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2021

Online at https://mpra.ub.uni-muenchen.de/106573/
MPRA Paper No. 106573, posted 22 Mar 2021 09:44 UTC

Sleep, Worker's Health, and Social Welfare

Ken Yoshida*

Abstract

The aims of this paper are to construct a theoretical model treating sleep as investment in health capital and to analyze how sleep affects a worker's health capital and social welfare. By regarding sleep as investment in health capital and including sleep in time constraint, it was revealed how workers decide hours slept. Of particular importance is the result that a worker stores their health capital by taking more sleep than the optimal value at a young age, and this leads to increased output. Moreover, we identified a complementary relation between consumption and labor supply. These results showed that policies to reduce hours worked could improve worker's health capital and social welfare.

JEL classification: I12; I18; I31; I38

Keywords: sleep; worker's health; social welfare; health investment; sleep–consumption model.

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1 Introduction

This study focuses on sleep as health investment, and verifies theoretically how workers decide their hours slept in the long run, and suggests the policies that can improve their health. Additionally, we examine whether improving worker's health ameliorates social welfare simultaneously.

Sleep is the most affordable and important health investment for all organisms. Nevertheless, in humans, sleep deprivation among workers has emerged as a serious problem. Hirshkowitz et al. (2015) claimed that adults need seven to nine hours of sleep daily to maintain health. Elementally, the required sleep time as well as the level of fatigue varies from person to person. For example, in USA, about thirty percent of workers sleep only less than six hours.[*1] For some people, this amount of sleep may be enough. However, there is assuredly a problem with lack of sleep. In response to this problem, the consumption of supplements and analgesics is also increasing to improve the ill-health resulting from lack of sleep (Kaufman et al., 2002).

It has long been known that sleep affects personal health and well-being, and it was reported that men who sleep more than eight hours feel happier than those who sleep fewer than six hours (Barry and Bousfield, 1935). Insufficient sleep engenders negative mental consequences including smoking and drinking (Strine and Chapman, 2005) as well as generalized physical discomfort and back pain (Haack and Mullington, 2005). In addition, retirees report improved physical and mental health from sleeping longer (Eibich, 2015).

It is also increasingly evident that individual health contributes to cumulative benefits to society (Braveman et al., 2014; Murayama et al., 2012). Improved health causes less morbidity from infectious diseases and greater labor productivity (Bloom, 2003). In other words, an increase in health capital indicates a positive impact on people and society (Becker, 2007).

As these previous studies showed, lack of sleep may hinder health and thereby reduce even labor productivity. The policies that governments implement to address these problems and improve workers' sleep are important not only for worker's health but also for social productivity. In order to do so, we must clearly determine how much sleep workers should get.

For clearing up this issue, we must create a new model that treats sleep as a health investment. Grossman (1972) modeled health investment, but the health investment factor introduced in this model is mainly medical consumption, such as medical services[*2] , and

[*1] NIH,https://www.nih.gov/news-events/news-releases/nih-offers-new-comprehensive-guide-healthy-sleep

[*2] Grossman (1972) states that medical care is all market goods that affect health, not only medical goods. In other words, all market goods can be called medical care. However, sleep is not included in medical care because the market does not treat sleep as goods.

sleep is not considered.

Besides, in the classic leisure and labor model, the consumer earns utility from consumer goods and leisure. Here, the time for consuming goods is not included in the time constraint. However, the consumption of goods usually takes time, so we include time of consumption in time constraint by using a similar method to Biddle and Hamermesh (1990) in the present paper. Besides, we refer to this behavior of consumption taking time as "time-consuming behavior," and define it as the entirety of time-consuming and money-spending behavior. Additionally, we include sleep into health investment and extend the Grossman model. By doing so, we can fit sleep into an econometric model as health investment. Moreover, we separate sleep from leisure and also include sleep in time constraint. This makes it possible to analyze the balance between health investment and consumption within the given time constraint.

The principal results from our model are as follows: (1) Workers sleep more at their younger age than at their older years; (2) Labor supply and consumption are complementary, and if workers want to increase consumption, labor supply must also increase at the same time.

Result (1) theoretically explains the transition in sleep duration with age reported in Kuroda (2008) and Ohayon et al. (2004). Workers take more sleep at their younger age to maintain their health throughout the future. Besides, when a firm's production function includes workers' health capital, their real wages are an increasing function of their health capital. This implies that workers increase their wages by sleeping more at a younger age to store their health capital, and they thereby increase their future consumption. This result contributes to clarifying the previously unexplained transition in sleep duration with age.

The most important result of this study is Result (2). In the case of firms imposing overtime-work on workers, the policies that reduce hours worked improve not only workers' health, but also workers' utility and the firms' profits. This means that social welfare is improved by the labor reduction policy. In situations where excess labor exists, workers have to sacrifice more sleep in order to increase consumption based on the complementary relationship between labor supply and consumption. Furthermore, workers' wages decrease due to lower health capital, and the increment in consumption is less than the increment in hours worked; thereby, they cannot consume enough. The policies to reduce hours worked force workers to sleep more by reducing the hours spent on work and consumption. This result is generated by our model including sleep, consumption, and labor in the time constraint. The improved health of workers due to increased hours slept increases production, workers' wages, and consumption. Labor supply and consumption increase simultaneously, but if workers' health improves, the increment in hours spent on labor is less than or equal to the increment in hours spent on consumption. This leads to workers getting enough sleep for maintaining their health and sufficient consumption. This also increases the firms' profits. Therefore, Result (2) provides the possibility that the policies to reduce hours worked improves social welfare.

The rest of this paper is organized as follows. Section 2 introduces, in more detail, empirical research on sleep and theoretical analysis of health investments for creating a new model. Section 3 constructs the model featuring hours slept. In it, we analyze budget constraint and hours spent on labor, in each of four cases. We also analyze how policies that reduce hours worked affects the workers' health and social welfare. Section 4 presents the results and compares them with seminal earlier studies. Section 5 concludes this paper.

2 Preliminary

This section presents empirical research on sleep and models of health investment for creating the new model.

2.1 Empirical Studies

Sleep is an essential factor for a healthy life, and humans sleep when they are tired and want to recover their health. Sleep is a physiological need, and if individuals ignore it and stay awake for a long time, it will reduce not only their health, but their well-being as well. As mentioned earlier, there is a relation between sleep and happiness. Supporting this, when comparing the hours slept and happiness levels of people in major developed countries, we can see that Japan, which has the lowest sleeping duration, has the lowest happiness level in these countries. In addition, the countries with longer sleeping duration have more highly ranked happiness levels.[*3] It is also pointed out that increasing sleep improves not only health but also well-being (Steptoe et al., 2008).

Quality of sleep is important for health and happiness (Pilcher et al., 1997, Sathyanarayana et al., 2016, Almojali et al., 2017), although people require variable quantities of sleep. Kuroda (2008) examined the differences in hours slept by age and education. The time-use survey by Kuroda (2008) observed hours worked and hours spent on leisure of Japanese workers spanning 1976-2001. Kuroda (2008) showed that sleep duration decreases with increasing age and leisure increases with age. This finding is not unique to Japan; Ohayon et al. (2004) also showed that sleep decreases with age. Besides, Mander et al., (2017) reported that older adults sleep for shorter duration than younger adults. The decrease in hours slept with aging is an interesting result. Intuitively, it seems that aging leads to physical weakness, which leads to increasing hours slept. As the Centers for Disease Control and Prevention (CDC) (2012) showed, other countries also show a similar result, and such results do not pertain exclusively to Japan.

Even though Japanese workers do not work comparably more hours in developed economies

[*3] Table 2.2 in Sachs et al. (2019)

(Figure 1[*4]), their hours slept is fewest hours among these countries (Figure 2[*5]); this is explained from a phenomenon explained as unpaid overtime working which is called "service-overtime-working," amounting to 300 to 350 hours per year (Ogata, 2015). [*6] Multiple

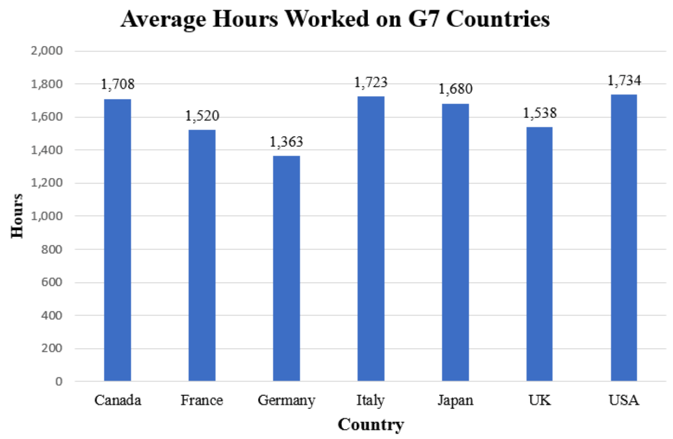

Figure 1 Average hours worked on G7 countries./ Created from OECD.Stat.

