Involuntary unemployment due to instability of the economy and fiscal policy for full-employment: A theoretical foundation for MMT (modern monetary theory)

Tanaka, Yasuhito

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Yasuhito Tanaka

Fasulty of Economics, Doshisha University, Kyoto, Japan.

Abstract

The existence of involuntary unemployment advocated by J. M. Keynes is a very important problem of the modern economic theory. Using a three-generations overlapping generations model, we show that the existence of involuntary unemployment is due to the instability of the economy. Instability of the economy is the instability of the difference equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment further reduces employment. This instability is due to the negative real balance effect that occurs when consumers' net savings (the difference between savings and pensions) are smaller than their debt multiplied by the marginal propensity to consume from childhood consumption. Also we present a discussion about fiscal policy by seigniorage to realize full-employment. We present a theoretical foundation for the so-called MMT (modern monetary theory).

Key Wods: overlappling generations model, involuntary unemployment, instability of the economy, negative real balance effect, fiscal policy by seigniorage, MMT (modern monetary theory)

JEL Classification: E12, E24.

1. Introduction

The existence of involuntary unemployment advocated by J. M. Keynes is a very important problem of the modern economic theory. It is a phenomenon that workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand.

In traditional Keynesian macroeconomics, the rigidity of the nominal wage rate was thought to be the cause of involuntary unemployment. The efficiency wage hypothesis is the most famous theory that provides a microeconomic basis for the existence of involuntary unemployment due to the rigidity of the nominal wage rate. The main references are Akerlof and Yellen (1986), Katz (1986), Shapiro and Stiglitz (1984), Yellen (1984) and Schlicht (2016). According to the efficiency wage hypothesis, workers may be more diligent if the wage they are earning is higher than the wage determined by the market, fearing that they will be fired for neglecting their work. Companies that are unable to perfectly monitor workers' laziness will pay higher wages than the market price, which will increase their own incentives to work. Firms will not reduce wages to the level where the labor market is in equilibrium. As wages remain high, rationing of jobs occurs and unemployment occurs. At this time, even if the unemployed person offers to work at a lower wage, no company will accept it for the reasons mentioned above. In addition, since the wage level set by a company is set relative to other companies, no one company will set a wage that is outstanding, and wages will converge to relatively the same level. In other words, under the efficient wage hypothesis, wages and prices will remain relatively stable despite the existence of unemployment in the labor market. Other theories include the insider-outsider theory, which assumes that the labor market is composed of two types of workers: employed workers (insiders) and unemployed workers (outsiders). Please see Blanchard and Summers (1986), (1987) and Lindbeck and Snower (1986), (1987).

Umada (1997), without assuming wage rigidity, derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. We also do not assume wage rigidity.

Otaki (2009) assumes indivisibility of labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and R. M. Solow (1981). The arguments of this paper do not depend on bargaining. As discussed by Otaki (2012) and Otaki (2015) (Theorem 2.3), if labor supply is divisible and very small, no unemployment exists. However, we show that even if labor supply is divisible, there may exist involuntary unemployment. Tanaka (2020a) and (2020b) analyzed involuntary unemployment under indivisible labor supply using a model similar to the one in this paper.

If labor supply is indivisible, it may be 1 or 0. On the other hand, in contrast if it is divisible, it takes a real value between 0 and 1. About indivisible labor supply also please see Hansen (1985). Hansen (1985) studies the existence of unemployed workers and fluctuations in the rate of unemployment over the business cycle with indivisible labor supply. To treat an indivisible labor supply in a representative agent model he assumes that people choose lotteries rather than hours worked. Each person chooses a probability of working, then a lottery determines whether or not he actually works. There is a contract between firms and individuals that commits the individual to work the predetermined number of hours with the probability which is chosen by an individual. The contract is being traded, so the individual is paid whether he works or not. The firm provides complete unemployment insurance to the workers.

In this paper we consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007), (2009), (2012) and (2015). We extend Otaki's model to a three-generations OLG model with a childhood period and pay-as-you-go pension system for the older generation consumers. We show that the existence of involuntary unemployment is due to the instability of the economy. Instability of the economy is the instability of the difference

[1] Lavoie (2001) presented a similar analysis.

equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment reduces employment. In the next section we explain the model and show the existence of involuntary unemployment when aggregate demand is insufficient. In Section 2.3 we will show the following results.

1. If the net savings (the difference between savings and pensions) is greater than debts (due to consumption in childhood period) of consumers, then the positive real balance effect kicks in, and involuntary unemployment will spontaneously dissipate because the decline in nominal wages and prices due to unemployment reduces unemployment.

2. If the net savings is smaller than debts of consumers, then the negative real balance effect kicks in, and involuntary unemployment does not spontaneously dissipate because the decline in the nominal wage and prices due to unemployment further increases unemployment.

