# Growth by technical progress and fiscal policy for full-employment: A theoretical foundation for MMT

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**Abstract**

We study the fiscal policy to maintain or realize full-employment in situations where there is involuntary unemployment with growth by technical progress under deflation or inflation. In a three-periods (generations) overlapping generations model of this paper consumptions in the childhood period are financed by borrowing money from the previous generation consumers, and these debts must be repaid in the next period. In such a model consumers have debts as well as savings. Mainly we show the following results. If the deflation rate is equal to the technical progress rate, in order to maintain a steady state with constant employment, including full-employment, a balanced budget is required. If the deflation rate is smaller than the technical progress rate or the inflation occurs and the savings of consumers (net of pay-as-you-o pensions) are larger (or smaller) than their debts, in order to maintain constant employment a budget deficit (or surplus) is required. Also we show that fiscal policy to realize full-employment in a situation with involuntary unemployment usually requires larger budget deficit (or smaller surplus). These budget deficits, including those in the full-employment state, should be financed by seigniorage not by public debt. If they are financed by public debts, they do not have to be repaid. Conversely, the budget surplus in some cases should not be returned to consumers as tax reduction.

**Key Words:** Involuntary unemployment, Three-periods overlapping generations model, Technical progress, Deflation or Inflation, Consumers' savings and debts, Pay-as-you-go pensions, Seigniorage.

**JEL Classification:** E12, E24.

## 1 Introduction

We study the fiscal policy to maintain or realize full-employment in situations where there is involuntary unemployment with growth by technical progress under deflation or inflation. In a three-periods (generations) overlapping generations (OLG) model of this paper consumptions in the childhood period are financed by borrowing money from the previous generation consumers, and these debts must be repaid in the next period. Also consumers receive pay-as-you-go pensions in their older (retired) period. In such a model consumers have debts as well as savings.

Involuntary unemployment is a phenomenon that workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. Umada(1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. Otaki(2009) assumes indivisibility of labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow(1985). The arguments of this paper do not depend on bargaining. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. As discussed by Otaki(2015) (Theorem 2.3) and Otaki(2012), if labor supply is divisible and very small, no unemployment exists[2]. However, we show that even if labor supply is divisible, unless it is so small, there may exist involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015). We extend Otaki's model to a three-periods (generations) OLG model with a childhood period, and also we consider pay-as-you-go pension system for the older generation consumers.

We will show the following results in Propositions 3, 4 and 5 in Section 3.

1. If the deflation rate is equal to the technical progress rate, in order to maintain a steady state with constant employment, including full-employment, a balanced budget is required.
2. If the deflation rate is larger than the technical progress rate or inflation occurs and the net savings of consumers is larger (smaller) than their debts, in order to maintain a steady state with constant employment a budget surplus (or deficit) is required.
3. If the deflation rate is smaller than the technical progress rate and the net savings of consumers is larger (smaller) than their debts, in order to maintain a steady state with constant employment a budget deficit (surplus) is required.

Also we show that the fiscal policy to realize full-employment in a situation with involuntary unemployment usually requires larger budget deficit or smaller budget surplus (Proposition 6) because by Propositions 3, 4 and 5, after realization of full-employment,

[1] Lavoie (2001) presented a similar analysis.

[2] About indivisible labor supply also please see Hansen (1985). In Tanaka (2020a, 2020b, 2020c) involuntary unemployment under indivisible labor supply is analyzed.

necessary budget deficit or budget surplus returns to the value which maintains the steady state with full-employment. These budget deficits, including those in the full-employment state, should be financed by seigniorage not by public debt. Even if it is financed by public debts, it does not have to be repaid. Conversely, the budget surplus in some cases should not be returned to consumers as tax reduction.

In the next section we explain the model and show the existence of involuntary unemployment due to deficiency of demand.

We present a theoretical foundatuion for MMT (Modern monetary theory, for exapme, Mitchell, Wray and Watts (2019)).

