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Abstract

Why are stock prices much more volatile than the underlying dividends? The excess volatility of prices can in principle be attributed to two different causes: time-varying discount rates for expected future dividends, arising from variation in risk premia; or the irrational exuberance of investors, bidding prices up and down even in the absence of changes in the underlying value of the asset. No consensus has so far emerged among economists as to the prevalence of one or the other source of price variation.

I propose in this paper a novel way to approach this problem, by identifying changes in the uncertainty faced by investors regarding the fundamental value of an asset and exploiting the different response in prices that such changes in uncertainty would generate through sentiments or risk premia. I then apply this framework to the S&P 500 index from 1872 till 2019: the positive correlation found between uncertainty and prices (or, equivalently, the negative correlation between uncertainty and implied risk premia) is not compatible with rational investors' behavior and suggests instead the presence of a significant sentiments component in stock prices.

Key words: uncertainty, risk premium, sentiments; information, financial markets.

JEL classification: D81, D83, G12, G14, G41.

1 Introduction and related literature

1.1 Introduction

Stock prices are notoriously volatile, much more than the underlying stream of future payments an asset entitles to. This fact has attracted the attention of economists for a long time, earning the status of a puzzle (see, e.g„ [42, Shiller (1981)] and [36, LeRoy and Porter (1981)]).

Two main approaches have been taken to explain the excess volatility of stock prices: one appeals to the notion of sentiments, or fads, as presented in [43, Shiller (1989)], and identifies the irrational behavior of investors at the basis of such excessive price variation; the other maintains instead the notion of investors' rationality and points to time-varying discount rates, due to changes in expected returns and risk premia, as the cause of excess price volatility, as argued by [23, Cochrane (1991)] in his review of Shiller's book.

The first approach appeals to irrationality, psychology, herd behavior and similar concepts to explain large variations in prices not accounted for by comparable variations in the underlying fundamental value. It rejects the idea that prices are efficient, in the sense that they incorporate all available information at any given time, and instead argues that their movements are often not warranted by changes in traders' information and should be attributed to irrational, or non-information based, behavior.

The second approach is grounded instead in the assumption that investors make rational decisions, given their available information. While this statement is easily formulated, its verification relies on the possibility to observe what available information investors have at any given time. Changes in information can impact on expected future dividends or on discount rates, in both cases affecting prices. Early tests of market rationality were based on constant discount rates and easily rejected the notion of rationality: there is simply too much variation in prices compared to the variation in dividends or earnings. It has since been acknowledged that time-varying risk premia, and thus discount rates, need to be accounted for if one wants to have any chance of settling the question regarding excess market volatility. For any time series of prices and dividends, it is always possible to derive a time-series of discount rates that justifies observed prices as fundamental values: the question then becomes what moves those discount rates, and whether their volatility is "excessive" compared to what would be expected in a market populated by rational investors.

Available models of time-varying discount rates, such as the consumption-based asset pricing model, don't seem to be able to account for the large variation in risk premia required to match the excess volatility in prices. Rejections of volatility tests imply that specific discount-rate models leave a residual unexplained: as [23, Cochrane (1991)] points out, one might decide to call such residual "sentiments", but it might just represent a shortcoming of our current models rather than a lack of rationality in investors' behavior.

Instead of trying to account for such residuals, or, equivalently, for the total volatility of prices, which seems an arduous task given the unobservability of much of agents' information, I propose to look instead at comovements between prices (or, equivalently, implied discount rates) and informational uncertainty, and assess the consistency of such comovements with either of the two theories: investors' sentiments or rational markets. This approach, thus, identifies uncertainty as the key variable that can help disentangle the two sources of price movements, by focusing on the differential effect that changes in uncertainty about future payoffs can be expected to have on prices through the two different channels, sentiments and risk premia.

To this end, I employ a novel way to model sentiments, first proposed in [16, Berardi (2020)]: sentiments are sparked by exogenous shocks, but they are amplified through uncertainty. This modelling device captures the intuitive feature that agents can only be optimistic or pessimistic about things they don't know for sure. If investors knew with certainty the future stream of dividends an asset entitles to, there could be no sentiments as no-one would be willing to pay a price different from its fundamental value. As uncertainty increases, agents can become more and more optimistic or pessimistic about the value of the asset and thus the price they are willing to pay for it can deviate more and more from the fundamental.

Without sentiments, an increase in uncertainty can be expected to always decrease prices, through an increase in risk premia: higher uncertainty about future payoffs, in fact, increases the risk for investors who decide to buy that asset, and thus requires higher risk premia. Different is instead the effect that uncertainty can be expected to have on prices through sentiments, depending on the nature of those sentiments: while an increase in uncertainty amplifies sentiments, such sentiments can result in higher or lower prices depending on their bullish or bearish nature. A decrease in uncertainty has the opposite effect, increasing prices through a lower risk premium (i.e., a higher discount rate) but again leading to an ambiguous effect through sentiments: it increases prices if sentiments are negative, and it decreases prices if sentiments are positive. Either way, a negative correlation between changes in uncertainty and changes in prices is consistent with both theories, while a positive correlation requires sentiments (in particular, positive sentiments) to be present. This new framework allows then for a clear identification of sentiments in the data: a positive correlation between uncertainty and prices requires (positive) sentiments to be accounted for.

The crucial issue, then, is how to measure investors' uncertainty. To this end, I compute a measure of agents' estimates about the fundamental value of the asset, and thus a measure of their uncertainty, based on one source of information only: dividends. In reality investors are likely to use many more sources of information, which can reduce their level of uncertainty. The measure I derive, thus, can be considered as an upper bound for the total uncertainty they face. This is not necessarily a problem for the empirical application of the model, since the identification strategy for sentiments that I suggest relies on correctly identifying movements in uncertainty, not its overall amount: as long as the direction of change is correctly identified, the testable implications of the model are valid. What is required, then, is that the informational content of other sources used by agents is not negatively correlated with the informational content of dividends. As long as that is the case, and there seems to be no reason to think otherwise, the measure of uncertainty I compute moves together with the overall uncertainty faced by investors, and the framework I propose can be used in order to identify (positive) sentiments in stock prices.

Another implicit assumption required for the proposed identification strategy to work is that investors' (absolute) risk aversion does not decrease when uncertainty increases (and viceversa). Without this assumption, an increase in uncertainty matched by a stronger decrease in risk aversion could generate a reduction in the risk premium and lead to wrong inference regarding the presence of sentiments on the market.

With these caveats in mind, the proposed framework provides testable restrictions that can help understand whether excessive movements in prices come from variations in discount rates (i.e., risk premia) or from investors' sentiments. Using data for the S&P 500 index and dividends from 1872 onwards, I find evidence that sentiments played a role in determining stock prices, since the observed positive comovements of prices and uncertainty cannot be explained by the rational response of risk averse investors to changes in uncertainty.