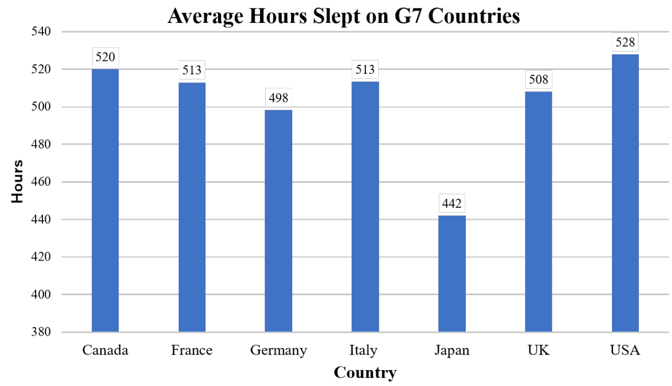

Figure 2 Average hours slept on G7 countries./ Created from OECD.Stat.

studies have reported that working overtime can lead to poor health, sleep deprivation, and sleep disorders (Liu et al., 2002; Dembe et al., 2005; Virtanen et al., 2009; Afonso et

[*4] This figure is created from OECD.Stat. https://stats.oecd.org/Index.aspx?DataSetCode=ANHRS(accessed 2020-12-05)

[*5] This figure is created from OECD.Stat. https://stats.oecd.org/Index.aspx?DataSetCode=ANHRS(accessed 2020-12-05)

[*6] Kuroda (2008) did not indicate the relation between sleep and health, but many studies attributed serious health problems to insufficient sleep (Van Dongen et al., 2003; Strine and Chapman, 2005, Grander et al., 2010a). Lack of sleep is also known to have a negative effect on adolescents (Fredriksen et al., 2004).

al., 2017). These results suggest that one of the most common causes of sleep deprivation is overtime. Additionally, Rosekind et al. (2010) shows that workers' short hours slept decreased employee productivity at a high cost to employers. Section 3.6 considers whether policies that reduce working hours can increase workers' health and social well-being when firms are forcing workers to work overtime.

As we have presented, sleep has a very important role in human health. Besides, hours slept is different among each individual by age, education, income, and so on. Therefore, it is an important issue to clarify how individuals decide the number of hours slept. However, the past empirical studies show that the differences in hours slept are not fully explained by age and income of workers. Every individual determines sleeping duration based on both their physiological and economic needs. For analyzing this, a theoretical model is needed.

2.2 Health Investment Model

Grossman (1972) introduced health capital as a way of thinking about health investment. Mushkin (1962), Becker (1964), and Ben-Porath (1967) included health as human capital and showed that investment in human capital increases personal income and labor productivity. In addition, Mincer (1984) and Becker et al. (1990) showed that investment in human capital enhances the society's economic growth. Investments in education and job training interact with each other, but investments in health pertain only to health and are unaffected by other factors. Their investigations showed that education exhibits no direct impact on health. Our investigation shows that education exhibits no direct impact on health.[*7]

The Grossman model analyzes health by focusing on health capital, but it disregards uncertainty, initial assets do not affect health investment, and the risk of death is not endogenous. Cropper (1977), Wolfe (1985), Dardnoni and Wagstaff (1987, 1990), Lijas (1998), and Jacobson (2000) extended the Grossman model to encompass uncertainty and included changes in mortality and morbidity risks attributable to occupation or education. However, their extensions of the Grossman model consider only medical services as health investment. Sleep is not included in this investment as it is not traded in the market.

Some studies regard sleep as leisure, and classical leisure-consumption models categorize leisure as a counter-concept to labor. Such categorizations reveal the conceptual shortcomings of earlier models. They cannot categorize sleep as a consumer good because it has no price and is not traded in markets. Therefore, former studies categorize sleep as leisure because it is neither a consumer good nor labor. Biddle and Hamermesh (1990) created a model that regarded sleep as another factor of time constraint that is separate from leisure, and whereby individuals can determine hours slept endogenously. However, their study did not treat sleep as a health investment.

Therefore, we need to create a model that introduces sleep into health investment.

[*7] Acquiring good education may enhance health through higher income as an indirect effect, but it cannot be said that education affects good health directly.

3 The Sleep-Consumption Model

Sleep is affected by physiological needs, but physiological needs alone cannot explain why hours slept are determined by differences between education, age, income, and so on. There is a need for a model that relates sleep to non–physiological factors.

Our model extends that of the Grossman model by separating sleep from leisure by a similar method of Biddle and Hamermesh (1990). The distinguishing feature is that our model regards hours slept as an investment in health subject to time constraint. In other words, sleep is not traded through the market so we treat sleep as an investment in health that entails no money-spending but only needs time. By doing so, we can analyze the effect of sleep on health. Further, we redefine consumption as the time-consuming and money-spending behavior, and this is called "time-consuming behavior." The time-consuming behavior should consider all of the time pertaining to consumption; for example, if we will eat at a restaurant, we do not only consider the meal itself as taking time, but rather the entire sequence of leaving home, heading to the restaurant, eating, and returning home. Then, the price of a time-consuming behavior is the cost of the entire behavior. By this definition of consumption, we can analyze workers' decision-making to determine their balance between sleep, consumption, and labor. Besides, depending on the definition of medical care in Grossman (1972), we assume that time-consuming behavior enhances health capital. This means that we only consider consumption to be a positive health investment or that the sum effects of the overall consumption behavior on health is positive.

3.1 The Model

Consider a worker who works and lives for T periods. Here, we do not consider the after retirement period of workers. We assume that people devote time only to sleep, time-consuming behavior, and labor. Under this condition, the time not spent working and consumption is sleep.

Let S_t, F_t, and H_t, denote the three variables of interest: sleep, time-consuming behavior, and health capital, respectively, at period $t = 1, ..., T$. Here, there are n goods, so time-consuming behavior is defined as

$$F_t = \sum_{i=1}^{n} \tau_i, \tag{1}$$

where τ_i represent the time required for each consumption, thus F_t denotes the sum of the time spent on consumption.[*8]

The utility function for workers is

[*8] In this model, leisure is included in time-consuming behavior.

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1}\left[u(S_t, F_t) + v(H_t)\right]\right], \tag{2}$$

where $\delta \in (0, 1)$ is a subjective discount rate. Total utility U is defined as the addition of the two utility functions $u(S_t, F_t)$ and $v(H_t)$. Here, $u(S_t, F_t)$ is the utility derived from the health investment itself. This part is the difference in utility function between our model and the Grossman model. As discussed in Sections 1 and 2, sleep is associated with personal well-being, so it is a natural assumption to introduce it into the utility function. We also explain the reason for splitting the utility function into two parts in the assumptions below.

Assumptions

(i) $\frac{\partial u}{\partial S} > 0,\ \frac{\partial u}{\partial F} > 0,\ \frac{\partial^2 u}{\partial S^2} < 0,\ \frac{\partial^2 u}{\partial F^2} < 0$;

(ii) There exists S^* such that it satisfies $S < S^*$ if $\frac{\partial u}{\partial S} < \frac{\partial u}{\partial F}$, and $S > S^*$ if $\frac{\partial u}{\partial S} > \frac{\partial u}{\partial F}$;

(iii) $\frac{\mathrm{d}v}{\mathrm{d}H} > 0$.

Assumption (i) indicates that the utility function is monotonic and concave. Under it, workers increase utility by sleeping more or consuming more, and marginal utility falls with additional time spent on sleep or time-consuming behaviors. Assumption (ii) assures that function f is single-peaked with $f(S) = u(S, \bar{T} - S)$. If $\frac{\partial u}{\partial S} < \frac{\partial u}{\partial F}$ (or $\frac{\partial u}{\partial S} > \frac{\partial u}{\partial F}$) holds, workers demand more (or less) time-consuming behavior because they improve (or sacrifice) utility at that point. By continuing this process, they can attain optimized hours spent on sleep and time-consuming behavior. This single-peaked assumption is that there is an optimal amount of sleep for each worker. Assumption (iii) implies that personal utility rises if personal health is improved. Besides, Assumption (i) and (iii) are the reason why the utility function is separated. When it comes to health, there is no effect of diminishing utility. Thus, we divide the utility function into two parts.

