The interaction of forward guidance in a two-country new Keynesian model

Ida, Daisuke and Iiboshi, Hirokuni

Faculty of Economics, Momoyama Gakuin University, Japan, Graduate School of Management, Tokyo Metropolitan University, Japan

22 March 2021

The interaction of forward guidance in a two-country new Keynesian model*

Daisuke Ida[†] and Hirokuni Iiboshi[‡]

March 22, 2021

Abstract

Using the method of Haberis and Lipińska (2020), this paper explores the effect of forward guidance (FG) in a two-country New Keynesian (NK) economy under the zero lower bound (ZLB). We simulate the effect of different length of FG or the zero interest rate policy under the circumstance of the global liquidity trap. We show that the size of the intertemporal elasticity of substitution plays an important role in determining the *beggar-thy-neighbor effect* or the *prosper-thy-neighbor effect* of home FG policy on the foreign economy. And in the former case, by targeting a minimum welfare loss of the individual country alone but not global welfare loss, two central banks can perform interesting FG bargaining in which they cooperatively adopt the same length of FG or strategically deviate from cooperation.

JEL codes: E52; E58; F41

Keywords : Forward guidance; Zero lower bound on nominal interest rates; Two country new-Keynesian model; Taylor rule

*We would like to thank Takashi Kano, Nao Sudo, and the conference participants at the autumn meeting of JEA 2020 for their helpful comments and suggestions. Ida was supported by JSPS KAKENHI Grant Number 16H03618 and 20K01784. Iiboshi was supported by JSPS KAKENHI Grant Numbers 18K01575 and 19H01494.

†Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. Tel.: +81-725-54-3131. E-mail: ida-dai@andrew.ac.jp

‡Graduate School of Management, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan. Tel.: +81-6-6381-8434. E-mail: iiboshi@tmu.ac.jp

1 Introduction

Since the late 1990s, central banks in advanced economies have conducted monetary policy based on the presumption that they would face the zero lower bound (ZLB) on nominal interest rates due to the observed disinflation and slowdown in productivity. In fact, since the global financial crisis triggered by the collapse of Lehman Brothers in 2008 and the European sovereign debt crisis in 2010, central banks have been implementing so-called unconventional monetary policies such as the zero interest rate policy (ZIP) and quantitative easing (QE) and have experienced the ZLB during this period. In the course of experiencing the ZLB for more than a decade, central banks have been considering when and how to exit the ZLB, as well as how long unconventional monetary policy should be continued for economic recovery. Recently, in order to properly understand and implement unconventional monetary policy, policymakers and central bankers have been focusing on forward guidance (FG) as a new monetary policy instrument. Moreover, due to the COVID-19 pandemic in 2020, the Fed announced that it will maintain the ZLB until 2023. As described, the study of FG policy is a serious issue of how to stimulate and recover the economy under the ZLB constraint.

This paper examines whether and to what extent FG policy is affected in a two-country New Keynesian (NK) model when both countries have different length of FG and ZIP policies in the context of a global liquidity trap. There is a growing body of literature that discusses the mechanism and effectiveness of FG in NK models in closed economies. On the other hand, there seems to be a limited number of papers that discuss how FG in one country spills over to other countries and how the international transmission mechanism of monetary policy through exchange rate changes works (Haberis and Lipińska, 2020; Jones, Kulish and Rees, 2018). In this paper, we use the NK model, which assumes two countries of roughly equal economic size, to analyze the international spillover effects of the adoption of FG policies when both countries face a zero interest rate constraint from the perspective of welfare comparison. The model used in this paper is the standard two-country NK model by Clarida, Galı and Gertler (2002), and we assume that the two countries face zero interest rate constraints simultaneously. As a numerical method, we adopt the Occasionally Binding constraint tool (OccBin) developed by Guerrieri and Iacoviello (2015), following the FG study by Haberis and Lipińska (2020). Thus, we will examine the international spillovers and welfare losses of FGs by using the latest numerical methods for the standard two-country NK model.

The contribution of this paper is to show the following new points in the literature dealing with ZLB and FG so far. First, in a two-country situation where two countries face a ZLB at the same time, we evaluate the policy in two cases: a) when both central banks adopt FG policy at the same time, and b) when the central bank of either country adopts FG policy. Next, we examine how the extension of the duration of FG policy in each country affects output, inflation, and nominal interest rates in the two countries through the international transmission mechanism of monetary policy. At this time, there will be bargaining between the central banks of the two countries as they seek to maximize their own welfare. We will examine what kind of bargaining might be envisioned. Again, the above discussion will add a new perspective to the growing body of FG literature.

The findings of this paper are as follows. First, the magnitude of the risk aversion coefficient of household consumption (CRRA) plays an important role in determining the beggar-thy-neighbor and prosper-thy-neighbor effects in foreign economies. In other words, in the case where households behave in a risk-averse manner (CRRA > 1), home FG has negative economic effects on the foreign country with symmetric economic structure. On the other hand, if households are risk averse, home FG has the opposite economic effect on the counterpart country. Second, if households are risk-averse, both countries would be better off adopting FG policies if the central bank's objective is to minimize individual country welfare losses rather than to minimize global welfare losses. Furthermore, if the home country extends the duration of FG over its counterpart, there will be a duration of FG that minimizes the welfare loss of the home country. Third, we point out the possibility of interesting FG bargaining in which either the central banks of the two countries adopt the same number of quarters of FG in a coordinated manner or, conversely, one of the central banks deviates from policy coordination. In sum, our study underscores the interesting result that bargaining about FG duration between two countries of roughly equal economic size is not a straightforward process.

This paper is organized as follows. Section 2 briefly reviews the literature related to this study, and Section 3 describes the two-country NK model. Section 4 describes the simulation method and calibration values of the occasional constraint used in this paper. Section 5 reports the main findings of this paper. We discuss the interaction of both FG and ZLB policies in a two-country NK economy and provide some robustness checks. Section 6 briefly concludes the paper.

2 Related literature

In this section, we briefly review the previous literature on the effects of FG in terms of the NK model and classify the contributions of this paper to previous studies. In particular, this paper is deeply related to the works of Fujiwara, Nakajima, Sudo and Teranishi (2013), Galí (2020), and Haberis and Lipińska (2020) on the effect of FG in the NK model.

The effectiveness of forward guidance

In recent years, the ZLB has been a serious problem in advanced economies such as the United States, the Eurozone, and Japan. As mentioned earlier, once faced with the ZLB, central banks are unable to manipulate short-term nominal interest rates as a policy instrument.[1] Within this framework, there have been a number of studies dealing with the expectations channel of monetary policy by managing private sector expectations. In particular, the effectiveness of this expectations channel is strongly supported by theoretical studies based on the NK model. For example, Eggertsson and Woodford (2003) and Jung, Teranishi and Watanabe (2005) argue that central banks can exert the effects of monetary policy even when they are faced with liquidity traps. According to their study, even in the case of a liquidity trap caused by a negative natural rate of interest, the central bank can manipulate private sector expectations through its commitment to keep zero interest rates for some time. According to Adam and Billi (2006) and Adam and Billi (2007), the cost of implementing discretionary policy is greater under the ZLB than under a model without the ZLB.[2]

Forward guidance in an open economy model

The effect of FG policy in an open economy has been examined in several studies. Galí (2020) showed that the difference between home and foreign interest rates captures the effect of FG policy in a small open economy model. He points out that the exchange rate dynamics are inconsistent with the interest rate parity hypothesis. He refers to this inconsistency as the

[1] See Bernanke and Reinhart (2004), English, López-Salido and Tetlow (2013), and Williamson et al. (2015) for a detailed discussion of unconventional monetary policy.

[2] Hirose, Iiboshi, Shintani and Ueda (2021) examine the effect of FG of behavioral NK model for the US economy, whereas Iiboshi, Shintani and Ueda (forthcoming) estimate the impact of FG policy for Japan's economy.

forward guidance exchange rate puzzle. Haberis and Lipińska (2020) also examine the international transmission mechanism of FG policy in a small open economy model. According to their analysis, foreign FG policies are effective if they improve social welfare in the home country. Jones et al. (2018) estimated the international spillover effects of FG policies in a two-country NK model. They showed that when a monetary easing FG shock occurs in the U.S. economy, the shock has a beggar-thy-neighbor effect on the Canadian economy. Thus, when a monetary easing FG shock occurs in the U.S., the output of the Canadian economy will be significantly reduced.

As mentioned earlier, central banks in advanced economies faced the ZLB after the 2008 global financial crisis and should have considered such a financial crisis as the cause of the negative global demand shock. Therefore, several papers have extended the issue of ZLBs in open economies (Cook and Devereux, 2011; Fujiwara et al., 2013; Ida, 2013, 2018; Nakajima, 2008). Fujiwara et al. (2013) examined the optimal commitment policy in a two-country NK model where the home country and the foreign country face a ZLB. Their analysis shows that when negative natural interest rate shocks in each country return to a steady state, the optimal commitment policy of each country becomes more complex than in a closed economy model. They also show the effectiveness of price level targeting in a two-country NK model where two countries face a ZLB.

Comparison between previous studies and this study

The contributions of this paper can be compared with four previous works in the literature. First, Galí (2020) examines the effects of FG policies in a small open NK model, whereas we examine the effects of FG policies in a two-country open NK model. Second, Haberis and Lipińska (2020) examine the impact of large foreign FG policies on a small home country. In their analysis, the large country's central bank follows the monetary policy rules imposed by the FG, while the small country's central bank is able to implement its optimal commitment policy. In contrast, our analysis assumes that both the home and foreign central banks of large countries implement monetary policy according to the Taylor rule with the FG under the ZLB and bargain with each other regarding policies. In this sense, this study focuses on the practical aspects of monetary policy rather than examining the optimal monetary policy. Third, while Haberis and Lipińska (2020) consider that negative shocks to natural rate of interest occur only in foreign countries, our study mainly considers the case of a global liquidity trap where the natural rates of interest decline simultaneously in the home country and foreign country. Finally, our paper is also close to that of Fujiwara et al. (2013), who examine the interaction of ZLB policies between the central banks in a two-country NK model with a global liquidity trap in terms of optimal commitment policy. In contrast to their work, this paper focuses on the case where the central banks of both countries adopt the FG and ZLB policies to follow monetary policy rules in a two-country NK model with a global liquidity trap.

3 A two-country new Keynesian model

In this paper, we derive a two-country NK model based on the framework of Clarida et al. (2002).[3] Consider an economy with two large symmetric countries: home and foreign. The population sizes of the home and foreign countries are $1 - \gamma$ and γ, respectively. There are two production sectors in each country: the final goods sector, which is characterized by perfect competition, and the intermediate goods sector, where firms face monopolistic competition and nominal price rigidities (Calvo, 1983). We allow the degree of price stickiness to vary across countries.

Assume that there is a complete market in both countries and that only final goods are traded. The number of producers of final goods is equal to the number of households in each country. The model also assumes that purchasing power parity is maintained according to Clarida et al. (2002). Thus, we consider the case of producer currency pricing.

Finally, unless otherwise noted, the same equation holds for foreign countries. Also note that variables for foreign countries are represented by asterisks.

3.1 Log-linearized two-country NK model

In this paper, we provide a log-linearized system of structural equations.[4] The structural equations derived in the previous section are log-linearized around the steady state. Here, lowercase

[3]This paper considers a two-country model in which the consumption basket consists of Cobb-Douglus aggregates. Pappa (2004) considers the costs without policy coordination in a two-country NK model where the consumption basket consists of CES aggregates

[4]The online Appendix provides a detailed derivation of the structural model. See also Clarida et al. (2002) and Walsh (2017).

variables are used to represent the logarithmic deviation from the steady state. Specifically, the log-linearized variables around the steady state are represented by $h_t = \log(H_t/\bar{H})$, where $\bar{H}$ represents the steady state value. The log-linearized structural equations can be summarized as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_1 x_t + \kappa_2 x_t^* + u_t, \tag{1}$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa_1^* x_t^* + \kappa_2^* x_t + u_t^*, \tag{2}$$

$$x_t = E_t x_{t+1} + \vartheta E_t \Delta x_{t+1}^* - (\sigma_0)^{-1}(r_t - E_t \pi_{t+1} - r_t^n), \tag{3}$$

$$x_t^* = E_t x_{t+1}^* + \vartheta^* E_t \Delta x_{t+1} - (\sigma_0^*)^{-1}(r_t^* - E_t \pi_{t+1}^* - (r_t^n)^*), \tag{4}$$

$$r_t \geq 0, \tag{5}$$

$$r_t^* \geq 0. \tag{6}$$

Here, variables with an asterisk represent foreign ones. π_t is the home country inflation, π_t^* is the foreign country inflation, x_t is the home country output gap, and x_t^* is the foreign country output gap. r_t is the home country nominal interest rate and r_t^* is the foreign country interest rate. u_t and u_t^* are home and foreign cost-push shocks, respectively. r_t^n is the home country natural interest rate and $(r_t^n)^*$ represents the foreign natural interest rate. The shock to natural rate of interest in each country follows an AR(1) process.

Also, structural parameters are defined as follows:

$$\kappa_1 = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\sigma + \eta - \gamma(\sigma - 1)), \ \kappa_2 = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\gamma(\sigma - 1)$$
$$\kappa_1^* = \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*}(\sigma + \eta - (1-\gamma)(\sigma - 1)), \ \kappa_2^* = \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*}(1-\gamma)(\sigma - 1),$$
$$\vartheta = \frac{\gamma(\sigma-1)}{\sigma - \gamma(\sigma-1)}, \ \vartheta^* = \frac{(1-\gamma)(\sigma-1)}{\sigma - (1-\gamma)(\sigma-1)}, \ \sigma_0 = \sigma - \gamma(\sigma - 1), \ \sigma_0^* = \sigma - (1-\gamma)(\sigma-1).$$

Equations (1) and (2) represent open economy NKPCs and show that the home country NKPC depends on the foreign output gap through both risk sharing and terms of trade channels. A similar mechanism can be seen in Equation (2). The coefficient $\gamma(\sigma - 1)$ represents this mechanism.[5] Equations (3) and (4) represent the open economy dynamic IS (DIS) curve. As in the case of NKPC, the home country DIS depends on the first difference in the foreign expected output gap through both the risk-sharing channel and the terms-of-trade channel. Finally, (5) and (6) represent the ZLB constraints on nominal interest rates for each country.

[5] See Clarida et al. (2002) for a detailed discussion of these channels.