1.2 Related literature

The role of information in financial markets has been a centre of attention of financial economists for a long time. [29, Fama (1970)] reviewed the theoretical and empirical literature on the efficient market hypothesis (EMH) in its various forms and concluded that, generally, stock prices reflect all available information. Fama's results generated a lot of interest and spurred further research. Attempts to find evidence in favour of either market efficiency or investors' irrationality require a theoretical framework that can generate falsifiable predictions using observable variables and two main approaches were taken: volatility tests (e.g., [42, Shiller (1981)], [20, Campbell and Shiller (1988)]), to assess whether prices are too volatile compared to the underlying fundamental value implied by rational investment decisions; and returns regressions (for an overview, [41, Rapach and Zhou (2013)]), to identify variables that could help predict future returns, thus disproving the EMH. The two approaches are, in fact, equivalent ([36, LeRoy and Porter (1981)], [23, Cochrane (1991)]), and both are tests of specific discount rate models. To various degrees, these tests have so far rejected the rationality hypothesis. While early volatility tests used to be carried out with constant discount rates, subsequent studies incorporated time-varying models of discount factors. For example, [20, Campbell and Shiller (1988)] construct volatility tests based on the consumption-based asset pricing model, and their tests continue to reject the rationality hypothesis.

Given the relative stability of risk-free rates, accounting for time-varying discount rates that can generate enough volatility in prices requires explaining substantial time variation in the equity risk premium (ERP).[1] Numerous attempts have been made to this end, using a variety of variables, from financial indicators such as dividend-price ratios, book-to-market ratios and various interest rates, to macroeconomic variables at business cycles frequencies, such as inflation rates, income, consumption and wealth. For example, [21, Campbell and Shiller (1998)] find that valuation ratios, such as the price-dividend ratio, have predictive power over long run stock market returns, and [37, Lettau and Ludvigson (2001)] find that the consumption-wealth ratio is a particularly good predictor of excess returns.[2] Opposite conclusions are instead reached by [33, Goyal and Welch (2008)], who find that none of these variables is robust to out-of-sample prediction and could not help investors improve their performance.

As said before, the residual left unexplained by these approaches can represent a shortcoming of current models, or genuinely capture irrational elements in stock markets. Attempts to analyse the impact of irrational exuberance and sentiments in stock prices include [46, White (1990)], [25, DeLong and Shleifer (1991)], [10, Barberis et al (1998)] and [11,

[1] ([28, Duarte and Rosa (2015)] conduct a review of models used to explain the ERP, classifying twenty different models into five categories, based on their underlying assumptions: models that use historical mean of realized returns, models based on discounted dividends, cross-sectional regressions, time-series regressions, and surveys.

[2] More in general, [24, Cochrane (2017)] discusses how macro factors can (or cannot) help solve anomalies in asset pricing.

Barberis et al (2015)].[3]

Given the difficulty in identifying sentiments, and in particular disentangling the direction of causality from psychological attitudes to actual outcomes, economists have tried to instrument for sentiments using a variety of variables. For example, [15, Benhabib and Spiegel (2019)] look for empirical evidence on the impact of sentiment on aggregate demand by using political outcomes as instruments and [35, Lagerborg et al. (2019)] try to capture psychological attitudes in U.S. consumer confidence using fatalities in mass shootings. An alternative approach has been to derive measures of sentiments using various surveys. For example, [19, Brown and Cliff (2005)] show how a direct measure of investor sentiments, the bull/bear spread from the Investor's Intelligence survey, can account for deviations of prices from intrinsic values. Evidence that survey measures capture at least some aspects of investors' psychological attitudes is also provided by [34, Lansing (2019)], who develops a real business cycle model with both fundamental shocks and an equity sentiment shock that captures fluctuations driven by animal spirits and shows that such model-identified sentiment shock is indeed highly correlated with survey-based measures of US consumer sentiments.

Scholars have also looked at historical evidence on particular market episodes to assess whether increases in stock prices were driven purely by the irrational exuberance of investors or were instead rooted in rising fundamental values. For example, [32, Freher et al (2013)], through cross-sectional analysis of stock prices, find that the 1720 boom was based on economic fundamentals, such as financial innovations and the increase in trans-Atlantic trade; [39, Nicholas (2008)] analyses cross-sectional data on patents in the 1920s and finds that firms with valuable patents did indeed rise relatively more than other firms prior to the 1929 crash; and [40, Pástor and Veronesi (2009)] look at cross-sectional historical data on the 19th century railroad boom in the United States and find that technological innovations had an important role in driving up stock prices of railroad companies.

Learning and bounded rationality have also been used to account for the large volatility of prices and for deviation of markets from fundamental values. Examples of such approaches include [12, Barsky and De Long (1993)], [18, Brennan and Xia (2001)], [17, Branch and Evans (2011)], [2, Adam et al (2016)] and [1, Adam et al (2016)]. Contrary to the sentiments literature, investors are not usually considered irrational in this approach, and the additional variation in prices required to match the data comes from changing beliefs over time due to

[3] For a comprehensive review of this approach, see [8, Baker and Wurgler (2007)]. More in general, behavioral approaches to asset pricing have been discussed in [9, Barberis and Thaler (2003)] and, more recently, in [45, Shiller (2014)]. For an earlier critical view, see [30, Fama (1998)].

limited information and learning. Heterogeneity of beliefs among traders can also give rise to speculative excesses, especially when short selling is limited (as in [38, Miller (1977)]).

Finally, terms such as irrational exuberance and sentiments should not be confused with the idea of bubbles, and in particular of rational bubbles. Rational bubbles, in fact, require strict conditions for their existence, as shown in [27, Diba and Grossman (1988b)], and [26, Diba and Grossman (1988a)] do not find evidence of their existence in stock prices.

2 The model

I start from a fundamental model of asset prices, where the fair price of an asset is given by the discounted value of expected future dividends, using a risk adjusted discount factor

$$f_t^{LI} = \sum_{i=1}^{\infty} \beta_{t,t+i} E_t d_{t+i}. \tag{1}$$

Here f_t^{LI} denotes the fundamental value of the asset under limited information for agents, as they don't know the future stream of dividends; $\beta_{t,t+i}$ represents the discount rate between period t and $t+i$, with its inverse given by the t-period risk free rate plus a risk adjustment factor, the risk premium. The expectational operator E_t is taken with respect to the information set available to agents. If the price of the asset, p_t, does not include sentiments, it should be equal to f_t^{LI}.

Equation (1) allows for time-varying discount rates and a terms structure of discount rates. I will assume, though, that at each period t, the i-period discount rate, $\beta_{t,t+i}$, is the compounded one period rate, $\beta_{t,t+1}$, simply denoted β_t; that is $\beta_{t,t+i} = \beta_t^i$. I will therefore abstract from the term structure of discount rates and focus on the time-varying aspect instead.

At each time t, in order to compute the fair price, given a certain discount rate β_t, agents need to form expectations about the future stream of dividends entitled to by the asset. I will now present assumptions about the dividends process and agents' information structure.

2.1 Dividends process and information

I assume that dividends include a permanent and a transient component. The permanent component is meant to represent the time-varying fundamental value of the firm, its ability to generate profits, while the transient component might capture temporary elements affecting dividends, such as variations in the dividends distribution policy of the firm. Agents only observe current and past dividends, but need to disentangle the two components in order to predict future dividends and thus compute the fundamental value of the asset.

