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Chan, Ying Tung and Zhao, Hong

The Research Institute of Economics and Management, Southwestern University of Finance and Economics, The School of Economics, Nankai University

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Ying Tung Chan* and Hong Zhao†

Abstract

Recessions associated with financial crises have become common in the US since 1990. This paper examines the importance of the financial frictions for US carbon emissions dynamics. Our empirical analysis reveals that financial market conditions have a substantial and nonlinear impact on carbon emissions dynamics. We build and estimate an environmental dynamic stochastic general equilibrium model that features financial frictions and a risk shock (a type of credit shock). The results show that: (i) the presence of financial frictions doubles the volatility of carbon emissions under positive TFP and government expenditure shocks; (ii) the risk shock generates counterfactual paths that can largely replicate the movements in emissions growth; (iii) the contribution share of the risk shock to emissions growth dynamics reaches a peak of around 50% after each recession; (iv) the optimal carbon tax rate response to shocks heavily depends on the Taylor rule specification.

JEL Classification Numbers: Q50, Q51, E32, E44

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*The Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, China (email: chanyt@swufe.edu.cn).

†The School of Economics, Nankai University (email: zhaohong42@gmail.com)

1 Introduction

As most recessions in the post-1990 era are associated with financial or banking system crises, the role of financial frictions in business cycles has received greater emphasis (e.g., Carlstrom and Fuerst, 1997; Bernanke et al., 1999; Christiano et al., 2014; Iacoviello, 2015; Duncan and Nolan, 2017). Since across all industries, borrowing to finance plant, equipment, and inventories is common and during the financial crisis, access to financial capital became difficult, investment declined. It is not surprising that investment declines lead to output declines. What is surprising is that almost all of the huge decline in output in the US economy following the crisis was confined to investment (Hall, 2010). The reduction in economic activity resulted in a reduction in energy usage and CO_2 emissions fell. How does the prosperity or failure of the financial sector, throughout the business cycle, affect carbon emissions in the economy? More important, when a distressed financial system becomes a source of serious macroeconomic contractions, how should environmental policy adapt to cyclical fluctuations?

Mainstream environmental macroeconomic models used for environmental policy analysis, such as the models used by Angelopoulos et al. (2010), Fischer and Springborn (2011), Heutel (2012), Annicchiarico and Di Dio (2015), Dissou and Karnizova (2016), and Khan et al. (2019) contain no role for financial frictions. One reason for the omission of financial frictions from standard models is that there is little consensus about their importance for emissions dynamics. Therefore, quantifying the importance of the credit market frictions is the primary objective of this paper. To this end, we first empirically examine the impact of credit market conditions on CO_2 emissions in the US. Then we investigate the above questions by incorporating financial frictions into a standard environmental dynamic stochastic general equilibrium (E-DSGE) model with carbon emissions. Specifically, we introduce the financial frictions developed by Bernanke et al. (1999) (henceforth, BGG) into the model developed by Annicchiarico and Di Dio (2015) that incorporates pollutant emissions and environmental policy. Furthermore, we consider the impact of a "risk shock" on carbon emissions. This risk shock, defined by Christiano et al. (2014) as a type of a credit market shock, is found to be the most important shock driving business cycles. By merging the two frameworks, we study the interplay between credit market frictions and the firms' environmental decisions.

At the core of this paper is the idea that if business cycles are financial rather than real, which are just the cases of at least two of the last three recessions in the US (the 1990-1991 recession and the Great Recession of 2007-2009), what are the impacts of financial frictions on the dynamics of emissions and on the environmental policy? Our study is, to the best of our knowledge, the first attempt to quantify the importance of the credit market frictions for carbon emissions dynamics. The main findings are as follows. First, the empirical results show that credit market conditions, as a nonlinear propagator of shocks, have a substantial impact on the dynamics of carbon emissions.

Second, by estimating our E-DSGE model with financial frictions over the sample period from 1985Q1 to 2019Q2, we find that the risk shock tightens the credit market and significantly reduces carbon emissions. In addition, the presence of financial frictions doubles the volatility of carbon emissions under positive TFP and government expenditure shocks. Most important, the risk shock generates counterfactual paths that perform the best in characterizing the movements in output and emissions growth. Among the four shocks (the TFP, monetary policy, government expenditure, and risk shocks) considered in this paper, the contribution share of the risk shock alone to the emissions growth rate dynamics reaches a peak of around 50% in 8 quarters after each recession. Third, the welfare analysis reveals that a 15% carbon tax would lead to a 32% decrease in carbon emissions and approximately 2.72 billion US dollars (measured in chained 2012 dollars) per quarter reduction in consumption to maintain the same level of household utility as when no tax is applied.

Finally, we find that the optimal carbon tax rate should be procyclical when considering financial frictions. Although the optimal carbon tax rate being procyclical is in line with the existing literature Heutel (2012) and Annicchiarico and Di Dio (2015), this result is highly conditional on the specification of the Taylor rule. In Annicchiarico and Di Dio (2015), the interest rate only responds to the inflation rate. But with a generalized Taylor rule, where the interest rate is also responsive to output gap, the procyclicality of the optimal carbon tax to the risk shock is reduced. Furthermore, it reverts to being countercyclical in response to a positive TFP shock. In the sense of entailing less volatility in emissions, the Taylor rule that reacts to output deviation may be more stabilizing.

We draw on two strands of literature. First, there is now a growing literature that incorporates pollutant emissions into standard macroeconomic business cycle models to address issues of environmental policy design.[1] The prominent works of Angelopoulos et al. (2010), Fischer and Springborn (2011), and Heutel (2012) use a real business cycle model to study environmental policy. Starting from these contributions, Annicchiarico and Di Dio (2015) extend the analysis on the relationship between business cycles and environmental policy in a standard NK model featuring rigidities and monopolistically competitive polluting firms. Regarding the procyclicality of emissions, these studies have emphasized that optimal emissions policies should also be procyclical. Typically, the drivers of business cycles in these environmental DSGE models are assumed to be TFP, monetary policy, and government expenditure shocks, while these models consider neither the financial sector nor credit shocks. Further, Annicchiarico and Di Dio (2017) investigate how optimal emissions respond to shocks in a similar framework, to that in Annicchiarico and Di Dio (2015), but with a generalized Taylor rule and conclude that a meaningful characterization of environmental policy is conditional on how the monetary policy reacts to shocks. Dissou and Karnizova (2016) use a multi-sector real business cycle model to rank the alternative policies

[1] Fischer and Heutel (2013) survey the literature on the macroeconomic approach to environmental policy issues. Shahiduzzaman and Layton (2015) also provides a comprehensive survey of recent empirical studies that examine the impacts of business cycle on emissions.

(taxes and permits) in the presence of shocks. Most recently, Khan et al. (2019) use a DSGE model and its counterpart VAR model to identify the shocks that drive emissions dynamics. They find that anticipated investment technology shocks account for 25% of the variation, while two-thirds of the variation in emissions appears to be due to an unidentified structural shock. To sum up, the relationship between the financial sector and carbon emissions is under-research.

Second, the modeling method in this paper used to characterize credit market imperfections is developed by BGG and how to model financial frictions is an important area of study in macroeconomics. This approach was established by Bemanke and Gertler (1989), Carlstrom and Fuerst (1997), and BGG. BGG capture the situation in which financial frictions propagate and amplify other shocks. Therefore, in addition to the intertemporal substitution and nominal rigidities, financial frictions act as the third shock propagator and amplifier in our model. The BGG model is proven to be important in accounting for the US business cycle (Christensen and Dib, 2008; Christiano et al., 2014) and has been widely applied in the macroeconomics literature.[2] Specifically, Christensen and Dib (2008) empirically show that BGG better captures the US data. Building on BGG, Christiano et al. (2014) explore the role of a risk shock (a type of credit shock) and find that the risk shock is the most important shock driving the business cycle.

Finally, for the sake of completeness, we highlight the literature related to the financial market and carbon emissions focusing on the pricing of carbon allowances according to asset pricing models and techniques.[3] Some papers suggest that the price of carbon allowances can be determined by the fundamentals, including electricity, gas, and coal prices (Aatola et al., 2013), temperature (Alberola et al., 2007), and economic growth and wealth conditions (Bredin and Muckley, 2011). Moreover, there is a proven, close linkage between carbon allowances and firm performance (Oestreich and Tsiakas, 2015). This and related works demonstrate the importance of the financial market in carbon pricing and emphasize the properties of a particular asset, namely, the carbon emission allowance. Our model differs by focusing on the interaction between macroeconomic variables and emissions in a general equilibrium framework. Rather than singling out the impact of a particular factor, we are interested in how the overall performance of the credit market affects the carbon emissions macroscopically.

The remainder of the paper proceeds as follows. Section 2 conducts an empirical study using a threshold vector auto-regressive (TVAR) model. Section 3 presents the main model. Section 4 estimates our model using a Bayesian approach and reports the results. Section 5 performs a numerical exercise to quantify the importance of financial frictions. Section 6 studies the welfare implications of different carbon tax regimes and how optimal environmental policy responds to shocks. Section 7 concludes.

[2]For example, Weber and Stern (2011) find that the difference in the US and European unemployment rates, regardless of their labor market similarity, can be explained by credit market imperfections. Céspedes et al. (2004) show that with financial imperfections, devaluation could be beneficial to countries with heavy foreign debts due to the higher profits generated by entrepreneurs.

[3]These models and techniques include the arbitrage-free model (Barrieu and Fehr, 2014), the high-frequency data technique (Conrad et al., 2012), and the market efficiency hypothesis and martingale properties (Daskalakis et al., 2009).

2 Empirical analysis

This section empirically tests the impacts of credit market conditions on CO_2 emissions in the US. Fig. 1 displays the HP-filtered log carbon emissions per capita and adjusted National Financial Conditions Index (ANFCI)[4] in the US from 1973Q1 to 2019Q2. Since the US economic and financial conditions tend to be highly correlated, different from NFCI, ANFCI isolates a component of financial conditions uncorrelated with economic conditions to provide an update on financial conditions relative to current economic conditions. From Fig. 1, it is evident that the emissions experience a substantial decline while ANFCI experiences a spike in every recession. How does the tightening of the financial sector and credit market contribute to the decline in emissions observed in the recession period? We investigate the relationship between credit market conditions and CO_2 emissions by applying a TVAR model:

$$y_t = A_1 y_{t-d}\mathbb{I}(\zeta_{t-d} \leq \bar{\zeta}) + A_2 y_{t-d}\mathbb{I}(\zeta_{t-d} > \bar{\zeta}) + U_t, \tag{1}$$

where y_t is a vector of variables that contains the real GDP growth rate $g_{Y,t}$, the inflation rate π_t, the Fed funds rate R_t, a measure of credit market condition ζ_t as the threshold variable, and the CO_2 emissions growth rate Z_t. $\mathbb{I}(.)$ is an indicator function that equals 1 when ζ_t is less than some optimal threshold value $\bar{\zeta}$ and 0 otherwise. $d \geq 1$ is the period lag of the TVAR variable. U_t is a structural disturbance term, which is assumed to be normally distributed. A_1 and A_2 reflect the contemporaneous relationships in the two regimes respectively. Following Balke (2000) and Bernanke et al. (1997), we assume that A_1 and A_2 have a recursive structure with the causal ordering of output growth, inflation, the Fed funds rate, a financial market variable, and carbon emissions. The choice of the credit proxy is controversial. Since we incorporate entrepreneurs' default and bankruptcy into the model and use the delinquency rate on commercial and industrial loans data to estimate the E-DSGE model later in sections 3 and 4, for the sake of consistency we also choose the delinquency rate on commercial and industrial loans as the credit proxy. In addition, we use ANFCI and NFCI as alternative measures to examine the credit's role in affecting carbon emissions.

2.1 Data description

The following quarterly time series were used to estimate the TVAR model 1: real GDP per capita, the inflation rate, the Fed funds rate, the delinquency rate on commercial and industrial loans, ANFCI, NFCI, and carbon dioxide emissions per capita in the US. The data span from 1985Q1 to 2019Q2,

[4] The Chicago Fed's National Financial Conditions Index (NFCI) provides a comprehensive update on US financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems. Positive values of the NFCI and ANFCI have been historically associated with tighter-than-average financial conditions, while negative values have been historically associated with the opposite.

including 138 quarterly observations. The length of the sample is constrained by the data availability of the delinquency rate on commercial and industrial loans. Real GDP per capita and carbon emissions per capita were log-transformed. Precise definitions of the data series can be found in appendix A. We verify whether the series were stationary on the ADF tests. Table 1 describes the results of these tests with and without intercept for the series in the level and in first differences. Given a 5% significance level, the results indicate that real GDP per capita, the delinquency rate, and carbon emissions have a unit root in the level, being stationary only in the first differences. We use the first difference of these variables. The inflation rate, the Fed rate, ANFCI, NFCI are stationary.

The first difference of the delinquency rate as a proxy for the unobserved credit market conditions is expressed as $\zeta_t = del_t - del_{t-1}$. A large value of ζ_t indicates a significant increase in the delinquency rate and so could be a signal of deteriorating credit conditions during recessions. We refer to the periods when $\zeta_{t-d} > \bar{\zeta}$ as a tight regime and the other periods as a normal regime. To determine the lag length d, we first estimate a linear VAR containing the five variables and select the optimal lags using the LR, FPE, AIC, SC, and HQ information criteria for each credit market proxy.[5] In the benchmark case of using the delinquency rate growth as the credit market proxy, the optimal lag is $d = 2$. FPE, AIC, and HQ all indicate a lag of 2, while LR and SC indicate the lags of 8 and 1, respectively. Then we set $d = 2$ and estimate the TVAR model 1. As robustness checks, we follow the same steps and use alternative measures of ANFCI and NFCI as the credit market proxies to estimate the TVAR model. The TVAR model (1) is able to capture the evolution of the variables in y_t, especially the changes in emissions and regime switches over time.

2.2 Empirical strategy

We follow the approach used by Balke (2000) to estimate the TVAR model 1. The optimal threshold value $\bar{\zeta}$ is not known in advance and needs to be estimated. Therefore, in the first step, the TVAR model is estimated by least squares for all threshold candidates ζ_t. The candidates ζ_t are set such that at least 15% of the observations plus the number of parameters in each equation are included in each regime to avoid overfitting. Then, the optimal threshold value is the one that produces the greatest (log) likelihood ratio, which equals $2(\ln L(\bar{\zeta}) - \ln L_0)$, where $L(\bar{\zeta})$ and $\ln L_0$ are the likelihoods of the TVAR model with threshold $\bar{\zeta}$ and the standard linear VAR model, respectively. The results show that the likelihood is maximized at $\bar{\zeta} = 0.00$ for the benchmark case, and at $\bar{\zeta} = -0.18$, $\bar{\zeta} = -0.31$ for the ANFCI and NFCI cases, respectively.[6]

[5] LR: sequential modified likelihood ratio test statistic. FPE: Akaike's final prediction error criterion. AIC: Akaike information criterion. SC: Schwarz information criterion. HQ: Hannan-Quinn Criterion.

[6] Fig. B1 in appendix B plots the likelihood ratio against the possible threshold values for the benchmark case, and the cases using ANFCI, NFCI as credit market proxies.

As in Hansen (2000), Afonso et al. (2018), and Balke (2000), three separate Wald statistics are constructed to test the null hypothesis of no threshold behavior. They are the maximum Wald statistic (sup-Wald), the average Wald statistic (avg-Wald), and the sum of the exponential Wald statistic (exp-Wald) over all threshold candidates. These three values are then compared with the critical values generated from simulating empirical distributions of sup-Wald, avg-Wald, and exp-Wald.[7] Table 2 reports the three Wald test results and the robustness checks with different credit proxies. In the benchmark case, with p-values very close to 0 when $d = 2$, we demonstrate that the data tend to reject the null hypothesis of no threshold behavior. That is, the TVAR model is able to capture the nonlinearities in the data. In addition, the results are robust to the cases of using alternative credit market proxies ANFCI and NFCI, since only the p-values for avg-Wald statistics are higher than 5%.

2.3 Results

Fig. 2 plots the time series of ζ_t and a line at the optimal threshold values in all three cases. For reference, the NBER recession periods are shaded. The threshold splits the data set such that the tight regimes are when the values are above the threshold and the normal regimes are otherwise. In the benchmark case (panel (a) of Fig. 2), approximately one-fourth of observations lie above the threshold. Moreover, all three NBER recessions are preceded or accompanied by a surge in ζ_t. This suggests that the first difference of the delinquency rate is a good indicator to capture the regime switching of the economy from normal to recession periods. In addition to the recession periods, the optimal threshold also identifies other two tight credit regimes in 1986Q4 and 1988Q4. For the most parts, the periods of tight regimes indicated by three alternative credit market proxies, the delinquency rate, ANFCI (panel (b)) and NFCI (panel (c)) coincide with one another, especially for the tight regimes in 2008. Of the three threshold variables, the delinquency rate performs the best on indicating the tight regimes before the other threshold variables and on indicating the NBER recessions.

