Loan portfolio loss distribution: Basel II unifactorial approach vs. Non parametric estimations

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October 2007

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2007

Abstract

This paper analyzes the measurement of credit risk capital requirements under the new Basel Accord (Basel II): the Internal Rating Based approach (IRB). It focuses in the analytical formula for its calculation, since its derivation to the main assumptions behind it. We also estimate the credit loss distribution for the Uruguayan portfolio in the period 1999-2006, using a non parametric technique, the bootstrap. Its main advantage is that we don't need to make any assumptions about the form of the distribution. Finally, we compare the requirements obtained using the IRB with the estimated ones, as an approximation of the application of the IRB in the Uruguayan financial system.

Index

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I. Introduction

Financial institutions, and particularly banks, are exposed to different risks, which are inherent to the nature of their activities. Taking the simplest definition, we can define a bank as an institution whose habitual operations consist in giving loans and taking deposits from the public. In this definition, one can observe that risk could derive from the counterparty as well as from the mismatch that emerges from the asset transformation that banks make. Principal risks can be resumed in: credit risk, market risk, liquidity risk and operational risk. This paper will focus on credit risk analysis, which derives from the probability that the borrower defaults on his obligations. It is necessary to require banks to maintain a minimum of capital to cover potential losses due to this risk, which leads to the need for a proper system to measure it.

In 1988, Basel Committee proposed some recommendations to improve banking regulation (Basel I Accord), which were adopted by most part of world regulators and were considered as "best practices". This Accord represented a first step towards capital requirement based on credit risk, as it established fixed weights according to the risk associated with every exposure[1]. Different categories of exposures were determined in a simple way, and they did not allow for a proper measure of credit risk. As an example, all borrowers from non financial sector had the same weight. Financial system changed dramatically since that first Basel Accord.

[1] Minimum regulatory capital is calculated as:
Regulatory Capital= Risk Weight x Exposure x 8% = Risk-weighted Assets x 8%

In 1996, the Accord incorporated an amendment to require capital to cover market risk, defined as *"the risk of losses in on and off-balance sheet positions arising from movements in market prices"*[2]. Basel Committee permits to choose between two broad methodologies: a standardized manner (which has been proposed in 1993) and internal models (VaR).

Despite this breakthrough, the restrictions of the agreement of 1988 require an adaptation of the former, which is intended to be carried out within Basel II. The principal purpose is to make capital requirement more risk-sensitive, and also the promotion of the use of internal models to measure it. The Committee has built the supervisory process around three pillars:

1. Minimum capital requirements
2. Supervisory review process
3. Market discipline

In relation to the first pillar, the Accord of 1988 opted for a standardized approach, in which different risk were weighted according to the borrower's category. Basel II incorporates important changes in this pillar, introducing capital requirements for operational risk and significantly modifying the measurement of credit risk. Although it maintains the capital adequacy ratio at 8%, the way banks measure capital requirements is different. The rest of the pillars are new; pillar two refers to the supervision process, which must ensure that banks have adequate capital to support all the risk in their business, and also encourage institutions to develop and use better risk management techniques in monitoring and managing risks. The Committee has identified four key principles of supervisory review. First, banks must be able to demonstrate that chosen internal capital targets are well founded and that these targets are consistent with their overall risk profile. Secondly, supervisors must review and evaluate banks' internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios. Third and fourth principles refer to the ability of supervisor to require banks to hold capital in excess of the minimum, and the early intervention to prevent capital from falling below the minimum levels required to support its risks. Lastly, pillar three aims to promote a more competitive and transparent market, which reinforces the two previous pillars. These three pillars work together towards ensuring the capital adequacy of institutions. They are more potent when working together within a common framework.

This paper will focus on the analysis of the first pillar, and particularly in capital requirements for credit risk. Related to this, the new accord has modified the risk-weighted assets definition: *'the new approaches for calculating risk-weighted assets are intended to provide improved bank assessments of risk and thus to make the resulting capital ratios more meaningful.'* [3] One of the main advantages of this new

[2]*Basel Committee on Banking Supervision (2005)*
[3] Overview of the new Basel capital accord, Consultative document; *Basel Committee on Banking Supervision (2003)*

accord is that it generates incentives for banks to develop more sophisticated risk management techniques. To assess credit risk, Basel II allows to choose between two methods: standardized approach and internal rating based approach IRB (foundation and advanced). In the standardized approach, which is similar to the current Accord, banks are required to slot their credit exposures into supervisory categories based on observable characteristics of the exposures (e.g. whether the exposure is a corporate loan or a residential mortgage loan. Fixed risk weights are established corresponding to each supervisory category and external credit assessments are used to enhance risk sensitivity. The IRB approach differs substantially from standardized approach, as banks internal assessments of key risk drivers are the principal inputs to measure credit risk. Capital requirements are determined by combining quantitative inputs provided by banks and formulas specified by the Committee. In Uruguay, regulation is based on the standardized approach. Capital requirements are detailed in the Article 14.1 of the Compilation of Central Bank norms (circulars) for the regulation and control of the financial system, where is stated that: "capital requirement for credit risk is equivalent to the 8% of risk-weighted assets". Risk weights for each category range from 0% to 125%.

The paper aims to serve as a first approximation to how the application of the foundation IRB could be in Uruguayan financial system, given that regulation has developed in line with Basel II spirit. During last years, and after financial crisis of 2002, the Superintendence of Financial Institutions (SFI) has substantially modified regulation with the purpose of giving more information to markets, thus having a more transparent and competitive banking sector as well as protecting agents when takings their decisions. Related to capital requirements, market risk has been incorporated in 2006, and there have been advances in credit risk measurement. Although the standardized method is used, regulation about credits classification introduced the analysis of debtors' cash flows, thus evaluating their ability to pay, and also requiring stress scenarios to assess it. This contributes to a better analysis of credit risk, and also provides a useful data base in case banks decide to use IRB approach. More recently, a regulation allows for the use of internal model to assess credit risk in small borrowers. All these changes make of the analysis an important tool to be aware of IRB implications when using it to measure capital requirements.

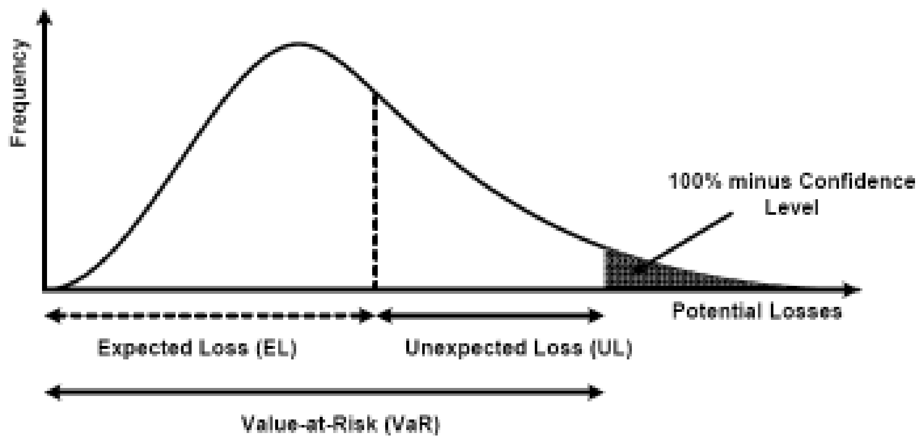
In the first part of the paper, we present IRB principal characteristics, emphasizing the analysis of the formula that Basel II proposed to calculate risk weights. Therefore, we focus on its main assumptions and its implications for developing financial systems

The second part will make use of a non parametric technique to estimate credit loss distribution of banking portfolio, to have a measure of expected and unexpected loses (VaR). Data covers private banking sector, during period 1999-2006, and we run different estimations distinguishing between corporate and retail portfolio. Having obtained these estimations, we compare them with the ones that would emerge in the case of using IRB approach formulas.

II. Basel II and the IRB approach

Banks activity could be seen as taking risks. During certain period of time, for example a year, it is common to observe that some borrowers do not pay their obligations. The bank can not exactly calculate the amount of such losses, but it can estimate the loss it expects to have at the end of the year. That measure is called *expected loss* (EL), and represents the amount of capital that the bank could lose as a result of its exposure to credit risk, given a time horizon. Such losses can be seen as the cost of doing banking business, and should be covered by provisions that banks make on each loan. However, losses could exceed that expected level, and more capital is thus needed to absorb them. These are known as *unexpected loss* (UL). Taking portfolio loss distribution, we can represent expected loss by its mean.

Figure 1



In Figure 1[4], unexpected loss is defined as the difference between the Value-at-Risk (VaR) and expected losses. The likelihood that losses will exceed the sum of expected Loss (EL) and Unexpected Loss (UL) - i.e. the likelihood that a bank will not be able to meet its own credit obligations by its profits and capital - equals the hatched area under the right hand side of the curve. 100% minus this likelihood is called the confidence level and the corresponding threshold is called Value-at-Risk (VaR) at this confidence level. VaR analysis constitutes an important tool when measuring risks, and given that Basel II aims to have more risk-sensitive capital requirements, this kind of analysis has been incorporated in the new accord.

In this section we present the main aspects related to IRB approach, and basic consideration one must take into account when applying it. We deepen in the analysis of the formula used to determine capital requirements for unexpected loss. Last part of this section analyses principal assumptions of the model, giving a better understanding of this approach, especially for emerging economies.

[4] Extracted from *An Explanatory Note on the Basel II IRB Risk Weight Functions,* 2005

1. IRB Fundamentals

The IRB approach is based in both expected and unexpected losses. Risk weights and capital requirements are determined by combining quantitative data from bank with formulas specified by the Committee. There are three key elements in IRB approach. The first are the **Risk Components**, which can be divided in:

- ✓ Default Probability (PD): quantifies the likelihood that the borrower will default in the coming twelve months.
- ✓ Loss Given Default (LGD): is the loss that the bank will suffer if the counterparty defaults. It is expressed as a percentage of the exposure.
- ✓ Exposure at Default (EAD): is the exposure of the bank at the moment the obligor goes into default.
- ✓ Maturity (M): the amount of time until the loan is fully due and payable.

The second element is **Risk-weight Functions**, in which risk components are transformed into risk-weighted assets and therefore capital requirements.

The last component is **Minimum Requirements**; these are minimum standards that must be met in order for a bank to use IRB approach. These standards are based fundamentally in rating and risk estimation systems and processes, which must provide for a meaningful assessment of borrower and transaction characteristics, a meaningful differentiation of risk; and reasonably accurate and consistent quantitative estimates of risk.

There are two types of IRB: foundation and advanced. In the first one, banks estimate PD and the rest of the parameters are established by the Committee. In the advanced IRB, all parameters are estimated by the financial institution.

Under the IRB approach, banks must categorize banking-book exposures into broad classes of assets with different underlying risk characteristics; these are: corporate, sovereign, bank, retail and equity. Within the corporate asset class, five sub-classes of specialized lending (SL) are separately identified, while for retail portfolio three sub-classes are determined. This is based in the fact that each exposure requires a different treatment because they have different risk-drivers. This work will focus on corporate and retail portfolios. Basel II allows for the inclusion in retail portfolios loans to small enterprises, if the exposure is below € 1 million.

Once the exposures are categorized and PD's are calculated, one should apply the formula proposed by the Committee to determine capital requirements. That formula will be analyzed in next sections, and can be expressed as follows:

$$K = \left\{ LGD * \left\langle N\left[\frac{1}{\sqrt{1-\rho}} N^{-1}[PD] + \frac{\sqrt{\rho}}{\sqrt{1-\rho}} N^{-1}[0.999] \right] - PD \right\rangle \right\} \frac{1+[M-2.5]b(PD)}{1-1.5\,b(PD)}$$

Where:

K = capital requirement

LGD = loss given default

PD = default probability

ρ = assets correlation

M = maturity

As we mentioned above, this formula generates capital requirements to cover unexpected loss, while expected loss is treated separately. Banks applying IRB must compare the total amount of eligible provisions with the total EL amount as calculated within the IRB approach (EL=PD*LGD). Where the calculated EL amount is lower than the provisions of the bank, its supervisors must consider whether the EL fully reflects the conditions in the market in which it operates before allowing the difference to be included in Tier 2 capital (with a maximum of 0.6% of risk-weight assets). Where the opposite is true, the difference is subtracted from capital (50% from Tier 1 and 50% from Tier 2).