## 2 Existence of involuntary unemployment

### 2.1 Consumers

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki(2007, 2009, 2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good $z$ is monopolistically produced by firm $z$ with constant returns to scale technology.
2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.
3. During Period 1, consumers supply $l$ units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.
4. During Period 2, consumers consume the goods using their savings carried over from their Period 1, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.
5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

Further we make the following assumptions

**Ownership of the firms** Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers.

**Zero interest rate** The interest rate will be determined so that the supply of funds from the savings of the younger generation plus government scholarships is equal to the consumption of the childhood generation, but without scholarships there is a large possibility that savings will be insufficient regardless of the interest rate, especially in the presence of a pay-as-you-go pension system. Since it is the scholarship that fills the gap, the interest rate can be controlled by determining the size of the scholarship. If the amount of scholarships is increased or decreased, or if they are made interest-bearing or interest-free, the interest rate will change, and this may change consumption. However, for example, a decline in the interest rate may increase consumption among the younger generation due to the substitution effect, but then consumption among the older generation will decline. The income effect is also ambiguous, since a fall in the interest rate reduces the debt associated with consumption in the childhood period, but lowers the value of savings. Therefore, the possibility that a change in the interest rate will significantly change aggregate demand is small, and it is not an important issue for the existence of involuntary unemployment, which is the theme of this paper. We assume here that the amount of the scholarship is determined so that the interest rate is zero. Repayment of the debts of consumers in their childhood period is assured. Consumers in the younger period are indifferent between lending money to childhood period consumers and savings by money.

**Notation** We use the following notation.

$C_i^e$: consumption basket of an employed consumer in Period $i$, $i = 1,2$.
$C_i^u$: consumption basket of an unemployed consumer in Period $i$, $i = 1,2$.
$c_i^e(z)$: consumption of good $z$ of an employed consumer in Period $i$, $i = 1,2$.
$c_i^u(z)$: consumption of good $z$ of an unemployed consumer in Period $i$, $i = 1,2$.
$D$: consumption basket of an individual in the childhood period, which is constant.
$P_i$: the price of consumption basket in Period $i$, $i = 1,2$.
$p_i(z)$: the price of good $z$ in Period $i$, $i = 1,2$.
$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
$W$: nominal wage rate.
$R$: unemployment benefit for an unemployed individual. $R = D$.
$\hat{D}$: consumption basket in the childhood period of a next generation consumer.
$Q$: pay-as-you-go pension for an individual of the older generation.
$\Theta$: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
$\Psi$: tax payment by an employed individual for the pay-as-you-go pension.
$\Pi$: profits of firms which are equally distributed to each consumer.
$l$: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
$L$: total employment.
$L_f$: population of labor or employment in the full-employment state.
$y$: labor productivity, which increases by technical change.

We assume that the population $L_f$ is constant. The technical progress rate is denoted by $\gamma$>1, and we write $y' = \gamma y$.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

Since the taxes for unemployed consumers' unemployment benefits are paid by employed consumers of the same generation, $D(= R)$ and $\Theta$ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, $D$ is the real value of the consumption in the childhood period of consumers.

Since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, $Q$ and $\Psi$ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over three periods is written as

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period $i$ are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1,2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1,2.$$

$\sigma$ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period $i$ is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},\ i = 1,2.$$

The budget constraint for en employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

The budget constraint for en unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q}$$

Since $R = D$,

$$P_1 C_1^u + P_2 C_2^u = \Pi + \hat{Q}.$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e},\ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}.$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, $\alpha$ is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1},$$

$$C_2^e = (1-\alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

and

$$C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1},\ C_2^u = (1-\alpha) \frac{\Pi + \hat{Q}}{P_2}.$$

Solving maximization problems in Step 2, the following demand functions of employed and unemployed consumers are derived[3].

