especially economic growth. Then, these expectations on aggregate demand are the ones that change the formation of expectations on current inflation and therefore, determine the pricing strategy of producers and sellers.

The paper is structured as follows: section II presents the general non-linear regression model for a single variable and explains the EM algorithm. Section III shows the structure for the Phillips curve and presents the main estimation results. Section IV describes the process of formation of inflationary expectations and section V concludes.

II. The regression model and the EM estimation

Because of the paramount importance of exogenous explanatory variables to describe the behavior of inflation in Venezuela, we consider the following general non-linear regression model suggested in Hamilton (1994):

$$y_t = z_t \beta_{si} + \varepsilon_{si,t} \qquad \text{for } si=1,2,\ldots,\text{N and } t=1,2,\ldots,\text{T} \tag{1}$$

y_t refers to the model's endogenous variable, z_t is a (1xk) vector that contains the explanatory variables (could include lagged values of y) , and β_{si} represents a (kx1) vector of coefficients associated to regime *si*, which by definition is unobservable. The error term $\varepsilon_{si,t}$ is also associated with regime *si* and is i.i.d. according to $\varepsilon_{si,t} \sim N\left(0, \sigma_{si}{}^2\right)$. The total number of possible regimes or hidden states is given by N, and the realizations of particular states are governed by the following first-order Markov process Q_t, such that:

$$
\begin{aligned}
&\Pr\left(q_t = sj \mid q_{t-1} = si\right) = p_{ij} \\
&\Pr\left(q_1 = si\right) = \pi_i \qquad\qquad 1 \leq si, sj \leq N \\
&\sum_{j=1}^{N} p_{ij} = 1 \quad , \quad \sum_{i=1}^{N} \pi_i = 1
\end{aligned}
\tag{2}
$$

These p_{ij}'s can be ordered in a so called transition probability matrix P, while the unconditional probabilities of hidden states (π_i) are represented with a column vector Π of initial probabilities, as follows:

$$
P = \begin{bmatrix} p_{11} & p_{21} & \ldots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \ldots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix} \quad , \quad \Pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_N \end{bmatrix}
\tag{3}
$$

The above description implies that, once a realization of a regime occurs at a given point in time, the observable variable y_t exhibits a conditional mean equal to $z_t \beta_{si}$. Then, the realization of the next hidden state is a random draw governed by the transition probabilities defined in P. The complete model can be characterized by the set of parameters $\Theta = \{P, \Pi, \mathrm{B}\}$, where $\mathrm{B} = \{\beta_{s1}, \beta_{s2}, \ldots, \beta_{sN}, \sigma^2_{s1}, \sigma^2_{s2}, \ldots \sigma^2_{sN}\}$ depicts the relationship between the endogenous and the explanatory variables of the model for all N different regimes.

The estimation of the above model is performed though the implementation of the EM algorithm, which finds the set of parameters that maximizes the likelihood function of the observed data through an iterative expectation process. We chose using the EM algorithm, as it is done in most of the non-economic literature, because "…this algorithm is quite robust with respect to poorly selected starting values and quickly moves to a reasonable region of the likelihood surface" (Hamilton 1990). This implies that for different starting values, the algorithm converges to the same solution with relatively few iterations and minimizes the problem of evaluating hundreds of initial values.

Given the structure of the model, the theoretical likelihood function for a sequence of observed data $Y^T = \{y_1, y_2, \mathrm{K}\ , y_T\}$ has to consider the possible sequence of hidden states that could have occurred, name it $S^T = \{q_1, q_2, \mathrm{K}\ , q_T\}$. This is the case because, hidden states condition the probability distributions of the endogenous variable, indicating that a joint probability of hidden states and observations must exist. Therefore, knowing the parameters of the model and a particular sequence S^T, this joint probability can be defined as:

$$\Pr\left(Y^T, S^T / \Theta\right) = \Pr(q_1) \prod_{t=2}^{T} \Pr(q_t / q_{t-1}) \prod_{t=1}^{T} \Pr\left(y_t / q_t, z_t\right) \tag{4}$$

And the theoretical likelihood function for the entire sample $L(Y/\Theta)$ can be simply written as:

$$L(Y/\Theta) = \sum_{\forall S} \Pr\left(Y^T, S^T / \Theta\right) = \sum_{\forall S} \Pr(q_1) \prod_{t=2}^{T} \Pr(q_t / q_{t-1}) \prod_{t=1}^{T} \Pr\left(y_t / q_t, z_t\right) \tag{5}$$

where, for instance, $\sum_{\forall S} g(S) = \sum_{q_T = s1}^{SN} \sum_{q_{T-1} = s1}^{SN} \mathrm{K} \sum_{q_1 = s1}^{SN} g(q_1, q_2, \mathrm{K}\ , q_T)$. That is, the likelihood function must consider all possible sequences of hidden states, and not only a particular sequence.

However, the expressions to implement the EM algorithm are derived, not by directly maximizing the likelihood function in (5), but by maximizing an alternative expression $Q\left(\Theta^{(l)}, \Theta^{(l-1)}\right)$ that makes explicit the fact that maximization is achieved iteratively by considering diverse parameter values for the model. The proof of this equivalence can be read either in Hamilton (1990) or Welch (2003). The particular form for this alternative expression is given by:

$$Q\left(\Theta^{(l)}, \Theta^{(l-1)}\right) = \sum_{\forall S} \ln \Pr\left(Y^T, S^T / \Theta^{(l)}\right) \Pr\left(Y^T, S^T / \Theta^{(l-1)}\right) \tag{6}$$

where the arguments of the function denote the existence of a sequence parameters $\left\{ \Theta^{(1)}, \Theta^{(2)}, \mathrm{K}\ , \Theta^{(l)} \right\}$ that are used in the different iterations of the maximization process. This function, according to Hamilton (1990) can be interpreted as the expected log-likelihood (for all sequences of hidden states) of the observable variable parameterized by $\Theta^{(l)}$, where the weights of the expectation operator are given by the joint probability of data and hidden states parameterized by $\Theta^{(l-1)}$.

