Equilibrium in the Production Process of a Two-Sector Model of the New Economy

Van, Germinal

George Washington University

March 2021

Equilibrium in the Production Process of a Two-Sector Model of the New Economy

By

GERMINAL G. VAN

The George Washington University
2121 I Street NW
Washington, D.C. 20052
germinalvan27@gwmail.gwu.edu

Equilibrium in the Production Process
of a Two-Sector Model of the New Economy

Abstract

The purpose of this paper is to provide a mathematical framework to investigate the method through which an equilibrium is met between two economic sectors. The equilibrium process being discussed in this paper focuses essentially on the production process of the goods and services in each sector within an economy that has been affected by COVID-19. This mathematical model helps explaining how two economic sectors reach an equilibrium when their production methods are significantly different and when they do not produce at the same pace. This paper is purely theoretical and does not concern itself with empirical verification.

Keywords: Mathematical Economics, Economic Theory, Production Theory, Input-Output Analysis, General Equilibrium Theory, Differential Calculus

I.

INTRODUCTION

This paper is essentially theoretical. It does not engage in any empirical research or verification. All theory depends on assumptions which are not quite true—this is what makes it a theory.[1] What this paper aims to show is the method through which an equilibrium is reached in a two-sector model. Of course, the idea of a two-sector model is not new. Generally speaking, when we discuss a two-model sector, we generally refer to two sectors wherein one sector is more technologically advanced than the other. Indeed, this remains the idea of the model we aim to develop in this paper. However, the structure is different.

Whereas it has been conventionally defined that a two-sector model consists of the agricultural sector and the manufacturing sector, as it was the case in the groundbreaking paper of Sir W. Arthur Lewis entitled *Economic Development with Unlimited Supply of Labour* published in 1954, in our paper, the two-sector model is divided between a physical sector and a digital sector. The reasoning for choosing to structure our two-sector model that way is due to the fact that COVID-19 has completely shaped our economies. If we could define the current structure of our modern economies, it is evident that our economies are divided between a sector that encompasses all the economic activities that take place in the physical realm and another sector where all economic activities take place remotely.

What does this paper seek to demonstrate? This is the main interrogation of this analysis. This paper is fundamentally concerned with the analysis of the production process in the new two-sector model. Our central idea claims that an equilibrium is not only reached at the distribution level but also at the production level. What we are mainly concerned with is not how the factors of production (capital stock and labor) determine the production process but how the production of the various industries within each economic sector interconnect with one another. By interconnecting with one another, the sum of the production of each industry leads the two economic sectors to ultimately reach an equilibrium. What is the significance of this equilibrium? It implies that the economy as a whole is at the peak of its production process, or in other words, it means that the economy is Pareto-efficient. This paper endeavors to propose a mathematical model to subsequently analyze the steps of the process to reach this equilibrium.

II.

A FRAMEWORK OF THE TWO-SECTOR MODEL

Let us say that an economy (Y_t) is composed of two sectors: the physical sector denoted as ($\sum X_{1i}$) and the digital sector denoted as ($\sum X_{2i}$). ($\sum X_{1i}$ and $\sum X_{2i}$) represent the sum of all the industries in each economic sector. Hence, (Y) is the result of the combination of the two sectors, and it could be written as the following equation:

$$Y_t = \alpha_t + \sum_{t=1}^{\infty} X_{1t} + \sum_{t=1}^{\infty} X_{2t} \qquad (1)$$

[1] Solow, Robert M. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics,* Vol. 70, No. 1 (1956), pp.65-94.

In this model, t is a discrete number representing time, and (∞) is the number of goods and services produced. (t = 1) is the year when the production process takes place, and the equilibrium is reached. Equation (1) can be re-written by a method of factorization. Then we will have:

$$Y_t = \sum_{t=1}^{\infty} \alpha_t + (X_{1t} + X_{2t}) \quad (1)$$

(α) represents in this model the fixed parameter that determines the production process.