In Section 3 we present discussions about fiscal policy in the presence of involuntary unemployment to realize full-employment. We show that the extra government expenditure to realize full-employment should be financed by seigniorage not by public debt because full-employment can be maintained by balanced budget after realizing full-employment by fiscal policy financed by seigniorage in the presence of involuntary unemployment. The additional government spending could be used as a benefit for the consumption of the older generation consumers, rather than for public investment. But, if benefits are paid to the younger generation consumers as tax cuts as well, the problem becomes more complicated because part of the benefits will be used to save for the next period. In that case, once full employment is achieved in Period t+1, taxes must be raised to maintain it in Period t+2, while tax cuts are needed to maintain full employment in Period t+3, taxes must be raised to maintain it in Period t+4, and so on, because a tax increase reduces savings, while a tax cut increases savings. However, if the marginal propensity to consume of the younger generation consumers is larger than $\frac{1}{2}$, the tax revenue converges to the steady state value, and the fiscal balance will converge to a balanced budget. The total fiscal balance over an infinite period of time is deficit, and the tax cut that is spent on consumption during the period in which the policy is implemented should be financed by seigniorage not by public debt. We present a theoretical foundation for the so-called MMT (modern monetary theory, for example, Mitchell, Wray and Watts(2019)).

2. The model and analysis

2.1 Consumers' utility maximization

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), (2009), and (2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with constant returns to scale technology.

2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply l units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.

5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

Further we make the following assumption.

Ownership of the firms Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers.

Zero interest rate The interest rate will be determined so that the supply of funds from the savings of the younger generation plus government scholarships is equal to the consumption of the childhood generation, but without scholarships there is a large possibility that savings will be insufficient regardless of the interest rate, especially in the presence of a pay-as-you-go pension system. Since it is the scholarship that fills the gap, the interest rate can be controlled by determining the size of the scholarship. If the amount of scholarships is increased or decreased, or if they are made interest-bearing or interest-free, the interest rate will change, and this may change consumption. However, for example, a decline in the interest rate may increase consumption among the younger generation due to the substitution effect, but then consumption among the older generation will decline. The income effect is also ambiguous, since a fall in the interest rate reduces the debt associated with consumption in the childhood period, but lowers the value of savings. Therefore, the possibility that a change in the interest rate will significantly change aggregate demand is small, and it is not an important issue for the existence of involuntary unemployment, which is the theme of this paper. We assume here that the amount of the scholarship is determined so that the interest rate is zero. Repayment of the debts of consumers in their childhood period is assured. Consumers in the younger period are indifferent between lending money to childhood period consumers and savings by money.

Notation We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1,2$.
C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1,2$.
$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1,2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1,2$.
D: consumption basket of an individual in the childhood period, which is constant.
P_i: the price of consumption basket in Period i, $i = 1,2$.
$p_i(z)$: the price of good z in Period i, $i = 1,2$.
$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\hat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
y: labor productivity, which is constant.

We assume that the population L_f is constant. We also assume that the nominal wage rate is constant in this section,. We examine the effects of a change in the nominal wage rate in Section 3.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

Since the taxes for unemployed consumers' unemployment benefits are paid by employed consumers of the same generation, $D(= R)$ and Θ satisfy $D(L_f - L) = L\Theta$. It means

$$L(D + \Theta) = L_f D.$$

The price index of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over three periods is

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \qquad i = 1,2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},\ i = 1,2.$$

The budget constraint for an employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

The budget constraint for an unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q} = \Pi + \hat{Q} \text{ (since } R = D).$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e},\ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}.$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

and

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1},$$

$$C_2^e = (1-\alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

$$C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1},$$

and

$$C_2^u = (1-\alpha)\frac{\Pi + \hat{Q}}{P_2}.$$

Solving maximization problems in Step 2 by standard calculations (please see Appendix), the following demand functions of employed and unemployed consumers are derived.

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+\hat{Q})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(\Pi+\hat{Q})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}, (1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2}, D\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi+\hat{Q}}{P_1}, (1-\alpha)\frac{\Pi+\hat{Q}}{P_2}, D\right).$$

Let $\omega = \frac{W}{P_1}$, $\rho = \frac{P_2}{P_1}$. Then, since the real value of D in the childhood period is constant, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}, \rho\right) - \Gamma(l),$$

and

$$V^u = \varphi\left(\frac{\Pi+\hat{Q}}{P_1}, \rho\right).$$

ω is the real wage rate. Denote

$$I = \omega l + \frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}.$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{1}$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial C_1^e} + (1-\alpha)\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate ω. Labor supply l may depend on the employment L. We assume that Ll is increasing in L.

2.2 Firms' profit maximization

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. *Net savings* is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$\widetilde{M} = M - L_fQ.$$

Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. It is financed by the tax on the younger generation consumers. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{Y}{P_1}, \tag{2}$$

where Y is the effective demand defined by

$$Y = \alpha\left(WLl + L_f\Pi - T - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits, and T is the tax revenue for the government expenditure. See Otaki (2007) and Otaki (2015) about this demand function.