2. Risk weight functions

A first characteristic of the IRB approach is that the model should be **portfolio invariant**; that is, the capital required for any loan should only depend on the risk of that loan and must not depend on the portfolio it is added to. Under this assumption, specific characteristics (PD, LGD and EAD) of each borrower are enough to calculate capital requirement for each loan.

It can be shown that[5] only Asymptotic Single Risk Factor (ASRF) models are portfolio invariant; these models are derived from traditional credit models by the law of large numbers. When a portfolio consists of a large number of small exposures, idiosyncratic risks associated with individual exposures tend to cancel out one-another and only systematic risks that affect many exposures have a significant effect on portfolio losses. *Vasicek (2002)* has demonstrated that under certain circumstances, Merton's model of 1974 can be adapted to an ASRF one. In this kind of models, all systematic risks that affect all borrowers, like industry or regional risks, are modeled with only one systematic risk factor.

Consider a portfolio consisting of n loans. The value of the assets of a borrower *i* can be described as a geometrical Brownian motion, as stated in equation 3.

$$dA_i = \mu\, A_i\, dt + \sigma A_i\, dz_i \qquad [3]$$

where dz_i is a Wiener process, $dz_i = z_i \sqrt{t}$, $z_i \sim N(0,1)$

Thus the value of assets at T can be expressed as:

$$\ln A_i(T) \approx N\left[\ln(A_i(0) + (\mu - \frac{\sigma^2}{2}); \sigma\sqrt{T}\right] \qquad [4]$$

so it can be established that:

[5] Gordy, 2003

$$\ln A_i(T) = \ln A_i(0) + (\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}\, z_i \qquad [5]$$

where z_i is a standard normal random variable.

Basel's II formula uses Merton's model interpretation of default probability of the borrower *i*, so an obligor will default if the value of his assets falls below the value of his debt:

$$p = p[A_i(T) < B_i] = p[z_i < c_i] = N[c_i] = N[-d_2]\, con\ c_i = \frac{\ln B_i - \ln A_i - \mu_i T + \frac{1}{2}\sigma_i^2 T}{\sigma\sqrt{T}} \qquad [6]$$

Remembering that N [-d_2] is default probability, and calling it by letter p, then N[c_i] =p, and therefore c_i=N^{-1}[p].

z_i is assumed to have the following expression:

$$z_i = b\, y + a\, \varepsilon_i \quad i = 1,2,3,.....n \qquad [7]$$

where *y* represents systemic risk affecting the entire portfolio and ε_i is the specific risk for borrower i. It is assumed that both of them follow standard normal distributions, and the values for b and a are given by:

$$b = \sqrt{\rho}\,; a = \sqrt{1-\rho}$$

ρ measures the asset correlation between borrowers' assets. It could be described as the dependence of the assets of a borrower on the general state of the economy; so all borrowers are linked to each other by the single risk factor. If we are talking about a dollarized economy, we can think in the exchange rate as the single risk factor, because a strong movement in that variable substantially affects loan portfolios.

Under those conditions, default probability of any loan, conditional on the single risk factor y, can be written as:

$$p[y] = P[by + a\varepsilon_i < c_i] = P\left[\varepsilon_i < \frac{c_i - by}{a}\right] \qquad [8]$$

From equation [6], can be noticed that $c_i = N^{-1}[p]$, so replacing in [8] we have

$$p[y] = P\left[\varepsilon_i < \frac{c_i - by}{a}\right] = P\left[\varepsilon_i < \frac{N^{-1}[p] - \sqrt{\rho}\, y}{\sqrt{1-\rho}}\right] = N\left[\frac{N^{-1}[p] - \sqrt{\rho}\, y}{\sqrt{1-\rho}}\right] \qquad [9]$$

Total portfolio consists on n identical individuals, with the same participation in total exposure. Being L_i the gross portfolio loss (before recoveries) for borrower *i,* so that L_i=1 if the obligor defaults and L_i=0 if the opposite is true, total gross portfolio loss can be expressed as:

$$L = \frac{1}{n} \sum_{i=1}^{n} L_i \qquad [10]$$

We can establish that the percentage of default on total portfolio equals the number of individuals that default on their obligations. If n is large enough, by the law of the large numbers it can be stated that the fraction of clients L that default on their debts is equal to the default probability conditional on y,

$$p[y] = P[L_i = 1/y] = N\left[\frac{N^{-1}[p] - \sqrt{\rho}\, y}{\sqrt{1-\rho}}\right] \qquad [11]$$

Thus the cumulative distribution function of loan losses on a very large portfolio is in the limit[6]:

$$N\left\{\frac{N^{-1}(x)\sqrt{1-\rho} - N^{-1}(p)}{\sqrt{\rho}}\right\} \qquad [12]$$

The VaR at 99.9% confidence level is thus:

$$x_{99.9\%} = N\left[\frac{\sqrt{\rho}\, N^{-1}(99.9\%) + N^{-1}(p)}{\sqrt{1-\rho}}\right] \qquad [13]$$

And default probability can be expressed like follows:

$$p_{99.9\%} = N\left[\frac{N^{-1}[p] + \sqrt{\rho}\, N^{-1}[0.999]}{\sqrt{1-\rho}}\right] \qquad [14]$$

Until now we have derived most part of Basel formula. The complete risk weight function for capital requirements to cover unexpected loss is:

$$K = \left\{ LGD * \left\langle N\left[\frac{1}{\sqrt{1-\rho}} N^{-1}[PD] + \frac{\sqrt{\rho}}{\sqrt{1-\rho}} N^{-1}[0.999]\right] - PD \right\rangle \right\} \frac{1 + [M - 2.5]b(PD)}{1 - 1.5b(PD)} \qquad [15]$$

[6] Convergence of the portfolio loss distribution to the limiting form actually holds even for portfolios with unequal weights. Let the portfolio weights are w_i, with $\sum_{i=1}^{n} w_i = 1$. The portfolio loss $L = \sum_{i=1}^{n} w_i L_i$ conditional on Y converges to its expectation p(Y) whenever $\sum_{i=1}^{n} w_i^{\,2} \to 0$ (this is a necessary and sufficient condition); inn other words, if the portfolio is not concentrated.

Calling *PD* the default probability previously defined as p, we can observe that the formula generates capital requirement for unexpected loss, given that the LGD is multiplied by the difference between the PD's VaR at 99.9% confidence level and the expected PD. That parameter is estimated by banks, in the foundation IRB as well as in the advanced.

It is worthy to comment some properties of this distribution function[7]. The cumulative distribution is given by the expression:

$$F(x;p;\rho) = N\left[\frac{\sqrt{1-\rho}\,N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right] \qquad [16]$$

Thus, if we want to obtain the density function we must calculate the derivative of expression [16],

$$f(x;\rho;p) = \sqrt{\frac{1-\rho}{\rho}}exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}N^{-1}(x) - N^{-1}(p)\right]^2 + [N^{-1}(x)]^2\frac{1}{2}\right\}$$

The measures of position for this density function are:

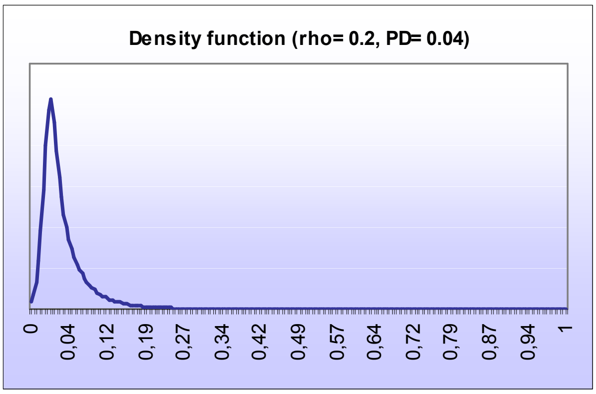
- $E(x) = p$
- $x_{0.5} = N\left[\frac{N^{-1}(p)}{\sqrt{1-\rho}}\right]$
- $L_{moda} = N\left[\frac{\sqrt{1-\rho}N^{-1}(p)}{1-2\rho}\right]$ if ρ<1/2

Variance is given by:

- $s^2 = N_2[N^{-1}(p), N^{-1}(p), \rho] - p^2$

being N_2 the bivariate normal distribution function.

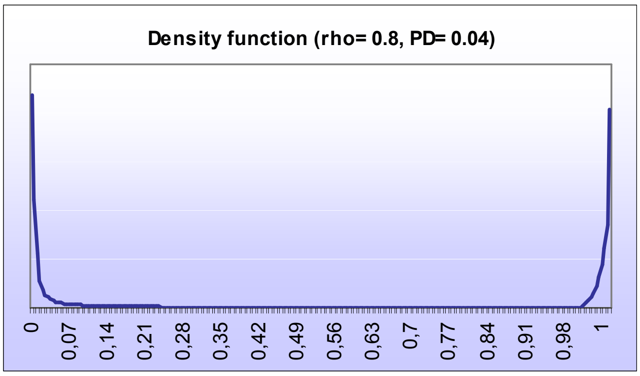
Graph 1



[7] Demonstrations are presented in Appendix A.

When ρ>1/2, density is U-shaped, which means that when correlation is high, bank result could be very good, in case all firms perform well, or really bad, in case all of them incur in default. Graph 2 illustrates this case.

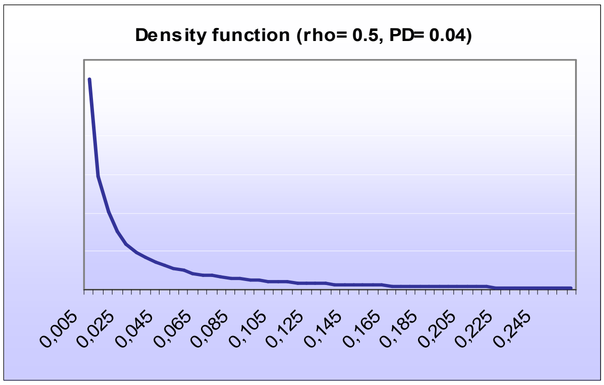
Graph 2



One can think about a dollarized economy, where banks do not have problems in case they face a stable exchange rate, but in case of a negative realization of this risk-driver, the financial system immediately would show bad results, thus being in the other extreme of the distribution.

In the particular case for ρ=1/2, function is monotone.

Graph 3



Lastly, when correlation is perfect, so that ρ=1, density tends to the binomial distribution, $f \rightarrow B(p; 1-p)$.

In Graph 1 we can see that loss distribution is asymmetrical. This pattern has implications when requiring capital, as it generates higher requirements that in the normal distribution's case.

Correlation coefficients were determined based on G10 data. Two main assumptions are made. The first one is that correlations have an inverse relationship with PD. The higher the PD is for a firm, the lower its correlation to the single risk factor. This is an empirical observation, as it is observed that when PD is high, it means that idiosyncratic risk prevails, so the risk is driven by its own characteristics and does not depend on the general state of the economy. The second assumption is that size of the firm is directly related to PD. The bigger the company, the higher the PD. As companies become bigger, they are more dependent on the state of the economy. Correlation is then calculated as follows:

$$correlacion\ (\rho) = 0.12x\ \frac{1-e^{-50xPD}}{1-e^{-50}} + 0.24\left[1-\frac{1-e^{-50xPD}}{1-e^{-50}}\right] - 0.04\ x\ \left(1-\frac{(S-5)}{45}\right)$$

Where 0.12 corresponds to maximum PD (100%) correlation, and 0.24 is the correlation for lowest PD (0%). Each of them is multiplied by exponential weights, which display the dependency on PD. Last term corresponds to a size adjustment, which affects borrowers with annual sales between €5 million and €50 million. For borrowers with €50 million annual sales and above, the size adjustment becomes zero, and for borrowers with €5 million or less annual sales, the size adjustment takes the value of 0.04, thus lowering the asset correlation from 24% to 20% (best credit quality) and from 12% to 8% (worst credit quality).