The estimated coefficients under the two regimes are reported in Table 3. The coefficients for the impact of the delinquency rate on emissions are significant in the two regimes.[8] But these two coefficients have the opposite signs. An increase in the delinquency rate increases the emissions in the normal regime, but reduces them in the tight regime. Moreover, the first (second) lag of interest rate has a positive (negative) and more significant impact on emissions in the tight regime than in the normal regime. That is, the monetary policy may have larger impacts on emissions during recessions than in the normal time. In general, the coefficients in the two regimes differ substantially, indicating a strong impact of credit regime switching on the correlations between the macroeconomic variables.

Due to the nonlinearity of the TVAR model, shocks to the system may lead to regime switches and

[7] Calculation procedures for the Wald statistics detail in appendix B.

[8] Except in the tight regime, the p-value for the coefficient before del_{t-1} on Z_t is higher than 10%.

hence create a nonlinear dynamic impact on the variables, i.e., their Wold decompositions do not exist. Consequently, unlike the linear model, the impulse response function (IRF) for the nonlinear model depends not only on the realized values of the disturbance U_t but also on the initial conditions of Y_t. Specifically, following Koop et al. (1996), the nonlinear impulse response functions (NIRFs) are defined as:

$$\mathbb{E}[y_{t+k}|\Omega_{t-1}, u_t] - \mathbb{E}[y_{t+k}|\Omega_{t-1}], \tag{2}$$

where Ω_{t-1} is the information set at time $t-1$. This equation computes the differences between the forecasted paths of variables with and without the realized value of the shock u_t based on the initial condition Ω_{t-1}.[9]

Fig. 3 plots the NIRFs of emissions to five types of shock conditional on the two regimes. The NIRFs behave quite differently in the two regimes. Except for the shock to the interest rate, all of the shocks have a larger impact on the emissions growth in the tight regime. This is particularly true for larger shocks (see the NIRF of two-standard-deviation shocks). In the normal regime, an increase in the interest rate reduces the emissions growth upon the impact of the shock, but emissions growth would increase thereafter above its steady state. However, in the tight regime, an increase in the interest rate would only reduce the emissions growth. It is notably interesting that the IRFs of the emissions growth to the shocks to delinquency rate growth have opposite signs in different regimes. Specifically, in the normal regime, a positive shock to ζ_t, i.e., an increase in the delinquency rate growth, would lead to an increase in the emissions growth. While in the tight regime, a positive shock to ζ_t would further decrease the emissions growth. Consistent with the findings of McCallum (1991) and Balke (2000), Fig. 4 suggests that output is more sensitive to the monetary policy shock in the tight regime.[10]

Asymmetric responses to different sizes and signs of the shocks can be clearly seen in Figs. 3 and 4, particularly to the first difference delinquency rate shock. In this regard, we explore which types of shock affect regime switching over time. $\mathbb{E}[\mathbb{I}(\zeta_{t+k-1} > \bar{\zeta})|\Omega_{t-1}, u_t]$ calculates the likelihood of being in the tight regime in period $t+k$, given that shock u_t is realized in period t. Conditional on starting from the normal regime, for both positive and negative two-standard-deviation shocks, we plot the probability of switching to the tight regime in Fig. 5. As expected, a positive shock to the delinquency rate has the most significant impact on increasing the probability of entering the tight regime. Negative shocks to the inflation rate and emissions also increase the transition probability.

The empirical study in this section demonstrates that credit market conditions play an important role as a nonlinear propagator of shocks in the dynamics of carbon emissions. This nonlinearity takes the form of credit regime switching when the measure of credit market conditions crosses a critical thresh-

[9] We follow the procedure in Balke (2000) to compute this NIRF, and the calculation details are shown in appendix B.

[10] Figs. C1 to C3 in appendix C depict the NIRFs of the inflation, interest, and delinquency rate to five types of shock conditional on the two regimes, respectively. In general, shocks in the tight regime have a larger impact on the variables concerned.

old. In particular, our results show that delinquency rate growth is a good indicator, compared to ANFCI and NFCI, to capture the regime switching of the economy from normal to recession periods. By examining the NIRFs, the emissions growth response asymmetrically to the delinquency rate growth shock in different regimes. Especially, an increase in delinquency rate growth would lead to a decrease in emissions growth in the tight regime. However, in the normal regime, an increase in delinquency rate growth would increase emissions growth. All of the shocks are more potent in the tight-credit regime, and emissions are more sensitive to monetary policy shocks in the tight regime. Therefore, to further investigate the impact of credit market conditions on carbon emissions, in the following sections we present an estimated structural model featuring credit market frictions. More important, we conduct policy analysis in this framework to investigate how environmental policy should adapt to cyclical fluctuations when these fluctuations are related to or even caused by financial frictions.

3 Model

This section presents a E-DSGE model that features financial market frictions, carbon emissions, and environmental policy. The model economy is populated by many identical infinitely-lived households who consume a final good that is a composite of a continuum of intermediate goods.

A continuum of intermediate polluting firms indexed by $i \in [0, 1]$ comprises monopolistic producers of differentiated intermediate goods, and these firms set prices à la Calvo (1983). Following Annicchiarico and Di Dio (2015) and Heutel (2012), CO_2 is emitted during the production process. The aggregate emissions stock accumulates and deteriorates the firms' productivity through a damage function. Since each firm is incapable of mitigating emissions due to its infinitesimal size, the government must intervene by levying an emission tax on firms. The representative final good producer packages the intermediate goods and sells them in a competitive market.

To model financial market frictions, by following BGG, three more classes of agents are introduced, namely, entrepreneurs, a financial intermediary, and capital producers. Entrepreneurs face an idiosyncratic capital return and determine capital investment. This investment can be funded either internally by entrepreneurs' net worth or externally by borrowing from the financial intermediary. The financial intermediary's loanable funds come from household deposits. Asymmetric information between entrepreneurs and financial intermediary creates financial frictions that amplify macroeconomic fluctuations. This amplification effect is called the financial accelerator effect. Capital producers build capital and sell it to the entrepreneurs. Moreover, the model also features a monetary authority, which sets the nominal interest rate according to an interest rate rule. In general, there are three types of rigidities in the model: sticky prices, capital adjustment costs, and credit frictions.

In addition to the total factor productivity (TFP) shock, the monetary policy shock, and the government expenditure shock, we follow Christiano et al. (2014) to incorporate the "risk shock," the shock to the volatility of the entrepreneurs' idiosyncratic return. The risk shock, by directly affects the quantity of net worth in the hands of entrepreneurs, produces immediate and prolonged impact on the delinquency rate. Christiano et al. (2014) show that the risk shock is the most important shock in driving the US business cycles.

3.1 Households

Consider an economy with infinitely many identical households. The representative household maximizes discounted lifetime expected utility as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \mu_L \frac{L_t^{1+\phi}}{1+\phi} \right), \tag{3}$$

where C_t and L_t are the household consumption and labor supply at time t, respectively. $0 < \beta < 1$ is a discount factor. The disutility of labor supply is controlled by a scale parameter $\mu_L > 0$ and the inverse of the Frisch elasticity $\phi > 0$. When maximizing lifetime utility Eq. (3), the household faces the following budget constraint in every period t:

$$P_t C_t + D_{t+1} = R_t D_t + W_t L_t + P_t V_t - T_t, \tag{4}$$

where P_t is the general price level at time t. V_t represents dividend income, or (real) profit received from the ownership of firms. D_t is deposits held at financial intermediaries at time t. R_t is a gross nominal interest rate. T_t represents a lump-tax (or transfers from the government). Each household earns income from supplying labor at a nominal wage rate W_t, the dividend $P_t V_t$, and the principal and interest $R_t D_t$. The household spends her after-tax income on consumption $P_t C_t$ and deposits D_{t+1}.

3.2 Firms

Production takes place in two stages. There are monopolistically competitive intermediate goods producing firms and perfectly competitive final goods producing firms that aggregate the intermediate goods into a homogenous final good Y_t. Following Heutel (2012) and Annicchiarico and Di Dio (2015), the intermediate goods producing firms emit pollutants during the production process. The stock of pollutants in turn negatively affects the firms' productivity. In particular, the production function is:

$$Y_t(i) = (1 - \Upsilon(M_t)) A_t K_t^{\alpha}(i) L_t^{1-\alpha}(i), \tag{5}$$

where $K_t(i)$ and $L_t(i)$ are the capital and labor employed by firm i, respectively. $0 < \alpha < 1$ denotes the share of capital. $\Upsilon(.)$ is a damage function, that accounts for the percentage reduction in production due to pollution. $\Upsilon(.)$ is assumed to positively depend on the aggregate emissions stock M_t of the economy. A_t represents the TFP of the firms and evolves according to an AR(1) process:

$$\ln(A_t/A) = \rho_A \ln(A_{t-1}/A) + \varepsilon_{A,t},$$

where $0 \leq \rho_A \leq 1$ is the persistence of the technology shock. $\varepsilon_{A,t}$ is normally distributed with mean 0 and standard deviation $\sigma_A > 0$. A denotes the steady-state value of A_t.

Pollutant $Z_t(i)$ is emitted during the production process. Intermediate firm i can reduce its pollution by exerting an abatement effort $U_t(i) \in [0,1]$. The abatement cost for firm i is $C_{A,t}(i)$. The emissions $Z_t(i)$, output $Y_t(i)$, abatement effort $U_t(i)$, and abatement cost $C_{A,t}(i)$ are related by the following two equations:

$$Z_t(i) = (1 - U_t(i))\varphi Y_t(i), \tag{6}$$

$$C_{A,t}(i) = \phi_1 U_t(i)^{\phi_2} Y_t(i), \tag{7}$$

where $\varphi > 0$ controls the marginal increase in CO_2 emissions given an additional increase in output when the abatement effort is 0. $\phi_1 > 0$ is a scale parameter, and $\phi_2 > 1$ determines the elasticity of the abatement cost with respect to the abatement effort. With $\phi_2 > 1$, the abatement cost is convex, so the marginal abatement is increasing in the abatement effort. This incentivizes the firm to divide its abatement effort across several periods.

Since each intermediate firm is infinitesimal, an increase in abatement effort by a particular firm has no impact on reducing the emissions stock M_t. Therefore, if there is no price for emissions, at the optimum, firms would not devote any effort to abatement, i.e., $U_t(i) = 0$ for any i. The government assesses a carbon tax $P_{Z,t} > 0$ on each unit of emissions. Under these assumptions, the optimal abatement effort satisfies the following equation:

$$\varphi \frac{P_{Z,t}}{P_t} = \phi_1 \phi_2 U_t(i)^{\phi_2 - 1}. \tag{8}$$

This equation states that the marginal cost of abatement effort (l.h.s) equals the marginal benefits of abatement (r.h.s).

In addition to the abatement decision, the intermediate firm i also chooses capital and labor to minimize its production cost, taking the real wage rate w_t and the rental cost of capital $r_{K,t}$ as given. The optimality conditions for the demand for labor and capital are as follows:

$$(1 - \alpha)\frac{Y_t(i)}{K_t(i)}\Psi_t = w_t, \tag{9}$$

$$\alpha \frac{Y_t(i)}{K_t(i)} \Psi_t = r_{K,t}, \tag{10}$$

where $\Psi_t > 0$ is the part of the firm's marginal cost and is related to capital and labor. Ψ_t can be expressed in terms of the factor prices w_t and $r_{K,t}$:

$$\Psi_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} A_t (1-\Upsilon(M_t))} w_t^{1-\alpha} r_{K,t}^{\alpha}. \tag{11}$$

From Eq.(11), the real marginal cost is increasing in both w_t and $r_{K,t}$. Moreover, a higher emissions stock M_t results in greater damage to productivity and hence would raise the marginal costs of firms.

For monetary policy to affect real economic activity in the short run, we introduce nominal rigidity in the firm sector. Staggered price setting is modeled à la Calvo (1983). Specifically, in each period, only a $(1-\xi) \in [0,1]$ portion of firms can adjust their prices. Firm i, which can reoptimize its price in period t, will choose $P_t^*(i)$ to maximize its discounted lifetime expected profit function:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k Q_{t,t+k} \left[P_t^*(i) Y_{t+k}(i) - MC_t Y_{t+k}(i) \right], \tag{12}$$

subject to the demand constraint $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$.[11] $Q_{t,t+k} = \beta^k (\lambda_{t+k}/\lambda_t)$ is the stochastic discount factor between time t and time $t+k$. $\xi^k > 0$ is the probability that firm i cannot adjust its good price from time t to time $t+k$. MC_t denotes the real marginal cost. For brevity's sake, the optimality condition for this problem is detailed in online appendix I.

3.3 Entrepreneur

As in BGG, a continuum of entrepreneurs, indexed by $j \in [0,1]$, borrows from a financial intermediary and decides the amount of capital to invest in each period. Entrepreneurs are risk neutral. In contrast to households, entrepreneurs only live over a finite horizon. Every period, a fraction γ of entrepreneurs will survive into the next period, such that their expected lifetime is $1/(1-\gamma)$. The finite lifetime assumption implies that entrepreneurs are unable to accumulate adequate wealth to fully self-finance, and thus need to borrow from the financial intermediary.

Suppose that entrepreneur j purchases capital $K_{t+1}(j)$ at the end of period t at price q_t. This capital acquisition is thereby financed by the entrepreneur's own net worth $N_{t+1}(j)$ and by borrowing $B_{t+1}(j)$ from a financial intermediary, which obtains its funds from household deposits at interest rate R_t. As a matter of fact, in equilibrium, the household deposits D_t at intermediaries equal the aggregate loanable

[11] This demand constraint comes from solving the final good producer's profit maximization problem.

funds B_t supplied to entrepreneurs. The budget constraint of entrepreneur j is :

$$K_{t+1}(j)q_t = N_{t+1}(j) + B_{t+1}(j). \tag{13}$$

After purchasing capital, each entrepreneur j experiences an idiosyncratic shock ω^j, which converts capital K_{t+1} into efficiency capital $\omega^j K_{t+1}$. The random variable ω^j is independent and identically distributed (i.i.d.) across time and entrepreneurs. In addition, ω^j follows a log-normal distribution with $\mathbb{E}[\omega^j] = 1$ and a standard deviation of $\ln(\omega^j)$ equals σ_t, i.e., $\omega \sim \ln N(-\sigma_t^2/2, \sigma_t)$. The random variable σ_t, which determines the dispersion of capital return for individual entrepreneur j, has been introduced in Christiano et al. (2014) as the risk shock. The risk shock σ_t follows an AR(1) process:

$$\ln(\sigma_t/\sigma) = \rho_\sigma \ln(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t},$$

where $\rho_\sigma > 0$ denotes the persistence of the risk shock. σ is the steady-state level of the risk shock and the term $\varepsilon_{\sigma,t}$ is an i.i.d innovation drawn from a normal distribution $N(0, \sigma_\sigma)$.

In the beginning of the next period $t+1$, entrepreneurs rent the efficiency capital to firms at the rate of $r_{K,t+1}$. After the production, entrepreneurs collect the remaining undepreciated capital $\omega^j K_{t+1}(j)(1-\delta_K)$ from firms and then sell them back to the capital producer at the price of q_{t+1}. Therefore, the gross return to capital averaged across entrepreneurs can be defined as:

$$R_{K,t+1} \equiv \frac{r_{K,t+1} + (1-\delta_K)q_{t+1}}{q_t}, \tag{14}$$

where δ_K is the depreciation rate. In this way, regardless of net worth, each entrepreneur j enjoys rate of return $\omega^j R_{K,t+1}$ in period $t+1$ on her capital purchase.

On the other hand, entrepreneurs and the financial intermediary adopt a standard debt contract (X_{t+1}, B_{t+1}) in period t. Here, $X_{t+1}(j)$ denotes the non-default loan rate. The optimal contract is characterized by X_{t+1} and $\bar{\omega}^j_{t+1}$, where $\bar{\omega}^j_{t+1}$ is a threshold value of the idiosyncratic shock ω^j_{t+1}. That is, given $q_t K_{t+1}(j)$, $B_{t+1}(j)$, and $R_{K,t+1}$, $\bar{\omega}^j_{t+1}$ separates the entrepreneurs who are able to repay the loan at the contractual rate X_{t+1} from those who have to declare bankruptcy.