Let us assume that both sectors in their production process, on a long-run basis, stimulates (Y) to its peak when each sector produces the same amount of goods and services at a given time. This process could be written by the following equation:

$$\sum_{t=1}^{\infty} Y_t \alpha_t + (X_{1t} + X_{2t}) = 0 \quad (2)$$

This same amount of goods and services produced by each sector within a given time is called the equilibrium of the production process. In order to understand how this equilibrium is reached, it is essential to analyze first how each sector is structured and how their production process occurs.

III.

A MODEL OF THE PHYSICAL SECTOR

What is the physical sector? As was aforementioned in the introduction, the physical sector is the sector that encapsulates all the industries that engage in economic activity in the physical realm. For example, the agricultural industry or the manufacturing industry would be based on the physical realm because they require physical interactions and effort. That being said, let us assume that the physical sector (X_{1t}) is mainly composed of four industries which are the agricultural industry (x), the manufacturing sector (x^2), the healthcare industry (x^3) and the transportation industry (x^4). The power raise after each variable x means that each of these industries produces x number of times what the agricultural industry produces. The aggregate output of all these industries is then denoted as $(\sum X_{1t})$. The particularity of this sector is that its production increases logarithmically. The reason why $(\sum X_{1t})$ grows logarithmically is that it relies on physical capital, and physical capital, over time, depreciates. Therefore, the physical sector could be modeled as the following equation:

$$\sum X_{1t} = \ln(x + x^2 + x^3 + x^4) \quad (3)$$

The model could be represented graphically as the following figure where Y is the output, and P is production:

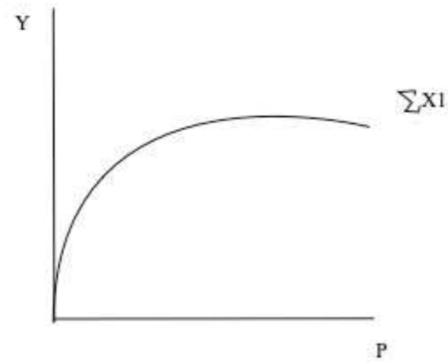

Figure 1

The aggregate output of ($\sum X_{1t}$) is formed between (X_1) and (X_1+h) before it commences to depreciate as we could see in figure 2.

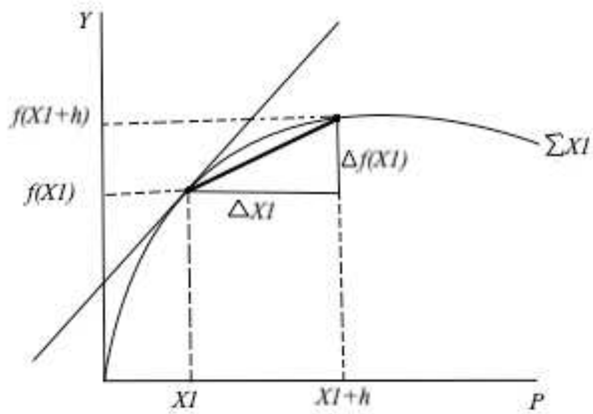

Figure 2

Hence, we can determine the distance between (X_1) and (X_1+h) by applying the derivative change of rate method.

$$\frac{dy}{dP} = \frac{f(X_1 + h) - f(X_1)}{X_1 + h - X_1}$$

$$\frac{dy}{dP} = \frac{f(X_1 + h) - f(X_1)}{h}$$

IV.

A MODEL OF THE DIGITAL SECTOR

The digital sector is the economic sector where all the economic activities are occurring remotely. In such a sector, economic transactions are effectuated through computers, and individuals are exchanging their goods and services through computers. Let us assume that the digital sector is equally composed of four industries as the physical sector. These industries are the e-commerce industry (y), the tech industry (y^2), the consulting industry (y^3), and the financial industry (y^4). The aggregate output of all of these industries is denoted as ($\sum X_{2t}$). Unlike the ($\sum X_{1t}$), ($\sum X_{2t}$) increases exponentially over time because in that sector, capital does not depreciate since it is based on digital factors. Therefore, the digital sector could be modeled as the following equation:

$$\sum X_{2t} = e^{(y+y^2+y^3+y^4)} \quad (4)$$

As the physical sector, the digital sector could be represented graphically where Y and P would represent the output and production, respectively.

