Empirical Testing of the Lucas Critique for the U.S. Economy, 1986 – 2005

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I. Introduction

Lucas critique suggests parameter instability in usual policy multiplier based models, where the multipliers are estimated by running a regression of output on the relevant/hypothesized policy variables. I aim to test this implication for the U.S. economy using a simple aggregate demand and supply model, with rational expectations as the mechanism of expectation formation, given a money supply specification. The relevant data set (for inferential purposes) is quarterly, 1986:1 – 2005:4, with the exact model specification chosen from Heijdra and Ploeg.

Lucas's suggestion of parameter instability is derived from the underlying instability in the relevant parameters that determine the policy multipliers. Therefore, based on one of the following four outcomes, one may conclude the following:

a. Parameter stability in both the final equation and the relevant equations[1] that determine the output: Lucas critique cannot be supported by this technique of analysis;
b. Parameter stability in the final equation, parameter instability in the relevant equations that determine the output: Lucas critique cannot be supported by this technique of analysis;
c. Parameter instability in the final equation, parameter stability in the relevant equations that determine the output: Lucas critique cannot be supported by this technique of analysis;
d. Parameter instability in the final equation, parameter instability in the relevant equations that determine the output: Lucas critique cannot be ruled out by this technique of analysis.

Further, the final estimated equation for output must satisfy certain parameter restrictions that are imposed by the structure of the model, since in this case it is estimated independently of other equations in the model, using tests of misspecification.

II. Data Set

The data set consists of quarterly time series on money, output and price level from 1985 to 2006. For the money supply, the relevant measure is the seasonally adjusted M1, and the data source is the Bureau of Economic Analysis, U.S. Department of Commerce. For the price level, the unchained Urban CPI (all items, U.S. city average) is taken as the proxy with 1982-84 as the base year. This series is not seasonally adjusted. The source is the U.S. Bureau of Labor Statistics. For output, the nominal GDP (in billions of dollars) is considered. The time series is seasonally adjusted. The source is the Bureau of Economic Analysis, U.S. Department of Commerce.

The original output series is quarterly, whereas the original price series and the money supply series are annual. Thus, to convert them into a quarterly series, I consider the corresponding value in the third, sixth, ninth and the twelfth months of each year.

To seasonally adjust the price time series, I consider two methods: Firstly, price (P) is regressed on variables D2, D3, D4 with an intercept, where D2 is the dummy that takes the value of 1 in the second quarter and 0 in the other quarters. Similarly, D3 and D4 are defined, so that the intercept term measures the estimated price level in the first quarter (Range: 1985:1 to 2005:4). However, this regression yields a very low Durbin-Watson (DW) Statistic of 0.007493. The residuals show a distinctly upward trend. Therefore, estimating the similar equation with an autoregressive error of first order, AR(1), I find absence of both heteroskedasticity (White's test) and autocorrelation (Breusch-Godfrey's LM test, DW statistic). Gujarati suggests that the residuals of such a regression must be added to the mean value of P to get the seasonally adjusted series. However, the problem with this procedure is that such a series does not show any upward trend, unlike the original series. If one applies the same procedure using the first equation, then although the trend is obtained, but the errors are autocorrelated. I therefore refrain from following this method, especially since in Gujarati's illustration, there was no specification error. It should be noted that whereas in the earlier case, coefficients of D2, D3 and D4 were statistically insignificant, in the second case, only the coefficient of D4 was statistically significant. This is another reason for not using the earlier equation. Further, this clearly points out that the price level in the fourth quarter is correlated with that in the next fourth quarter, and so on. Therefore, my second method for seasonal adjustment is to regress P on P_{-4}. In this case, the DW statistic was very low (0.228075). Therefore, I regressed P on P_{-4} assuming AR(1) error term. In this case, the DW statistic is 1.901727. The equation passes both the tests of autocorrelation and White's test of heteroskedasticity assuming cross-terms. To arrive at the seasonally adjusted values, I obtain a static forecast of this equation. The sample for regression is 1986:1 to 1995:4. For the second period, the DW statistic for this equation is 2.148295. However, this equation does not pass the White's test of heteroskedasticity, assuming cross-terms. Whereas the Obs*R-squared is statistically significant, the coefficients of the regressor and its square are insignificant:

White Heteroskedasticity Test:

F-statistic	4.080233	Probability	0.025284
Obs*R-squared	7.206857	Probability	0.027230

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares

Sample: 1996:2 2005:4
Included observations: 39

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	151.1953	106.3820	1.421249	0.1638
P(-4)	-1.843817	1.245163	-1.480784	0.1474
P(-4)^2	0.005628	0.003630	1.550346	0.1298

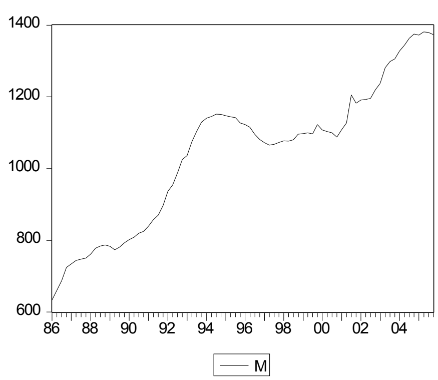
R-squared	0.184791	Mean dependent var	1.210286
Adjusted R-squared	0.139502	S.D. dependent var	2.742927
S.E. of regression	2.544423	Akaike info criterion	4.779488
Sum squared resid	233.0671	Schwarz criterion	4.907454
Log likelihood	-90.20002	F-statistic	4.080233
Durbin-Watson stat	2.609302	Prob(F-statistic)	0.025284