For most retail loans,[8] correlation is determined as stated below:

$$correlacion\ (\rho) = 0.03x\ \frac{1-e^{-35xPD}}{1-e^{-35}} + 0.16\left[1-\frac{1-e^{-35xPD}}{1-e^{-35}}\right]$$

Last term of equation [15] is a maturity adjustment[9], as it was assumed that all loans had a one-year maturity. Empirical evidence shows that long-term loans are riskier that short-term ones, so capital requirements should be higher for longer maturities. This could be seen as the additional requirements that would emerge for possible credit downgrades, which are more likely to happen in long-term loans.

The adjustment has the following form:

$$FA(M,PD) = \frac{1+[M-2.5]b(PD)}{1-1.5b(PD)}$$

Where $b(PD)$ is:

$$b(PD) = [0.11852 - 0.05478\ \ln(PD)]^2$$

Adjustments are linear and increasing in M. The reason is that the lower de PD, the loan is more likely to be downgraded, so it is riskier. The slope of the adjustment function with respect to M decreases as the PD increases. Next Figure presents a matrix which contains the values of the adjustment for different PD and M.

[8] Except for mortgage and revolving loans, where correlations are fixed, taking values of 0.15 and 0.04 respectively.

[9] This adjustment does not apply for retail exposures

Figure 2: Maturity adjustment

	Probabilidad de default									
M	**1%**	**2.00%**	**3.00%**	**4.00%**	**5.00%**	**6.00%**	**7.00%**	**8.00%**	**9.00%**	**10.00%**
1	1	1	1	1	1	1	1	1	1	1
2	1.1732	1.1328	1.1128	1.1000	1.0908	1.0837	1.0780	1.0732	1.0692	1.0658
3	1.3464	1.2657	1.2256	1.1999	1.1815	1.1673	1.1559	1.1465	1.1385	1.1315
4	1.5196	1.3985	1.3384	1.2999	1.2723	1.2510	1.2339	1.2197	1.2077	1.1973
5	1.6928	1.5314	1.4512	1.3999	1.3630	1.3346	1.3118	1.2929	1.2769	1.2630
6	1.8660	1.6642	1.5640	1.4999	1.4538	1.4183	1.3898	1.3662	1.3461	1.3288
7	2.0392	1.7971	1.6768	1.5998	1.5445	1.5020	1.4678	1.4394	1.4154	1.3946
9	2.3857	2.0627	1.9024	1.7998	1.7260	1.6693	1.6237	1.5859	1.5538	1.5261
10	2.5589	2.1956	2.0152	1.8998	1.8168	1.7529	1.7016	1.6591	1.6230	1.5918
B(PD)	0.13749	0.11077	0.09648	0.08694	0.07988	0.07433	0.06980	0.06599	0.06271	0.05986

It should be noticed that for M=1, $FA(1, PD) = 1, \forall\, PD$. From Figure 2, it can also be seen that there exists a negative relationship between the adjustment and PD for any given M.

3. Principal assumptions behind the risk weight function

3.1 The two main assumptions

There are two strict assumptions behind Basel's II formula: bank's credit portfolio is infinitely fine-grained, and there is a single systematic risk factor which drives all dependence across credit losses in the portfolio.

First assumption implies that any single obligor represents a very small share of the portfolio's total exposure. For the traditional Merton's model to be adapted to an ASRF model, *Vasicek (2002)* has demonstrated that it is necessary that the loan portfolio consists of a large number of small exposures. If this assumption is not met, then there will be an undiversified idiosyncratic risk, which results in an underestimated capital requirement. In view of this kind of situation, *Vasicek (2002)* proponed a granularity adjustment which can be applied when the portfolio is not sufficiently large for the law of large numbers to take hold.

In equation [14], granularity adjustment could take place by taking $\rho + \delta\,(1-\rho)$ instead of ρ.

Being $\delta = \sum_{i=1}^{n} w_i^2$

The second main assumption is the existence of a single systematic risk factor. Diversification effects, like regional or industry branch, are not considered. This failure to recognize the diversification effects could result in an overestimation of portfolio risk, especially for banks that have properly diversified its risks. Industries are subject to different kind of risks and cycles, and they should be separate modeled. Some recent papers suggest the use of multi-factor frameworks to reflect diversification effects. *Céspedes et al (2005)* estimate a model based on what the called 'diversification factor', which is a function of two parameters that broadly capture size concentration and the average cross-sector correlation. *Tasche (2005)* incorporates diversification effects by including a diversification index, where VaR contributions of each factor are calculated. Capital requirements can be substantially reduced when the portfolio is well-diversified.

3.2 Asset correlations

Another crucial factor that affects capital requirements is the calculation of asset correlation. That formula assumes that there exists a positive relationship between asset correlation and size of the firm. This is based on the idea that smaller firms have a higher component of idiosyncratic risk, thus having less correlation with the general state of the economy. Most research in this area confirms this assumption[10]. However, an argument against this evidence could be found in *Bernanke et al (1996)*. They claim that bigger firms have access to financial markets in case of negative shocks, while small and medium companies do not have it, thus being more exposed to state of the economy. Debtors with higher agency costs in credit markets (small firms) will burden the costs of economic recessions; the so called *flight to quality*.

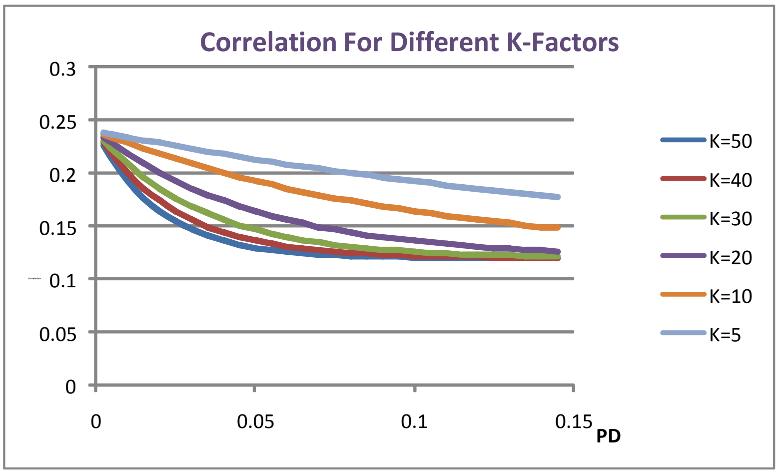
To calculate correlations, it is also assumed that the relationship between PD and asset correlation is negative. In contrast with the previous assumption, there is no consensus about this. *Düllmann y Scheule (2003)* analyzed asset correlation as a measure of systematic credit risk from a database of balance sheet information of German companies, finding that it increases with firm size but the relationship with PD is not unambiguous. *Dietsch y Petey (2003)* estimated asset correlations in two large populations of French and German SMEs. They found that, on average, the relationship between PDs and correlations is not negative as assumed by Basel II. It is U shaped in France, and positive in Germany. They conclude that capital requirements could be too high for SMEs, where correlation is lower.

Critics also come from PD's modellization. *Rosch (2002)* argues that PD is not constant over time, and that it depends on macro economical conditions, so when calculating PD's one must incorporate proxy variables for the business cycles. That would reduce uncertainty about PD, and also correlations and capital requirements. *Hamerle et al (2003)* also introduce macro economical factors in their estimation, and propose a time-dependent PD, thus obtaining lower correlations.

In conclusion, capital requirements are said to be highly sensitive to asset correlations; if they are lower than the ones that result from Basel II, capital requirements are much lower and vice versa. Parameter's calibration is therefore crucial; Basel II has used data from the group of ten major supervisors (G-10), which may not be suitable for emerging economies. In particular, exponential weights could be too high. The pace of the exponential function is determined by the "k-factor", which is set at 50, generating a fast decline. From Graph 4, it is noticeable how changes in k-factor can smooth the function, thus enlarging the range between minimum and maximum PD.

[10] Dietsch and Petey (2003), Düllmann and Scheule (2003)

Gaph 4



3.3 Loss given default

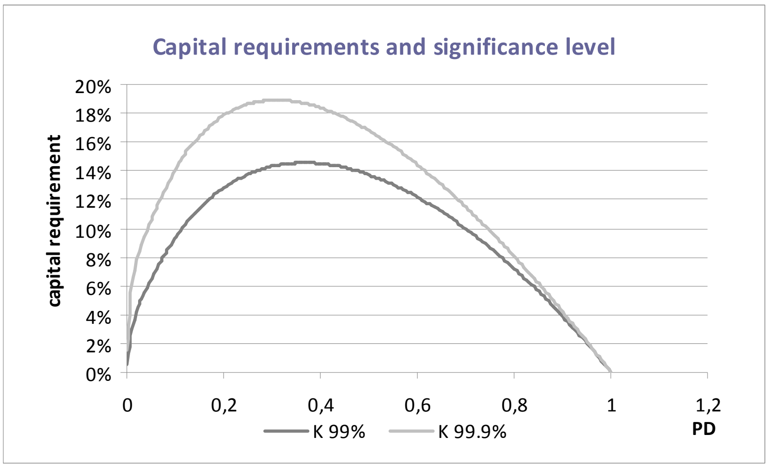
To determine the LGD, the Committee proposed a determinist value, when one can think that LGD is a random variable which take values between 0 and 1, and also that there may exists a dependence relationship between LGD and PD. *Altman et al (2002)* found a positive relationship, and they argue that it must be considered in the analysis, as it increases capital requirements. The same factors that affect PD also affect LGD. *Hillebrand (2006)* estimated a model for the portfolio loss including dependence of PD and LGD on the economic cycle. He states that LGD depends on the general state of the economy, and it has to be incorporated when measuring credit risk. In a recession, for example, the value of collateral decreases considerably, and therefore the LGD is much higher.

Criticism also relates to the LGD estimation, as banks may have different measures according to the model they use. This applies also for PD's estimation.

3.4 Confidence level

Confidence is set at 99.9%, recognizing that it may result too high. Basel's argument is that this exigency covers possible measurement errors in estimation (institutions with well-specified and calibrated models will be punished with higher capital requirements that they may not need).

Graph 5



As it can be observed in Graph 5, if the confidence level is reduced by 0.9%, capital requirement is significantly lower.

This conservative criterion is also observed when calculating market risk capital requirement, where the VaR is multiplied by a minimal factor of 3, which also pretends to reflect measurement error in estimations.

Previous graph also shows how from certain levels of PD, capital requirement starts decreasing rather fast, as a consequence of the increase of expected loss component, which becomes zero when PD equals unity (that is, when the borrower is in default), as there is not unexpected loss.

III. Non parametric estimation of credit loss distribution and comparison with IRB approach

When estimating capital requirements to cover unexpected losses due to credit risk it is necessary to obtain the parameters of the distribution function of loss portfolio. There exists different methodologies to calculate them; in this paper we opt for the procedure proposed by *Carey (1998, 2002)*, where he estimated portfolio loss distribution using a non parametric technique, known as *bootstrap*. The main idea is that the sample itself is the best guide to infer the distribution. The principal advantage of this method is that we do not need to make assumptions about the functional shape and the parameters of that distribution; the only assumption is that the sample is representative enough. The bootstrap consists on simulating a large number of portfolios, extracting, with reposition, the loss rate for each of them. The frequency distribution of that loss rates is the estimation of the relevant distribution function.

Majnoni, Miller y Powell (2004) conducted a boostrapping exercise for three countries (Argentina, Mexico and Brazil) to replicate the distribution of credit losses prevailing at a specific period of time. Their paper concludes that capital requirements that emerge from IRB are lower that the estimated ones for the cases of Argentina and Mexico, while the opposite is true for Brazil. It should be noticed that results are based in date from only one year, while this technique requires having a larger period, to cover economical cycles. *Jacobson, Lindé y Roszbach (2005)* also applied Carey's nonparametric method to two banks' complete loan portfolios, to compare the risk associated with small and medium enterprises (SME) with big corporate portfolios. Lastly, the paper of *Gutiérrez Girault (2007)* estimates conditional and unconditional loss distributions for loan portfolios of argentine banks, to compare with Basel's II requirements. His exercise controlled by type of borrower and type of bank, and covered the period 1999-2004.