Total time, $\bar{T}$, is the sum of hours slept, spent on time-consuming behavior and spent on labor, then time constraint is

$$\bar{T} = S_t + F_t + L_t. \tag{3}$$

Health capital at period t is the sum of the health capital during the previous period and time spent on sleep and time-consuming behavior minus hours spent on labor:

$$H_t = \phi_{t-1} H_{t-1} + \alpha S_{t-1} + \beta F_{t-1} - \gamma L_{t-1}, \tag{4}$$

where $\alpha > \beta > 0$[*9] and $\gamma > 0$. This assumption means that the relation of health through sleep is greater than that from time-consuming behavior.[*10][*11] In addition, ϕ_t denotes a

[*9] As we discussed earlier, consumption sometimes restores and sometimes degrades worker' s health. Therefore, parameter β could be positive or negative. For simplicity, we assume the former–that is, $\beta > 0$.

[*10] In this model, time-consuming behavior is supposed to be relaxation, eating, travel, and so on.

[*11] Appendix considers the case of $\beta > \alpha$.

depreciation rate for physical deterioration at period t. Following Grossman's model and by nature, we assume that health deteriorates at a certain rate per period t. Assuming $\phi_t \leq 1$ for any t, it is a natural assumption because physical health declines with age. Moreover, we assume that initial health capital, H_1, is given. $H_t = 0$ signifies death.

The budget constraint is

$$A_t = (1 + r)A_{t-1} + wL_t - pF_t, \tag{5}$$

where A_t, r, w, and p, respectively, are the assets during period t, interest rate, wage rates, and price of time-consuming behavior. Besides, wL_t represents an income during period t. The term pF_t is time-consuming behavior expressed in units of price. For simplicity, the interest rate and prices are constant and we suppose that there is only one consumer good. We assume that initial assets are equal to zero:

$$A_0 = 0.$$

Finally, the maximization problem of workers can be formulated as follows:

$$\max_{\{S_t\}_{t=1}^T, \{F_t\}_{t=1}^T} \qquad U = E\left[\sum_{t=1}^{T} \delta^{t-1} \left[u(S_t, F_t) + v(H_t)\right]\right]$$

$$\begin{aligned} \text{subject to} \qquad \mathrm{A_t} &= (1 + r)A_{t-1} + wL_t - pF_t \\ H_{t+1} &= \phi_t H_t + \alpha S_t + \beta F_t - \gamma L_t \\ \bar{T} &= L_t + F_t + S_t. \end{aligned}$$

3.2 Constant Labor Supply with No Budget Constraint

Workers cannot always determine how many hours they work because they are set by firms, labor-management agreements, and statutes. Here, we assume that hours worked are set exogenously; therefore, workers can decide the ratio of hours spent on sleep and time-consuming behavior. We also assume in this section that workers face no budget constraint. They have sufficient wealth or get sufficiently high enough wages to ignore the budget constraint and can access sufficient goods for consumption.

Since labor supply (hours worked) is an externally determined constant, $L_t = \bar{L}$, we have

$$\bar{T} - \bar{L} \equiv \bar{T}' = F_t + S_t, \tag{6}$$

where $\bar{T}'$ indicates the amount of time available for sleep and time-consuming behavior. Equation (6) enables workers' utility function to be rewritten as a function of S_t. That is,

$$U = E\left[\sum_{t=1}^{T}\delta^{t-1}\left[u(S_t, \bar{T}' - S_t) + v(H_t)\right]\right]. \tag{7}$$

Here, $u(S_t, F_t)$ is concave, so we treat $u(S_t, \bar{T} - S_t)$ as a concave function.*12

By differentiating Equation (7) with respect to S_t, we find the optimal value of sleep at period t by

$$\frac{\partial U}{\partial S_t} = \delta^{t-1}\cdot\frac{\mathrm{d}u}{\mathrm{d}S} + \frac{\mathrm{d}v}{\mathrm{d}H}\cdot(\alpha-\beta)\left[\delta^t + \sum_{j=t+1}^{T-1}\delta^j\prod_{k=t+1}^{j}\phi_k\right] = 0. \tag{8}$$

The following relation is used in Equation (8):

$$H_T = \prod_{t=1}^{T-1}\phi_t H_1 + \sum_{t=1}^{T-1}\prod_{k=1}^{T-t+1}\phi_{T-k}\left(\alpha S_t - \beta F_t - \gamma L_t\right). \tag{9}$$

Since ϕ_t, δ^t, and $(\alpha-\beta)$ are positive in Equation (8) and Assumption (ii), the following inequality holds between the optimal value, S^*, and the optimal hours slept for each period, S_t^*, except the final period:

$$S_t^* > S^*.$$

Assumption (ii) reveals the optimal value of hours slept, S^*. In addition, Equation (8) shows that the optimal number of hours slept during the final period coincides with the optimal value S^*:*13

$$S_T^* = S^*,$$

thereby resulting in the following inequality:

$$S_t^* > S_T^* = S^*, \tag{10}$$

for all t.

Equation (10) indicates that the optimal number of hours slept during the final period, S_T^*, is fewer than that during all other periods. We obtain Proposition 1 from these results.

Proposition 1. *Without a budget constraint, the optimal number of hours slept, S_1^*, is more than that during other periods and $S_t^* > S_{t+1}^*$ holds for each period. Therefore, the following inequalities hold:*

$$S_1^* > S_2^* > \cdots > S_{T-1}^* > S_T^* = S^*. \tag{11}$$

*12 See Appendix, Lemma 1.

*13 This means that $\frac{\partial u}{\partial S} = \frac{\partial u}{\partial F}$ holds in this point.

Proof. See Appendix. □

Proposition 1 is expressed in Figure 3, which shows that optimal hours slept during each period excluding period T is shifted farther to the right side than the optimal value, S^*. This means that workers are trying to maintain their health by taking more sleep at a younger age.

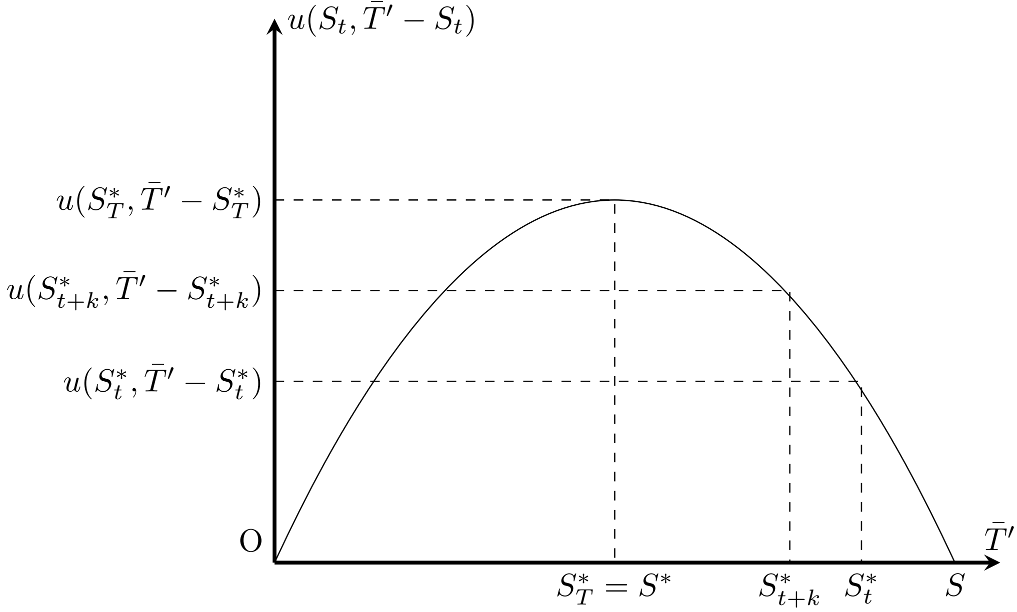

Figure 3 Transition of the optimal hours slept.

3.3 Constant Labor Supply with Budget Constraint

This section introduces a budget constraint into the model in Section 3.2. The budget constraint is

$$A_t = (1 + r)A_{t-1} + w\bar{L} - pF_t. \tag{12}$$

Here, the income is constant inasmuch as wage rates and labor supply are constant.