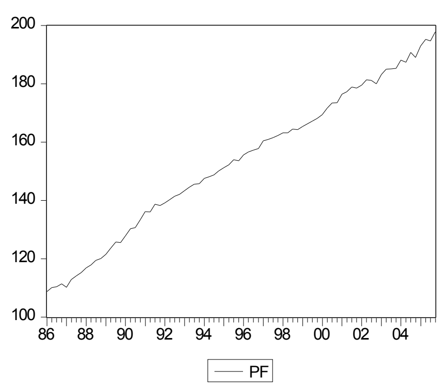
This implies that the error term may be correlated with the regressor. However, this is not the case, since the Ramsey's RESET test is passed. Both the F-statistic and the log- likelihood ratio suggest that the fitted values of P (to two terms) are not correlated with P. Therefore, a final implication is that the linear specification of the model may be incorrect. In any case, I follow this equation to maintain homogeneity in the techniques for seasonal adjustment.

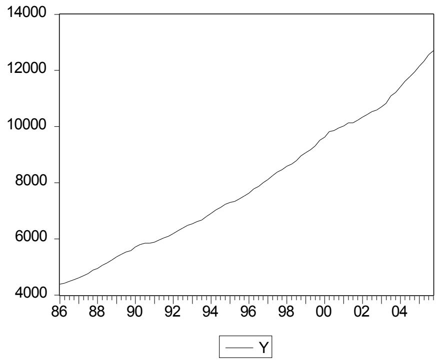
Two notes on forecasting the equation are important: Firstly, I perform a static forecast, i.e. the previous values of P used for the forecast, are actual and not the estimated values. Second, I include the observations for the year 1985 to get the estimates for the year 1986. This does not provide the forecast for 1986:1 due to absence of the value for 1984:4, so I substitute its actual value. Values for the year 1985 are not seasonally adjusted. In both the time periods, the equation forecast is very good, with very low bias proportion and variance proportion. The seasonally adjusted series is labeled PF.

III. Detecting Structural Change

The following are the graphs for money (M), seasonally adjusted prices (PF) and output (Y) for the time period 1986:1 to 2005:4:







There is no evidence of structural break in these graphs. The graphs of PF and Y show an upward trend. The graph of M shows a decline in the money supply from around 1994:1 but the decline is not abrupt and money supply starts rising from 1997:1. However, this shift in money supply after 1994:1 may account for evidence of greater parameter instability in the second period.

Therefore, tests for parameter instability may be used to detect shifts in policy multipliers, knowing that they are not caused by a structural break in M, PF or Y. (Johnston and Dinardo)

IV. Evaluation of the data set

For the first time period, 1986:1 – 1995:4, I regress log(Y) on log(Y(-1)), log(M) and log(M(-1)) assuming an intercept term, since this is the equation that determines the relevant multipliers in the model. The method used is OLS. It is observed that the equation passes both the tests of autocorrelation and heteroskedasticity. The DW statistic is 1.71073, and from the White's test of heteroskedasticity, the Obs*R-squared has a p-value of 0.922156. However, the F-statistic and the log likelihood ratio in the Ramsey's RESET test (null hypothesis: coefficients of fitted log(Y) raised to powers are zero) have a p-value lower than 0.05 suggesting some variable endogeneity. This is true for both the cases, i.e., when the order of predicted log(Y) is 1 and when it is 2. Thus, I perform Hausman test for the three variables, log(Y(-1)), log(M) and log(M(-1)) separately. This was done by regressing the suspect variable on its instrument (I take the lagged value as the instrument) and the other exogenous variables, assuming an intercept term. The residual series of this OLS regression was one of the regressors in the original equation. According to the test, the coefficient of the residual must be insignificant for the variable to be considered exogenous, at a given significance level (Here, 0.05). The results are summarized for the three variables below:

Variable	t-statistic of the coefficient of the corresponding residual series	p-value
log(Y(-1))	-0.995695	0.3264
log(M)	-2.091756	0.0442
log(M(-1))	-1.310059	0.1992

This suggests that only log(M) should be considered endogenous. Therefore, I use the two stage least squares method to estimate the original equation, using log(M(-2)) as the instrument for log(M). The new equation satisfies the Breusch-Godfrey's test of serial correlation and White's test of heteroskedasticity. The results from the estimation are given below:

Dependent Variable: LOG(Y)
Method: Two-Stage Least Squares

Sample(adjusted): 1986:3 1995:4
Included observations: 38 after adjusting endpoints
Instrument list: LOG(Y(-1)) LOG(M(-2)) LOG(M(-1))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.216690	0.059334	3.652012	0.0009
LOG(Y(-1))	0.951166	0.016696	56.96944	0.0000
LOG(M)	-0.071880	0.077696	-0.925151	0.3614
LOG(M(-1))	0.104543	0.078451	1.332588	0.1915

R-squared	0.999189	Mean dependent var	8.685645
Adjusted R-squared	0.999118	S.D. dependent var	0.152944
S.E. of regression	0.004543	Sum squared resid	0.000702
F-statistic	13969.77	Durbin-Watson stat	1.765163
Prob(F-statistic)	0.000000		

White's test: the Obs*R-squared has a p-value of 0.871991 and the F statistic has a p-value of 0.911653. Breusch-Godfrey test: Obs*R-squared has a p-value of 0.724011.