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(\Pi + \hat{Q})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(\Pi + \hat{Q})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, (1-\alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2}, D\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha \frac{\Pi + \hat{Q}}{P_1}, (1-\alpha) \frac{\Pi + \hat{Q}}{P_2}, D\right).$$

Let

$$\omega = \frac{W}{P_1},\ \rho = \frac{P_2}{P_1}.$$

Then, since the real value of $D$ in the childhood period is constant, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, \rho\right) - \Gamma(l),$$

[3] About calculations of the maximization problems in Step 2 please see Appendix.

$$V^u = \varphi\left(\frac{\Pi+\hat{Q}}{P_1}, \rho\right),$$

$\omega$ is the real wage rate. Denote

$$I = \omega l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}.$$

The condition for maximization of $V^e$ with respect to $l$ given $\rho$ is

$$\frac{\partial \varphi}{\partial I}\omega - \Gamma'(l) = 0, \quad (1)$$

where

$$\frac{\partial \varphi}{\partial I} = \alpha \frac{\partial u}{\partial c_1^e} + (1-\alpha)\frac{\partial u}{\partial c_2^e}.$$

Given $P_1$ and $\rho$ the labor supply is a function of $\omega$. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial \varphi}{\partial I} + \frac{\partial^2 \varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2 \varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate $\omega$.

## 2.2 Firms

Let $d_1(z)$ be the total demand for good $z$ by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - LD - L\Theta + L_f\hat{Q} - L\Psi)}{P_1}$$

$$= \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good $z$ in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good $z$ by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q})}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of $\alpha$ for the older generation. $Q$ is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\bar{\alpha})(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between $M$ and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ.$$

Their demand for good $z$ is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good $z$ is written as

$$d(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{Y}{P_1}, \tag{2}$$

where $Y$ is the effective demand defined by

$$Y=\alpha\left(WLl+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ\right)+G+L_f\widehat{D}+M.$$

Note that $\widehat{D}$ is consumption in the childhood period of a next generation consumer. $G$ is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits (see Otaki(2007), Otaki(2015) about this demand function).

Let $L$ and $Ll$ be employment and the "employment $\times$ labor supply" of firm $z$. The total employment and the total "employment $\times$ labor supply" are

$$\int_0^1 Ldz=L,\ \int_0^1 Lldz=Ll.$$

The output of firm $z$ is $Lly$. At the equilibrium $Lly=d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial(Ll)}=y.$$

From (2)

$$\frac{\partial p_1(z)}{\partial d(z)}=-\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial(Ll)}=-\frac{p_1(z)y}{\sigma d(z)}=-\frac{p_1(z)y}{\sigma Lly}.$$

The profit of firm $z$ is

$$\pi(z)=p_1(z)Lly-LlW.$$

The condition for profit maximization is

$$\frac{\partial\pi(z)}{\partial(Ll)}=p_1(z)y-Lly\frac{p_1(z)y}{\sigma Lly}-W=p_1(z)y-\frac{p_1(z)y}{\sigma}-W=0.$$

Therefore, we obtain

$$p_1(z)=\frac{1}{(1-\frac{1}{\sigma})y}W.$$

Let $\mu=\frac{1}{\sigma}$. Then,

$$p_1(z)=\frac{1}{(1-\mu)y}W.$$

This means that the real wage rate is

$$\omega=(1-\mu)y.$$

Since all firms are symmetric,

$$P_1=p_1(z)=\frac{1}{(1-\mu)y}W. \tag{3}$$

### 2.3 Involuntary unemployment

The (nominal) aggregate supply of the goods is equal to

$$WL+L_f\Pi=P_1Lly.$$

The (nominal) aggregate demand is

$$\alpha\left(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\widehat{D} + M$$
$$= \alpha\left[P_1Lly - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\widehat{D} + M.$$

Since they are equal,

$$P_1Lly = \alpha\left[P_1Lly - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\widehat{D} + M. \tag{4}$$

In real terms[4]

$$Lly = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\widehat{D} + M}{(1-\alpha)P_1}.$$

The equilibrium value of $Ll$ cannot be larger than $L_fl$. However, it may be strictly smaller than $L_fl$. Then, we have $L < L_f$ and involuntary umemployment exists.

