One Suggestion for Microfoundation of Non-Walrasian Disequilibrium Macroeconomics: Matching Theory with Dual Decision

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Abstract

In this study, we present a canonical static disequilibrium model which is microfounded. The model includes standard optimization, stochastic quantity rationing, and market friction. In temporary equilibrium, every agent makes a decision under perceived quantity signals of all markets. This specifies market spill-over effect which could be interpreted as dual-decision effect. In temporary equilibrium, market friction mixes classical and Keynesian mechanism for unemployment so that the non-Walrasian regime dividing becomes obscure.

1 Introduction

Microfoundation of Keynesian macroeconomics has been an important issue of macroeconomics.

We have two types of models to treat (notional) disequilibrium in general equilibrium framework.[1] The first one comes from Bénassy (1975). His work extends Barro and Grossman (1971), which present a mathematical interpretation of Clower (1965). It highlights *dual decision hypothesis*, in which the expressed demand (and supply) depends on the realized transaction in the other markets. Suppose that individuals exchange N commodities. The utility function is $U(z)$, where z is the excess demand vector. The expressed excess demand z^B or, *Bénassy demand*,[2] is derived from the following problem:

$$\forall i, \quad \max_{z_i} U(z_i, z_{-i}) \text{ subject to } z_{-i} \in B(z_{-i})$$

where z_{-i} is the set of the excess demands excluding z_i and $B(z_{-i})$ is the (perceived) quantity constraint for them. This formulation implies that the individuals determines the demand expression of one good following the signals of quantity constraints in the other markets but that they do not perceive any constraint on that good. *Bénassy demand* specifies the spill-over effect among markets, but it is not consistent with standard

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[1]For the survey of these approaches, see Grandmont (1977) and Svensson (1980).

[2]It has various namings such as Barro-Grossman-Bénassy demand and Clower demand. We utilize the name Bénassy since the rigorous formulation is written for the first in Bénassy (1975).

microeconomics. We should solve the different maximization problems for N times, and the expressed excess demand could violate the budget connstraint, that is, $p \cdot z_B > 0$.

The second one comes from Drèze (1975). The central aim of this work seems the extension of general equilibrium framework. The incomplete price mechanism causes quantity constraints, and therefore the individuals should maximizes the utility function with these constraints:

$$\max_z U(z) \text{ subject to } p \cdot z \leq 0, \quad z \in B(z)$$

where $B(z)$ is quantity constraint. Usually, $B(z)$ is described with the upper and lower bounds of transaction: $B(z) = \Pi_{i=1}^{N}[\underline{z}_i, \bar{z}_i]$. This *Drèze demand* is a simple extension of excess demands in general equilibrium framework. In this model, however, the realized transaction and the expressed excess demand always match, which means that disequilibrium is not observed and how quantity constraints are generated is not specified.

These defects are generated since the framework is deterministic; the economic individuals decide the demand and supply under determinate quantity constraint. Following the determinate quantity constraints and usual microfoundation (optimizing under perceived constraint) could not coexist properly, as shown above.[3] Therefore we should use a stochastic framework to built microfoundation which is consistent with standard microeconomic models.

In the stochastic rationing framework, the quantity rationings on individuals stochastically happen. The individuals use quantity signals such as market tightness to expect the rationing on their demand or supply. Using this framework loosens the interconnection between the realized transactions and the perceived quantity constraints so that we could overcome the defects in deterministic disequilibrium models.

The stochastic rationing framework is pioneered by Gale (1979) and Svensson (1980), both of which are written as drafts in 1977. Although Svensson used non-manipulable rationing scheme, in which the rationed individual could not change the realized transaction quantity by overbidding their transaction offers, Green (1980) proved that the manipulable rationing scheme is desirable to attain the feasible stochastic equilibrium. Weinrich (1984) also shows that manipulable rationing scheme is compatible with nontrivial equilibrium in a continuum economy. As these works are too abstract, it is difficult to derive some intuition about macroeconomic features such as unemployment. For application to macroeonomics, Ioannides (1983) and Honkapohja and Ito (1985) constructed simple three commodities (produced goods, labor and money) models.

However, Honkapohja and Ito's work seems to have a problem to describe an economy with uncertainty: they utilize the short-side rule to individuals' rationings. The short-side rule argues that the transaction quantity is determined by the short side so that the long side should be rationed. They assumed that the short-sided individuals expect no quantity rationing on their demand or supply. In a situation with unemployment (the aggregate labor demand is smaller than supply), for instance, every firm believes it could hire the planed amount of workers. This assumption is unrealistic when we consider about matching process, in which every agent faces uncertainty.[4]

Matching process, on the other hand, has been modeled and applied to macroeconomic analyses, such as Mortensen and Pissarides (1994). However, these analyses are restricted

[3]Weinrich (1984) supplies a concise explanation.

[4]Of course, the market frictions and the searching process are not the resource of Keynesian unemployment; see Negishi (1979, Chapter 2). This paper presents a disequilibrium model which includes Keynesian, classical and frictional unemploymemt.

by equilibrium models, which mean the quantity constraint does not play a significant role. These days, Michaillat and Saez (2013, 2015) uses a frictional matching model in order to explain Keynesian unemployment. They assume that the transactions in goods and labor markets are determined by the matching function with transaction costs. In their simple model, the market tightness in each market could be different from the efficient level, and this difference could be interpreted as a kind of disequilibrium. That is, their model has four inefficient regimes depending on the tightness of each market, e.g., product-tight and labor-slack. Although they explain the regime-dividing mechanisms and refer to the basic general disequilibrium model of Barro and Grossman (1971), it is hard to justify that their model is an extension of Keynesian disequilibrium model.

The most important issue in disequilibrium school is the quantity constraints on individuals' demand and supply. The market "disequilibrium" means not only the gap between expressed demand and expressed supply but also the discordance between planed (expected) transaction and realized transaction. To model disequilibrium economics, we should describe how markets are correlated with spill-over effect, which is specified by dual-decision mechanism. That is, the demand or supply of one individual must depend on the (expected) realized transaction quantities in other markets, as well as other economic variables, e.g., price.

In this paper, we present a simple micro-founded model which treats dual-decision effect. The model has the following features. (1) Each agent solves their optimization problem so that the model is "micro-founded." (2) The good and labor markets are directly and completely connected by the dual-decision hypothesis. (3) Market friction is included in that the both side could be quantity constrained even if the one is on short side. The model has two aspects. First, it is a re-interpretation of Green (1980) from macroeconomic perspective. We specify how quantity signals affect the agents' decision problem. Second, it is an extension of Michaillat and Saez (2015). We will show how markets are interconnected by dual-decision system without any wedge, e.g., access costs.

This paper is organized as follow. In section 2, we present a canonical static model. Every agent is stochastically rationed and makes her decision using quantity signals. After the mathematical analysis, we discuss some possible extensions in section 3. In section 4, we discuss dynamic analysis. There are obstacles to proceed dynamic model which treats non-Walrasian concepts, so we clarify what problem is left for future analyses. In section 5, we conclude our analysis.

2 The model

In this section, we present a simple model in which the transactions of goods and labor are determined by matching functions. In our model, households indexed i and firms indexed j deal with goods Y and labor L.