1. Data and methodology

1.1 Non parametric estimation

The approximation to estimate credit loss distribution follows the work of *Carey (1998, 2002)*. In his paper of 1998 he estimates the credit risk associated with private debt portfolios, reporting non parametric estimates of the size of losses in the bad tail, using Monte Carlo resampling methods. Portfolios are simulated extracting different assets from total sample, repeating this procedure 50.000 times. Empirical losses are computed for each drawn portfolio, and the frequency distribution of such losses is the relevant loss distribution.

The term *bootstrap* was introduced in 1979 by Efron, although the type of methodology was being used since time before. It is a computer-intensive method, which allows to make statistical inference without making hypothesis about population distribution (*F*). This method starts from the concept of *bootstrap sample*. Taking $\hat{F}$ as the empirical distribution, a bootstrap sample is defined as a random sample of size n, obtained from $\hat{F}$,

$x^*=(x_1^*, x_2^* \dots x_n^*)$

$\hat{F} \rightarrow (x_1^*, x_2^* \dots x_n^*)$

The supra index * means that we are not dealing with the original data set, as it is a random version or resampling of x. This values are a sample of size n extracted with replacement from the original sample ($x_1, x_2, \dots, x_n$), so each bootstrap sample consists of n values of the original sample, where some of them could appear more than once or even not be included. For every bootstrap sample, the same function applied to the original data, s (.) , is then calculated.

$$\widehat{\theta^*} = s(x^*)$$

For example, if s(*x*) is the sample mean $\bar{X}$, then s(x*) is the bootstrap sample mean,

$$\overline{X^*} = \sum_{i=1}^{n} \frac{x_i^*}{n}.$$

Both the construction of the bootstrap sample and the calculus of the relevant statistic are repeated B times, constructing a frequency distribution which will constitute the relevant probability function.

1.2 Data

Data is obtained from the Credit Registry of the Superintendence of Financial Institutions (SIIF) - Central Bank of Uruguay. Banks and other financial intermediaries send to the Credit Registry information about transactions, which include data about borrowers (name, business activity, document) and about their debts (amount, type of facility, collateral). From the total database, we only consider those credits that were performing at the beginning of every period[11], excluding those that were in non performing categories. Then we observe the performance of each loan, particularly if it has been included in non performing categories. To determine if the credit is non-performing, we follow the same criterion of the SIIF's Accounting Scheme; that is, if the loan payments are past due by 60 days or more. If the credit has been written off, it is also considered as default, while if the credit has been cancelled it is considered as a recovery.

The analysis will cover the period 1999–2006, thus covering the deep crisis that the Uruguayan economy suffered in 2002, as well as its following recovery. By this way, unconditional distribution will be representative. Data come from private banks only, thus excluding the activity of public banks.

Portfolio will be classified according to the type of borrower, following SIIF regulations. It allows for the distinction between corporate and retail sectors. This segmentation aims to match Basel's II categorization.

Exposure at default is determined by total risks of a borrower, minus the coverage of liquid collaterals (as they are totally recovered after a default event). It is also assumed

[11] As starting point of each period we took December of every year.

that the rate of recovery for mortgage collaterals is 30% of the balance sheet value. Lastly, LGD is assumed to be 50%.

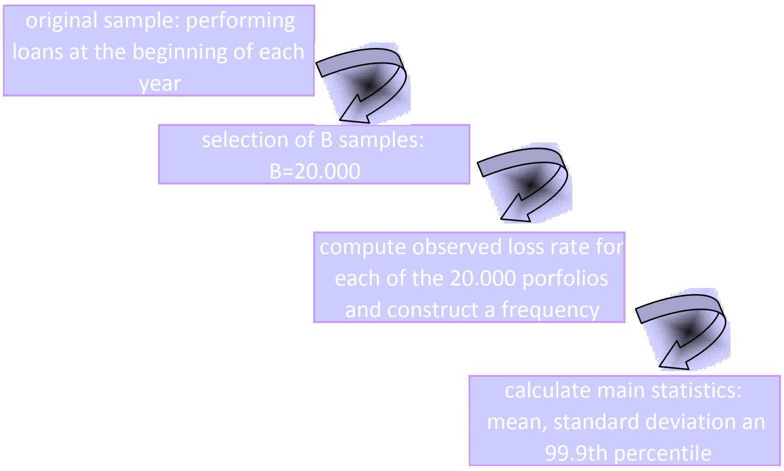
We defined a variable L_i which take the value $L_i=1$ in case of default and $L_i=0$ if the loan remains performing or has been cancelled.

$$loss.rate = \frac{\sum_{i=1}^{n} L_i \times Exposure_i \times 50\%}{\sum_{i=1}^{n} Exposure_i}$$

Defining the exposure as follows:

Exposure = Total Risks– Liquid Collaterals – 30% Mortgage Collaterals

Non parametric estimations were done for each year during the period 1999-2006, and for each type of borrower. Simulations included 20.000 repeats for conditional as well as unconditional distribution. The size of the portfolios was determined according to the observed data for every year, taking the average size for private financial system. For the unconditional distribution, the average of 1999-2006 was taken[12]. The process can be summarized in next picture.



2. Results

2.1 Corporate portfolio

Next we present the results obtained from the bootstrapping for corporate portfolio.

Table 1 reports the main indicators of loss distribution: expected loss, standard deviation and the 99.9th percentile (VaR measure). The first three columns show estimations in million dollars, but we are interested in losses relative to the total exposure, so the next three columns calculate that. Last column measures the

[12] The process was also calculated taking the size of the portfolio as the total number of observations for every year and results were not significantly different.

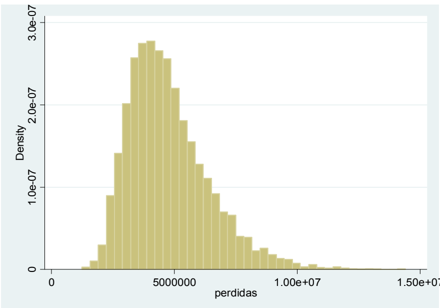
unexpected loss, which is the difference between the 99.9th percentile and the expected loss.

Table 1: Estimation for corporate portfolio

	Losses in million USD			Loss rate			Unexpected loss
	Expected	St. Dev.	99.9th percentile	Expected	St. Dev.	99.9th percentile	
1999-2000	4,89	1,47	10,90	2,12%	0,67%	4,93%	2,81%
2000-2001	4,84	1,44	10,50	1,91%	0,60%	4,35%	2,44%
2001-2002	35,40	6,57	61,10	11,81%	2,40%	20,81%	8,99%
2002-2003	2,49	0,81	5,65	1,47%	0,51%	3,68%	2,21%
2003-2004	11,60	9,65	59,30	4,40%	3,29%	17,64%	13,24%
2004-2005	5,84	6,28	40,10	2,51%	2,49%	13,45%	10,94%
2005-2006	1,95	1,69	10,70	1,18%	1,00%	6,23%	5,05%
incondicional	**11,20**	**4,40**	**34,80**	**4,72%**	**1,73%**	**13,22%**	8,50%

Looking at the period 1999-2000, expected loss of portfolio is USD 4,89 million. The credit VaR at 99,9% confidence level is USD 10,9 million. The frequency distribution is presented below, where it can be noticed the asymmetrical shape that characterizes credit risk distributions[13]. As we mentioned before, this asymmetry implies that the likelihood of high losses is higher than in the case of a normal distribution.

Graph 6- Credit loss distribution for period 1999-2000



From Table 1 one can also extract statistics for the loss rate, which has an expected value of 2,12%, the VaR at 99.9% is 4,93% and the unexpected loss is thus 2,81%.

It is noticeable from Table 1 that financial crisis of 2002 had an important impact in our estimations. Expected loss increased significantly, as well as the volatility of the distribution. For the next periods we observe a higher standard deviation too, which could be indicating that financial system became more fragile. It should be mentioned that the low values obtained for period 2002-2003 could be attributed to the fact that after financial crisis the loans that remain in the sample were only the good ones. To illustrate this argument, in 2001-2002 there were 12.000 observations while for the years 2002-2003 they reduced by almost a half.

Once obtained the non parametric estimations, we compared estimated requirements with IRB approach ones. To calculate IRB formula, it is assumed that LGD is 45% (defined in Basel Committee final document) and the values for PD are the ones that emerge from bootstrap estimation. In other words, taking into account that expected loss can be defined as:

[13] Appendix B presents estimations for each year.

*EL = PD*LGD*

Using EL estimations from Table 1, one can obtain the value of PD consistent with a LGD of 45%.

Basel II also proposed a size adjustment to be included in the formula. When analyzing the pertinence of doing so for Uruguayan companies, we observed that for May and June of 2007, more than a half of firms were small and medium enterprises (SMEs), whose annual sales are below USD 5 million. So it is necessary to reflect this situation when calculating capital requirements. To do that, we consider that before the crisis of 2002, the percentage of SMEs was similar to actual rates, while in the two years after crisis, only big companies ‘survived’.

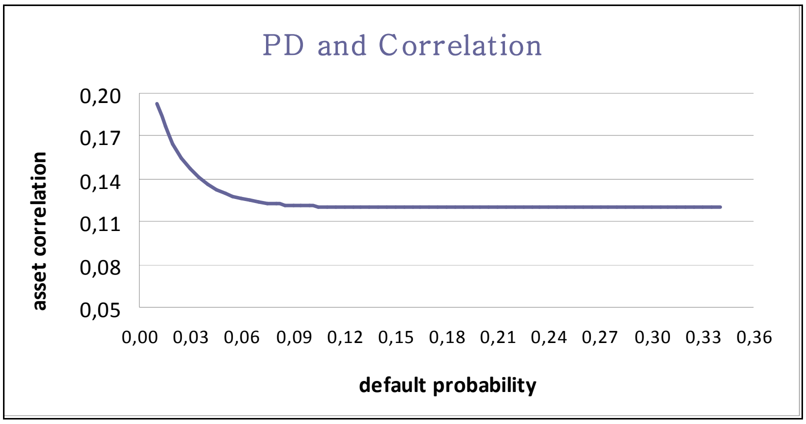
A maturity adjustment also needs to be done, to reflect the fact that during all the period of analysis most part of loans had a maturity of less of one year. Results are reported in Table 2.

Table 2: Comparison of IRB and Non parametric estimations for corporate portfolio

period	K (IRB)	IRB/Estim.
1999-2000	8,57%	3,04
2000-2001	8,23%	3,38
2001-2002	16,54%	1,84
2002-2003	7,49%	3,38
2003-2004	13,93%	1,05
2004-2005	11,02%	1,01
2005-2006	6,95%	1,38
unconditional	**12,17%**	**1,43**

First column in Table 2 reports the capital requirement generated from IRB formulas, while in the second column that requirement is divided into the estimated one by the bootstrapping process. As it can be observed, results are not homogeneous. Before 2003, Basel’s II requirements are much higher than estimated ones, while after that year IRB requirements seems to be closer to the non parametric estimation. A factor that may be explaining those differences could be the measurement of asset correlations. The formula proposed by Basel depends on PD, and as mentioned before, it is assumed that correlation with systematic risk has a negative relationship with PD. Next graph shows that relationship, and also how correlation arrives at its minimum for low values of PD (thus the so called k-factor results too high). For low values of PD, correlation is too high and the range of intermediate correlations is too short. Therefore, it is indicating that parameters’ calibration was thought for developed economies, where low PDs prevail.

Graph 7 - PD and correlation in IRB approach



The differences we obtain thus indicate that for years previous to the financial crisis, correlations calculated from IRB are too high, therefore generating a higher capital requirement than the estimated one. However, during last years of analysis results show that firms are more dependent on the general state of the economy, as the capital requirements is closer to estimations. Given that is hard to obtain data to estimate asset correlations for Uruguayan firms, we used bootstrapping estimations to infer the asset correlations implicit in them. That is, taking the estimated capital requirement, we use Basel's formula to determine which correlation is required to yield that result.