When the intertemporal substitution of consumption becomes unity, as in Clarida et al. (2002), the international dependence between home and foreign countries disappears. Thus, when $\sigma = 1$, the two-country NK model becomes a closed one.

3.2 Monetary policy rules and welfare criteria

Here, we derive a second-order approximation of the household utility function weighted by the degree of openness. The welfare criterion is used to evaluate the performance of FG policies in each country. The derivation of the central bank's loss function is carried out in the case of policy coordination. The following derivation is based on Clarida et al. (2002).

The utility function of the planner is given by

$$W_t = (1-\gamma)\left[u\left(C_t, \frac{M_t}{P_t}\right) - v(N_t)\right] + \gamma\left[u\left(C_t^*, \frac{M_t^*}{P_t^*}\right) - v(N_t^*)\right]. \tag{7}$$

To obtain the well-defined loss function, we need to eliminate distortions created by monopolistic competition and real money balances. The first distortion is eliminated by an optimal subsidy rate that eliminates price markup created by monopolistic competition in each country. Fiscal authorities choose an optimal subsidy rate that restores the natural level of output to the efficient level of one in a zero-inflation steady state. As noted earlier, such an optimal subsidy is given as follows:

$$(1-\tau)\mu = 1, \ (1-\tau^*)\mu^* = 1,$$

where τ denotes an optimal subsidy rate and μ denotes the steady-state mark-up. Accordingly, as shown in Clarida et al. (2002), the optimal subsidy rate leads to the efficient steady state in terms of the distortion caused by monopolistic competition.

However, we also consider the second distortion, which is a result of an opportunity cost of holding money. As shown in Woodford (2003), this opportunity cost should be considerably smaller in steady state to obtain a well-defined loss function of the central bank. In particular, Woodford (2003) argues that real money balances are sufficiently close to being satiated in the optimal steady state. To obtain the loss function, we can eliminate the distortion produced by the opportunity cost of money.[6]

[6]See Chapter 6 in Woodford (2003) for a detailed discussion of this issue.

Under these assumptions, we derive the central bank's loss function by implementing the second-order approximation of Equation (7) as follows:[7]

$$\sum_{t=0}^{\infty} W_t \approx -\sum_{t=0}^{\infty} \beta^t L_t^w + t.i.p. + O(||\xi||^3), \tag{8}$$

Here $t.i.p$ represents the terms that are independent of monetary policy and $O(||\xi||^3)$ includes the higher order terms for the Taylor approximation. For any period t, the central bank's loss function under policy coordination (L_t) is given as follows:

$$L_t^w = (1-\psi)L_t + \psi L_t^* - 2\Lambda x_t x_t^*. \tag{9}$$

The loss function for the home country (L_t) and that for the foreign country (L_t^*) are given as follows:

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_r r_t^2, \tag{10}$$

$$L_t^* = (\pi_t^*)^2 + \lambda_x^*(x_t^*)^2 + \lambda_r^*(r_t^*)^2. \tag{11}$$

Also, the structural parameters in Equation (9) are defined as follows:

$$1-\psi = \frac{(1-\gamma)\varpi^{-1}}{\Omega},\ \Omega = (1-\gamma)\varpi^{-1} + \gamma(\varpi^*)^{-1},\ \lambda_x = \frac{\kappa_1}{\theta},\ \lambda_r = \frac{\eta_r}{\bar{v}\theta},$$
$$\lambda_x^* = \frac{\kappa_1^*}{\theta},\ \lambda_r^* = \frac{\eta_r^*}{\bar{v}^*\theta},\ \Lambda = \frac{2(1-\gamma)\gamma(1-\sigma)}{\varpi\theta}.$$

Also,

$$\varpi = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha},\ \varpi^* = \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*}.$$

Equation (10) consists of three objectives. The first term of the right-hand side denotes the stabilization of home inflation. The second term of the right-hand side represents the stabilization of the output gap. Finally, due to the presence of real money balances, the interest rate stabilization is captured by the third term of the right-hand side. The corresponding objectives hold for the foreign central bank's loss function. In Equation (9), the presence of the third-term of the right-hand side is associated with the international spillover effect through both risk-sharing and the terms of trade channels. Because this term disappears when $\sigma = 1$, the above loss function simply becomes a weighted average of the home and foreign loss functions like a closed economy model.

[7] The online Appendix provides a detailed derivation of the central bank's loss function. See also Clarida et al. (2002).

3.3 Monetary policy rules, specification of FG and structural shocks

Monetary policy rules

We specify monetary policy rules. In contrast to Fujiwara et al. (2013) and Haberis and Lipińska (2020), we consider that central banks follow a simple instrument rule instead of implementing a targeting rule derived from optimal monetary policy. Following Taylor (1993), we assume that both home and foreign central banks adopt the following simple monetary policy rule with the ZLB constraints on nominal interest rates:

$$r_t = max\{0,\ (1-\psi_r)(\psi_\pi \pi_t + \psi_x x_t) + \psi_r r_{t-1} + e_t\}, \tag{12}$$

$$r_t^* = max\{0,\ (1-\psi_r^*)(\psi_\pi^* \pi_t^* + \psi_x^* x_t^*) + \psi_r^* r_{t-1}^* + e_t^*\}. \tag{13}$$

Here, ψ_π denotes the inflation stabilization in the Taylor rule, ψ_x denotes the output gap stabilization in the rule, and ψ_r is the term for interest rate smoothing. e_t represents an exogenous monetary policy shock. It follows from Equation (13) that the foreign monetary policy rule follows the same specification as the home monetary policy rule.

Specification of FG policies

Let us explain how FG is specified in this paper. Following Haberis and Lipińska (2020), the FG policies adopted by both countries is referred to as a calendar-based FG. This FG specification implies that the central bank commits to a ZIP for longer than the ZLB periods suggested by the standard Taylor rule. In this paper, we regard the terminology of FG policy adopted in Haberis and Lipińska (2020) as the terminology of fixed length FG suggested by Eggertsson, Egiev, Lin, Platzer and Riva (2020). Following Haberis and Lipińska (2020), we label several specifications of FG policies. More specifically, in our simulation, we focus on four types of FG policies: with two extra quarters, four extra quarters, five extra quarters, and ten extra quarters. In addition, we consider the following four policy options for the foreign central bank. First, the foreign central bank can conduct its monetary policy in an economy without the ZLB. Second, similarly to the home country, the foreign country also faces the ZLB. Third, the foreign central bank adopts the same length of FG policy as is employed by the home central bank. Fourth, while the home central bank can choose from four types of FG policies, the foreign central bank fixes the number of quarters of the FG policy at five.

As mentioned earlier, Haberis and Lipińska (2020) explored the international FG spillover effect of how a large country's FG policy affects an optimal commitment policy adopted by a small country. In contrast to their study, we consider that a global liquidity trap shock matters when both countries are large. This is because a negative shock of natural rate of interest in one country causes an adverse effect on the other country. Therefore, even the Taylor rule can induce the adverse effect of a negative natural interest rate in one country on the other country. Accordingly, we consider a global liquidity trap shock that simultaneously creates a negative impact on the natural rate of interests in both countries. We postulate that this shock makes the natural interest rate in each country undergo a decline of the same size.

Structural shocks

The structural shocks considered in this paper are two shocks: a shock to natural rate of interest that occur in only the home country, and a global liquidity trap shock in both the home country and foreign countries. Although the latter shock is characterized by the former shocks, the latter feature is set to occur simultaneously in both countries. These two shocks are assumed to be persistent and can be expressed as first-order auto regressive (AR) processes as follows:

$$r_t^n = \rho_r r_{t-1}^n + e_t^{NR} + e_t^{GL},$$

$$(r_t^n)^* = \rho_r (r_{t-1}^n)^* + (e_t^{GL})^*,$$

where ρ_r is the coefficient of AR processes, e_t^{NR} is the independent shock to the natural rate of interest generated in the home country, and e_t^{GL} and$(e_t^{GL})^*$ are the independent shocks to the global liquidity traps in the home and foreign countries, respectively.

4 Solution methods

4.1 Parameterizations

This section summarizes the calibrated values used in this study. The calibrated value of structural parameters are summarized in Table 1. We set the Calvo lottery to 0.9.[8] The

[8]This value seems to be slightly higher value compared to previous studies. We confirmed that the main results quantitatively and qualitatively remains unaffected any change of this parameter.

discount factor is set to 0.985. The degree of openness is set to 0.5. Thus, we consider the case where the country size of the home country is equal to that of the foreign country.

[Insert Table 1 around here]

We set the relative risk aversion coefficient for consumption to 2.0 as a benchmark calibration. The selection of this value is based on the calibrated value in Pappa (2004). This value is crucial in a two-country NK model. Clarida et al. (2002) showed that when this value is less than unity, the international risk-sharing effect works negatively. Thus, if this value takes unity, the model corresponds to the closed economy one because the open economy effect vanishes. In particular, Fujiwara et al. (2013) and Nakajima (2008) addressed the role of this parameter in a two-country NK model with the ZLB. Naturally, even in the case for simple monetary policy rules, the effect of FG policies in a two-country economy is affected by the value of the risk aversion coefficient. We select several values of this parameter in the robustness check.

Finally, we select the parameters for monetary policy rules and the natural rate of interest. We assume that the parameterization for the monetary policy rule is common to both countries. Inflation stabilization ψ_π is set to 1.25 and the output gap stabilization is set to 0.5. We set the term for interest rate smoothing to 0. If we take interest rate smoothing into account, we conjecture that the reaction of the interest rate gradually changes even when an FG policy terminates. The AR (1) coefficient for the natural interest rate for each country is set to 0.8 with a variance of 1.0.

4.2 Numerical approach

In this paper, we follow Haberis and Lipińska (2020) and use a piecewise linear perfect-foresight algorithm by adopting the OccBin toolkit using the Dynare platform, which is a toolkit developed by Guerrieri and Iacoviello (2015) to solve equilibrium under the occasionally binding constraint. Let us give a brief summary of the numerical method for the occasionally binding model used to solve the model equilibrium under the ZIP.

This type of method can be roughly divided into two categories as follows. The first is called the global method, which has extremely high computational accuracy but is extremely computationally demanding and unsuitable for large models. The second is the extended perfect-foresight path (EPFP) by Ray and Taylor (1983). The advantage of the EPFP is that it simplifies the method of calculating the expected value, so it can be applied to medium or large-scale models even on an ordinary personal computer.

OccBin belongs to the second category, EPFP. Atkinson, Richter and Throckmorton (2019) used a Monte Carlo method to compare the numerical results of the global method and EPFP. Atkinson et al. (2019) compared the numerical results of the global method and EPFP using the Monte Carlo method, and reported that the difference between the two methods was not significant, although the results of the former were higher. Therefore, if our research follows this latest research report, the numerical results of this paper will be justified even if the global method is not used.

5 Quantitative results

This section provides the main results of the paper. We investigate the effect of FG policies on the international spillover mechanism of monetary policy under a global liquidity trap shock. Section 5.1 examines the effect of a home FG on a two-country economy when the ZLB is binding in the home country, which faces a negative shock to natural rate of interest. Section 5.2 reports the impulse response analysis under a negative global liquidity trap shock. We focus on how several specifications of home FG policies affect the international transmission mechanism of monetary policy. Section 5.3 reports the welfare losses in terms of the degree of FG policies of both countries. We calculate the time series transition of welfare losses in terms of home, foreign and global losses.

5.1 FG policy under a liquidity trap in the home country

We focus on how a negative shock to natural rate of interest changes the impact on the home country and how it affects foreign macroeconomic variables through international spillovers. The international spillover effect is captured by σ, γ, κ_1, κ_2, and ϑ as well as the counterparts in the foreign country. In particular, the sign of ϑ is the threshold for the existence of open economy effects: it is non-negative when $\sigma > 1$, and negative when $\sigma < 1$.

As pointed out by Clarida et al. (2002) and Fujiwara et al. (2013), the parameter σ plays an important role in the two-country NK model when international spillover effects are considered. On the other hand, the above structural parameters are unaffected by changes in the parameter γ. Therefore, before going to analysis of a global liquidity trap, we consider different cases based on attitudes toward risk: avoidance, neutrality, and affection, corresponding to σ being one of the three values 2.0, 1.0, and 0.5, and show the contrasting effect of home country FG policies on the two-country NK model by.

Panel (a) of Figure 1 shows the impulse response to a negative shock to natural rate of interest occurring in the home country for $\sigma = 2$. The solid line shows the impulse response to a negative shock to natural rate of interest occurring in the home country. The solid line shows the impulse response in the case of a two extra quarters FG policy and the dashed line shows the impulse response in the case of a five extra quarters FG policy. The dashed line also shows the case of the nine extra quarters FG policy. Finally, the dotted line shows the impulse response when the ZLB is not introduced. The left side of the figure shows the impulse response of the home country and the right side shows the impulse response of the foreign country. The lower left part of the graph shows that the natural rate of interest is declining in the home country.

[Insert Figure 1 (a)-(c) around here]

According to this figure, after a liquidity trap shock in the home country, a stronger FG policy implies that a more extended ZIP is adopted. In particular, a nine-extra-quarter FG policy strongly raises both inflation and the output gap. Thus, a home country central bank that adopts a strong FG policy strongly stimulates inflation and the output gap. On the other hand, the response of foreign macroeconomic variables does not seem to be affected by the decline in the home country's natural interest rate shock.

Table 2(a) reports the welfare losses for each country in this scenario. The numbers with a star indicate that the welfare loss is minimized in this simulation. From this table, we can see that the welfare loss in the foreign country is minimized when the welfare loss in the home country is minimized. As long as the negative natural interest rate occurs only in the home country, the extended FG policy adopted by the home central bank can improve social welfare in both countries. Thus, in the case of a shock to natural rate of interest in the home country, the home country's enhanced FG policy will not be inconsistent with foreign monetary policy.

[Insert Table 2 (a)-(d) around here]

Figure 1(b) shows the case where $\sigma = 1$. It immediately turns out that ϑ and κ_2 (and the foreign counterparts) are zero when $\sigma = 1$. When $\sigma = 1$, the response on the left side of this graph is the same as that obtained in Figure 1(a). On the other hand, macroeconomic variables in the foreign economy do not respond to shocks to natural rate of interest in their home country, simply because there are no international spillover effects. Therefore, the welfare loss is naturally null in the foreign country, as shown in Table 2(b). This result is consistent with that of Fujiwara et al. (2013), who examined the impact of a global liquidity trap shock in a two-country NK model in which the home and foreign central banks cooperate to implement an optimal commitment policy.