Since labor supply is constant, the time constraint is

$$\bar{T}' = S_t + F_t.$$

From the time constraint above, Equation (12) can be rewritten as

$$\bar{T}' - \frac{(1 + r)A_{t-1} - A_t + w\bar{L}}{p} = S_t. \tag{13}$$

Here, optimal hours slept for workers holds when $A_t = 0$. If $A_t > 0$ holds, then workers can improve their utility by increasing consumption until $A_t = 0$ from the monotonicity of u. Besides, if $A_t < 0$ holds, then workers can increase consumption at this period, but they must forgo more consumption in the following periods than the increased consumption in the current period, because the wage rate is constant and there is an interest rate. Thus, workers chose $A_t = 0$ for maximizing their total utility, U. Then, Equation (13) is rewritten as

$$\bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} = S_t. \tag{14}$$

To analyze the difference between Section 3.2 and 3.3, we exclude the case that

$$S_t^* \geq \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p}. \tag{15}$$

If Equation (15) holds, then optimal hours slept coincides with the case in Section 3.2.*[14] This is a special case that holds when initial assets are sufficiently large or wage rate, w, is sufficiently large compared to price, p. By this discussion, the following inequality holds in the case with budget constraint:

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p}. \tag{16}$$

Thus, it is sufficient to consider only the following inequality for each period:

$$S_t^* < S_t', \tag{17}$$

for all periods $t = 1, 2, ..., T$.

This section defines S_t' as optimal hours slept given a budget constraint, S_t^{b*}, if S_t' satisfies $\bar{T}' + \frac{(1+r)A_{t-1}+w\bar{L}}{p} = S_t'$. We analyze the difference in optimal value of hours slept given in Section 3.2 and 3.3 by this argument.

From Equations (16) and (17), there is S_t' such that:

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} \leq S_t'.$$

From the monotonicity of u, hours slept by workers are consistent with the budget constraint. Workers' utility would improve if hours slept moved slightly left from a point on the right side of the budget constraint, but workers cannot select a point left of the budget constraint because hours worked are constant. Thus, we have S_t^{b*} such that

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} = S_t^{b*}.$$

*[14] See Appendix, Theorem 1.

From Equation (12) and concavity, the following inequality holds for each period $t = 1, 2, ..., T$:

$$A_{t+1} > A_t. \tag{18}$$

Therefore, we obtain Proposition 2.

Proposition 2. *The following inequalities hold in the presence of a budget constraint.*

$$S_1^{b*} > S_2^{b*} > \cdots > S_{T-1}^{b*} > S_T^{b*}.$$

Proof. See Appendix. □

In this section, even though labor is constant under the budget constraint, the results of the transition of hours slept were similar to those in Section 3.2. However, in terms of total hours slept, hours slept in the case with the budget constraint is longer than the case without. This indicates that when labor is constant and there is a budget constraint, hours slept increases because time cannot be carried over to the next period and workers cannot take time-consuming behavior fully.

The discussion thus far reveals the following relation for all periods $t = 1, 2, ..., T$:

$$S_t^* < \bar{T}' - \frac{(1+r)A_{t-1} + w\bar{L}}{p} = S_t^{b*}.$$

Therefore, from Propositions 1 and 2, and the above, we obtain the proposition below.

Proposition 3. *Under a budget constraint, either of the following inequalities holds for all periods $t = 1, ..., T$:*

$$S_t^* \leq S_t^{b*}. \tag{19}$$

Proof. See Appendix. □

Figure 4 illustrates the above relation for some period, t. As shown in Figure 4, optimal hours slept under a budget constraint exceed those without a budget constraint. This means that S^* is bounded by budget constraints(12). In addition, the amount of time-consuming behavior declines under the time constraint. Further, total utility under a budget constraint is smaller than that without one. [*15]

[*15] Thia is trivial. From Propositions 1, 2, and 3, $\sum_{t=1}^{T} S_t^{b*} > \sum_{t=1}^{T} S_t^*$ and $\sum_{t=1}^{T} F_t^{b*} < \sum_{t=1}^{T} F_t^*$ hold. From Assumption (ii), at the point where $\frac{\mathrm{d}u}{\mathrm{d}S} < \frac{\mathrm{d}u}{\mathrm{d}F}$, utility obtained from time-consuming behavior is greater than that obtained from sleep. Further, utility obtained from time-consuming behavior near optimal point $S^* = F^*$ exceeds utility earned from time-consuming behavior farther from the optimal position. This statement holds.

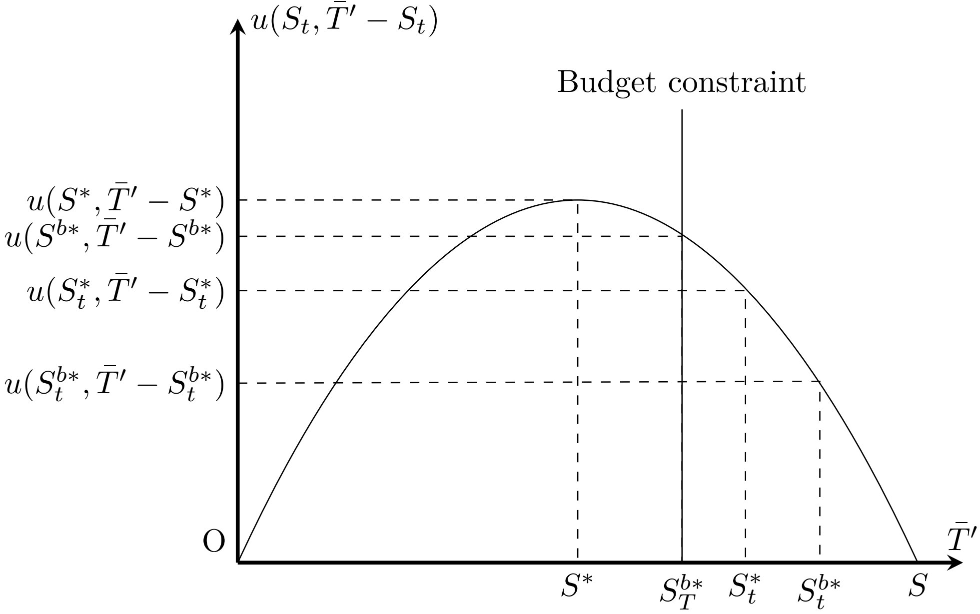

Figure 4 Transition of the optimal hours slept under the budget constraint.

3.4 Flexible Labor Supply with No Budget Constraint

As stated in Section 3.2, workers generally cannot alter the socially-determined labor supply at their discretion. However, new modes of working (e.g., freelance and remote work) grant some workers greater discretion to alter the labor that they supply. This section considers that labor supply can be changed, but there is no budget constraint. Although no budget constraint seems unrealistic, it is important to analyze all cases.

In this case, workers have no will to work because they have ample money, and working negatively affects their health as indicated by the definition of accumulated health. Declining to work raises their utility, and the quantity of labor supplied during each period must be zero. That is,

$$L_t = 0.$$

Then, utility of workers is

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1} \left[u(S_t, \bar{T} - S_t) + v(H_t)\right]\right].$$

Unlike in Section 3.2, workers' accumulated health is represented as

$$H_{t+1} = \phi_t H_t + \alpha S_t + \beta(\bar{T} - S_t). \tag{20}$$

Equation (20) shows that work no longer affects health, and we find the optimal value of sleep in this setting.

Differentiating U with respect to S_t, we obtain the optimal hours slept for each period as:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} + \frac{\mathrm{d}v}{\mathrm{d}H} \cdot (\alpha - \beta) \left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k \right] = 0. \tag{21}$$

The value of Equation (21) is identical to that in Section 3.2. Therefore, the discussion here generalizes Section 3.2. If $L_t = \bar{L}$ holds, workers' total utility, hours slept, and hours spent on time-consuming behavior are equal to the results in Section 3.2. If $L_t < \bar{L}$ holds, total utility, hours slept, and hours spent on time-consuming behavior obviously differ. We obtain Proposition 4.

Proposition 4. *Without the budget constraint, workers' utility and health decline if they increase labor supply.*

Proof. See Appendix. □

From Proposition 4, we can recast the character of labor as inciting not just disutility but also detrimental effects on health. We can confirm that working fewer hours improves health and utility, but we cannot surmise how many more hours workers devote to sleep or time-consuming behavior.

3.5 Flexible Labor Supply with Budget Constraints

This section compares the most realistic case to the outcomes in previous sections. Workers earn income from supplying labor and live by this income. Sometimes they work overtime with an order from a firm. As Section 3.4 mentioned, there are now more and more jobs available where workers can choose working time freely. We analyze such a case.