Table 3: Asset correlations

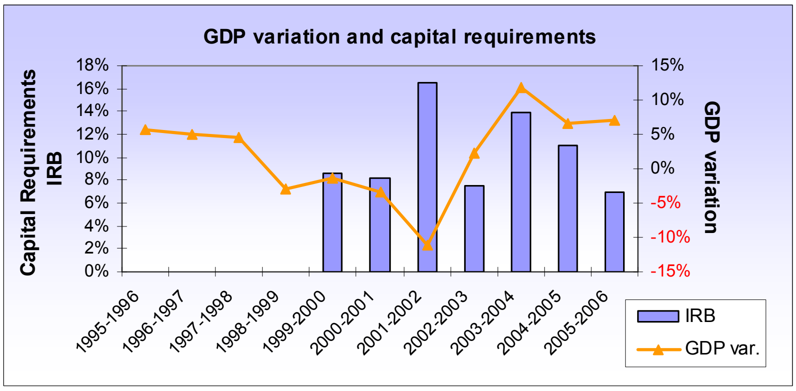
	estimated ρ	Basel's ρ
1999-2000	2,5%	13,1%
2000-2001	2,0%	13,4%
2001-2002	3,0%	12,0%
2002-2003	2,3%	14,3%
2003-2004	11,5%	12,1%
2004-2005	12,7%	12,7%
2005-2006	8,5%	15,2%

From Table 3 it is clear that correlation for years before 2003 are too high, while they tend to converge at the end of the period, coinciding with a better adjustment of IRB requirements in relation to estimated ones. Asset correlation thus results time dependent; previous to the crisis, firms' performance depended on themselves, so correlation is low. After the crisis of 2002, as the economy was recovering, companies depend more on the realization of the systematic factor; therefore, a higher correlation is expected[14]. In view of that, it is necessary to adjust the coefficient according to the business cycle phase. Again, IRB's formulas are thought for developed economies where macro economical fluctuations are not as usual as in developing ones, and where PD is much lower.

[14] During the period 2001-2002, correlation implicit in capital requirement is low, which can be attributed to the high value of PD for that period (giving Basel's formula, correlation is much lower). It could also be attributed to the fact that increases in correlation have certain lags, thus appearing some years after the crisis took place.

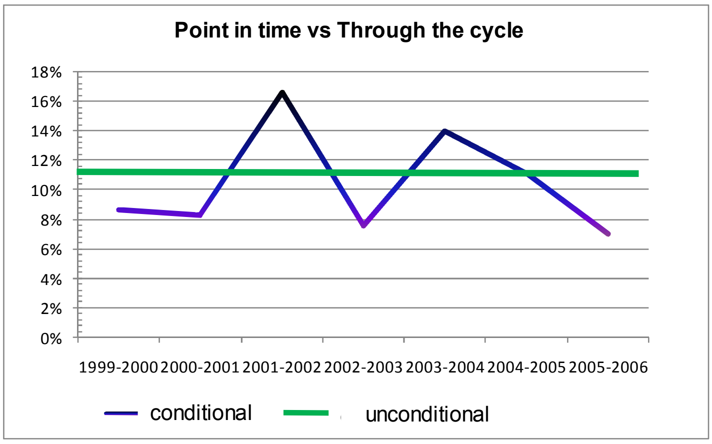
Another important pattern observed in IRB's capital requirements is their procyclicality. In recession times, it increases significantly, while in expansions is lower.

Graph 8



In Graph 8 one can see how the requirement increases as the GDP variation is lower, reaching its maximum in the period 2001-2002 when GDP fell by 11%. It should be remembered that period 2002-2003 is not representative, as it has fewer observations. As the economy recovers showing positive variations of GDP, capital requirements fall. Having reached this point, it is pertinent to distinguish between estimation point in time (PIT) and estimation through the cycle (TTC). The former emerges from using the PD in each period (conditional estimations), while the TTC estimation attempts to measure credit quality in a long time horizon, thus incorporating cyclical aspects of the economy. Therefore, the procyclicality could be reduced when using TTC estimations of PD.

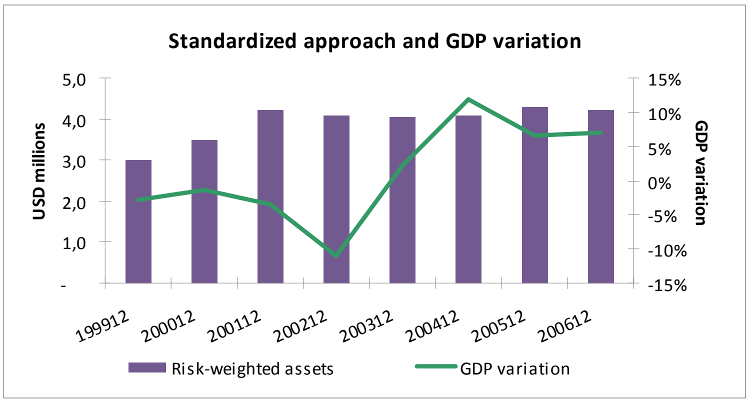
Graph 9



Graph 9 presents the point-in-time requirements, which are the ones that have been calculated in conditional distributions. Horizontal line represents the unconditional estimation, which could be interpreted as the through-the-cycle requirement.

If we want capital requirements to reflect the risk profile of institutions, then procyclicality should not be a problem. It is reasonable that in recession, when credit quality deteriorates, the risk associated is higher and so the capital requirement is. There is a trade-off between procyclicality and risk measurement. If we calculate requirements with the standardized approach (with fixed risk weights), we observe that they do not vary with economic cycle, thus being useless as an indicator of risk.

Graph 10



Lastly, as mentioned in Part I, a risk-insensitive requirement does not allow for inferring about the economic capital of a bank, and does not reflect changes in portfolio's risk profile, thus making that market agents unable to monitor institutions and impeding the implementation of risk management policies.

2.2 Retail portfolio

The retail portfolio was not segmented by type of facility. This could lead to differences as some parameters are fixed for mortgage and revolving loans. The former have an asset correlation of 15% while for the last ones it is set at 4%. Table 7 presents estimations for this kind of borrower, and graphs for each distribution function could be found in Appendix C.

Table 7: Estimation for retail portfolio

	Losses in million USD			Loss rate			Unexpected loss
	Expected	St. Dev.	99.9th percentile	Expected	St. Dev.	99.9th percentile	
1999-2000	4,13	0,79	7,86	3,19%	0,56%	5,62%	2,43%
2000-2001	6,18	0,58	8,26	3,84%	0,37%	5,11%	1,27%
2001-2002	57,90	3,11	69,00	17,86%	0,91%	20,71%	2,85%
2002-2003	3,54	0,28	4,47	2,20%	0,19%	2,82%	0,62%
2003-2004	2,52	0,31	3,66	2,69%	0,34%	3,88%	1,19%
2004-2005	2,49	0,21	3,19	2,70%	0,23%	3,47%	0,77%
2005-2006	0,85	0,11	1,23	1,66%	0,22%	2,43%	0,77%
incondicional	**13,10**	**1,35**	**18,40**	**6,78%**	**0,64%**	**9,19%**	**2,41%**

In contrast with corporate portfolio, distribution for retail has a major component of expected loss, which results in substantially lower capital requirements. By looking at graphs corresponding to each kind of debtor[15] it is clear that corporate portfolio has a more asymmetrical distribution, thus indicating more volatility and higher requirements due to unexpected loss.

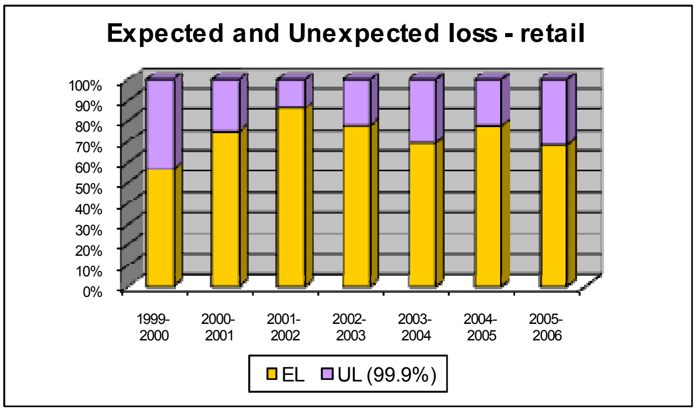
[15] Presented in Appendix B and C.

Table 8: Comparison of IRB and Non parametric estimations for retail portfolio

period	K (IRB)	IRB/Estim.
1999-2000	5,55%	2,29
2000-2001	5,78%	4,56
2001-2002	9,57%	3,35
2002-2003	5,30%	8,53
2003-2004	5,42%	4,56
2004-2005	5,42%	7,04
2005-2006	5,16%	6,72
unconditional	**7,10%**	**2,94**

Previous analysis summarized in Tables 7 and 8 is indicating that there is an important difference between IRB's requirements and estimated ones. From Graph 13 it can be seen that unexpected loss is a minor part of total retail's losses.

Graph 11



Correlations were computed in the same way we did for corporate, also resulting in higher coefficients in case of applying Basel's formula.

Table 91: Asset Correlation

	estimated ρ	Basel's ρ
1999-2000	2,0%	4,1%
2000-2001	0,5%	3,7%
2001-2002	0,5%	3,0%
2002-2003	0,3%	5,4%
2003-2004	0,7%	4,6%
2004-2005	0,3%	4,6%
2005-2006	0,6%	6,6%

It can be thought that the performance of this kind of borrower is less depending on the general state of the economy, and that's why the correlation is much higher in Basel's formula. Again, calibration is crucial. It should also be mentioned that the confidence level of 99.9% is too high for this portfolio. If we set it at 95%, IRB's requirements would be closer to our estimations. In conclusion, calibration of the formula is fundamental as we may overestimate unexpected loss of retail portfolio.

IV. Conclusions

IRB approach implies an important advance in credit risk measurement and also contributes to make capital requirements more close to the economic capital of the bank. Uruguayan banking regulation has developed recently in line with making requirements more risk-sensitive. In this sense, it could be possible to apply the Foundation IRB, where institutions estimate PD's, and the SIIF establish the rest of parameters. Once the procedures and statistical models are validated, the Advanced IRB could be implemented, although it would be a long-term project.

This paper serve as a first approximation to understand the IRB approach, and the consequences it could have if applied to Uruguayan institutions. From a regulator's point of view, the capital requirement that emerge from IRB's formulas seems to overestimate unexpected loss for some type of borrowers. In view of results presented in Part II, the risk weight functions should be smoothed, to count for the observed characteristics of the economy. As mentioned before, calibration of parameters has been done for the group of the tenth major supervisors, where portfolio's characteristics are far different from emerging banking institutions. According to our estimations, asset correlations and confidence level should be modified. The method also help in accomplishing with pillars 2 and 3, as it provides a measure that reflects the risk assumed by an institution, and the main determinants of it. Market agents' thus can monitor bank's risk strategies, having more information when making decisions. Actual capital requirements based in the standardized approach do not allow to infer about the risk profile, as risk weights are fixed for different credit facilities (as an example, all loans to non-financial sector denominated in local currency have a 100% weight)[16].

In case of applying the IRB, banks will have to develop internal models to estimate PD's, which implies having a large historical database, as well as procedures to validate results (back testing). Being PD's estimation the most important element for the approach, the supervisor's role is crucial, as it will validate models, in terms of their accuracy and predictability power. Related to this aspect, non parametric estimation becomes a useful tool to validate models, as it gives an empirical measure of portfolio's losses that can be compared with the ones that result from internal models. The main advantage of non parametric estimation is the lack of assumptions about the form of the distribution.

The method of estimation also helps when adjusting capital requirements of Basel's formula to emerging economies' characteristics. In that sense, parameters such as asset correlation could be estimated by bootstrap techniques, thus reflecting more properly different realities of banking systems.

To conclude, it seems necessary to have capital requirements that reflect the risk profile of institutions. The IRB method offers an excellent opportunity to establish better techniques of risk measurement, with more sophisticated tools and therefore a better understanding of the risk that a bank is taking. There is no doubt about the suitability of IRB measures for monitoring risk assumed by banks, as it could be seen as

[16] With the exception of mortgage loans, whose weight is 75%.

the economic capital of an institution. However, regulators should be careful about parameter's calibration as they may not reflect the reality of some banking systems. Another issue for regulators to take into account is the potential procyclicality behind this approach.

