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Time-varying dependence structure between oil and agricultural commodity markets: A dependence-switching CoVaR copula approach

Satish Kumar ^{a*}, Aviral K. Tiwari^b, Ibrahim D. Raheem^c and Erik Hille^d
^a IBS Hyderabad (ICFAI Foundation for Higher Education), India
^b Rajagiri Business School, Rajagiri Valley Campus, Kochi, India
^c University of Kent, Canterbury, United Kingdom
^d HHL Leipzig Graduate School of Management, Leipzig, Germany
* Corresponding author: : satishwar1985@gmail.com

Abstract

We examine the energy-food nexus using the dependence-switching copula model. Specifically, we look at the dependence for four distinct market states, such as, increasing oil–increasing commodity, declining oil–declining commodity, increasing oil–declining commodity, as well as declining oil–increasing commodity markets. Our results support the argument that the crash of oil markets and agricultural commodities happen at the same time, especially during crisis period. However, the same is not true during times of normal economic conditions, implying that investors cannot make excess profits in both agricultural and oil markets at once. Furthermore, our analysis suggests that the return chasing effect dominates for all commodities on maximum occasions. The CoVaR and Δ CoVaR results indicate important risk spillover from oil to agricultural markets, especially around the financial crisis.

Keywords: Agricultural commodities; Oil; CoVaR; Dependence-switching copula; Tail dependence.

JEL Classification: C58, C63, G11, Q1, Q4.

1. Introduction

Recently, the linkages among oil and agricultural markets have increased either due to the replacement of fossil fuels with bio-fuels or hedging strategies used to combat oil price driven inflation (Jebabli et al., 2014; Ahmadi et al., 2016). Moreover, the financialization of commodities has resulted in higher level of association among oil and agricultural markets (Tang and Xiong, 2012). In order to effectively manage the risks in the portfolio, it is crucial for investors to measure dependence and tail dependence. Depending upon the direction of dependence between oil and agricultural markets, one could plausibly argue for a positive correlation regime (return chasing effect) or a negative correlation regime (portfolio rebalancing effect) (Wang et al., 2013). On the one hand, return chasing effect occurs when booming oil prices attract investors in the commodity markets, resulting in the rise of commodity prices. Hence booming oil prices coexist with high commodity prices, generating a positive correlation between them. On the other hand, a portfolio rebalancing effect is observed when crash in oil prices encourages investors to shift to agricultural commodity markets, resulting in higher commodity prices. In this case, lower oil market prices are associated with higher agricultural commodity prices, resulting in a negative correlation regime.

When the dependence between markets is analyzed, using the traditional Pearson coefficient of correlation may not be a right choice since it may underestimate the risk in case of joint severe outcomes (Tastan, 2006). To address these issues, various other models have been used such as, multivariate GARCH (M-GARCH) models (Abdelradi and Serra, 2015; Kang et al., 2017), mixture of multivariate normal distributions (Han et al., 2015). However, these models are not able to capture the asymmetric tail dependence among markets because they believe that returns are symmetrically distributed (Garcia and Tsafack, 2011). Another technique adopted to address this issue is the extreme-value approach (e.g., see, Poon et al., 2004), however, it considers asymptotic dependence between the returns, leading to a severely overestimating risk.

We analyze the time-varying dependence framework between oil and agricultural commodities using a dependence-switching CoVaR copula approach. Prior analyses have argued that a time invariant copula cannot model the real relationships, hence, they have allowed the parameters to vary in a copula function (Lourme and Maurer, 2017) or allowed the copula function to vary with time (Okimoto, 2008). Given the former framework is not indicative of the dependence switching between positive and negative correlation regimes, the latter framework is preferred.

Our work differs from Rodriguez (2007) and Okimoto (2008), who examine the contagion effect on stock market during crisis and co-movement of stock returns across countries respectively. Both suffer from two major limitations. First, they classify two different financial markets in the same regime, implying that two countries under consideration are either in good or bad condition simultaneously. This supposition might be acceptable if the co-movements in the same markets are analyzed across countries. However, it is unlikely to hold if two different markets considered, as agricultural commodities and oil

in our case. Second, their approach allows the state variables to impact factors in marginal distribution and copula function, making it hard to estimate the maximum likelihood function (Patton, 2006). Moreover, Mensi et al. (2017) study the dependence structure amid oil and food prices with a wavelet-based copula approach to analyse the transformations in the tail dependence at many investment horizons. However, they allow the coefficients in the copula function to vary over period, which is again not indicative of the dependence changing in the negative and positive correlation regimes.

Our main contribution to the literature is in showing that the correlation among oil and agricultural commodities changes in the positive and negative correlation regimes based on whether the return chasing or the portfolio rebalancing effect dominates. To do so, we use the dependence-switching copula model, proposed by Wang et al. (2013). The switching in a positive and negative correlation regime is determined by a latent conditional variable. A negative correlation regime signifies that bull (bear) oil market is coupled with a bear (bull) agricultural market. On the contrary, a positive correlation between oil and agricultural commodity markets signifies bull (bear) oil market coupled with bull (bear) agricultural commodity market. In this regard, we argue for a switch in copula functions rather than allowing the coefficients in a single copula function.