Figure 1(c) plots the impulse responses for the case of $\sigma = 0.5$. As mentioned earlier, the structural parameters captured by the international spillover effect are negative values. The results show that the response of the home country is symmetric to the case of Figure 1(a) and (b). However, a decrease in the home country's natural interest rate leads to a decrease in both the foreign interest rate and the output gap.

Summing up, we would like to stress the case for $\sigma = 2.0$. On the one hand, in the case of $\sigma = 0.5$, the responses of foreign macroeconomic variables are symmetric to those of the home country in which the shock to natural rate of interest occurs. On the other hand, in the case of $\sigma = 2.0$, the responses of foreign macroeconomic variables are asymmetric to those in the home country.

5.2 The interaction of FG under a global liquidity trap

In this and the following sections, we focus on the world economy under a positive international risk sharing channel, say $\sigma = 2.0$. And we examine the impulse response to a global liquidity trap shock that simultaneously induces a negative shock to natural rate of interest in both countries. We classify the impulse response to a global liquidity trap shock in accordance with several specifications of foreign monetary policy. These exercises are described by four cases shown in Figure 2 (a) to (d).

First, Figure 2 (a) illustrates the impulse response to a global liquidity trap shock in the case where the foreign central bank conducts its monetary policy without the ZLB. The left-hand side of the figure shows the response of the home country, whereas the right-hand side of the figure corresponds to that of the foreign country. The first, second, and third rows of the figure represent the response of inflation, the output gap, and the nominal interest rate, respectively. It follows from this figure that the extended announcement of the home FG policy prolongs the termination date of a zero interest rate policy. Since the ZLB is not binding in the foreign country, the foreign nominal interest rate seems to be unaffected by changes of the home FG policy. In addition, both inflation and the output gap in the foreign country remains unaffected by the extension of the home FG policy. On the contrary, the figure shows that home ten extra quarters FG policy creates a boom in both inflation and the output gap in its country. As shown in Table 2, however, the effect of the extended FG policy in the home country might not be desirable in terms of worldwide social welfare. Indeed, Table 3 (a) indicates that the extension of FG in the home country worsens social welfare in the foreign country.

[Insert Figure 2 (a)-(d) around here]

Second, Figure 2 (b) portrays the impulse response when the ZLB is binding in the foreign country. The home central bank extends a ZIP in accordance with the prolonged home FG policy, whereas the foreign policy rate faces the ZLB. However, the number of quarters during which the foreign policy rate faces the ZLB is much less than in the home country. In addition, when the foreign central bank terminates the ZIP, the foreign policy rate is characterized by an inertial manner in response to changes of the home FG policy. Concretely, the foreign policy rate gradually increases as the home FG policy is extended in the case where the foreign central bank decides to terminate the ZIP. Regardless of this reaction of the foreign policy rate, inflation and the output gap remain unchanged by changes of the home FG policy.

[Insert Table 3 (a)-(d) around here]

On the other hand, as shown in Figure 2 (a), the home central bank that extends the ZIP for nine extra quarters can create a boom in both inflation and the output gap in its country. Figure 2 (a) and (b) indicates that the adoption of the home FG policy deteriorates the welfare loss of the foreign country, whereas it enhances the social welfare of the home country. This result contrasts starkly with the result of Haberis and Lipińska (2020). More precisely, in their study, the adoption of the FG policy in the large country can counteract welfare losses of the home country as long as the large country can enhance social welfare in its country. In other words, the adoption of the FG policy in the large country causes the prosper-thy-neighbor effect on the home country in their study. On the other hand, we demonstrate that when the size of the home country is equal to that of the foreign country ($\gamma = 0.5$), the extension of the home

FG policy produces the beggar-thy-neighbor effect on the foreign country. This observation is also confirmed in Table 3 (a) and (b).

Third, Figure 2 (c) shows that the foreign central bank employs the same size of FG policy as the home central bank does. Interestingly, in contrast to the above two cases, it is obvious that booms in both inflation and the output gap are created in both countries when home and foreign central banks adopt the nine extra quarters FG policy. The impulse response is symmetric in both countries. This result is associated with the assumption that the shock size is common to both countries and $\gamma = 0.5$. Importantly, as shown in Table 3 (c), the minimum worldwide welfare loss is attained when both home and foreign central banks follow an FG policy fixed at six quarters. This might indicate the gain from policy coordination when the ZLB is binding in both home and foreign countries.

Finally, Figure 2 (d) illustrates the impulse response to a global liquidity trap shock when the foreign central bank implements the FG policy fixed at five quarters. We have shown that home and foreign central banks that cooperatively implement the five-extra-quarters FG policy can minimize the worldwide welfare loss derived from a global liquidity trap shock. In this simulation we now assume that the foreign central bank adopts a five-extra-quarters FG policy. It follows from Figure 2 (d) that the extended termination period of the home FG policy causes the beggar-thy-neighbor effect on the foreign country. Thus, when the home central bank considerably extends the implementation period of the FG policy, then it can create booms in inflation and the output gap in the home country and also bring recession in foreign country. This result is consistent with the cases for Figure 2 (a) and (b).

5.3 FG bargaining in a global liquidity trap

In this section we consider the international cooperation bargaining between home and foreign central banks in terms of minimizing each central bank's welfare loss. More precisely, we focus on the condition of the gain from international policy coordination when both countries faces the possibility of the ZLB. To do this, we calculate the time series properties of each central bank's welfare losses based on the results of the impulse response analysis.[9]

Figure 3 shows the time-series properties of each central bank's welfare loss when a global liquidity trap shock occurs in both countries. These calculations are based on the specifications

[9] In this simulation, we focus on the case for $\sigma = 2$.

of each central bank's loss function, namely (10) and (11). The sum of welfare losses of both countries is calculated by the welfare criterion (9). In this figure the description in each panel corresponds to that in Figure 2. The first row of the figure illustrates the global welfare loss, which is a weighted sum of each central bank's loss function. The second and third rows describe the home and foreign welfare losses, respectively. It follows from this figure that while both countries experience larger welfare losses when a global liquidity trap shock has occurred in the first quarter, these losses converge in the second quarter. Also, this figure shows that a nine extra quarters FG policy, which is described by the blue line, causes a persistent welfare losses in both countries. We observe that in this case the persistent losses disappear in the eleventh quarter. In particular, as shown in panels (b) and (c), when the global liquidity trap shock has occurred in the first quarter, each central bank's welfare loss under the nine extra quarters FG policy is smaller than that under the five-extra-quarters FG policy. However, we note that if the welfare loss in each country is assessed over the total number of quarters, the performance of the nine-extra-quarters FG policy leads to poorer outcomes than the five-extra-quarters FG policy. This result appears to be an important point for evaluating the effect of the FG policy in a two-country NK model.

[Insert Figure 3 (a)-(d) around here]

As expressed in Table 3, we evaluate the effect of FG policies by calculating welfare losses. The welfare losses calculated in this table represent the sum of welfare losses in all periods in accordance with the cases (a) to (d) in Figure 3. The fourth and fifth columns represent the home and foreign welfare losses, respectively. The third column again denotes the global welfare loss, which is the weighted average of home and foreign welfare losses. In each panel, we mark the minimized value with a star. In these four panels, the global welfare losses are minimized when the six-extra-quarters FG policies are adopted by the home and foreign central banks.

However, we note that the termination period of the FG policy is chosen not continuously, but discretely. In addition, we cannot find the optimal length of the FG policy in all periods we simulate because this might be a technical limitation associated with an EPFP methods in an economy with occasionally binding constraints. In particular, we have difficulty in finding the exact optimal FG length in the given range where the responses of home and foreign inflation reverse from negative to positive. Therefore, as we cannot find the optimal FG solution that globally minimizes the worldwide welfare loss, we also acknowledge that our analysis is constrained by the limitation that we cannot numerically and explicitly demonstrate the subgame Nash equilibrium under an international monetary policy game framework. Summing up, with several technical limitations to our analysis, these results lead to policy implications regarding the effect of forward guidance in a two-country NK model with the ZLB as follows.

In the cases (a), (b), and (d) in Table 3, when the home central bank adopts the FG policy with six-extra-quarters, it reduces the welfare loss in its country. On the other hand, in these three cases, since the foreign central bank cannot freely select the strength of the FG policy, the extension of the home FG policy worsens the welfare loss in the foreign country. Table 3 also shows that when the home central bank conducts the FG policy that terminates the zero interest rate after ten-extra-quarters, the home welfare loss again increases. Therefore, as long as the home central bank seeks to minimize the welfare loss in its country, it would never adopt the zero interest rate policy after ten-extra-quarters. We summarize the above discussion as follows:

Remark 1 *Regardless of whether or not the ZLB is faced in the foreign country, the home FG policy causes the beggar-thy-neighbor effect on the foreign country if the home and foreign countries are of the similar economic scale under a positive international risk sharing. This result holds as long as the termination date of the foreign FG policy is shorter than that of the home FG policy.*

As shown in Table 3 (a), (b), and (d), the more extra quarters the FG is adopted for, the smaller the welfare loss in both countries. This indicates that as long as both central banks evaluate their welfare losses based on the measure of their own loss function, both central banks would adopt the FG policy. Moreover, these tables show that the home central bank achieves poorer outcomes when the number of FG quarters exceeds six in the home country, and the foreign central bank also faces the same situation. As shown in Figure 2, extension of the termination date of the FG policy creates a boom in both inflation and the output gap. However, Figure 3 illustrates that since such an FG policy produces a persistent welfare loss, it results in poorer outcomes in that neither central bank can enhance social welfare either domestically or internationally. This becomes the source of the time inconsistency problem of monetary policy. Thus, after the central bank declares an extension of the FG policy to combat the recession in its country, it has an incentive to renege on such a commitment. We summarize this result as follows:

Remark 2 *Under a positive international risk sharing, both home and foreign central banks can improve social welfare by adopting FG policies as long as they assess the welfare loss based on their own loss function. In addition, the home central bank minimizes its welfare loss when it employs the FG policy for more quarters than the foreign country. However, in this case the home central bank has an incentive to renege on such a commitment. Thus, there exists the problem of time inconsistency associated with FG policies.*

As shown in Table 3 (c), there is a gain from policy coordination if the home and the foreign central bank cooperatively select the same number of quarters for their FG policies. Indeed, both central banks attain their minimum welfare losses when they simultaneously adopt the six extra quarters FG policies. On the other hand, Table 3(d) shows that when the foreign FG policy is fixed at five quarters, the home welfare loss is 9.62 if the home central bank employs the six-extra-quarters FG policy. This implies that the home central bank can obtain a gain of 0.31 if it extends the FG policy by one quarter once the foreign country has announced a five-extra-quarters FG policy. However, the foreign central bank cannot achieve the minimum welfare loss in this case. Furthermore, the worldwide welfare loss in this case is 20.83, and this value is larger than the case where both central banks cooperatively fix the number of FG quarters at five. Therefore, both central banks achieve their minimum welfare loss if they cooperatively select FG policies of the same size. Nevertheless, each central bank has an incentive to escape the international monetary policy cooperation. This result is summarized as follows:

Remark 3 *Under a positive international risk sharing, home and foreign central banks achieve their minimum welfare losses when they conduct FG policies cooperatively. However, they always have an incentive to renege on the framework of international monetary policy cooperation.*

Finally, in this section we argue the case where σ is above unity. We note the case for $\sigma = 0.5$. While the result of this case is reported in the online Appendix, the result shows that the home FG policy now leads to the prosper-thy-neighbor effect on the foreign country. We would easily conjecture that even in this case, both the home and the foreign central bank jointly select FG policies of the same size.

5.4 Discussion: Relevance to FG puzzle

Finally, we remark on the implication regarding the impact of the FG policy on the international monetary transmission mechanism in a two-country NK model. Our model has shown that the extension of the duration of the FG policy in each country affects output, inflation, and nominal interest rates in the two countries through the international transmission mechanism of monetary policy. However, the FG effect obtained in the paper might be stronger or weaker than that obtained in a closed economy.

Several studies argued that the effect of FG is much stronger in the standard NK model than the actual policy effects. For instance, Carlstrom, Fuerst and Paustian (2015) showed that the effect of an FG policy unrealistically increases both inflation and the output gap when the duration of FG exceeds a threshold value. Del Negro et al. (2012) labelled this inconsistency as the forward guidance (FG) puzzle. In particular, McKay, Nakamura and Steinsson (2016) pointed out that the extension of the duration of commitment of the central bank to the zero interest rate renders this FG puzzle more severe.

While the effectiveness of the FG policy as one tool for non-conventional monetary policy is theoretically supported in terms of the NK model, its power seems to be problematic. Several recent studies focus on how the FG puzzle is ameliorated by extending the standard NK model. To do this, Del Negro, Giannoni and Patterson (2012) incorporated the perpetual-youth model into the standard NK model. Also, Gabaix (2020) showed that the FG puzzle is resolved once the bounded rationality takes the standard NK model into consideration. McKay et al. (2016) attenuated the effect of FG policy by incorporating incomplete markets into the standard NK model.

Moreover, Bodenstein, Hebden and Nunes (2012) and Haberis, Harrison and Waldron (2019) showed that the power of an FG policy is weakened if the central bank's commitment associated with the FG policy is imperfectly credible.[10] Campbell, Ferroni, Fisher and Melosi (2019) showed the limitation of an FG policy, and argued that an imperfect communication strategy of the central bank causes macroeconomic fluctuations. Nakata, Ogaki, Schmidt and Yoo (2019) examined the effect of an FG policy in a model in which the forward-looking structural equations, such as the new Keynesian Phillips curve and the dynamic IS curve, are discounting.

[10] Boneva, Harrison and Waldron (2018) showed that a threshold-based FG policy outperforms a calendar-based one.

Nakata et al. (2019) showed that in the case of the attenuated FG effect the central bank can implement the optimal monetary policy by extending the ZIP for a while.

While these caveats are important, extending the current model to take into account the above points is beyond the scope of this paper. While recognizing the limitations of comparing the results with those obtained in the above papers, this paper addresses the importance of considering the role of international spillovers of FG policies in a two-country NK framework.