Therefore, the application of the EM algorithm entails finding a sequence of estimated parameters $\left\{\hat{\Theta}^{(1)}, \hat{\Theta}^{(2)}, \mathrm{K}, \hat{\Theta}^{(l)}\right\}$ such that $L\left(\hat{\Theta}^{(l)}\right) \geq L\left(\hat{\Theta}^{(l-1)}\right)$ is always satisfied for any l^{th} iteration of the algorithm. The recursive application of this procedure leads eventually to find a fixed point where $\hat{\Theta}^{(l)} = \hat{\Theta}^{(l-1)}$ is satisfactorily approximated, and $\hat{\Theta}^{(l)} = \arg\max L\left(\hat{\Theta}\right)$, that is, $\hat{\Theta}^{(l)}$ is the maximum likelihood estimator.

In the EM algorithm, the analytical functional forms for the parameter estimates are obtained by solving the first order conditions that maximize expression (6) respect to $\Theta^{(l)}$. Among these FOCs, Hamilton (1990) shows that the estimation of the regression parameters in (1) satisfies:

$$\sum_{\forall S} \left. \frac{\partial \ln \Pr\left(Y^T, S^T / \Theta^{(l)}\right)}{\partial \mathrm{B}^{(l)}} \right|_{\hat{\mathrm{B}}^{(l)}} \Pr\left(Y^T, S^T / \hat{\Theta}^{(l-1)}\right) = 0 \tag{7}$$

Since the sequences of hidden states are not directly observed by the econometrician, then they are inferred from the sequence of realizations of the observed variable Y^T, which entails to re-writing $\Pr\left(Y^T, S^T / \hat{\Theta}^{(l-1)}\right) = \Pr\left(S^T / Y^T, \hat{\Theta}^{(l-1)}\right) \Pr\left(Y^T / \hat{\Theta}^{(l-1)}\right)$. After several algebraic manipulations, and a change of representation of the sequences of hidden states, Hamilton (1990) shows that the maximum likelihood estimator $\hat{\mathrm{B}}^{(l)}$ must satisfy:

$$\sum_{t=1}^{T} \sum_{q_t = s1}^{SN} \left. \frac{\partial \ln f\left(y_t / q_t = si, z_t, \mathrm{B}^{(l)}\right)}{\partial \mathrm{B}^{(l)}} \right|_{\hat{\mathrm{B}}^{(l)}} \Pr\left(q_t = si / Y^T, Z^T, \hat{\Theta}^{(l-1)}\right) = 0 \tag{8}$$

where $f\left(y_t / q_t = si, z_t, \mathrm{B}\right)$ is the density function of y_t conditional on the parameters of the regression model, on the assumed hidden state q_t, and on z_t, which is a row vector of dimension k containing information on the lagged endogenous variable and on the exogenous variables of the model (x_t), such that $z_t = \{y_{t-1}, y_{t-2}, \ldots\, y_{t-p}, x_{1t}, x_{2t}, \ldots, x_{(k-p)t}\}$ and p is the number of lags for the endogenous variable. On the other hand, $\Pr\left(q_t = si / Y^T, Z^T, \hat{\Theta}^{(l-1)}\right)$ is the probability that the hidden state si has occurred at time t, conditional on the entire data sample: $Y^T = \{y_1, y_2, \ldots\, y_T\}$ and $Z^T = \{z_1, z_2, \ldots\, z_T\}$, evaluated in the parameter estimates from the preceding iteration. In our model, as already stated, we assume that the conditional density of y_t is normal, such that $f(y_t / q_t = si, z_t, \mathrm{B}) = \frac{1}{\sqrt{2\pi\sigma_{si}^2}} \exp\left\{\frac{-(y_t - z_t\beta_{si})^2}{2\sigma_{si}^2}\right\}$. The specific form of the EM algorithm used in the estimation process is presented in appendix A. All the econometric programming is carried out in Gauss.

III. A Phillips curve estimation with Markov-switching

In this section, inflation is analyzed through the estimation of a two-regime New Keynesian Phillips curve. We model inflation strictly as a function of lagged inflation, indicating that only inflationary inertia (and not inflationary expectations) determines the level of the structural or underlying inflation. Statistically, this simplifying assumption will allow fitting the model within the class of models presented in (1), and will also enable showing the length of the impact of shocks hitting the economy in each regime. Theoretically, the existence of inflationary inertia is related to the existence of a staggered price setting, which means that, if firms change prices at different times, adjustment of the aggregate price level to shocks takes longer, even when individuals change prices frequently (Ball et al. 1988). This is equivalent to stating that during periods of high inflationary (or price level) inertia, shocks have larger and longer lasting effects. Furthermore, an increase in inflationary inertia would imply a higher dispersion in the timing of price adjustments by individual firms, or equivalently, a larger coordination failure between firms in acknowledging the occurrence of aggregate demand shocks.

The pressures of aggregate demand on inflation are summarized by the inclusion of the output gap (the IS component) and a variable that measures the quantity of money created by the public sector (the LM component) as explanatory variables. This money variable represents the main source of money supply in the economy and it is the result of combining the state monopoly of the oil activity with the fact that an important size of domestic public expenditures is financed with oil resources. Its inclusion as an additional aggregate demand factor tries to find out if an excess of money supply respect to the size of the nominal output will impinge a positive pressure on the inflation rate. Inflation also depends on the nominal depreciation of the domestic currency, as a way to acknowledge the potential impact of cost-push elements (supply shifters) on the inflation dynamic.