The dependence-switching copula model offers several advantages. One, we combine the Clayton copula with the Survival Clayton copula that allows for asymmetry in the tail dependence¹. Two, we attempt to capture the actual market situation, where the dependence varies over time. To do so, we let the dependence among the oil and agricultural commodities switch during positive and negative correlation regimes. Whereas, the conventional copula models examine the conditional correlation across markets only for the full sample period and, therefore, provide no room for dependence switching. Three, we can compute the dependence in tail for four varied outcomes such as rising oil–rising agricultural commodity, falling oil–falling agricultural commodity, rising oil–falling agricultural commodity, and falling oil–rising agricultural commodity prices.

Ours is the only study to have used the dependence-switching copula to examine dependence framework among oil and agricultural commodity markets². Further, based on Adrian and Brunnermeier (2016), we make use of conditional value-at-risk (CoVaR) and Δ CoVaR analysis for capturing the tail risk dependence between agricultural commodities and oil. It offers more flexibility in assessing the asset having the maximum risk as well as measuring the risk spillover among asset classes. Several empirical models have been applied to study the said relationship, such as GARCH-type models, the conventional copula models, the Markov-switching models and the extreme-value models. However, these models fail to capture the tail dependence in those four distinct market states. Moreover, none of these approaches appropriately captures whether the bear oil/bull agricultural markets and bull

¹ The Clayton and the Survival Clayton copula model the left-side and the right side tail dependence, respectively.

 $^{^{2}}$ The only exception to our study is Wang et al. (2018) which used the dependence-switching copula to understand the return–volume dependence across major international equity markets.

oil/bear commodity markets in the negative correlation regime are asymmetric; such information is crucial for investors.

Estimating a dependence-switching copula model, as done in this analysis, tends to be rather complicated, because the latent conditional variable impacts the copula and the marginal model. To address this issue, we use the instrument variable as an alternative for the latent conditional variable. We compute the marginal models using quasi-maximum likelihood approach and fit the dependence switching copula to the innovations from marginal models.

In this paper, we analyze the dependence and tail dependence for the four distinct economic scenarios mentioned above using the daily futures prices for five major agricultural commodity futures (corn, soybean oil, oats, soybeans, and wheat) and oil prices from January 3, 2002 to June 29, 2017. While we acknowledge that the variation in the different agricultural commodity futures is large, we chose those commodities that were considered by several other researchers. This allows us to evaluate the results of our paper with those of previous studies and to show how we are able to offer additional insights. Our results indicate significant left tail dependence, implying that the crash of oil and agricultural market happen almost simultaneously. On the contrary, the right tail dependence is found to be small and insignificant, implying that investors, long in oil and agricultural markets, cannot make excess profits at the same time. The analysis of the smoothing probability suggests that majority of commodities remain in the positive correlation regime for most of the time, which points out that the return chasing effect tends to be dominant. Moreover, the CoVaR and Δ CoVaR analysis provides the evidence of considerable risk spillover from oil to agricultural commodities during the crisis periods. Our results indicate that cross-market relationships must be analyzed using a time-variant copula framework which in turn has imperative implications for risk management across markets.

The remaining article is structured as. Next section describes the methodology. Section 3 highlights the data and preliminary statistics whereas Section 4 discusses the main findings. Section 5 concludes.

2. Methodology

2.1.Copula specification

This study is intended to explore the dependence pattern between the returns of oil (X_I) and agricultural commodity price indices (X_2) by a regime-switching copula approach. The bi-variate copula models are used to evaluate the combined distribution for variables having homogeneous marginal distributions (Sklar, 1959). If the combined distribution function of oil and agricultural commodity price index may be written as $F_{X_1X_2}(x_1, x_2)$, their joint distribution would be represented in the form a copula function *C* as:

$$F(X_{1,t}, X_{2,t}; \delta_1, \delta_2; \theta^c) = C(F_1(X_{1,t}, \delta_1), F_2(X_{2,t}, \delta_2); \theta^c).$$
(1)

where $F_K(X_{K,t}; \delta_K)$, K is the marginal cumulative distribution function while θ^c are the parameter sets of $F_K(X_{K,t}; \delta_K)$ and C.

Upon differentiating the cumulative distribution:

$$f(X_{1,t}, X_{2,t}; \,\delta_1, \delta_{2;} \,\theta^c) = c(u_{1,t}, u_{2,t}; \,\theta^c) \prod_{k=1}^2 f_k(X_{k,t}; \,\delta_k), \tag{2}$$

where $f(X_{1,t}, X_{2,t}; \delta_1, \delta_2; \theta^c) = \partial F^2(X_{1,t}, X_{2,t}; \delta_1, \delta_2; \theta^c) / \partial X_{1,t} \partial X_{2,t}$ is the joint density of $X_{1,t}$ and $X_{2,t}$. $u_{k,t}$ is the integral transformation of $X_{K,t}$ conditional on $F_K(X_{K,t}; \delta_K)$. $F_K(X_{K,t}; \delta_K)$ is the marginal density of $X_{k,t}$.