Let L and Ll be employment and the "employment × labor supply" of firm z. The output of firm z is Lly. At the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial (Ll)} = y.$$

From (2) $\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}$. Thus

$$\frac{\partial p_1(z)}{\partial (Ll)} = -\frac{p_1(z)y}{\sigma d(z)} = -\frac{p_1(z)y}{\sigma Lly}.$$

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial (Ll)} = p_1(z)y - Lly\frac{p_1(z)y}{\sigma Lly} - W = p_1(z)y - \frac{p_1(z)y}{\sigma} - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{(1-\frac{1}{\sigma})y}W = \frac{1}{(1-\mu)y}W,\ \mu = \frac{1}{\sigma}.$$

This means that the real wage rate is

$$\omega = (1-\mu)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)y}W. \tag{3}$$

2.3 Involuntary unemployment due to instability of the economy

Consider an economy at Period t. The (nominal) aggregate supply of the goods is equal to

$$W^tL^t + L_f\Pi^t = P_1^tL^tl^ty.$$

The (nominal) aggregate demand is

$$\alpha\left(W^tL^t + L_f\Pi^t - T^t - L_fD^t + L_f\hat{Q}^t - L_fQ^t\right) + G^t + L_f\widehat{D}^t + M^t$$
$$= \alpha\left(P_1^tL^tl^ty - T^t - L_fD^t + L_f\hat{Q}^t - L_fQ^t\right) + G^t + L_f\widehat{D}^t + M^t.$$

The superscript t denotes variables at Period t. Since the aggregate demand and supply are equal in the equilibrium,

$$P_1^tL^tl^ty = \alpha\left(P_1^tL^tl^ty - T^t - L_fD^t + L_f\hat{Q}^t - L_fQ^t\right) + G^t + L_f\widehat{D}^t + M^t.$$

We obtain L^tl^t as follows:

$$L^tl^t = \frac{\alpha\left(-T^t - L_fD^t + L_f\hat{Q}^t - L_fQ^t\right) + G^t + L_f\widehat{D}^t + M^t}{(1-\alpha)P_1^ty}. \tag{4}$$

L^tl^t cannot be larger than $L_fl(L_f)$, where $l(L_f)$ is the labor supply at full-employment. However, it may be strictly smaller than $L_fl(L_f)$. Then, we have $L^t < L_f$ and involuntary umemployment exists. We assume balanced budget $G^t = T^t$. In the full-employment equilibrium without excess demand $L^tl^t = L_fl(L_f)$, $P_1^{t+1} = P_1^t$, $\hat{Q}^t = Q^t$, $\widehat{D}^t = D^t$. Denote the variables in the full-employment equilibrium by a superscript *. Then,

$$L_fl(L_f) = \frac{\alpha\left(-G^* - L_fD^* + L_fQ^* - L_fQ^*\right) + G^* + L_fD^* + M^*}{(1-\alpha)P_1^*y} = \frac{(1-\alpha)(G^* + L_fD^*) + M^*}{(1-\alpha)P_1^*y}.$$

Let us denote the real values of G^t, $\widehat{D}^t$ and Q^t, respectively, by g, d and q. We assume that the real values of these variables are maintained even if the prices change. Then,

$$L_fl(L_f) = \frac{(1-\alpha)(P_1^*g + L_fP_1^*d) + M^*}{(1-\alpha)P_1^*y}$$

This means

$$M^* = (1-\alpha)P_1^*(L_f l(L_f)y - g - L_f d).$$

Suppose that when there exists involuntary unemployment, the nominal wage rate falls. Then, the prices of the goods also fall at the same rate because of the constant returns to scale according to (3). This relation is expressed by the following difference equation.

$$P_1^{t+1} = \gamma\left(L^t l^t - L_f l(L_f)\right) + P_1^t,\ \gamma > 0.$$

Let us denote

$$P_1^{t+1} = f(P_1^t).$$

We assume $f'(P_1^t) > 0$. Since $L_f l(L_f)$ is constant,

$$f'(P_1^t) = \gamma \frac{dL^t l^t}{dP_1^t} + 1. \tag{5}$$

According to Chap. 4 of Schreiber, Smith and Getz (2014) the stability condition for the full-employment equilibrium is

$$f'(P_1^t) < 1 \ \text{at}\ P_1^t = P_1^*.$$

The total savings or total consumption of the older generation M^t is not constant nor predetermined, but the net savings

$$\widetilde{M}^t = M^t - L_f P_1^t q$$

is predetermined. Also note that $\hat{Q}^t = P_1^{t+1} q$ in (4). From (4) with $G^t = T^t = P_1^t g$, $Q^t = P_1^t q$, $\hat{Q}^t = P_1^{t+1} q$ and $\widehat{D}^t = P_1^t d$,

$$f'(P_1^t) = \gamma \frac{L_f P_1^t q - \alpha\left(-L_f D^t + L_f P_1^{t+1} q\right) - M^t}{(1-\alpha)\left(P_1^t\right)^2 y} + \gamma \frac{\alpha L_f P_1^t f\prime(P_1^t) q}{(1-\alpha)\left(P_1^t\right)^2 y} + 1,$$

where $\frac{\partial M^t}{\partial P_1^t} = \frac{\partial L_f P_1^t q}{\partial P_1^t} = L_f q$. At $P_1^t = P_1^*$,

$$f'(P_1^*) = \gamma \frac{(1-\alpha)L_f P_1^* q + \alpha L_f P_1^* d - M^*}{(1-\alpha)(P_1^*)^2 y} + \gamma \frac{\alpha L_f P_1^* f\prime(P_1^*) q}{(1-\alpha)(P_1^*)^2 y} + 1,$$