6 Conclusions

This paper studied the effect of FG of monetary policy in a two-country NK model. While the effectiveness of FG has been discussed in the closed-economy NK model, it is unclear how FG affects the international transmission mechanism of monetary policy through a change in the exchange rate. How does the effect of FG in one country lead to spillovers in other countries? This possibility is not fully explained in the previous literature. This paper filled this theoretical gap in the NK model. Also, after the turmoil of the financial crisis originated in the US in 2008, it appears that the central banks in advanced countries adopted the zero interest rate policy as one of the tools of unconventional monetary policy. Recently, the large shocks associated with the COVID-19 pandemic force the central banks to again adopt the ZIP to recover the economy from a severe recession. These facts motivated this study.

The findings of this paper are threefold as follows. First, the magnitude of the CRRA of household consumption plays an important role in determining the beggar-thy-neighbor and prosper-thy-neighbor effects in foreign economies. On the one hand, when households engage in risk-averse behavior, home FG has a negative economic effect on a foreign country with a symmetric economic structure. On the other hand, when households engage in risk-averse behavior, it has the opposite economic effect. Second, if households are risk-averse and the central bank's objective is to minimize the welfare loss of individual countries, then both countries would be better off adopting FG policies. Moreover, it is more likely that the home country will adopt a policy of extending the duration of FG than the foreign country. Third, there is a possibility of an FG bargaining situation in which the central banks of each country cooperate to adopt the same FG extension period or, conversely, one of the central banks deviates from policy coordination.

Finally, we would like to mention some extensions of this study as future works. As we mentioned earlier, we again note that we cannot explicitly derive the optimal length of the FG policy under international policy coordination because our analysis is based on a numerical exercise. As a future work, we would like to consider how the optimal FG policy is analytically derived in a two-country framework. In addition, we show the possibility of the gain from policy coordination through the case where home and foreign central banks jointly select the same size of FG policy. This result is based on the assumption that home and foreign central banks that follow a simple policy rule with FG jointly select the same size of FG policies. However, strictly speaking, we need to check whether our results correspond to the optimal cooperative policy, which implies that home and foreign central banks jointly minimize the worldwide loss function.

References

Adam, K., Billi, R.M., 2006. Optimal monetary policy under commitment with a zero bound on nominal interest rates. Journal of Money, credit and Banking , 1877–1905.

Adam, K., Billi, R.M., 2007. Discretionary monetary policy and the zero lower bound on nominal interest rates. Journal of monetary Economics 54, 728–752.

Atkinson, T., Richter, A.W., Throckmorton, N.A., 2019. The zero lower bound and estimation accuracy. Journal of Monetary Economics .

Bernanke, B.S., Reinhart, V.R., 2004. Conducting monetary policy at very low short-term interest rates. American Economic Review 94, 85–90.

Bodenstein, M., Hebden, J., Nunes, R., 2012. Imperfect credibility and the zero lower bound. Journal of Monetary Economics 59, 135–149.

Boneva, L., Harrison, R., Waldron, M., 2018. Threshold-based forward guidance. Journal of Economic Dynamics and Control 90, 138–155.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of monetary Economics 12, 383–398.

Campbell, J.R., Ferroni, F., Fisher, J.D., Melosi, L., 2019. The limits of forward guidance. Journal of monetary economics 108, 118–134.

Carlstrom, C.T., Fuerst, T.S., Paustian, M., 2015. Inflation and output in new keynesian models with a transient interest rate peg. Journal of Monetary Economics 76, 230–243.

Clarida, R., Galı, J., Gertler, M., 2002. A simple framework for international monetary policy analysis. Journal of monetary economics 49, 879–904.

Cook, D., Devereux, M.B., 2011. Optimal fiscal policy in a world liquidity trap. European Economic Review 55, 443–462.

Del Negro, M., Giannoni, M.P., Patterson, C., 2012. The forward guidance puzzle. FRB of New York Staff Report .

Eggertsson, G., Woodford, M., 2003. The zero bound on interest rates and optimal monetary policy. Brookings Papers on Economic Activity 1, 139–233.

Eggertsson, G.B., Egiev, S.K., Lin, A., Platzer, J., Riva, L., 2020. A Toolkit for Solving Models with a Lower Bound on Interest Rates of Stochastic Duration. Technical Report. National Bureau of Economic Research.

English, W.B., López-Salido, J.D., Tetlow, R.J., 2013. The federal reserve's framework for monetary policy—recent changes and new questions .

Fujiwara, I., Nakajima, T., Sudo, N., Teranishi, Y., 2013. Global liquidity trap. Journal of Monetary Economics 60, 936–949.

Gabaix, X., 2020. A behavioral new keynesian model. American Economic Review 110, 2271–2327.

Galí, J., 2020. Uncovered Interest Parity, Forward Guidance, and the Exchange Rate. Technical Report. National Bureau of Economic Research.

Guerrieri, L., Iacoviello, M., 2015. Occbin: A toolkit for solving dynamic models with occasionally binding constraints easily. Journal of Monetary Economics 70, 22–38.

Haberis, A., Harrison, R., Waldron, M., 2019. Uncertain policy promises. European Economic Review 111, 459–474.

Haberis, A., Lipińska, A., 2020. A welfare-based analysis of international monetary policy spillovers at the zero lower bound. Journal of Money, Credit and Banking 52, 1107–1145.

Hirose, Y., Iiboshi, H., Shintani, M., Ueda, K., 2021. Estimating a behavioral new keynesian model with the zero lower bound. mimeo .

Ida, D., 2013. Optimal monetary policy rules in a two-country economy with a zero bound on nominal interest rates. The North American Journal of Economics and Finance 24, 223–242.

Ida, D., 2018. The role of money and optimal monetary policy in a two-country model. Momoyama Gakuin University Discussion Paper Series .

Iiboshi, H., Shintani, M., Ueda, K., forthcoming. Estimating the nonlinear new keynesian model with the zero lower bound for japan. Journal of Money, Credit and Banking .

Jones, C., Kulish, M., Rees, D.M., 2018. International Spillovers of Forward Guidance Shocks. International Monetary Fund.

Jung, T., Teranishi, Y., Watanabe, T., 2005. Optimal monetary policy at the zero-interest-rate bound. Journal of Money, credit, and Banking 37, 813–835.

McKay, A., Nakamura, E., Steinsson, J., 2016. The power of forward guidance revisited. American Economic Review 106, 3133–58.

Nakajima, T., 2008. Liquidity trap and optimal monetary policy in open economies. Journal of the Japanese and international Economies 22, 1–33.

Nakata, T., Ogaki, R., Schmidt, S., Yoo, P., 2019. Attenuating the forward guidance puzzle: implications for optimal monetary policy. Journal of Economic Dynamics and Control 105, 90–106.

Pappa, E., 2004. Do the ecb and the fed really need to cooperate? optimal monetary policy in a two-country world. Journal of Monetary Economics 51, 753–779.

Ray, F., Taylor, J., 1983. Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models,". Econometrica 51, 1169–85.

Taylor, J.B., 1993. Discretion versus policy rules in practice, in: Carnegie-Rochester conference series on public policy, Elsevier. pp. 195–214.

Walsh, C.E., 2017. Monetary theory and policy. MIT press.

Williamson, S.D., et al., 2015. Monetary policy normalization in the united states. Federal Reserve Bank of St. Louis Review 97, 87–108.

Woodford, M., 2003. Interest and prices: Foundations of a theory of monetary policy. princeton university press.

Table 1: Calibrated parameters

Parameter	Description	Value
α	Calvo pricing for home country	0.9
α^*	Calvo pricing for foreign country	0.9
β	Discount factor	0.985
σ	Relative risk aversion coefficient	2.0
γ	Degree of openness	0.5
η	Inverse of Frish labor elasticity	1.5
θ	Elasticity of substitution for individual goods	10.0
ψ_r	Interest rate smoothing in the Taylor rule for Country H	0
ψ_π	Inflation stabilization in the Taylor rule for Country H	1.25
ψ_x	Output gap stabilization in the Taylor rule for Country H	0.5
ψ_r^*	Interest rate smoothing in the Taylor rule for Country F	0
ψ_π^*	Inflation stabilization in the Taylor rule for Country F	1.25
ψ_x^*	Output gap stabilization in the Taylor rule for Country F	0.5
ρ_r	AR(1) coefficient for natural interest rate shock in Country H	0.8
ρ_r^*	AR(1) coefficient for natural interest rate shock in Country F	0.8
e_t^{NR}	Size of shock to natural rate of interest in Country H	-0.05
e_t^{GL}	Size of shock to global liquidity trap in Country H	-0.04
$(e_t^{GL})^*$	Size of shock to global liquidity trap in Country F	-0.04

Table 2: Welfare losses in a natural rate shock of only Country H

(a) High degree of CRRA ($\sigma = 2$)

H policy	F policy	World loss	H loss	F loss
ZLB	non ZLB	23.94	23.50	0.44
FG 2 extra qtrs	non ZLB	22.03	21.64	0.39
FG 4 extra qtr	non ZLB	17.30	16.97	0.33
FG 5 extra qtr	non ZLB	**14.77***	**14.46***	**0.31***
FG 9 extra qtr	non ZLB	19.94	19.42	0.52
FG 10 extra qtr	non ZLB	29.72	29.04	0.68

(b) Medium degree of CRRA ($\sigma = 1$)

H policy	F policy	World loss	H loss	F loss
ZLB	non ZLB	26.95	26.95	0.00
FG 2 extra qtr	non ZLB	25.30	25.30	0.00
FG 4 extra qtr	non ZLB	19.26	19.26	0.00
FG 5 extra qtr	non ZLB	**15.84***	**15.84***	0.00
FG 9Q Extra	non ZLB	24.54	24.54	0.00

(c) Low degree of CRRA ($\sigma = 0.5$)

H policy	F policy	World loss	H loss	F loss
ZLB	non ZLB	33.27	32.67	0.60
FG 2 extra qtr	non ZLB	28.28	27.72	0.57
FG 4 extra qtr	non ZLB	19.01	18.44	**0.56***
FG 5 extra qtr	non ZLB	**15.65***	**15.06***	0.59
FG 9 extra qtr	non ZLB	55.31	54.18	1.13

Note : In Table 2, we compute the impulse responses for the case where only the home country experiences a negative shock of natural rate of interest and the foreign country does not. The H policy column indicates the type of monetary policy taken by the home country; similarly, the F policy column indicates the type of monetary policy taken by the foreign country; Non ZLB indicates monetary policy without the ZLB constraint of the normal linear model; on the other hand, ZLB indicates the zero lower bound constraint. The welfare loss values shown in the third and fifth columns are calculated from equations (9) through (11) described in Section 3.2. Specifically, the value of the home country's welfare loss is calculated from equation (10), while the foreign country's loss is calculated from equation (11). In addition, the loss of the world economy is calculated from equation (9).

Table 3: Welfare losses under the home FG policy

(a) Country F without ZLB constraint

H policy	F policy	World loss	H loss	F loss
ZLB	non ZLB	27.98	13.75	**14.24***
FG 4 extra qtr	non ZLB	26.22	11.40	14.82
FG 5 extra qtr	non ZLB	**25.17***	**10.17***	15.00
FG 10 extra qtr	non ZLB	36.44	21.67	14.76

(b) Country F with ZLB constraint

H policy	F policy	World loss	H loss	F loss
ZLB	ZLB	27.20	13.60	**13.60***
FG 2 extra qtr	ZLB	26.94	13.19	13.75
FG 4 extra qtr	ZLB	25.38	11.27	14.11
FG 5 extra qtr	ZLB	**24.33***	**10.06***	14.27
FG 10 extra qtr	ZLB	35.59	21.80	13.79

(c) Both central banks adopt the same length of FG

H policy	F policy	World loss	H loss	F loss
ZLB	ZLB	27.20	13.60	13.60
FG 2 extra qtr	FG 2 extra qtr	26.73	13.37	13.37
FG 4 extra qtr	FG 4 extra qtr	23.82	11.91	11.91
FG 6 extra qtr	FG 6 extra qtr	**19.86***	**9.93***	**9.93***
FG 10 extra qtr	FG 10 extra qtr	33.00	16.50	16.50

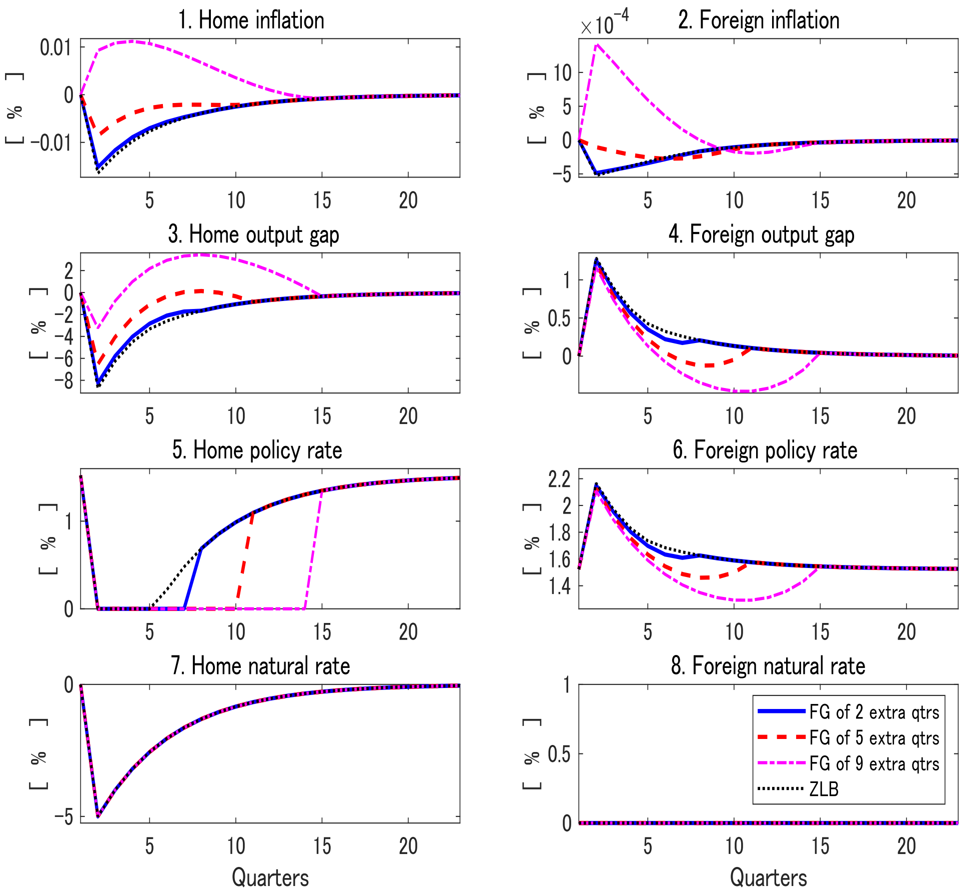
(d) Country F adopt fixed 5 extra quarters FG