To test for stability, I conduct the Chow's breakpoint test and the Chow's forecast test. In the first test, the null hypothesis is that the corresponding parameters in the equations of the separated data sets are equal. In the second test, the regression from the first sub data set is used to forecast the values of the dependent variable in the second sub data set. A significant deviation in prediction suggests parameter instability. For the breakpoint test, this requires rejection of the null hypothesis. Both tests use the F statistic (formulas seen in the E-Views help). I take the separating date as 1991:1. The result of the breakpoint test is:

Chow Breakpoint Test: 1991:1			
F-statistic	0.580445	Probability	0.679095

For the forecast test, the result is:

Chow Forecast Test: Forecast from 1991:1 to 1995:4			
F-statistic	0.339498	Probability	0.986319

Both the tests provide evidence for parameter stability, at 5% level of significance. The F-statistic for the null hypothesis of the equality of error variances is 1.9244. It is not significant at the 5% significance level with degrees of freedom, 18, 18. Thus, the results from the two tests are accurate at this significance level.

For the second time period, 1996:1 – 2005:4, I regress log(Y) on log(Y(-1)), log(M) and log(M(-1)) by OLS, assuming an intercept term. In this case, the estimated equation satisfies the White's test of heteroskedasticity and the Breusch-Godfrey's test of serial correlation at the 5% level of significance. The DW statistic is 2.024726. However, the Ramsey's RESET test gives some conflicting results. If the order of fitted log(Y) is 1, i.e., the estimated log(Y) is raised to the power of 2, then there is no evidence of misspecification/endogeneity at 5% level of significance. The p-value of the F statistic is 0.805711, and the p-value of the log likelihood ratio is 0.791141. However, if the order of fitted log(Y) is raised to 2 and the test is carried out again, then we find that the p value of the log likelihood ratio is only marginally above 0.05. It is 0.51732. The p-value of the F statistic is 0.080666. Looking at the significance of the coefficients, the p-value of the coefficient of the fitted log(Y) raised to the power of 3 is 0.0268, and for the lower order of the power of 2 is 0.0267. This test result is more compelling than the previous one because it suggests significance of the coefficients of log(Y(-1)), log(M) and log(M(-1)), unlike in the previous test in which these were all not significant. Therefore, I again perform the Hausman test for the three variables log(Y(-1)), log(M) and log(M(-1)) separately. The results are:

Variable	t-statistic of the coefficient of the corresponding residual series	p-value
log(Y(-1))	-2.438942	0.0199
log(M)	-1.399726	0.1704
log(M(-1))	-1.870767	0.0698

Therefore, at the 5% level of significance, only log(Y(-1)) can be considered endogenous. Thus, I estimate the original equation using the two stage least squares method, using log(Y(-2)) as the instrument for log(Y(-1)). The Breusch-Godfrey test for serial correlation gives a bit conflicting result: the p-value of the F statistic is 0.000001; however, the p-value of the Obs*R-squared is 0.168462. Further, the coefficients of resid(-1) and resid(-2) are both non-significant at the 5% level when the residual series is regressed on all the regressors, resid(-1) and resid(-2), assuming an intercept term. Therefore, it can be concluded that autocorrelation is not of at least the second order. The same significance results are obtained when the order of residual lags is extended to 3, and to 4. Thus, although a higher order autocorrelation cannot be ruled out at the 0.05 significance level, the bias in the estimates of the coefficients in the original equation may not be too large. For White's test of heteroskedasticity, the p-value of Obs*R-squared is 0.372014. In fact, all the coefficients in the regression of residual series on the cross-products of the regressors are non-significant at the 0.05 significance level. The results from the estimation of the original equation using the two stage least squares method are:

Dependent Variable: LOG(Y)
Method: Two-Stage Least Squares

Sample: 1996:1 2005:4
Included observations: 40
Instrument list: LOG(Y(-2)) LOG(M(-1)) LOG(M)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.074814	0.056601	-1.321793	0.1946
LOG(Y(-1))	0.974653	0.010825	90.03681	0.0000
LOG(M(-1))	0.074482	0.048229	1.544353	0.1312

LOG(M)	-0.029079	0.050860	-0.571744	0.5710
R-squared	0.999138	Mean dependent var		9.196576
Adjusted R-squared	0.999066	S.D. dependent var		0.143996
S.E. of regression	0.004401	Sum squared resid		0.000697
F-statistic	13891.68	Durbin-Watson stat		2.024463
Prob(F-statistic)	0.000000			

To test for parameter stability, I again conduct the Chow's breakpoint test and Chow's forecast test. The date used for separation is 2001:1. The result from the Chow's breakpoint test is:

Chow Breakpoint Test: 2001:1			
F-statistic	11.32406	Probability	0.000008

The result from the Chow's forecast test is:

Chow Forecast Test: Forecast from 2001:1 to 2005:4			
F-statistic	3.297722	Probability	0.009429

In both the cases, the hypothesis of parameter stability is rejected at the 0.05 significance level. The F-statistic for the null hypothesis of the equality of error variances is 1.18604 approx. It is not significant at the 5% significance level with degrees of freedom, 20, 20. Thus, the results from the two tests are accurate at this significance level. As said earlier, this difference in the conclusion on parameter stability for the two sub-periods may be due to a fall in the money supply around the year 1994, though this does not suggest a structural break, based on the graph of the money supply for the whole period.