The maximization problem for workers is

$$\max_{\{S_t\}_{t=1}^{T}\{F_t\}_{t=1}^{T}} \qquad U = E\left[\sum_{t=1}^{T} \delta^{t-1} \left[u(S_t, F_t) + v(H_t)\right]\right]$$

$$\begin{aligned} \text{subject to} \qquad \mathrm{A_t} &= (1+r)A_{t-1} + wL_t - pF_t \\ H_{t+1} &= \phi_t H_t + \alpha S_t + \beta F_t - \gamma L_t \\ \bar{T} &= L_t + F_t + S_t. \end{aligned}$$

Substituting the time constraint, $\bar{T} = L_t + F_t + S_t$, into the budget constraint, we have

$$A_t = (1+r)A_{t-1} + w(\bar{T} - F_t - S_t) - pF_t, \tag{22}$$

where assuming the initial asset is as follow:

$$A_0 = 0.$$

Then, the budget constraint is rewritten as

$$A_t = (1+r)A_{t-1} + w(\bar{T} - F_t - S_t) - pF_t \tag{23}$$

$$\Leftrightarrow S_t + (1 + \frac{p}{w})F_t + \frac{A_t - (1+r)A_{t-1}}{w} = \bar{T}. \tag{24}$$

From time constraint and budget constraint, we can rewrite the budget constraint as follows:[*16]

$$\bar{T} = S_t + (1 + \frac{p}{w})F_t. \tag{25}$$

From Equation (25), by substituting time-consuming behavior into utility function and health accumulation, then we have

$$U = E\left[\sum_{t=1}^{T} \delta^{t-1}\left[u\left(S_t, \frac{\bar{T} - S_t}{(1 + \frac{p}{w})}\right) + v(H_t)\right]\right] \tag{26}$$

and

$$H_{t+1} = \phi_t H_t + \left(\alpha - \left(\frac{1}{(1+\frac{p}{w})}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right)S_t + \left(\frac{1}{(1+\frac{p}{w})}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\bar{T}. \tag{27}$$

Thus, we can analyze the maximization problem in this section as the optimization of one variable.

As in previous sections, differentiating U with respect to S_t produces

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} + \frac{\mathrm{d}v}{\mathrm{d}H} \cdot \left[\alpha - \left(\frac{1}{(1+\frac{p}{w})}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0. \tag{28}$$

From Equation (28), we obtain Proposition 5.

Proposition 5. *If hours worked are flexible under a budget constraint, the following inequalities hold:*

$$S_1^* > S_2^* > \cdots > S_{T-1}^* > S_T^* = S^*.$$

Proof. See Appendix. □

[*16] See Appendix, Theorem 3

In this case, the result of the transition of hours slept remained the same as the previous sections, with workers taking more sleep at a younger age. As in the previous sections, Proposition 5 indicates that workers try to maintain their health throughout their lives by taking more sleep at a younger age.

Regarding the optimal time-consuming behavior and labor, the following relations hold by the ratio between wage rate and price:

$$\begin{aligned} L_t^* > F_t^* \quad &\text{if} \quad p > w \\ L_t^* = F_t^* \quad &\text{if} \quad p = w \\ L_t^* < F_t^* \quad &\text{if} \quad p < w. \end{aligned} \tag{29}$$

From Proposition 5 and Equation (29), we obtain Proposition 6.

Proposition 6. *If L is flexible under a budget constraint, workers' optimal hours slept is less than or equal to when L is constant.*

Proof. See Appendix. □

Finally, we obtain Proposition 7.

Proposition 7. *If L is flexible and there is budget constraint, then more time-consuming behavior also increases labor supply and vice versa.*

Proof. See Appendix. □

Proposition 7 asserts that since labor and time-consuming behavior are complementary, increasing time-consuming behavior also increases labor and vice verse. Due to this fact and time constraints, reducing labor will decrease consumption and also increase sleep and improve health. This fact is important. If a firm forces its workers to work excessively, then the workers increase their consumption because they get more income by supplying labor more. But this reduces sleep and also increases the damage from labor, which reduces health capital. The increase in consumption increases the utility function of u, but the decrease in health capital decreases the utility function of v. Here, we do not know the impact of increasing labor on U. However, if labor supply decreases, then sleep increases and health improves, which lead to increases in both u and v. If this effect is large, then U will increase. Therefore, we should consider the effect of labor reduction policy to workers' health and utility. How policies that reduce working time affect the health and social welfare of workers is discussed in the next section.

3.6 Effectiveness of Labor Reduction Policy

Proposition 7 shows that reducing labor also reduces time-consuming behavior, and also that workers' health could be improved by increasing sleep. Here, we consider the impact of a policy of decreasing labor on social welfare when firms force workers to work excessively.

In this case, the firm's production function is

$$F(K_t, H_tL_t) = Y_t, \tag{30}$$

where K_t and Y_t are a capital at period t and an output at period t. Assume that a capital is constant, which is $K_t = K = 1$. Then, (30) is

$$F(H_tL_t) = Y_t \tag{31}$$

To analyze simply, we specify the production function as follows:

$$Y_t = (H_tL_t)^{(1-\alpha)} \tag{32}$$

From the definition of time-consuming behavior (1), there is a relationship between tY_t, which is the output Y_t multiplied by the time t to consume it, and time-consuming behavior F_t, as follows

$$tY_t = F_t. \tag{33}$$

Equation (33) requires that all of the production is fully consumed by the worker in their time of consumption F_t. That is, as production increases, there is a corresponding increase in the amount of time spent on time-consuming behavior within the time constraint. Indeed, time-consuming behavior is a function of labor and real wages. This result is also obvious from the below: [*17]

$$F_t = \frac{w}{p}L_t. \tag{34}$$

Using Equation (33), the market clearing condition with $t = 1$ is

$$Y_t = F_t. \tag{35}$$

The profit maximization problem of the firm is

$$\begin{aligned} \max_{\{L_t\}_{t=1}^{T}} \quad & \Pi = \sum_{t=1}^{T} \pi_t = \sum_{t=1}^{T} (pY_t - wL_t) \\ \text{subject to} \quad & Y_t = (H_tL_t)^{(1-\alpha)}. \end{aligned} \tag{36}$$

We assume that $\pi'_{L_t} > 0$, $\pi''_{L_t} < 0$. Differentiating Π with respect to L_t, we have

[*17] See Appendix, Corollary 2.

$$\frac{\partial \Pi}{\partial L_t} = (1-\alpha)\frac{H_t^{(1-\alpha)}}{L_t^{\alpha}}p - w = 0$$

$$\Leftrightarrow \frac{w}{p} = (1-\alpha)\left(\frac{H_t^{(1-\alpha)}}{L_t^{\alpha}}\right) \tag{37}$$

$$\Leftrightarrow L_t = \left[\left(\frac{p}{w}\right)(1-\alpha)H_t^{(1-\alpha)}\right]^{\frac{1}{\alpha}}. \tag{38}$$

Equation (38) indicates a labor demand function, $L^D = L^D(\frac{w}{p}, H_t)$, which is the decreasing function of real wage and the increasing function of health capital. From Equation (37), real wages fall as workers' health declines. Besides, since real wages are the decreasing function of labor and health capital is also the decreasing function of labor, firms can decrease real wages by imposing excess labor on workers. If the effect of the demand for labor from lower real wages exceeds the effect of improved health capital, then the firm's demand for labor will increase, according to Equation (38). That is, from this result, there is an incentive for firms to impose excess labor on workers.

Since the optimal labor supply L_t^* is given by the worker's utility-maximization problem, we obtain the equilibrium price as below:

$$p^* = w\left[\frac{L_t^*}{((1-\alpha)H_t)^{(\frac{1-\alpha}{\alpha})}}\right]^{(\frac{1+\alpha}{1+2\alpha})}. \tag{39}$$

The equilibrium quantity of the output Y_t^* is determined under the equilibrium price p^*.

Now suppose that a firm thinks only of its own profit and forces its workers to work excessively, that is, $L_t' > L_t^*$. Then the profit in this period will increase from the assumption $\pi'_{L_t} > 0$. Moreover, from the production function, production in this period Y_t' increases as well.

