Technical progress and involuntary unemployment under deflation with real balance effect and fiscal policy for full-employment

Tanaka, Yasuhito

19 March 2021

Technical progress and involuntary unemployment under deflation with real balance effect and fiscal policy for full-employment

Yasuhito Tanaka
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
E-mail: yatanaka@mail.doshisha.ac.jp

Abstract

We study the steady state with involuntary unemployment and fiscal policy to realize full-employment in a situation with technical progress. Under involuntary unemployment the nominal wage rate may decline. Then, the prices of the goods also decline, and the real balance effects work. In a three-generations OLG model of this paper consumptions in the childhood period are financed by borrowing money from the previous generation consumers, and these debts must be repaid in the next period. In such a model there may exist positive or negative real balance effect of decline of the nominal wage rate and the prices. Among others we show the following results. If the deflation (nominal wage rate decline) rate is equal to the technical progress rate, in order to maintain a steady state with constant employment a balanced budget is required. If the deflation rate is smaller than the technical progress rate and there exists a positive (or negative) real balance effect, in order to maintain a steady state with constant employment a budget deficit (surplus) is required. Also we show that fiscal policy to realize full-employment usually requires larger budget deficit. These budget deficits, including those for maintaining full-employment, should be financed by seigniorage not by public debt. If they are financed by public debts, they do not have to be repaid. Conversely, the budget surplus in some cases should not be returned to consumers as tax reduction.

Key Words: Involuntary unemployment, Three-periods overlapping generations model, Technical progress, Deflation, Real balance effect.

JEL Classification: E12, E24.

1 Introduction

Involuntary unemployment is a phenomenon that workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. Umada(1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. Otaki(2009) assumes indivisibility of labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow(1985). The arguments of this paper do not depend on bargaining. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. As discussed by Otaki(2015) (Theorem 2.3) and Otaki(2012), if labor supply is divisible and very small, no unemployment exists[2]. However, we show that even if labor supply is divisible, unless it is so small, there may exist involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015). We extend Otaki's model to a three-generations OLG model with a childhood period, and also we consider pay-as-you-go pension system for the older generation consumers.

In this paper we study the steady state with involuntary unemployment and fiscal policy to realize full-employment in a situation with technical progress. Under involuntary unemployment the nominal wage rate may decline. Then, the prices of the goods also decline, and the real balance effects (or Pigou effects) work. In our three-generations OLG model consumptions in the childhood period are financed by borrowing money from the previous generation consumers, and these debts must be repaid in the next period. In such a model there may exist positive or negative real balance effect of decline of the nominal wage rate and the prices. If the savings of the older generation consumers net of the pay-as-you-go pensions are larger (or smaller) than the debts due to consumptions in the childhood period, there exists positive (or negative) real balance effect. We will show the following results in Propositions 2, 3 and 4 in Section 3.

1. If the deflation (nominal wage rate decilen) rate is equal to the technical progress rate, in order to maintain a steady state with constant employment a balanced budget is required.
2. If the deflation rate is larger than the technical progress rate and there exists a positive (or negative) real balance effect, in order to maintain a steady state with constant employment a budget surplus (or deficit) is required.
3. If the deflation rate is smaller than the technical progress rate and there exists a positive (or negative) real balance effect, in order to maintain a steady state with constant employment a budget deficit (surplus) is required.

[1] Lavoie (2001) presented a similar analysis.

[2] About indivisible labor supply also please see Hansen (1985). In Tanaka (2020a, 2020b, 2020c) involuntary unemployment under indivisible labor supply is analyzed.

Also we show that fiscal policy to realize full-employment usually requires larger budget deficit or smaller budget surplus (Proposition 5). These budget deficits, including those for maintaining full-employment, should be financed by seigniorage not by public debt. If they are financed by public debts, they do not have to be repaid. Conversely, the budget surplus in some cases should not be returned to consumers as tax reduction. From Propositions 2, 3 and 4, after realization of full-employment, necessary budget deficit or budget surplus returns to the value which maintains the steady state with full-employment.

In the next section we explain the model and show the existence of involuntary unemployment due to deficiency of demand.

2 Existence of involuntary unemployment

2.1 Consumers

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki(2007, 2009, 2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with constant returns to scale technology.
2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.
3. During Period 1, consumers supply l units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.
4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.
5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

Further we make the following assumptions

Ownership of the firms Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers.