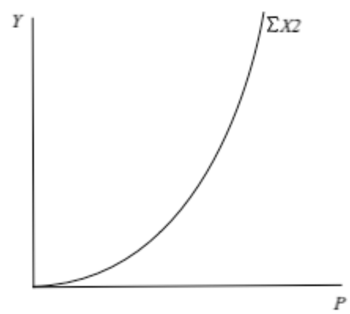

Figure 3

The aggregate output of ($\sum X_{2i}$) occurs between (X_2) and ($X_2 + h$) and it keeps increasing over time as we could see in figure 4.

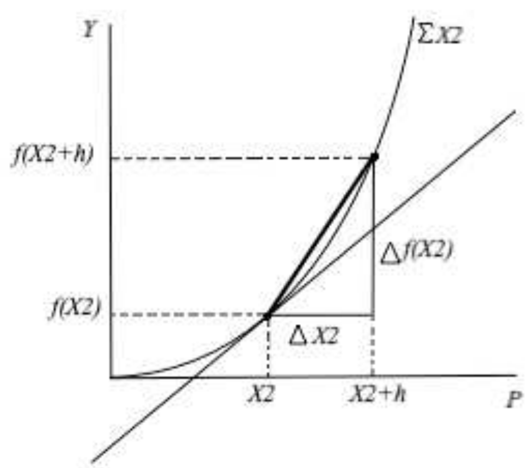

Figure 4

Let us follow the same process by calculating the distance between (X_2) and ($X_2 + h$) by applying the same derivative change of rate method:

$$\frac{dy}{dP} = \frac{f(X_2 + h) - f(X_2)}{X_2 + h - X_2}$$

$$\frac{dy}{dP} = \frac{f(X_2 + h) - f(X_2)}{h}$$

V.

AN EQULIBRIUM MODEL OF THE PRODUCTION PROCESS

In the development of the framework of our equilibrium model, we determined that equation (2) was the equation that would establish the equilibrium between the two sectors. What is important to fathom in this part of our analysis is that the equilibrium process occurs through the interdependency of each industry with one another. This interdependency implies that what is produced in the physical sector, would be considered as added value to the digital sector. In our economic model, the whole economy is structured as a service-based economy. Consumer demand has fueled technological progress and technological progress has engendered the ability of people to engage remotely in their various economic activities. To understand how the equilibrium takes place in the production process between ($\sum X_{1i}$) and ($\sum X_{2i}$), let us first rewrite equations (3) and (4) as a system of equations:

$$\sum_{t=1}^{\infty} Y_t \alpha_t + \begin{pmatrix} \ln(x+x^2+x^3+x^4) \\ + \\ e^{(y+y^2+y^3+y^4)} \end{pmatrix} = 0 \qquad (5)$$

Let us first calculate the derivative of equations (3) and (4) in order to determine their interdependency in the production process. Let us first start solving equation (3). The derivative of equation (3) will be equation (6).

We know that equation (3) is:

$$\sum X_{1t} = \ln(x + x^2 + x^3 + x^4) \qquad (3)$$

$$\sum X_{1t} = lnx + lnx^2 + lnx^3 + lnx^4$$

Let us substitute the (*ln*) by the (*e*). We will have:

$$\sum X_{1t} = e^{lnx} + e^{lnx^2} + e^{lnx^3} + e^{ln^4}$$

$$\sum X_{1t} = x + e^{2lnx} + e^{3lnx} + e^{4lnx}$$

Let us now apply the derivative of ($\sum X_{1i}$):

$$f'(\sum X_{1t}) = \frac{d}{dx}(x + e^{2lnx} + e^{3lnx} + e^{4lnx})$$

$$f'(\sum X_1) = \left(\frac{d}{dx}x\right) + \left(\frac{d}{dx}2lnx \times e^{2lnx}\right) + \left(\frac{d}{dx}3lnx \times e^{3lnx}\right) + \left(\frac{d}{dx}4lnx \times e^{4lnx}\right)$$

$$f'(\sum X_{1t}) = 1 + \left(\tfrac{2}{x} \times e^{2lnx}\right) + \left(\tfrac{3}{x} \times e^{3lnx}\right) + (\tfrac{4}{x} \times e^{4lnx})$$

$$f'(\sum X_{1t}) = 1 + \left(2x^{-1} \times e^{lnx^2}\right) + \left(3x^{-1} \times e^{lnx^3}\right) + (4x^{-1} \times e^{lnx^4})$$

$$f'(\sum X_{1t}) = 1 + (2x^{-1} \times x^2) + (3x^{-1} \times x^3) + (4x^{-1} \times x^4)$$

$$f'(\sum X_{1t}) = 4x^3 + 3x^2 + 2x + 1 \qquad (6)$$

Now we have determined the derivative of equation (3), let us use the same method to find the derivative of equation (4). The derivative of equation (4) will be equation (7).