From Proposition 7, workers increase their time-consuming behavior due to their increased labor supply, but this reduces their sleep, which in turn reduces their health capital H_{t+1} in the next period. Thereby, from the production function, the output in the next period Y_{t+1}' will be less than the equilibrium output Y_{t+1}^* even if workers supply the same amount of labor as L_{t+1}^*. However, as real wages decrease due to the decline in the health of workers, the firms' demand for labor increases, and firms force workers to put in more work. Thereby, supply increases more, but because workers' real wages are decreased by two effects: decreasing hours slept and decreasing hours spent on time-consuming behavior. An increase in labor causes a decrease in real wages (Equation (37)), and subsequently reduces consumption as well as sleep. These results lead to an excess supply, and firms cannot maximize their profits.

Besides, we consider if firms force workers to work excessively for only one period and then allow workers to freely choose their hours worked from the next period onwards. Output in the first t period increases over equilibrium output, but output in the next period decreases due to lower workers' health capital. Even if the firm tries to return production to equilibrium in the period $t+2$, the necessary hours slept for recovering workers' health capital needed to do so will require more than the initially increased hours spent on labor and time-consuming behavior due to existence of the physical depreciation rate ϕ_t. Thus, the output at period $t+1$ decreases by more than the incremental amount of production in period t. Comparing the amount of output over these three periods, it holds that

$$Y_t^* + Y_{t+1}^* + Y_{t+2}^* > Y_t' + Y_{t+1}' + Y_{t+2}'. \tag{40}$$

Thus, we can see that firms cannot maximize profits by forcing excessive work to workers.

From Equation (40), when L increases, p is an increasing function of L and a decreasing function of H, so p increases by more than the incremental amount of L. Additionally, since w is an increasing function of H_t and a decreasing function of L_t, increasing L decreases wage by more than the incremental amount of L. From these results, real wage decreases drastically.

From Equation (29), $L_t > F_t$ holds in this state, and the only way to consume Y_t' is to borrow money.*18

If workers get into debt, they will have to put in more labor to repay it because of the existence of the interest rate r, but if they do, their health will continue to decline and their wages will continue to decrease. That way, even if workers are able to pay off their debts, they will still need more sleep to restore their health after this period. Then the total utility of the worker will be reduced because they will not be able to supply labor enough; therefore, they cannot consume enough in the future.

Thus, when the firm forces workers to work in excess, the economy is in a state of excess supply, and furthermore, workers are unable to maximize utility. This implies that there is an externality in which an increase in work and time-consuming behavior time results in a decrease in sleep.

Hence, from these results, social welfare is not maximized when firms force workers to work excessively.

In this case, suppose that the government enforces a policy of fixing the working hours to $\bar{L}$. Then, at least the state in Section 3.3 can be achieved. Here, workers' health is improved due to the increase in sleep. From the production function, this will increase the output of the firm. Also, real wages increase due to improved health. An increase in real wages leads to an increase in consumption, which in turn increases the utility of workers. Furthermore, the increase in consumption has a positive effect on health, so workers' health is further improved. An increase in consumption also implies an increase in the firm's profits. Of course, in the situation in Section 3.3, the problem of excess supply still remains. However, if the increase in real wages due to health improvements is sufficiently high, the excess supply will be infinitesimal.*19

*18 This means that increasing labor to increase time-consuming behavior results in excess output, Y', which cannot be consumed within the time constraint.

*19 If wages increase until $p < w$, then it will be the same situation as in Section 3.2.

Therefore, the policies that reduce labor not only improves the health of workers, but also social welfare.

4 Results

This section summarizes three results obtained from the previous sections.

First, we review the relation between age and hours slept. Some empirical studies found that both men and women sleep less as they age. Propositions 1, 3, and 6 from our study generated results consistent with this statistical evidence. Our propositions can explain the phenomenon of diminishing sleep with increasing age as follows. Workers are trying to maintain their health throughout their lives by sleeping more than the optimal value at a young age. This is because workers take into account the presence of physical attrition rates. Additionally, from Equation (37), workers intend to earn higher wages over their lifetime and consume more in the future by storing their health while they are young.

It is also interesting to note that similar results were obtained in the cases with and without budget constraint. These two cases can be interpreted as differences in initial assets. Even if the initial assets are sufficient, workers get more sleep when they are young because they get utility from their health. In other words, they choose to continue consuming under good health throughout their life rather than consuming more when they are young and gaining utility from it.

Finally, Sections 3.2, 3.5, and 3.6 suggested conventions and statutory considerations that affect hours spent on sleep and time-consuming behavior by reducing hours worked. If sleep supports health more than time-consuming behavior does, workers can improve their health capital by sleeping. From Propositions 2 and 5, when the labor supply is constant, workers get more sleep than when their work is flexible. Besides, from Proposition 7, we know that if workers try to increase consumption, then labor is also increased and this leads to decrease in hours slept. From these results, at least workers' health is found to improve by maintaining constant hours worked. Furthermore, from the discussion in Section 3.6, if firms force workers to supply labor excessively, the health of the workers will decline from the damage to the body caused by labor and, furthermore, the output will decrease. When the health of workers declines, from Equation (39), prices increase while wages decrease, leading to an oversupply of goods because they cannot consume all goods within the time constraint. Then, the third inequality in Equation (29) is established and workers will sacrifice more sleep in order to consume goods. Besides, they cannot consume goods enough within their time constraint by their real wages decrease.

In this paper, we assumed that time-consuming behavior improves workers' health, so decreasing hours slept and spent on time-consuming behavior lead to decrease in their health and utility. If workers try to maintain sleep while increasing consumption by borrowing money, they will have to increase their labor in the next period to pay it off, and they will be in a similar situation sacrificing sleep to consume goods. Moreover, once they have borrowed money, workers will have to either increase their labor or reduce their consumption in order to repay the debt, and in either case, their welfare will not be maximized. If the government implements the policies that reduce hours worked, then workers increase their hours slept from Proposition 7. Besides, the labor reduction policy then increases not only the health of workers but also their consumption within their time constraint because the relation between consumption and labor approaches to the second equation in Equation (29). Workers can consume enough goods by their increased wages as their health improves. Firms can also increase their profits due to the improved health of their workers. Thus, when labor is in excess supply, policies that reduce working hours will improve social welfare and not only workers' health.

5 Conclusions

The main purpose of this paper was to construct a new health investment model that includes sleep, and to analyze how sleep affects worker's health.

The first health investment model was constructed by Grossman (1972). The Grossman model analyzed health investment by health investment goods treated in the market, with a predominant focus on health services. However, sleep was not included within the health investment goods in the model. Sleep does not fit into Grossman's model because it is not traded through the market. In addition, sleep required only time and could not be valued in monetary terms, making theoretical analysis difficult. In our paper, we solved this problem by introducing sleep into time constraint. Moreover, by assuming that consumption requires both time and money, we were able to include consumption in time constraint, allowing us to analyze the balance between consumption, sleep, and labor.

The two main results from the model are as follows. The first result is that workers take more hours slept when they are young and decrease their hours slept as they age. This result is consistent with Ohayon et al. (2004), Kuroda (2008), and the NIH's recommendations for sleep duration. However, these studies did not explain why sleep duration followed such a trend. Our model provides a theoretical explanation for why. We found that workers try to stock up on health capital by sleeping more at a younger age in order to maintain health throughout their lives. Workers conserve consumption and stockpile health capital at a younger age, and as they age, they try to maximize utility by decreasing their hours slept and increasing their consumption. We also found that labor productivity and worker income are an increasing function of health capital. Furthermore, we similarly found that price is a decreasing function of health capital. These results can also explain the transition in sleep. That is, workers increase their own income and production by storing health at a younger age, thereby allowing workers to consume more within the time constraint. The second result is that labor and consumption are complementary. In other words, if workers increase consumption, labor also increase at the same time, and if they decrease labor, consumption decreases as well. This result shows that if a firm imposes work excessively on its workers, policies that reduce or keep hours worked constant can improve workers' health. When hours worked is reduced, workers are forced to consume less but instead sleep more, so their health improves. However, as health improves, production and workers' real wages increases, then workers can increase consumption, which in turn improves their utility. Besides, consumption improves workers' health also, thus health is improved through the effects of both increased sleep and consumption by labor reduction policy. Additionally, increasing consumption leads to improving the firm's profits. Thus, we find that policies that reduce hours worked can improve not only the health of workers, but also social welfare. This result is important in that it shows that health and sleep are essential to social welfare.