As explained above, the co-movement of the return series of oil price and a commodity price index could be positive (return chasing effect) or negative (portfolio rebalancing effect). One of effects can potentially be dominant at many occasions in the given sample implying that the two series under consideration may switch from the positive to negative dependence regime or vice-versa. Our framework help examine this dependence switching behaviour. Assume the below model:

$$C_{S,t}(u_{1,t}, u_{2,t}; \theta_1^C \theta_0^C) = \begin{cases} C_1(u_{1,t}, u_{2,t}; \theta_1^C), & if S_t = 1\\ C_0(u_{1,t}, u_{2,t}; \theta_0^C), & if S_t = 0 \end{cases}$$

where S_t is an latent conditional variable while $C_1(u_{1,t}, u_{2,t}; \theta_1^C)$ is the positive while $C_0(u_{1,t}, u_{2,t}; \theta_0^C)$ denote the negative dependence. This copula function combines the C^c with the C^{SC} copula as:

$$C_1(u_{1,t}, u_{2,t}; \theta_1^C) = C^C(u_{1,t}, u_{2,t}; \alpha_1) + C^{SC}(u_{1,t}, u_{2,t}; \alpha_2),$$
(3)

$$C_0(u_{1,t}, u_{2,t}; \theta_0^C) = C^C(1 - u_{1,t}, u_{2,t}; \alpha_3) + C^{SC}(1 - u_{1,t}, u_{2,t}; \alpha_4),$$
(4)

where $\theta_1^C = (\alpha_1, \alpha_2)^{\prime}, \theta_0^C = (\alpha_3, \alpha_4)^{\prime}; \ C^C(u, v, \alpha) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}, \ C^{SC}(u, v, \alpha) = (u + v - 1) + C^C(1 - u, 1 - v, \alpha) \text{ and } \alpha \in (0, \infty).$ The shape parameter, α_1 , could be changed as Kendall's τ_i , as: $\tau_i = \frac{\alpha_i}{(2 + \alpha_i)}, \rho_1 = \sin(\pi * \tau_i/2)$ and $\varphi_i = 0.5 * 2^{-1/\alpha_i}$.

 $\rho_2(\rho_3)$ computes the dependence of high oil price with rising (falling) agricultural commodity prices while $\rho_1(\rho_4)$ computes the dependence of falling oil price with falling (rising) agricultural commodity prices. Consequently, $\varphi_2(\varphi_3)$ computes the dependence of exceptionally rising oil prices with exceptionally rising (falling) agricultural commodity prices. $\varphi_1(\varphi_4)$ computes the dependence of exceptionally falling oil prices with exceptionally falling oil prices with exceptionally falling (rising) agricultural commodity prices.

The transition probability matrix for latent variable S_t may be expressed as follows:

$$p = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix}$$

where $p_{ij} = 1 - p_{11} \Pr[S_t = j | S_{t-1} = i |]$ for I, j = 0, 1 indicating that the S_t may switch from positive to negative regimes and vice-versa. Its density function in bivariate framework may be expressed as follows:

$$f(\eta_1, \eta_2, \delta_1^1, \delta_1^0, \delta_2^1, \delta_2^0, \theta_c^1, \theta_c^0) = \{ \sum_{j=0}^{1} \Pr(S_t = j) C^j(u_{1,t}, u_{2,t}; \theta_c^j) \prod_{k=1}^{2} \{ \sum_{j=0}^{1} \Pr(S_t = j) f_k(\eta_k, \delta_k^j, S_t = j) \} \}$$
(5)

Transforming Equation (5) into log-likelihood is presented as:

$$L(\vartheta) = L_c(\varphi_1) + \sum_{k=1}^2 L_k(\varphi_{2,k})$$
(6)

Where $\vartheta = (\theta_c^1, \theta_c^0, \delta_1^1, \delta_1^0, \delta_2^1, \delta_2^0, p_{11}, p_{00})$. The densities of the copula are shown as:

$$L_{c}(\varphi_{1}) = \log[\Pr(S_{t} = 1) c^{1}(u_{1}, u_{2}; \theta_{c}^{1}) + (1 - \Pr(S_{t} = 1) c^{0}((u_{1}, u_{2}; \theta_{c}^{0})],$$

$$L_{K}(\varphi_{2,k}) = \log[\Pr(S_{t} = 1) f_{k}(\eta_{k}: \delta_{k}^{1}.S_{t} = 1) + (1 - \Pr(S_{t} = 1) f_{k}(\eta_{k}, \delta_{k}^{0}.S_{t} = 0)]$$

where $\varphi_1 = (\theta_c^1, \theta_c^0, p_{11}, p_{00}).$

2.2. Marginal models

We used a Glosten, Jagannathan, and Runkle (GJR, 1993)-GARCH (p,q) specification to model the log-return time-series. Suppose a time series may be expressed as:

$$r_t = \phi + \epsilon_t$$
 ,

where ϕ and ϵ_t , respectively, are the expected return and an IID error term. Particularly, it may be believed that $\epsilon_t \sim GJR - GARCH$ if we can write $\epsilon_t = \sigma_t z_t$, where z_t is standard Gaussian.

$$\sigma_t^2 = \omega + \sum_{i=1}^q (\alpha_i \epsilon_{t-i}^2 + \gamma_i I_{t-i} \epsilon_{t-i}^2) + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$
(7)

where ϵ_{t-i} is the ARCH term, σ_{t-i}^2 is the GARCH component and

$$I_{t-i} = \begin{cases} 0 \quad if, \qquad r_{t-1} \geq \phi \\ 1 \quad if, \qquad r_{t-1} < \phi \end{cases}$$

The estimation methodology and the procedure to calculate risk and its spillover has been provided in the supplementary material to review.