We know that equation (4) is:

$$\sum X_{2t} = e^{(y+y^2+y^3+y^4)}$$

$$\sum X_{2t} = e^{y} + e^{y^2} + e^{y^3} + e^{y^4}$$

$$\sum X_{2t} = e^{lny} + e^{lny^2} + e^{lny^3} + e^{lny^4}$$

$$\sum X_{2t} = y + e^{2lny} + e^{3lny} + e^{4lny}$$

Let us now apply the derivative of equation (4):

$$f'(\sum X_{2t}) = \tfrac{d}{dx}(y + e^{2lny} + e^{3lny} + e^{4lny})$$

$$f'(\sum X_{2t}) = \tfrac{d}{dx}y + \tfrac{d}{dx}e^{2lny} + \tfrac{d}{dx}e^{3lny} + \tfrac{d}{dx}e^{4lny}$$

$$f'(\sum X_{2t}) = 1 + \left(\tfrac{d}{dx}2lny \times e^{2lny}\right) + \left(\tfrac{d}{dx}3lny \times e^{3lny}\right) + (\tfrac{d}{dx}4lny \times e^{4lny})$$

$$f'(\sum X_{2t}) = 1 + \left(\tfrac{2}{y} \times e^{2lny}\right) + \left(\tfrac{3}{y} \times e^{3lny}\right) + (\tfrac{4}{y} \times e^{4lny})$$

$$f'(\sum X_{2t}) = 1 + \left(2y^{-1} \times e^{lny^2}\right) + \left(3y^{-1} \times e^{lny^3}\right) + (4y^{-1} \times e^{lny^4})$$

$$f'(\sum X_{2t}) = 1 + (2y^{-1} \times y^2) + (3y^{-1} \times y^3) + (4y^{-1} \times y^4)$$

$$f'(\sum X_{2t}) = 4y^3 + 3y^2 + 2y + 1 \qquad (7)$$

Now that we have determined the derivative of equations (3) and (4) which have now become equations (6) and (7) respectively, let us establish the equilibrium process. We established at the outset of our analysis that equation (2) is the model that determines the equilibrium of our two-sector economy. Hence, since equation (2) is based on two independent variables which are equation (3) and (4), and these two equations have been differentiated through the application of the power rule, let us re-write equation (2) by substituting equations (3) and (4) with equations (6) and (7). Thus, we will have the following equation:

$$\sum_{t=1}^{\infty} Y_t \alpha_t + [f'(\sum X_{1t}) + f'(\sum X_{2t})] = 0 \tag{8}$$

To now analyze the interdependency between the industries of the physical and the digital sectors, let us write equation (8) in a matrix form:

$$\sum_{t=1}^{\infty} y_t \alpha_t + \left(\begin{bmatrix} x^3 \\ x^2 \\ x \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y^3 \\ y^2 \\ y \end{bmatrix} \right) = 0 \tag{9}$$

Then we will have:

$$\sum_{t=1}^{\infty} Y_t \alpha_t + \left(\begin{bmatrix} x^3 \\ x^2 \\ x \end{bmatrix} \begin{bmatrix} 4+4 & 3+3 \\ 2+2 & 1+1 \end{bmatrix} \begin{bmatrix} y^3 \\ y^2 \\ y \end{bmatrix} \right) = 0$$

$$\sum_{t=1}^{\infty} Y_t \alpha_t + \left(\begin{bmatrix} x^3 \\ x^2 \\ x \end{bmatrix} \begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} y^3 \\ y^2 \\ y \end{bmatrix} \right) = 0 \tag{10}$$

Now that we have added the values that represent the interdependency of the industries of the two economic sectors, let us determine the equilibrium in the production process by applying the matrix inverse method.