If the government collects a lump-sum tax $T$ from the younger generation consumers, (4) is rewritten as

$$P_1Lly = \alpha\left[P_1Lly - T - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\widehat{D} + M. \tag{5}$$

## 3 Growth by technical progress and fiscal policy for full-employment

### 3.1 Constant employment and output under deflation or inflation without technical progress

First as a benchmark we consider a steady state where employment, which may be full-employment, and output are constant under deflation or inflation without technical progress. Suppose that the prices of the goods change at the rate $\rho - 1$, and the price change is correctly predicted by consumers and the government. If $\rho - 1<0$, deflation occurs, and if $\rho - 1>0$, inflation occurs. Even when the prices of the goods change, we can assume that the real values of $D$ and $Q$ do not change. Thus, we assume $\widehat{D} = \rho D$, $\hat{Q} = \rho Q$, and then (5) is rewritten as

$$P_1Lly = \alpha\left[P_1Lly - T - L_fD + L_f\rho Q - L_fQ\right] + G + L_f\rho D + M.$$

From this

$$(1-\alpha)\left[P_1Lly - T - L_fD + (\rho - 1)L_fQ\right] = G - T + (\rho - 1)L_fQ + (\rho - 1)L_fD + M.$$

This is the savings of the younger generation consumers. It should be equal to $\rho M$ to maintain the steady state under deflation or inflation. Therefore,

$$G - T = (\rho - 1)[M - L_fQ - L_fD].$$

We get the following result.

**Proposition 1**

1. **(When net savings is larger than debts under deflation)** If $M > L_fD + L_fQ$, in order to maintain a state where the output and the employment are constant with falling prices ($\rho < 1$), a budget surplus ($G < T$) is required. .
2. **(When net savings is smaller than debts under deflation)** If $M < L_fD + L_fQ$, in order to maintain a state where the output and the employment are constant with falling

[4] $\frac{1}{1-\alpha}$ is a multiplier.

prices ($\rho < 1$), a budget deficit ($G > T$) is required.

**Proposition 2**

1. **(When net savings is larger than debts under inflation)** If $M > L_f D + L_f Q$, in order to maintain a state where the output and the employment are constant with rising prices ($\rho > 1$), a budget deficit ($G > T$) is required.
2. **(When net savings is smaller than debts under inflation)** If $M < L_f D + L_f Q$, in order to maintain a state where the output and the employment are constant with rising prices ($\rho > 1$), a budget surplus ($G < T$) is required.

### 3.2 Ongoing technical progress under deflation or infaltion

Suppose that the prices of the goods change at the rate $\rho - 1$, the labor productivity increases at the rate $\gamma - 1 > 0$; and the employment $L$ is constant. These changes are correctly predicted by consumers and the government. We assume $\widehat{D} = \gamma\rho D$, $\widehat{Q} = \gamma\rho Q$, and then (5) is rewritten as

$$P_1 L l y' = \alpha\left[P_1 L l y' - T - L_f D + L_f \gamma\rho Q - L_f Q\right] + G + L_f \gamma\rho D + M,$$

where $y' = \gamma y$. From this

$$(1-\alpha)\left[P_1 L l y' - T - L_f D + (\gamma\rho - 1)L_f Q\right] = G - T + (\gamma\rho - 1)L_f Q + (\gamma\rho - 1)L_f D + M. \quad (6)$$

It should be equal to $\gamma\rho M$ to maintain the steady state. Therefore,

$$G - T = (\gamma\rho - 1)[M - L_f Q - L_f D].$$

If the prices are constant, we have

$$G - T = (\gamma - 1)[M - L_f Q - L_f D].$$

From these analyes we obtain the following results.

**Proposition 3** ***(Equal deflation rate and technical progress rate, or constant prices without technical progress)*** *If $\gamma\rho = 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), or constant pries without technical progress ($\rho = \gamma = 1$, ), a balanced budget ($G = T$) is required. In the case where $\gamma\rho = 1$, we say that the rate of deflation and the rate of technical progress are equal.*

**Proposition 4**

1. **(When net savings is larger than debts under deflation)**

*Assume $M > L_f D + L_f Q$. We have two cases.*

(1). If $\gamma\rho < 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget surplus ($G < T$) is required. In this case the rate of deflation is larger than the rate of technical progress.