Denoting dividends with d_t, the permanent component with θ_t and the transient component with e_t:

$$d_t = \theta_t + e_t. \tag{2}$$

The transient component e_t is a zero mean process following a normal distribution $N\left(0, \sigma_{e,t}^2\right)$ while the permanent component evolves according to the equation

$$\theta_t = \rho_{t-1}\theta_{t-1} + v_{t-1}, \tag{3}$$

with v_t a zero mean process (in particular, independent of e_t) following a normal distribution $N\left(0, \sigma_{v,t}^2\right)$. The AR(1) parameter is allowed to be time-varying and modelled as

$$\rho_t = \rho_{t-1} + z_{t-1}, \tag{4}$$

with z_t a zero mean process following a normal distribution $N\left(0, \sigma_{z,t}^2\right)$ .The noise processes v_t and z_t can be (contemporaneously) correlated with each other, but independent of e_t.

Variances and covariances of all shocks are allowed to be time-varying: both the precision of agents' information and the volatility of the state variables can change over time, giving rise to time-varying uncertainty in agents' expectations.

Estimates for θ_t and ρ_t will be derived by agents through a non-linear Kalman filter, which allows them to use their information optimally in this context.

2.2 Fundamental value with limited information

If investors were to have full information, observing θ_t and ρ_t, the rational expectations, full information solution for the fundamental value, for $\beta_t\rho_t < 1$, would be

$$f_t^{FI} = \frac{\theta_t}{1 - \beta_t\rho_t}, \tag{5}$$

where the superscript FI denotes full information. Agents, though, do not have full information, as they are uncertain about θ_t and ρ_t and can only observe dividends.

Given the assumptions stated above about the discount rate, equation (1) can be rewritten as

$$f_t^{LI} = \sum_{i=1}^{\infty} \beta_t^i E_t d_{t+i},$$

where $E_t = E[\bullet \mid I_t]$, with I_t the information set of agents at time t, now given by $\{d_j\}_{j \leq t}$.

In general, because of imperfect information (that is, θ_t,$\rho_t \notin I_t$), $f_t^{LI} \neq f_t^{FI}$.

The framework presented in (2.1) implies that

$$E_t d_{t+i} = E_t \theta_{t+i} = E_t \rho_t^i \theta_t = E_t \rho_t^i E_t \theta_t + cov(v_t, z_t). \tag{6}$$

In order to compute such expectations, I will use the approximation $E_t [f(\rho_t)] \simeq f(E_t[\rho_t])$, so $E_t[\rho_t^i] \simeq E_t[\rho_t]^i$, using a first order Taylor expansion around $E_t[\rho_t]$, with the linear term being equal to zero. This leads to

$$E_t[\rho_t^i \theta_t] = E_t \rho_t^i E_t \theta_t + cov(v_t, z_t) \simeq E_t[\rho_t]^i E_t[\theta_t] + cov(v_t, z_t).$$

Given then the assumed model for dividends and information structure, the optimal estimates for θ_t and ρ_t, $E_t[\rho_t]$ and $E_t[\theta_t]$, can be derived through an extended Kalman filter (EKF) procedure, which I will outline below. For the moment, denoting $\hat{\theta}_t$ and $\hat{\rho}_t$ such EKF estimates:

$$E_t d_{t+i} \simeq \hat{\rho}_t^i \hat{\theta}_t + cov(v_t, z_t), \tag{7}$$

with $cov(v_t, z_t)$ also provided by the EKF algorithm.

Assuming $\beta_t \hat{\rho}_t < 1$, this leads to

$$f_t^{LI} = \sum_{i=1}^{\infty} \beta_t^i \hat{\rho}_t^i \hat{\theta}_t = \frac{\hat{\theta}_t}{1 - \beta_t \hat{\rho}_t} + \frac{cov(v_t, z_t)}{1 - \beta_t}. \tag{8}$$

The uncertainty related to such estimate for the fundamental value is denoted $\sigma_{I,t}^2$ and given by (disregarding uncertainty about $cov(v_t, z_t)$, which is negligible in comparison)

$$\sigma_{I,t}^2 = E_t \left(\frac{\theta_t}{1 - \beta_t \rho_t} - \frac{\hat{\theta}_t}{1 - \beta_t \hat{\rho}_t} \right)^2. \tag{9}$$

3 Beliefs and uncertainty

3.1 Formation of beliefs: extended Kalman filter

Given the assumed state-space model for dividends, presented in (2), (3) and (4), the optimal way for agents to form expectations about future dividends, and thus the fundamental value of the asset, is through an EKF, which allows for the estimation of current values of the unobservable θ_t and ρ_t based on the history of dividends up to time t.

For convenience, I rewrite here the state-space system describing the evolution of the state variables θ_t and ρ_t and the observation/measurement equation for dividends here:

$$\begin{aligned} \theta_t &= \rho_{t-1}\theta_{t-1} + v_{t-1} && (10) \\ \rho_t &= \rho_{t-1} + z_{t-1} && (11) \\ d_t &= \theta_t + e_t, && (12) \end{aligned}$$

with (10) and (11) representing the state or transition equations and (12) the measurement equation.

It is worth noting here that since the only information used by agents is common (i.e., dividends), realised prices do not represent an additional source of information above and beyond the exogenous information conveyed by dividends.

Since the system (10)-(11) is nonlinear, the EKF implements an approximation around the current estimates of means and covariances. For a detailed derivation of the equations below, see [22, Chui and Chen (2009)].

Defining

$$x_t = \begin{bmatrix} \theta_t \\ \rho_t \end{bmatrix}, \varepsilon_{t-1} = \begin{bmatrix} v_{t-1} \\ z_{t-1} \end{bmatrix},$$

the system can be represented as

$$\begin{aligned} x_t &= F(x_{t-1}) + \varepsilon_{t-1} && (13) \\ d_t &= h(x_t) + e_t. && (14) \end{aligned}$$

This system is then approximated on the basis of a linear Taylor approximation of $F(x_{t-1})$

around the latest estimate $\hat{x}_{t-1}$ and $h(x_t)$ around the prediction $\hat{x}_t^-$.[4] Defining

$$\begin{aligned}
A_{t-1} &= \frac{dF}{dx_{t-1}}(\hat{x}_{t-1}) = \begin{bmatrix} \hat{\rho}_{t-1} & \hat{\theta}_{t-1} \\ 0 & 1 \end{bmatrix} \\
B &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
C_t &= \frac{dh}{dx_t}(\hat{x}_t^-) = \begin{bmatrix} 1 & 0 \end{bmatrix} \equiv C \\
D &= 1,
\end{aligned}$$

with $E\varepsilon_t\varepsilon_t^{'} = Q_t$, $Ee_te_t^{'} = R_t$ and given appropriate initial conditions $\hat{x}_0, \hat{\rho}_0, \hat{\theta}_0, \Sigma_0$, the EKF algorithm is given by:

1. State estimates time update:

$$\hat{x}_t^- = F(\hat{x}_{t-1}).$$

2. Error covariance time update (prior error covariance):

$$\Sigma_t^- = A_{t-1}\Sigma_{t-1}A_{t-1}' + BQ_{t-1}B'.$$

3. Output estimate

$$\hat{d}_t = C\hat{x}_t^-.$$

4. Kalman gain

$$L_t = \Sigma_t^- C' [S_t]^{-1}$$

with innovation covariance

$$S_t = C\Sigma_t^- C' + DR_tD'.$$

5. State estimate measurement update: defining the measurement innovation $\kappa_t = d_t - \hat{d}_t$,

$$\hat{x}_t = \hat{x}_t^- + L_t\kappa_t. \tag{15}$$

6. Error covariance measurement update (posterior error covariance)

$$\Sigma_t = (I - L_tC)\Sigma_t^-.$$

[4] To be precise, in the present system function h does not need to be approximated, as it is already in linear form.