It must be noted that this paper includes a number of simplified assumptions, all of which should be relaxed in future studies. Firstly, this paper does not consider goods that do not take time to consume. However, we cannot ignore the existence of health investments that can be made with little or no time consumption such as supplements. It may be expected that introducing such goods into the model will allow us to analyze their balance with sleep. Besides, by including saving in our model, the balances between sleep, consumption, and labor may be analyzed more precisely.

Another limitation is the possibility of introducing uncertainty. Even if healthy, death can be possible from causes such as sudden heart attacks. Thus, we need to construct a model in which the probability of death is introduced. The positive correlation between sleep deprivation and mortality is widely known, but some statistics show that sleep of excessive duration may actually increase mortality (Youngstedt and Kripke, 2004; Steptoe et al., 2006; Grandner et al., 2010b). However, as we discussed above, it is also true that sleep improves health. Therefore, a new model that identifies an optimal sleeping time, where sleep contributes both to recovering health and reduced mortality risk, should be considered.

Besides, this paper only focused on the long-term decision-making of workers for sleep. However, in the short term, workers may be to temporarily sacrifice sleep to work or to increase their consumption. In order to analyze such behaviors, a model for analyzing short-term sleep duration should also be considered.

Lastly, this model could be extended to a life cycle model. In this paper, it is assumed that workers work until their final term and then die, which is somewhat unrealistic. Essentially, one of the incentives for workers to try to stay healthy would be the desire to stay healthy during retirement. By making modifications with respect to these points, this model would allow for a more general analysis.

However, irrespective of the modifications made, it would be erroneous to neglect the essential features of the model that we have presented in this paper. The most important aspect of this paper is that we have provided a constraint on sleep, a factor that is theoretically difficult to deal with, which allows for analysis in the theoretical model, and it is important to consider how to deal with goods that cannot be handled through the market, rather than ignoring such goods.

Acknowledgment

We are grateful to Rie Ono Yoshida, Hiromi Nagane, and Takayuki Tamura for their suggestions and encouragement. The author is responsible for any errors.

Appendix

This appendix considers the case that time-consuming behavior has a higher health-promotional effect than sleep and provides proof of our proposition, and of selected theorems and lemmas.

When Time-Consuming Behavior Has Highly Health-Promoting Effects

In Section 3, we assumed that sleep has higher effect on promoting health than time-consuming behavior. However, there may be consumption with a health-promotional effect that is higher than sleep, as described by $\beta > \alpha$.

If $\beta > \alpha$ holds, then in Sections 3.2 and 3.4, sleep approaches the optimal value from a lesser point then the optimal value. On the other hand, in such a case, time-consuming behavior approaches the optimum value from a higher point then the optimal point. This is the exact opposite of the result in the model of Section 3. Indeed, it seems natural to sacrifice sleep and increase time-consuming behavior in the era of young age if there are no budget constraints. We may also be able to reduce sleep by consumption that is more health-improving than sleep. For example, entering a hot spring, receiving a massage, or resting in an oxygen capsule may be more effective for health than sleeping.

If an individual is not restricted by budget constraints, they take the minimum sleep for living but they can further reduce sleep in order to increase time-consuming behavior, which has a better health-promoting effect than sleep. However, in the case where budget constraint exists, we can confirm that even if $\beta > \alpha$ is satisfied, we cannot necessarily obtain such a result. In the case where the labor supply is constant, as in Section 3.3, it is not possible to increase consumption beyond the budget. If such a point could be chosen, it was only when the initial assets were large enough, wage rates were large enough, or prices were low enough. This is no different from the case without budget constraints.

In the case of Section 3.5, from the following equations:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} + \frac{\mathrm{d}v}{\mathrm{d}H} \cdot \left[\alpha - \left(\frac{1}{(1+\frac{p}{w})}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0,$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}F} + \frac{\mathrm{d}v}{\mathrm{d}H} \cdot \left[\beta - \left(\alpha\left(1+\frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] = 0,$$

even if $\beta > \alpha$, then we cannot obtain the result that an individual increases time-consuming behavior by reducing sleep. If the labor is flexible and there is budget constraint, we basically get the same results as in Section 3.5, even if $\beta > \alpha$. In order to earn the result that workers increase consumption by reducing sleep, the following condition must be satisfied:

$$\alpha\left(1+\frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right) < \beta. \tag{41}$$

If this condition holds, then we obtain Proposition 8.

Proposition 8. *Suppose that labor supply is flexible, and there is a budget constraint. If $\beta > \alpha$ and the following inequality hold;*

$$\alpha\left(1+\frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right) < \beta,$$

then, the following inequalities hold:

$$S_1^* < S_2^* < \cdots < S_{T-1}^* < S_T^* = S^*;$$

$$F_1^* > F_2^* > \cdots > F_{T-1}^* > F_T^* = F^*;$$

$$L_1^* > L_2^* > \cdots > L_{T-1}^* > L_T^* = L^*$$

Proof. See Appendix. □

However, Inequality (41) is a relatively strict condition because if such goods exist, we can consider that the price of these goods would be $p \geq w$ relative to the wage rate of a typical worker. As Proposition 7 shows, increasing time-consuming behavior means more labor supply, and if the effect of restoring health from consumption is not greater than the damage to health from work, we cannot reduce sleep. Generally, goods that greatly restore health are very expensive, and increasing the labor required to purchase them may result in poor health. Therefore, we need not consider the case wherein prices are extremely low. Thus, we will consider a case where $\beta > \alpha$ and the wage rates are sufficiently large. Then, we can rewrite the above two equations as follows:

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} - \frac{\mathrm{d}v}{\mathrm{d}H} \cdot [\beta - \alpha] \left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k \right] = 0,$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}F} + \frac{\mathrm{d}v}{\mathrm{d}H} \cdot [\beta - \alpha] \left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k \right] = 0.$$

In this case, We also obtain Proposition 8.

Proposition 8 is consistent with the results for sleep and time-consuming behavior in Sections 3.2 and 3.4 under the assumption $\beta > \alpha$, but its interpretation differs. If there is budget constraint and labor is flexible, increasing consumption requires more labor. However, the disutility from labor, (γ), no longer applies if the wage rate is sufficiently large.

This situation might arise among people who work as a pastime. They seek to recover their health not just to increase hours spent on time-consuming behavior but to increase hours worked. This is because, as Proposition 7 shows, labor supply, and time-consuming behavior are complementary. In summary, if β is large, then there are more efficient ways of restoring health than sleeping. Those who like to work can derive more benefits. If $\alpha > \beta$ holds, then they sleep to recover health, since they must work more to secure more consumption. Thus, they reduce hours slept by refreshing with hours spent on time-consuming behavior, which increases the number of hours that they can work. However, as Proposition 8 shows, the optimal time of time-consuming behavior and labor in each period gradually approaches the optimal value from the higher point than the optimal value, even in individuals who like to work. We can interpret this as the age-related deterioration of the body, ϕ_t. This means that such a person cannot recover completely unless their sleep is gradually increased. Moreover, if individuals can offset their labor-induced physical damage with effective consumption within a short time period, then the health decrease with age increase can be ignored. However, the existence of such health-promoting time-consuming behavior and goods within a short time period is not realistic.

The discussion so far has shown that not only $\alpha > \beta$ but also the assumption of $\beta > \alpha$ can be considered. Apart from individuals that get disutility from labor supply, we should not forget that there are also individuals who like to work.