3. Data and descriptive statistics

We analyze global daily futures prices for five agricultural commodities (corn, soybean oil, oats, soybeans, and wheat) and the WTI oil price³. The sample period spans from January 3, 2002 to June 29, 2017 covering instability of oil markets in summer, 2008, and

³ In contrast, many prior studies examined the relations between markets using sub-sample of countries (Hau and Roy, 2006). The findings of these studies tend to be restricted to the selected countries.

mid-2014, the 2008-2009 sub-prime crisis, and the 2010-2012 European sovereign debt crisis. The data have been extracted from Bloomberg. Fig. 1 plots the time variations of all the series under consideration.

We examine the continuous returns compounded daily as is majorly done in the financial literature. Table 1 exhibits the descriptive statistics for all time-series under consideration. The greater intensity of risks in these markets is reflected in the standard deviations, which are more than their mean for all returns series. Except for wheat and oil, the values of skewness are less than zero for every time-series. The time-series also display excess kurtosis, implying tails are asymmetric and fat. Moreover, the Jarque-Bera test significantly rejects the normality hypothesis. We use Dickey-Fuller (1979), Phillips and Perron (1988) unit root tests and the Kwiatkowski et al. (1992) stationarity test to test the stationarity for selected series. The results indicate that all return series are stationary.⁴ The correlations between oil and agricultural returns are significantly positive.

4. Main findings

4.1. Findings for the marginal distribution models

We examine various lags of GJR-GARCH (1,1) framework having *t*-distribution. The findings, shown in Table 2, reveal that the mean and the slope parameters are usually significant at the traditional levels. The Ljung–Box tests for the correlation in the standardized error terms (Q(5) and Q(10)) and the squared standardized error terms ($Q^2(5)$ and $Q^2(10)$) provide no evidence of autocorrelation. Similarly, the ARCH-LM statistic shows that ARCH effects in the estimated error terms are generally absent. Largely, it could be argued that the GJR-GARCH (1,1) is a correct specification to understand the dependence structure between agricultural commodities and oil.

4.2. Estimating the copula models

In this section, we examine many models of single-copula including the normal Gaussian copula, the Student-*t* copula, and four types of Clayton copula – the Clayton copula, the survival Clayton copula, the rotated Clayton copula, and the rotated survival Clayton copula. The results are presented in Table 3. For the Gaussian and the Student-*t* copula, the estimates (ρ) are significant for all agricultural commodities. However, in case of the single Clayton copula, either all the copula ρ 's are significant (Clayton and rotated Clayton copula or all are insignificant (half rotated Clayton copula). Out of all copulas estimated, the half rotated Clayton copula generates the highest log likelihood (LL) values, however, the Student-*t* copula generates the smallest AIC and BIC.

One of the fundamental conjectures of the student-*t* copula model lies in its symmetry in tail dependence. This implies that the tail dependence between agricultural commodity and oil market remains same when both markets are rising and when both are declining. However, such an assumption may be too restrictive. Therefore, we allow for asymmetry and observe

⁴The results are not reported here but they are available upon request.

the dependence structure between agricultural commodities and oil market using the dependence-switching copula model.

4.3. Dependence-switching copula model between agricultural commodities and oil

The findings, shown in Table 4, reveal that the parameters (α_i) are considerable for all commodities in the negative correlation regime, whereas, they are insignificant in the positive correlation regime on maximum occasions. A key outcome here is that all the log-likelihood functions are more compared to those reported in Table 3. Likewise, all the AIC and BIC estimates in Table 4 are lesser compared to single copula models. Such results further confirm our argument of employing a dependence-switching copula instead of a single copula model, in capturing the actual dependence framework between agricultural commodities and oil market. The transition probabilities (P_{11} and P_{00}) are large and significant; suggesting that both the regimes last long. Therefore, a transition among them does not happen regularly.

An additional gain from the considered copula framework lies in that it lets us study the dependence framework between two markets for four varied outcomes: namely bear oil markets coupled with bull agricultural markets; bull oil markets coupled with bear agricultural markets; bear oil markets coupled with bear agricultural markets and bull oil markets coupled with bull agricultural markets. If we observe considerable dependences in the tail, measured by φ_i , it implies greater likelihood of acute outcomes, which in turn is an indication of a larger VaR estimate compared to the normal distribution. In such a case, ignoring the tail dependence might result in underestimation of risk of losses and therefore, information about tail dependencies is crucial to compute the real VaR, subsequently, superior risk management.

A positive correlation regime represents cases when both oil and agricultural markets are either falling or rising simultaneously. While the dependence in the left tail (φ_3) represents the likelihood of big loss (bear oil markets coupled with bear agricultural markets), while the dependence in the right tail (φ_4) shows the likelihood of high gains (bull oil markets coupled with bull agricultural markets). If φ_3 (φ_4) is more, going long (short) in oil and agricultural markets will make investors witness high losses (gains). Thus, tail dependence is essential for better portfolio management.