We know that the inverse method for a matrix is:

$$A^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

Therefore, we will have:

$$A^{-1} = \frac{1}{(8 \times 2) - (6 \times 4)} \times \begin{bmatrix} 8 & -6 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{8} \times \begin{bmatrix} 8 & -6 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{8}(8) & -\frac{1}{8}(-6) \\ -\frac{1}{8}(-4) & -\frac{1}{8}(2) \end{bmatrix} = \begin{bmatrix} -1 & \frac{3}{4} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

Hence, the definitive equilibrium model could be written as:

$$\sum_{t=1}^{\infty} Y_t \alpha_t + \left(\begin{bmatrix} -1 & \frac{3}{4} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}\right) = 0 \tag{11}$$

The equilibrium that takes place in the production process between $(\sum X_1)$ and $(\sum X_2)$ can be represented by the following figure:

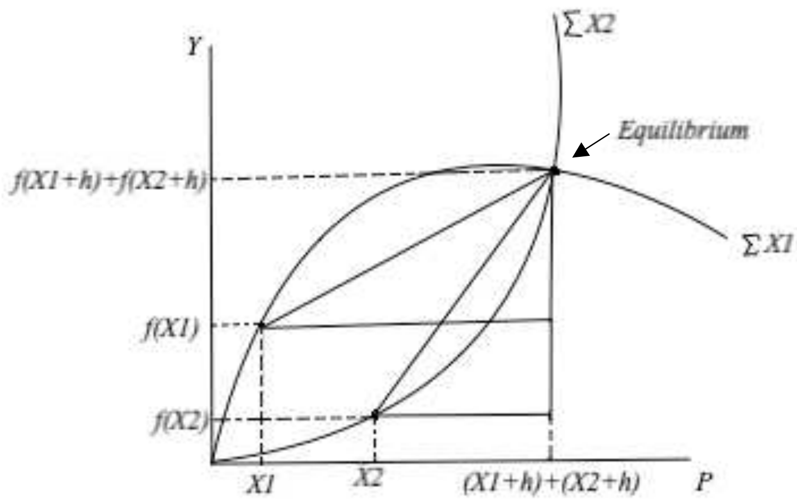

Figure 5

As we could see, an equilibrium is reached when both sectors produce the same amount of goods and services at $(X_1 + h) + (X_2 + h)$ in the production process. Once this equilibrium is reached, we could assume that the economy is Pareto-efficient because no criterion preference can be better off for one economic sector without making the other economic sector worst off.[2]

VI.

CONCLUSION

Throughout this analysis, we have seen that for an equilibrium to be reached in a two-sector model, both sectors must produce sectors the same amount of goods and services at a given time. Hence, at the moment the equilibrium is reached, we could consider the economy to be Pareto-efficient because, beyond the moment at which the equilibrium is reached, the production of the physical sector declines sharply while the production of the digital sector increases more and more exponentially. But what does it mean exactly? More people are shifting sectors as more employment is being created in the digital economy. The economy we tried to model in this paper is the reflection of what the new economy could potentially look like. We founded to divide into two sectors, between the physical sector and the digital because these are the two realms where economic events and activities are taking place. More people have realized that they do not need

[2] Mas-Colell, A.; Whinston, Michael,; Green, Jerry, R. "Chapter 16: Equilibrium and its Basic Welfare Properties." *Microeconomic Theory.* Oxford University Press. (1995). ISBN: 978-0-19-5102680.

to physically move around to conduct their economic activities on a daily basis. The internet and digitalization have made this possible for people to remain economically efficient while staying in their homes. It is clear that the pandemic has totally changed the way economic activities are now pursued, and the creation of many more industries in the digital world will expand its production.

References

[1] Solow, Robert M. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics,* Vol. 70, No. 1 (1956), pp.65-94.

[1] Mas-Colell, A.; Whinston, Michael,; Green, Jerry, R. "Chapter 16: Equilibrium and its Basic Welfare Properties." *Microeconomic Theory.* Oxford University Press. (1995). ISBN: 978-0-19-5102680.