If an entrepreneur receives a large idiosyncratic shock $\omega^j \geq \bar{\omega}^j$, after repaying the promised principal and interest $X_{t+1}(j)B_{t+1}(j)$, it earns profits, equal to $\omega^j_{t+1} R_{K,t+1} q_t K_{t+1}(j) - X_{t+1}(j)B_{t+1}(j)$. However, if a low idiosyncratic shock $\omega^j \leq \bar{\omega}^j$ is realized, it is unable to repay the debt and declares bankruptcy. In this case, the financial intermediary takes over the entrepreneur's property and pays a monitoring cost. Specifically, the financial intermediary obtains $(1-\mu)\omega^j_{t+1} R_{K,t+1} q_t K_{t+1}(j)$, where $\mu \in [0,1]$ is the monitoring cost rate. Hence, the threshold value $\bar{\omega}^j_{t+1}$ should satisfy the zero-profit condition:

$$\bar{\omega}^j_{t+1} R_{K,t+1} q_t K_{t+1}(j) = X_{t+1}(j) B_{t+1}(j). \tag{15}$$

In equilibrium, the expected return from lending funds to the entrepreneur j should equal the opportunity cost of obtaining the loan. In particular, the loan contract satisfies the following condition:

$$\int_{\bar{\omega}^j_{t+1}}^{\infty} \bar{\omega}^j_{t+1} R_{K,t+1} q_t K_{t+1}(j) dF_t(\omega) + \int_0^{\bar{\omega}^j_{t+1}} (1-\mu)\omega^j_{t+1} R_{K,t+1} q_t K_{t+1}(j) dF_t(\omega) = R_t B_{t+1}(j), \tag{16}$$

where the first and second terms on the left-hand side represent the expected gross return when entrepreneur j survives and goes bankruptcy, respectively.

Therefore, entrepreneur j decides how much capital $K_{t+1}(j)$ to buy from the capital producer and the value of the threshold $\bar{\omega}^j_{t+1}$ in order to maximize its profit function in the following:

$$\max_{\bar{\omega}^j_{t+1}, K_{t+1}(j)} \int_{\bar{\omega}^j_{t+1}}^{\infty} \omega^j_{t+1} R_{K,t+1} q_t K_{t+1}(j) dF_t(\omega) - X_{t+1}(j) B_{t+1}(j), \tag{17}$$

which states that entrepreneurs j's profit equals the expected return when the realized idiosyncratic shock $\omega^j \geq \bar{\omega}^j_{t+1}$, less the borrowing cost. To maximize this problem subject to Eq. (16) and Eq. (13) yields the optimal capital purchase K_{t+1} and the optimal threshold $\bar{\omega}_{t+1}$.

As mentioned earlier in this section, in each period, a fraction γ of entrepreneurs survive into the next period and the rest leave the market. Those who leave the market obtain zero net worth. The evolution of the aggregate entrepreneurial net worth N_{t+1} satisfies:

$$N_{t+1} = \gamma(1-\Gamma_t) R_{K,t} q_{t-1} K_t. \tag{18}$$

where Γ_t represents the share of the average expected return $R_{K,t} q_{t-1} K_t$ that the financial intermediary obtains from bankrupt entrepreneurs. Naturally, $(1-\Gamma_t) R_{K,t} q_{t-1} K_t$ denotes the average expected return received by entrepreneurs.

3.4 Capital producers

The capital production market is assumed to be perfectly competitive. Capital producers use existing capital and investment goods, I_t to produce new capital. Therefore, their profit maximization problem is to decide how much to invest in each period subject to the capital stock evolution equation:

$$\begin{aligned} &\max_{I_t} q_t K_{t+1} - I_t - \frac{\gamma_I}{2}(\frac{I_t}{K_t} - \delta_K)^2 K_t, \\ &s.t. \quad K_{t+1} = (1-\delta_K)K_t + I_t, \end{aligned} \tag{19}$$

where $\gamma_I \geq 0$ controls the scale of the capital adjustment cost. The profit of the capital producer equals the revenue from selling capital to entrepreneurs minus the cost, which comprises the purchasing cost I_t and the adjustment cost $\gamma_I(\frac{I_t}{K_t} - \delta_K)^2 K_t/2$. Since the investment good is expressed in terms of the consumption good, the price of the investment good equals one. Solving this maximization problem yields the optimal investment condition:

$$q_t - 1 - \gamma_I \left(\frac{I_t}{K_t} - \delta_K \right) = 0. \tag{20}$$

Note that when the adjustment cost is absent, i.e., when $\gamma_I = 0$, q_t becomes 1. Hence, the price of capital equals the price of the investment good. In this situation, the average gross capital return would be equal to the marginal product of capital after depreciation (see Eq. (14) above). With the capital adjustment cost, the time-varying capital price could amplify the volatility of the capital return and thus the net worth of the entrepreneurs.

3.5 Monetary policy rule and government budget constraint

We assume that the central bank follows a standard Taylor rule that adjusts the nominal interest rate R_t in response to deviations in both inflation Π_t and output Y_t from their steady-state values. Then, the monetary policy rule is expressed as:

$$R_t = R \left(\frac{\Pi_t}{\Pi} \right)^{\iota_\pi} \left(\frac{Y_t}{Y} \right)^{\iota_Y} \eta_t, \tag{21}$$

where ι_π and $\iota_Y > 0$ control the elasticity of the nominal interest rate with respect to gross inflation and output, respectively. R, Y, and Π denote their steady-state values. This specification of the Taylor rule is different from the policy rule in Annicchiarico and Di Dio (2015), who assume that the nominal interest rate reacts only to changes in inflation. η_t is a shock to monetary policy, which characterizes the short-term deviation of the policy from the rule. Assume that the logarithm of η_t follows an AR(1) process:

$$\ln \eta_t = \rho_\eta \ln \eta_{t-1} + \varepsilon_\eta, \tag{22}$$

where $\rho_\eta \in [0,1]$ is the persistence of the monetary policy shock and ε_u is normally distributed with mean zero and standard deviation $\sigma_\eta > 0$.

For the public sector, we assume that the government balances its budget every period:

$$T_t + P_{Z,t} Z_t = P_t G_t. \tag{23}$$

Here, the total income of the government, namely, the lump-sum tax T_t collected from households and the emission tax revenue $P_{Z,t} Z_t$, equals its expenditure $P_t G_t$. For simplicity, assume that the logarithm of G_t is exogenous and follows an AR(1) process:

$$\ln(G_t/G) = \rho_G \ln(G_{t-1}/G) + \varepsilon_G, \tag{24}$$

where $\rho_G \in [0,1]$ is the persistence of the government policy shock and ε_G is normally distributed with mean zero and standard deviation $\sigma_G > 0$.

3.6 Aggregation and equilibrium

Most of the market equilibrium conditions are obvious. Here, we focus on those about carbon emissions. For the abatement effort, since it only depends on the carbon tax rate and is thus identical across all firms, that is, $U_t(i) = U_t$. By Eq. (6), the aggregate emissions are as follows:

$$Z_t = \int_0^1 Z_t(i)di = (1 - U_t)\phi \int_0^1 Y_t(i)di = (1 - U_t)\phi Y_t D_{p,t}. \tag{25}$$

where $D_{p,t}$ denotes price dispersion. Similarly, by aggregating the abatement cost in Eq. (7) yields:

$$C_{A,t} = \int_0^1 C_{A,t}(i)di = \phi_1 U_t^{\phi_2} Y_t D_{p,t}. \tag{26}$$

Finally, as in Annicchiarico and Di Dio (2015), the aggregate emissions stock M_t is assumed to evolve according to the following equation:

$$M_t = (1 - \delta_M) M_{t-1} + Z_t + Z_t^*, \tag{27}$$

where δ_M denotes the decay rate of emissions stock. Z_t^* is the rest of the world's emissions which contribute to the home country's emissions stock accumulation.

The model is closed by the model economy's resource constraint:

$$Y_t = C_t + I_t + G_t + C_{A,t} + \frac{\gamma_I}{2}\left(\frac{I_t}{K_t} - \delta_K\right)^2 K_t + d_t. \tag{28}$$

Beyond consumption C_t, investment I_t, and government expenditure G_t, the aggregate goods demand includes the resources spent due to market frictions, which include average abatement costs $C_{A,t}$, adjustment costs $\gamma_I (I_t/K_t - \delta_K)^2 K_t/2$, and monitoring costs $d_t = \mu R_{K,t} q_{t-1} K_t \int_0^{\bar{\omega}_{t+1}} \omega f_t(\omega) d\omega / P_t$.

4 Estimation

4.1 Data

The model is estimated using 138 quarterly observations of real GDP per capita, real consumption per capita, real investment per capita, the Fed funds rate, and the delinquency rate on commercial and industrial loans in the US. We use the same data series as in the empirical analysis section and add series on real consumption per capita and real investment per capita. These two additional data series are obtained from the FRED database available at the Federal Reserve Bank of St. Louis website. Further details are in data appendix A. The first four data series are commonly used in the structural estimation of NK DSGE models, while the delinquency rate measures the bankruptcy rate $F(\bar{\omega}_{t+1})$ in the model with a financial accelerator. Again, due to the availability constraint with the delinquency rate series, the maximum time length of our dataset is from 1985Q1 to 2019Q2.

One difficulty in the estimation is that the number of time series we used (5 series) is greater than the number of shocks (4 shocks) in the model. This leads to stochastic singularity.[12] To avoid this problem, we introduce measurement errors in each of the observable variables.[13] Specifically, let Y_t^{obs}, C_t^{obs}, I_t^{obs}, R_t^{obs}, and $\mathscr{F}_t^{obs}$ be the data on GDP, consumption, investment, the nominal interest rate, and the delinquency rate, respectively. Denote by Y_t^{err}, R_t^{err}, and $\mathscr{F}_t^{err}$ the measurement errors at time t. The observable variables and measurement errors are related by the following equations:

$$\begin{bmatrix} \ln Y_t^{obs} - \ln Y_{t-1}^{obs} \\ \ln C_t^{obs} - \ln C_{t-1}^{obs} \\ \ln I_t^{obs} - \ln I_{t-1}^{obs} \end{bmatrix} = \begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln I_t - \ln I_{t-1} \end{bmatrix} + \ln Y_t^{err}, \tag{29}$$

[12] Stochastic singularity problem is that there may exist a deterministic linear combination of observed variables if the number of shocks is less than the number of observed variables (Ruge-Murcia, 2007).

[13] In general, to avoid stochastic singularity, the number of shocks should be at least equal to the number of observable variables. This can either take the form of introducing measurement errors to each of the observable variables or introducing enough shocks to the model.

$$\ln R_t^{obs} = \ln R_t + \ln R_t^{err}, \tag{30}$$

$$\ln \mathscr{F}_t^{obs} = \ln \mathscr{F}_t + \ln \mathscr{F}_t^{err}, \tag{31}$$

Following Smets and Wouters (2007), the observed GDP, consumption, and investment share a common measurement error Y_t^{err}, which is interpreted as a common quarterly trend growth rate for the three variables. Moreover, assume that each measurement error follows an AR(1) process as follows:

$$\ln Y_t^{err} - \ln Y^{tr} = \varepsilon_{Y,t}^{err}, \tag{32}$$

$$\ln R_t^{err} - \ln R^{tr} = \varepsilon_{R,t}^{err}, \tag{33}$$

$$\ln \mathscr{F}_t^{err} - \ln \mathscr{F}^{tr} = \varepsilon_{\mathscr{F},t}^{err}, \tag{34}$$

where $\varepsilon_{Y,t}^{err}$, $\varepsilon_{R,t}^{err}$, and $\varepsilon_{\mathscr{F},t}^{err}$ are i.i.d. and follow normal distributions with mean 0 and standard deviation σ_Y^{err}, σ_R^{err}, and $\sigma_{\mathscr{F}}^{err}$, respectively. For simplicity, we set $\sigma_Y^{err} = \sigma_R^{err} = \sigma_{\mathscr{F}}^{err} = 0.001$ in the estimation. The value Y^{tr} is the long-run gross growth rate of output. R^{tr} and $\mathscr{F}^{tr}$ capture the means of the observed interest rate and bankruptcy rate, respectively. Note that from Eq. (32) to (34), the measurement errors are constructed such that they randomly fluctuate around the averages of their corresponding observable variables.

4.2 Calibrated parameters

We partition the model parameters into two sets. The first set contains parameters that have been well established elsewhere in the literature. We simply calibrate these parameters. Following common practice in the macroeconomic literature, the Frisch elasticity ϕ is set at 1. The household discount factor β is set at 0.99, implying an annual return on the riskless bond of approximately 4% ($(1/0.99-1)\times 4 \approx 4\%$). On the production side, we set the share of capital α at $1/3$. The capital depreciation rate $\delta_K = 0.025$, equivalent to a 10% annual depreciation rate ($1-(1.025)^4 \approx 0.1$). Following Christensen and Dib (2008), we fix the capital adjustment cost γ_I at 0.5882. The elasticity of substitution between any two intermediate goods θ equals 6, which is common in the literature.

Regarding monetary policy, we set the parameters ι_π and ι_Y in the Taylor rule at 3 and $1/4$, respectively, which are standard in the literature. We simply set the carbon tax rate p_Z to be 0 as a benchmark, while it is set at different values in the numerical analysis thereafter. In the data, from 1985Q1 to 2019Q2, the real government expenditure share in real GDP is 22.2%. We thus set the steady-state government expenditure to be 22.2% of the steady-state output, i.e., $G/Y = 0.222$. The steady-state values of the technology and monetary policy shocks are simply assumed to be 1. Further, we follow Christiano et al. (2014) and set the steady-state value of the risk shock σ at 0.2588.

The parameters related to carbon emissions and abatement effort are taken from Annicchiarico and Di Dio (2015). The damage function is assumed to be quadratic. Specifically, we have $\Gamma(M) = \gamma_0 + \gamma_1 M + \gamma_2 M^2$, where γ_0, γ_1, and γ_2 are set at $1.395e^{-3}$, $-6.6722e^{-6}$, and $1.4647e^{-8}$, respectively. The global emissions Z^* are set at 1.3299. The parameter ϕ_2 in the abatement cost function is fixed at 2.8 used by Nordhaus (2008). We follow the procedure in Annicchiarico and Di Dio (2015) to pin down the parameter ϕ_1. The average carbon emissions per unit of output in the US from 1985 to 2019, expressed in kilos per PPP dollars of GDP, is 0.53. Starting from this average carbon intensity, i.e., $Z/Y = 0.53$ at $p_z = 0$, we search for the carbon tax rate, hence the steady-state value of abatement effort U, to meet a 20% reduction in carbon intensity. Then, such an abatement effort is substituted into the abatement cost function Eq. (7). ϕ_1 is calculated from Eq. (7) by imposing the constraint that the abatement cost to output ratio equals 0.15%. This yields $\phi_1 = 0.1761$. It is fairly close to the 0.1850 value used by Annicchiarico and Di Dio (2015). The calibrated parameter values are reported in Table 5.

4.3 The Choice of Priors

The second set of the 14 parameters to be estimated are listed in Table 4. We first discuss the choice of the prior means of these parameters. In Calvo pricing, it is usually assumed that only one-fourth of the intermediate firms can adjust their price every period, i.e., the prior mean of ν is $3/4$. Following Christiano et al. (2014), the prior mean of the monitoring cost rate μ is set to 0.21, which implies that the monitoring cost is 21% of the average return of the entrepreneurs. We set the prior mean of the survival rate of entrepreneurs γ at 0.97, such that the failure rate of entrepreneurs is approximately 3% every quarter. The prior mean of the decay rate of the emissions stock δ_M is set at 0.0021, and that of the marginal emission of production φ is 0.53, which is just the emissions (in kilograms) per PPP dollars of GDP in the US mentioned above. The prior mean of the scale of the labor supply disutility μ_L is simply set at 1.

For prior means of the shock processes, we follow Smets and Wouters (2007) to assume that the prior means of the shock persistence ρ_A, ρ_G, ρ_η, and ρ_σ all equal 0.5. For the standard deviation, we assume the prior mean of σ_A, σ_G, σ_σ, and σ_η to be 0.005. The prior mean of σ_σ is set at 0.2588, as discussed by Christiano et al. (2014). The standard deviation of the prior distributions are carefully chosen according to the prior mean. The choice of the prior distributional form closely follows those in the literature, such as Smets and Wouters (2007) and Christiano et al. (2014).

4.4 Estimation results

Table 4 reports the modes and standard deviations of the prior and posterior distributions. Following Christiano et al. (2014) and conventions in the DSGE literature, we use the posterior modes as the estimates of the parameter values. A summary of the parameter values is provided in Table 5. After obtaining the parameters, the deterministic steady state is computed, and the model is solved by first-order perturbation around the deterministic steady state.