On the one hand, the panel C shows that the parameters ρ_3 are noteworthy for corn, soybeans and wheat. Though, the corresponding tail dependence (φ_3) are very large, they are noteworthy only for oats and soybeans. On the other hand, the estimates of the right dependence ρ_4 and tail dependence φ_4 in Panel D are insignificant for all agricultural commodities, except for ρ_4 for corn. The significant left tail dependence estimates are in line with the argument that the collapse of oil markets and agricultural commodities happen simultaneously during a financial crisis period. This finding is similar to Wang et al. (2013). They show that around turmoil, the equity markets and foreign exchange markets collapse at the same time. Similarly, the largely insignificant right tail dependencies suggest that investors, long in oil and agricultural market, cannot make excess profits at the same time. We now turn to the negative correlation regime where either falling oil markets are coupled with rising agricultural markets, or rising oil markets are coupled with falling agricultural markets. The left tail dependence φ_1 represents the case of suffering large losses in oil markets but having large gains in agricultural commodity markets. The opposite is represented by the right tail dependence φ_2 . Hence, finding large $\varphi_1(\varphi_2)$ indicates the likelihood of suffering big losses for a being long (short) in oil market while short (long) in the agricultural commodity market. Panel A reveals that the left dependence estimates ρ_1 and the tail dependence estimates φ_1 are significant for each commodity. While ρ_1 ranges between 0.379 to 0.454; φ_1 lies between 0.173 to 0.223. In Panel B the right dependence values ρ_2 are significant for all commodities and the tail dependence values φ_2 are significant for soybean oil, soybeans and wheat. Thereby, ρ_2 ranges between 0.135 and 0.400, and φ_2 between 0.013 and 0.188.

With exception of soybeans, our results indicate that the values of tail dependence are larger when both oil and agricultural markets are collapsing than when both of them are rising. This suggests that if an investor is long in both the markets, the likelihood of suffering high losses (gains) at once in the oil and agricultural commodity markets tends to be larger (smaller). Out of these four market conditions, wheat (soybeans) exhibits the minimum tail dependence in case a rising (falling) oil market is coupled with rising (falling) agricultural commodity market. Hence, for the wheat (soybeans) market the systemic risk is the lowest when it is advancing (crashing) simultaneously. Furthermore, oats (corn) exhibits the least tail dependence in case a falling (rising) oil market is coupled with rising (falling) agricultural commodity markets. This implies that investors have the smallest systemic risk for oats (corn), when they are long (short) in the oil market, however, short (long) in the agricultural commodity market.

Figure 3 shows smoothing probability for a positive correlation regime, which suggests that the commodities tend to stay in the regime for a longer period of time. A partial exception is wheat. The figure also indicates the general importance of return chasing effect for the commodities. We see at least two explanations for this. First, a rise in oil prices caused by an unexpected growth of the global economy may be associated with increasing prices of agricultural commodities, because the economic expansion also entails a higher demand for food (Baumeister and Kilian, 2014). Second, increasing oil prices raise the manufacturing costs of commodities through higher carrying costs and increased prices of fertilizers, resulting in increase in the price of agricultural commodities also (Tyner, 2010). Many studies examine the relations between markets using a sub-sample (Hau and Rey, 2006). The findings in their study have a tendency to be biased to the sample choices which is not the case with our methodology. Our methodology is superior in that it investigates the correlations among oil and agricultural commodities without any restriction on the sample choice.

In particular, Figure 3 shows a positive correlation regime during times of crisis. This is in line with the reality, when, for example, during the sub-prime crisis, a collapse of both the oil and agricultural commodity prices could be observed. Similarly, Figure 3 exhibits a high positive correlation regime during the European sovereign debt crisis between 2010 and

2012. In contrast to that, the more recent plunge in oil prices around mid-2014 was not strongly coupled with decreasing agricultural commodity prices. Consequently, Figure 3 reveals a negative correlation regime for every commodity in the considered time. Overall, our dependence-switching copula model seems to correctly capture the relationship between oil and agricultural commodity markets for periods of financial turbulence.

4.4. Measuring risk spillover

After discussing the tail dependencies, we now shift the focus of the analysis to risk spillovers between the agricultural commodities and oil using the CoVaR and Δ CoVaR. Figure 4 shows the plots of downside VaR, CoVaR, and Δ CoVaR for all commodities' returns⁵. We find that the downside CoVaR is smaller in absolute values than the corresponding VaR for each commodity. This suggests the agricultural markets are more risky when extreme events occur jointly in both the agricultural and the oil market. In addition, the downside CoVaRs of the agricultural commodities tend to have downward spikes around the GFC and the ESDC. In this regard, the largest downward spike can be found for soybeans around the financial crisis. Such a finding is similar to Algieri and Leccadito (2017) who show that contagion effect of crude oil on food prices is more pronounced during crisis periods. The downward spike for corn between 2013 and 2014 may be explained by the crash in prices by almost 30 percent compared to the previous year. These findings are supported by generally larger and positive values of the Δ CoVaR for all commodities.

In Table 5, the descriptive statistics of the above mentioned risk spillover parameters are presented. The average values generally verify the results from Figure 4 that news from oil markets may add risk to the agricultural commodities. Out of all the commodities considered, oil-corn has the largest average downside CoVaR in absolute values, followed by oil-wheat. On the contrary, oil-soybean oil exhibits the smallest magnitude of the average downside CoVaR, implying the least market risk compared to other commodities. Moreover, the descending order of the Δ CoVaR values is fairly similar compared to the one of the CoVaR, particularly for corn and wheat that has the largest Δ CoVaR.