H policy	F policy	World loss	H loss	F loss
ZLB	FG 5 extra qtr	24.33	14.27	**10.06***
FG 5 extra qtr	FG 5 extra qtr	21.72	10.86	10.86
FG 6 extra qtr	FG 5 extra qtr	**20.83***	**9.62***	11.20
FG 10 extra qtr	FG 5 extra qtr	32.89	20.79	12.10

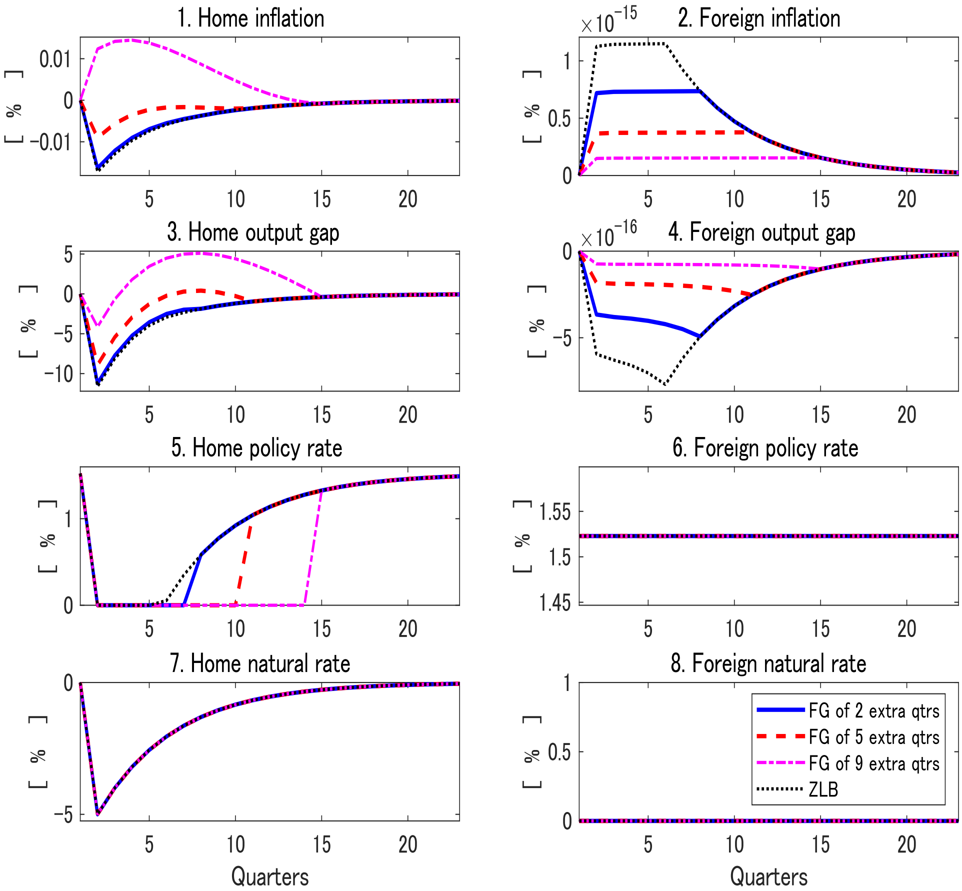
Note: In Table 3, we compute the impulse responses for the case where both the home country and the foreign country experience negative shocks of natural rate of interest with the same magnitude at the same time. However, unlike in Table 2, the size of the intertemporal elasticity of substitution is fixed at 2. The H policy column indicates the type of monetary policy taken by the home country. Similarly, the F policy column indicates the type of monetary policy taken by the foreign country. The notation for each item is as in Table 2.

Figure 1: The impulse response to a natural rate shock of only country H

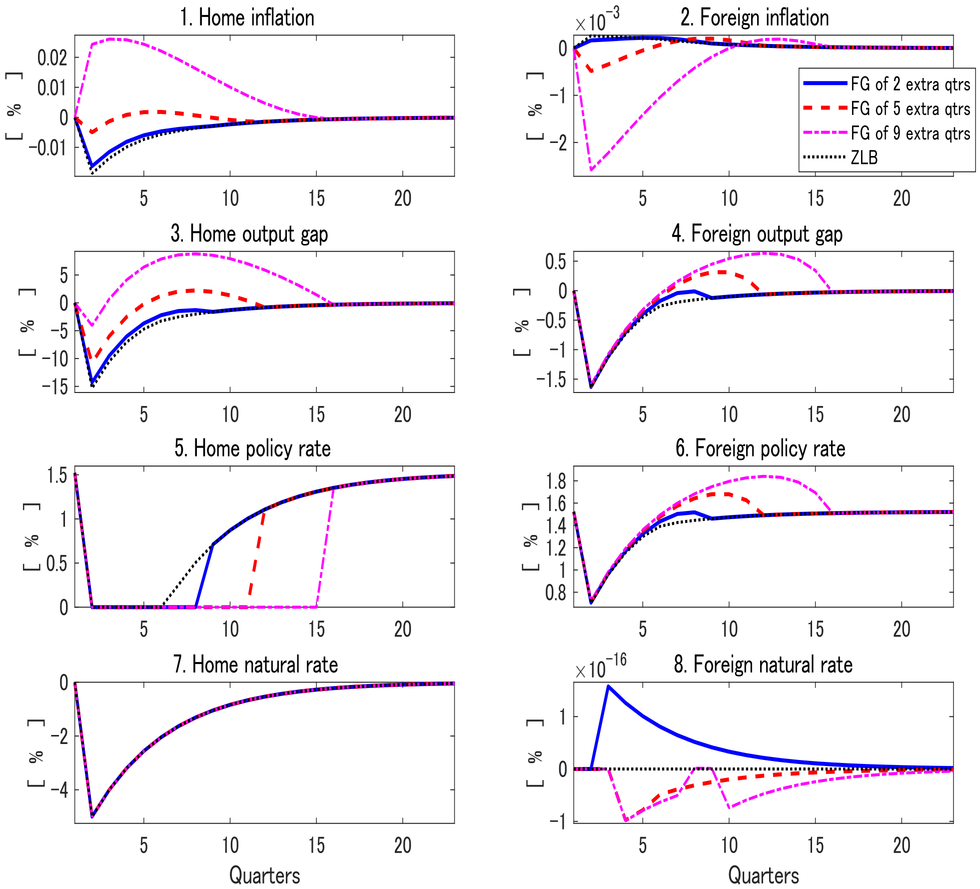
(a) High degree of CRRA ($\sigma = 2$)



(b) Medium degree of CRRA ($\sigma = 1$)



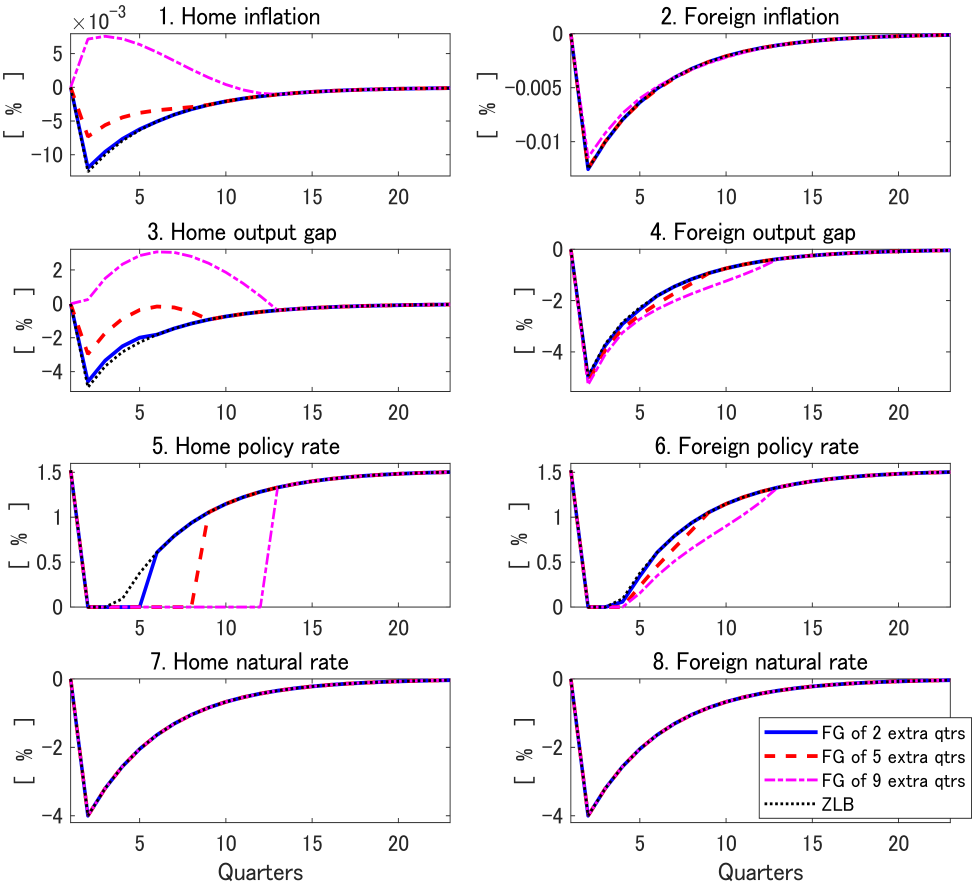
(c) Low degree of CRRA ($\sigma = 0.5$)



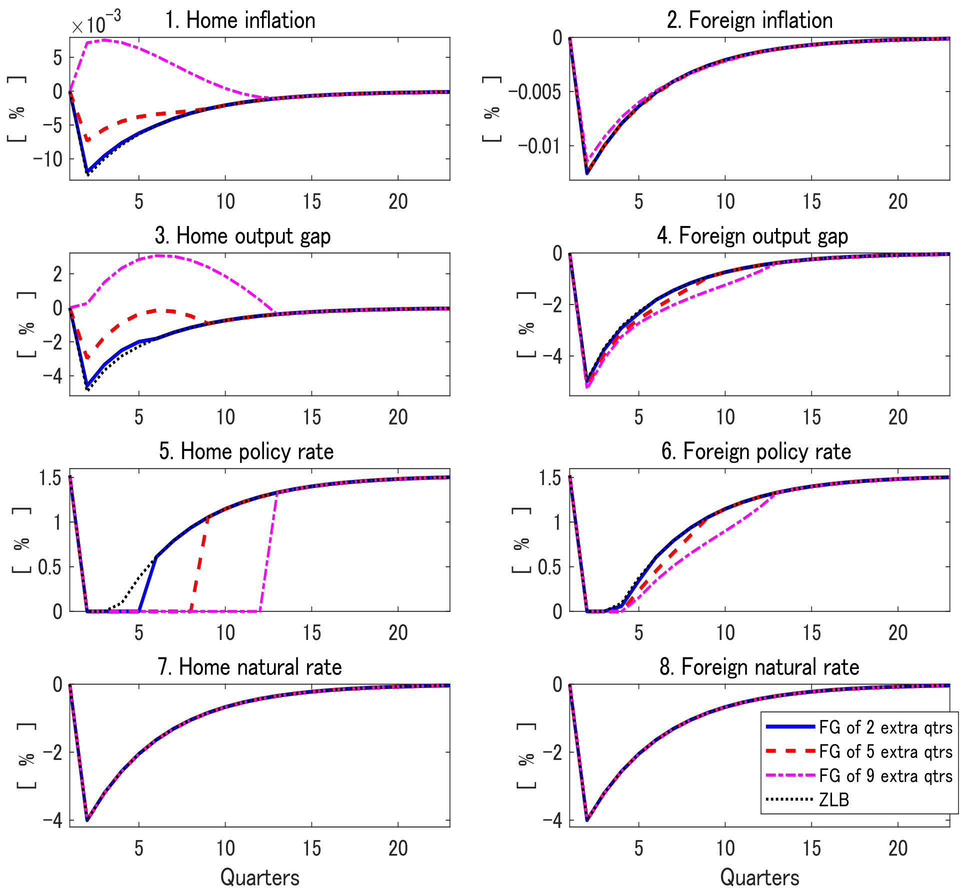
Note: The impulse responses are calculated for the case where negative shocks of natural rate of interest occur only in the home country and no shocks occur in the foreign country. Panels a, b, and c show the case where the intertemporal elasticity of substitution is 2, 1, and 0.5, respectively. It should be noted that in all three cases, no zero lower bound constraint is imposed on the foreign monetary policy.