All shocks exhibit a high degree of persistence. We find that $\rho_\sigma = 0.99$, $\rho_A = 0.99$, $\rho_G = 0.918$, and $\rho_\eta = 0.646$. The monetary policy shock has a standard deviation of 1.23%, which is larger than those of the other three shocks. The risk shock has a standard deviation of 0.386%. The standard deviations of the TFP shock and government expenditure shock are 0.602% and 0.751%, respectively. The estimated long-run common quarterly gross growth rate to real output Y^{tr} (as well as to consumption and investment) is 1.000, which is very close to the average value, 1.0037, computed from the data.

Concerning the parameters related to emissions, the estimate of the pollution decay rate δ_M is 0.003, which is very close to the value 0.0021 used in Annicchiarico and Di Dio (2015). This implies that approximately 0.3% of the CO_2 emissions stock decays naturally every quarter. The coefficient ϕ, measuring marginal emissions of production, is estimated to be 0.348, which is slightly less than the value of 0.45 calibrated in Annicchiarico and Di Dio (2015).

Finally, for other structural parameters, the monitoring cost rate μ is estimated to be 0.185, which is similar to the value of 0.21 estimated in Christiano et al. (2014). This means that when the financial intermediary takes over the assets of bankrupt entrepreneurs, 18.5% of their value is spent to cover the monitoring cost. The firms' survival probability γ is estimated to be 0.954, which indicates that there are about 4.6% $(1 - 0.954)$ of firms will leave the market every quarter. Our estimate of the price stickiness parameter ν is 0.926, which is slightly larger than the estimate of 0.81 in Christiano et al. (2014). Both of these estimates are larger than the conventional value of 0.75 used in the literature. The disutility weight on labor μ_L is estimated to be 0.993, which is similar to the common values employed in the literature.

5 Impulse response analysis

5.1 The impact of risk shock on emissions

Fig. 6 depicts the IRFs of the main macroeconomic variables to a one-standard-deviation increase in the risk shock. Specifically, we consider a one-standard-deviation increase in innovation $\epsilon_{\sigma,t}$. Risk shock affects the cyclical movement of carbon emissions through its impact on the output. Specifically, the positive risk shock generates a more dispersed distribution of idiosyncratic returns for the entrepreneurs. That is, the occurrence of either a higher or a lower return in the two extremes becomes more likely. This situation increases the probability that entrepreneurs will receive a low return rate. Although the default threshold decreases, the bankruptcy rate nevertheless increases. In response to a higher bankruptcy rate, entrepreneurs reduce their capital demand, i.e., to save more and to borrow less. On one hand, less loanable funds demand indicates a decrease in the nominal interest rate. The decreased nominal interest rate in turn discourages household savings.

On the other hand, a lower capital demand implies a lower equilibrium capital level. This drives down capital prices and thereby discourages investment. Furthermore, the substantial decrease in capital translates into a decline in output. Carbon emissions decrease accordingly. As a result, the emissions stock accumulates more slowly. Since the abatement effort is assumed to depend monotonically on the carbon tax rate according to Eq. (8), the constant carbon tax rate implies a constant abatement effort.

The wage rate, which is proportional to the marginal product of labor, is positively associated with output. The decline in output would result in decreases in the wage rate and household income. A lower wage rate is followed by a lower labor supply and further discourage household savings. Moreover, due to intertemporal substitution, the representative household shifts more share of its income to consumption under a lower nominal interest rate. As a result, consumption initially increases.

In sum, a positive risk shock, that is, an increase in the dispersion of the entrepreneurs' potential return, would lead to a decrease in carbon emissions. This decrease is attributed to the fact that entrepreneurs are not willing to invest a large amount of capital when facing a surge in return uncertainty. Consequently, the lower capital level leads to lower output and carbon emissions. Therefore, this section can provide an explanation for the observed phenomenon in Fig. 1 that carbon emissions move in the opposite direction as the financial condition index during the recessions. In other words, if the recessions are related to or even driven by a positive risk shock, carbon emissions would experience a substantial decline.

5.2 The impact of the financial accelerator on emissions

This section examines how the dynamics of carbon emissions differ in the presence of a financial accelerator. To this end, we compute the IRFs of carbon emissions and other macroeconomic variables of interest under three types of shocks, namely, TFP, government expenditure, and monetary policy shocks. We compare these IRFs with and without a financial accelerator in Figs. 7 to 9. The solid lines represent the IRFs with a financial accelerator, while the dashed lines are the IRFs without a financial accelerator. All of the results are reported as percentage deviations from the steady state spanning 40-quarter periods.

Fig. 7 displays the impulse responses to one standard deviation of a positive TFP shock. As shown, with a financial accelerator, the increase in emissions Z is substantially greater than that in the case without a financial accelerator. The reason is as follows. In both scenarios, a positive TFP shock implies an increase in the marginal product of labor and hence an increased wage rate. This leads to higher output and household income. In the scenario without a financial accelerator, consumption increases. The capital price is driven up by a higher capital demand from the intermediate firms because of the higher marginal product of capital.

However, with a financial accelerator, fluctuations in the capital return $R_{k,t}$ create an additional channel of influence. Specifically, an increase in the marginal product of capital raises the capital return $R_{k,t}$, which further drives up the capital price. As a result, entrepreneurs increase their capital supply. Therefore, investment and output increase more in the presence of a financial accelerator. On the demand side, the increased capital return also brings about an increase in capital demand, and entrepreneurs borrow more from the financial intermediary, which leads to an increased interest rate. Moreover, with a generalized Taylor rule whereby the interest rate also responds to output gap, the interest rate initially increases in response to the expansion in output. However, the interest rate subsequently decreases once it adjusts to the decline in inflation. Since the financial accelerator amplifies the responses of both output and inflation, the response of the interest rate is also amplified. Due to intertemporal substitution, consumption initially increases less than in the case without a financial accelerator, but it increases more as nominal interest rates are further reduced. Consider now the dynamics of carbon emissions. With a financial accelerator, the significant increase in output leads to a corresponding substantial increase in emissions. Specifically, the increase in emissions is more than doubled in the case without a financial accelerator. The emissions stock thus accumulates faster than in the case without financial accelerator.

Under a positive government expenditure shock, emissions also increase twice as large as in the case without a financial accelerator. Fig. 8 displays the IRFs of macroeconomic and envrionment-related variables to a one-standard-deviation government expenditure shock. In both cases, a higher government expenditure raises the goods demand. Consequently, output increases and consumption is crowded out. Since carbon emissions are positively correlated with output, it is obvious that the emissions jump up. Note that with a financial accelerator, the initial jump in emissions is higher, owing to the larger response in output explained above.

Fig. 9 displays the IRFs of the main variables in the model to a one-standard-deviation increase in the monetary policy shock. The IRFs with and without a financial accelerator are much more similar, compared to the results of the TFP and government expenditure shocks. This result is mainly due to the relatively low persistence of the monetary policy shock. In general, a higher nominal interest rate encourages households to defer their consumption. Output and investment fall sharply on impact as expected. As a result, carbon emissions decline.

Overall, the impact of financial accelerator mechanism via the production of investment goods on macroeconomic variables and carbon emissions, operates to amplify the volatility of emissions more than doubled under positive technology and positive government expenditure shocks. However, this mechanism has a limited influence on emissions under a contractionary monetary policy shock due to the low persistence of the monetary policy shock. Under all the three shocks, including financial accelerator mechanism would suggest a higher emissions level than otherwise without the financial accelerator.

5.3 Shock decomposition

Having studied the short-run impact of each shock on the dynamics of the emission-related variables, we wish to investigate the importance of the risk shock in fitting the emissions throughout the business cycles in our data. The risk shock affects the economy through the mechanism of the financial accelerator, and hence the explanatory power of the risk shock in turn reveals how necessary the financial accelerator mechanism is in explaining the significant decline in emissions during recessions. Following the approach in Christiano et al. (2014), we perform a shock decomposition of our data series for each of the four shocks. In Fig. 10, the solid lines plot the actual data series on output growth (in the left panel) and the actual data series on the carbon emissions growth (in the right panel). The dotted lines plot the corresponding counterfactual series when only one of the shocks is presented. Taking the right column as an example, from the top row to the bottom row, each dotted line represents a counterfactual series of the carbon emissions when only the risk, TFP, monetary policy, and government expenditure shocks are, in turn, fed into the model.

By comparing the counterfactual and actual data series, it is clear that the risk shock generates counterfactual paths that perform the best in characterizing the movements in output and emissions growth. Especially, the decline in emissions during the 2001 and 2008 recessions can reasonably be explained by the risk shock alone. In contrast, the counterfactual series generated by the TFP and monetary shocks are too volatile compared to the actual data. And the counterfactual series generated by the government shock is not volatile enough to explain the data. Moreover, it moves in the opposite direction to the data. For example, the counterfactual output and emissions series rise slightly during the recent 2008 financial crisis, while the two data series decrease during the period. Shock decompositions of the rest of the data series are depicted in Figs. C4 to C5. They all show that all of the data series (consumption growth, the bankruptcy rate, and investment growth) can better be explained by the risk shock than by the other three macroeconomic shocks, individually.

Fig 11 displays the contribution share of the risk shock in all four shocks to the emissions growth rate dynamics through time. Specifically, the historical values of the emissions growth rate are decomposed into the accumulated effects of current and past 4 shocks, i.e., the risk, monetary policy, TFP, and government expenditure shocks. The contribution share of the risk shock is calculated by the contributions of the risk shock over the contributions of all four shocks at each time point during the sample period. The bars indicate this contribution share of the risk shock. The grey shading identifies NBER recessions. For most of the time, 80 out of 138 sample data, the contribution share of the risk shock is smaller than 10%. However, the contribution from the risk shock gains in importance after each recession, which can account for around 50% and indicate the accumulated effects of the risk shock. Among the four shocks considered, the risk shock contributes the largest share. This is particularly true after the 2008 recession, the contribution share of the risk shock reaches 55.31%.

Overall, our results suggest that the risk shock generates counterfactual paths that perform the best in characterizing the movements in output, consumption growth, investment growth, the bankruptcy rate, and emissions growth. More important, among the four shocks considered in this paper, the risk shock alone contributes a large share of the fluctuations in the emissions growth rate, around 50%, in 8 quarters after each recession.

6 Carbon taxation

The previous sections have shown that the dramatic decline in carbon emissions during recessions is substantially driven by the risk shock operating through the financial accelerator. In addition, the presence of a financial accelerator mechanism implies a significant amplification and propagation of the TFP and government expenditure shocks. This section examines the impacts of a carbon tax on the dynamics of emissions in the presence of a financial accelerator. To this end, we conduct a welfare analysis to derive the welfare implications of different carbon tax regimes. Moreover, to gain insights into how a carbon tax should respond to business cycles, we consider a Ramsey problem to determine the optimal carbon tax on emissions.

6.1 Welfare comparison

This section studies the welfare implications of different carbon taxation regimes in the presence of a financial accelerator. The most direct approach for a welfare comparison is to calculate the magnitude of lifetime utility gains when the carbon tax rate switches from one value to another. However, measuring the gains in terms of utility levels is difficult to interpret. To compare the welfare differences implied by different carbon tax rates, we employ a measure called compensating variation to express the welfare difference in terms of consumption. Compensating variation measures what percentage of consumption must be given up (compensated) such that households' value function under different carbon tax regimes achieves the same value. For completeness, the definition of compensating variation and its derivation is described below.[14]

Models with and without carbon taxation are considered. We compare the model when the carbon tax rate is 5% (model 1) and the model without a carbon tax (model 2). As shown in Table 6, a 5% carbon tax leads to a 6.4% decline in steady-state carbon emissions stock.

Given the two models $i \in \{1, 2\}$, denote the expected present discounted value of utility in model i in

[14] The appendix in Lester et al. (2014) provides more details of derivations of compensating variation.

a particular state at time t as $V_{i,t}$:

$$V_{i,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \left(\ln C_{i,t+j} - \mu_L \frac{L_{i,t+j}^{1+\phi}}{1+\phi} \right), \tag{35}$$

where $C_{i,t}$ and $L_{i,t}$ are the equilibrium consumption and labor in model i, respectively.

The conditional compensating variation λ_C for the two models is defined implicitly in the following equation:

$$V_{1,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \ln\left[(1+\lambda_C) C_{2,t+j}\right] - \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \mu_L \frac{L_{2,t+j}^{1+\phi}}{1+\phi}. \tag{36}$$

Using the definition of $V_{i,t}$ and simplifying yields:

$$V_{1,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \ln(1+\lambda_C) + V_{2,t}. \tag{37}$$

Solving the equation above provides the explicit expression for λ_C:

$$\lambda_C = \exp\left[(1-\beta)(V_{1,t} - V_{2,t})\right] - 1. \tag{38}$$

Similarly, the unconditional compensating variation λ_U is:

$$\lambda_U = \exp\left[(1-\beta)(\mathbb{E}(V_{1,t}) - \mathbb{E}(V_{2,t}))\right] - 1, \tag{39}$$

where the expectation is operated on the (unconditional) distribution of the state variables, measuring the welfare difference in the long run. Based on these specifications, a positive value of λ_C indicates that the model with taxation (model 1) yields higher welfare.

Fig. 12 displays the IRFs of conditional compensating variation between models 1 and 2 under the four shocks. As shown, a higher carbon tax leads to an initial increase in welfare under the contractionary monetary policy, positive TFP, and government expenditure shocks, but leads to a decrease in welfare under the positive risk shock.

Note that the IRFs of compensating variation are simultaneously driven by the responses of consumption and labor in the presence of shocks, which can be seen from the construction of compensating variation. A lower carbon tax rate generates a higher consumption level, which is thus crowded out more by the same government expenditure shock. Consequently, a higher carbon tax increases the conditional compensating variation under the positive government expenditure shock. Similarly, given the same TFP shock, the goods producers who face a low carbon tax rate earn more profits and thus expand their production scale to a greater extent. Both labor and consumption increase more under a lower carbon tax rate. The initial increases in the conditional compensating variation reveal that the former effect dominates under the TFP shock, while the latter effect takes charge as time passes. Thus, compensating variation turns to negative in response to the TFP shock after a few periods.

Finally, the IRF of conditional compensating variation to a risk shock initially decreases and then increases gradually. As observed in Fig. 6, consumption initially increases and then declines to negative values in response to a positive risk shock. A higher carbon tax rate dampens the response of consumption such that the representative household is worse off by having a lower initial increase in consumption but better off since consumption decreases to a lesser extent. In sum, the welfare comparison of different carbon tax regimes should be treated separately based on the occurrence of the different types of shock. We find that a higher carbon tax rate could be welfare improving initially under positive government expenditure, TFP, and contractionary monetary policy shocks, while this is not the case under a positive risk.

The unconditional compensating variation calculates the difference between the unconditional expectation of the two value functions. Table 6 reports the unconditional compensating variation λ_U of moving from the model without a carbon tax to the models with different values of carbon tax rates and the steady-state values of the selected variables. As expected, a higher carbon tax rate reduces the consumption, capital in production, output, and hence the emissions, emissions stock, and the mean value of welfare. For example, under a 5% carbon tax, the most a consumer would forgo to have no carbon tax is approximately 0.00750% of mean consumption. That is, consumers would like to give up approximately 664.71 million US dollars per quarter (in chained 2012 US dollars) in exchange for the same level of utility when no tax is applied.[15] As the carbon tax rate increases from 0 to 15%, the carbon emissions and emissions stock decrease by 32% (from 0.793 to 0.537) and 12% (from 683.654 to 600.932), respectively. This result reveals that carbon taxation at effective in combating the air pollution problem in the long run. However, the steady-state values of both consumption and output decrease by approximately 5%, from 1.449 to 1.379 and from 2.277 to 2.161, respectively and consumers would like to give up approximately 2.27 billion US dollars per quarter (0.0256% of mean consumption). These numbers imply that a higher carbon tax rate could substantially dampen economic activity and lead to a consumption loss in the long run.

6.2 Ramsey problem

Although a higher carbon tax rate could help dampen the emissions volatility caused by various shocks and ensure a low level of emissions, it could reduce output and depress economic activity. In this regard,

[15] The sample mean of the real quarterly personal consumption expenditure is 8862.86 billion dollars (in chained 2012 US dollars) over our sample period from 1985Q1 to 2019Q2. A 0.00750% reduction is equivalent to 8862.86 × 0.00750% = 0.66471 billion US dollars (in chained 2012 US dollars).

we follow Annicchiarico and Di Dio (2015) to examine the optimal time-varying carbon tax rate. In particular, this section considers a Ramsey problem that features a social planner who maximizes the expected lifetime utility of the household by choosing a carbon tax rate p_Z, subject to the equilibrium conditions of the decentralized economy.[16] Fig. 13 displays the IRFs of the optimal carbon tax rate and the corresponding emissions to the risk, TFP, monetary policy, and government expenditure shocks. As shown, except under a positive TFP shock, the Ramsey social planner should reduce the carbon tax rate for all the examined periods, and the IRFs move across both the positive and negative regions for the other three shocks. In particular, the carbon tax rate should initially increase and then decrease gradually under positive risk and monetary policy shocks, while it should increase gradually after an initial reduction under a positive government expenditure shocks.