2.1 Rationing and matching mechanism

First, we formulate the transactions of goods x, $x = Y, L$. According to the standard matching theory, the aggregate quantity of realized transcation X is determined by the function g^X:

$$X = g^X(X^d, X^s), \tag{2.1}$$

where the subscript d is the expressed demand of X and s is expressed supply, respectively. We utilize the following assumptions.

Assumption 1. The matching function $g^X : \mathbb{R}^2_+ \to \mathbb{R}_+$ has the following properties:

1. g^X is an increasing function in both X^d and X^s.
2. $g^X(X^d, X^s) \leq \min\{X^d, X^s\}$ always holds.
3. g^X is at least twice continuous differentiable.
4. g^X is linear homogeneous.

The first one is a natural but slightly strict assumption. When the expressed (and announced) demand and supply increase, the realized transaction also increases. We can insteadly use a non-decreasing fucntion, which is more general, but we set the strictly increasing function for the simplicity. The second one is explained by *voluntary exchange* property in disequilibrium economics, which ensures that the individuals are not forced to buy (or sell) the more goods than they would like to. The third and fourth ones are assumed for mathematical simpicity.

2.2 Basic framework

There are two types of agents: households and firms. The households are employed by the firms and buy the goods to consume. The firms use the employed workforce to produce the goods and sell them.

Each agent k receives the quantity signals for aggregate transactions (Y^d, Y^s, L^d, L^s) and they determine the quantities of expressed demand and supply. For instance, the household i expresses C_i^d and L_i^s depending the aggregated quantity signals. Note that these quantities are only signals so that they are not always same as the expected transaction quantities.

Each agent k is supposed to know the probability distributions of the realized transactions of goods and labor $X_k, X = L, Y$ which depend on the aggregate quantity signals and the bidden quantities they express.

$$X_k = X_k(\tilde{X}_k^d, \tilde{X}_k^s; X_k^\xi), \xi = d, s, \tag{2.2}$$

where X_k is a random variable and $\tilde{x}_k$ is the quantity signal of x which the agent k receives. For the simplicity, we use the following assumptions:[5]

Assumption 2. X_k is proportional to X_k^ξ and homogeneous with degree zero with X^d and X^s:

$$X_k = \phi_k^X(\tilde{\theta}_k^X) X_k^\xi,\ \phi_k^X > 0,\ (\phi_k^X)' \begin{cases} < 0 & \text{if } \xi = d \\ > 0 & \text{if } \xi = s \end{cases} \tag{2.3}$$

where $\theta_k = X^d/X^s$ is a market tightness for X.

[5]The proportional rationing is a natural formulation under standard assumptions; see Green (1980); Weinrich (1984).

Assumption 3. The quantity signal is homogenous and is a function of the actural aggregate quantity θ^X:

$$\tilde{\theta}_k^X = \tilde{\theta}^X(\theta^X) \in [0,1],\ \tilde{\theta}^{X\prime}(\theta^X) > 0. \tag{2.4}$$

The rationing function ϕ_k^X has the following properties:

1. $E_k[\phi_k^X(\tilde{\theta}^X)] \in (0,1]$, where E_k is the expectation operator for k.
2. If $h(\phi_k^X)$ is monotonically increasing in ϕ_k^X, then $E_k[h(\phi_k^X)]$ is increasing in ϕ_k^X when individual k supplies X and decreasing in ϕ_k^X when individual k demands X.

Households are indexed by i and they are unifromly distributed on I with positive measure. They initially hold money M_{0i}. Their real income flow consists of the dividend π_i and labor income wL_i where w is real wage. They consume C_i and reserve the money M_{1i}. Therefore the realized transactions of L_i and C_i always satisfies the following equation:

$$M_{1i}/P + C_i = wL_i + \pi_i + M_{0i}/P. \tag{2.5}$$

However, each household does not know the quantities of realized transactions when they determine their expression C_i^d, L_i^s, and M_{1i}. We suppose that they make decision to satisfy the following expected budget constraint:

$$M_{1i}/P + E_i[C_i] = wE_i[L_i] + E_i[\pi_i] + M_{0i}/P, \tag{2.6}$$

where E_i is an expectation operator for i. It means that the households choose the expressed quantities to satisfy the budget constraint with expected quantities.

Following Assumption 2, the expected budget constraint is revised as follows:

$$M_{1i}/P + E_i[\phi_i(\tilde{\theta}^Y)]C_i^d = wE_i[\phi_i(\tilde{\theta}^L)]L_i^s + E_i[\pi_i(\tilde{\theta}^Y, \tilde{\theta}^L)] + M_{0i}/P. \tag{2.7}$$

The preference of each household is represented by a von Neumann-Morgenstein utility function u_i which satisfies expected utility hypothesis. u_i is supposed twice continuous differentiable, increasing in C_i and M_{1i}/P, and decreasing in L_i. Therefore, the household i solves the following expected utility maximization problem:

$$\max_{C_i^d, \bar{L}-L_i^s, M_{1i}/P} E_i[u_i(C_i, \bar{L} - L_i, M_{1i}/P)] \text{ subject to equation (2.7) and } L_i^s \leq \bar{L}, \tag{2.8}$$

where $\bar{L}$ is the upper limit of labor supply.

To make the solution simple, we use the following assumption;

Assumption 4. The von-Neumann Morgenstein utility function u_i is separable, that is,

$$u_i(C_i, \bar{L} - L_i, M_{1i}/P) = \tilde{u}_i(C_i, \bar{L} - L_i) + \bar{u}_i(M_{1i}/P) \tag{2.9}$$

holds. u_i is increasing in every factors and the marginal utility is decreasing in them:

$$\frac{\partial \tilde{u}_i}{\partial X} > 0,\ \frac{\partial^2 \tilde{u}_i}{\partial X^2} > 0,\ X = C_i, \bar{L} - L_i,\ (\bar{u}_i)' > 0,\ (\bar{u}_i)'' < 0.$$

Suppose that first order conditions hold when the expected utility is maximized under the expected budget constraint. The conditions could be rearranged as follows:

$$C_i^d : \frac{\partial E_i[\tilde{u}_i]}{\partial C_i^d} - E_i[\phi_i^C](\bar{u}_i)'(wE_i[\phi_i^L(\tilde{\theta}^L)]L_i^s + E_i[\pi_i(\tilde{\theta}^Y, \tilde{\theta}^L)] + M_{0i}/P - E_i[\phi_i^C(\tilde{\theta}^Y)]C_i^d) = 0,$$
$$L_i^s : \frac{\partial E_i[\tilde{u}_i]}{\partial L_i^s} + wE_i[\phi_i^L](\bar{u}_i)'(wE_i[\phi_i^L(\tilde{\theta}^L)]L_i^s + E_i[\pi_i(\tilde{\theta}^Y, \tilde{\theta}^L)] + M_{0i}/P - E_i[\phi_i^C(\tilde{\theta}^Y)]C_i^d) = 0.$$