I now conduct parameter stability tests for the entire time period, 1986:1 – 2005:4. I regress log(Y) on log(Y(-1)), log(M), log(M(-1)) using OLS and assuming an intercept term. This equation passes the White's test of heteroskedasticity at the 5% level: the p-value of Obs*R-squared is 0.803076. If the residual lag is taken as 1, then for the Breusch-Godfrey's test of serial correlation, the p-value of the F-statistic is 0.069499, and for the Obs*R-squared is 0.062840. This suggests absence of AR(1) error. Further, the coefficient of resid(-1) is non-significant at the 0.05 significance level when the residual series is regressed on the regressors and resid(-1) assuming an intercept term. However, if the residual lag is extended to 2, then for this test, we find p-values lower than 0.05 for the F-statistic and the Obs*R-squared. Further, the coefficient of resid(-2) has a p-value of 0.0243, suggesting the presence of AR(2) error term. For the Ramsey's RESET test, the p-values of the F-statistic and the log likelihood ratio are less than 0.05 for both the order of fitted terms, 1 and 2. However, in the first case, when log(Y) is regressed on all the regressors and the fitted value of log(Y) raised to the power of 2, the coefficients are non-significant at the 0.05 significance level. In the second case, the p-values of the F-statistic and the log likelihood ratio are higher than in the previous case, but still are significant at the 0.05 significance level. However, the coefficients of the fitted values are still not significant at the 0.05 significance level. Thus, to check for endogeneity, I perform the Hausman's test separately for log(Y(-1)), log(M) and log(M(-1)). The results are shown below:

Variable	t-statistic of the coefficient of the corresponding residual series	p-value
log(Y(-1))	-1.868934	0.0655
log(M)	-0.760112	0.4496
log(M(-1))	-0.823885	0.4126

Thus at the 0.05 significance level, no variable can be considered endogenous and the estimation method remains of the OLS. However, note that it was earlier concluded that in this case, the error term is AR(2). Thus, I consider two cases: Firstly, I regress log(Y)-log(Y(-2)) on the corresponding second order differenced regressors, assuming OLS. Secondly, I regress log(Y)-log(Y(-1))-log(Y(-2)) on the correspondingly differenced regressors, assuming OLS. Note that the intercept term vanishes in both the cases, but our estimates of the regression coefficients must reflect the true estimates of the policy multipliers since they do not get changed in this process (assuming that the requisite specification tests are passed). The results are summarized below:

CASE 1	CASE2
White's test: p-value of Obs*R-squared: 0.401899; in the regression of the square of the residual series on the regressors, no coefficient is significant at the 0.05 level.	White's test: p-value of Obs*R-squared: 0.089836; in the regression of the square of the residual series on the regressors, some coefficients are significant at the 0.05 level. However, at best, the bias in the estimates is not large since the Obs*R-squared is not significant at the 0.05 level.
DW statistic: 1.676031, Breusch-Godfrey test: For residual lag of 1, the Obs*R- squared has a p-value	DW statistic: 1.597687. Breusch-Godfrey test: For residual lag of 1, the Obs*R- squared has a p-value

of 0.217558. For the lag of 2, it has a p-value of 0.000874. Also, the coefficient of resid(-2) is significant at the 0.05 significance level.	of 0.092107. For the lag of 2, it has a p-value of 0.132655. Also, the coefficients of resid(-1) and resid(-2) are not significant at the 0.05 significance level, suggesting absence of an AR(2) error.
Ramsey's RESET test: When the order of the fitted terms is 1, the p-values of both the F-statistic and the log likelihood ratio are less than 0.05. When the order of the fitted terms is 2, the p-values further decline for both the F-statistic and the log likelihood ratio.	Ramsey's RESET test: When the order of the fitted terms is 1, the p-value of the F-statistic is 0.157880 and of the log likelihood ratio is 0.146033. When, the order of the fitted terms is 2, the p-values decline for both but are still higher than 0.05, suggesting that the F-statistic and the log likelihood ratio are not significant. Also, the coefficients of the fitted values are not significant at the 0.05 level.

Note that in the comparison for the White's test, the cross-products of the regressors are not taken into account, due to the differencing of the variables[2]. Therefore, I use the second equation to test for parameter stability. Its regression estimates and related results are shown below:

Dependent Variable: LOG(Y)-LOG(Y(-1))-LOG(Y(-2))
Method: Least Squares

Sample: 1986:1 2005:4
Included observations: 80

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(Y(-1))-LOG(Y(-2))-LOG(Y(-3))	0.990420	0.007205	137.4586	0.0000
LOG(M)-LOG(M(-1))-LOG(M(-2))	-0.012619	0.036113	-0.349425	0.7277
LOG(M(-1))-LOG(M(-2))-LOG(M(-3))	0.026951	0.036154	0.745449	0.4583
R-squared	0.999419	Mean dependent var		-8.893266
Adjusted R-squared	0.999404	S.D. dependent var		0.307619
S.E. of regression	0.007509	Akaike info criterion		-6.908627
Sum squared resid	0.004342	Schwarz criterion		-6.819301
Log likelihood	279.3451	Durbin-Watson stat		1.597687