Proof of Lemma 1

Lemma 1. *One variable function $u(x, \bar{T} - x)$ is concave.*

Proof. We put $G(x) = u(x, \bar{T} - x)$. We will show that the following equation holds for all variables $x, y \in \mathbb{R}_{++}$ and $\lambda \in [0, 1]$:

$$G(\lambda x + (1 - \lambda) y) \geq \lambda G(x) + (1 - \lambda) G(y).$$

Since by concavity of u, we have

$$
\begin{aligned}
G\left(\lambda x+(1-\lambda) y\right) &= u(\lambda x+(1-\lambda) y, \bar{T}-(\lambda x+(1-\lambda) y)) \\
&= u(\lambda x+(1-\lambda) y,((\lambda+(1-\lambda))-(\lambda(\bar{T}-x)+(1-\lambda)(\bar{T}-y))) \\
&= u(\lambda(x, \bar{T}-x)+(1-\lambda)(y, \bar{T}-y)) \\
&\geq \lambda u(x, \bar{T}-x)+(1-\lambda) u(y, \bar{T}-y) \\
&= \lambda G(x)+(1-\lambda) G(y).
\end{aligned}
$$

Thus, we obtain

$$
G\left(\lambda x+(1-\lambda) y\right) \geq \lambda G(x)+(1-\lambda) G(y).
$$

□

Proof of Proposition 1

Proof. In this model, parameters δ^t, ϕ_t, α, and β are positive and the inequality $\alpha > \beta$ holds. In addition $\delta \cdot [\alpha - \beta]$ must be positive. From Assumption (iii), $v(H_t) > 0$, the derivative of the utility function at the health capital is positive, and via Equation (8), we have this inequality:

$$
\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} = -\frac{\mathrm{d}v}{\mathrm{d}H} \cdot (\alpha - \beta)\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right] < 0,
$$

for all periods $t = 1, 2, ..., T-1$. From Assumption (ii), the following relation holds for hours slept during final period T and any prior period t:

$$
S_t^* > S_T^* = S^*,
$$

Finally, concavity means that the slope of this curve always decreases. Thus, the following inequality holds for all periods t:

$$
S_t^* > S_{t+1}^*.
$$

Hence, we conclude that

$$
S_1^* > S_2^* > \cdots > S_{T-1}^* > S_T^* = S^*.
$$

□

Proof of Theorem 1

Theorem 1. *If the following equations hold during all periods before T, that is,*

$$
\begin{aligned}
S_1^* &= \bar{T}' - \frac{w\bar{L}}{p} \\
S_2^* &= \bar{T}' - \frac{(1+r)A_1}{p} - \frac{w\bar{L}}{p} \\
&\vdots \\
S_T^* &= \bar{T}' - \frac{(1+r)A_{T-1} - A_T}{p} - \frac{w\bar{L}}{p},
\end{aligned}
$$

then the optimal values of sleep and time-consuming behavior are consistent with having no budget constraint even if one exists.

Proof. On such a point, a person can choose optimal time-consuming behavior F_t^* during each period by choosing S_t^*. Thus, the theorem is proven. □

Proof of Proposition 2

Proof. From inequality (18), we have

$$
\bar{T}' - \frac{(1+r)A_{t-1} - A_t + w\bar{L}}{p} > \bar{T}' - \frac{(1+r)A_t - A_{t+1} + w\bar{L}}{p}.
$$

From the definition of S_t^{b*}, we have

$$
\bar{T}' - \frac{(1+r)A_{t-1} - A_t + w\bar{L}}{p} = S_t^{b*} > S_{t+1}^{b*} = \bar{T}' - \frac{(1+r)A_t - A_{t+1} + w\bar{L}}{p}.
$$

The proposition is proven. □

Proof of Proposition 3

Proof. Suppose $S_t^* > S_t^{b*}$ holds. From Assumption (ii), workers can improve utility by sleeping less and enjoying more time-consuming behavior. Thus, we obtain

$$
S_t^* \leq S_t^{b*},
$$

for all periods $t = 1, 2, ..., T$. □

Proof of Theorem 2

Theorem 2. *The optimal values in the cases with and without budget constraints do not coincide if the following inequalities hold during all periods:*

$$
\begin{aligned}
S_1^* &< \bar{T}' - \frac{w\bar{L}}{p} \\
S_2^* &< \bar{T}' - \frac{(1+r)A_1}{p} - \frac{w\bar{L}}{p} \\
&\vdots \\
S_T^* &< \bar{T}' - \frac{(1+r)A_{T-1} - A_T}{p} - \frac{w\bar{L}}{p}.
\end{aligned}
$$

Proof. From Propositions 2 and 3, the following inequalities hold:

$$S_t^* \leq S_t^{b*} < S_{t-1}^{b*},$$

for all periods $t = 1, 2, ..., T$. If $S_t^* = S_t^{b*}$ holds, then Theorem 1 holds at this point. Thus, it can be ignored. From the monotonicity of u, the budget constraint is established as an equality. Then we have

$$\bar{T}' - \frac{(1+r)A_{t-1} - A_t}{p} - \frac{w\bar{L}}{p} = S_t^{b*}.$$

Therefore, we obtain

$$S_t^* < S_t^{b*} = \bar{T}' - \frac{(1+r)A_{t-1} - A_t}{p} - \frac{w\bar{L}}{p},$$

for all periods $t = 1, 2, ..., T$. □

Proof of Proposition 4

Proof. Suppose that hours spent on sleep and time-consuming behavior are zero at period t. This means

$$\bar{T} = L_t.$$

Accumulated health at period t is

$$H_{t+1} = \phi_t H_t - \gamma L_t.$$

Thus, health is decreased as labor supply increases. Moreover, utility is represented as the sum of $u(S_t, F_t)$ and $v(H_t)$. By working fewer hours, people can take more time-consuming behavior and/or sleep. Their utility is improved by working less. □

Proof of Theorem 3

Theorem 3. *If the utility function satisfies monotonicity, then the budget constraint can be written for all $t = 1, 2, ..., T$ as:*

$$\bar{T} = S_t + (1 + \frac{p}{w})F_t.$$

Proof. In general, the assets at period t are

$$A_t = w\left[\bar{T}\sum_{j=1}^{t} {}_tC_j r^{j-1} - \sum_{j=1}^{t}(1+r)^{j-1}(S_j + (1+\frac{p}{w})F_j)\right].$$

The above equation can be rewritten as

$$\bar{T}\sum_{j=1}^{t} {}_tC_j r^{j-1} = \sum_{j=1}^{t}(1+r)^{j-1}(S_j + (1+\frac{p}{w})F_j) + A_t.$$

From the monotonicity of the utility function, they obtain the maximum utility by $A_t = 0$, then the above equation is rewritten as:

$$\bar{T}\sum_{j=1}^{t} {}_tC_j r^{j-1} = \sum_{j=1}^{t}(1+r)^{j-1}(S_j + (1+\frac{p}{w})F_j).$$

This is none other than that the following formulas are satisfied in each period:

$$\bar{T} = S_t + (1 + \frac{p}{w})F_t.$$

□

In addition, we obtain the following corollary from Theorem 3.

Corollary 1. *Sleep, time-consuming behavior, and labor satisfy below for all periods $t = 1, 2, ..., T$:*

$$S_t > 0, F_t > 0, L_t > 0.$$

Corollary 2. *There is the following relation between time-consuming behavior and labor:*

$$L_t = \frac{p}{w}F_t.$$

Proof of Proposition 5

Proof. This is the same as the proof for Proposition 1. □

Proof of Proposition 6

Proof. Workers can increase consumption by supplying labor more and sleeping less if labor is flexible. If $w \geq p$ holds, workers eventually can attain optimal time-consuming behavior in Section 3.2, F^*, then this result coincides with Proposition 3. Besides, if $w < p$ holds, then the optimal hours slept at period t in this case is equal to S_t^{b*}. Additionally, from Proposition 2 and 5, this result holds at optimal hours slept for each period. Thus, the proposition is proven. □

Proof of Proposition 7

Proof. From Theorem 3 and Corollary 2, if labor supply increases, then time-consuming behavior also increases. Thus, this proposition is proven. □

Proof of Proposition 8

Proof. Since Inequality (41) holds, then we have

$$\frac{\partial U}{\partial S_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}S} = -\frac{\mathrm{d}v}{\mathrm{d}H} \cdot \left[\alpha - \left(\frac{1}{(1+\frac{p}{w})}\right)\left(\beta - \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right];$$

$$\frac{\partial U}{\partial F_t} = \delta^{t-1} \cdot \frac{\mathrm{d}u}{\mathrm{d}F} = -\frac{\mathrm{d}v}{\mathrm{d}H} \cdot \left[\beta - \left(\alpha\left(1+\frac{p}{w}\right) + \gamma\left(\frac{p}{w}\right)\right)\right]\left[\delta^t + \sum_{j=t+1}^{T-1} \delta^j \prod_{k=t+1}^{j} \phi_k\right].$$

From the above, we then obtain the following inequalities:

$$S_1^* < S_2^* < \cdots < S_{T-1}^* < S_T^* = S^*;$$

$$F_1^* > F_2^* > \cdots > F_{T-1}^* > F_T^* = F^*.$$

In addition, from Theorem 3 and Corollary 2, we have

$$\frac{w}{p}L_1^* > \frac{w}{p}L_2^* > \cdots > \frac{w}{p}L_{T-1}^* > \frac{w}{p}L_T^* = \frac{w}{p}L^*.$$

Dividing this by $\frac{w}{p}$, we obtain

$$L_1^* > L_2^* > \cdots > L_{T-1}^* > L_T^* = L^*.$$

Thus, Proposition 8 is proven. □

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