Zero interest rate We assume zero interest rate, and that repayment of the debts of consumers in their childhood period is assured. Consumer borrowing in childhood period is

constant. If the savings of consumers in the younger period are insufficient for the borrowing, the government lends the scholarship to consumers in the childhood period. Consumers in the younger period are indifferent between lending money to childhood period consumers and savings by money.

Due to the existence of pay-as-you-go pension the savings is likely to be insufficient for borrowing when consumption by consumers in the childhood period is not so small.

Notation We use the following notation.

C_i^e: consumption basket of an employed consumer in Period $i,\ i = 1,2$.
C_i^u: consumption basket of an unemployed consumer in Period $i,\ i = 1,2$.
$c_i^e(z)$: consumption of good z of an employed consumer in Period $i,\ i = 1,2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period $i,\ i = 1,2$.
D: consumption basket of an individual in the childhood period, which is constant.
P_i: the price of consumption basket in Period $i,\ i = 1,2$.
$p_i(z)$: the price of good z in Period $i,\ i = 1,2$.
$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\widehat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\widehat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
y: labor productivity, which increases by technical change.

We assume that the population L_f is constant. We also assume that the nominal wage rate is constant in this section,. We examine the effects of a change in the nominal wage rate in Section 3.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

Since the taxes for unemployed consumers' unemployment benefits are paid by employed consumers of the same generation, $D(=R)$ and Θ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over three periods is written as

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \ i = 1,2,$$

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \ i = 1,2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}, \ i = 1,2.$$

The budget constraint for en employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

The budget constraint for en unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q}$$

Since $R = D$,

$$P_1 C_1^u + P_2 C_2^u = \Pi + \hat{Q}.$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e}, \ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}.$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

$$1-\alpha=\frac{P_2C_2^e}{P_1C_1^e+P_2C_2^e}=\frac{P_2C_2^u}{P_1C_1^u+P_2C_2^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e=\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},$$

$$C_2^e=(1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2},$$

and

$$C_1^u=\alpha\frac{\Pi+\hat{Q}}{P_1},\ C_2^u=(1-\alpha)\frac{\Pi+\hat{Q}}{P_2}.$$

Solving maximization problems in Step 2, the following demand functions of employed and unemployed consumers are derived[3].

$$c_1^e(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1},$$

$$c_2^e(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_2},$$

$$c_1^u(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+\hat{Q})}{P_1},$$

and

$$c_2^u(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(\Pi+\hat{Q})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e=u\left(\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},(1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2},D\right)-\Gamma(l),$$

and

$$V^u=u\left(\alpha\frac{\Pi+\hat{Q}}{P_1},(1-\alpha)\frac{\Pi+\hat{Q}}{P_2},D\right).$$

Let

$$\omega=\frac{W}{P_1},\ \rho=\frac{P_2}{P_1}.$$

Then, since the real value of D in the childhood period is constant, we can write

$$V^e=\varphi\left(\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},\rho\right)-\Gamma(l),$$

$$V^u=\varphi\left(\frac{\Pi+\hat{Q}}{P_1},\rho\right),$$

ω is the real wage rate. Denote

$$I=\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}.$$

The condition for maximization of V^e with respect to l given ρ is

[3] Calculations of the maximization problems in Step 2 are standard using Lagrange multiplier method. They are available upon request.

$$\frac{\partial \varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{1}$$

where

$$\frac{\partial \varphi}{\partial I} = \alpha \frac{\partial u}{\partial c_1^e} + (1-\alpha)\frac{\partial u}{\partial c_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial \varphi}{\partial I} + \frac{\partial^2 \varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2 \varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate ω.

2.2 Firms

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - LD - L\Theta + L_f\hat{Q} - L\Psi)}{P_1}$$
$$= \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ.$$

Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{Y}{P_1}, \tag{2}$$

where Y is the effective demand defined by

$$Y = \alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits (see Otaki(2007), Otaki(2015) about this demand function).

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are

$$\int_0^1 Ldz = L,\ \int_0^1 Lldz = Ll.$$

The output of firm z is Lly. At the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial(Ll)} = y.$$

From (2)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial(Ll)} = -\frac{p_1(z)y}{\sigma d(z)} = -\frac{p_1(z)y}{\sigma Lly}.$$

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial\pi(z)}{\partial(Ll)} = p_1(z)y - Lly\frac{p_1(z)y}{\sigma Lly} - W = p_1(z)y - \frac{p_1(z)y}{\sigma} - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{(1-\frac{1}{\sigma})y}W.$$

Let $\mu = \frac{1}{\sigma}$. Then,

$$p_1(z) = \frac{1}{(1-\mu)y}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)y}W. \tag{3}$$