We assume that the cross term effect on marginal utility is small enough to be ignored. In other words, the absolute value of $\frac{\partial^2 \tilde{u}}{\partial C_i \partial(\bar{L}-L_i)}$ is sufficiently small.[6] Furthermore, we assume that the expected dividend income is not sensitive to the current quantity signals. Then, the two conditions above could be revised as follows:

$$C_i^d = C_i^d(\underbrace{L_i^s}_{\oplus}; \underbrace{\tilde{\theta}^Y}_{?}, \underbrace{\tilde{\theta}^L}_{\oplus}, \underbrace{w}_{\oplus}, \underbrace{M_{0i}/P}_{\oplus}) \tag{2.10}$$
$$L_i^s = L_i^s(\underbrace{C_i^d}_{\oplus}; \underbrace{\tilde{\theta}^Y}_{\ominus}, \underbrace{\tilde{\theta}^L}_{?}, \underbrace{w}_{?}, \underbrace{M_{0i}/P}_{\ominus}) \tag{2.11}$$

The goods demand and the labor supply is positively correlated since buying goods needs sufficient (labor) income. The first equation is about goods demand. When the labor market is tight, the employment is expected secure so that C_i^d is increasing in $\tilde{\theta}^L$. Since the ample budget promotes consumption, C_i^d is increasing in w and M_{0i}.

The second condition is about labor supply. Large $\tilde{\theta}^Y$ makes the households difficult to buy the planned amount of consumption goods, which would reduce the realized expense. This implies that the households do not need labor income so much, and therefore L_i^s is decreasing in $\tilde{\theta}^Y$. The effect of w is ambiguous since it has substitution effect and income effect on labor supply. The other ways to secure the budget reduce necessity of labor income so that L_i^s is decreasing in M_{0i}/P.[7]

The ambiguity of the effect of market tightness on its demand or supply, e.g., the sign of $\frac{\partial C_i^d}{\partial \tilde{\theta}^Y}$ must be discussed. This ambiguity of "own-market effect" comes from manipulability of rationing scheme. Our stochastic rationing scheme shown in Assumption 2 is manipulable in that the large expression of demand or supply increases the expected transaction. Let us change our perspective to the rationing coefficient ϕ_i^C to see how the manipulability works on the first order conditions. Large ϕ_i^C induces two different effects. First, the expected transaction of C_i ($\phi_i^C C_i^d$) increases at given C_i^d, which lowers the marginal utility of consumption. On the other hand, the complimentarity of ϕ_i^C lowers the expression of goods demand C_i^d, which would increase the marginal utility of consumption. These conflicting effects make the sign of $\frac{\partial C_i^d}{\partial \tilde{\theta}^Y}$ ambiguous. In addition to it, the dividend income also works on the ambiguity. If the household expects high dividend income because of active market (high $\tilde{\theta}^Y$), it would increases the consumption demand.

Figure 1 shows the two dual decision process. The crossing point of the curves is the point on which the household determines the expressions of goods demand and labor supply simultaneously.

[6] When the utility from consumption is separable from that from leisure, the cross term becomes zero. For instance, a log linear utility function satisfies this condition.

[7] If the devidend income is not a stochastic variable, then the labor supply is decreasing in π_i.

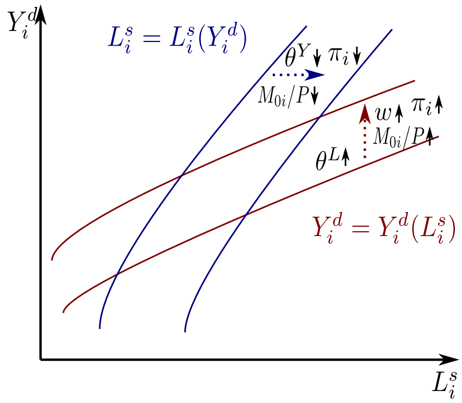

Figure 1: First order conditions of household i

The determination of goods demand and labor supply is described as follows:

$$C_i^d = C_i^d(\underbrace{\tilde{\theta}^Y}_{?}, \underbrace{\tilde{\theta}^L}_{\oplus}, \underbrace{w}_{\oplus}, \underbrace{M_{0i}/P}_{\oplus})$$

$$L_i^s = L_i^s(\underbrace{\tilde{\theta}^Y}_{\ominus}, \underbrace{\tilde{\theta}^L}_{\ominus}, \underbrace{w}_{?}, \underbrace{M_{0i}/P}_{\ominus})$$

As we introduce the utility of leisure (disutility of working), the existence of another source of revenue declines the labor supply. In ordinal microeconomic theory of household, this is intuitive. The ambiguity of effect of wage on labor supply is also a common problem which is caused by the substitution effect and income effect.

Note that the determinations of the expressions of goods demand and labor supply overcomes the defects of *Drèze demand* and *Bénassy demand*. Households determines only expressions, not the realized transaction. It implies that the quantity gap between the expressed demand (or supply) and the realized transaction could occur for each household. What is important is that the expressions are consistently affected by the quantity signals not only of own market but also of the other market, which means they are derived through the dual-decision path.

We should discuss the ambiguity of the effect of market tightness, e.g., the sign of $\frac{\partial C_i^d}{\partial \tilde{\theta}^Y}$. Michaillat and Saez (2015) gains the goods demand decreasing in the goods market tightness using market friction. When the goods market is tight (demand is relatively large), the buyers must access the market many times since the matching probability is low for them. It means the cost of purchasing goods is high, which declines the goods demand.

In this paper, we utilize the stochastic rationing instead of market friction as an extension of general disequilibrium model. The market tightness, which works as a quantity signal, affects the goods demand expression through several paths. The tight goods market decreases the expected realized goods transaction. As mentioned above, the own-market effect is ambiguous without market friction. The spill-over effect is obvious: the small expected transaction declines the other expression, as $\frac{\partial L_i^s}{\partial \tilde{\theta}^Y} < 0$. The decline of labor supply lowers the goods demand expression, so that the spill-over effect indirectly decreases the goods demand.

Figure 2 shows how the final result of $\frac{\partial C_i^d}{\partial \tilde{\theta}^Y}$ comes.

In this paper, we suppose that the consumption demand is not sensitive decreasing in the goods market tightness. This is a Keynesian stability condition which is described in

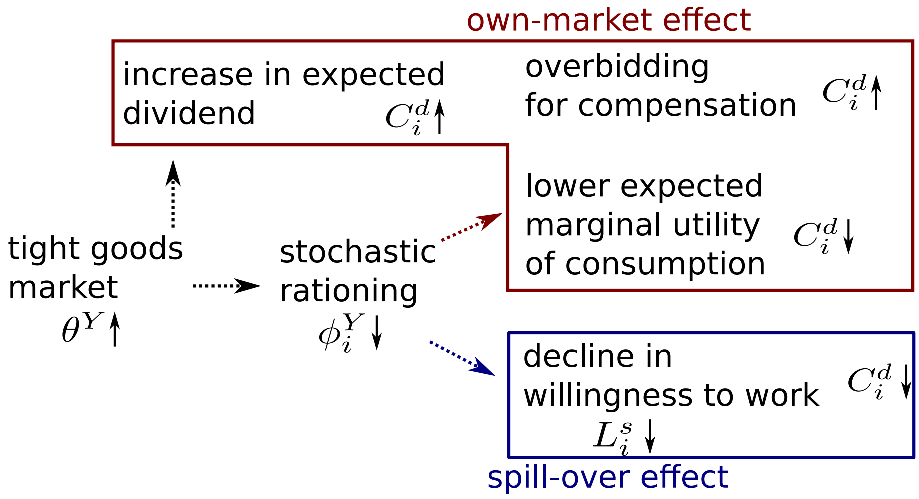

Figure 2: How goods market tightness affects the goods demand expression

the following subsection. Following empirical studies and usual macroeconomic studies, furthermore, we suppose that the expression of labor supply is increasing in real wage rate.[8]