Figure 2: The impulse response to a global liquidity trap shock

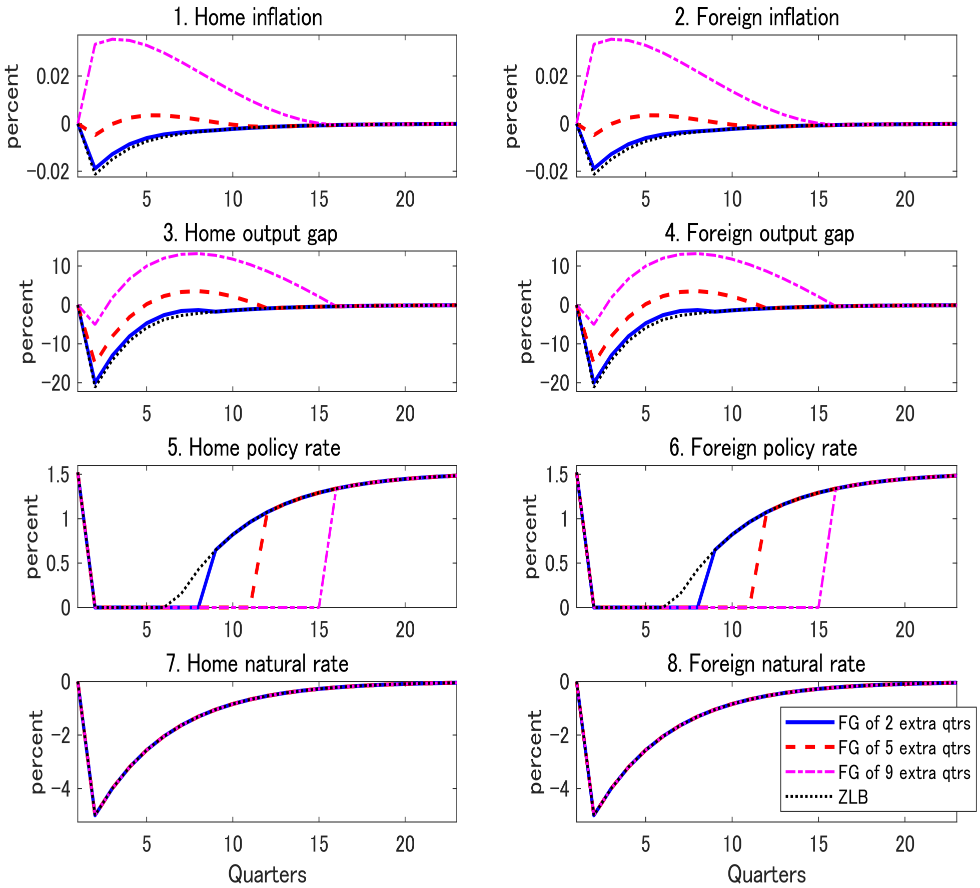
(a) Country F without the ZLB constraint



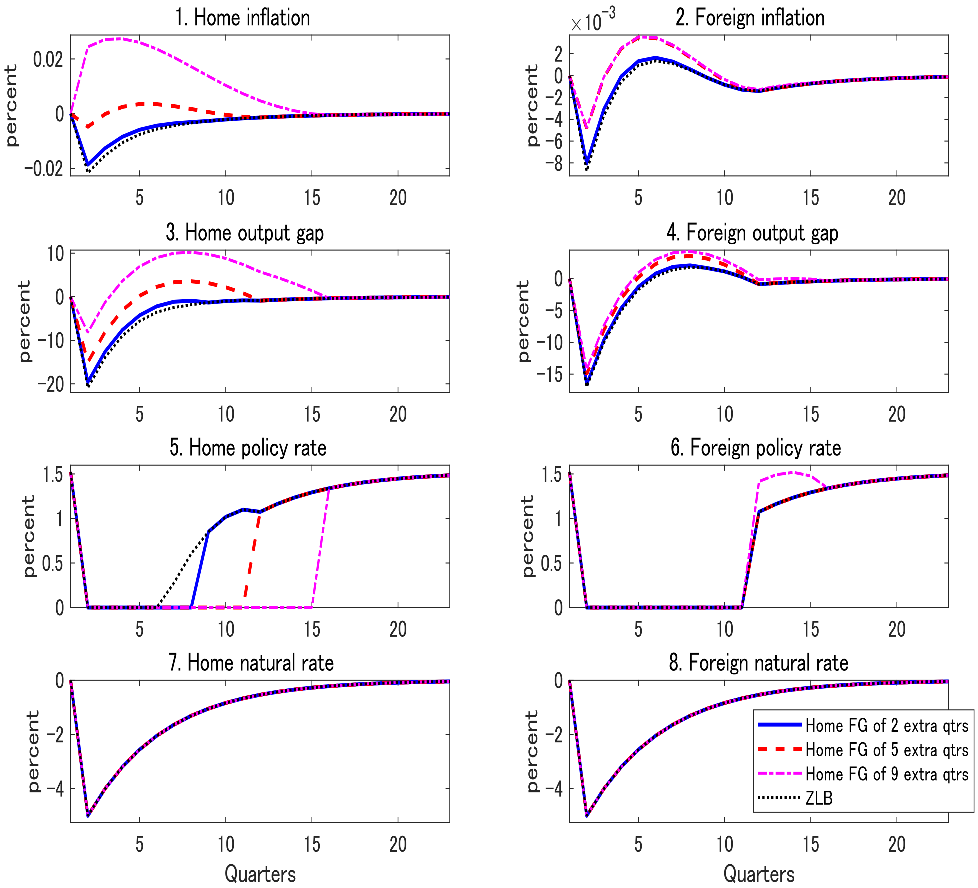
(b) Country F with the ZLB constraint



(c) Both central banks adopt the same length of FG



(d) Country F adopts fixed 5 extra quarters FG

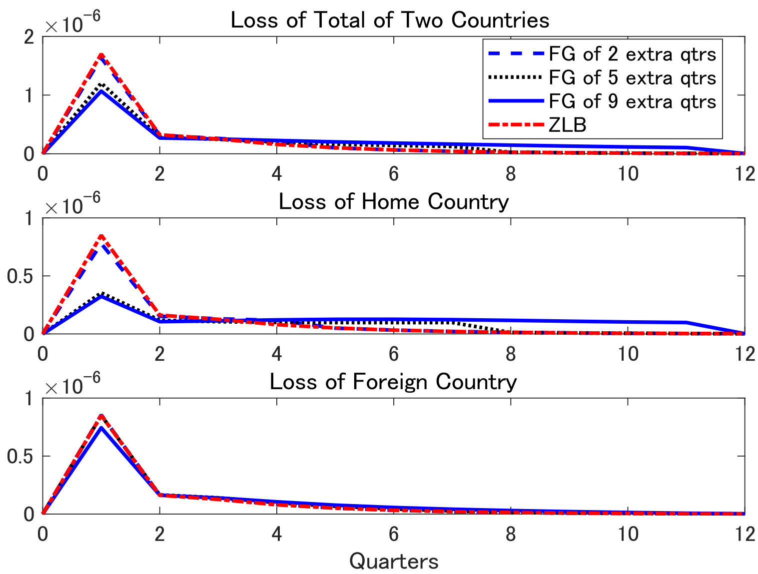


Note: We compute the impulse response in the case where both home and foreign countries experience a negative shock of natural rate of interest with the same magnitude at the same time. However, unlike in Figure 1, the size of the intertemporal elasticity of substitution is fixed at 2. Panels a, b, c, and d show the case where the type of monetary policy taken by the foreign country is no zero lower bound constraint, the case where the type of monetary policy taken by the foreign country is zero lower bound constraint, the case where the two countries implement forward guidance policies for the same period, the case where a foreign country implements a forward guidance policy extended by five quarters, respectively.

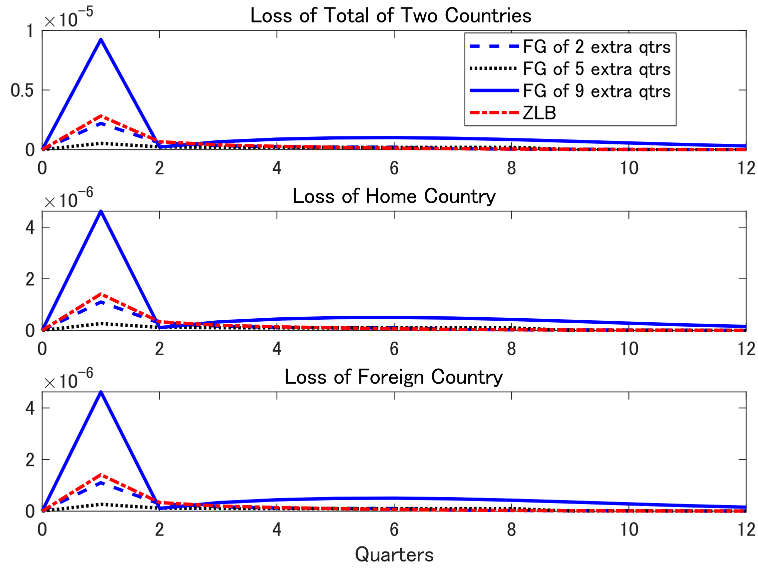
Figure 3: The effect of FG on social welfare

(a) Country F without the ZLB constraint

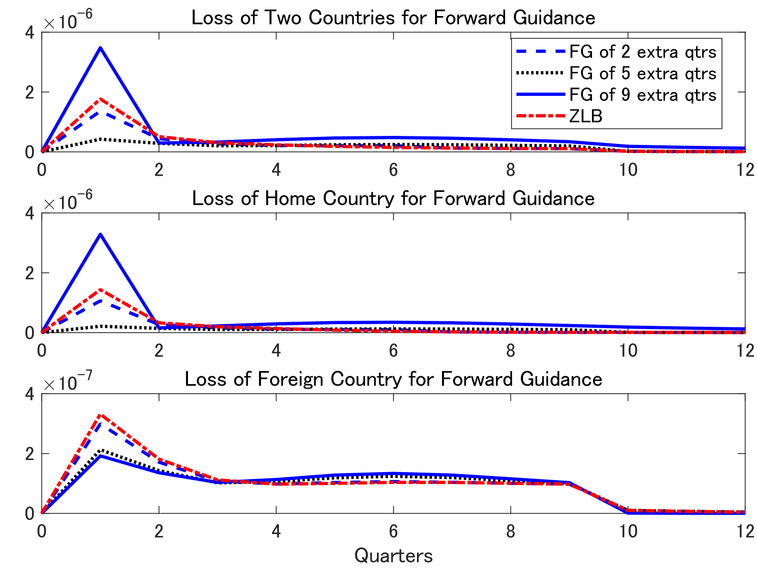
(b) Country F with the ZLB constraint



(c) Both central banks adopt the same length of FG



(d) Country F adopt fixed 5 extra quarters FG



Note: We calculate the value of the welfare loss in the case where both the home country and the foreign country simultaneously experience a negative shock of natural rate of interest with the same magnitude. This value is calculated from equations (9) through (11), which are explained in Section 3.2. Panels a, b, c, and d show the case where the type of monetary policy taken by the foreign country is no zero lower bound constraint, the case where the type of monetary policy taken by the foreign country is zero lower bound constraint, the case where the two countries implement forward guidance policies for the same period, the case where a foreign country implements a forward guidance policy extended by five quarters, respectively. Specifically, the world loss in the upper graph of each panel is calculated from equation (9); the value of the home country's welfare loss in the second graph is calculated from equation (10); and the foreign country's loss in the third graph is calculated from equation (11).

The interaction of forward guidance in a two-country new Keynesian model: Technical appendix

Daisuke Ida[*]and Hirokuni Iiboshi[†]

March 22, 2021

Abstract

In this note, we provide a detailed derivation of a two-country New Keynesian (NK) economy. We derive the structural model and the loss function of the central bank in a two-country NK model. We then provide some additional results that are not reported in the main text.

Keywords: Zero lower bound; Optimal monetary policy; Inflation persistence; Two-country model

JEL classification: E52; E58; F41

[*]Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. E-mail: ida-dai@andrew.ac.jp

[†]Graduate School of Management, Tokyo Metropolitan University, 1-1 Minami-osawa, Hachioji-City, Tokyo 192-0397, Japan. E-mail: iiboshi@tmu.ac.jp

A Model

In this note, we derive a two-country new Keynesian model between the two countries. This model is based on the framework developed by Clarida *et al.* (2002). We consider an economy with two symmetric large countries, a home country and a foreign country. The sizes of the home and foreign economies are $1-\gamma$ and γ, respectively.

There are two production sectors in each country. The final goods sector, which is characterized by perfect competition. The intermediate goods sector faces monopolistic competition and Calvo (1983) type nominal price rigidities. We acknowledge that the degree of price stickiness varies across countries. The number of final goods producers is equal to the number of households in each country. We also assume that there is a complete market in both countries and that only final goods are traded. The case of producer currency prices is assumed, which implies complete pass-through of the exchange rate.

Finally, unless otherwise noted, similar equations hold for foreign countries. Also, note that foreign variables are denoted with an asterisk.

A.1 Households

A.1.1 Preferences

Preferences for consumption in the home country are given by

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{A1}$$

where $C_{H,t}$ is the consumption of domestic goods and $C_{F,t}$ is the consumption of foreign goods. The price index in the home country is given by:

$$P_t = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} = k^{-1} P_{H,t} S_t^{\gamma}, \tag{A2}$$

where $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$, $P_{H,t}$ is the price of domestic goods and $P_{F,t}$ is the price of foreign goods. Also, S_t represents the terms of trade, which is given by

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{A3}$$

A.1.2 Household's optimization problem

The intertemporal utility of an infinitely lived representative household is

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(C_t, \frac{M_t}{P_t} \right) - V(N_t) \right\},$$

where C_t is consumption and N_t is the household's labor supply. We assume that the utility function, $u(\cdot)$, is strictly concave and continuously differentiable, and the disutility of labor supply, $V(\cdot)$, is strictly convex and continuously differentiable. We assume that the utility function of households is separable into consumption and real money balances.

The representative household maximizes the above utility function subject to the following budget constraint:

$$P_t C_t + M_t + E_t[Q_{t,t+1} B_{t+1}] = B_t + M_{t-1} + W_t N_t + \Gamma_t - T_t,$$

where B_t is nominal bonds held for one period, M_t denotes nominal money supply, and W_t and Γ_t are the nominal wage and dividend, respectively, earned from domestic firm. Also, T_t denotes the lump-sum tax.

We assume that a complete market is present in both countries, and introduce the following stochastic discount factor:

$$E_t(Q_{t,t+1}) = \frac{1}{1 + r_t}, \tag{A4}$$

where $Q_{t,t+1}$ denotes a stochastic discount factor and r_t is the risk free short-term nominal interest rate.

We assume that the purchasing power parity condition holds for this economy:

$$P_t = \mathcal{E}_t P_t^*, \tag{A5}$$

where $\mathcal{E}_t$ is the nominal exchange rate and P_t^* is the price level in the foreign country.

The first order conditions of this household's optimization problem are as follows:

$$Q_{t,t+1} = \beta \frac{u_c(C_{t+1}, Z_{t+1})}{u_c(C_t, Z_t)} \frac{P_t}{P_{t+1}}, \tag{A6}$$

$$\frac{u_m(C_t, Z_t)}{u_c(C_t, Z_t)} = \frac{r_t}{1 + r_t}, \tag{A7}$$

$$-\frac{V_n(N_t)}{u_c(C_t, Z_t)} = \frac{W_t}{P_t}, \tag{A8}$$

where $Z_t = M_t/P_t$ denotes real money balances.

Taking the expectation for Eq.(A6),

$$E_t[Q_{t,t+1}] = \frac{1}{1+r_t} = \beta E_t\left[\frac{u_c(C_{t+1}, Z_{t+1})}{u_c(C_t, Z_t)} \frac{P_t}{P_{t+1}}\right]. \tag{A9}$$

In the subsequent discussion, we assume a separable utility function between consumption and real money balances.