There are two opposite forces determining whether a higher carbon tax rate is beneficial to the economy. On the one hand, as discussed in the previous section, a positive risk shock could naturally lead to a higher delinquency rate for entrepreneurs. In a difficult business environment, it is optimal for the government to reduce the carbon tax rate to reduce operating expenses for firms. A lower carbon tax rate stimulates capital demand from good producers and hence drives up the return to capital. This eventually encourages entrepreneurs to make more capital expenditure. On the other hand, the initial decline in output due to the positive risk shock leads to a reduction in the nominal interest rate according to the Taylor rule (21). A lower interest rate implies lower borrowing costs for entrepreneurs. To balance the surge in capital demand induced by the lower borrowing cost, the optimal carbon tax rate should increase. Since the latter effect dominates in the first few periods, the carbon tax rate increases in response to a positive risk shock. As the first effect comes to prevail, the carbon tax rate declines rapidly a few periods later.

Turning to the IRFs to a positive TFP shock, the optimal carbon tax is countercyclical, which is in contrast to the findings in Annicchiarico and Di Dio (2015) and other studies (e.g., Angelopoulos et al., 2013 and Heutel, 2012). As explained in Annicchiarico and Di Dio (2015), a procyclical carbon tax helps reduce the procyclicality of the carbon cycle, as emissions are cyclically more volatile if the tax rate does not increase during expansions. Our countercyclical results are mainly due to the different specifications of the Taylor rule. Different from the setting in Annicchiarico and Di Dio (2015), where the central bank adopts a sole inflation targeting rule, we assume that the central bank also responds to the output gap as shown in Eq. (21). Specifically, the central bank would raise the interest rate if a positive output gap occurs to cool the economy. Although such monetary policy would stabilize output and emissions over the business cycle, it would also introduce an additional channel for the carbon tax to react during business cycles. That is, when a positive TFP shock increases output, this signals the central bank to raise the interest rate. Then, triggered by the higher borrowing cost for entrepreneurs, capital demand decreases, and this ultimately translates into a decline in output. In such circumstances,

[16]The detailed formulation of the problem is described in online appendix II.

the optimal carbon tax rate should be reduced to stimulate capital demand, thereby hedging against the higher borrowing cost and stabilizing conditions in the financial market.

The IRFs of the optimal carbon tax rate to a contractionary monetary policy shock are in line with the results in Annicchiarico and Di Dio (2015). The carbon tax is reduced to offset the contractionary effect on output, and emissions increase. Moreover, a positive shock to the interest rate induces a higher borrowing cost. The Ramsey planner would reduce the carbon tax to stimulate the financial market. Finally, as shown in the last row of Fig. 13, a rise in government expenditure crowds out private consumption and investment. It is thus optimal to reduce the carbon tax to mitigate such impacts.

The right panel of Fig. 13 compares the emissions under the optimal carbon tax rate and the constant tax rate of $p_Z = 1\%$ and 15%, respectively. Compared to the paths of emissions with constant carbon tax rates, the optimal carbon tax rate generates lower emissions under all four shocks. This reveals that emissions are likely to be lower under the optimal carbon tax, although the objective of the Ramsey social planner is to maximize the lifetime utility of households, without considering carbon mitigation as her objective.

A major difference from the Ramsey problem in Annicchiarico and Di Dio (2015) is the negative response of the optimal carbon tax to the TFP shock that we observe. To examine the role of Taylor rule specifications in explaining the difference, the left panels of Fig. 14 plot the IRFs of the optimal carbon tax rate to the four shocks when ι_Y takes values $1/4$ (solid line) and 0 (dashed line). Note that in the left panels the left axes are for the IRFs with $\iota_Y = 1/4$, while the right axes are for the IRFs with $\iota_Y = 0$. In all cases, we keep the parameter of response to inflation deviations ι_Π at 3.

$\iota_Y = 0$ indicates that the interest rate rule only reacts to inflation deviations. In this pure inflation-targeting regime, the IRF of the carbon tax rate to TFP shock becomes positive: that is, the carbon tax rate is procyclical. This result is consistent with the findings of Annicchiarico and Di Dio (2015). Note that the IRF of the carbon tax rate to the risk shock does not initially increase and that the optimal carbon tax rate exhibits a larger decline in response to the monetary policy shock. As explained above, this type of interest rate rule turns off the additional channel of interest rate reactions to the output gap. For example, in response to the monetary policy shock, the optimal carbon tax (dashed line) has to decrease further to counteract the negative impact of the increased interest rate on the financial market. The more stable dynamics of the tax rate (dashed line) following a positive government expenditure shock can also be attributed to the absence of the channel where the interest rate reacts to the output gap.

The above analysis naturally leads to the following question: which of the Taylor rule specifications is more environmentally improving? The right panels of Fig. 14 report the IRFs of carbon emissions to the four shocks, while the optimal carbon tax rate applies when ι_Y takes $1/4$ (solid line) or 0 (dashed line). As expected, the responses of emissions are negatively associated with the tax rate. Although the Taylor rule with $\iota_Y = 0$ always results in a smaller initial increase in emissions except in the case of the government expenditure shock, the Taylor rule in Eq. (21) might be a better choice if we consider which of the rules would produce less volatile emissions. While under the Taylor rule in Eq. (21) a continued downward trend in the tax rate over the examined periods following the risk and monetary policy shocks should be welcomed by entrepreneurs, the improved business environment also encourages production activities, leading to more carbon emissions. To conclude, the choice of monetary policy has a significant impact on emissions, even if the carbon tax policy is set at its optimal level. In the sense of entailing less volatility in emissions, the Taylor rule that reacts to output deviation may be more stabilizing.

7 Conclusion

We examine the role of credit market frictions in explaining US carbon emissions over business cycles. While recent literature (e.g., Fischer and Springborn, 2011; Heutel, 2012; Annicchiarico and Di Dio, 2015; and Khan et al., 2019) has already employed E-DSGE models for environmental policy analysis, these models do not include credit market frictions, which are considered to be among the most important drivers of the business cycles in the macroeconomic literature. To this end, we present a E-DSGE model that features credit market frictions à la Bernanke et al. (1999) and Christiano et al. (2014).

Our results are as follows. First, we empirically demonstrate that credit market conditions, as a nonlinear propagator of shocks, have a substantial impact on the dynamics of carbon emissions. In particular, by using a threshold VAR model, we find that the first difference of the delinquency rate is a good indicator of the regime switching of the economy from normal to recession periods. By examining the NIRFs, we find that emissions are more sensitive to monetary policy shocks in a tight regime.

Second, our model simulation results suggest that the risk shock, a sudden increase in the dispersion of the entrepreneurs' potential return, tightens the credit market and significantly reduces carbon emissions. In addition, the presence of a financial accelerator amplifies the volatility of carbon emissions under positive TFP and government expenditure shocks more than doubled. By studying the US macroeconomic and emissions data over the period from 1985Q1 to 2019Q2, the shock decomposition shows that the risk shock generates counterfactual paths that perform the best in characterizing the movements in output and emissions growth. Most important, among the four shocks considered in this paper, the contribution share of the risk shock alone to the emissions growth rate dynamics reaches a peak of around 50% in 8 quarters after each recession.

Third, the policy analysis reveals that increasing the carbon tax rate is helpful in reducing emissions volatility during recessions. However, in the long run, a 15% carbon tax, which leads to a 32% decrease in carbon emissions, would result in approximately a per quarter consumption decline of approximately 2.27 billion US dollars (measured in chained 2012 dollars) to maintain the same level of utility for households as when no tax is applied. In the short run, the welfare impact of increasing the carbon tax rate is shock-dependent: it is welfare improving under positive government expenditure, TFP, and contractionary monetary policy, while it is welfare deteriorating under a positive risk shock.

Finally, we solve for the optimal time-varying carbon tax rate from a Ramsey planner problem. In line with the existing literature (e.g., Heutel, 2012 and Annicchiarico and Di Dio, 2015), the optimal carbon tax rate should be procyclical even in the presence of a financial accelerator. However, this result is highly conditional on the Taylor rule's specification. In Annicchiarico and Di Dio (2015), the interest rate only responds to inflation rate. With a generalized Taylor rule in which the interest rate is also responsive to output gap, the procyclicality of the optimal carbon tax to the risk shock is greatly reduced. Furthermore, the reaction becomes countercyclical in response to a positive TFP shock. In the sense of entailing less volatility in emissions, Taylor rule that reacts to output deviation may be more stabilizing.

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Tables and figures

Table 1
ADF unit root test (p value).

		Series in the level		In first difference	
Variables	Parameter	With intercept	W/o intercept	With intercept	W/o intercept
Real GDP per capita	y_t	0.6019	0.9997	0.0000***	0.0001***
Inflation rate	π_t	0.0238**	0.1912	-	-
Interest rate	r_t	0.4282	0.0440**	-	-
Delinquency rate	del_t	0.0724	0.1204	0.0000***	0.0006***
CO_2 emissions	z_t	0.9761	0.0649*	0.0000***	0.0000***
ANFCI	$anfci_t$	0.0017***	0.0001***	-	-
NFCI	$nfci_t$	0.0176**	0.0197**	-	-

Notes: *, **, and *** represent $p < 0.10$, $p < 0.05$, and $p < 0.01$, respectively.

Table 2
Wald tests for TVAR.

Threshold Variables	Estimated threshold value	Wald statistics		
		Sup-	Avg-	Exp-
Delinquency rate, $d = 2$	$\bar{\zeta} = 0.00$	156.85	109.13	75.14
		(0.00)	(0.00)	(0.00)
ANFCI, $d = 3$	$\bar{\zeta} = -0.18$	182.58	107.49	87.57
		(0.05)	(0.264)	(0.05)
NFCI, $d = 3$	$\bar{\zeta} = -0.31$	204.92	111.52	98.58
		(0.012)	(0.194)	(0.012)

Notes: The delay for the threshold variable is given by d. P-values are displayed in parentheses.

Table 3
Regression table of TVAR model.

	Normal regime					Tight regime				
	Y_t	R_t	π_t	Z_t	del_t	Y_t	R_t	π_t	Z_t	del_t
Y_{t-2}	0.0727	-0.00767	0.0163	-0.439*	0.0136	-0.548**	-0.00516***	-0.0839**	-0.117	0.137
	(0.46)	(0.69)	(0.16)	(0.08)	(0.55)	(0.01)	(0.00)	(0.02)	(0.75)	(0.76)
Y_{t-1}	-0.0451	0.00699	0.0370	0.207**	0.0216	-0.570**	0.0162	-0.0366***	1.177***	0.0201**
	(0.65)	(0.36)	(0.16)	(0.03)	(0.88)	(0.01)	(0.32)	(0.00)	(0.00)	(0.02)
R_{t-2}	-0.379	0.172*	0.0924	-1.52	-0.340	-7.50	-0.174	0.170	-23.5	2.34
	(0.45)	(0.08)	(0.21)	(0.72)	(0.60)	(0.77)	(0.40)	(0.56)	(0.19)	(0.59)
R_{t-1}	-0.902	0.390***	0.193	5.68**	-0.184	-2.174	0.247	-0.0906	20.2	-0.053
	(0.49)	(0.00)	(0.20)	(0.03)	(0.74)	(0.37)	(0.17)	(0.59)	(0.70)	(0.29)
π_{t-2}	-0.496	-0.0612	-0.576***	-0.138	0.129	-1.89**	0.0395	-0.677***	-4.82	0.113
	(0.27)	(0.52)	(0.00)	(0.17)	(0.73)	(0.04)	(0.72)	(0.00)	(0.68)	(0.14)
π_{t-1}	0.722**	0.0654	1.543***	0.197	0.146	2.67	0.0391	1.56***	4.82	-0.386**
	(0.01)	(0.18)	(0.00)	(0.31)	(0.21)	(0.39)	(0.83)	(0.00)	(0.28)	(0.04)
Z_{t-2}	0.0380	-0.000797	-0.00435	0.126	-0.0153	0.0172	0.00569***	0.00423**	-0.300*	-0.0230
	(0.33)	(0.73)	(0.95)	(0.20)	(0.35)	(0.85)	(0.00)	(0.04)	(0.06)	(0.90)
Z_{t-1}	0.0443	0.00449	-0.00318	-0.199**	-0.0152	0.193*	0.00165***	0.0106*	-0.00170	-0.0417
	(0.65)	(0.20)	(0.93)	(0.04)	(0.60)	(0.07)	(0.00)	(0.06)	(0.99)	(0.51)
del_{t-2}	-0.145	0.0131	-0.0124	1.05***	0.186***	-0.181*	-0.0264***	-0.168	-0.220	0.168
	(0.58)	(0.15)	(0.28)	(0.00)	(0.00)	(0.08)	(0.00)	(0.70)	(0.12)	(0.44)
del_{t-1}	-0.0396	-0.00908	0.0495	0.631***	0.207***	-1.41	0.0543	-0.240	-1.20**	0.162
	(0.38)	(0.45)	(0.25)	(0.00)	(0.00)	(0.80)	(0.95)	(0.22)	(0.02)	(0.47)

Notes: Standard errors are displayed in parentheses. *, **, and *** represent $p < 0.10$, $p < 0.05$, and $p < 0.01$, respectively. The normal regime is a regime where the threshold variable $\zeta_t \leq \bar{\zeta}$, while the tight regime is when $\zeta_t > \bar{\zeta}$.

Table 4
The priors and posteriors for the estimated parameters.

		Prior distribution			Posterior distribution	
Parameter name	Parameter	Prior dist	Mean	SD	Mode	SD
Calvo price stickiness	ν	beta	0.75	0.1	0.926	0.000304
Disutility weight on labor	μ_L	normal	1	0.2	0.872	0.000164
Pollution decay rate	δ_M	beta	0.0021	0.002	0.003	1.71e-06
Marginal emissions of output	φ	normal	0.53	0.3	0.348	0.000485
Survival rate of entrepreneurs	γ	beta	0.97	0.03	0.954	2.22e-05
Monitoring cost	μ	normal	0.21	0.05	0.185	2.27e-05
Real GDP gross growth rate	Y^{tr}	normal	1.0037	0.5	1	9.77e-05
SS. nominal interest rate	R^{tr}	normal	1.0268	0.5	1.052	0.00127
SS. bankruptcy rate	F^{tr}	normal	4.0673	2	2.832	0.00605
Persistence of TFP shock	ρ_A	beta	0.5	0.5	0.99	7.49e-05
Persistence of government shock	ρ_G	beta	0.5	0.2	0.918	0.000431
Persistence of monetary shock	ρ_η	beta	0.5	0.2	0.646	0.001473
Persistence of risk shock	ρ_σ	beta	0.5	0.2	0.99	0.000164
SD of TFP shock	σ_A	invg2	0.005	0.003	0.00386	5.82e-06
SD of government shock	σ_G	invg2	0.005	0.003	0.00602	5.01e-06
SD of monetary shock	σ_η	invg2	0.005	0.003	0.00751	4.79e-06
SD of risk shock	σ_σ	invg2	0.005	0.003	0.01231	1.67e-05

Notes: Normal, beta, and invg2 stand for normal, beta, and type 2 inverse gamma distributions, respectively.

Table 5
The calibrated and estimated parameter values used for numerical analysis.

Calibrated parameter	Value	Description
α	$1/3$	Share of capital in production
β	0.99	Discount factor
δ_K	0.025	Depreciation rate of capital
γ_I	0.5882	Parameter of capital adjustment cost
ϕ	1	Inverse of Frisch elasticity
θ	6	Elasticity of substitution within goods sectors
ϕ_1	0.1761	Parameter of abatement cost
ϕ_2	2.8	Parameter of abatement cost
γ_0	$1.395e^{-3}$	Parameter of damage function
γ_1	$-6.6722e^{-6}$	Parameter of damage function
γ_2	$1.4647e^{-8}$	Parameter of damage function
Z^*	1.3299	Foreign emissions
ι_π	3	Parameter of inflation gap
ι_Y	1/4	Parameter of output gap
A	1	Steady state of TFP level
η	1	Steady state of monetary policy shock
σ	0.2588	Steady state of risk shock
Estimated parameter	**Value**	**Description**
ν	0.926	Parameter of Calvo pricing adjustment
μ_L	0.872	Parameter of labor disutility
δ_M	0.003	Depreciation rate of emissions stock
φ	0.348	Marginal emissions of production
γ	0.954	Survival rate of entrepreneurs
μ	0.185	Monitoring cost rate
ρ_A	0.99	Persistence of TFP shock
ρ_G	0.918	Persistence of government expenditure shock
ρ_η	0.646	Persistence of monetary policy shock
ρ_σ	0.99	Persistence of risk shock
σ_A	0.00386	Standard deviation of TFP shock
σ_G	0.00602	Standard deviation of government expenditure shock
σ_η	0.00751	Standard deviation of monetary policy shock
σ_σ	0.01231	Standard deviation of risk shock

Table 6
Unconditional compensating variations and the steady-state values of the selected variables with different carbon tax rates.