Therefore,

$$f'(P_1^*) = \frac{\gamma\left[(1-\alpha)L_f P_1^* q + \alpha L_f P_1^* d - M^*\right] + (1-\alpha)(P_1^*)^2 y}{(1-\alpha)(P_1^*)^2 y - \gamma\alpha L_f P_1^* q} = -\frac{\gamma\left(M^* - L_f P_1^* q - \alpha L_f P_1^* d\right)}{(1-\alpha)(P_1^*)^2 y - \gamma\alpha L_f P_1^* q} + 1.$$

Then, since $f'(P_1^*) > 0$, $f'(P_1^*) < 1$ is equivalent to

$$M^* - L_f P_1^* q - \alpha L_f P_1^* d > 0.$$

On the other hand, in contrast $f'(P_1^*) > 1$ is equivalent to

$$M^* - L_f P_1^* q - \alpha L_f P_1^* d < 0. \tag{6}$$

From (5), if $f'(P_1^*) > 1$, then $\left.\frac{dL^t l^t}{dP_1^t}\right|_{P_1^t = P_1^{t+1} = P_1^*} > 0$, which implies that a fall in the price of the goods decreases the employment, and involuntary unemployment won't go away naturally. This and (6) mean that due to the fact that consumer debt multiplied by the marginal propensity to consume is greater than net savings, the negative real balance effect works.

We have shown the following results.

Proposition 1 1. If the net savings (the difference between savings and pensions) is greater than debts (due to consumption in childhood period) of consumers, then the positive real balance effect kicks in, and involuntary unemployment will spontaneously dissipate because the decline in nominal wages and prices due to unemployment reduces unemployment.

2. If the net savings is smaller than debts of consumers, then the negative real balance effect kicks in, and involuntary unemployment does not spontaneously dissipate because the decline in the nominal wage and prices due to unemployment further increases unemployment.

3. Fiscal policy by seigniorage to decrease involuntary unemployment

Assume that in Period t, $L^t l^t$ expressed by the following equation;

$$L^t l^t = \frac{(1-\alpha)L_f P_1^t d - \alpha T^t + G^t + M^t}{(1-\alpha)P_1^t y}$$

is smaller than $L_f l(L_f)$ because G^t or M^t is insufficient and there exists involuntary unemployment. The savings of the younger generation consumers in this period is

$$M^{t+1} = (1-\alpha)(P_1^t L^t l^t y - T^t - P_1^t L_f d).$$

With $T^t = T^*$ and $P_1^t = P_1^*$,

$$M^{t+1} < M^*$$

or with $\frac{T^t}{P_1^t} = \frac{T^*}{P_1^*}$

$$\frac{M^{t+1}}{P_1^t} < \frac{M^*}{P_1^*}$$

because $L^t l^t < L_f l(L_f)$.

Suppose that in Period $t+1$ the full-employment equilibrium is realized by the government expenditure G^{t+1}. If $P_1^{t+1} = P_1^t$,

$$L_f l(L_f) = \frac{(1-\alpha)L_f P_1^{t+1} d - \alpha T^{t+1} + G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1} y}.$$

We assume

$$\frac{T^{t+1}}{P_1^{t+1}} = \frac{T^t}{P_1^t}.$$

Then,

$$L_f l(L_f) = \frac{(1-\alpha)L_f d - \alpha\frac{T^t}{P_1^t} + \frac{G^{t+1}}{P_1^{t+1}} + \frac{M^{t+1}}{P_1^{t+1}}}{(1-\alpha)y}. \tag{7}$$

In the steady state full-employment equilibrium with balanced budget, we have

$$L_f l(L_f) = \frac{(1-\alpha)L_f P_1^* d - \alpha G^* + G^* + M^*}{(1-\alpha)P_1^* y} = \frac{(1-\alpha)L_f d - \alpha\frac{G^*}{P_1^*} + \frac{G^*}{P_1^*} + \frac{M^*}{P_1^*}}{(1-\alpha)y}. \tag{8}$$

If $P_1^{t+1} = P_1^t = P_1^*$ and $T^t = T^* = G^*$, from (7), (8) and $M^{t+1} < M^*$, we obtain $G^{t+1} > G^*$.