Therefore, comparative statics of determination of household is clarified as follows:

$$C_i^d = C_i^d(\underbrace{\tilde{\theta}^Y}_{?}, \underbrace{\tilde{\theta}^L}_{\oplus}, \underbrace{w}_{\oplus}, \underbrace{\pi_i}_{\oplus}, \underbrace{M_{0i}/P}_{\oplus}), \tag{2.12}$$

$$L_i^s = L_i^s(\underbrace{\tilde{\theta}^Y}_{\ominus}, \underbrace{\tilde{\theta}^L}_{\oplus}, \underbrace{w}_{\oplus}, \underbrace{\pi_i}_{\ominus}, \underbrace{M_{0i}/P}_{\ominus}), \tag{2.13}$$

where the absolute value of $\frac{\partial C_i^d}{\partial \tilde{\theta}^Y}$ is not large.

Firms, which are indexed j and uniformly distributed on J with positive measure, produce goods using employed workforce and sell them. The production technology is described as the following production function:

$$\hat{Y}_j = F_j(L_j), \quad (F_j)' > 0, \quad (F_j)'' < 0, \tag{2.14}$$

where $\hat{Y}_j$ is the amount of produced goods with the realized employment L_j. Each firm j initially holds the inventory Inv_{0j}. We suppose that the firm evaluate the inventory at the end of the term Inv_{1j} with function v_j. Therefore, the real net return on the production and employment r_j consists of the real profit and the real value of inventory:

$$r_j = \pi_j + v_j(Inv_{1j}) = Y_j - wL_j + v(F_j(L_j) - Y_j + Inv_{0j}). \tag{2.15}$$

The firm choose the expressions of goods supply Y_j^s and labor demand L_j^d to maximize the expected real net return so that the problem of firm j is

$$\max_{Y_j^s, L_j^d} E_j[r_j] \text{ subject to } Inv_{1j} \geq 0 \text{ with probability 1.} \tag{2.16}$$

The first order conditions are as follows:

$$Y_j^s : E_j[\phi_j^Y]Y_j^s - E_j[(v_j)'; \tilde{\theta}^Y, \tilde{\theta}^L] = 0, \tag{2.17}$$

$$L_j^d : wE_j[\phi_j^L]L_j^d - E_j[(v_j)'F'; \tilde{\theta}^Y, \tilde{\theta}^L] = 0. \tag{2.18}$$

[8]Frisch elasticity is estimated positive in both microeconomic and macroeconomic studies; see Peterman (2016).

These conditions are rearranged as follows:

$$Y_j^s = Y_j^s(\underbrace{L_j^d}_{\oplus}; \underbrace{\tilde{\theta}^Y}_{?}, \underbrace{\tilde{\theta}^L}_{\ominus}) \tag{2.19}$$

$$L_j^d = L_j^d(\underbrace{Y_j^s}_{\oplus}; \underbrace{\tilde{\theta}^Y}_{\ominus}, \underbrace{\tilde{\theta}^L}_{?}, \underbrace{w}_{\ominus}) \tag{2.20}$$

First, note that the goods supply and the labor demand inversely work on the marginal value of inventory so that they are positively correlated in the first order conditions.

The first condition is about the determination of goods supply. The large labor demand should accord with the large goods supply so that Y_j^s is increasing in L_j^d. However, the tight labor market makes the firms difficult to secure the workforce, which is needed for goods production and supply. This is why Y_j^s is decreasing in $\tilde{\theta}^L$, which shows the spillover effect. The signal of own-market ambiguously works on goods supply determination since the realized production inversely works on the profit and the inventory value.

The second equation is the first order condition for labor demand. The large goods supply plan calls for large employment, so that L_j^d is increasing in Y_j^s and $\tilde{\theta}^Y$. $\tilde{\theta}^L$ ambiguously works on the labor demand. These are same as the conditions for goods supply. On this equation, the real wage w decreases the labor demand. This is intuitive for profit maximization problem.

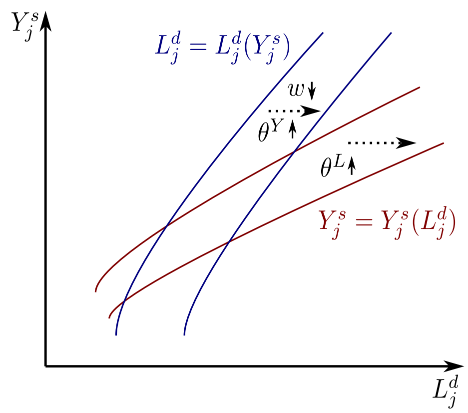

Figure 3: First order conditions of firm j

As supposed above, the overbidding for compensation is not strong. Then, The firm's determination of goods supply and labor demand could be described as follows:

$$Y_j^s = Y_j^s(\underbrace{\tilde{\theta}^Y}_{\oplus}, \underbrace{\tilde{\theta}^L}_{\ominus}, \underbrace{w}_{\ominus}), \tag{2.21}$$

$$L_j^d = L_j^d(\underbrace{\tilde{\theta}^Y}_{\oplus}, \underbrace{\tilde{\theta}^L}_{\ominus}, \underbrace{w}_{\ominus}). \tag{2.22}$$

The both variables could be interpreted as the firm's activity level. The firm becomes active (a) when the goods demand is relatively abundant, (b) when the labor supply is relatively abundant, and (c) the real wage is low enough to secure its profit.

2.3 Temporary equilibrium

We are ready to describe aggregate transactions of goods and labor. In the beginning, we define the aggregate demand and supply variables as follows:

$$Y^d(\theta^Y, \theta^L; \lambda) = \int_I C_i^d(\tilde{\theta}^Y(\theta^Y), \tilde{\theta}^L(\theta^L); \lambda)\, di + G, \tag{2.23}$$

$$Y^s(\theta^Y, \theta^L; \lambda) = \int_J Y_j^s(\tilde{\theta}^Y(\theta^Y), \tilde{\theta}^L(\theta^L); \lambda)\, dj, \tag{2.24}$$

$$L^d(\theta^Y, \theta^L; \lambda) = \int_J L_j^d(\tilde{\theta}^Y(\theta^Y), \tilde{\theta}^L(\theta^L); \lambda)\, dj, \tag{2.25}$$

$$L^s(\theta^Y, \theta^L; \lambda) = \int_I L_i^s(\tilde{\theta}^Y(\theta^Y), \tilde{\theta}^L(\theta^L); \lambda)\, di, \tag{2.26}$$

where λ denotes the exogenous variables.[9] Hereinafter, we assume that the comparative statics of individuals hold in the aggregate variables; for instance, the aggregated consumption demand increases in the wage rate.