Appendix A – Properties of the loss distribution

1. Density function

Starting from the cumulative distribution function,

$$F(x;p;\rho) = N\left[\frac{\sqrt{1-\rho}\,N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right]$$

Calling *A* the term between brackets, density function can be expressed as:

$$\frac{dF}{dx} = f(A)\,\frac{dA}{dx}$$

Where $f(A) = \frac{1}{\sqrt{2\pi}} e^{-A^2/2}$

$$\frac{dA}{dx} = \sqrt{\frac{1-\rho}{\rho}}\frac{dN^{-1}(x)}{dx} \qquad [1]$$

Being $z(x) = N^{-1}(x)$, then $x = N(z) = x(z)$. As we can observe, $z(x)$ and $x(z)$ are inverse expressions, so we can state that:

$$z'(x) = \frac{1}{x'(z)} \rightarrow z'(x) = \frac{dN^{-1}(x)}{dx} = \frac{1}{\frac{dN(z)}{z}} = \frac{1}{f(z)} = \frac{1}{\frac{1}{\sqrt{2\pi}}e^{-z^2/2}} = \sqrt{2\pi}\,e^{[N^{-1}(x)]^2/2} \qquad [2]$$

Using [2] in equation [1], we have:

$$\frac{dA}{dx} = \sqrt{\frac{1-\rho}{\rho}}\,\sqrt{2\pi}\,e^{[N^{-1}(x)]^2/2}$$

Thus,

$$\frac{dF}{dx} = f(x) = \frac{1}{\sqrt{2\pi}}\,e^{-A^2/2}\sqrt{\frac{1-\rho}{\rho}}\,e^{[N^{-1}(x)]^2/2}\sqrt{2\pi} \rightarrow \sqrt{\frac{1-\rho}{\rho}}\,e^{\frac{-A^2+[N^{-1}(x)]^2}{2}}$$

Taking previous results, density function has the following expression:

$$f(x;\rho;p) = \sqrt{\frac{1-\rho}{\rho}} exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}N^{-1}(x) - N^{-1}(p)\right]^2 + [N^{-1}(x)]^2\frac{1}{2}\right\}$$

2. The mean

It will be demonstrated that the mean of the loss distribution is p; in analytical terms, that is:

$$\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx=p$$

To do that, consider the next function of p:

$$J(p)=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx$$

Demonstration will take three steps:

1) $f[x,\rho,p]=\sqrt{\frac{1-\rho}{\rho}}\exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}$ is a densities' distribution family for values of ρ є (0;1) and also for different values of p є (0;1).

2) The derivative of J(p) with respect to p equals 1; analytically:

$$\frac{dJ(p)}{dp}=1$$

3) Having shown 1) and 2), J(p) will have the following generic form:
$J(p)=\ p+k$, where k is a constant.
If we prove that $J(\frac{1}{2})=\frac{1}{2}$, then we know that k=0.

Taking into account 2) and 3), we can conclude that $J(p)\ =\ p$, which is what we want to demonstrate.

1st step: Observe that $f[x,\rho,p]\geq 0$ for x є (0;1) and for ρ є (0;1). Then, to show that it is a density function it is only necessary to prove that the total probability for all possible values of x is 1;

$$\int_0^1 \sqrt{\frac{1-\rho}{\rho}} \exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx=$$

$$=\int_0^1 \sqrt{\frac{1-\rho}{\rho}} \exp\left\{-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)}{\sqrt{\rho}}\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx$$

Consider next change of variable:

$$u=\frac{\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)}{\sqrt{\rho}} \Rightarrow du=\sqrt{\frac{1-\rho}{\rho}}\,\sqrt{2\pi}\,\exp\left[\frac{1}{2}\left[N^{-1}(x)\right]^2\right]dx$$

Rearranging terms:

$$\int_0^1 \sqrt{\frac{1-\rho}{\rho}} \exp\left\{-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)}{\sqrt{\rho}}\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx=$$

$$\frac{1}{\sqrt{2\pi}}\int_0^1 \exp\left[-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)}{\sqrt{\rho}}\right]^2\right]\sqrt{\frac{1-\rho}{\rho}}\sqrt{2\pi}\,\exp\left[\frac{1}{2}\left[N^{-1}(x)\right]^2\right]dx=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}u^2\right]du=1$$

The suggested change of variable make the standard normal density function appears, so we know that the area under that function is 1.

2nd step: $\frac{dJ(p)}{dp}=1$

Previously, let's make the same change of variable we did before:

$$J(p)=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}} \exp\left\{-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)- N^{-1}(p)}{\sqrt{\rho}}\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx$$

$$u=\frac{\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)}{\sqrt{\rho}}\Rightarrow du=\sqrt{\frac{1-\rho}{\rho}}\ \sqrt{2\pi}\ \exp\left[\frac{1}{2}\left[N^{-1}(x)\right]^{2}\right]dx\Rightarrow$$

$$x=N\left[\frac{u\sqrt{\rho}\ +N^{-1}(p)}{\sqrt{1-\rho}}\right]$$

Then:

$$J(p)=\int_{0}^{1}x\sqrt{\frac{1-\rho}{\rho}}\exp\left\{-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)}{\sqrt{\rho}}\right]^{2}+\frac{1}{2}\left[N^{-1}(x)\right]^{2}\right\}dx=$$

$$\frac{1}{\sqrt{2\pi}}\int_{0}^{1}x\ \exp\left[-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(p)}{\sqrt{\rho}}\right]^{2}\right]\sqrt{\frac{1-\rho}{\rho}}\sqrt{2\pi}\ \exp\left[\frac{1}{2}\left[N^{-1}(x)\right]^{2}\right]dx=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}u^{2}\right]N\left[\frac{u\sqrt{\rho}+N^{-1}(p)}{\sqrt{1-\rho}}\right]du$$

Using Leibnitz result, we can take derivatives with respect to p inside the integral:

$$\frac{d\,J(p)}{dp}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}u^{2}\right]\frac{\delta}{\delta p}\left\{N\left[\frac{u\sqrt{\rho}\ +N^{-1}(p)}{\sqrt{1-\rho}}\right]\right\}dx=$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}u^{2}\right]\frac{1}{\sqrt{2\pi}}\exp\left[\left[-\frac{1}{2}\right]\left[\frac{u\sqrt{\rho}\ +N^{-1}(p)}{\sqrt{1-\rho}}\right]^{2}\right]\sqrt{\frac{1}{1-\rho}}\sqrt{2\pi}\exp\left[\frac{1}{2}\left[N^{-1}(p)\right]^{2}\right]dx=$$

$$=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{1}{1-\rho}}\exp\left[\frac{1}{2}\left[N^{-1}(p)\right]^{2}\right]\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}u^{2}\right]\exp\left\{\left[-\frac{1}{2}\right]\left[\frac{u\sqrt{\rho}\ +N^{-1}(p)}{\sqrt{1-\rho}}\right]^{2}\right\}dx=$$

Forgetting for a while the integral, we evaluate the terms related to the exponential function:

$$\left\{\left[-\frac{1}{2}\right]\left[\frac{u\sqrt{\rho}+N^{-1}(p)}{\sqrt{1-\rho}}\right]^2+u^2\right\}=\left\{\left[-\frac{1}{2}\right]\left[\frac{\rho}{1-\upsilon}u^2+2\frac{\sqrt{\rho}}{1-\rho}N^{-1}(p)u+\frac{\left[N^{-1}(p)\right]^2}{1-\rho}+u^2\right]\right\}=$$

$$\left\{\left[-\frac{1}{2}\right]\left[\frac{u^2}{1-\rho}+2u\frac{\sqrt{\rho}}{1-\rho}N^{-1}(p)+\frac{\left[N^{-1}(p)\right]^2}{1-\rho}\right]\right\}=\left\{\left[-\frac{1}{2}\right]\left[\left[\frac{u}{\sqrt{1-\rho}}+\frac{\sqrt{\rho}}{1-\rho}N^{-1}(p)\right]^2+\left[N^{-1}(p)\right]^2\right]\right\}$$

Now back to the integral we have:

$$\frac{d\,J(p)}{dp}=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{1}{1-\rho}}\exp\left[\frac{1}{2}\left[N^{-1}(p)\right]^2\right]\int_{-\infty}^{+\infty}\exp\left\{\left[-\frac{1}{2}\right]\left[\frac{u}{\sqrt{1-\rho}}+\frac{\sqrt{\rho}}{\sqrt{1-\rho}}N^{-1}(p)\right]^2\right\}\exp\left[-\frac{1}{2}\left[N^{-1}(p)\right]^2\right]dx=$$

$$\frac{1}{\sqrt{2\pi}}\sqrt{\frac{1}{1-\rho}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}\left[\frac{u}{\sqrt{1-\rho}}+\frac{\sqrt{\rho}}{\sqrt{1-\rho}}N^{-1}(p)\right]^2\right]dx$$

Making the change of variable:

$$z=\frac{u}{\sqrt{1-\rho}}+\frac{\sqrt{\rho}}{\sqrt{1-\rho}}N^{-1}(p)\Rightarrow dz=\frac{1}{\sqrt{1-\rho}}du$$

The following is true:

$$\frac{d\,J(p)}{dp}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}[z]^2\right]dz=1$$

So it has been demonstrated that derivative of J(p) equals unity.

3th step: $J\left[\frac{1}{2}\right]=\frac{1}{2}$

Remembering that $N^{-1}\left[\frac{1}{2}\right]=0$ and substituting in J(p) we have:

$$J(\frac{1}{2})=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left\{-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x)-\ N^{-1}(\frac{1}{2})}{\sqrt{\rho}}\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right\}dx=$$

$$\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[-\frac{1}{2}\left[\frac{\sqrt{1-\rho}\,N^{-1}(x))}{\sqrt{\rho}}\right]^2+\frac{1}{2}\left[N^{-1}(x)\right]^2\right]=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx$$

Then we can establish:

$$J(\frac{1}{2})=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx=\frac{1}{2}\ \Leftrightarrow$$

$$\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx=\frac{1}{2}\int_0^1\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]$$

As we know from the first step that the integral of the second term equals 1. Here we took that result applying for the special case of p=1/2.

Continuing with the expression:

$$J(\frac{1}{2})=\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx=\frac{1}{2}\ \Leftrightarrow$$

$$\int_0^1 x\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx=\frac{1}{2}\int_0^1\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]\Leftrightarrow$$

$$\int_0^1\left[x-\frac{1}{2}\right]\sqrt{\frac{1-\rho}{\rho}}\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]dx=0$$

Calling G(x) to the function inside the integral:

$$G(x)=\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\left[x-\frac{1}{2}\right]\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]=h(x)\ p(x)\ \ where:$$

$$h(x)=\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\left[x-\frac{1}{2}\right]\ \ and\ \ p(x)=\exp\left[\ \frac{2\rho-1}{2\rho}\left[N^{-1}(x)\right]^2\right]$$

h(x) is an odd function for x=1/2; and p(x) is an even function for x=1/2.

So it can be stated that:

$$h(\frac{1}{2}+a)=-h(\frac{1}{2}-a)\;\; y \;\; p(\frac{1}{2}+a)=p(\frac{1}{2}-a) \quad a\in\left[0;\frac{1}{2}\right]$$

As the product of an odd function by an even function is odd, then G(x) is odd for x=1/2.

Then:

$$G(\frac{1}{2}+a)=-G(\frac{1}{2}-a) \quad a\in\left[0;\frac{1}{2}\right]$$

That is, to make that the average point between two values x and x' be ½, G(x) must take opposite values. This implies that:

$$\int_0^1 G(x)dx=0$$

What we wanted to demonstrate.