If $P_1^{t+1} = P_1^t \neq P_1^*$ and $\frac{T^{t+1}}{P_1^{t+1}} = \frac{T^t}{P_1^t} = \frac{T^*}{P_1^*} = \frac{G^*}{P_1^*}$, then (7), (8) and $\frac{M^{t+1}}{P_1^t} < \frac{M^*}{P_1^*}$ mean $\frac{G^{t+1}}{P_1^{t+1}} > \frac{G^*}{P_1^*} = \frac{T^{t+1}}{P_1^{t+1}}$. Therefore, we need budget deficit to realize full-employment in Period $t+1$. The savings of the younger generation consumers in Period $t+1$ is

$$M^{t+2} = (1-\alpha)(P_1^{t+1}L_f l(L_f)y - T^{t+1} - P_1^{t+1}L_f d)$$
$$= (1-\alpha)P_1^{t+1}\left(L_f l(L_f)y - \frac{T^{t+1}}{P_1^{t+1}} - L_f d\right).$$

This means that when $P_1^{t+2} = P_1^{t+1}$,

$$\frac{M^{t+2}}{P_1^{t+2}} = \frac{M^*}{P_1^*}. \tag{9}$$

Next, assume that $P_1^{t+1} < P_1^t$, $\frac{T^t}{P_1^t} = \frac{T^*}{P_1^*} = \frac{G^*}{P_1^*}$ and $\frac{T^{t+1}}{P_1^{t+1}} = \frac{T^*}{P_1^*}$. Then, (7) and (8) imply

$$\frac{G^{t+1}}{P_1^{t+1}} + \frac{M^{t+1}}{P_1^{t+1}} = \frac{G^*}{P_1^*} + \frac{M^*}{P_1^*}.$$

Since $\frac{T^t}{P_1^t} = \frac{T^*}{P_1^*}$, we have

$$\frac{M^{t+1}}{P_1^{t+1}} = \frac{(1-\alpha)P_1^t\left(L^t l^t y - \frac{T^*}{P_1^*} - L_f d\right)}{P_1^{t+1}}.$$

On the other hand,

$$\frac{M^*}{P_1^*} = \frac{(1-\alpha)P_1^*(L_f l(L_f)y - \frac{T^*}{P_1^*} - L_f d)}{P_1^*} = (1-\alpha)\left(L_f l(L_f)y - \frac{T^*}{P_1^*} - L_f d\right).$$

Even though $P_1^{t+1} < P_1^t$, if $P_1^{t+1}L_f l(L_f)y > P_1^t L^t l^t y$, that is, the nominal output in Period $t+1$ is larger than that in Period t, we get

$$\frac{M^{t+1}}{P_1^{t+1}} < \frac{M^*}{P_1^*}.$$

Therefore, we have $\frac{G^{t+1}}{P_1^{t+1}} > \frac{G^t}{P_1^t} = \frac{G^*}{P_1^*} = \frac{T^{t+1}}{P_1^{t+1}}$, and we need budget deficit.

After realizing full-employment in Period $t+1$, to maintain full-employment in Period $t+2$ with $P_1^{t+2} = P_1^{t+1}$ we need

$$L_f l(L_f) = \frac{(1-\alpha)L_f d - \alpha\frac{T^*}{P_1^*} + \frac{G^{t+2}}{P_1^{t+1}} + \frac{M^{t+2}}{P_1^{t+1}}}{(1-\alpha)y}.$$

From (9) we find $\frac{G^{t+2}}{P_1^{t+2}} = \frac{G^*}{P_1^*}$. Thus, after realizing full-employment in Period $t+1$, we need balanced budget to maintain full-employment in Period $t+2$. Therefore, the extra government expenditure in Period $t+1$ to realize full-employment should be financed by seigniorage not by public debt.

We summarize the results in the following proposition.

Proposition 2

1. We need budget deficit to realize full-employment in the presence of involuntary unemployment by fiscal policy.

2. The extra government expenditure to realize full-employment should be financed by seigniorage not by public debt because full-employment can be maintained by balanced budget after realizing full-employment by fiscal policy financed by seigniorage in the

presence of involuntary unemployment.

Money suppy

Money aupply in a period is carried over to the next period as the net savings of the younger generation consumers. A change in the money supply is equal to the difference between the net savings of the younger generation consumers and the net savings of the older geneartion consumers carried over from the previous period. Since the pay-as-you-go pension for an individual consumer is constant, we have$M^{t+2}-M^{t+1}=G^{t+1}-T^{t+1}$. Thus, an incraese in the money supply is equal to the budget deficit. However, the savings will eventually be spent, and the national income will increase. The money supply will not be excessive. When $P_1^{t+1}=P_1^t$,

$$L_f l(L_f)y-L^t l^t y=\frac{1}{(1-\alpha)P_1^t}(G^{t+1}-T^{t+1}).$$

This means that the increase in the income equals the budget deficit multiplied by the multiplier. When $P_1^{t+1}<P_1^t$, we have

$$P_1^{t+1}L_f l(L_f)y-P_1^t L^t l^t y=\frac{M^{t+2}-M^{t+1}}{1-\alpha}+P_1^{t+1}\frac{T^{t+1}}{P_1^{t+1}}-P_1^t\frac{T^t}{P_1^t}+(P_1^{t+1}-P_1^t)L_f d.$$

If $\frac{T^{t+1}}{P_1^{t+1}}=\frac{T^t}{P_1^t}$, the increase in the nominal income is smaller than the increase in the money supply multiplied by the multiplier. A chage in the real income is

$$L_f l(L_f)y-L^t l^t y=\frac{1}{1-\alpha}\left(\frac{M^{t+2}}{P_1^{t+1}}-\frac{M^{t+1}}{P_1^t}\right).$$

The increase in the real income is equa to the change in the real money supply multiplied by the multiplier.