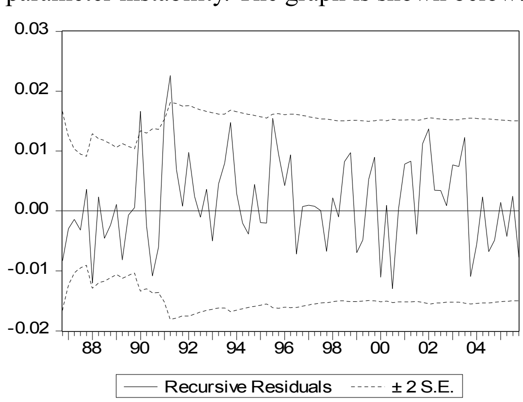
I use the date 1996:1 as the separation in this data set for the Chow's tests. The result of the Chow's breakpoint test is:

Chow Breakpoint Test: 1996:1			
F-statistic	0.380278	Probability	0.767494
Log likelihood ratio	1.223927	Probability	0.747271

The result of the Chow's forecast test is:

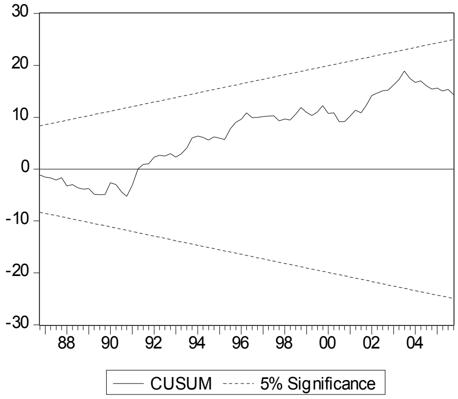
Chow Forecast Test: Forecast from 1996:1 to 2005:4			
F-statistic	0.724123	Probability	0.840944
Log likelihood ratio	46.25640	Probability	0.229734

Thus, both tests indicate parameter stability over the 20 year period at the 5% level of significance. Next, I use the recursive residuals test for stability. In this test, the recursive residuals are computed from t=4 to t=20, since 3 observations are the minimum required to estimate the three regression coefficients. E-Views gives a plot of these residuals along with a +2 S.E., -2S.E. band. Residuals lying outside this band indicate parameter instability. The graph is shown below:

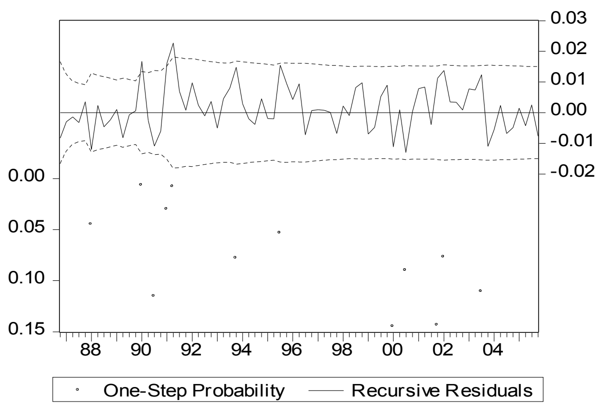


This suggests parameter instability in the period 1990:2 - 1991:1. Next, I perform the CUSUM test. E-

Views plots the cumulative sum of squares of the recursive residuals divided by the standard error of regression for the sample data set. It also plots the 5% critical value lines. If the values of this statistic go beyond these lines, then parameter instability is concluded for those time-periods. The graph is:



The above graph does not provide evidence of parameter instability for the 20 year period. Finally, I use the One-step forecast test. In this test, the recursive residuals are plotted for the entire period as before. Further, the graph shows the p-values for those points where the hypothesis of parameter stability would be rejected at the 0.05, 0.10 and 0.15 levels of significance. For points whose p-value is lower than 0.05, we observe that the graph goes outside the standard error bounds. This test is thus important in that it clearly identifies periods of instability. The graph is shown below:



The qualitative results on parameter instability are similar as in the test of recursive residuals.

After performing the tests for parameter stability, the estimated equation must now be checked for parameter constraints. Let C(1) be the regression coefficient of log(Y(-1))-log(Y(-2))-log(Y(-3)). Let C(2) be the regression coefficient of log(M)-log(M(-1))-log(M(-2)). Let C(3) be the regression coefficient of log(M(-1))-log(M(-2))-log(M(-3)). Then the estimated equation must satisfy the following constraints: μ_1*C(2)+C(3) = 0 and μ_2*C(2)+C(1) = 0, where μ_1 and μ_2 are the regression coefficients of log(M(-1)) and log(Y(-1)) in the regression of log(M) on log(M(-1)) and log(Y(-1)) that yields consistent and unbiased estimates of these parameters.