Temporary equilibrium is defined as the state in which (a) the aggregate quantity signals accord with individuals' expression in real markets, (b) the aggregate money and dividend are completely allocated, and (c) the aggregate transaction is determined by the aggregate quantity signals:

$$\theta^Y = \frac{Y^d(\theta^Y, \theta^L; \lambda)}{Y^s(\theta^Y, \theta^L; \lambda)} \tag{2.27}$$

$$\theta^L = \frac{L^d(\theta^Y, \theta^L; \lambda)}{L^s(\theta^Y, \theta^L; \lambda)} \tag{2.28}$$

$$\int_I M_{0i}\, di = \int_I M_{1i}\, di \tag{2.29}$$

$$\int_I \pi_i di = \int_J \pi_j dj \tag{2.30}$$

$$Y = g^Y(Y^d(\theta^Y, \theta^L; \lambda), Y^s(\theta^Y, \theta^L; \lambda)) \tag{2.31}$$

$$L = g^L(L^d(\theta^Y, \theta^L; \lambda), L^s(\theta^Y, \theta^L; \lambda)) \tag{2.32}$$

In a static model, the rationings on individuals is not determined since the ex post stock variables are not important in our model.

We focus on the transactions of goods and labor. First, the partial equilibria in the two markets are described in equations (2.27) and (2.28). These recursive definitions shows that the aggregate quantity signals accord with the realized market tightnesses in the both markets. Figure 4 shows how θ^Y is determined in the goods market as θ^L and the exogenous variables given. We could illustrate a similar figure for equantion (2.28). We could interpret this figure as a special case of Keynesian cross since the expressed demand and supply depend on the realized (or perceived) market tightness. However, our cross is different from the ordinal Keynesian Cross in two points. First, supply as well as demand is determined by the realized quantity signals. It comes from the dual decision rule, in which every economic individual perceives quantity signals to determine the expressions of demand and supply in every market. Ordinal Keynesian models adopt this rule only to goods demand (households who determine consumption

[9] Note that if the rational expectation hypothesis holds, we can conclude that $\tilde{\theta}^X = \theta^X$.

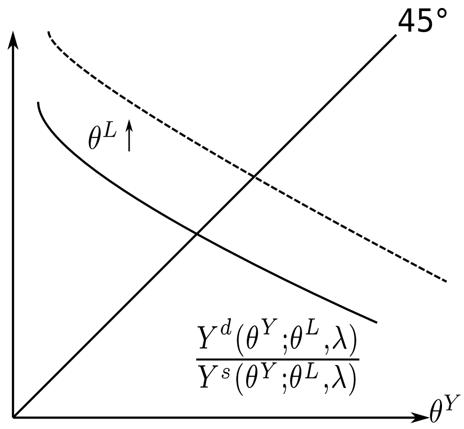

Figure 4: Determination of θ^Y with given θ^L

demand depending on the realized income), but we adopt it to every individuals. Second, the spill over effect is specified so that both the goods and labor markets are affected by each other. In Michaillat and Saez (2013, 2015), goods demand is independent of the (expected) employment and the market access cost plays an important role to connect the markets; the relationship among markets is not completely described. In our model, the economic individuals are affected by all quantity signals without any friction so that the Keynesian cross in figure 4 shifts when the labor market signal θ^L changes.

The slope of the curve in figure 4 could be negative since it reflects partial (dis)equilibrium. We could revise it by including labor market; by substituting equation (2.28) into equation (2.28), we gain the general temporary equilibrium of (θ^Y, θ^L) as $\theta^Y = Y^d(\theta^Y, \theta^L(\theta^Y))/Y^s(\theta^Y, \theta^L(\theta^Y))$. When the slope is positive and less than unity, the fiscal multiplier would be larger than one.

Solving equations (2.27) and (2.28), we gain two equations $\theta^Y = \theta^Y(\theta^L; \lambda)$ and $\theta^L = \theta^L(\theta^Y; \lambda)$ as shown in figure 5. The crossing point of the two curves is temporary equilibrium of (θ^Y, θ^L).

How the two curves cross is important for comparative statics and uniqueness of temporary equilibrium. Hereinafter, let η^x_z denote z-elasticity of x, or $\frac{\partial x}{\partial z}\frac{z}{x}$.

Lemma 1. If the spill-over effect is not too strong, the two curves in figure 5 uniquely cross.

Proof. The slopes of two curves (about Y market and L market) are calculated as follows:

$$Y : \frac{d\theta^Y}{d\theta^L} = \frac{\theta^Y}{\theta^L} \frac{\eta^{Y^d}_{\theta^L} - \eta^{Y^s}_{\theta^L}}{1 - \eta^{Y^d}_{\theta^Y} + \eta^{Y^s}_{\theta^Y}}, \tag{2.33}$$

$$L : \frac{d\theta^Y}{d\theta^L} = \frac{\theta^Y}{\theta^L} \frac{1 - \eta^{L^d}_{\theta^L} + \eta^{L^s}_{\theta^L}}{\eta^{L^d}_{\theta^Y} - \eta^{L^s}_{\theta^Y}} \tag{2.34}$$

Both of them are positive. The spill-over effect is detected as the absolute value of $\eta^{x^\xi}_{\theta^z}$ where $\xi = d, s$ and $x \neq z$. If the values of them are sufficiently small, the slope conditions are satisfied. □

Figure 5 also shows how exogenous variable w affects θ^Y and θ^L in temporary equilibrium. The increase in w makes goods market tight and labor market slack in the first

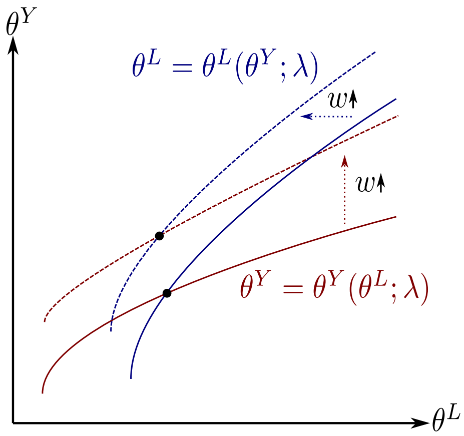

Figure 5: Temporery equilibrium of θ^Y and θ^L

place, as shown above. However, the first changes in θ^Y and θ^L induce the changes in them through the spill-over effect. Therefore the comparative statics of w is ambiguous.