4. Tax reduction for full employment

The additional government spending could be used as a benefit for the consumption of the older generation consumers, rather than for public investment. However, if benefits are paid to the younger generation consumers as tax cuts as well, the problem becomes more complicated because part of the benefits will be used to save for the next period. In that case, once full employment is achieved in Period t+1, taxes must be raised to maintain it in Period t+2, while tax cuts are needed to maintain full employment in Period t+3, taxes must be raised to maintain it in Period t+4, and so on, because a tax increase reduces savings, while a tax cut increases savings.

Suppose that in Period $t+1$ the full-employment equilibrium is realized by the tax $T^{t+1}<T^t$. Consider a case of $P_1^{t+1}=P_1^t$. We assume

$$\frac{G^{t+1}}{P_1^{t+1}}=\frac{G^t}{P_1^t}=\frac{G^*}{P_1^*}.$$

Then,

$$L_f l(L_f)=\frac{(1-\alpha)L_f d-\alpha\frac{T^{t+1}}{P_1^{t+1}}+\frac{G^t}{P_1^t}+\frac{M^{t+1}}{P_1^{t+1}}}{(1-\alpha)y}. \quad (10)$$

If $P_1^{t+1}=P_1^t=P_1^*$ and $G^t=G^*=T^*$, from (8), (10) and $M^{t+1}<M^*$, we obtain $T^{t+1}<$

T^*.

If $P_1^{t+1} = P_1^t \neq P_1^*$ and $\frac{G^{t+1}}{P_1^{t+1}} = \frac{G^t}{P_1^t} = \frac{G^*}{P_1^*} = \frac{T^*}{P_1^*}$, then (8), (10) and $\frac{M^{t+1}}{P_1^t} < \frac{M^*}{P_1^*}$ mean $\frac{T^{t+1}}{P_1^{t+1}} < \frac{T^*}{P_1^*} = \frac{G^{t+1}}{P_1^{t+1}}$. Therefore, we need budget deficit to realize full-employment in Period $t+1$.

Next, assume that $P_1^{t+1} < P_1^t$, $\frac{G^t}{P_1^t} = \frac{G^*}{P_1^*} = \frac{T^*}{P_1^*}$ and $\frac{G^{t+1}}{P_1^{t+1}} = \frac{G^*}{P_1^*}$. Then, (8) and (10) imply

$$-\alpha\frac{T^{t+1}}{P_1^{t+1}} + \frac{M^{t+1}}{P_1^{t+1}} = -\alpha\frac{T^*}{P_1^*} + \frac{M^*}{P_1^*}.$$

Since $\frac{T^t}{P_1^t} = \frac{T^*}{P_1^*}$, we have

$$\frac{M^{t+1}}{P_1^{t+1}} = \frac{(1-\alpha)P_1^t\left(L^t l^t y - \frac{T^*}{P_1^*} - L_f d\right)}{P_1^{t+1}}.$$

On the other hand,

$$\frac{M^*}{P_1^*} = \frac{(1-\alpha)P_1^*(L_f l(L_f)y - \frac{T^*}{P_1^*} - L_f d)}{P_1^*} = (1-\alpha)\left(L_f l(L_f)y - \frac{T^*}{P_1^*} - L_f d\right).$$

Even though $P_1^{t+1} < P_1^t$, if $P_1^{t+1}L_f l(L_f)y > P_1^t L^t l^t y$, that is, the nominal output in Period $t+1$ is larger than that in Period t, we get

$$\frac{M^{t+1}}{P_1^{t+1}} < \frac{M^*}{P_1^*}.$$

Therefore, we have $\frac{T^{t+1}}{P_1^{t+1}} < \frac{T^*}{P_1^*} = \frac{G^{t+1}}{P_1^{t+1}}$, and we need budget deficit.