A.2 International risk-sharing

Next, we consider a risk-sharing condition between countries. The Euler equation for foreign consumption denominated in home currency is

$$\frac{1}{1+r_t^*} = \beta E_t\left[\frac{u_c(C_{t+1}^*, Z_{t+1}^*)}{u_c(C_t^*, Z_t^*)} \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}}\right]. \tag{A10}$$

As in Clarida *et al.* (2002), we assume that the first order conditions are symmetric across countries and the power parity condition holds. Under the separable utility function between consumption and real money balances, as shown in Clarida *et al.* (2002), we obtain the following result:

$$C_t = C_t^*, \tag{A11}$$

for all t.

A.3 Firms

A.3.1 Final goods firm

The final goods sector is perfectly competitive and producers use inputs that are produced in the intermediate goods sector. In particular, final goods are produced according to the following CES aggregate:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{A12}$$

where Y_t is aggregate output, $Y_t(i)$ is demand for intermediate goods produced by firm i, and θ is the elasticity of substitution. Note that both variables are normalized by $1-\gamma$.

Under the CES aggregate, the demand function is given by

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} Y_t, \tag{A13}$$

and the domestic price level is defined as:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \tag{A14}$$

where $P_{H,t}(i)$ is the prices for intermediate goods produced by the firm i. Note that these variables are also normalized by $1-\gamma$.

A.3.2 The intermediate goods sector

The intermediate goods sector is characterized by monopolistic competition, and each firm produces a differentiated intermediate good. Firm i's production function is given by

$$Y_t(i) = A_t N_t(i), \tag{A15}$$

where A_t denotes an aggregate productivity disturbance.

As in Clarida *et al.* (2002), the intermediate firm's real marginal cost is given as follows:

$$\varphi_t = (1-\tau)\frac{W_t}{P_{H,t}}\frac{1}{A_t}. \tag{A16}$$

Using the household's first order conditions, we can rewrite Eq. (A16) as follows:

$$\varphi_t = \frac{1-\tau}{kA_t}\frac{V_n(N_t)}{u_c(C_t, m_t)}S_t^{\gamma}. \tag{A17}$$

Eq. (A17) reveals that the home real marginal cost depends on the terms of trade in an open economy compared to the closed economy model.

Following Calvo (1983), we assume that price rigidity is present in the intermediate goods sector. The following explanation focuses on the home country. A fraction $1-\alpha$ of all firms adjusts their price while the remaining fraction of firms α do not.

We now consider the intermediate firms that can adjust their price. When revising their prices, these firms take into account uncertainty concerning when they will be able to adjust prices next. In this case, the intermediate firm's optimization problem for the home country is given by

$$E_t \sum_{t=0}^{\infty} (\alpha\beta)^j Q_{t,t+j} Y_{t+j}(i)(P_{H,t}^{opt} - P_{H,t+j}\varphi_{t+j}). \tag{A18}$$

where $P_{H,t}^{opt}$ is the firm's optimal price. The first order condition of this maximization problem is as follows:

$$E_t \sum_{t=0}^{\infty} (\alpha\beta)^j Q_{t,t+j} Y_{t+j}(i)(P_{H,t}^{opt} - (1+\mu)P_{H,t+j}\varphi_{t+j}) = 0. \tag{A19}$$

where the variable $\mu = 1/(\theta - 1)$ is the price mark-up. In particular, when $\alpha = 0$, this equation takes the following form:

$$\frac{P_{H,t}^{o}(i)}{P_{H,t}} = (1+\mu)\varphi_t. \tag{A20}$$

Finally, the price level in the intermediate goods sector is defined as:

$$P_{H,t} = \left[\alpha(P_{H,t-1})^{1-\theta} + (1-\alpha)(P_{H,t}^{opt})^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{A21}$$

A.4 Equilibrium

We now describe the equilibrium conditions in an open economy. The equilibrium conditions for the goods market are given as follows:

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C_{H,t}^*, \tag{A22}$$

$$\gamma Y_t^* = (1-\gamma)C_{F,t} + \gamma C_{F,t}^*. \tag{A23}$$

Since we assume that the elasticity of substitution between home and foreign goods is one, purchasing power parity holds. In this case the real exchange rate is one:

$$\frac{\mathcal{E}_t P_t^*}{P_t} = 1.$$

Furthermore, under the assumptions that the consumption index follows a Cobb-Douglas specification and that the purchasing power parity condition holds, current accounts in both countries always equalize because the ratio of home income to foreign income is constant. As this implies that the trade balance is zero, the following conditions hold:

$$P_{H,t}Y_t = P_t C_t, \tag{A24}$$

$$P^*_{F,t}Y^*_t = P^*_t C^*_t. \tag{A25}$$

In turn, substituting Eq. (A24) into Eq. (A2), we obtain the following equation:

$$Y_t = k^{-1} C_t S_t^{\gamma}. \tag{A26}$$

At this point, the home terms of trade are represented by the ratio of home output to foreign output:

$$S_t = \frac{Y_t}{Y^*_t}. \tag{A27}$$

Eq. (A27) indicates that holding domestic output constant, an increase in foreign output leads to an appreciation of the home terms of trade.

On the other hand, due to complete risk-sharing in both countries, we also obtain the following equation:

$$C_t = k(Y_t)^{1-\gamma}(Y^*_t)^{\gamma}. \tag{A28}$$

According to Eq. (A28), holding home output constant, a rise in foreign output induces an increase in home consumption. Home consumption increases less than a rise in home output because complete risk-sharing leads to consumption smoothing of households. Using the assumption of separable utility between consumption and real balances and substituting Eq. (A28), we can rewrite Eq. (A17) as follows:

$$\varphi_t = \frac{1-\tau}{A_t} \frac{v_n(Y_t/A_t)}{u_c((Y_t)^{1-\gamma}(Y^*_t)^{\gamma})} \left(\frac{Y^*_t}{Y_t}\right)^{\gamma}. \tag{A29}$$

It follows from Eq. (A29) that the home real marginal cost depends not only on domestic output, but also on foreign output. For instance, from Eq. (A27), the terms of trade improve when foreign output increases. The improvement in the terms of trade leads to a decline in the home real marginal cost. Consequently, the decline in home marginal cost induces a decrease in home inflation. This mechanism is referred to as the terms of trade externality. On the other hand, an increase in foreign output pushes the home real marginal cost up due to consumption risk-sharing between countries. As pointed out in Clarida *et al.* (2002), whether which of two effects dominates movements in the home real marginal cost depends on the value of the relative risk aversion coefficient for consumption.

Equilibrium under flexible prices

First of all, $\bar{H}$ represents the value of steady state, H_t^n is the value of efficient level. Also, we define $h_t = \log(H_t/\bar{H})$ as the deviation of H_t from steady state. The log-linearization of the efficient level of domestic output is given by

$$[\sigma + \eta - \gamma(\sigma - 1)]y_t^n + \gamma(\sigma - 1)y_t^{n*} = (1 + \eta)a_t, \tag{A30}$$

$$[\sigma + \eta - (1 - \gamma)(\sigma - 1)]y_t^{n*} + (1 - \gamma)(\sigma - 1)y_t^n = (1 + \eta)a_t^*, \tag{A31}$$

where $\sigma \equiv -u_{cc}\bar{C}/u_c$ and $\eta \equiv -V_{yy}\bar{y}/V_y$. y_t^n denotes the efficient level of domestic output and y_t^{n*} represents the efficient level of foreign output.

Equilibrium under sticky prices: Log-linearilzation

The structural equations in a two-country model are summarized as follows. In this derivation, we define the output gap for both countries as $x_t = y_t - y_t^n$ and $x_t^* = y_t^* - y_t^{n*}$, respectively.

- New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_1 x_t + \kappa_2 x_t^* + u_t, \tag{A32}$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa_1^* x_t^* + \kappa_2^* x_t + u_t^*. \tag{A33}$$

- Dynamic Investment-Saving (IS) curve

$$x_t = E_t x_{t+1} + \vartheta[E_t \Delta x_{t+1}^*] - \sigma_0^{-1}(r_t - E_t \pi_{t+1} - r_t^n), \tag{A34}$$

$$x_t^* = E_t x_{t+1}^* + \vartheta^*[E_t \Delta x_{t+1}] - (\sigma_0^*)^{-1}(r_t^* - E_t \pi_{t+1}^* - (r_t^n)^*). \tag{A35}$$

- Money demand function[1]

$$m_t = \eta_y x_t + \eta_y^* x_t^* - \eta_r r_t, \tag{A36}$$

$$m_t = \eta_y^* x_t^* + \eta_y x_t - \eta_r^* r_t^*. \tag{A37}$$

where

$$\begin{aligned}
\kappa_1 &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\sigma+\eta-\gamma(\sigma-1)),\\
\kappa_2 &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\gamma(\sigma-1),\\
\kappa_1^* &= \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*}(\sigma+\eta-(1-\gamma)(\sigma-1)),\\
\kappa_2^* &= \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*}(1-\gamma)(\sigma-1),\\
\vartheta &= \frac{\gamma(\sigma-1)}{\sigma-\gamma(\sigma-1)}, \vartheta^* = \frac{(1-\gamma)(\sigma-1)}{\sigma-(1-\gamma)(\sigma-1)},\\
\sigma_0 &= \sigma-\gamma(\sigma-1), \sigma_0^* = \sigma-(1-\gamma)(\sigma-1).
\end{aligned}$$

Finally, from the definition of the terms of trade, the nominal exchange rate evolves as follows:

$$\epsilon_t = \epsilon_{t-1} + s_t - s_{t-1} + \pi_t - \pi_t^*, \tag{A38}$$

where ϵ_t ($= \log \mathcal{E}_t$) denotes the logarithm of the exchange rate.

B Central bank's loss function

We now derive the second-order approximation of the household's utility function weighted by degree of openness. The derivation of the central bank's loss function is implemented in the case of policy coordination. The following derivation is based on Clarida *et al.* (2002).

[1]The money demand function is redundant in this paper. As mentioned earlier, this is because the household's utility function is assumed to be separable between consumption and real money balances.

The utility function of the planner is given by

$$W_t = (1-\gamma)\left[u\left(C_t, \frac{M_t}{P_t}\right) - V(N_t)\right] + \gamma\left[u\left(C_t^*, \frac{M_t^*}{P_t^*}\right) - V(N_t^*)\right], \tag{A39}$$

In order to obtain a well-defined loss function, it is necessary to eliminate the distortions caused by monopolistic competition and real money balances. The first distortion is eliminated by an optimal subsidy rate that eliminates the price markup caused by monopolistic competition in each country. At an efficient level,

$$\varphi(Y_t^n, Y_t^n, Y_t^{n*}; A_t) = 1,$$

where $\varphi(Y_t^n, Y_t^n, Y_t^{n*}; A_t)$ represents the real marginal cost under efficient output. The fiscal authority chooses the optimal subsidy rate that restores natural output to an efficient level at zero inflation. As mentioned earlier, such an optimal subsidy rate is given by

$$(1-\tau)\mu = 1, (1-\tau^*)\mu^* = 1,$$

and, therefore, we obtain $u_c\bar{C} = V_n\bar{N}$.

The second distortion is a result of an opportunity cost of holding money. As shown in Woodford (2003), this opportunity cost should be considerably small in steady state to obtain a well-defined loss function of the central bank. In particular, Woodford (2003) argues that real money balances are sufficiently close to being satiated in the optimal steady state. To do so, we can eliminate the distortion produced by the opportunity cost of money.[2]

Before deriving the loss function, we define some notations. First of all, $\bar{H}$ represents the value of steady state, H_t^n is the value of efficient level. Also, we define $h_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state. In addition to these notations, we introduce the following equation:

$$H_t - \bar{H} = \bar{H}\left(\frac{H_t}{\bar{H}} - 1\right) \simeq h_t + \frac{1}{2}h_t^2.$$

[2]See Chapter 6 in Woodford (2003) for a detailed discussion of this issue.

The second-order approximation of the first term of the right hand side is given by:

$$u\left(C_t, \frac{M_t}{P_t}\right) \simeq \quad u_c\bar{C}\left[c_t + \frac{1}{2}(1-\sigma)c_t^2 + s_m m_t + \frac{1}{2}(1-\sigma_m)m_t^2\right] + t.i.p + O(\|\xi\|^3), \tag{A40}$$

where $t.i.p.$ represents the terms that are independent of monetary policy, and $O(\|\xi\|^3)$ indicates that we neglect terms of third or higher order. Also, $m_t = \log(Z_t/\bar{Z})$. In addition,

$$\sigma_m = \frac{u_{mm}\bar{Z}}{u_m}, s_m = -\frac{u_m\bar{Z}}{u_c\bar{C}}$$

and

$$s_m\sigma_m = -(\bar{v}\eta_r)^{-1},\ \eta_y = \bar{v}\chi\eta_r,\ \chi = \frac{u_{cm}\bar{Z}}{u_m}.$$

where $\bar{v}$ is the velocity of money and $\bar{Z} = \bar{M}/\bar{P}$.

Substituting the log-linearization of Eqs. (A28) and (A36) into Eq. (A40), we obtain

$$u\left(C_t, \frac{M_t}{P_t}\right) \simeq u_c\bar{C}\Bigg\{(1-\gamma)y_t + \gamma y_t^* + \frac{1}{2}(1-\sigma)\left[(1-\gamma)^2 y_t^2 + \gamma^2 y_t^{*2} + 2(1-\gamma)\gamma y_t y_t^*\right]$$
$$+ (1-\gamma)(s_m\eta_y((1-\gamma)y_t + \gamma y_t^*) - \eta_r s_m r_t - \eta_i(\bar{v})^{-1}r_t^2 - \chi^2\eta_y^2 c_t^2\Bigg\} + t.i.p. + O(\|\xi\|^3), \tag{A41}$$

Next, the second-order approximations of the second and third terms of the right side of the utility function are given by:

$$V(N_t) = V_n(\bar{N})\bar{(}N)\left[y_t - a_t + \frac{1}{2}(1+\eta)(y_t - a_t)^2 + \frac{\theta}{2}p_{H,t}\right] + t.i.p. + O(||\xi||^3), \tag{A42}$$

$$V(N_t^*) = V_n(\bar{N})\bar{(}N)\left[y_t^* - a_t^* + \frac{1}{2}(1+\eta)(y_t^* - a_t^*)^2 + \frac{\theta}{2}p_{F,t}^*\right] + t.i.p. + O(||\xi||^3), \tag{A43}$$

where $p_{H,t} = \int_0^1 (P_{H,t}(i)/P_{H,t})^{-\theta}\mathrm{di}$ and $p_{F,t}^* = \int_0^1 (P_{F,t}^*(i)/P_{F,t}^*)^{-\theta}\mathrm{di}$.