Using definitions of θ^Y and θ^L in equations (2.27) and (2.28), we could calculate how each market tightness changes when the exogenous variable λ changes:

$$\eta_\lambda^{\theta^Y} = \frac{\frac{\eta_\lambda^{Y^d} - \eta_\lambda^{Y^s}}{\eta_{\theta L}^{Y^d} - \eta_{\theta L}^{Y^s}} + \frac{\eta_\lambda^{L^d} - \eta_\lambda^{L^s}}{1 - \eta_{\theta L}^{L^d} + \eta_{\theta L}^{L^s}}}{\frac{1 - \eta_{\theta Y}^{Y^d} + \eta_{\theta Y}^{Y^s}}{\eta_{\theta L}^{Y^d} - \eta_{\theta L}^{Y^s}} - \frac{\eta_{\theta Y}^{L^d} - \eta_{\theta Y}^{L^s}}{1 - \eta_{\theta L}^{L^d} + \eta_{\theta L}^{L^s}}}, \quad \eta_\lambda^{\theta^L} = \frac{\frac{\eta_\lambda^{Y^d} - \eta_\lambda^{Y^s}}{1 - \eta_{\theta Y}^{Y^d} + \eta_{\theta Y}^{Y^s}} + \frac{\eta_\lambda^{L^d} - \eta_\lambda^{L^s}}{\eta_{\theta Y}^{L^d} - \eta_{\theta Y}^{L^s}}}{\frac{1 - \eta_{\theta L}^{L^d} + \eta_{\theta L}^{L^s}}{\eta_{\theta Y}^{L^d} - \eta_{\theta Y}^{L^s}} - \frac{\eta_{\theta L}^{Y^d} - \eta_{\theta L}^{Y^s}}{1 - \eta_{\theta Y}^{Y^d} + \eta_{\theta Y}^{Y^s}}} \tag{2.35}$$

When the spill-over effect is not strong, the denominator of right hand sides of both equations are positive.

For macroeconomic analysis, the aggregate quantity of realized transactions Y and L must be specified. Using Assumption 1, the realized transaction of $X = Y, L$ is described as follows:

$$X = f_X(\theta^X)X^s, \quad \text{where } f_X(\theta^X) = \frac{g^X(X^d, X^s)}{X^s} = g^X(\theta^X, 1) \tag{2.36}$$

f_X could be interpreted as the average rationing on supply.

Then, the change in X due to the change in exogenous variable λ is calculated as follows:

$$\eta_\lambda^X = \eta_\lambda^{\theta^X}(\eta_{\theta^X}^{X^s} + \eta_{\theta^X}^{f_X}) + \eta_\lambda^{X^s} \tag{2.37}$$

The change in exogenous variable λ affects both the market tightness $\theta^X, X = Y, L$ and the supply X^s. Suppose that the goods demand strongly responds to the wage income so that the realized θ^Y is increasing in w. The increase in θ^Y increases the realized transaction Y as Y^s given. However, this increase in θ^Y is accompanied by the decrease in Y^s so that the change in Y is ambiguous.

2.4 Stability of quantity adjustment

In our fixed price economy, the expression of quantity should be adjusted. We describe an example of this process, which is a counterpart of Walrasian *tâtonnement* process. An auctioneer observes the expressed demand and supply and informs the individuals of aggregate quantity signal (market tightness). The auctioneer adjusts the quantity signals θ^Y and θ^L, and the adjustments respond to the gap between the expressed market tightness in markets and the signals the auctioneer announces. This adjustment could be described as follows:

$$\dot{\theta}^Y = F_Y\left(\frac{Y^d(\theta^Y, \theta^L; \lambda)}{Y^s(\theta^Y, \theta^L; \lambda)} - \theta^Y\right), \tag{2.38}$$

$$\dot{\theta}^L = F_L\left(\frac{L^d(\theta^Y, \theta^L; \lambda)}{L^s(\theta^Y, \theta^L; \lambda)} - \theta^L\right), \tag{2.39}$$

where $F_X(0) = 0$ and $F_X' > 0$, $X = Y, L$. The Jacobian matrix around the steady state $(\theta^{Y*}, \theta^{L*})$ of the dynamical system above $\mathbf{J}$ is

$$\mathbf{J} = \begin{pmatrix} F_Y'(\eta_{\theta^Y}^{Y^d} - \eta_{\theta^Y}^{Y^s} - 1) & F_Y'\frac{\theta^{L*}}{\theta^{Y*}}(\eta_{\theta^L}^{Y^d} - \eta_{\theta^L}^{Y^s}) \\ F_L'\frac{\theta^{Y*}}{\theta^{L*}}(\eta_{\theta^Y}^{L^d} - \eta_{\theta^Y}^{L^s}) & F_L'(\eta_{\theta^L}^{L^d} - \eta_{\theta^L}^{L^s} - 1) \end{pmatrix}. \tag{2.40}$$

The stability condition of quantity *tâtonnement* process is as follows.

Proposition 1. If the following two inequalities hold, then the temporary equilibrium is locally asymtotically stable under the non-Walrasian *tâtonnement* process.

$$\eta_{\theta^X}^{X^d} - \eta_{\theta^X}^{X^s} < 1, \ X = Y, L \tag{2.41}$$

$$(\eta_{\theta^Y}^{Y^d} - \eta_{\theta^Y}^{Y^s} - 1)(\eta_{\theta^L}^{L^d} - \eta_{\theta^L}^{L^s} - 1) - (\eta_{\theta^L}^{Y^d} - \eta_{\theta^L}^{Y^s})(\eta_{\theta^Y}^{L^d} - \eta_{\theta^Y}^{L^s}) > 0. \tag{2.42}$$

Proof. If inequality (2.41) holds, then the trace of $\mathbf{J}$ is negative. If inequality (2.42) holds, then the determinant of $\mathbf{J}$ is positive. These condition show that the dynamical system linearized around the steady state is asymptotically stable. □

The stability conditions say that the slope of curve illustrated in figure 4 must be under unity and the spill-over effect must not be too strong so that the slope conditions in Lemma 1 hold.

Note that the adjustment described above is only a counterpart of Walrasian adjustment process and that we have not approach the brand new model which includes both equilibrium and disequilibrium schools. For further dynamic adjustment studies, we should adopt what is called non-*tâtonnement* model such as Hahn and Negishi (1962) and Hahn (1978) to frictional economy. This is a future issue.

3 Macroeconomic interpretation of the static model

3.1 Fiscal multiplier

For Keynesian studies, fiscal multiplier is an important issue. By definition, the fiscal multiplier is calculated as follows:

$$\eta_G^Y \frac{Y}{G} = \frac{\eta_{\theta^Y}^{Y^s} + \eta_{\theta^Y}^{f_Y}}{1 - \eta_{\theta^Y}^{Y^d} + \eta_{\theta^Y}^{Y^s} - \frac{(\eta_{\theta^L}^{Y^d} - \eta_{\theta^L}^{Y^s})(\eta_{\theta^Y}^{L^d} - \eta_{\theta^Y}^{L^s})}{1 - \eta_{\theta^L}^{L^d} + \eta_{\theta^L}^{L^s}}} \frac{Y}{Y^d} \tag{3.1}$$

As the goods demand increases without change in goods supply, the goods transaction itself increases. The scale of change, however, is ambiguous. The term of elasticities is positive, but the denominator would be large since the spill-over effect (e.g., $\eta^{L^d}_{\theta^Y}$) is small. When the multiplier becomes large? We should focus on the term $\frac{Y}{Y^d}$. Note that it tends large when θ^Y is small, that is, the multiplier would be large when the goods demand is relatively small.