The particular regression model for the Phillips curve is given by:

$$Inf_t = a_{si} + \rho_{si}\ Inf_{t-1} + \alpha_{si}\left(Gdp_{t-1} - Gdp^*_{t-1}\right) + \gamma_{si}\ M_{t-2} + \delta_{si}\ \hat{E}_{t-1} + \varepsilon_{t,si} \qquad \text{for } si\text{=1,2} \qquad (9)$$

where *Inf* represents the annual average inflation rate, $Gdp_{t-1} - Gdp^*_{t-1}$ is the output gap computed as the difference between the log of the annual real GDP and its Hodrick-Prescott tendency, *M* is the ratio between the money created by the public sector[2] in a year span and the nominal GDP, and $\hat{E}$ is the rate of depreciation of the domestic currency, measured as the log difference of the yearly average of the nominal exchange rate (Bs per U.S. dollar)[3].

The lag structure of the regression model in (9) was chosen by running several linear regressions for the complete estimation period. The recursive procedure implied starting with a general model of four lags (for all explanatory variables) and reducing all non significant variables until obtaining a parsimonious model that contained only significant lags. The estimation period is defined from 1990:2 to 2008:04, for a total of 75 quarterly observations. This estimation period was selected to incorporate the longest quarterly series available for the variables chosen.

---

[2] For this case, the public sector is defined as the sum of the Central Government, the state oil industry (PDVSA) and the Central Bank.

[3] During periods of exchange rate controls (1994-1996 and 2003 to the present), this exchange rate refers to the value of the dollar in the non-official market.

The number of regimes or hidden states was selected using a mixed criterion: a statistical one and an economic one. First, we evaluated the value of the likelihood function for two and three regimes respectively. Second, given that the differences in the likelihood functions seemed statistically insignificant, we observed the classification of regimes provided by each model. A two-state model was preferred over a three-state model because of the few time periods classified in the third regime (barely three) and the lower power of the three-state model to distinguish among diverse regimes.

Initial values for the $\beta^{(0)}$ parameters to implement the EM algorithm were chosen by imposing, in each regime, variations to the estimated linear regression parameters. Such variations were constructed taking into account that each regime might contain extreme values of the parameters, but within their expected theoretical range. In this way, OLS estimates are simply interpreted as average estimates of the true two underlying regimes prevailing in the economy. Additionally, we use a grid search to discover the combination of initial values for the transition matrix ($P^{(0)}$) that converged to the maximum value of the empirical expected log-likelihood function. Initial unconditional probabilities ($\Pi^{(0)}$) were set as the ergodic probabilities of the Markov process, as suggested by Hamilton (1994).

After applying the EM algorithm, estimation results are summarized in table1.

Estimated coefficients in regime 1 show that inflation responds significantly to all the explanatory variables of the model in the expected magnitude and direction. The autoregressive component of the inflation is positive and strictly less than one, describing inflation as a stationary autoregressive process. Among of aggregate demand factors, the output gap has the greatest explanatory power. The pass-through coefficient indicates that a 10% depreciation of the domestic currency will cause 2 p.p. of increase in the rate of inflation in the first quarter, and 6.1 p.p. in a year span. Regarding the public money supply, an increase of this variable in 10 points of the nominal GDP, will boost inflation in 2.8 p.p. the first quarter and, 8.5 p.p. in a year span. According to the relationship established between inflation and the explanatory variables, one could characterize this regime as the “normal” state of the economy, or at least, as a regime in which inflation is appropriately described by the theory.

Table 1.- Two-Regime Coefficient Estimates for the Phillips Curve

Dependent Variable: *Inf*
Estimation Method: EM
Sample (adjusted): 1990Q2 2008Q4
Included observations: 75 after adjustments
Iterations: 70

Regime	Parameters	Estimates	Std. Error	t-Statistc	Prob.
1	a_1	-0.055658	0.000368	-2.901602	0.0050
	ρ_1	0.818271	0.000319	45.80546	0.0000
	α_1	0.438148	0.013587	3.75882	0.0004
	γ_1	0.279732	0.016161	2.200452	0.0311
	δ_1	0.202377	0.000523	8.852313	0.0000
2	a_2	-0.037591	0.000023	-7.872397	0.0000
	ρ_2	1.191053	0.000388	60.438563	0.0000
	α_2	0.649072	0.004672	9.495787	0.0000
	γ_2	-0.081859	0.001226	-2.337526	0.0223
	δ_2	0.034802	0.000106	3.379271	0.0012

R-squared	0.9913	Log likelihood	295.1499
Adjusted R-squared	0.9908	F-statistic	1987.0159
S.E. of regression	0.0134	Prob (F-statistic)	0.0000
Sum squared resids	0.0126		

$\hat{\Pi}_1$	0.4421	$\hat{\Pi}_2$	0.5579
$\hat{p}_{11}$	0.8945	$\hat{p}_{22}$	0.8950
$\hat{\sigma}_1$	0.0166	$\hat{\sigma}_2$	0.0091

Source: own calculations.

On the contrary, at a first glance, estimated coefficients in regime 2 seem not to conform to the results anticipated by the theory. The most striking characteristic of this regime is that the autoregressive coefficient of inflation (ρ_2), although positive, is strictly greater than one[4]. From the statistical point of view, this implies that inflation is an explosive stochastic process. In a two regimes dynamic, this does not seem a real problem, since the whole stochastic process could be bounded by the piece-wise stationarity of the series under regime 1. Indeed, this result is close to the results found in the literature in which the inflation follows a random walk process in one regime, and an autoregressive process in the other regime (Evans and Watchel 1993, and Simon 1996). Nonetheless, since the literature defines that there exists inflationary inertia when the coefficient accompanying lagged inflation is positive but smaller than one, the difficult task is to theoretically understand, if in this case, we can still interpret the lagged value of inflation as inertia or if we need to look for an alternative interpretation of the phenomenon.