The standard derivation of the EKF filter is based on the assumption that the time-varying variance covariance matrices of state and observation noise, Q_{t-1} and R_t, are known. When this is not the case, such matrices can be estimated adaptively over time. To this end, I will follow the methodology outlined in [4, Akhlaghi et al (2017)]:

1. Residual based adaptive estimation of R_t:

Defining the residual $\eta_t = d_t - C\hat{x}_t = d_t - \hat{\theta}_t$, it can be shown that $R_t = E[\eta_t^2] + C\Sigma_t^- C'$. This term is then approximated adaptively using the observed η_t^2 and a forgetting factor $0 < \alpha \leq 1$:

$$\hat{R}_t = \alpha \hat{R}_{t-1} + (1-\alpha)\left(\eta_{t-1}^2 + C\Sigma_t^- C'\right).$$

The update can be implemented from $t = 2$ on, for an appropriate guess $\hat{R}_1$.

2. Innovation based adaptive estimation of Q_{t-1}:

From (13),

$$\varepsilon_{t-1} = x_t - F\left(x_{t-1}\right)$$

and using (15) one can write,

$$\hat{\varepsilon}_{t-1} = \hat{x}_t - F\left(\hat{x}_{t-1}\right) = \hat{x}_t - \hat{x}_t^- = L_t \kappa_t.$$

It follows that

$$E\hat{\varepsilon}_{t-1}\hat{\varepsilon}_{t-1}^{'} = E\left[L_t \kappa_t \kappa_t^{'} L_t^{'}\right] = L_t E\left[\kappa_t \kappa_t^{'}\right] L_t^{'}$$

which can be estimated adaptively, using time averages and a forgetting factor α:

$$\hat{Q}_t = \alpha \hat{Q}_{t-1} + (1-\alpha)\left(L_t \kappa_t \kappa_t^{'} L_t^{'}\right)$$

for an appropriate guess $\hat{Q}_0$.

3.2 Measuring uncertainty

Uncertainty is the key variable in the proposed methodology to disentangle sentiments from variations in risk premia and its measurement is thus particularly important. In particular, (9) defines uncertainty as the variance of the information based fundamental value. Having defined the structure of the system and the information available to agents, it is now possible to compute such value.

The EKF procedure provides estimates for the posterior variance and covariance of the estimates $\hat{\theta}_t$ and $\hat{\rho}_t$, that is:

$$\begin{aligned}
\sigma^2_{\theta,t} &= E_t\left(\theta_t - \hat{\theta}_t\right)^2 \\
\sigma^2_{\rho,t} &= E_t\left(\rho_t - \hat{\rho}_t\right)^2 \\
\sigma^2_{\theta\rho,t} &= E_t\left(\theta_t - \hat{\theta}_t\right)\left(\rho_t - \hat{\rho}_t\right)
\end{aligned}$$

where $\hat{\theta}_t$ and $\hat{\rho}_t$ are the EKF estimates for θ_t and ρ_t respectively.

In order to compute $\sigma^2_{I,t}$ from these variances and covariances, I derive a second order Taylor expansion for $\sigma^2_{I,t}$ around current estimates $\hat{\theta}_t$ and $\hat{\rho}_t$. Denoting $\left(\frac{\theta_t}{1-\beta\rho_t} - \frac{\hat{\theta}_t}{1-\beta\hat{\rho}_t}\right)^2 \equiv f\left(\theta_t, \rho_t\right)$:

$$\sigma^2_{I,t} \equiv E_t\left(\frac{\theta_t}{1-\beta_t\rho_t} - \frac{\hat{\theta}_t}{1-\beta_t\hat{\rho}_t}\right)^2 \approx E_t\left[\begin{array}{c}
\left(\frac{\hat{\theta}_t}{1-\beta_t\hat{\rho}_t} - \frac{\hat{\theta}_t}{1-\beta_t\hat{\rho}_t}\right)^2 + f'_\theta|_{\hat{\theta}_t,\hat{\rho}_t}\left(\theta_t - \hat{\theta}_t\right) + \\
+ f'_\rho\big|_{\hat{\theta}_t,\hat{\rho}_t}\left(\rho_t - \hat{\rho}_t\right) + \frac{1}{2}\, f''_{\theta,\theta}\big|_{\hat{\theta}_t,\hat{\rho}_t}\left(\theta_t - \hat{\theta}_t\right)^2 + \\
+\frac{1}{2}\, f''_{\rho,\rho}\big|^2_{\hat{\theta}_t,\hat{\rho}_t}\left(\rho_t - \hat{\rho}_t\right)^2 + f''_{\theta,\rho}\big|^2_{\hat{\theta}_t,\hat{\rho}_t}\left(\theta_t - \hat{\theta}_t\right)\left(\rho_t - \hat{\rho}_t\right)
\end{array}\right].$$

Given that, from the EKF, $E_t\theta_t = \hat{\theta}_t$ and $E_t\rho_t = \hat{\rho}_t$, all linear terms are equal to zero and

$$\begin{aligned}
E_t\left(\frac{\theta_t}{1-\beta_t\rho_t} - \frac{\hat{\theta}_t}{1-\beta_t\hat{\rho}_t}\right)^2 &\approx \frac{1}{2}\, f''_{\theta,\theta}\big|_{\hat{\theta}_t,\hat{\rho}_t} E_t\left(\theta_t - \hat{\theta}_t\right)^2 + \frac{1}{2}\, f''_{\rho,\rho}\big|_{\hat{\theta}_t,\hat{\rho}_t} E_t\left(\rho_t - \hat{\rho}_t\right)^2 + \ldots \\
&\quad \ldots + f''_{\theta,\rho}\big|_{\hat{\theta}_t,\hat{\rho}_t} E_t\left(\theta_t - \hat{\theta}_t\right)\left(\rho_t - \hat{\rho}_t\right),
\end{aligned}$$

where

$$\begin{aligned}
f''_{\theta,\theta}\big|_{\hat{\theta}_t,\hat{\rho}_t} &= \frac{2}{\left(1-\beta_t\hat{\rho}_t\right)^2} \\
f''_{\rho,\rho}\big|_{\hat{\theta}_t,\hat{\rho}_t} &= \frac{2\left(\beta_t\hat{\theta}_t\right)^2}{\left(1-\beta_t\hat{\rho}_t\right)^4} \\
f''_{\theta,\rho}\big|_{\hat{\theta}_t,\hat{\rho}_t} &= \frac{2\beta_t\hat{\theta}_t}{\left(1-\beta_t\hat{\rho}_t\right)^3}.
\end{aligned}$$