3. The mode

We must obtain the maximum of the density function.

$$\frac{df}{dx}=\sqrt{\frac{1-\rho}{\rho}}\,exp\left\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}N^{-1}(x)-N^{-1}(p)\right]^2+[N^{-1}(x)]^2\frac{1}{2}\right\}\frac{d\{.\}}{dx}$$

To simplify, let's call *h* the term between braces, so that:

$$\frac{df}{dx}=\sqrt{\frac{1-\rho}{\rho}}\,exp\{h\}\left\{-\frac{1}{2\rho}2\left[\sqrt{1-\rho}N^{-1}(x)-N^{-1}(p)\right]\sqrt{1-\rho}\frac{dN^{-1}(x)}{dx}+N^{-1}(x)\frac{dN^{-1}(x)}{dx}\right\}$$

Simplifying the previous expression and equaling to zero, we have:

$$\frac{df}{dx}=\sqrt{\frac{1-\rho}{\rho}}\,exp\{h\}\,\frac{dN^{-1}(x)}{dx}\left\{-\frac{\left[\sqrt{1-\rho}N^{-1}(x)-N^{-1}(p)\right]\sqrt{1-\rho}}{\rho}+N^{-1}(x)\right\}=0$$

The first three terms are positive[17], so to obtain the mode it is enough to equal to zero the last term,

$$(1-\rho)N^{-1}(x) - \sqrt{1-\rho}\,N^{-1}(p) = \rho N^{-1}(x)$$

$$N^{-1}(x)\,[1-2\rho] = \sqrt{1-\rho}\,N^{-1}(p) \rightarrow N^{-1}(x) = \frac{\sqrt{1-\rho}}{[1-2\rho]}\,N^{-1}(p) \rightarrow$$

$$x = N\left[\frac{\sqrt{1-\rho}}{[1-2\rho]}\;N^{-1}(p)\right], \rho \neq 1/2$$

4. The median

The value of x that accumulates the 50% of the distribution is given by:

$$N\left[\frac{\sqrt{1-\rho}\,N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right] = 0.5$$

$$\frac{\sqrt{1-\rho}N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}} = 0 \rightarrow \sqrt{1-\rho}N^{-1}(x) = N^{-1}(p) \rightarrow N^{-1}(x) = \frac{N^{-1}(p)}{\sqrt{1-\rho}}$$

So the median is:

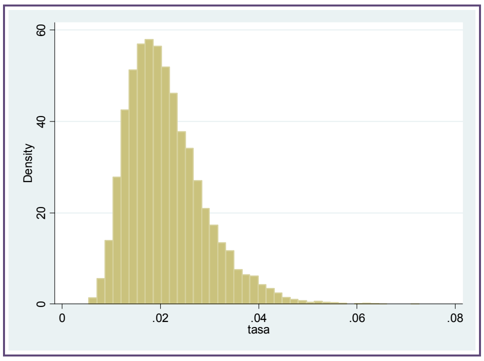
$$x_{0.5} = N\left[\frac{N^{-1}(p)}{\sqrt{1-\rho}}\right]$$

[17] Remember that $\frac{dN^{-1}(x)}{dx} = \sqrt{2\pi}\, e^{[N^{-1}(x)]^2/2}$, a positive number.

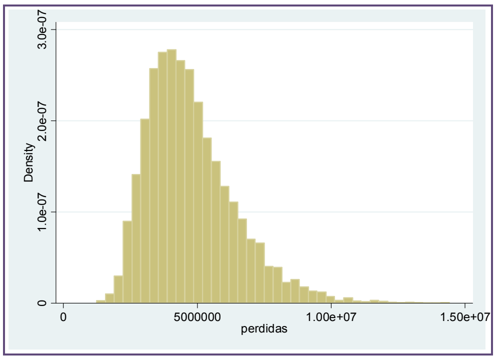
Appendix B – Estimations for corporate portfolio