		Value			
Variable	Description	$p_Z = 0$	$p_Z = 0.01$	$p_Z = 0.05$	$p_Z = 0.15$
C	Consumption	1.449	1.445	1.425	1.379
K	Capital in production	11.926	11.830	11.476	10.738
Y	Output	2.277	2.268	2.233	2.161
Z	Emissions	0.793	0.740	0.657	0.537
M	Emissions stock	683.654	666.375	639.617	600.932
λ_U	Unconditional CV	-	-0.00750%	-0.0162%	-0.0256%

Notes: Unconditional compensating variations of moving from the model with no carbon tax $p_z = 0$ to different values of carbon tax rates.

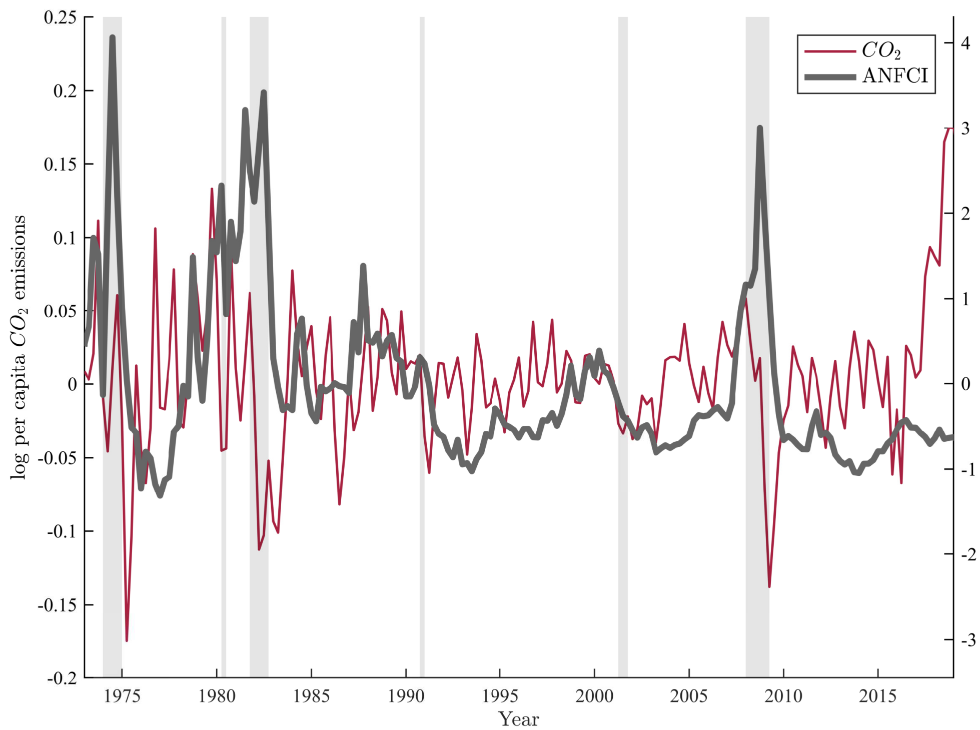
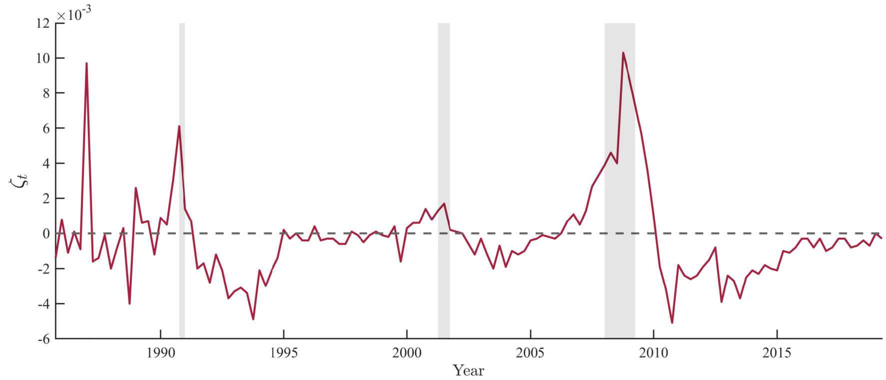
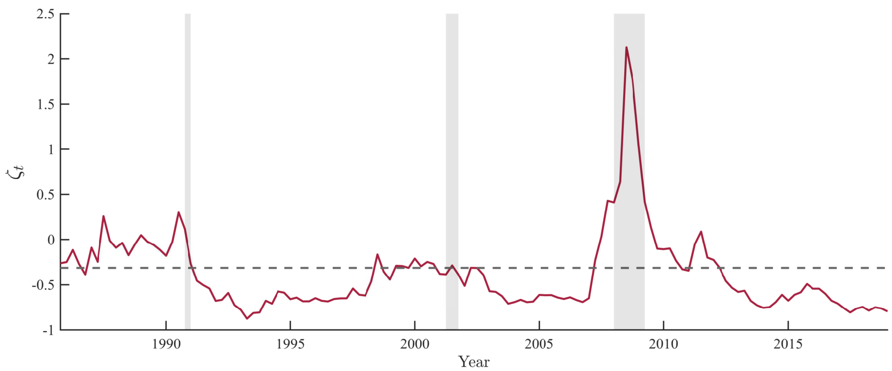


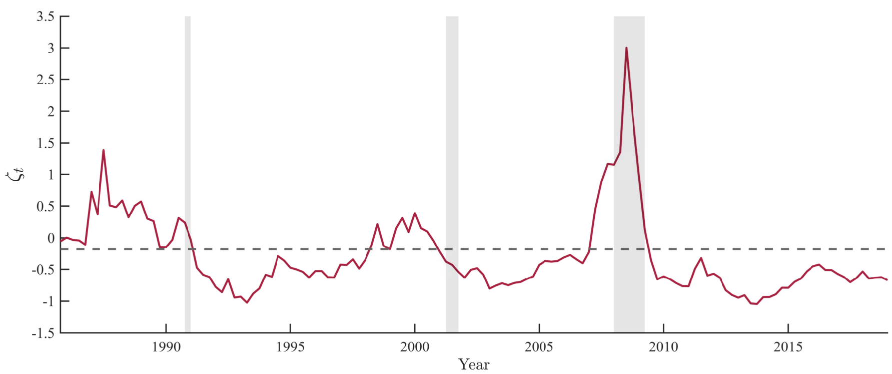
Fig. 1. The HP-filtered log CO_2 emissions per capita in the US from 1973Q1 to 2019Q2. *Notes*: Shaded areas represent the periods of National Bureau of Economic Research (NBER) recession.



(a) The time series of delinquency rate growth



(b) The time series of ANFCI



(c) The time series of NFCI

Fig. 2. Alternative threshold variable and estimated threshold values. *Notes*: The dashed horizontal line is at $\bar{\zeta} = 0$, -0.18, and -0.31 for delinquency rate growth, ANFCI, and NFCI, respectively, where the likelihood ratio is maximized. Shaded areas represent the periods of NBER recession.

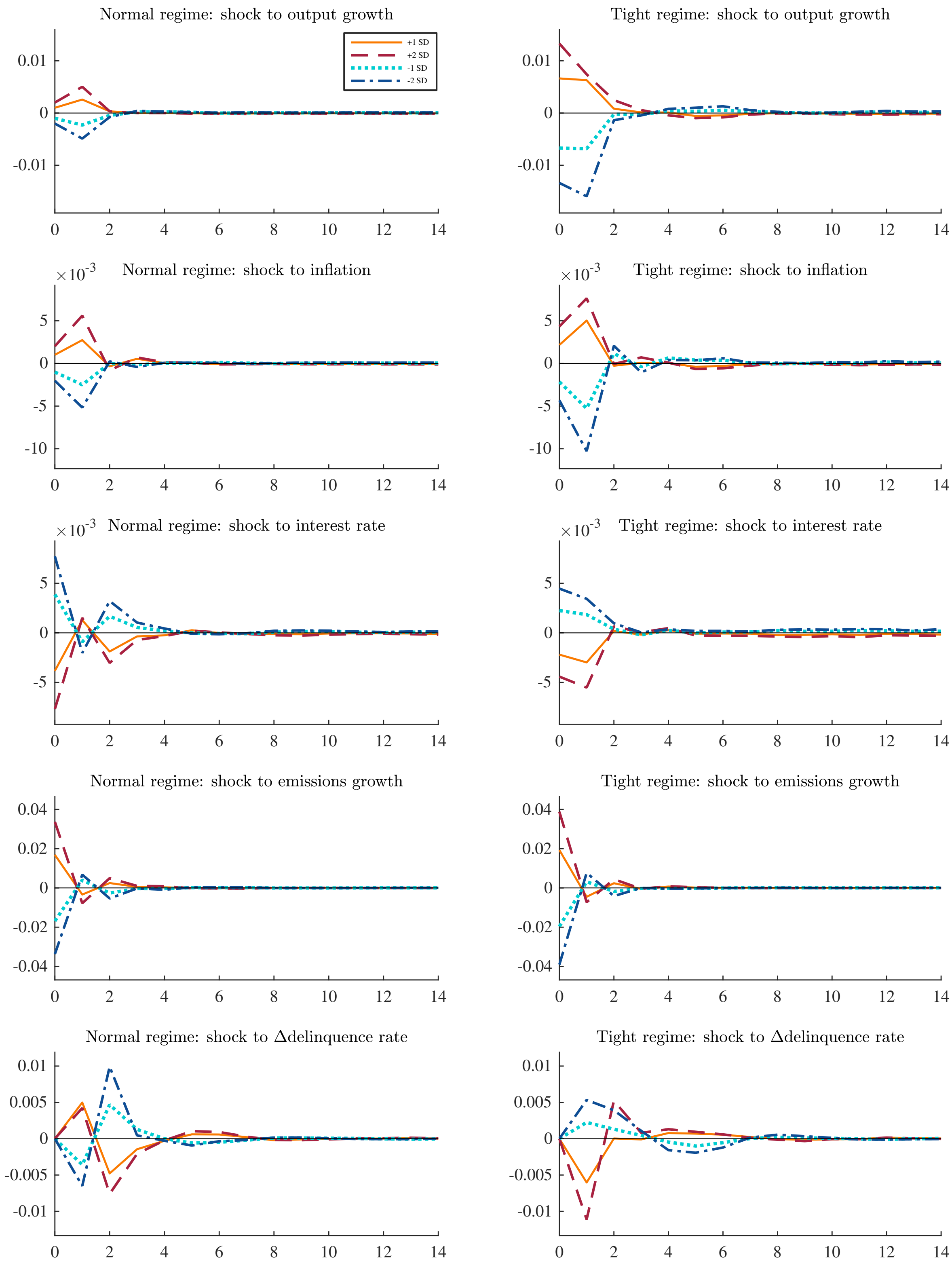


Fig. 3. IRFs of the emissions growth to different shocks in different regimes. *Notes*: The left panels are the IRFs in the normal regime where the delinquency rate $\zeta_t \leq \bar{\zeta}$, while the right panels are the IRFs in the tight regime. The solid, dashed, dotted, and dashed-dotted line represent the IRFs under a +1, +2, -1, and -2 standard deviation of shocks, respectively.

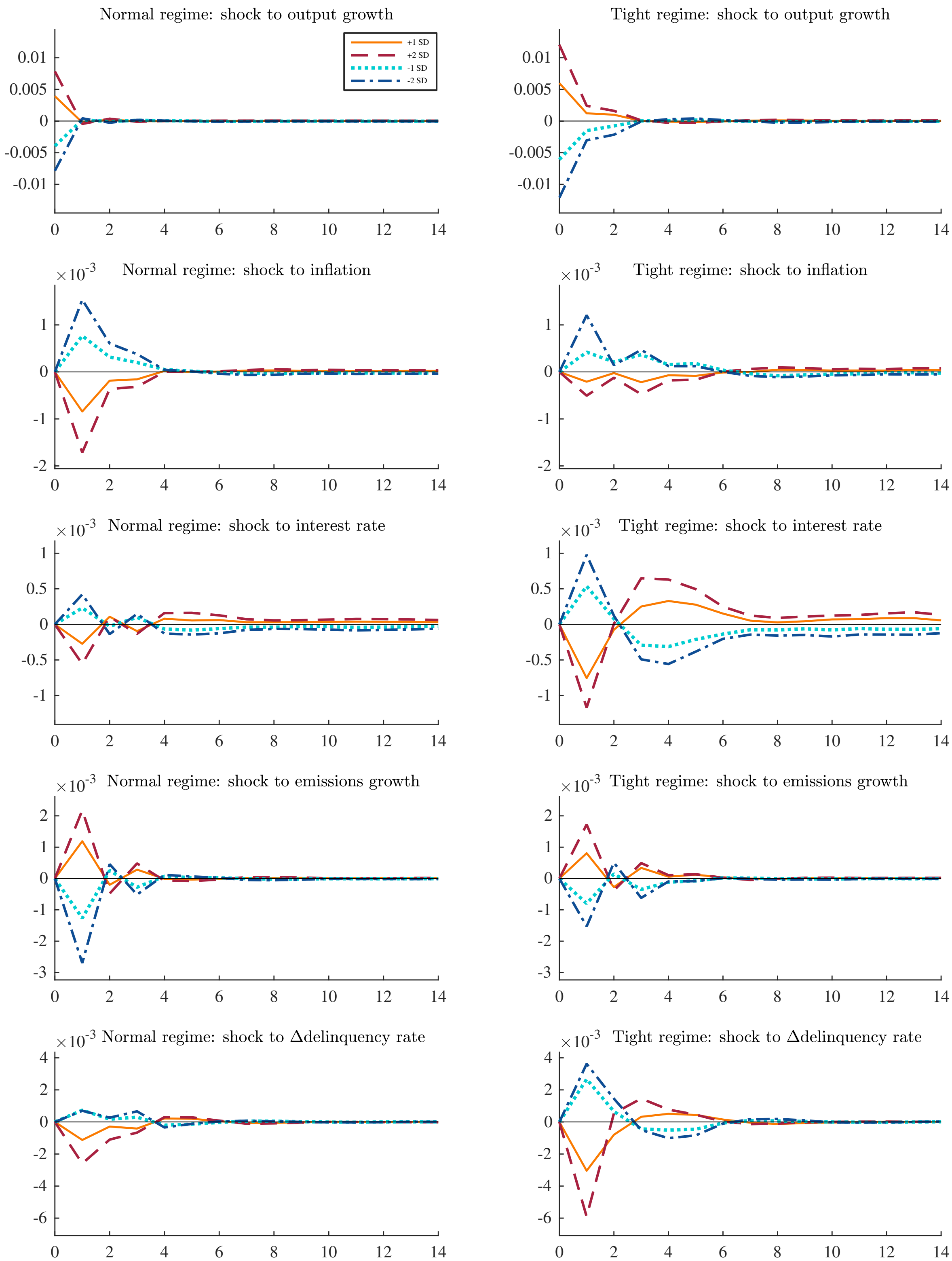


Fig. 4. IRFs of output the growth to different shocks in different regimes. *Notes*: The left panels are the IRFs in the normal regime where the delinquency rate $\zeta_t \leq \bar{\zeta}$, while the right panels are the IRFs in the tight regime. The solid, dashed, dotted, and dashed-dotted line represent the IRFs under a +1, +2, -1, and -2 standard deviation of shocks, respectively.