Several works have analyzed inflation in Venezuela, but only three of them have explicitly referred to the problem of inflationary inertia. Dorta et al. (1998) in their analysis of the inflation for the period 1970 to 1997, state that inflationary inertia has increased since 1984 mainly due to the reduced credibility of agents in the performed economic policy. Álvarez et al. (2002), in their analysis of the period 1984-2002, using a Kalman filter estimation, show that the coefficient of lagged inflation has increased in a piece-wise fashion, first during 1989-1997 and then during 1998-2002. However this coefficient has always fluctuated in the range between 0.5 and 0.8, and its behavior is basically explained by the process of price indexation and the own volatility of inflation. Additionally, Guerra and Pineda (2004), when studying the implementation of a bound system for the exchange rate (1997 to 2002) claim that, although the inflation rate had shown a descending path during the whole period, a further decreased was precluded, exactly because of the existence of a greater inflationary inertia. This empirical evidence, although it could relate intuitively to our findings, it does not provide yet an alternative interpretation to having an estimated coefficient on lagged inflation that is greater than one.

[4] A standard contrast of hypothesis rejected the null that the $\rho_2 \leq 1$.

IV. The formation of inflationary expectations

A different manner to proceed for interpreting the estimates obtained, particularly in regime 2, is to relate the magnitude of the autoregressive coefficient (ρ) to the way in which agents use the information on variables to form their expectations on current inflation.

First of all, consider that we can represent inflation as the particular solution of the implicit first-order difference equation estimated in each regime, so that its current level can be described by the dynamic of the explanatory variables of the model. After obtaining the particular solution of the regression model in (9) according to the two estimated sets of parameters, take its expected value based on the information set available at time *t*-1. Additionally, assume that all the moments of the error term of order equal or greater than 2 are negligible.

In regime 1, inflationary expectations can be characterized as:

$$E(Inf_t) = 0.44\left[\sum_{m=0}^{\infty} 0.82^m \left(Gdp_{t-m-1} - Gdp^*_{t-m-1}\right)\right] + 0.28\left[\sum_{m=0}^{\infty} 0.68^m M_{t-m-2}\right] + 0.20\left[\sum_{m=0}^{\infty} 0.68^m \hat{E}_{t-m-1}\right] \tag{10}$$

Instead, in regime 2, after solving the difference equation forward, inflationary expectations can be described by:

$$E(Inf_t) = 0.2 - 0.55\left[\sum_{m=0}^{\infty} 0.84^m E\left(Gdp_{t+m} - Gdp^*_{t+m}\right)\right] + 0.07\left[\sum_{m=0}^{\infty} 0.84^m E\left(M_{t+m-1}\right)\right] - 0.03\left[\sum_{m=0}^{\infty} 0.84^m E\left(\hat{E}_{t+m}\right)\right] \tag{11}$$

where $E(\circ)$ denotes the expectation operator given the set of information available at *t*-1.

In regime 1, the expected inflation responds to the past values of the output gap, money creation and currency depreciation, and changes in these variables might significantly affect the inflation rate for approximately 24 quarters[5]. In regime 2, expected inflation depends on agent's expectations about the output gap, money creation and currency depreciation. The effect of expected changes in any of the explanatory variables will last for approximately 26 quarters, similarly as in regime 1.

In regime 2, of all the relevant information about the future state of the economy, the most important piece to form expectations about inflation is the output gap. For instance, if the economy is expected to grow above its potential level, then expected inflation will tend to drop below 20%, while if the economy is expected to be in a recession, expected inflation will tend to rise above 20%. Regarding the other variables, an expected increase in the quantity of money will have a positive impact on expected inflation. On the contrary, an expected increase in the exchange rate will diminish the current expected rate of inflation[6]. This finding, although unusual, can be related to situations of real exchange rate appreciation, where nominal depreciations can be perceived as a mechanism to reduce the miss-alignment of the real exchange rate, and therefore slow down the overall rate of inflation[7].

Theoretically, the fact that in regime 2 current inflationary expectations depend on agents' expectations on other variables, can be supported by the premise that rational agents use all the relevant information available to form their expectations, which in this case is the subjective information on hand about key variables such as growth, exchange rate and quantity of money. This way of forming expectations allows labeling this regime 2 as a "rational expectation" regime, as opposed to the other estimated regime in which expectations are formed in a "backward looking" manner.

If we presume that agents modify their behavior according to their expectations, then, using subjective information regarding the future performance of the economy presumably brings about adjusting the pricing strategy on goods. Regarding this point, we can look for support in

[5] This intuitive form of characterizing the duration of a change in any explanatory variable results from assuming that the effect over inflation disappears when the factor $\rho^m = 0.01$.

[6] In this case the annual pass-through is -0.20.