Conditional distribution: period 1999-2000

Loss rate

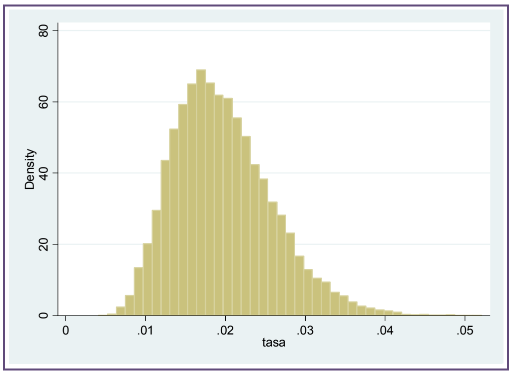


Loss in USD

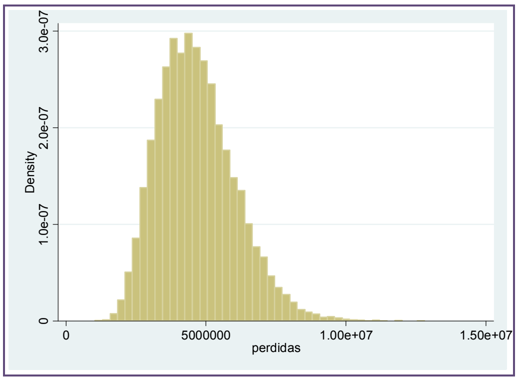


Conditional distribution: period 2000-2001

Loss rate

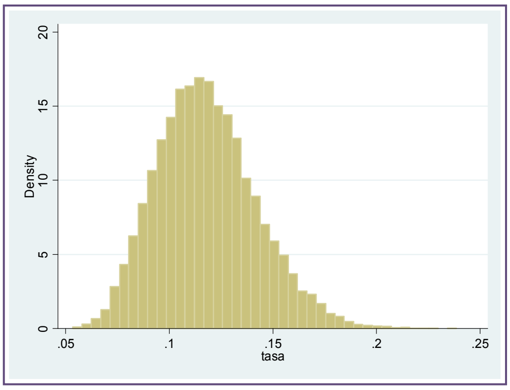


Loss in USD

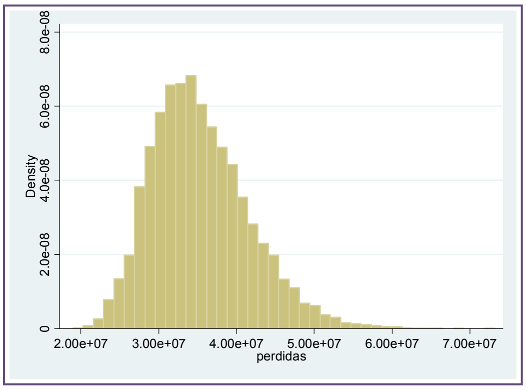


Conditional distribution: period 2001-2002

Loss rate

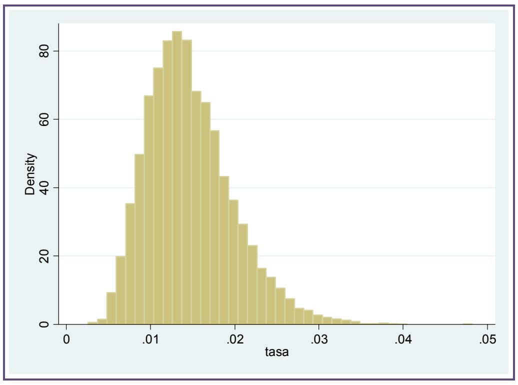


Loss in USD

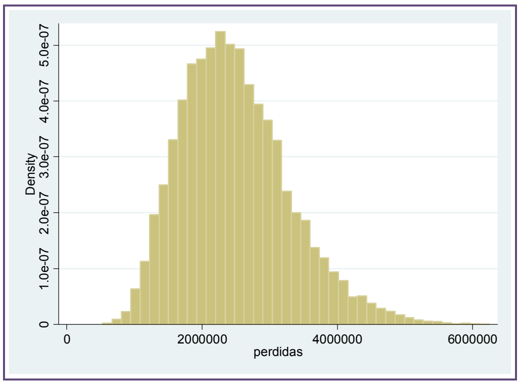


Conditional distribution: period 2002-2003

Loss rate

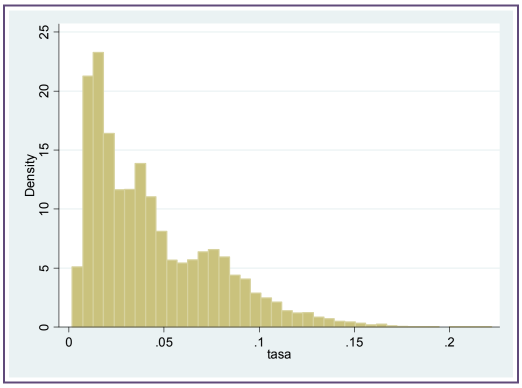


Loss in USD

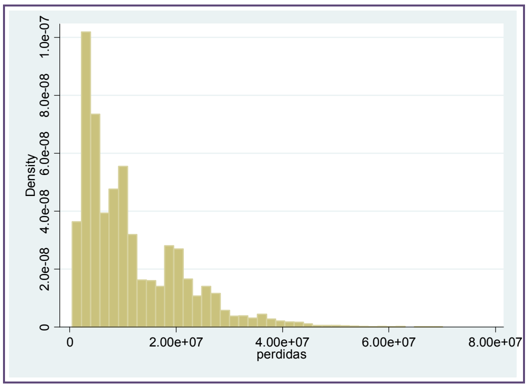


Conditional distribution: period 2003-2004

Loss rate

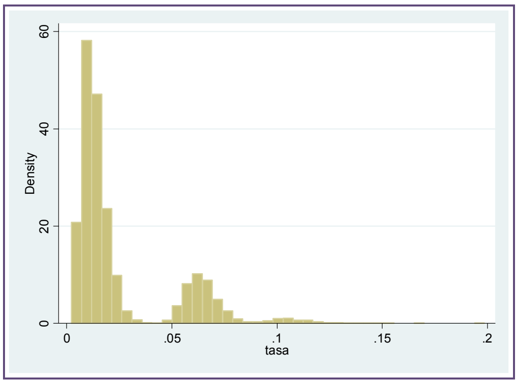


Loss in USD

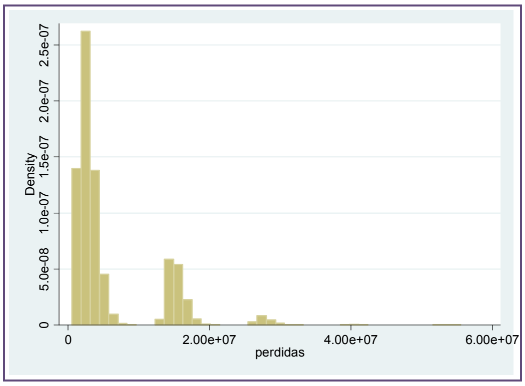


Conditional distribution: period 2004-2005

Loss rate

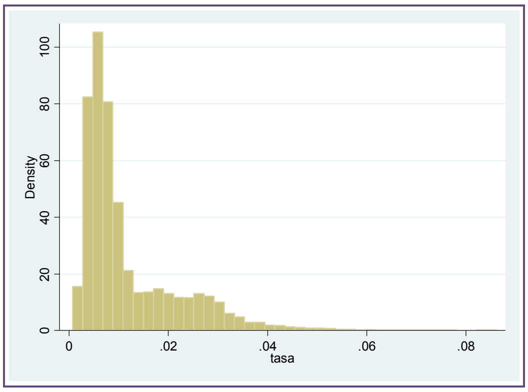


Loss in USD

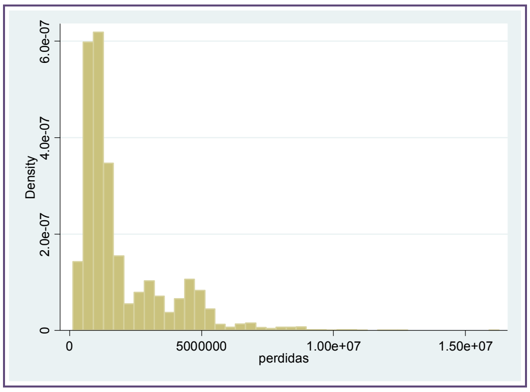


Conditional distribution: period 2005-2006

Loss rate

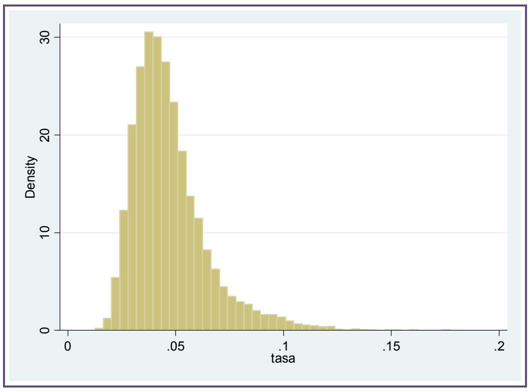


Loss in USD

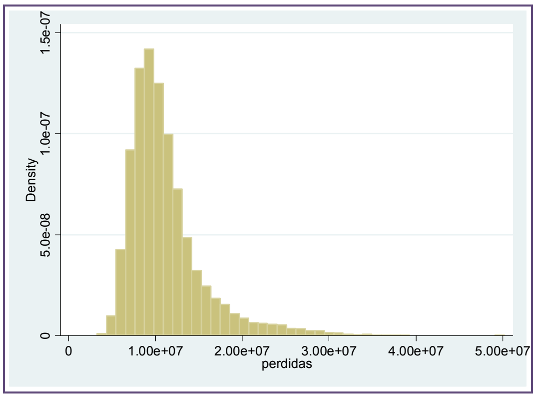


Unconditional distribution: period 1999-2006

Loss rate



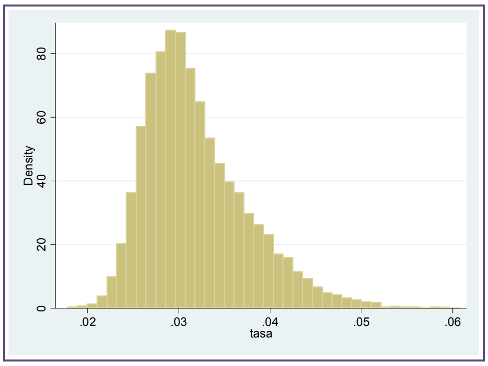
Loss in USD



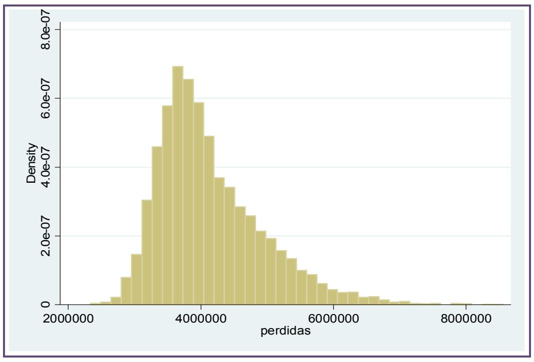
Appendix C – Estimations for retail portfolio

Conditional distribution: period 1999-2000

Loss rate

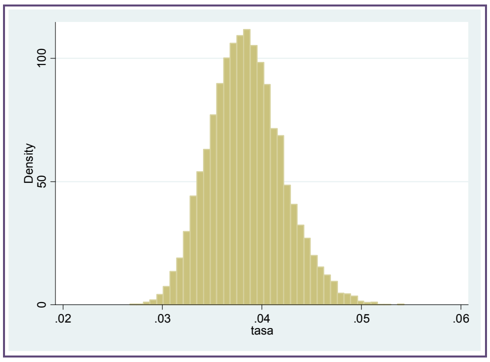


Loss in USD

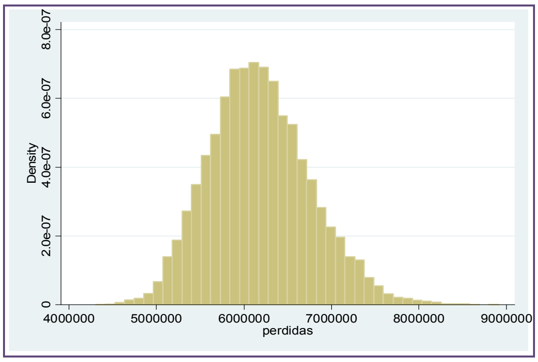


Conditional distribution: period 2000-2001

Loss rate

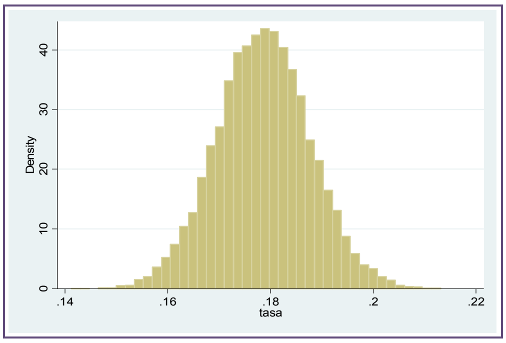


Loss in USD

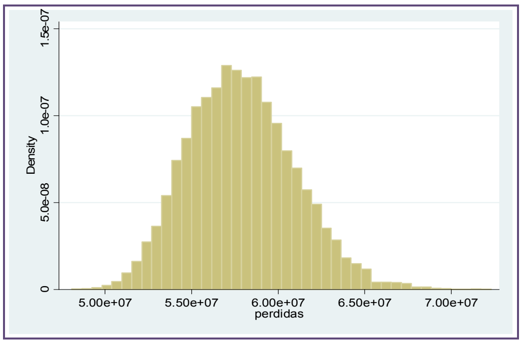


Conditional distribution: period 2001-2002

Loss rate

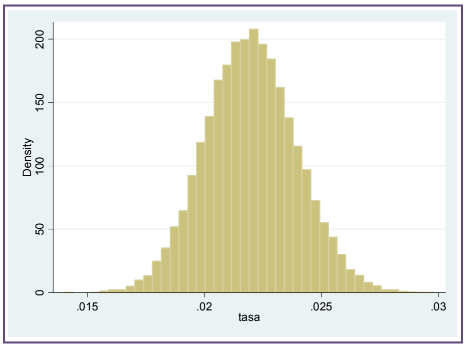


Loss in USD

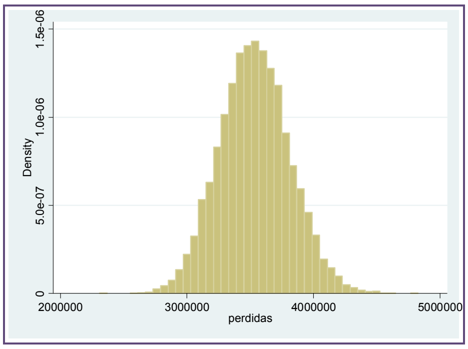


Conditional distribution: period 2002-2003

Loss rate

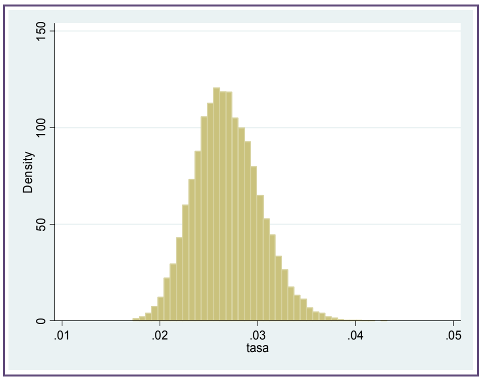


Loss in USD

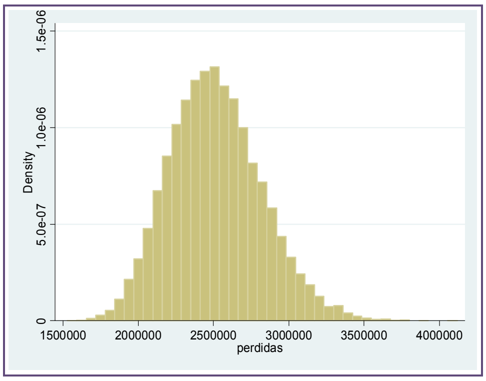


Conditional distribution: period 2003-2004

Loss rate

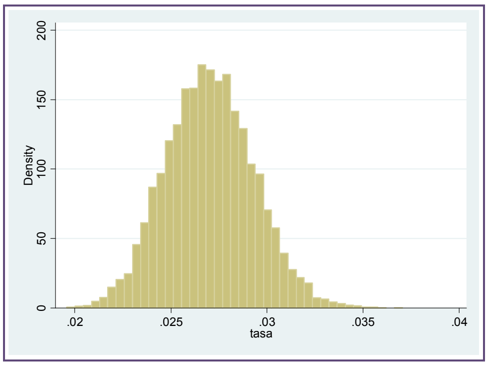


Loss in USD

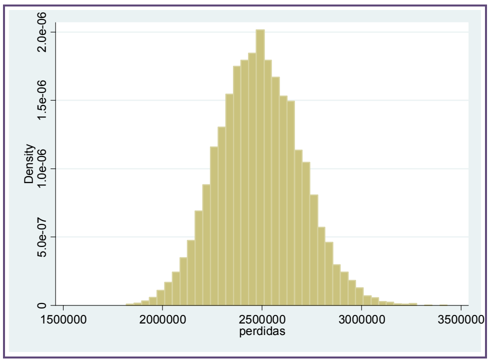


Conditional distribution: period 2004-2005

Loss rate

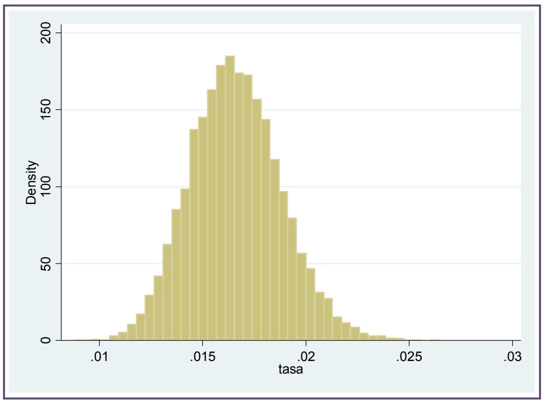


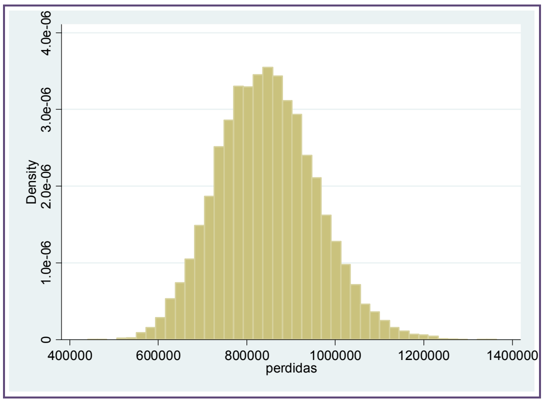
Loss in USD



Conditional distribution: period 2005-2006

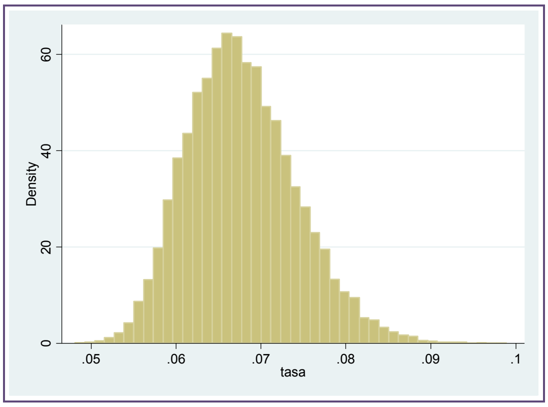
Loss rate *Loss in USD*

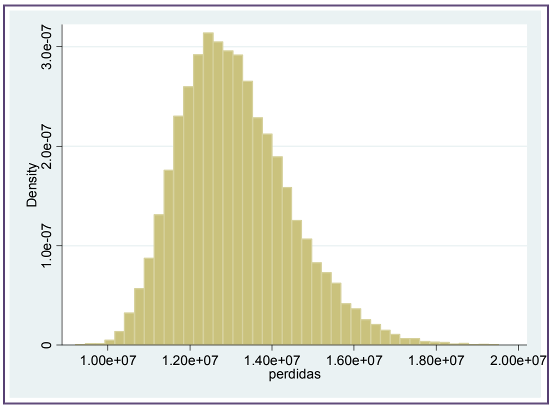




Unconditional distribution: period 1999-2006

Loss rate *Loss in USD*





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