(2). If $\gamma\rho > 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget deficit ($G > T$) is required. In this case the rate of deflation is smaller than the rate of technical progress. This case includes a case with technical progress and constant prices.

2. **(When net savings is smaller than debts under deflation)**

*If* $M < L_f D + L_f Q$, *we obtain the following results.*

(1). If $\gamma\rho < 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget deficit ($G > T$) is required.

(2). If $\gamma\rho > 1$, in order to maintain a state where the employment is constant with falling prices ($\rho < 1$) and technical progress ($\gamma > 1$), a budget surplus ($G < T$) is required. This case includes a case with technical progress and constant prices.

**Proposition 5**

1. **(When net savings is larger than debts under inflation)**

*Assume* $M > L_f D + L_f Q$. *We have two cases. Since* $\gamma\rho > 1$ in this case, in order to maintain a state where the employment is constant with rising prices ($\rho > 1$) and technical progress ($\gamma > 1$), a budget deficit ($G > T$) is required.

2. **(When net savings is smaller than debts under inflation)**

*If* $M < L_f D + L_f Q$, *we obtain the following results.* Since $\gamma\rho > 1$ in this case, in order to maintain a state where the employment is constant with rising prices ($\rho > 1$) and technical progress ($\gamma > 1$), a budget surplus ($G < T$) is required.

### 3.3 Fiscal policy for full-employment under deflation or inflation with technical progress

Suppose that full-employment is realized in period. We assume $\hat{D} = \gamma\rho D$ and $\hat{Q} = \gamma\rho Q$. Let $G'$ and $T'$ be the government expenditure and the tax in this case. Then, (5) is rewritten as

$$P_1 L_f l y' = \alpha\left[P_1 L_f l y' - T' - L_f D + L_f \gamma\rho Q - L_f Q\right] + G' + L_f \gamma\rho D + M.$$

From this

$$(1-\alpha)\left[P_1 L_f l y' - T' - L_f D + (\gamma\rho - 1) L_f Q\right]$$
$$= G' - T' + (\gamma\rho - 1) L_f Q + (\gamma\rho - 1) L_f D + M. \quad (7)$$

Note that $M$ is the savings of the older generation consumers. Comparing (6) and (7), we find that if

$$(1-\alpha)\left[\left(P_1 L_f l y' - T'\right) - \left(P_1 L l y' - T\right)\right] > 0, \quad (8)$$

we have

$$G' - T' > G - T.$$

We have shown the following proposition.

**Proposition 6** *If (8) holds, we need larger budget deficit (or smaller budget surplus) to realize full-employment than that to maintain constant employment.*

$P_1 L_f l y' - T'$ and $P_1 L l y' - T$ are the disposable income after and before full-employment. Thus, (8) holds when the disposable income increases by realization of full-employment and the deflation rate is not so large.

From Propositions 3, 4 and 5, after realization of full-employment, necessary budget deficit or budget surplus returns to the value which maintain the steady state with full-employment. Therefore, the extra budget deficit for full-employment should be financed by seigniorage not by public debt.

### 3.4 About budget deficits and surpluses

In the folloiwing cases we need budget deficit.

1. The net savings is smaller than debts under deflation without technical progress. (Proposition 1. 2).
2. The net savings is larger than debts under inflation without technical progress. (Proposition 2. 1).
3. The net savings is larger than debts under deflation with growth by technical progress and the rate of deflation is smaller than the rate of technical progress. (Proposition 4. 1 (2)). This case include a case with technical progress and constant prices.
4. The net savings is smaller than debts under deflation with growth by technical progress and the rate of deflation is larger than the rate of technical progress. (Proposition 4. 2 (1)).
5. The net savings is larger than debts under inflation with growth by technical progress . (Proposition 5. 1).