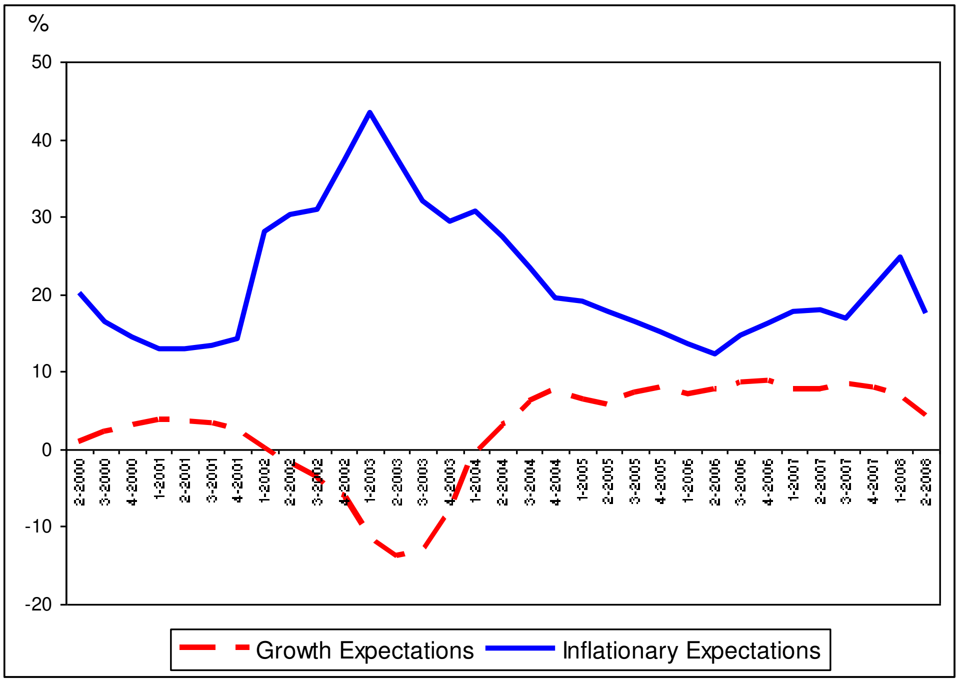
[7] This could happen if the reduction in agent's real income caused by the depreciation lessens more than proportionally the demand in non-tradable goods, which are the main boosters of inflation in situations of real exchange rate appreciation.

Woodford (1991) when explaining that, without requiring any objective change in economic circumstances, the degree of optimism of economic actors can have an important role in explaining recurrent cyclical fluctuations of the business activity, and consequently inflation. However, a more challenging task is to justify why the expected output gap is the variable that agents mostly take into consideration for forming their expectations and ultimately for establishing their pricing strategy. One could argue that, in regime 2, the expected output gap becomes the best proxy for the size of the demand that sellers of goods would face in the future. Therefore, as demand is expected to rise, revenues will be obtained by increasing the amount of goods sold or produced and prices could be allowed to increase less. At the micro level, as in Stiglitz (1991) and Rotemberg and Saloner (1991), this could imply that the downward-slopping demand faced by sellers and producers would shift outward and become more elastic during phases of economic expansion as competition in the market is expected to kick in.

Empirically, the frequent occurrence of regime 2, i.e. 56% of the times according to our estimations, would imply that agents' expectations on inflation are inversely related to the expected economic growth. In fact, looking at the polls on economic outlook collected by the Central Bank, we verified that, on average, there is a significant negative correlation (-0.81) between inflationary expectations and growth expectations. This can be verified by eyeballing figure 1.

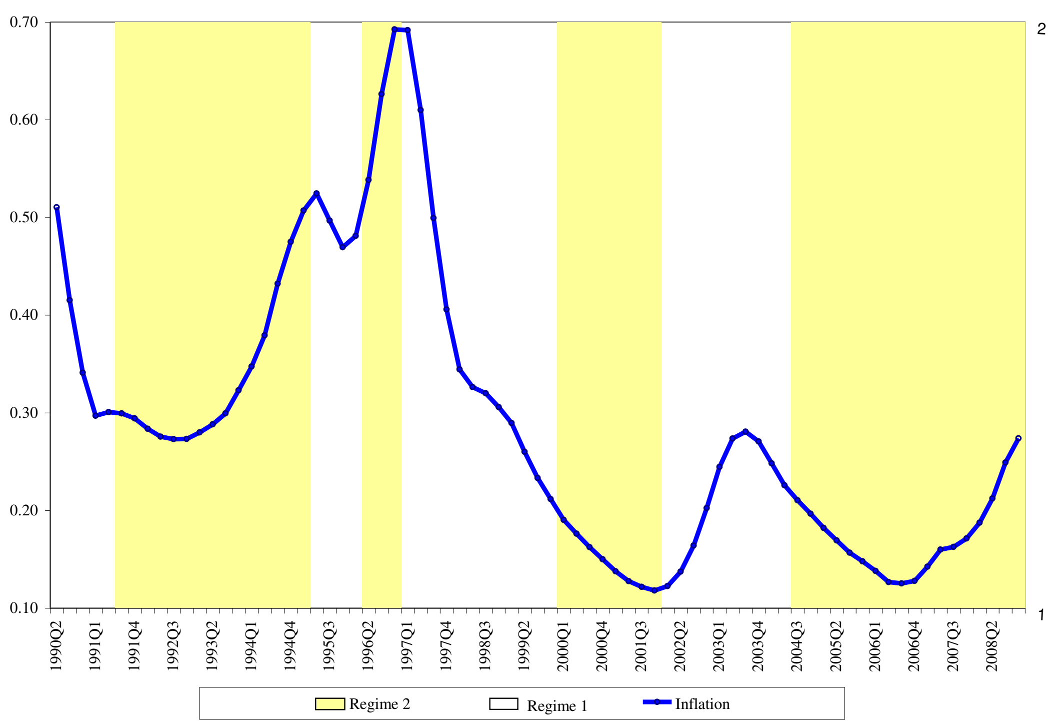
One important output of the estimation performed is the (filtered and smooth) probabilities computed for each observation of the dependent variable. These probabilities reflect the likelihood that each hidden state has occurred, allowing classifying each quarter of the estimation period according to one of the regimes, as shown in figure 2. Then, this classification along with the main economic historical events provides a notion of what circumstances were present during the occurrence of each regime.

Figure 1.- Annual Inflationary and Growth Expectations



Source: Venezuelan Central Bank Surveys on Inflationary and Growth Expectations.