2.3 Involuntary unemployment

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly.$$

The (nominal) aggregate demand is

$$\alpha\left(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M$$
$$= \alpha\left[P_1Lly - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M.$$

Since they are equal,

$$P_1Lly = \alpha\left[P_1Lly - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M. \tag{4}$$

In real terms[4]

$$Lly = \frac{\alpha(-L_f D + L_f \hat{Q} - L_f Q) + G + L_f \hat{D} + M}{(1-\alpha)P_1}.$$

The equilibrium value of Ll cannot be larger than $L_f l$. However, it may be strictly smaller than $L_f l$. Then, we have $L < L_f$ and involuntary umemployment exists.

If the government collects a lump-sum tax T from the younger generation consumers, (4) is rewritten as

$$P_1 Lly = \alpha\left[P_1 Lly - T - L_f D + L_f \hat{Q} - L_f Q\right] + G + L_f \hat{D} + M. \quad (5)$$

3 Steady states and fiscal policy

3.1 Constant employment and output under ongoing deflation

If there exists involuntary unemployment, the nominal wage rate may decline. Then, from (3) the prices of the goods also decline.

First as a benchmark we consider a steady state where employment and output are constant under *ongoing* deflation (nominal wage rate decline). Suppose that the prices of the goods decline at the rate $\rho - 1 < 0$, and the price change is correctly predicted by consumers and the government. Then, we can assume that $\hat{D} = \rho D$, $\hat{Q} = \rho Q$, and (5) is rewritten as

$$P_1 Lly = \alpha\left[P_1 Lly - T - L_f D + L_f \rho Q - L_f Q\right] + G + L_f \rho D + M.$$

From this

$$(1-\alpha)\left[P_1 Lly - T - L_f D + (\rho - 1)L_f Q\right] = G - T + (\rho - 1)L_f Q + (\rho - 1)L_f D + M.$$

This is the savings of the younger generation consumers. It should be equal to ρM to maintain the steady state. Therefore,

$$G - T = (\rho - 1)[M - L_f Q - L_f D].$$

We get the following result.

Proposition 1

1. **(Positive real balance effect case)** If $M > L_f D + L_f Q$, in order to maintain a state where the output and the employment are constant with falling prices ($\rho < 1$), a budget surplus ($G < T$) is required. In this case there exists positive real balance effect because the net savings of the older generation consumers ($M - L_f Q$) is larger than the debts of the younger generation consumers ($L_f D$).
2. **(Negative real balance effect case)** If $M < L_f D + L_f Q$, in order to maintain a state where the output and the employment are constant with falling prices ($\rho < 1$), a budget deficit ($G > T$) is required. In this case there exists negative real balance effect because the net savings of the older generation consumers ($M - L_f Q$) is smaller than the debts of the younger generation consumers ($L_f D$).

[4] $\frac{1}{1-\alpha}$ is a multiplier.

3.2 Ongoing technical progress under ongoing deflation

Suppose that the prices of the goods decline at the rate $\rho - 1 < 0$, the labor productivity increases at the rate $\gamma - 1 > 0$; and the employment L is constant. These changes are correctly predicted by consumers and the government. We can assume that $\widehat{D} = \gamma\rho D$, $\hat{Q} = \gamma\rho Q$, and (5) is rewritten as

$$P_1 Lly' = \alpha\left[P_1 Lly' - T - L_f D + L_f \gamma\rho Q - L_f Q\right] + G + L_f \gamma\rho D + M,$$

where $y' = \gamma y$. From this

$$(1-\alpha)\left[P_1 Lly' - T - L_f D + (\gamma\rho - 1)L_f Q\right] = G - T + (\gamma\rho - 1)L_f Q + (\gamma\rho - 1)L_f D + M. \quad (6)$$

It should be equal to $\gamma\rho M$ to maintain the steady state. Therefore,

$$G - T = (\gamma\rho - 1)[M - L_f Q - L_f D].$$

We obtain the following results.

Proposition 2 (Equal deflation rate and technical progress rate) *If $\gamma\rho = 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a balanced budget ($G = T$) is required. In this case we say that the rate of deflation and the rate of technical progress are equal.*

Proposition 3 (Positive real balance effect case) *Assume $M > L_f D + L_f Q$, that is, there exists positive real balance effect. We have two cases.*

1. If $\gamma\rho < 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget surplus ($G < T$) is required. In this case the rate of deflation is larger than the rate of technical progress.
2. If $\gamma\rho > 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget deficit ($G > T$) is required. In this case the rate of deflation is smaller than the rate of technical progress.