The uncertainty associated with the estimated fundamental value for the asset is thus given by

$$\sigma^2_{I,t} \approx \frac{\sigma^2_{\theta,t}}{\left(1-\beta_t\hat{\rho}_t\right)^2} + \frac{\left(\beta_t\hat{\theta}_t\right)^2\sigma^2_{\rho,t}}{\left(1-\beta_t\hat{\rho}_t\right)^4} + \frac{2\beta_t\hat{\theta}_t\sigma^2_{\theta\rho,t}}{\left(1-\beta_t\hat{\rho}_t\right)^3}. \tag{16}$$

The variable $\sigma^2_{I,t}$ thus captures the total uncertainty of the estimated fundamental value. It depends on the EKF variances $\sigma^2_{\theta,t}$, $\sigma^2_{\rho,t}$ and $\sigma^2_{\theta\rho,t}$, and on the estimated $\hat{\theta}_t$ and $\hat{\rho}_t$, besides the (known) β_t. This dependency is due to the nonlinearities in f^{LI}_t.

In the empirical analysis, I will also use the EKF variances $\sigma^2_{\theta,t}$ and $\sigma^2_{\rho,t}$ as a measure for uncertainty: the reason is that, since β_t is computed as a residual from actual prices, a positive correlation of $\sigma^2_{I,t}$ with prices might arise mechanically and it might not necessarily be driven by changes in uncertainty. The measure $\sigma^2_{\theta,t}$ and $\sigma^2_{\rho,t}$ are instead completely exogenous from prices and necessarily reflect changes in uncertainty.

4 The role of uncertainty

Sentiments and time-varying risk premia are two ways to explain the residual movement in prices not justified by changes in expected dividends or payoffs, one attributing it to irrational behavior of investors and the other to their rational decision making.

For any given series of prices and dividends, one can come up with an appropriate series of risk premia which justifies prices as the fundamental value of the asset. The question then becomes whether the implied variation in risk premia is reasonable or not, according to some specific model of rational investor behavior.

I propose in this work a method to distinguish the two possible sources of price variation, sentiments and risk premia, through their relationship with uncertainty. This strategy amounts to assess the comovements between implied risk premia and uncertainty: if sentiments are not present, risk premia should respond to changes in perceived risk only; if prices instead include a sentiments component, this will be captured by the implied risk premium (since it is computed as a residual) and can thus make such variable move in ways that are not consistent with rational risk averse behavior.

4.1 Time-varying risk premia

Rates used to discount future dividends depend on two components: the real risk-free rate, i_t, and the risk premium (erp_t):

$$\beta_t^{-1} = r_t = i_t + erp_t. \tag{17}$$

As the risk premium compensates for risk, it should depend positively on the perceived riskiness of the asset, and therefore on the uncertainty of its estimated fundamental value.

If there was no uncertainty and the fundamental value of the asset was known for sure, there would be no risk: investors would know precisely the payoffs they were going to receive from their investment. The larger is the uncertainty about the future stream of dividends, and thus about the fundamental value, the riskier is the asset and therefore the higher is the compensation required by investors, i.e., the risk premium.

Besides on the amount of risk, the risk premium should depend also on the price of risk, that is the degree of (absolute) risk aversion of investors. I abstract from this element here, effectively treating risk aversion as constant. This semplification will not create problems in my empirical application of the model, as long as risk aversion does not systematically move in the opposite direction of risk. If this was the case, an observed increase in risk, matched by an even stronger reduction in risk aversion, could lead to a decrease in the risk premium, rather than an increase, and lead to wrong conclusions regarding the presence of sentiments on the market.

I formalize the dependence of the risk premium on uncertainty by denoting $erp_t = erp\left(\sigma^2_{I,t}\right)$, with $\sigma^2_{I,t}$ given by (9). (17) thus becomes

$$r_t = i_t + erp\left(\sigma^2_{I,t}\right). \tag{18}$$

Since the risk premium increases with uncertainty,[5] that is, $\frac{d\ erp\left(\sigma^2_{I,t}\right)}{d\ \sigma^2_{I,t}} > 0$, it follows that $\frac{dr_t}{d\sigma^2_{I,t}} > 0$ and $\frac{d\beta_t}{d\sigma^2_{I,t}} < 0$. This implies that $\frac{df^{LI}_t}{d\sigma^2_{I,t}} < 0$: an increase in uncertainty should always decrease prices through the risk premium channel.

4.2 Sentiments

Besides reflecting the limited information fundamental value f^{LI}_t, prices can include also an irrational component, sentiments, determined by the psychological attitudes of investors. That is, agents' beliefs about the value of an asset can deviate from information based estimates, and such beliefs are reflected into prices.

Denoting sentiments as S_t, prices are made up of two components:

$$p_t = f^{LI}_t + S_t. \tag{19}$$

The key modelling device I introduce at this point is to explicitly link sentiments to

[5]This terminology is not consistent with the classical distinction between risk and uncertainty, depending on known or unknown probabilities of future events, as the *risk* premium depends here on the amount of *uncertainty*, as measured by the variance of the estimate fundamental value.

uncertainty about the fundamental value. In particular, I model sentiments as[6]

$$S_t = h\left(\sigma^2_{I,t}, s_t\right), \tag{20}$$

where s_t is a stochastic variable that represents a sentiment shock (not necessarily i.i.d.) deriving from investors' psychological attitudes. A positive value for s_t captures investors' optimism about the value of the asset, while a negative value represents pessimism. The function $h\left(\sigma^2_{I,t}, s_t\right)$ combines those attitudes with uncertainty, determining the magnitude of sentiments, and has the following properties:

1. $h(0,.) = 0$: there can be no sentiments when there is no uncertainty, as one cannot be optimistic or pessimistic about something it is known for sure;
2. $sign\left(h(., s_t)\right) = sign(s_t)$: whether investors are optimistic or pessimistic is determined by the psychological attitudes of investors, and does not depend on the level of uncertainty;
3. $sign\left(\frac{dh(.,s_t)}{d\sigma^2_{I,t}}\right) = sign(s_t)$: an increase in uncertainty amplifies sentiments, in either direction.

An example of a function that satisfies these restrictions is $h\left(\sigma^2_{I,t}, s_t\right) = \left(\sigma^2_{I,t}\right)^\gamma s_t$, $\gamma > 0$, though I will not impose such restriction in the empirical analysis.