Finally, to confirm the robustness of the asymmetric risk spillover, we employ the Kolmogorov–Smirnov (K–S) test as discussed in Section 2.4. Specifically, we examine the asymmetric reaction of positive and negative commodity returns to high uncertainty in oil prices. The results are presented in Table 6. Similar to Ji et al. (2018b), for downside and upside spillover from oil to commodity markets, we consider the upside severe risk of uncertain fluctuations in oil prices. Therefore, the tests in the second and third columns of Table 6 show whether the VaR and the CoVaR for commodity returns are significantly different, restricted to the upside VaR for uncertain changes in oil prices.

⁵ We do not show the plots of Δ CoVaR for Oats and Soybeans due to the reason that its value emerges out to be very high, making the value of VaR and CoVaR insignificant in the figure. The plots of Δ CoVaR for these two commodities are available separately upon request.

As can be seen, the downside and upside CoVaRs for all commodity returns are considerably lower compared to their respective downside and upside VaRs. This implies that commodity returns are more responsive to large uncertain changes in the oil price, i.e., rising uncertainty will affect the commodity returns negatively. The asymmetric measures of the upside and downside CoVaRs based on the upside severe fluctuation of oil price changes, suggest that the extent of the downside risk spillover of commodity returns is considerably higher compared to the upside risk spillover for corns, soybean oil and wheat.

5. Concluding remarks

We analyse the dependence structure between the oil and five agricultural commodities through the dependence-switching copula. It advances prior research as it facilitates the analysis of dependence and tail dependence for four distinct states, i.e., bull and bear oil markets associated with bull and bear agricultural commodity markets. We estimate these dependencies between oil and agricultural commodities using the daily prices from January 3, 2002 to June 29, 2017. Furthermore, we compute the risk spillover from oil to agricultural commodities using the CoVaR framework.

The results point out considerable dependence in the left tail, which is in line with the argument that the collapse of oil markets and agricultural commodities happen at the same time during turmoil periods. During times of normal economic conditions, the estimated insignificant right tail dependencies suggest that it is highly unlikely to earn excess returns in these two markets concurrently. The plots of smoothing probability indicate that, with the exception of wheat, the commodities generally remain in the positive correlation regime, implying that the return chasing effect is highly relevant for the analyzed commodities. The CoVaR and Δ CoVaR show that the intensity of spillover from oil to commodities is higher during the turbulent times of the 2008 sub-prime crisis, when the CoVaRs sharply declined for all commodities. Hence, the contagious effect of oil on commodities is highly dominant during the periods of financial and economic shocks. Interestingly, with only few exceptions, the CoVaRs and Δ CoVaRs of all commodities show a similar plot. Overall, ignoring the significance of tail dependence might result in underestimation of the risk of losses. Therefore, the tail dependence are central for the evaluation of actual risk and a better portfolio management.

Using regime-dependence copula we could offer several interesting results that were not, or only, partly uncovered in previous studies. For instance, Mensi et al. (2014) show a significant linkage between oil and cereal markets (barley, sorghum, Wheat and corn) while we discover the significant evidence of left tail dependence. Moreover, Koirala et al. (2015) detect a linear positive relationship and high correlation in the nexus, while we further explore the issue in our analysis and find that crash of oil markets and agricultural commodities happen simultaneously during a financial crisis period. Jiang et al. (2018) estimated that oil prices lag agricultural prices and that the lower dependencies between the markets are more significant than the upper dependencies. We contribute to the literature by finding the evidence of positive correlation regime for most of the agricultural commodities and thus, a largely dominant return chasing effect. Our results offer significant useful implications for risk management across markets. We recommend that when both agricultural and oil commodity markets collapse, market participants should consider other commodities. Put differently, when oil and agricultural markets are crashing, investors ought to expand their portfolio to other markets than these two. On the flip side, the likelihood of earning more returns, when these two markets are booming, are practically absent. Finally, when oil and agricultural commodity markets are either rising or declining, the likelihood of suffering losses is more than achieving gains simultaneously.

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Table	1
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Summary s	statistics
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	Mean	Med	Max	Min	SD	Skew	Kurt	J-B	Correlation	Ν
Corn	0.014	0.000	12.76	-26.86	1.91	-0.58	15.40	25151.4***	0.22***	3892
Soybean Oil	0.019	0.000	14.36	-18.82	1.64	-0.46	14.75	22522.3***	0.33***	3892
Oats	0.007	0.000	25.69	-28.19	2.49	-0.81	22.04	59245.1***	0.12***	3892
Soybeans	0.020	0.087	20.32	-25.02	1.77	-1.43	28.14	103814***	0.25***	3892
Wheat	0.014	0.000	13.56	-11.95	2.06	0.17	5.286	865.78***	0.20***	3892
Oil	0.020	0.059	16.41	-13.07	2.39	0.08	6.928	2506.6***		3892

Note: J-B stands for the Jarqe-Bera test of normality. *** indicates significant at 1% level of significance.