Proposition 4 (Negative real balance effect case) *If $M < L_f D + L_f Q$, that is, there exists negative real balance effect, we obtain the following results.*

1. If $\gamma\rho < 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget deficit ($G > T$) is required.
2. If $\gamma\rho > 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget surplus ($G < T$) is required.

3.3 Fiscal policy for full-employment under deflation with technical progress

Suppose that full-employment is realized in Period 1. We assume again that $\widehat{D} = \gamma\rho D$, however, about $\hat{Q}$ we assume $\hat{Q} = \gamma Q$ because the prices dot not fall after realization of full-employment. Let G' and T' be the government expenditure and the tax in this case. Then, (5) is rewritten as

$$P_1 L_f l y' = \alpha\left[P_1 L_f l y' - T' - L_f D + L_f \gamma Q - L_f Q\right] + G' + L_f \gamma \rho D + M.$$

From this

$$(1-\alpha)\left[P_1 L_f l y' - T' - L_f D + (\gamma - 1) L_f Q\right] = G' - T' + (\gamma - 1) L_f Q + (\gamma\rho - 1) L_f D + M. \quad (7)$$

Note that M is the savings of the older generation consumers. Comparing (6) and (7), we find that if

$$(1-\alpha)\left[\left(P_1 L_f l y' - T'\right) - \left(P_1 L l y' - T\right)\right] - \alpha\gamma(1-\rho) L_f Q > 0, \quad (8)$$

we have

$$G' - T' > G - T.$$

We have shown the following proposition.

Proposition 5 *If (8) holds, we need larger budget deficit (or smaller budget surplus) to realize full-employment than that to maintain constant employment.*

$P_1 L_f l y' - T'$ and $P_1 L l y' - T$ are the disposable income after and before full-employment. Thus, (8) holds when the disposable income increases by realization of full-employment and the deflation rate is not so large. The term $-\alpha\gamma(1-\rho) L_f Q$ exists because the consumers correctly predict that the prices of the goods no longer fall after full-employment is realized . If they have the expectation that prices will continue to fall, this term disappears.

These budget deficits, including those for maintaining full-employment in Proposition 3 and 4, should be financed by seigniorage not by public debt. If they are financed by public debts, they do not have to be repaid. Conversely, the budget surplus in some cases should not be returned to consumers as tax reduction. From Propositions 2, 3 and 4, after realization of full-employment, necessary budget deficit or budget surplus returns to the value which maintain the steady state with full-employment.

4 Concluding Remark

We have examined the steady state with involuntary unemployment and fiscal policy to realize full-employment under deflation due to involuntary unemployment with technical progress. We assumed that the goods are produced by only labor. In future research, we want to analyze involuntary unemployment and fiscal policy in a situation where goods are produced by capital and labor, and there exist investments of firms.

Acknowledgment

This work was supported by the Japan Society for the Promotion of Science KAKENHI (Grant Number 18K01594).

References

G. D. Hansen. Indivisible labor and business cycle. *Journal of Monetary Economics*, 16:309–327, 1985.

M. Lavoie. Efficiency wages in Kaleckian models of employment. *Journal of Post Keynesian Economics*, 23:449–464, 2001.

I. M. McDonald and R. M. Solow. Wage bargaining and employment. *American Economic Review*, 71:896–908, 1981.

M. Otaki. The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 96:23–29, 2007.

M. Otaki. A welfare economics foundation for the full-employment policy. *Economics Letters*, 102:1–3, 2009.

M. Otaki. The aggregation problem in the employment theory: The representative individual model or individual employees model? (https://www.scirp.org/journal/paperinformation.aspx?paperid=25920). *Theoretical Economics Letters*, 2:530–533, 2012.

M. Otaki. *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer, 2015.

Y. Tanaka. Indivisible labor supply and involuntary unemployment: monopolistic competition model. *The Singapore Economic Review*, forthcoming, 2020.

Y. Tanaka. Involuntary unemployment and fiscal policy for full-employment. *Theoretical Economics Letters*, 10(4):745–757, 2020.

Y. Tanaka. Involuntary unemployment with indivisible labor supply under perfect competition. *Economics Bulletin*, 40:1914–1923, 2020.

T. Umada. On the existence of involuntary unemployment (hi-jihatsuteki-shitsugyo no sonzai ni tsuite (in Japanese)). *Yamaguchi Keizaigaku Zasshi*, 45(6):61–73, 1997.