Seeing figure 5, we notice that the slopes of the curves are steep when θ^Y is (or equivalently, θ^L is small). This is because the slope of them are multiplied by $\frac{\theta^Y}{\theta^L}$. The increase in G shifts up the curve $\theta^Y = \theta^Y(\theta^L)$. How the crossing point moves is affected by the slopes of the curves. When they are steep (θ^Y is small), the increase in G induces the large increase in θ^Y at the crossing point.

Therefore, we attain a simple proposition: the fiscal multiplier would ba large when the goods market is slack.

We should note that the spill-over (dual-decision) effect plays an important role in the multiplier process. We have assumed that the own-market effect on demand is not large so that the quantity adjustment process is stable. That is, the aggregate goods demand could negatively responds to the aggregate goods market tightness under a given labor market tightness. Therefore, *partial equilibrium* in figure 4 shows that the curve Y^d/Y^s has a negative slope, which implies the fiscal multiplier is under unity. In general temporary equilibrium, however, the upward pressure on goods market tightness also increases the labor market tightness since the firms revise their sales expectations to optimistic. They increase the labor demand, and then the consumption demand increases which generates further upward pressure on the goods market tightness. This positive feedback through the both markets is the multiplier process.

3.2 Regimes and source of unemployment

Ordinal non-Walrasian macroeconomic models have regimes, such as Keynesian unemployment and repressed inflation. These regimes are defined by demand-supply gaps in the goods and labor market, e.g., both excess supplies in the two market defines Keynesian unemployment. Our model could use a similar definition; for instance, Keynesian unemployment could be defined as the state with $\theta^Y < 1$ and $\theta^L < 1$. However, we should care about the name of regime. The situation is different from the deterministic models.

Note that the cause of unemployment is always mixed; that is, all the shortage of goods demand, relative high wage, and market friction induce the unemployment. In other words, the possible unemployment is (old or neo-) Keynesian, classical, and new Keynesian.

Suppose that $\phi^L_i < 1$ for some household i (unemployed). Then the temporary equilibrium (θ^Y, θ^L) must lie in the interior of $[0,1] \times [0,1]$. First, suppose that the government purchase G increases to stimulate goods demand Y^d. Then, the curve $\theta^Y = \theta^Y(\theta^L)$ shifts up without any change of labor supply so that the equilibrium value of θ^L increases, which means the unemployment is cured. Second, suppose that the fixed nominal wage W goes down by some policy. According to the firms' first order conditions and positive Frishc elasticity, the curve $\theta^L = \theta^L(\theta^Y)$ shifts to right side. However, the curve $\theta^Y = \theta^Y(\theta^L)$ shifts down due to the decrease in consumption demand. If the former is stronger than the latter, the employment rate $f_L(\theta^L)$ rises. Third, suppose that the market friction is partly eliminated so that the matching process is developed. This is expressed as the increase in $G(Y^d, Y^s)$ without changes of Y^d and Y^s. Since the realizations of transactions grow without any conditions, the expected rationing function $E_k[\phi^X_k(\theta_k)]$ goes up. For this optimistic change, the goods and labor demand increases and the two curves of θ^Y

and θ^L shifts same as the former two cases and the labor supply increases. Therefore, the employment increases.

As described above, the strict distinctions of regimes are not suitable for our model. The existence of friction not only induces unemployment but also makes the boundaries of regimes obscure; see figure 6. Note that the effectiveness of each policy varies as the values of (θ^Y, θ^L) changes. As shown above, the expansin of G makes more increase in θ^Y which the values of (θ^Y, θ^L) are low. In this sense, the economy with $\theta^Y < 1$ and $\theta^L < 1$ has a strong Keynesian feature.

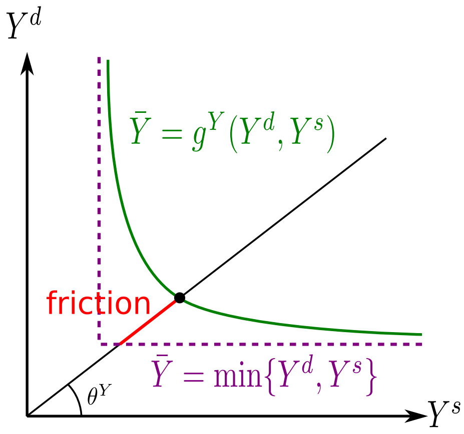

Figure 6: The frictionless economy (dashed line) and the economy with friction (solid line)

3.3 Multiple equilibria and indeterminancy

We have assumed that the aggregate expressed demand and supply are well-behaved so that the temporary equilibrium uniquely exists as shown in figure 5. However, these assumptions seem a little strict since we have not employed market frictions and specific intertemporal maximization problems. Our model has the possibility to generate multiple equilibria or indeterminate equilibrium as shown in figure 7.[10]

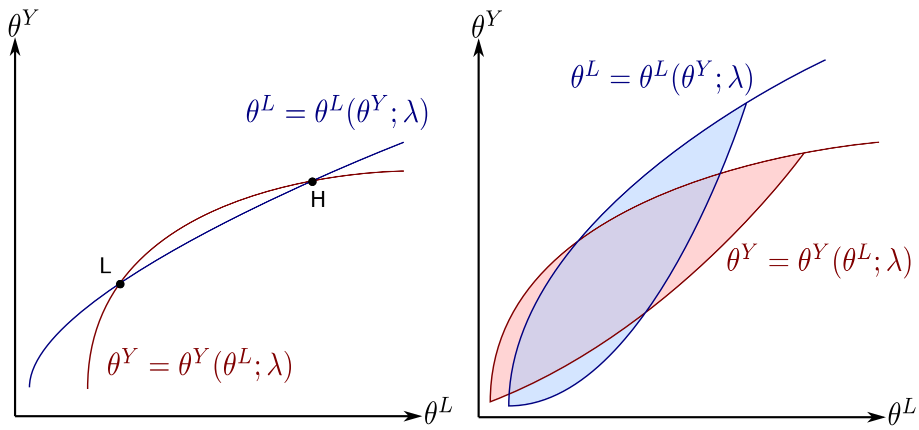

Figure 7: Multiple equilibria (left) and indeterminate equilibrium (right)

[10] Green (1980) shows that the demand or supply correspondences in an exchange economy could be upper hemi-continuous with standard assumptions.

The left one in figure 7 shows the case there are two equilibria; the one with low market tightness (L) and the other with high (H). The example of multiple equilibria shows that whether the economy is optimistic or pessimistic determines the realization of temporary equilibrium (H or L), even though the fundamental variables λ is unchanged.[11] This feature is related to sun-spot equilibria; see Madden (1992) for non-Walrasian framework.

3.4 Information misspecification

The quantity signal perception of agent k is described as the equation $\tilde{\theta}_k^X = \tilde{\theta}_k^X(\theta^X)$. If the agent could use the correct information ($\tilde{\theta}_k^X = \theta^X$) and the subjective expectation on rationing is same as the objective one, their expression of demand and supply follows the rational expectation hypothesis.

Suppose that the firms' expectations for current transaction become pessimistic so that the quantity constraint on goods supply is overestimated, that is, $E_j[\phi_j^Y]$ is smaller than the objective expectation.