Results of the mai	igiliai uisuin	Jution models				
	Corn	Soybean Oil	Oats	Soybeans	Wheat	Oil
Mean constant	0.007	0.029	0.035	0.050**	0.007	0.010
	(0.800)	(0.193)	(0.305)	(0.025)	(0.811)	(0.738)
Variance constant	0.041***	0.042***	0.232***	0.030***	0.023**	0.022**
	(0.002)	(0.005)	(0.006)	(0.001)	(0.026)	(0.024)
ARCH	0.069***	0.045***	0.103***	0.076***	0.039***	0.020***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)
GARCH	0.927***	0.932***	0.895***	0.943***	0.963***	0.948***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
GJR	-0.010	0.009	-0.054***	-0.053***	-0.016**	0.058***
	(0.492)	(0.418)	(0.006)	(0.000)	(0.044)	(0.000)
Asymmetry	0.018	0.077***	-0.012	-0.064***	0.092***	-0.069***
	(0.438)	(0.001)	(0.523)	(0.003)	(0.000)	(0.003)
Tail	5.039***	8.486***	3.546***	5.024***	7.918***	10.013***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
AIC	3.904	3.584	4.328	3.626	4.145	4.298
Shibata	3.904	3.584	4.328	3.626	4.145	4.298
SIC	3.915	3.595	4.339	3.637	4.156	4.309
Hannan-Quinn	3.908	3.588	4.332	3.630	4.149	4.302
Q(5)	(0.782)	(0.527)	(0.010)**	(0.568)	(0.934)	(0.371)
Q(10)	(0.223)	(0.839)	(0.222)	(0.349)	(0.790)	(0.621)
$Q^{2}(5)$	(0.662)	(0.503)	(0.212)	(0.012)*	(0.161)	(0.145)
Q ² (10)	(0.976)	(0.090)*	(0.546)	(0.126)	(0.307)	(0.197)
ARCH-LM	(0.813)	(0.300)	(0.628)	(0.105)	(0.143)	(0.201)

Table 2Results of the marginal distribution models

Note: The numbers in parentheses are p-values. *, ** and *** indicates significance at the 10%, 5% and 1% level of significance.

Single-copula	mouchs. On and agi		ing futures		
	Corn	Soybean Oil	Oats	Soybeans	Wheat
Normal copula	ı				
ρ	0.201***	0.315***	0.122***	0.228***	0.152***
SE	(0.015)	(0.014)	(0.016)	(0.015)	(0.016)
LL	-80.293	-202.77	-29.144	-104.27	-45.258
AIC	-158.59	-403.53	-56.289	-206.55	-88.516
BIC	-152.32	-397.26	-50.022	-200.28	-82.250
Student-t copu	la				
ρ	0.204***	0.320***	0.124***	0.231***	0.153***
SE	(0.016)	(0.015)	(0.017)	(0.015)	(0.016)
DoF	31.695***	13.561***	12.119***	25.760***	15.653***
SE	(16.540)	(3.178)	(2.819)	(10.131)	(4.360)
LL	-82.300	-214.02	-39.664	-107.43	-52.858
AIC	-162.60	-426.04	-77.327	-212.86	-103.72
BIC	-156.33	-419.78	-71.061	-206.59	-97.449
Clayton(u, v)					
α	0.218***	0.383385***	0.136***	0.258***	0.157***
SE	(0.022)	(0.024)	(0.020)	(0.022)	(0.021)
LL	-62.260	-165.68	-27.787	-85.897	-35.401
AIC	-122.52	-329.36	-53.573	-169.79	-68.802
BIC	-116.25	-323.10	-47.307	-163.53	-62.536
Rotated Clayto	on copula (with tail de	ependence in upper	r rather than the l	ower tail): Clay	ton(1-u, 1- v)
α	0.206***	0.3614***	0.121***	0.236***	0.161***
SE	(0.022)	(0.024)	(0.020)	(0.022)	(0.021)
LL	-55.708	-147.58	-22.211	-70.781	-37.457
AIC	-109.42	-293.15	-42.421	-139.56	-72.914
BIC	-103.15	-286.89	-36.155	-133.30	-66.648
Rotated Clayto	on copula (half rotated	d): Clayton(1 - u, v	,)		
α	0.000	0.000	0.000	0.000	0.000
SE	(0.020)	(0.022)	(0.018)	(0.021)	(0.019)
LL	0.060	0.089	0.034	0.070	0.045
AIC	2.120	2.178	2.067	2.139	2.090
BIC	8.387	8.445	8.334	8.406	8.357
Rotated Clavto	on copula (half rotated	d): Clavton(u. 1 - v	,)		
α	0.000	0.000	0.000	0.000	0.000
SE	(0.018)	(0.020)	(0.017)	(0.018)	(0.017)
LL	0.057	0.088	0.032	0.062	0.039
AIC	2.114	2.177	2.063	2.125	2.078
BIC	8 381	8 443	8 330	8 391	8 345
	0.501	0.115	0.550	0.571	0.545

 Table 3

 Single-copula models: oil and agricultural commodity futures

Note: DoF stands for the degrees of freedom. *, ** and *** indicates significance at 10%, 5% and 1% level.