Firstly, I regress log(M) on log(M(-1)) and log(Y(-1)), using the OLS method and assuming an intercept term. In this case, the most important problem comes with the tests for autocorrelation. The DW statistic is 1.213367. This is less than the lower bound of the DW statistic at the 0.05 significance level. Thus, the null hypothesis of no autocorrelation is rejected. Further, using the residual lag of 1, the Obs*R-squared statistic is significant at the 0.05 significance level. Further, at the residual lag of 2, this statistic is again significant. The coefficients of resid(-1) and resid(-2) are also significant. Therefore, I estimate this equation assuming an AR(2) error term to get consistent estimates of the regression coefficients. In the actual model, it is assumed that autocorrelation is absent. Therefore, this regression is carried out only to obtain consistent estimates. The results from this regression are presented below:

Dependent Variable: LOG(M)
Method: Least Squares

Sample: 1986:1 2005:4
Included observations: 80
Convergence achieved after 8 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.220828	0.100815	2.190428	0.0316
LOG(M(-1))	0.940943	0.041029	22.93380	0.0000
LOG(Y(-1))	0.022150	0.026645	0.831289	0.4084
AR(2)	0.422821	0.121826	3.470697	0.0009
R-squared	0.995370	Mean dependent var		6.928926
Adjusted R-squared	0.995187	S.D. dependent var		0.203518
S.E. of regression	0.014119	Akaike info criterion		-5.633830
Sum squared resid	0.015151	Schwarz criterion		-5.514729
Log likelihood	229.3532	F-statistic		5445.816
Durbin-Watson stat	1.497954	Prob(F-statistic)		0.000000
Inverted AR Roots	.65	-.65		

Thus, μ_1=0.940943, μ_2=0.022150. Now, we can test the restrictions 0.940943*C(2)+C(3) = 0 and 0.022150*C(2)+C(1) = 0. The method I use is that of Wald's test. The statistic is taken from Maddala. For the first restriction, the result is:

Wald Test:
Equation: M4

Null Hypothesis:	0.940943*C(2)+C(3) = 0		
F-statistic	3111.081	Probability	0.000000
Chi-square	3111.081	Probability	0.000000

For the second restriction, the result is:

Wald Test:
Equation: M4

Null Hypothesis:	0.022150*C(2)+C(1) = 0		
F-statistic	5.808607	Probability	0.018366
Chi-square	5.808607	Probability	0.015948

Thus, both the restrictions are not satisfied at the 0.05 significance level. Another restriction that can be tested is (μ_1+μ_2)*C(2) + C(1) + C(3) = 0. In this case, it is equivalent to 0.963093*C(2) + C(1) + C(3) = 0. The result is shown below:

Wald Test:
Equation: M4

Null Hypothesis:	0.963093*C(2) + C(1) + C(3) = 0		
F-statistic	182.0827	Probability	0.000000
Chi-square	182.0827	Probability	0.000000

Even in this case, the restriction is not satisfied at the 0.05 significance level.

From these tests, there is a strong implication that the estimated equation may not have come from this model. This problem is a disadvantage of this analysis, though the Lucas critique holds for any model that suggests output to be a function of a policy parameters. However, this does imply that the following analysis of stability of this equation and the aggregate demand equation may not necessarily provide strong conclusions regarding the stability of the original equation, i.e., it will be difficult to link stability of the parameters in the original equation with that of the given structural equations of the model, in this case, the AD equation, and the money specification equation (AS equation may be avoided, if AD equation is considered – see note 1). Since policy multipliers are determined by the coefficients of these equations and are central to the explanation of the Lucas critique, I still continue with analyzing the stability of these equations.

The AD equation is $y_t = \beta_0 + \beta_1(m_t-p_t) + \beta_2 E_{t-1}(p_{t+1}-p_t) + v_t$. The lower case letters denote the log of the original variable. $p_{t+1}-p_t$ is defined as $\prod_t$, where $\prod_t$ is the inflation rate in period t. Thus, by the rational expectations hypothesis, $E_{t-1}(\prod_t) = \prod_t - \varepsilon_t$ where $\{\varepsilon_t\}$ is a white noise. Therefore, the AD equation may be written as $y_t = \beta_0 + \beta_1(m_t-p_t) + \beta_2\prod_t + v_t - \beta_2 \varepsilon_t$. The new residual term is also a white noise. Performing the OLS regression, the estimated equation has a very low value of DW statistic: 0.052065. The Obs*R-squared and the coefficients of resid(-1) and resid(-2) are also significant in the LM test with order 2. Thus, I re-estimate the equation assuming AR(1) error. This is not assumed in the model and is done only for consistent and unbiased estimation (see note 3). The DW statistic is 1.537226. Thus, the null hypothesis of no autocorrelation is rejected. The equation passes the White's test at the 0.05 significance level, but not the Ramsey's test or the Breusch-Godfrey's test at the same significance level. Yet, the DW statistic is highest for this order of autoregression, considering the orders from 1 to 4.

Thus, I estimate the model using an ARMA(1,2) error term. To see the reason, I present the correlograms and the autocorrelation functions of the error term from simple OLS regression in levels and the first differences using 36 lags in the appendix. The basic reason is that the peaks in the autocorrelation function dampen over time in the first differences correlogram, unlike in the levels correlogram, suggesting AR(1). However, the peaks dampen in an oscillatory fashion, i.e. with both positive and negative values, indicating an MA component. I check for both ARMA(1,1) and ARMA(1,2) errors and found better results with the latter. The new estimated equation has a DW statistic of 1.590498. It passes the Breusch-Godfrey's LM test: the Obs*R-squared is not significant at the 0.05 significance level: its p-value is 0.083475. Further, it passes White's test for heteroskedasticity at the 0.05 significance level: the Obs*R-squared has a p-value of 0.122727. Further, the coefficients of all the variables in the regression of the square of the residual on the regressors and their cross-products are not significant at the 0.05 level. For the Ramsey's RESET test, the p-values of the F-statistic and the log likelihood ratio are greater than 0.01 and less than 0.05. Thus, there may be evidence of endogeneity of some variable, given that the p-values of the coefficients of the fitted log(Y(-1)) to the powers of 2 and 3 are both significant. However, I still proceed with inference from this equation, assuming that the bias in the standard errors of the coefficients of the original equation is not large. To check for stability of the parameters, I follow the Chow's tests. The recursive residual tests cannot be carried out in this case. Like previously, I use 1996:1 as the date to separate the data set into two sub-sets. The result of the Chow's breakpoint test is:

Chow Breakpoint Test: 1996:1			
F-statistic	0.374597	Probability	0.864474
Log likelihood ratio	2.112425	Probability	0.833380

The result of the Chow's forecast test is:

Chow Forecast Test: Forecast from 1996:1 to 2005:4			
F-statistic	0.854594	Probability	0.686067
Log likelihood ratio	54.51344	Probability	0.062724

Thus, at the 0.05 significance level, both the tests do not reject the hypothesis of parameter stability of the aggregate demand equation.

For the money specification equation, the results from the estimation have been shown previously. To test for parameter stability, I again use the two Chow's tests using 1996:1 as the reference date, like previously. Note that the recursive residual tests cannot be performed in this case since the error term is AR(2). The result of the Chow's breakpoint test is:

Chow Breakpoint Test: 1996:1			
F-statistic	1.746770	Probability	0.149141
Log likelihood ratio	7.409459	Probability	0.115769

The result of the Chow's forecast test is:

Chow Forecast Test: Forecast from 1996:1 to 2005:4			
F-statistic	1.181072	Probability	0.307789
Log likelihood ratio	67.05951	Probability	0.004671

Thus, Chow's breakpoint test does not reject the hypothesis of parameter stability at the 0.05 significance level. For the Chow's forecast test, the results are conflicting: the F-statistic is not significant and the log likelihood ratio is significant at the 0.05 level. Note that the difference in p-values is quite high. Further, the log likelihood is a large sample test. Therefore, parameter stability may be concluded at the 0.05 significance level, though the results are not as conclusive as for the AD equation.

V. Conclusion

Evidence of parameter stability in the original equation is conflicting. For 1986:1 – 1995:4, both the Chow's tests do not reject the hypothesis of parameter stability at the 0.05 level. This conclusion is reversed for the period 1996:1 – 2005:4 from both the tests at the same significance level. To get a better picture, I still performed both the tests for the twenty year period, 1986:1 – 2005:4. In this case, both the tests indicated parameter stability for the entire period at the 0.05 significance level. However, the results from the one step forecast error test and the recursive residuals test suggested parameter instability around 1990:2 – 1991:1. But, this instability is not reflected in the CUSUM test for the same period. It was suggested that parameter instability in the second sub-period, 1996:1 – 2005:4 may have been due to a fall in the money supply around the year 1994. Based on all these tests, the least strict interpretation for the entire period may be that inference on parameter stability for the twenty year period depends on the tests performed, and is thus inconclusive.

A further problem is that the original output equation did not satisfy any of the constraints imposed by the model. This suggests that although the fiscal and monetary policy multipliers may in fact change with time, the method of using the given structural equations to establish a connection between the stability of the structural parameters and the policy multipliers may not yield reliable results. However, the explanation of Lucas critique is based on the fact that these multipliers are a function of the structural parameters which change with time, leading to change in the multipliers themselves. Thus, the critique asserts a causal connection between the stability of these two sets of multipliers. Thus, even though the inferences on this connection will be most probably unreliable, it is still essential to do so.

For the aggregate demand equation, both of the Chow's tests do not reject the hypothesis of parameter stability at the 0.05 level of significance, for the time period 1986:1 – 2005:4. For the money specification equation, the results from the Chow's breakpoint test and the Chow's forecast test are conflicting: the latter rejects the hypothesis of parameter stability at the 0.05 level of significance, unlike the former.

Thus, based on these results and the criteria for testing the Lucas critique given in the introduction, it appears that the existence of the Lucas critique for this data set is inconclusive. Assuming parameter stability in all the equations concerned, one may say that it cannot be denied. However, given that the parameter constraints are not satisfied at the 0.05 significance level, this causal relation is questionable. There is also some ambiguity regarding parameter stability in the original equation.

VI. NOTES AND TABLES

[1] It is sufficient to investigate parameter instability in the AD equation and the money specification equation to try to explain the parameter instability, or otherwise, in the original output equation.
[2] Since the variables are differenced, some cross-product can lead to the problem of perfect multicollinearity. E-Views does not perform the White's test of heteroskedasticity assuming cross-terms in this case.