Figure 2.- Regime Classification for Inflation



Source: Venezuelan Central Bank statistics and own calculations.

According to the classification of periods provided by the model, in many cases, the “rational expectation” regime coincides with episodes of macroeconomic instability or with the last part of non-floating exchange rate systems that usually ended up with speculative attacks and reforms. In fact, the first long period of regime 2 detected by the model (1991:03 to 1995:01) corresponds to a period of general (political and economic) instability coupled with a financial crisis[8]. Also, as a response to the recurrent speculative attacks to the system of managed devaluations applied since 1993, at the end of this period (second quarter of 1994), an exchange rate control was implemented. The second period of regime 2 (1996:02 to 1996:04) corresponds to the end of the exchange rate control started in 1994 and the beginning of the implementation of a system of exchange rate bounds in July of 1996. This system consisted on establishing an upper and a lower bound to the trajectory of the exchange rate, such that deviations of the exchange rate outside these bounds triggered additional interventions of the Central Bank in the market[9]. The “rational expectation” state is again detected by the model at the end of the system of bounds (2000:01 to 2001:04), just before the implementation of a floating exchange rate in March 2002.

On the other hand, the longest episode classified by the model as belonging to the “rational expectation” regime (2004:03 to 2008:04) does not coincide with the occurrence of any speculative attack that led to the abandonment of the current exchange rate system. Moreover, in this period, the economy exhibited high rates of growth based on a large and long increase in oil prices. Nonetheless, we could state that this growth has attached a high level of uncertainty, since the duration and intensity of the oil boom can not be accurately forecasted with any past information. In fact, most empirical evidence suggests that oil prices can be regarded as a random walk process, and only can be viewed as a stationary autoregressive process if analyzed in a very long time span[10]. In this line of reasoning, since the growth of the economy is highly dependent on the future draw of external shocks, agents stop looking at the past

---

[8] In 1992, the government in charge confronted a military cup, and during the outset of the financial crisis in 1994, the president of the Central Bank resigned as the result of existing contradictory policy intentions between the Central Bank and the Executive Power.

[9] In practice, this period was a type of fixed exchange rate since the chosen distance between the bounds was relatively small.

[10] Another source of uncertainty in the sustainability of such growth can be attributed to the important institutional changes

information of this variable and revert to using the available subjective information on its future performance. Then, these expectations on aggregate demand are the ones that change the formation of expectations on current inflation and therefore, determine the pricing strategy of producers and sellers. As a matter of fact, since the end of 2006 and particularly the last quarter of 2007, the economic growth has exhibited a clear tendency to slow down while most indicators of forecasted and current inflation show higher levels.

Succinctly, the above analysis shows the way to characterize regime 2 as being consistent with episodes of high uncertainty regarding the performance of the economy, either due to propitious conditions for the collapse of non-floating exchange rate systems, or due to conditions of high vulnerability to external shocks. However, it is still a question, what is the source of the subjective information that replaces past information on variables and becomes the focal point of economic actors in their business decisions.

V. Conclusions

In this paper we have analyzed the dynamic of inflation in Venezuela in the last twenty years through a Markov-switching estimation of a Phillips curve.

From the point of view of the inflation dynamic, the model recognizes an explosive stochastic process and a stationary autoregressive process, both of them with equal expected duration once occurred. Since the existence of an explosive stochastic process is non compatible with the standard characterization granted to the phenomenon of inflationary inertia, we restore to interpret these results in terms of their implications for the process of formation of inflationary expectations.

---

implemented by the government to achieve a "socialist economy", particularly since 2004.

From the point of view of the expectations, the model distinguishes between a "normal or backward looking" regime and a "rational expectation" regime. In the "backward looking" regime, agents form their expectations looking at the past values of the variables that typically determine inflation: output gap, money creation and currency depreciation. In the "rational expectation" regime, agents model their expectations mainly based on the subjective information available on the future growth of the economy, supporting the empirically observed notion that, in Venezuela, situations of economic contraction are, on average, associated with episodes of higher inflation.

Given the assumptions that build up Markov-switching models, it is clear that this type of models can only offer a statistical interpretation of what drives switching between regimes. However, in this paper, the characterization of inflationary expectations along with the main economic events occurred during each regime has provided us with an economic interpretation of which factors govern the inflationary dynamics. In particular, we find that the "rational expectation" regime is consistent with episodes of high uncertainty regarding the performance of the economy and this uncertainty seems to have two different sources: the conditions that anticipate the collapse of non-floating exchange rate systems, and the conditions that signal vulnerability of the economy to external (oil) shocks.

This result, extrapolated to a more general context, may contribute to build a connection between models of crises driven by fundamentals (those in which significant economic variables are explained by the evolution of other relevant variables, called fundamentals) and models in which outcomes seem to be driven either by self-fulfilling expectations or any other focal point of pertinent information. This connection what seems to point at is that both types of models might be relevant to explaining the behavior of economic agents. However, what essentially triggers a modification in such behavior is some form of materialization of the uncertainty about future times, which ultimately changes the information set used by agents to form their expectations. However, what this approach can not answer is where the information used in these critical situations comes from.