After realizing full-employment in Period $t+1$, to maintain full-employment in Period $t+2$ with $P_1^{t+2} = P_1^{t+1}$ we need

$$L_f l(L_f) = \frac{(1-\alpha)L_f d - \alpha\frac{T^{t+2}}{P_1^{t+1}} + \frac{G^*}{P_1^*} + \frac{M^{t+2}}{P_1^{t+1}}}{(1-\alpha)y}. \quad (11)$$

The savings of the younger generation consumers in Period $t+1$ is

$$M^{t+2} = (1-\alpha)P_1^{t+1}\left(L_f l(L_f)y - \frac{T^{t+1}}{P_1^{t+1}} - L_f d\right). \quad (12)$$

Since $\frac{T^{t+1}}{P_1^{t+1}} < \frac{T^*}{P_1^*}$, we have

$$\frac{M^{t+2}}{P_1^{t+1}} > \frac{M^*}{P_1^*}.$$

Then, from (11)

$$\frac{T^{t+2}}{P_1^{t+2}} > \frac{T^*}{P_1^*} = \frac{G^*}{P_1^*}. \quad (13)$$

Therefore, we can maintain full-employment in Period $t+2$ by budget surplus. The savings of the younger consumers in Period $t+2$ is

$$M^{t+3} = (1-\alpha)P_1^{t+2}\left(L_f l(L_f)y - \frac{T^{t+2}}{P_1^{t+2}} - L_f d\right). \quad (14)$$

From (13)

$$\frac{M^{t+3}}{P_1^{t+2}} < \frac{M^*}{P_1^*}. \quad (15)$$

To maintain full-employment in Period $t+3$ with $P_1^{t+3} = P_1^{t+2} = P_1^{t+1}$, we need

$$L_f l(L_f) = \frac{(1-\alpha)L_f d - \alpha\frac{T^{t+3}}{P_1^{t+3}} + \frac{G^*}{P_1^*} + \frac{M^{t+3}}{P_1^{t+2}}}{(1-\alpha)y}. \quad (16)$$

(15) implies

$$\frac{T^{t+3}}{P_1^{t+3}} < \frac{T^*}{P_1^*} = \frac{G^*}{P_1^*}.$$

Thus, we need budget deficit to maintain full-employment in Period $t+3$. The same applies hereafter.

(11), (12), (14) and (16) mean

$$\frac{T^{t+3}-T^{t+2}}{P_1^{t+1}} = -\frac{1-\alpha}{\alpha}\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right). \quad (17)$$

In general

$$\frac{T^{t+i+2}-T^{t+i+1}}{P_1^{t+1}} = -\frac{1-\alpha}{\alpha}\left(\frac{T^{t+i+1}-T^{t+i}}{P_1^{t+1}}\right), \quad (18)$$

for $i \geq 1$. If $\alpha > \frac{1}{2}$, that is, the marginal propensity to consume of the younger generation consumers is larger than $\frac{1}{2}$, we obtain

$$\lim_{i\to+\infty}\frac{T^{t+i+2}-T^{t+i+1}}{P_1^{t+1}} = 0.$$

Since

$$\frac{T^{t+i+1}}{P_1^{t+1}} > \frac{T^*}{P_1^*} > \frac{T^{t+i}}{P_1^{t+1}}$$

for i, which is an odd number greater than or equal to 1, if $\alpha > \frac{1}{2}$,

$$\lim_{i\to+\infty}\frac{T^{t+i}}{P_1^{t+1}} = \frac{T^*}{P_1^*}.$$

Therefore, if the marginal propensity to consume of the younger generation consumers is larger than $\frac{1}{2}$, the tax revenue converges to the steady state value, and the fiscal balance will converge to a balanced budget. From (17) and (18)

$$\frac{T^{t+i+1}-T^{t+i}}{P_1^{t+1}} = \left(-\frac{1-\alpha}{\alpha}\right)^{i-1}\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right).$$

Thus,

$$\frac{T^{t+i+1}}{P_1^{t+1}} = \frac{T^{t+1}}{P_1^{t+1}} + \sum_{j=1}^{i}\left(-\frac{1-\alpha}{\alpha}\right)^{j-1}\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right) = \frac{T^{t+1}}{P_1^{t+1}} + \frac{1-\left(-\frac{1-\alpha}{\alpha}\right)^i}{1-\left(-\frac{1-\alpha}{\alpha}\right)}\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right) \quad (19)$$

$$= \frac{T^{t+1}}{P_1^{t+1}} + \alpha\left(1-\left(-\frac{1-\alpha}{\alpha}\right)^i\right)\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right).$$

Since

$$\lim_{i\to+\infty}\frac{T^{t+i+1}}{P_1^{t+1}} = (1-\alpha)\frac{T^{t+1}}{P_1^{t+1}} + \alpha\frac{T^{t+2}}{P_1^{t+1}} = \frac{T^*}{P_1^*}, \quad (20)$$

(19) means

$$\frac{T^{t+i+1}}{P_1^{t+1}} = \frac{T^*}{P_1^*} - \alpha\left(-\frac{1-\alpha}{\alpha}\right)^i\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right).$$

Then,

$$\sum_{i=1}^{n}\left(\frac{T^{t+i+1}}{P_1^{t+1}}-\frac{T^*}{P_1^*}\right)=\alpha(1-\alpha)\left(1-\left(-\frac{1-\alpha}{\alpha}\right)^n\right)\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right).$$

From (20)

$$\frac{T^{t+2}}{P_1^{t+1}}-\frac{T^{t+1}}{P_1^{t+1}}=\frac{1}{\alpha}\left(\frac{T^*}{P_1^*}-\frac{T^{t+1}}{P_1^{t+1}}\right).$$