Data tables:

Date	M	P	PF	Y
1985:1	566.6000	106.4000	106.4000	4119.500
1985:2	582.2000	107.6000	107.6000	4178.400
1985:3	603.3000	108.3000	108.3000	4261.300
1985:4	619.8000	109.3000	109.3000	4321.800
1986:1	633.5000	108.8000	108.8000	4385.600
1986:2	660.6000	109.5000	110.1425	4425.700
1986:3	687.4000	110.2000	110.3781	4493.900
1986:4	724.7000	110.5000	111.3901	4546.100
1987:1	733.8000	112.1000	110.2117	4613.800
1987:2	743.2000	113.5000	112.8471	4690.000
1987:3	747.5000	115.0000	114.1818	4767.800
1987:4	750.2000	115.4000	115.1939	4886.300
1988:1	761.7000	116.5000	116.9277	4951.900
1988:2	778.4000	118.0000	117.8743	5062.800
1988:3	783.7000	119.8000	119.4724	5146.600
1988:4	786.7000	120.5000	120.1123	5253.700
1989:1	783.0000	122.3000	121.5075	5367.100
1989:2	773.5000	124.1000	123.6566	5454.100
1989:3	781.0000	125.0000	125.7422	5531.900
1989:4	792.9000	126.1000	125.5986	5584.300
1990:1	801.6000	128.7000	127.7982	5716.400
1990:2	808.9000	129.9000	130.3268	5797.700
1990:3	820.0000	132.7000	130.6609	5849.400
1990:4	824.7000	133.8000	133.4872	5848.800
1991:1	838.7000	135.0000	136.1396	5888.000
1991:2	856.7000	136.0000	136.0370	5964.300
1991:3	870.2000	137.2000	138.7122	6035.600
1991:4	896.9000	137.9000	138.3197	6095.800
1992:1	936.6000	139.3000	139.1649	6196.100
1992:2	954.4000	140.2000	140.3429	6290.100

1992:3	988.0000	141.3000	141.4622	6380.500
1992:4	1024.800	141.9000	142.0593	6484.300
1993:1	1036.600	143.6000	143.3940	6542.700
1993:2	1075.100	144.4000	144.5532	6612.100
1993:3	1104.200	145.1000	145.5721	6674.600
1993:4	1129.700	145.8000	145.7978	6800.200
1994:1	1140.200	147.2000	147.6260	6911.000
1994:2	1145.200	148.0000	148.1308	7030.600
1994:3	1151.900	149.4000	148.8301	7115.100
1994:4	1150.700	149.7000	150.1649	7232.200
1995:1	1146.800	151.4000	151.2284	7298.300
1995:2	1144.200	152.5000	152.2844	7337.700
1995:3	1141.800	153.2000	153.9776	7432.100
1995:4	1127.400	153.5000	153.6135	7522.500
1996:1	1122.500	155.7000	155.7000	7624.100
1996:2	1115.100	156.7000	156.6797	7776.600
1996:3	1096.000	157.8000	157.3158	7866.200
1996:4	1081.400	158.6000	157.8676	8000.400
1997:1	1072.300	160.0000	160.4387	8113.800
1997:2	1065.700	160.3000	160.9170	8250.400
1997:3	1067.300	161.2000	161.5815	8381.900
1997:4	1072.800	161.3000	162.2562	8471.200
1998:1	1077.400	162.2000	163.2317	8586.700
1998:2	1076.800	163.0000	163.1969	8657.900
1998:3	1079.800	163.6000	164.4346	8789.500
1998:4	1095.900	163.9000	164.3307	8953.800
1999:1	1097.200	165.0000	165.3800	9066.600
1999:2	1099.600	166.2000	166.3136	9174.100
1999:3	1096.700	167.9000	167.1715	9313.500
1999:4	1123.000	168.3000	168.1723	9519.500
2000:1	1107.600	171.2000	169.3595	9629.400
2000:2	1103.500	172.4000	171.7226	9822.800
2000:3	1099.700	173.7000	173.4465	9862.100
2000:4	1087.700	174.0000	173.5733	9953.600
2001:1	1108.700	176.2000	176.4773	10021.50
2001:2	1126.700	178.0000	177.2132	10128.90
2001:3	1205.200	178.3000	178.9103	10135.10
2001:4	1182.000	176.7000	178.5576	10226.30
2002:1	1191.300	178.8000	179.5950	10333.30
2002:2	1192.700	179.9000	181.3413	10426.60
2002:3	1195.900	181.0000	181.1721	10527.40
2002:4	1219.500	180.9000	180.0373	10591.10
2003:1	1237.300	184.2000	183.1761	10705.60
2003:2	1281.000	183.7000	185.0367	10831.80
2003:3	1298.100	185.2000	185.1244	11086.10
2003:4	1305.500	184.3000	185.2594	11219.50
2004:1	1328.200	187.4000	188.1346	11405.50
2004:2	1343.700	189.7000	187.4398	11610.30
2004:3	1363.000	189.9000	190.7758	11779.40
2004:4	1375.200	190.3000	188.9973	11948.50
2005:1	1371.400	193.3000	193.0225	12154.00
2005:2	1380.800	194.5000	195.2668	12317.40
2005:3	1379.400	198.8000	194.7312	12558.80
2005:4	1373.200	196.8000	197.7583	12705.50

For the definition of the variables and the data sources, see Section II.

VII. BIBLIOGRAPHY

1. G.S. Maddala: Introduction to Econometrics (Third edition), Chapters 4, 5, 6, 9, 10, 13.
2. Jack Johnston and John Dinardo: Econometrics, Chapters 2, 3.
3. Damodar Gujarati: Essentials of Econometrics (Third edition), Chapter 10.
4. Ben J. Heijdra, Frederick van der Ploeg, The Foundations of Modern Macroeconomics, pp. 60-70.

Software used for estimation: E-Views Version 3.1

Note: The correlograms of the levels and first difference of the error term of the AD equation estimated by the OLS method are attached.