Table	4
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Results of the	e dependence-swi	itching copula me	odel.		
	Corn	Soybean Oil	Oats	Soybeans	Wheat
A negative cor	relation regime – l	Panel A and Panel	B		
Panel A: Bear	oil markets couple	ed with bull agricu	ltural markets		
α.	0.657***	0.858***	0.653***	0.746***	0.744***
u1	(0.226)	(0.090)	(0.129)	(0.103)	(0.217)
0	0.379***	0.454***	0.377***	0.414***	0.413***
p_1	(0.093)	(0.031)	(0.053)	(0.039)	(0.082)
$arphi_1$	0.174***	0.223***	0.173***	0.197***	0.197***
	(0.063)	(0.019)	(0.036)	(0.025)	(0.053)
Panel B: Bull o	oil markets coupled	d with bear agricu	ltural markets		
a	0.189**	0.709***	0.354***	0.471***	0.538***
u_2	(0.080)	(0.100)	(0.114)	(0.055)	(0.165)
	0.135***	0.400***	0.234***	0.295***	0.327***
$ ho_2$	(0.052)	(0.039)	(0.063)	(0.027)	(0.076)
φ_2	0.013	0.188***	0.071	0.115***	0.138***
	(0.020)	(0.026)	(0.044)	(0.020)	(0.054)
A positive corr	relation regime – P	Panel C and Panel	D		
Panel C: Bear	oil markets couple	ed with bear agrici	ultural markets		
~	-0.092	-0.046	-0.018	-0.161***	-0.131***
u_3	(0.078)	(0.051)	(0.043)	(0.000)	(0.042)
0	-0.076*	-0.037	-0.015	-0.137***	-0.110***
P_3	(0.067)	(0.042)	(0.034)	(0.000)	(0.038)
φ_3	935.14	1701923	9.81E+15***	37.107***	98.398
	(5962.0)	(28566918)	(0.432)	(0.001)	(168.10)
Panel D: Bull	oil markets couple	d with bull agricul	tural markets		
<i>a</i>	-0.113*	-0.090*	-0.028	-0.047	0.032
uų	(0.058)	(0.054)	(0.042)	(0.068)	(0.048)
04	-0.094*	-0.074	-0.022	-0.038	0.025
P4	(0.051)	(0.047)	(0.034)	(0.056)	(0.037)
$arphi_4$	233.80	1121.4	4.28E+10	1112306	0.000
	(745.14)	(5229.1)	(1.64E+12)	(23411588)	(0.000)
Panel E: Regime Switching					
P_{11}	0.999***	0.999***	0.999***	0.998***	0.998***
P_{00}	0.994***	0.997***	1.000***	0.995***	0.995***
LL	8358	8175	8387	8315	8381
AIC	-16756	-16391	-16815	-16669	-16801
BIC	-16881	-16516	-16940	-16794	-16927

Results of the dependence-switching copula model

Note: P_{11} and P_{00} are the two transition probabilities. *, ** and *** show the significance at the 10%, 5% and 1% level.



Figure 1: The returns series for oil and five agricultural commodities from January 3, 2002 to June 29, 2017





Figure 2: Smoothing probability during positive correlation regime





Figure 3: Smoothing correlation coefficients of agricultural commodities with oil.





Soybean Oil





Figure 4:CoVaRs and Δ CoVaRs of agricultural commodities with oil

	Corn	Soybean Oil	Oats	Soybeans	Wheat
	-3.0673	-2.5091	-4.0127	-2.6734	-3.3101
VaR	(-199.00)	(-184.25)	(-205.59)	(-179.44)	(-256.31)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
	-11.4538	-5.5709	-7.8140	-6.7365	-8.2935
CoVaR	(-82.336)	(-64.432)	(-73.220)	(-55.025)	(-93.041)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
	17.846	12.833	-24571	2.100	17.368
∆CoVaR	(183.35)	(88.90)	(-1.0043)	(0.0400)	(159.41)
	[0.0000]	[0.0000]	[0.3153]	[0.9681]	[0.0000]

Fable 5	
Descriptives of the VaR, CoVaR and △CoVaF	2

Note: The numbers in the brackets () and square brackets [] are *t*-statistics and *p*-values, respectively.

Table 6

Results for the risk spillover using the CoVaRs

	Downside risk spillover	Unside risk spillover	Asymmetry conditional
	Downside fisk spinover	Opside fisk spillover	on extreme movement
			$H \cdot \frac{CoVaR_{1 2U}^{D,\alpha}}{CoVaR_{1 2U}^{D,\alpha}} - \frac{CoVaR_{1 2U}^{U,\alpha}}{CoVaR_{1 2U}^{U,\alpha}}$
	H_{01} : $CoVaR_{1 2U}^{D,\alpha} = VaR_1^{D,\alpha}$	H_{02} : $CoVaR_{1 2U}^{U,\alpha} = VaR_{1}^{U,\alpha}$	$\frac{1}{VaR_1^{D,\alpha}} = \frac{1}{VaR_1^{U,\alpha}}$
	$H_{11}: CoVaR_{1 2U}^{D,\alpha} < VaR_1^{D,\alpha}$	$H_{12}: CoVaR_{1 2U}^{U,\alpha} < VaR_1^{U,\alpha}$	$CoVaR_{1 2U}^{D,\alpha} CoVaR_{1 2U}^{U,\alpha}$
			$H_{13}: \frac{VaR_1^{D,\alpha}}{VaR_1} > \frac{VaR_1^{U,\alpha}}{VaR_1}$
Com	0.6259***	0.6405***	0.0308**
Com	[0.0000]	[0.0000]	[0.0247]
Soybean	0.7251***	0.7811***	0.1842***
Oil	[0.0000]	[0.0000]	[0.0000]
Oats	0.8368***	0.8371***	0.0036
Oats	[0.0000]	[0.0000]	[0.9509]
Soubeans	0.5758***	0.5504***	0.0000
Soybeans	[0.0000]	[0.0000]	[1.0000]
Wheat	0.7079***	0.7518***	0.1570***
wneat	[0.0000]	[0.0000]	[0.0000]

Note: ** and *** denote 5% and 1% level of significance, respectively.