Appendix A: The EM algorithm

In order to estimate the model (1)-(3), the implementation of the EM algorithm in any l^{th} iteration implies following the next four steps:

1. Given the estimated parameters in the preceding iteration $\left(\hat{\Theta}^{(l-1)}\right)$ and the sequence of the observable variable until time *t* (Y^t), estimate the probability that each possible state *si* has occurred at time *t*, computing recursively, from *t*=1 through *t*=T, the following expressions:

(a) $\xi_{1/0} = \hat{\Pi}^{(l-1)}$

(b) $\xi_{t/t} = \dfrac{\eta_t \circ \xi_{t/t-1}}{j_N'\left(\eta_t \circ \xi_{t/t-1}\right)}$

(c) $\xi_{t+1/t} = \hat{P}^{(l-1)} \; \xi_{t/t}$

(d) $f\left(y_t / z_t, \hat{B}^{(l-1)}\right) = j_N'\left(\xi_{t/t-1} \circ \eta_t\right)$

where η_t, $\xi_{t/t}$, $\xi_{t+1/t}$ and j_N are column vectors of dimension *N*, defined as:

$$\eta_t = \begin{bmatrix} f\left(y_t / q_t = s1,\, z_t,\, \hat{B}^{(l-1)}\right) \\ \vdots \\ f\left(y_t / q_t = sN,\, z_t,\, \hat{B}^{(l-1)}\right) \end{bmatrix}, \quad \xi_{t/t} = \begin{bmatrix} \Pr\left(q_t = s1 / \, \Omega^t,\, \hat{\Theta}^{(l-1)}\right) \\ \vdots \\ \Pr\left(q_t = sN / \, \Omega^t,\, \hat{\Theta}^{(l-1)}\right) \end{bmatrix}, \quad j_N = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

$f\left(y_t / q_t, z_t, \hat{B}^{(l-1)}\right)$ is the conditional density for a given time period evaluated in parameters estimates from the preceding iteration, and the operator ($\circ$) indicates an element by element multiplication. Notice that because of the recursive nature of $\xi_{t/t}$, the set of information used is $\Omega^t \equiv Y^t \cup Z^t$, which also includes the sequence of realizations of the lagged endogenous and exogenous variables of the model until time t (Z^t). This must be the case, because at each time t, the algorithm needs to evaluate the likelihood that a particular hidden state has occurred, but taking into consideration that its transition could have taken place from any possible sequence of t-1 hidden states.

2. Use the complete sequence of the observable variable (Y^T instead of Y^t) to re-estimate the probabilities that each possible state *si* has occurred at time *t.* These new probabilities are computed with the Kim's algorithm and are referred by Hamilton (1990, 1994) as *smooth probabilities*. This algorithm is applied recursively, from t+1=T backward to t=1, calculating the following expressions:

(a) $\xi_{t+1/T} = \xi_{t+1/t}$ for $t+1=T$

(b) $\xi_{t/T} = \xi_{t/t} \circ \left\{ \hat{P}^{(l-1)\prime} \left(\xi_{t+1/T} \div \xi_{t+1/t} \right) \right\}$

(c) $\varsigma_{i,t/T} = \Pr\left(q_t = si / \Omega^t, \hat{\Theta}^{(l-1)}\right) \left\{ \hat{P}_i^{(l-1)} \circ \left(\xi_{t+1/T} \div \xi_{t+1/t} \right) \right\}$ i=1,…,N

where ($\div$) indicates an element by element division, $\varsigma_{i,t/T}$ is a column vector of dimension N, and $\hat{P}_i$ is the estimated i^{th} column of matrix P:

$$\varsigma_{i,t/T} = \begin{bmatrix} \Pr\left(q_t = si, q_{t+1} = s1 / \Omega^T, \hat{\Theta}^{(l-1)}\right) \\ \vdots \\ \Pr\left(q_t = si, q_{t+1} = sN / \Omega^T, \hat{\Theta}^{(l-1)}\right) \end{bmatrix}, \qquad \hat{P}_i = \begin{bmatrix} \hat{p}_{i1} \\ \hat{p}_{i2} \\ \vdots \\ \hat{p}_{iN} \end{bmatrix}$$

3. Re-estimate the model parameters $\hat{\Theta}$ for this l^{th} iteration, by solving the different FOCs that maximize (6). According to Hamilton (1990, 1994) this procedure is equivalent to computing:

(a) Transition probabilities using the equations:

$$\hat{P}_i^{(l)} = \left(\sum_{t=2}^{T} \varsigma_{i,t/T} \right) \div \left(\sum_{t=2}^{T} \xi_{t/T} \right) \qquad \text{for} \quad i=1,\ldots,N$$

(b) Unconditional probabilities of being at each state following:

$$\hat{\Pi}^{(l)} = \frac{\sum_{t=1}^{T} \xi_{t/T}}{T}$$

(c) Parameters of the regression model in (1), by solving the FOCs stated in (8), such that:

$$\hat{\beta}_{si}^{(l)} = \left(z' \, \hat{\Gamma}_{si} \; z \right)^{-1} z' \, \hat{\Gamma}_{si} \; y \qquad \text{for} \quad i=1,\ldots,N$$

$$\hat{\sigma}_{si}{}^{2} = \frac{\left(y - z\hat{\beta}_{si}^{(l)} \right)' \, \hat{\Gamma}_{si} \left(y - z\hat{\beta}_{si}^{(l)} \right)}{j_T \, ' \, \hat{\Gamma}_{si} \; j_T} \qquad \text{for} \quad i=1,\ldots,N$$

$$V\hat{a}r\left(\hat{\beta}_{si} \right) = \hat{\sigma}_{si}^{2} \left(z' \hat{\Gamma}_{si} \; z \right)^{-1} \qquad \text{for} \quad i=1,\ldots,N$$

where:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}, \qquad z = \begin{bmatrix} z_{11} & z_{21} & \cdots & z_{k1} \\ z_{12} & z_{22} & \cdots & z_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{1T} & z_{2T} & \cdots & z_{kT} \end{bmatrix}, \quad \beta_{si} = \begin{bmatrix} \beta_{1,si} \\ \beta_{2,si} \\ \vdots \\ \beta_{k,si} \end{bmatrix}, \quad \varepsilon_{si} = \begin{bmatrix} \varepsilon_{1,si} \\ \varepsilon_{2,si} \\ \vdots \\ \varepsilon_{T,si} \end{bmatrix}, \quad j_T = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix},$$

$$\hat{\Gamma}_{si} = \begin{bmatrix} \Pr\left(q_1 = si / \Omega^T, \hat{\Theta}^{(l-1)} \right) & 0 & \cdots & 0 \\ 0 & \Pr\left(q_2 = si / \Omega^T, \hat{\Theta}^{(l-1)} \right) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \Pr\left(q_T = si / \Omega^T, \hat{\Theta}^{(l-1)} \right) \end{bmatrix}$$

4. Evaluate if the parameter estimates have attained a fixed point, that is $\left| \hat{\Theta}^{(l)} - \hat{\Theta}^{(l-1)} \right| \leq 10^{-8}$.