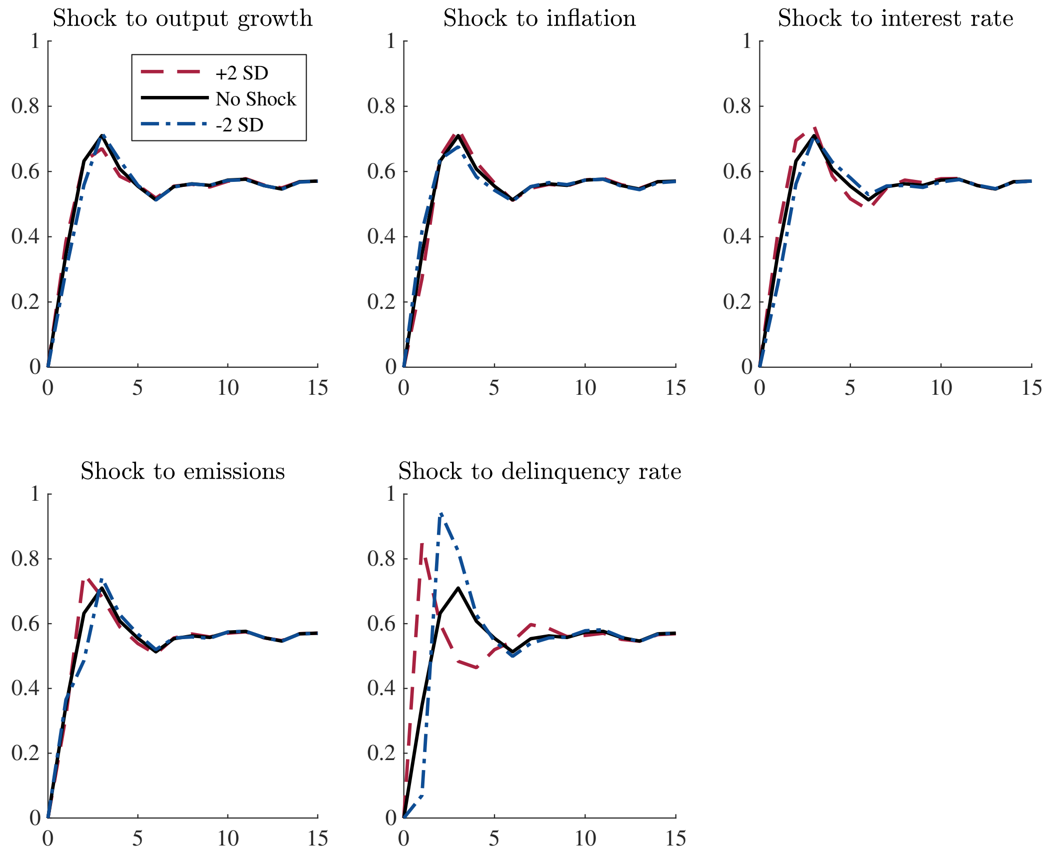


Fig. 5. Probability of being in a tight regime, conditional on staring in the normal-credit regime. *Notes*: The dashed and dotted lines represent the cases where there are +2 and -2 standard deviation of shocks in the initial period, respectively. The solid lines represent the benchmark case where there is no shock initially.

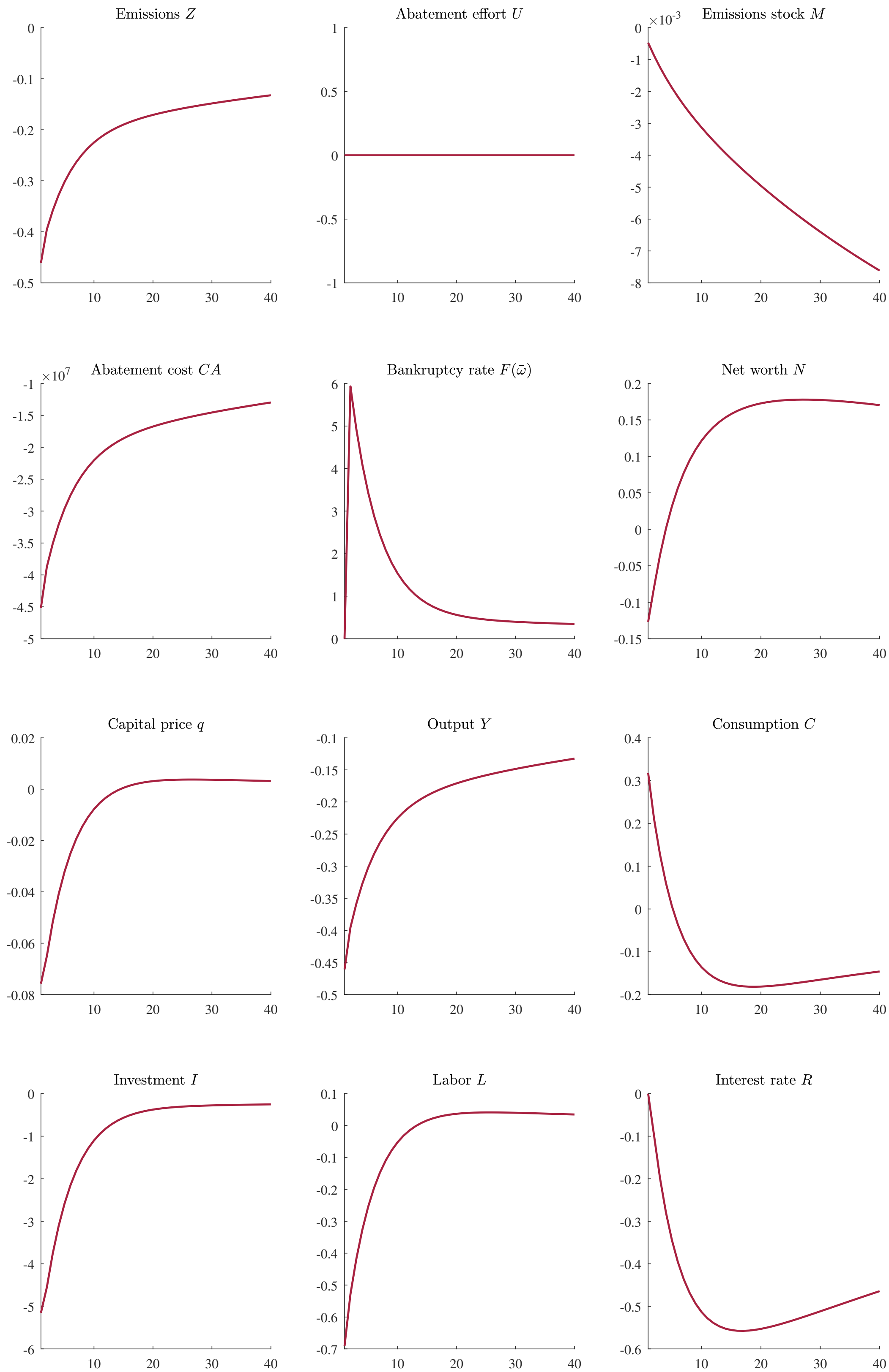


Fig. 6. IRFs to a one-standard-deviation of risk shock. *Notes*: The value of the IRFs are scaled up by 100 times.

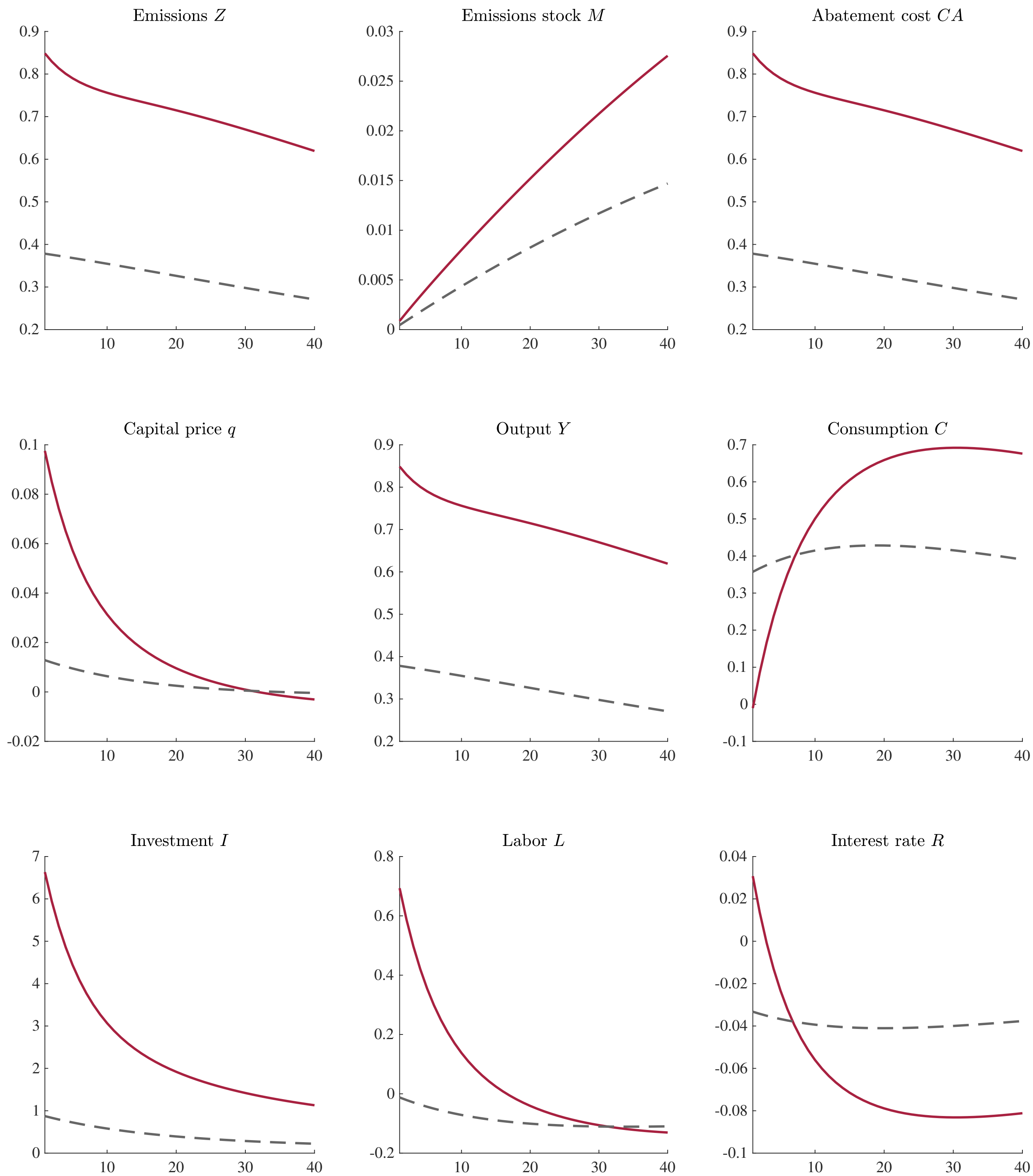


Fig. 7. IRFs to a one-standard-deviation of TFP shock. *Notes*: The solid lines represent the IRFs with a financial accelerator, while the dashed lines represent the IRFs without a financial accelerator. The value of the IRFs are scaled up by 100 times.

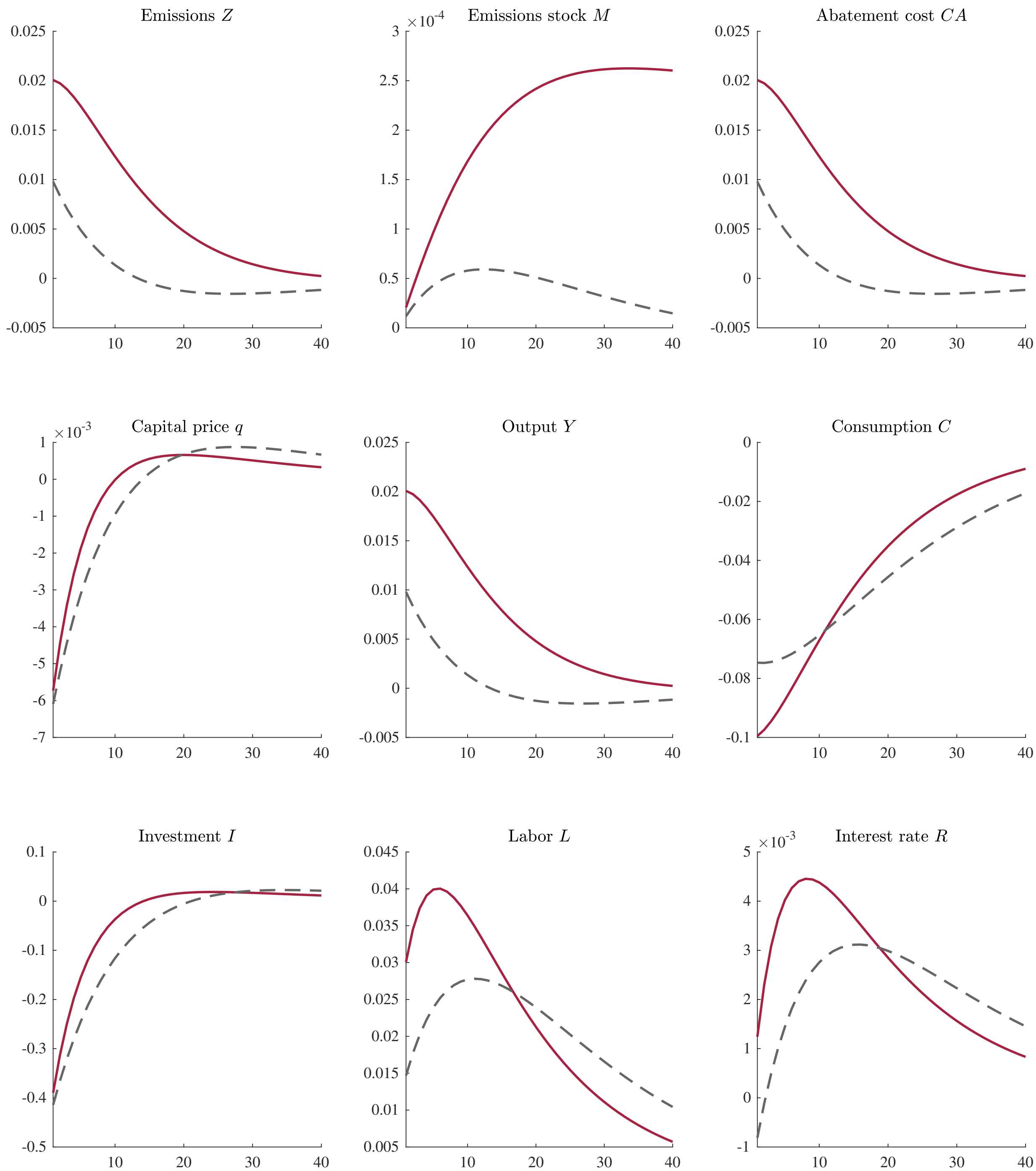


Fig. 8. IRFs to a one-standard-deviation of government expenditure shock. *Notes*: The solid lines represent the IRFs with a financial accelerator, while the dashed lines represent the IRFs without a financial accelerator. The value of the IRFs are scaled up by 100 times.

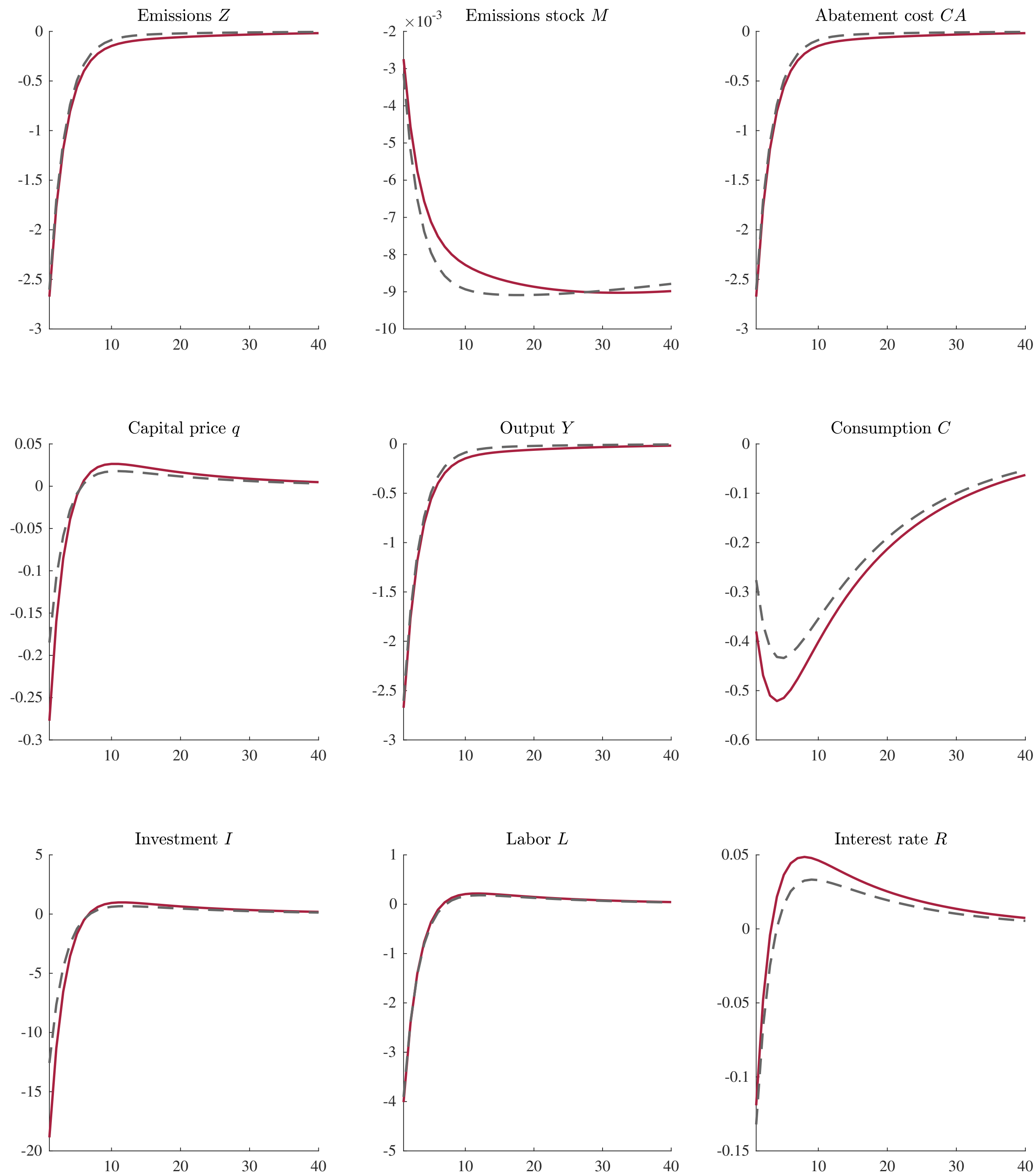


Fig. 9. IRFs to a one-standard-deviation of monetary policy shock. *Notes*: The solid lines represent the IRFs with a financial accelerator, while the dashed lines represent the IRFs without a financial accelerator. The value of the IRFs are scaled up by 100 times.