These budget deficits are necessary to maintain the steady state with full-employment under ongoing deflation or inflation with or without growth by technical progress. Therefore, these budget deficits should be financed by seigniorage not by public debts. Even if it is financed by public debts, it does not have to be repaid.

Proposition 5 means that we need larger budget deficits to realize full-empoyment in a situtaion where there exists involuntary unemployment. These extra budget deficits also should be finaced by seignirrage.

On the other hand, we need budget surplus in the following cases.

1. The net savings is larger than debts under deflation without technical progress. (Proposition 1. 1).
2. The net savings is smaller than debts under inflation without technical progress. (Proposition 2. 2).
3. The net savings is smaller than debts under deflation with growth by technical progress and the rate of deflation is smaller than the rate of technical progress. (Proposition 4. 1 (1)).
4. The net savings is larger than debts under deflation with growth by technical progress and the rate of deflation is larger than the rate of technical progress. (Proposition 4. 2 (2)).
5. The net savings is smaller than debts under inflation with growth by technical progress . (Proposition 5. 2).

These budget deficits are necessary to maintain the steady state with full-employment under ongoing deflation or inflation with or without growth by technical progress. Therefore, these budget surpluses should not be returned to consumers as tax reduction.

## 4 Concluding Remark

We have examined the steady state with involuntary unemployment and fiscal policy to realize full-employment under deflation due to involuntary unemployment with technical progress. We assumed that the goods are produced by only labor. In future research, we want to analyze involuntary unemployment and fiscal policy in a situation where goods are produced by capital and labor, and there exist investments of firms.

## Appendix: Calculations of Step 2 of consumers' utility maximization

Lagrange functions in the second step for employed and unemployed consumers are

$$\mathcal{L}_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} \tag{A.1}$$
$$-\lambda_1^e\left[\int_0^1 p_1(z)c_1^e(z)dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$-\lambda_2^e\left[\int_0^1 p_2(z)c_2^e(z)dz - (1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^u\left[\int_0^1 p_1(z)c_1^u(z)dz - \alpha(\Pi + \hat{Q})\right],$$

and

$$\mathcal{L}_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^u\left[\int_0^1 p_2(z)c_2^u(z)dz - \alpha(\Pi + \hat{Q})\right].$$

$\lambda_1^e$, $\lambda_2^e$, $\lambda_1^u$ and $\lambda_2^u$ are Lagrange multipliers.

The first order condition for (A.1) is

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda_1^e p_1(z) = 0. \tag{A.2}$$

From this

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = (\lambda_1^e)^{1-\sigma}p_1(z)^{1-\sigma}.$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = (\lambda_1^e)^{1-\sigma}\int_0^1 p_1(z)^{1-\sigma}dz = 1,$$

It means

$$\lambda_1^e\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}} = 1,$$

and so

$$P_1 = \frac{1}{\lambda_1^e}.$$

From (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \lambda_1^e p_1(z)c_1^e(z).$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$= C_1^e = \lambda_1^e \int_0^1 p_1(z)c_1^e(z)dz = \frac{1}{P_1}\int_0^1 p_1(z)c_1^e(z)dz.$$

Therefore,

$$\int_0^1 p_1(z)c_1^e(z)dz = P_1C_1^e.$$

Similarly,

$$\int_0^1 p_2(z)c_2^e(z)dz = P_2C_2^e.$$

Thus,

$$\int_0^1 p_1(z)c_1^e(z)dz + \int_0^1 p_2(z)c_2^e(z)dz = P_1C_1^e + P_2C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi,$$

and we obtain

$$P_1C_1^e = \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi).$$

By (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} c_1^e(z)^{-1} = C_1^e c_1^e(z)^{-1} = (\lambda_1^e)^\sigma p_1(z)^\sigma = \left(\frac{p_1(z)}{P_1}\right)^\sigma.$$

From this we get

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1}.$$

$c_2^e(z)$, $c_1^u(z)$ and $c_2^u(z)$ are similarly obtained.

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