If $\alpha>\frac{1}{2}$,

$$\lim_{n\to+\infty}\sum_{i=1}^{n}\left(\frac{T^{t+i+1}}{P_1^{t+1}}-\frac{T^*}{P_1^*}\right)=\alpha(1-\alpha)\left(\frac{T^{t+2}-T^{t+1}}{P_1^{t+1}}\right)=(1-\alpha)\left(\frac{T^*}{P_1^*}-\frac{T^{t+1}}{P_1^{t+1}}\right).$$

From this we obtain

$$\lim_{n\to+\infty}\sum_{i=0}^{n}\left(\frac{T^{t+i+1}}{P_1^{t+1}}-\frac{T^*}{P_1^*}\right)=\frac{T^{t+1}}{P_1^{t+1}}-\frac{T^*}{P_1^*}+(1-\alpha)\left(\frac{T^*}{P_1^*}-\frac{T^{t+1}}{P_1^{t+1}}\right)=-\alpha\left(\frac{T^*}{P_1^*}-\frac{T^{t+1}}{P_1^{t+1}}\right)<0.$$

This is equal to the portion of the tax cut that will be spent on consumption during Period $t+1$. Therefore, the total fiscal balance over an infinite period of time is deficit, and the tax cut that will be spent on consumption during Period $t+1$ should be financed by seigniorage not by public debt.

We summarize the results in the following proposition.

Proposition 3

1. We need budget deficit to realize full-employment in the presence of involuntary unemployment by tax cuts, and we need budget surplus in the next period, budget deficit in the next period, and so on, to maintain full-employment.

2. However, if the marginal propensity to consume of the younger generation consumers is larger than $\frac{1}{2}$, the tax revenue converges to the steady state value, and the fiscal balance will converge to a balanced budget.

3. The total fiscal balance over an infinite period of time is deficit, and the tax cut that will be spent on consumption during Period $t+1$ should be financed by seigniorage not by public debt.

4. Conclusion

In this paper we sdudied the problem of the existence of involuntary unemployment due to instability of the economy. Instability of the economy is the instability of the difference equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment further reduces employment. This instability is due to the negative real balance effect. Also we have shown that the extra government expenditure to realize full-employment in a state with involuntary unemployment should be financed by seigniorage not by public debt.

In this paper, we assume that the production of goods is done only by labor, so there is no capital and no investment, and debt arises only from consumption in the childhood period, but a more general model in which the production of goods is done by capital and labor would allow us to deal with cases in which firms or capitalists have debt. This is an issue for the future research.

Appendix: Calculations of Step 2 of consumers' utility maximization

Lagrange functions in the second step for employed and unemployed consumers are

$$\mathcal{L}_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \qquad \text{(A.1)}$$
$$-\lambda_1^e\left[\int_0^1 p_1(z)c_1^e(z)dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$-\lambda_2^e\left[\int_0^1 p_2(z)c_2^e(z)dz - (1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^u\left[\int_0^1 p_1(z)c_1^u(z)dz - \alpha(\Pi + \hat{Q})\right],$$

and

$$\mathcal{L}_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^u\left[\int_0^1 p_2(z)c_2^u(z)dz - \alpha(\Pi + \hat{Q})\right].$$

λ_1^e, λ_2^e, λ_1^u and λ_2^u are Lagrange multipliers.

The first order condition for (A.1) is

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda_1^e p_1(z) = 0. \qquad \text{(A.2)}$$

From this

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{-1} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = (\lambda_1^e)^{1-\sigma} p_1(z)^{1-\sigma}.$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{-1} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz = (\lambda_1^e)^{1-\sigma}\int_0^1 p_1(z)^{1-\sigma}dz = 1,$$

It means

$$\lambda_1^e\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}} = 1,$$

and so

$$P_1 = \frac{1}{\lambda_1^e}.$$

From (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \lambda_1^e p_1(z) c_1^e(z).$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}$$

$$= C_1^e = \lambda_1^e \int_0^1 p_1(z) c_1^e(z) dz = \frac{1}{P_1} \int_0^1 p_1(z) c_1^e(z) dz.$$

Therefore,

$$\int_0^1 p_1(z) c_1^e(z) dz = P_1 C_1^e.$$

Similarly,

$$\int_0^1 p_2(z) c_2^e(z) dz = P_2 C_2^e.$$

Thus,

$$\int_0^1 p_1(z) c_1^e(z) dz + \int_0^1 p_2(z) c_2^e(z) dz = P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi,$$

and we obtain

$$P_1 C_1^e = \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi).$$

By (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} c_1^e(z)^{-1} = C_1^e c_1^e(z)^{-1} = (\lambda_1^e)^\sigma p_1(z)^\sigma = \left(\frac{p_1(z)}{P_1}\right)^\sigma.$$

From this we get

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1}.$$

$c_2^e(z)$, $c_1^u(z)$ and $c_2^u(z)$ are similarly obtained.

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