According to the first order conditions, the firms decrease both the goods supply and the labor demand. Firms become inactive. Then, the curve $\theta^Y = \theta^Y(\theta^L)$ shifts up and the curve $\theta^L = \theta^L(\theta^Y)$ shifts to left in figure 8. Note that in these shifts the households observe the change in market tightness so that they revise their decisions. The labor market tightness and realized employment decrease since the firms restrict employment for pessimistic expectation on sales. The change in goods market tightness θ^Y is, however, ambiguous since the firms decrease supply but the households also decreases goods demand due to unemployment. When θ^Y does not increase, the realized goods transaction should decrease since the goods supply decreases.

This simple experiment shows how pessimistic expectation is enhanced in quantity signal observations and that the expectation has a kind of self-filling feature in disequilibrium model.

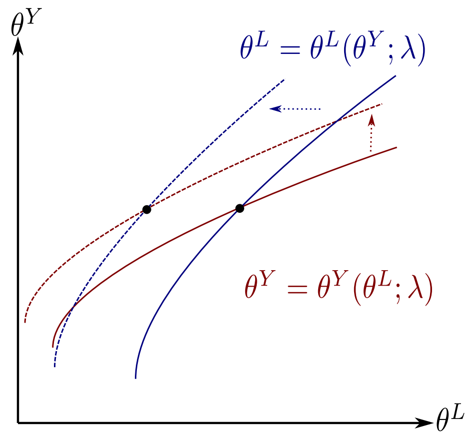

Figure 8: How pessimistic firm affects temporary equilibrium

[11]Note that if we utilize the quantity *tâtonnement* process, then the equilibrium H in the figure would be locally asymptotically stable.

4 Toward dynamic analysis

We have constructed the temporary equilibrium model in the static framework. The most important issue left is to adopt our stochastic model to dynamic perspective to construct Dynamic Stochastic General Disequilibrium (DSGD) model. The mathematical formulation of model is beyond the scope of the present paper, and let us discuss it instead.

4.1 Expectation and information

In dynamics, the agents must make decisions considering the future. To accord with the existing equilibrium models, suppose that the agents optimize their inter-temporal objective functions. For instance, the household maximize the expected sum of utility functions:

$$\max_{\{C_t, L_t\}} \sum_{t=0}^{\infty} E_0[\beta^t u(C_t, L_t)] \text{subject to } \forall \tau (C_\tau, L_\tau) \in E_0[\Gamma_\tau], \tag{4.1}$$

where E_0 is the expectation operator at $t = 0$ and Γ_τ is a quantity constraint on the household at $t = \tau$. The crucial problem for disequilibrium theory is E_0. How do agents form the expectations on future quantity constraint (disequilibrium)? In the static model above, the realized transaction for agent k, or x_k, could hardly match both the expected transaction $E_k[x_k]$ and the expressed demand or supply x_k^λ. Then, the agents must revise the budget constraints (and then the expected quantity constraints) in all succeeding periods so that the problem of time inconsistency is intrinsic in disequilibrium models.[12]

Furthermore, the economic agents do not necessarily know their stochastic rationing mechanisms. It is difficult to justify that they know the correct relationship between the perceived signals and the realized quantities. Therefore, the agents must update the expectation of rationing function ϕ_k^X in every period. Let us describe a simple dynamics.[13] In each passed period t, agent k has expressed its demand and supply $\boldsymbol{x}_{kt}^\lambda$ and has been rationed the quantity $\boldsymbol{x}_{kt}$. Let Θ_{kt} denote the observed information set which the agent k observes at period t. This includes the aggregate quantity signal and expressions and transactions of other agents; $(\boldsymbol{x}_{lt}^\lambda, \boldsymbol{x}_{lt})$ where $l \neq k$. Suppose that the agent use the information set which has been generated since the period $-s$, that is, $\{\Theta_{kt}, \boldsymbol{x}_{kt}^\lambda, \boldsymbol{x}_{kt}\}_{t=-1,-2,\cdots,-s}$.

The agent k decides the current ($t = 0$, no time descriptions below) expressions of demand and supply $\boldsymbol{x}_k^\lambda$ depending on the past information set and the currently observed information Θ_k. Note that the current information is affected aggregate quantity signal and that the quantity signal is directly affected by $\boldsymbol{x}_k^\lambda$. The quantity adjustment converges in each period so that the aggregate transaction quantity is determined. The agent updates their information set with the observed information, the expressions and the realized transactions to revise their expectations on quantity rationings in the following periods.

The main question of learning dynamics is whether the economy converges into REE (rational expectation equilibrium), which is same as the canonical static model above.

[12]If all the agents know all the rationings in future, the dynamic model would be a sticky-price model and no unexpected unemployment occurs; see Ginsburgh et al. (1985) for the deterministic growth model. The growth path could be interpreted as a Nash equilibrium so that only the price stickiness matters. It is difficult to justify that optimal growth path as Keynesian unemployment.

[13]This is a kind of learning dynamics; see Evans and Honkapohja (1999) for summary.

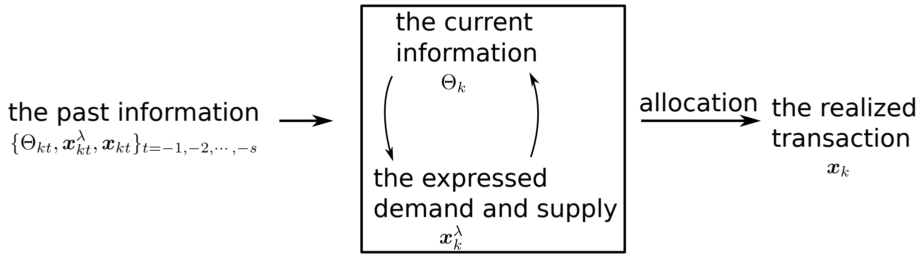

Figure 9: The process of expectation revision

This is a difficult but interesting issue of modern non-Walrasian dynamic model. The characteristic problem of non-Walrasian dynamics is that the agents should learn the recursive structures of effective demand and supply. That is, the agents learns not only the stochastic rationing mechanism on individuals (described as ϕ) but also how expressions determine the final aggregate transaction (described as g^X). As analyzed in the former section, pessimistic underestimation makes the realized transaction small. This positive feedback might bring about instability of learning dynamics and the economy moves in such divergent path which describes secular stagnation (or recession) dynamics.

5 Conclusions

In this paper, we have constructed a simple canonical model which treats stochastic rationing. The mathematical structure itself is not novel so much, but we have learned interesting features. The most important one is that the market friction could not be an alternative to Keynesian unemployment mechanism (goods demand shortage). It is another resource of unemployment and combines classical and Keynesian unemployment mechanisms. This is natural for non-Walrasian economics, but we explicitly described it with the model.

We have also revealed how expectation works in non-Walrasian economics. The quantity signals and individuals' expectations are closely interconnected, and the quantity signals play important roles in non-Walrasian economics in which quantity adjustment is a main issue. The pessimistic expectation might induce unemployment without any change in fundamental variables. To adopt our model to dynamic analysis, we should first consider about expectation. We should note that the most of existing studies on expectations are on equilibrium assumptions, in that the quantity adjustment is not important. To explore recession and stagnation, it is desirable to accumulate dynamic analysis in disequilibrium framework.

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