Then, verify that the empirical expected log-likelihood function of the dependent variable (for all hidden states), has also achieved a maximum.

This verification implies observing, for:

$$E\left[g\left(y/\hat{\Theta}^{(l)}\right)\right]=\sum_{t=1}^{T}\ \sum_{q_t=s1}^{SN}\ln f\left(y_t/q_t=si,\, z_t,\, \hat{\Theta}^{(l)}\right)\ \Pr\left(q_t=si/\Omega^T,\, \hat{\Theta}^{(l-1)}\right),$$

that $E\left[g\left(y/\hat{\Theta}^{(l)}\right)\right]-E\left[g\left(y/\hat{\Theta}^{(l-1)}\right)\right]\leq tolerance value$. The maximization of this expected log-likelihood function should be easily confirmable since it is a by-product of the estimation process, and in particular, of the imposition of the FOCs stated in (8).

References

Álvarez F., Dorta M. y Guerra J. (2002). "Persistencia Inflacionaria en Venezuela: evolución, causas e implicaciones". *Estudios sobre la Inflación en Venezuela*, editor: Guerra, J., Departamento de Publicaciones del BCV, Caracas, Venezuela.

Arreaza A., Blanco E. y Dorta M. (2003). "A Small Macroeconomic Scale Model for Venezuela". Serie de Documentos de Trabajo del BCV, 43.

Ball L., Mankiw G. and Romer D. (1988). "The New Keynesian Economics and the Output-Inflation Tradeoff". *Brookings Papers on Economic Activity*, 1, 1-82.

Blix, M. (1999). "Forecasting Swedish Inflation with a Markov Switching VAR". Working Paper Series of the Sveriges Riksbank (Central Bank of Sweden), 76.

Diebold F. and Rudebush G. (1996). "Measuring Business Cycles: a Modern Perspective". *Review of Economics and Statistics*, 78, 67-77.

Dorta M. (2006). "La Función de Producción, el Producto Potencial y la Inflación en Venezuela 1950-2005". Serie de Documentos de Trabajo del BCV, 87.

Dorta M., Guerra J. and Sánchez G. (1998). “Credibilidad y Persistencia de la Inflación en Venezuela”. *Revista del BCV*, XII.2, 135-156.

Dorta M., Álvarez F. and Bello O. (2002) “Determinantes de la Inflación en Venezuela: un Análisis Macroeconómico para el Período 1986-2002” *Revista del BCV*, XVI.2, 93-130.

Evans M. and Wachtel P. (1993). “Inflation Regimes and the Sources of Uncertainty”. *Money, Credit and Banking* 25, 3, 475-511.

Guerra J. and Pineda J. (2004). “Trayectoria de la Política Cambiaria en Venezuela”. *Temas de Política Cambiaria en Venezuela*, editors: Guerra, J. and Pineda J., Departamento de Publicaciones del BCV, Caracas, Venezuela.

Hamilton J. (1989). “A New Approach to the Economic Analysis of Non-Stationary Times Series and the Business Cycle”. *Econometrica*, 57, 357-384.

Hamilton J. (1990). “Analysis of Time Series subject to Changes in Regime”. *Journal of Econometrics*, 45, 39-70.

Hamilton J. (1994). *Time Series Analysis.* Princeton University Press, Princeton, New Jersey, U.S.A.

Kahn J. and Rich R. (2007). “Tracking the New Economy: using Growth Theory to Detect Changes in Trend Productivity”. *Journal of Monetary Economics*, 54, 1670-1701.

Kim C. and Murray C. (2002). “Permanent and Transitory Components of Recessions”. *Empirical Economics*, 27, 163-183.

Mendoza O. (2006). “Las asimetrías del pass-through en Venezuela”. *Monetaria*, XXIX, 2, 151-183.

Rotemberg J. and Saloner G.(1991). “A Supergame-Theoretic of price Wars during Booms”. *New Keynesian Economics*, *Volume 2*, *Coordination Failures and Real Rigidities*, editors: Mankiw, G and Romer, D., MIT Press, Cambridge, Massachusetts, 387-415.

Sargent T. (1987). *Macroeconomic Theory*. Academic Press, INC, San Diego, California, U.S.A.

Simon J. (1996). “A Markov-Switching Model of Inflation in Australia”. Reserve Bank of Australia, Research Discussion Paper 9611.

Stiglitz J. (1991). “Price Ridigities and Market Structure”. *New Keynesian Economics*, *Volume 2*, *Coordination Failures and Real Rigidities*, editors: Mankiw, G and Romer, D., MIT Press, Cambridge, Massachusetts, 377-386.

Welch L. (2003). Hidden Markov Models and the Baum-Welch Algorithm. *IEEE Information Theory Society Newsletter*, 53, 1, 10-13.

Woodford M. (1991). “Self-Fulfilling Expectations and Fluctuations in Aggregate Demand”. *New Keynesian Economics*, *Volume 2*, *Coordination Failures and Real Rigidities*, editors: Mankiw, G and Romer, D., MIT Press, Cambridge, Massachusetts, 77-110.