The proposed representation (20) captures the idea that sentiments are sparked by exogenous shocks to beliefs, represented by investors' psychological attitudes, but can only propagate to prices through uncertainty. There can be no sentiments when agents know with certainty the value of an asset, as no-one would be willing to buy the asset at a higher price or sell it at a lower price than its fundamental value. It is the uncertainty about the intrinsic value of the asset that makes sentiments possible, and the larger is that uncertainty, the more prices can deviate from the rational, information based, fundamental value.

Equation (19) then implies that

$$\frac{dp_t}{d\sigma^2_{I,t}} = \frac{df^{LI}_t}{d\sigma^2_{I,t}} + \frac{dS_t}{d\sigma^2_{I,t}}. \tag{21}$$

The first component on the right hand side captures the impact of uncertainty on prices through changes in the discount factor, i.e., through risk premia, and the second component captures the impact through sentiments.

[6] This characterization of sentiments has been originally proposed in [16, Berardi (2020)].

This framework then allows one to derive testable implications for the presence of sentiments in asset prices, which I will present in the following Section (4.3).

4.3 Testable restrictions for identifying sentiments

According to the model of risk premium developed in Sections (4.1), the first component of (21), $df_t^{LI}/d\sigma_{I,t}^2$, is always negative: an increase (decrease) in uncertainty increases (decreases) the risk premium, decreasing (increasing) β_t and thus having a depressing (expansionary) effect on prices. Without sentiments (i.e., $S_t \equiv 0$), thus, one would expect to see negative comovements between prices and uncertainty, since $\frac{dp_t}{d\sigma_{I,t}^2} = \frac{df_t^{LI}}{d\sigma_{I,t}^2} < 0$.

With sentiments, instead, an additional effect of uncertainty on prices is present. The second component in (21), $dS_t/d\sigma_{I,t}^2$, can be either positive or negative, depending on the sign of s_t, according to the framework developed in Section (4.2). With positive psychological attitudes (optimism), i.e.„ $s_t > 0$, increased (reduced) uncertainty leads to larger (smaller) sentiments, as $dS_t/d\sigma_{I,t}^2 > 0$: if strong enough, this effect can dominate the negative impact on prices of the increased risk premium and lead to an overall increase in prices. With negative psychological attitudes (pessimism), i.e., $s_t < 0$, increased (reduced) uncertainty leads instead to smaller (larger) sentiments (that is, larger in absolute value, but negative) since $dS_t/d\sigma_{I,t}^2 < 0$: sentiments now act in the same direction as risk premia, depressing prices. Positive comovements between changes in uncertainty and changes in prices, thus, require sentiments (and in particular, positive sentiments) to be present, as they cannot be generated by changes in risk premia alone. Negative comovements instead can be generated by risk premia alone, though they do not rule out sentiments.

These are the model restrictions that I will be using in order to identify sentiments in stock prices and distinguish between the two alternative explanations for price movements. Equivalently, one could look at changes in the implied risk premium after a change in uncertainty. If $\frac{derp_t}{d\sigma_{I,t}^2} < 0$, this means that after an increase (decrease) in uncertainty, a lower (higher) risk premium is required to match prices with the limited information fundamental value, implying that prices have increased (decreased). Again, this would be inconsistent with the theory of rational investors, and would require sentiments to be present. Looking at implied risk premia rather than prices has the advantage to clean out the effect of changes in the risk free rate, though there is no reason to expect them to be related to changes in uncertainty.

In order to test these restrictions in the data, I will look at the relation between uncertainty on one side, and prices and implied risk premia on the other side, using S&P 500 data from 1872 till 2019. A positive correlation between uncertainty and prices would require sentiments to be explained, while a negative one would be consistent with rational investors' behavior (while not necessarily ruling out sentiments). Similarly, a negative correlation between implied risk premia and uncertainty would point to the presence of sentiments in stock prices.

5 Empirical analysis

I now apply the theoretical framework developed so far to historical financial data and see if it possible to identify the presence of sentiments in stock prices. In particular, I use historical data for the S&P 500 index and dividends, constructed by Schiller and available on his webpage at: http://www.econ.yale.edu/~shiller/data.htm. I use annual data, inflation adjusted, covering the period 1872-2019.

First, from the historical series of dividends, I compute investors' real time estimates of first and second moments $\hat{\theta}_t$, $\hat{\rho}_t$, $\sigma^2_{\theta,t}$, $\sigma^2_{\rho,t}$ and $\sigma^2_{\theta\rho,t}$ through the EKF procedure outlined before. For the smoothing parameter in the adaptive estimation of variance covariance matrices, I set $\alpha = .75$. This value generates enough variability in estimated uncertainty while still guaranteeing good behavior of the whole algorithm. I also initialize all variance covariance matrices according to their long-run mean.

With estimates from the EKF procedure, I then use historical data for the S&P 500 index and equate it to the limited information fundamental value according to equation (8), thus computing the implied series for the discount rate β_t, and therefore for $r_t = \beta_t^{-1}$. Using then Shiller's historical data for the real long term government bond yield, as a proxy for the real risk-free rate i_t, I use (18) to derive the series for the implied risk premium erp_t.

As noted before, in addition to the measure of uncertainty represented by $\sigma^2_{I,t}$, computed through (16), I will also use the EKF variances $\sigma^2_{\theta,t}$ and $\sigma^2_{\rho,t}$.[7] These estimates are an additional measure for investors' uncertainty, and have the desirable property of being exogenous to the pricing model used. This means that, contrary to $\sigma^2_{I,t}$, they are independent of β_t, which is derived as a residual. While movements in $\sigma^2_{I,t}$ might come entirely from movements in β_t required to match observed prices, thus introducing spurious correlation between $\sigma^2_{I,t}$ and p_t, movements in $\sigma^2_{\theta,t}$ and $\sigma^2_{\rho,t}$ are solely due to changes in the precision of the EKF estimates, thus ensuring exogeneity with respect to prices.

[7] Note that $\sigma^2_{\theta,t}$ and $\sigma^2_{\rho,t}$ are strongly correlated with each other, given that the filtering procedure is based on a unique common observable, dividends.

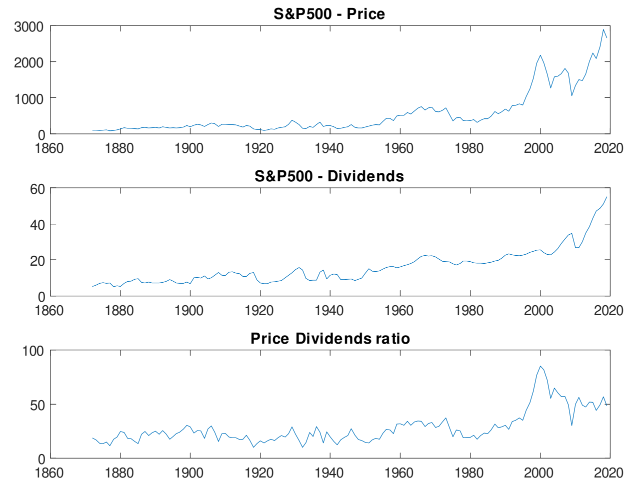

Figure 1: S&P 500 index, dividends and P/D ratio. Source for prices and dividends: http://www.econ.yale.edu/~shiller/data.htm

Before presenting results on the comovements between uncertainty and prices, I present a visual summary of the data and the outcomes from the EKF procedure.