Combining Eqs.(A41), (A42), and (A43) and using the definition of the natural rate of output for both countries, we obtain

$$U_t \simeq -\frac{u_c\bar{C}}{2}\Bigg\{(1-\gamma)[(\sigma+\eta-\gamma(\sigma-1))(y_t - y_t^n)^2 + (\bar{v})^{-1}\eta_r r_t^2 + \theta p_{H,t}]$$
$$\gamma[(\sigma+\eta-(1-\gamma)(\sigma-1))(y_t^* - y_t^{n*})^2 + (\bar{v})^{-1}\eta_r (r_t^*)^2 + \theta p_{F,t}^*]$$
$$- 2\gamma(1-\gamma)(1-\sigma)(y_t - y_t^n)(y_t^* - y_t^{n*})\Bigg\} + t.i.p. + O(\|\xi\|^3). \tag{A44}$$

In this derivation, we used the relationship $u_c\bar{C} = V_n\bar{N}$, which is held in the efficient steady state. Also, following Woodford (2003), we assumed that the distortion derived from money holding cost is eliminated in this derivation.

Regarding the term for price dispersion, following Woodford (2003), we obtain

$$\sum_{t=0}^{\infty}\beta^t p_{H,t} = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p. + O(\|\xi\|)^3, \tag{A45}$$

$$\sum_{t=0}^{\infty}\beta^t p_{F,t}^* = \frac{\alpha^*}{(1-\alpha^*)(1-\alpha^*\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t^*)^2 + t.i.p. + O(\|\xi\|)^3. \tag{A46}$$

Substituting Eq. (A45) and Eq. (A46) into Eq. (A44), the central bank's loss function under policy coordination is given by

$$\sum_{t=0}^{\infty} W_t \approx -\Omega\sum_{t=0}^{\infty}\beta^t L_t^w + t.i.p. + O(||\xi||^3), \tag{A47}$$

Here, the periodic loss function L_t^w in Equation (A47) is given by

$$L_t^w = (1-\psi)\left[\pi_t^2 + \lambda_x x_t^2 + \lambda_r r_t^2\right] + \psi\left[(\pi_t^*)^2 + \lambda_x^*(x_t^*)^2 + \lambda_r^*(r_t^*)^2\right] - 2\Lambda x_t x_t^*, \tag{A48}$$

where

$$\varpi = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha},\ \varpi^* = \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*},$$

and

$$\begin{aligned}
&1-\psi = \frac{(1-\gamma)\varpi^{-1}}{\varpi},\\
&\varpi = (1-\gamma)\varpi^{-1} + \gamma(\varpi^*)^{-1}\\
&\lambda_x = \frac{\kappa_1}{\theta},\ \lambda_r = \frac{\eta_r}{\bar{v}\theta},\ \lambda_{\Delta\pi} = \frac{(1-\omega)}{\omega\alpha}\\
&\lambda_x^* = \frac{\kappa_1^*}{\theta},\ \lambda_r^* = \frac{\eta_r^*}{\bar{v}^*\theta},\ \lambda_{\Delta\pi}^* = \frac{(1-\omega^*)}{\omega^*\alpha^*}\\
&\Lambda = \frac{2(1-\gamma)\gamma(1-\sigma)}{\varpi\theta}
\end{aligned}$$

C Quantitative results under negative international risk sharing

In this section, we report the results of a simulation of the impact of the home country's monetary policy on foreign economies in the case of a global liquidity trap shock, assuming that households in both countries are risk-lovers. Thus, both countries have a negative international risk-sharing channel. Again, as in our main manuscript, we examine four cases; (a) the foreign country does not face a ZLB constraint, (b) the foreign country faces a ZLB constraint, (c) the foreign country has an FG of the same length as the home country's FG, and (d) the foreign country has an FG of five extra quarters.

The relative risk aversion coefficient is set at 0.5, and the other parameters and the size of the global shock are the same as in our main manuscript. Compared to the case in our main manuscript where households in both countries are risk-averse, the first thing to notice from Table A1 is that the decline in welfare in both countries is more significant in all four cases. Second, as the size of the difference between the responses of ZLB and FG policies can be shown in Figure A1 (a) to (d), the impact of the home country's monetary policy on the foreign country seems to be smaller in the risk-loving case than in the risk-averse case.

Panel (a) of Table A1 shows that when the home country's central bank adopts FG policy instead of ZLB policy, it contributes to improving the foreign country's welfare in the form of *prosper-thy-neighbor effect*. In this case, if the home country's central bank implements a five-extra-quarters of FG policy, the foreign country's loss (welfare) is minimized (maximized). The other three cases shown in Panels (b), (c), and (d) are similar to Panel (a), where the foreign country's loss (welfare) is minimum (maximum) if the home country's central bank adopts a five-extra-quarters of FG policy.

Thus, even when households in both countries behave as risk-lover, both central banks can improve their countries' welfare by adopting FG policies when a global liquidity trap occurs. Moreover, since both countries' interests are aligned, the choice of monetary policy will become monotonic. In this case, bargaining and betrayal of determining the length of FG are not beneficial. The coordination of monetary policies is sustained in both countries, unlike the conclusion of our main manuscript.

References

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, **12** (3), 383–398.

Clarida, R., Gali, J. and Gertler, M. (2002). A simple framework for international monetary policy analysis. *Journal of monetary economics*, **49** (5), 879–904.

Woodford, M. (2003). *Interest and prices: Foundations of a theory of monetary policy*. princeton university press.

Table A1: Welfare Losses by a Global Liquidity Trap Shock

(a) Country F without the ZLB constraint

H Policy	F Policy	World Losses	H Loss	F Loss
ZLB	non ZLB	43.53	20.58	22.95
FG 2 extra qrts	non ZLB	40.96	18.24	22.72
FG 4 extra qrts	non ZLB	35.42	12.91	22.51
FG 5 extra qrts	non ZLB	**33.84**	**11.28**	**22.55**
FG 6 extra qrts	non ZLB	34.96	12.23	22.73

(b) Country F with the ZLB constraint

H Policy	F Policy	World Losses	H Loss	F Loss
ZLB	ZLB	47.12	23.56	23.56
FG 2 extra qrts	ZLB	44.08	20.89	23.20
FG 4 extra qrts	ZLB	37.70	14.75	22.95
FG 5 extra qrts	ZLB	**35.62**	**12.57**	**23.05**
FG 6 extra qrts	ZLB	36.20	12.87	23.32

(c) Countries H and F adopt the Same Length of FG

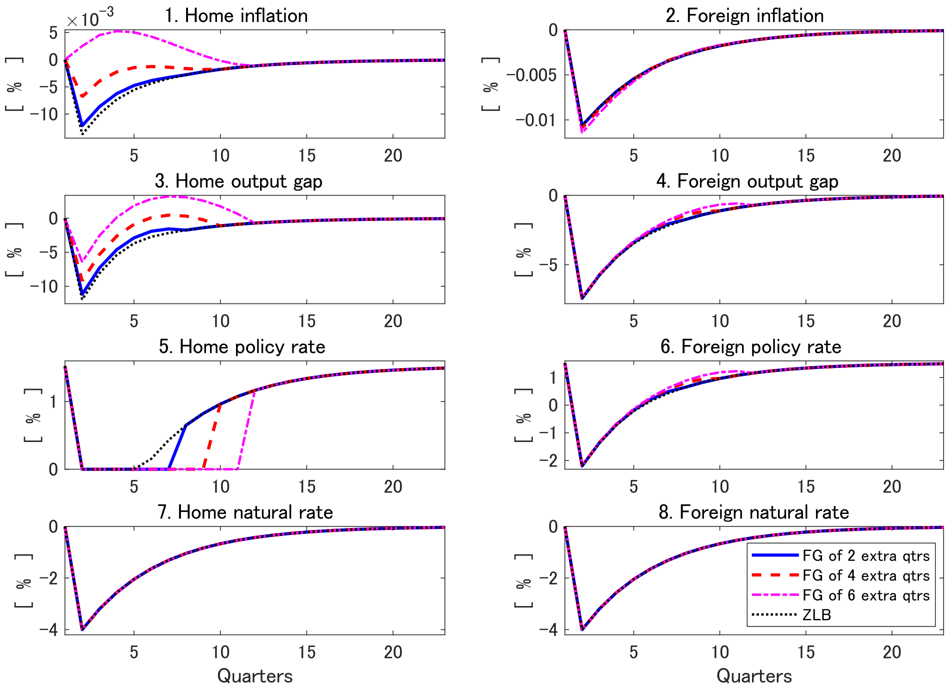
H Policy	F Policy	World Losses	H Loss	F Loss
ZLB	ZLB	47.12	23.56	23.56
FG 2 extra qrts	FG 2 extra qrts	39.75	19.87	19.87
FG 4 extra qrts	FG 4 extra qrts	25.11	12.56	12.56
FG 5 extra qrts	FG 5 extra qrts	**23.29**	**11.65**	**11.65**
FG 6 extra qrts	FG 6 extra qrts	32.05	16.03	16.03

(d) Country F takes fixed 6 extra qrts FG

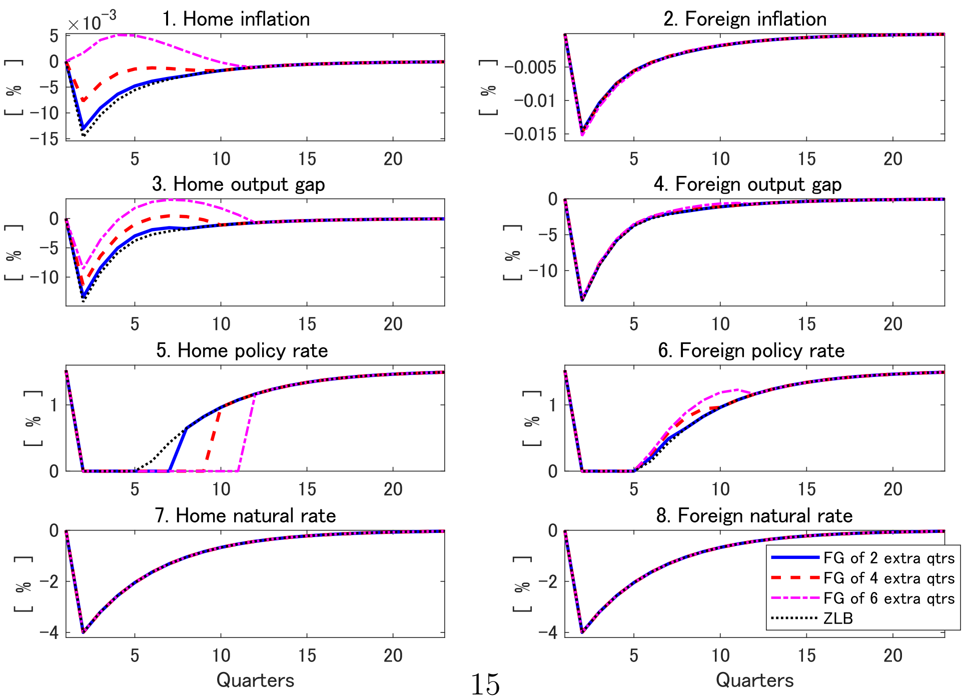
H Policy	F Policy	World Losses	H Loss	F Loss
ZLB	FG 5 extra qrts	35.62	23.05	12.57
FG 2 extra qrts	FG 5 extra qrts	31.04	18.87	12.17
FG 4 extra qrts	FG 5 extra qrts	23.84	12.17	11.67
FG 5 extra qrts	FG 5 extra qrts	**23.29**	**11.65**	**11.65**
FG 6 extra qrts	FG 5 extra qrts	26.30	14.63	11.67

Figure A1: IRFs to the Global Liquidity Trap Shock under negative international risk sharing

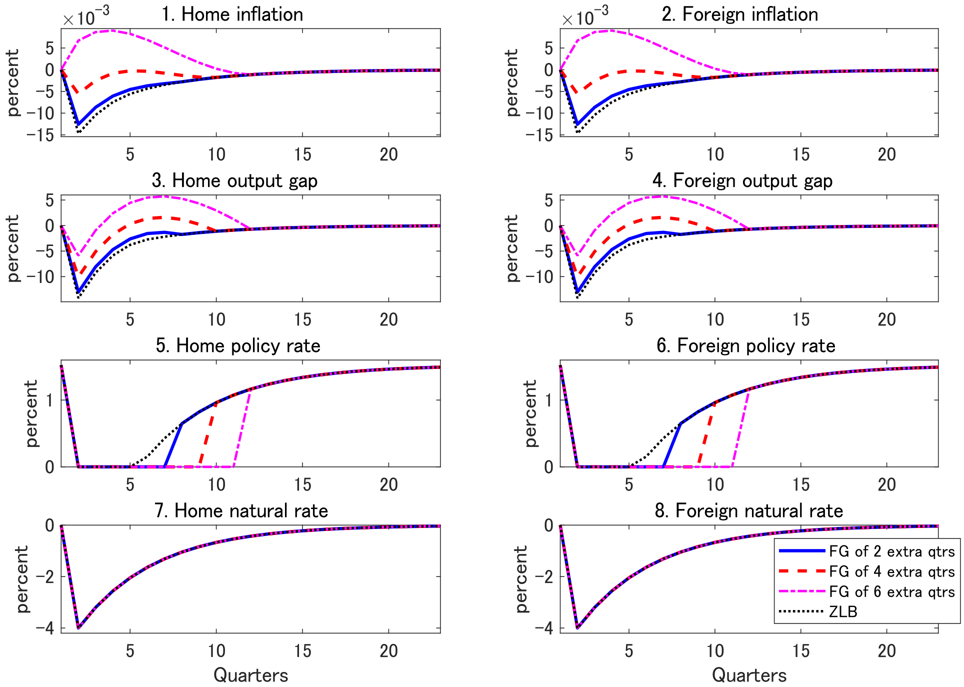
(a) Country F without the ZLB



(b) Country F with the ZLB



(c) Country H and F adopt the same length of FG



(d) Country F takes fixed 6 extra qrts FG