Fig. 1 displays the S&P 500 index, dividends and the price dividends ratio. It can be seen that particularly from the second half of the 1990', prices and dividends diverge, with a significant increase in the price dividends ratio.

Fig. 2 presents the evolution of investors' estimates for $\hat{\theta}_t$ and $\hat{\rho}_t$ coming from the EKF filtering procedure based on current and past dividends at each point in time.

Fig. (3) then presents the evolution of uncertainty, as $\sigma^2_{\theta,t}$, $\sigma^2_{\rho,t}$ and $\sigma_{I,t}$.

Finally, Fig. 4 presents estimates for the implied equity risk premium, erp_t: the overall average for the whole period is 3.82% and it can be seen that the volatility of the implied risk premium has gone down substantially from the late 1980', with also a somewhat smaller average. As the volatility of prices requires much less variation in implied risk premia to be accounted for in recent years, sentiments that were computed from the residual variation in the risk premium not accounted for by some model of discount rates would show less volatility.

I now turn to the analysis of comovement patterns between uncertainty and prices. Without sentiments, one would expect to find a negative correlation between prices and uncertainty measures, as discussed above. A positive correlation, instead, would point to the presence of sentiments in S&P 500 prices.

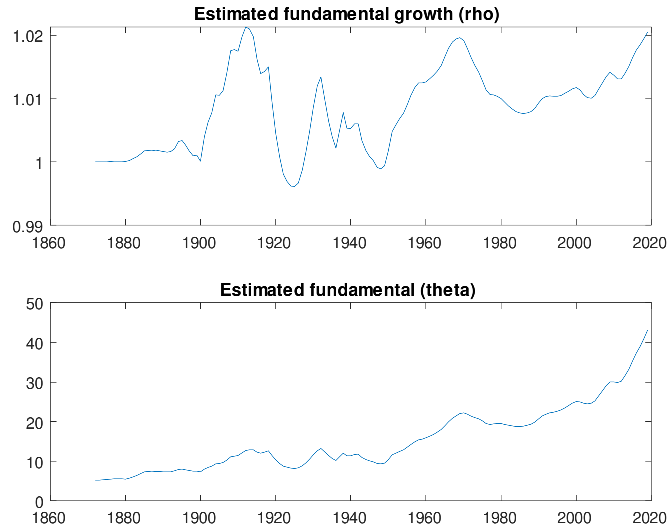

Figure 2: Estimates for $\hat{\theta}_t$ and $\hat{\rho}_t$ computed using the EKF procedure and equation (16).

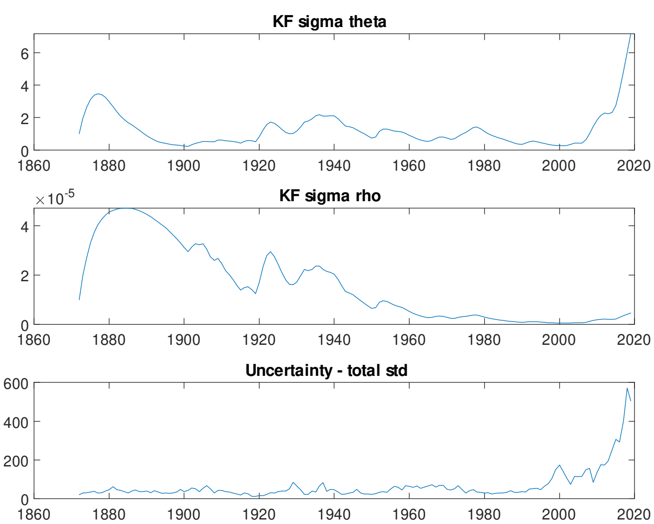

Figure 3: Measures of uncertainty: $\sigma^2_{\theta,t}$, $\sigma^2_{\rho,t}$ and $\sigma_{I,t}$

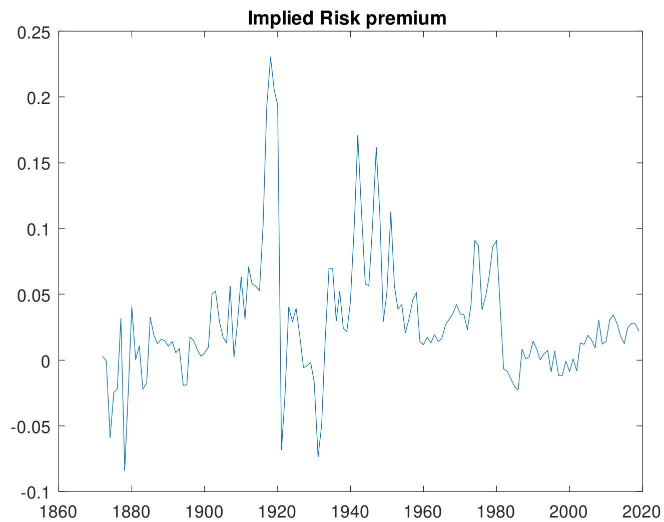

Figure 4: Implied risk premium (erp_t).

As said, I measure uncertainty as the variance of agents' estimated value for the asset, based only on information represented by the stream of past and current dividends at each time, which can be assumed to be common knowledge for investors. This measure of uncertainty, thus, is greater than or equal to the total uncertainty agents really faced, as other sources of information (both public and private) might have been used that would have reduced the uncertainty of agents' estimates. This limitation, though, does not represent a problem for my analysis, as long as the precision of unobserved information is not negatively correlated with the precision of the information from dividends. That is, it must not be the case that, when uncertainty from dividends increased (decreased), the informational content of other variables increased (decreased) and compensated for it. There is no reason to believe that this would be the case.

I start with the regression,

$$p_t = b_0 + b_1 \log(\sigma^2_{I,t}),$$

whose results are in Table 1.

	Estimate	SE	tStat	pValue
(Intercept)	-2486.1	131.7	-18.876	5.2318e-41
log_tu	390.79	16.722	23.37	3.4082e-51

Table 1: Output for regression $p_t = b_0 + b_1 \log(\sigma^2_{I,t})$

It can be seen that the coefficient on the measure of uncertainty $\log(\sigma^2_{I,t})$ is positive and significant: prices increase with uncertainty. This result, as explained above, is not consistent with rational investors, and points to the presence of sentiments in the S&P 500 index.

For robustness, I run the additional regression

$$p_t = b_0 + b_1 \log(\sigma^2_{I,t}) + b_2 \hat{\rho}_t + b_3 \hat{\theta}_t,$$

where now estimates $\hat{\rho}_t$ and $\hat{\theta}_t$ are included among the regressors. Results are reported in Table 2. As it can be seen, findings from the previous regression are confirmed here, with the coefficient on measured uncertainty still positive and significant. In addition, the coefficient on $\hat{\theta}_t$ has the expected sign, while the coefficient on $\hat{\rho}_t$ seems to go against intuition.

	Estimate	SE	tStat	pValue
(Intercept)	9533.5	2981.9	3.1972	0.0017063
log_tu	197.06	15.745	12.515	8.7899e-25
rho	-11152	2964.1	-3.7625	0.00024416
theta	47.499	3.2336	14.689	1.9939e-30

Table 2: Output for regression $p_t = b_0 + b_1 \log(\sigma^2_{I,t}) + b_2 \hat{\rho}_t + b_3 \hat{\theta}_t$

As said, the use of $\sigma^2_{I,t}$ as measure of uncertainty might introduce spurious correlation with prices. I therefore run the following regression next:

$$p_t = b_0 + b_1 \sigma^2_{\rho,t} + b_2 \sigma^2_{\theta,t} + b_3 \hat{\rho}_t + b_3 \hat{\theta}_t.$$

Results are reported in Table 3: the coefficient on $\sigma^2_{\rho,t}$ is positive and highly significant, confirming the previous positive correlation between uncertainty and prices. The coefficient on $\sigma^2_{\theta,t}$ is instead negative, but not significant. The sign on $\hat{\rho}_t$ is still puzzling.

	Estimate	SE	tStat	pValue
(Intercept)	15265	3970.7	3.8444	0.00018131
rho	-16158	3966.8	-4.0734	7.6454e-05
theta	92.377	4.5351	20.369	4.1105e-44
sigmarho	1.1856e+07	2.0703e+06	5.7266	5.8157e-08
sigmatheta	-25.884	20.756	-1.247	0.21442

Table 3: Output for regression: $p_t = b_0 + b_1\sigma^2_{\rho,t} + b_2\sigma^2_{\theta,t} + b_3\hat{\rho}_t + b_3\hat{\theta}_t$

I now investigate the correlation between implied risk premia and uncertainty. Looking at the movements in implied risk premia, in addition to prices, is important because prices could be reacting to a larger set of unobserved factors, while risk premia, by definition, should depend only on risk. I first run the regression

$$erp_t = b_0 + b_1 \log(\sigma^2_{I,t}),$$

whose results are in Table 4. It can be seen that the coefficient on measured uncertainty is negative and highly significant: increases in uncertainty correspond to decreases in implied risk premia. This result means that when uncertainty increases, implied risk premia have to decrease in order for the expected dividends discount model to match observed prices: as this goes against our understanding of the behavior of risk premia, it suggests that sentiments were present and compensated for the depressing effect that risk premia had when uncertainty increased.

	Estimate	SE	tStat	pValue
(Intercept)	0.092576	0.022431	4.1272	6.1465e-05
log_tu	-0.0082503	0.002848	-2.8969	0.0043492

Table 4: Output for regression: $erp_t = b_0 + b_1 \log(\sigma^2_{I,t})$

I then use the alternative measures of uncertainty, $\sigma^2_{\rho,t}$ and $\sigma^2_{\theta,t}$ in the same regression on risk premia:

$$erp_t = b_0 + b_1\sigma^2_{\rho,t} + b_2\sigma^2_{\theta,t}.$$

Results are reported in Table 5: coefficients on both $\sigma^2_{\rho,t}$ and $\sigma^2_{\theta,t}$ are negative, confirming results from the previous regression, though only the one on $\sigma^2_{\rho,t}$ is significant at 95% confidence level.

	Estimate	SE	tStat	pValue
(Intercept)	0.040161	0.0069575	5.7724	4.5696e-08
sigmarho	-537.94	262.68	-2.0479	0.042378
sigmatheta	-0.0024675	0.0037911	-0.65087	0.51616

Table 5: Output for regression $erp_t = b_0 + b_1\sigma^2_{\rho,t} + b_2\sigma^2_{\theta,t}$

Results from this empirical analysis thus suggest that sentiments systematically affected S&P 500 prices over the sample period. The fact that I find evidence of a positive impact of uncertainty on prices (and of a negative impact on implied risk premia) implies that investors' sentiments were largely positive over the sample period: when uncertainty increased, investors' optimism about the value of the asset would drive up prices despite the contemporaneous increase in risk premia. This does not necessarily imply that investors would systematically expect prices to rise, as expectations are formed here on the value of the asset rather than on its future price, though it would seem reasonable to expect some strong correlation between the two. Evidence from the American Association of Individual

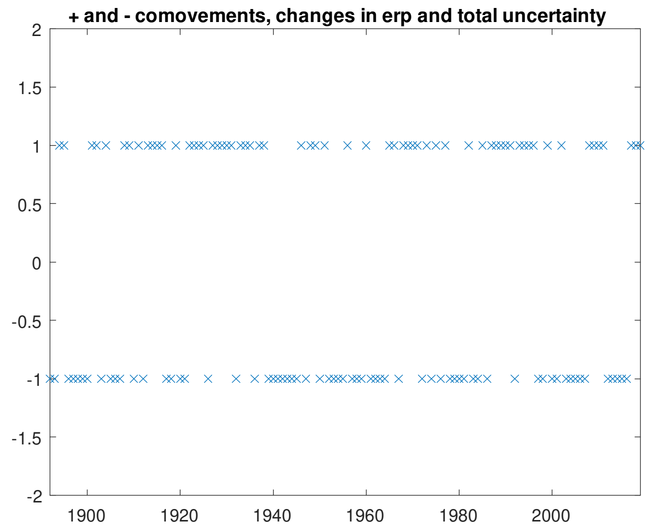

Figure 5: Years with negative comovements between erp_t and $\sigma^2_{I,t}$.

Investors survey indeed shows that over the period 1987 - 2020, the proportion of bullish investors has been 7.4% higher than that of bearish ones, and out of those 34 years, the annual average bull-bear spread has been positive for 30 years, with only four years in which investors have been more bearish than bullish. This evidence suggests that indeed investors tend to be more optimistic than pessimistic, on average, regarding stock prices.

In order to try and identify specific periods where sentiments affected prices, I select years in which changes in uncertainty and changes in implied risk premia had the opposite sign. Using $\sigma^2_{I,t}$ as a measure of uncertainty, negative comovements with implied risk premia were observed in 79 years (out of 148). This is reported in Fig (5). Using instead $\sigma^2_{\rho,t}$ and $\sigma^2_{\theta,t}$ as measures of uncertainty, in 57 occasions both measures of uncertainty moved in the same direction and opposite to the implied risk premium while only in 33 years the three moved in the same direction.

6 Conclusions

I have proposed in this paper a new way to look at the sources of volatility in stock prices. By modelling sentiments as uncertainty amplified shocks (psychological attitudes), and isolating the effect that changes in uncertainty should have on prices through sentiments and risk premia, I am able to derive testable implications that can help detect the presence of sentiments on financial markets. Looking at historical data for the S&P 500 index and dividends through the lenses of this new framework, I find evidence that investors' psychological attitudes have played an important role in determining movements in stock prices over the last 150 years. The observed positive comovements between uncertainty and prices could not have been generated by the rational response of investors to changes in risk, and require instead the presence of an irrational component to be explained.

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