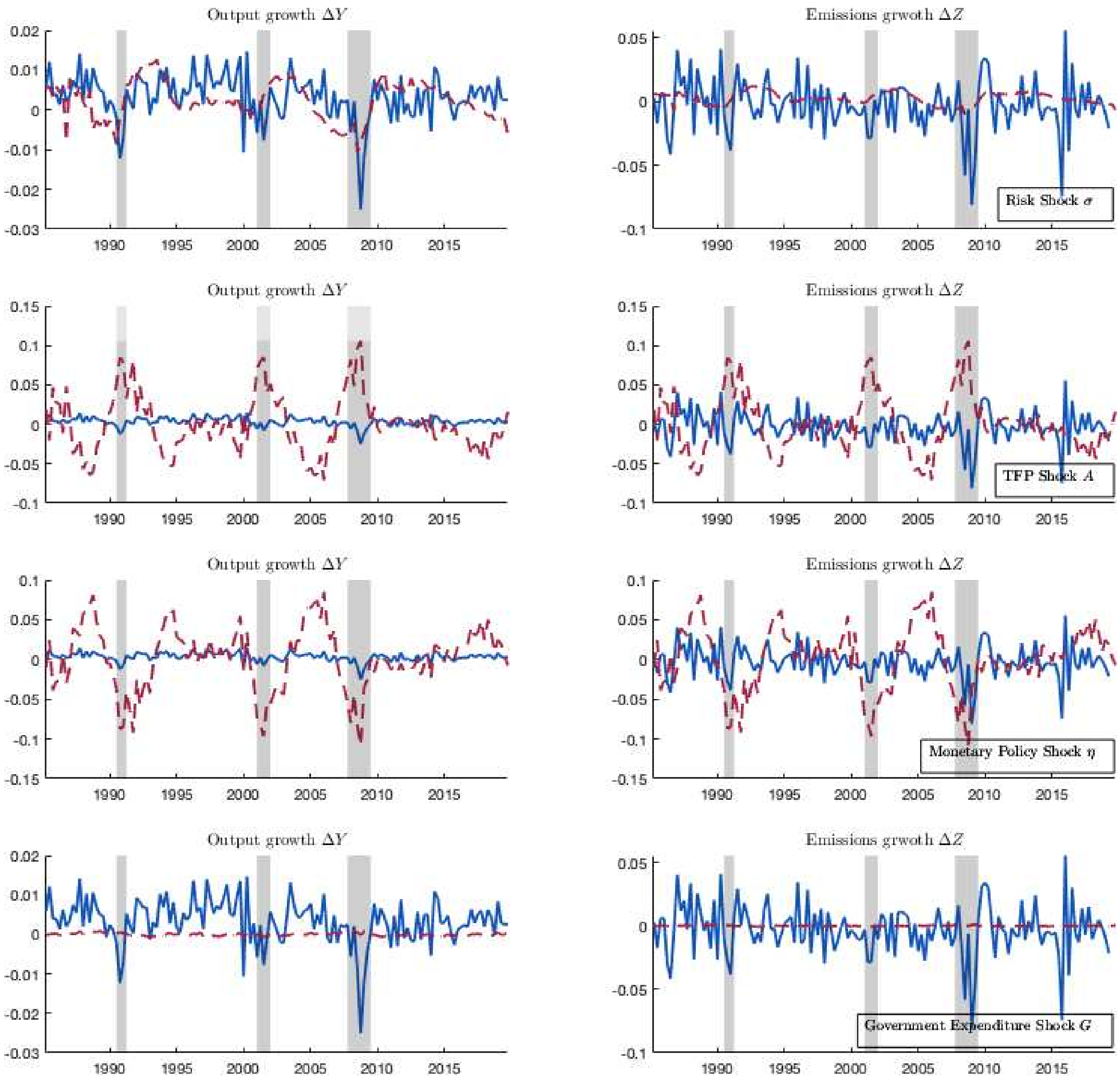


Fig. 10. Shock decompositions of output and emissions. *Notes*: The solid lines are the data series on output (left panel) and emissions (right panel). The dashed lines are the counterfactual series on output and emissions when only one shock is fed. In particular, in each of the four rows, only the risk, TFP, monetary policy, and government expenditure shocks are fed to the model. The shaded areas are NBER recession periods.

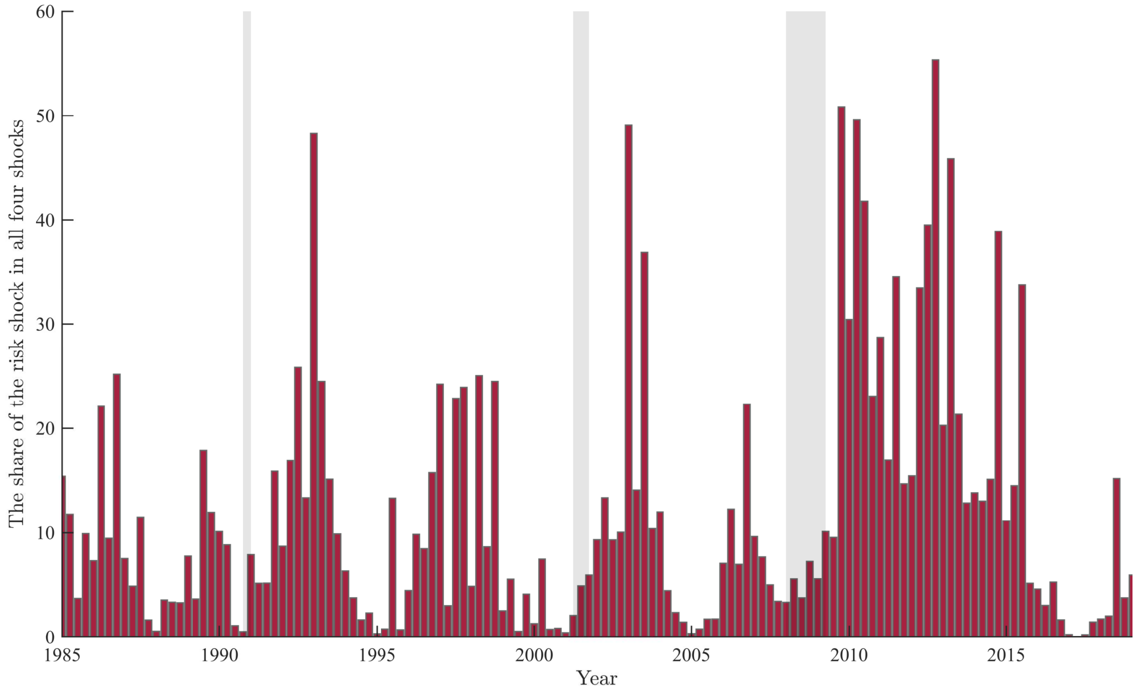


Fig. 11. The contribution share of the risk shock in the all four shocks to the emissions growth rate dynamics historically. *Notes*: The bars indicate the share of the risk shock in all four shocks in explaining the observed emissions growth rate dynamics through time. The shaded areas are NBER recession periods.

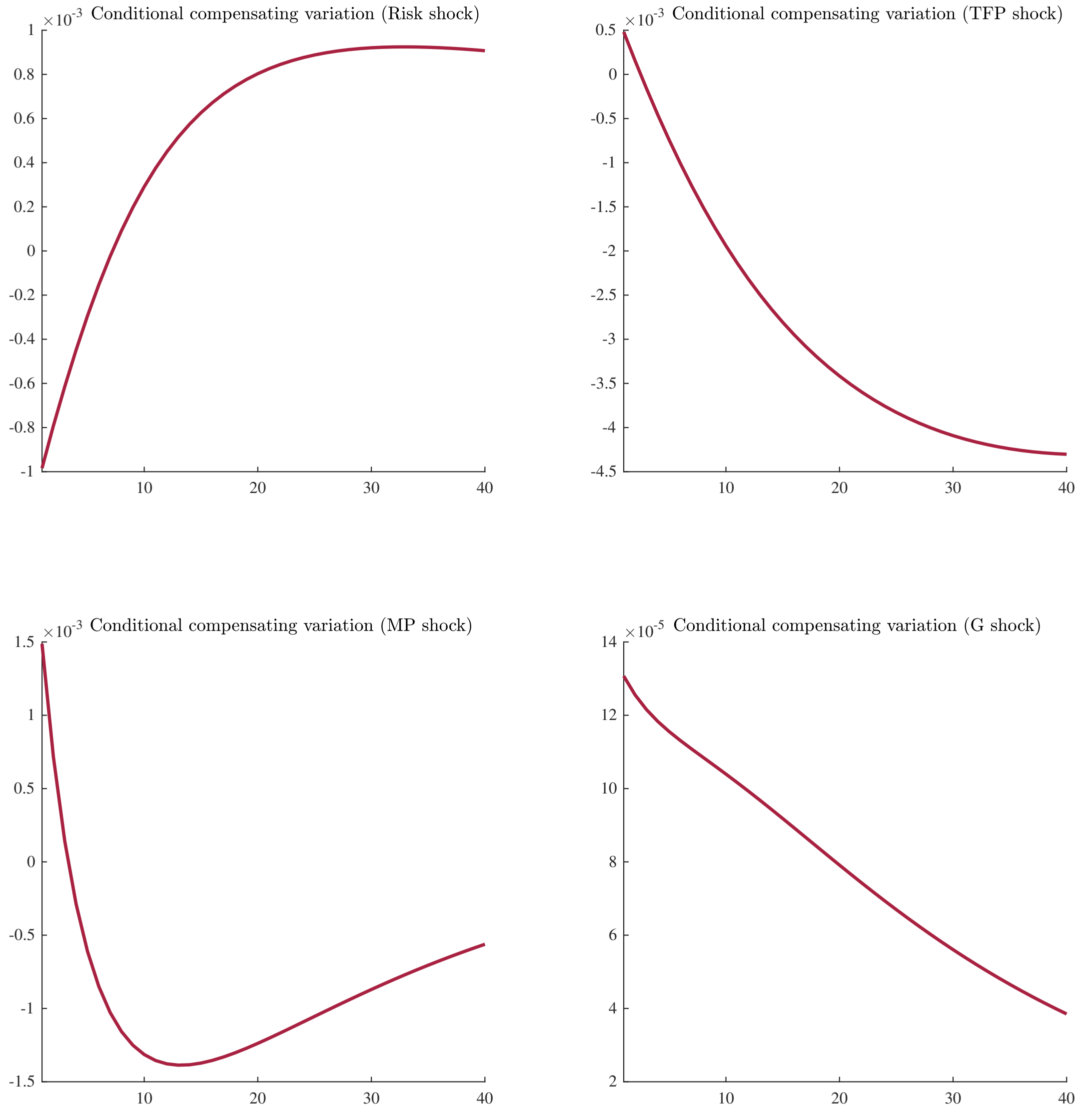


Fig. 12. Conditional compensating variation under the four shocks moving from model 1 (a 5% carbon tax) to model 2 (no tax). *Notes*: The values are scaled up by 100 times.

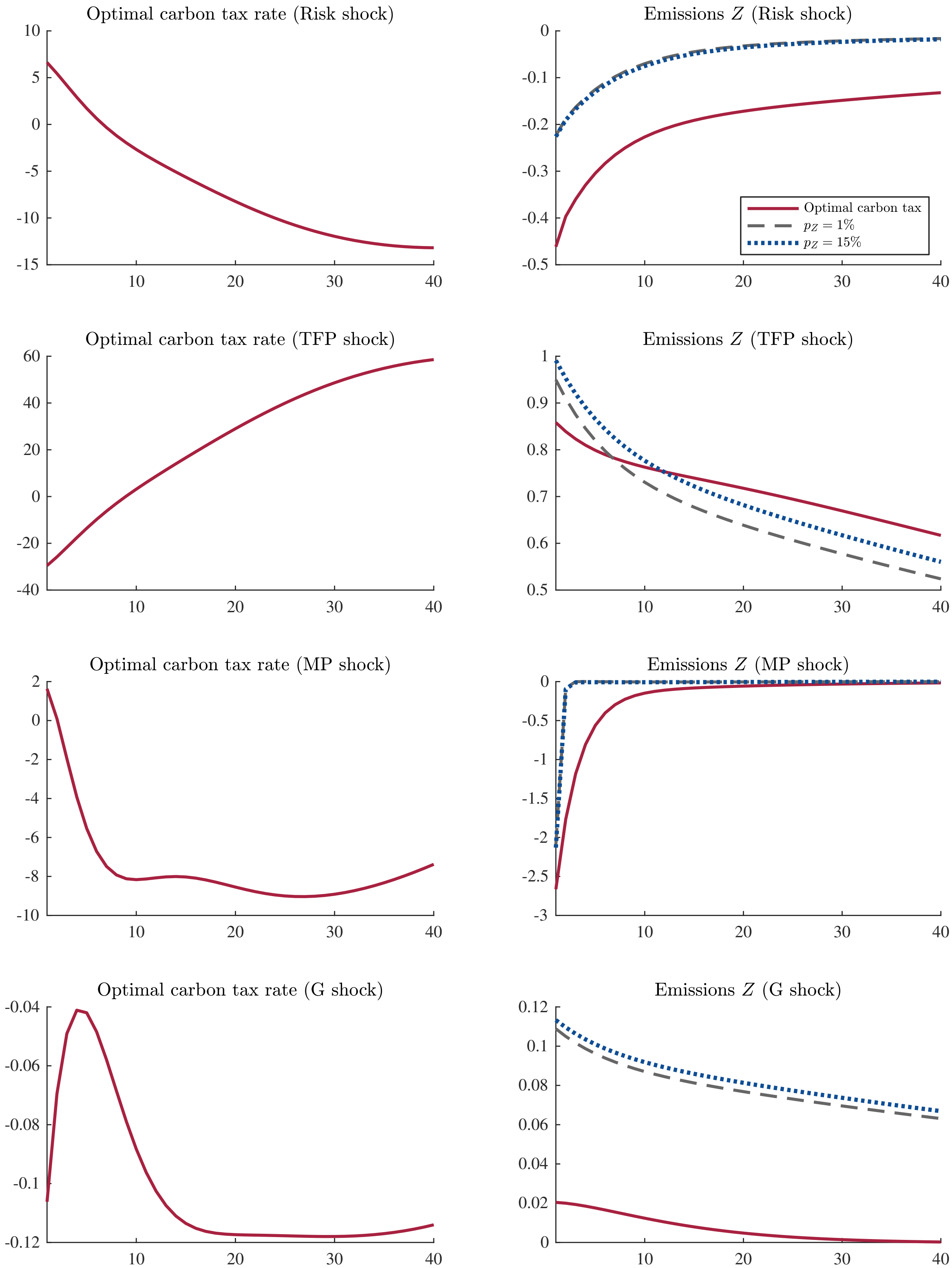


Fig. 13. IRFs of the optimal carbon tax rate and emissions to a one-standard-deviation of the four shocks in the model. *Notes*: The 'MP shock' stands for the monetary policy shock. The 'G shock' stands for the government expenditure shock. The value of the IRFs are scaled up by 100 times.

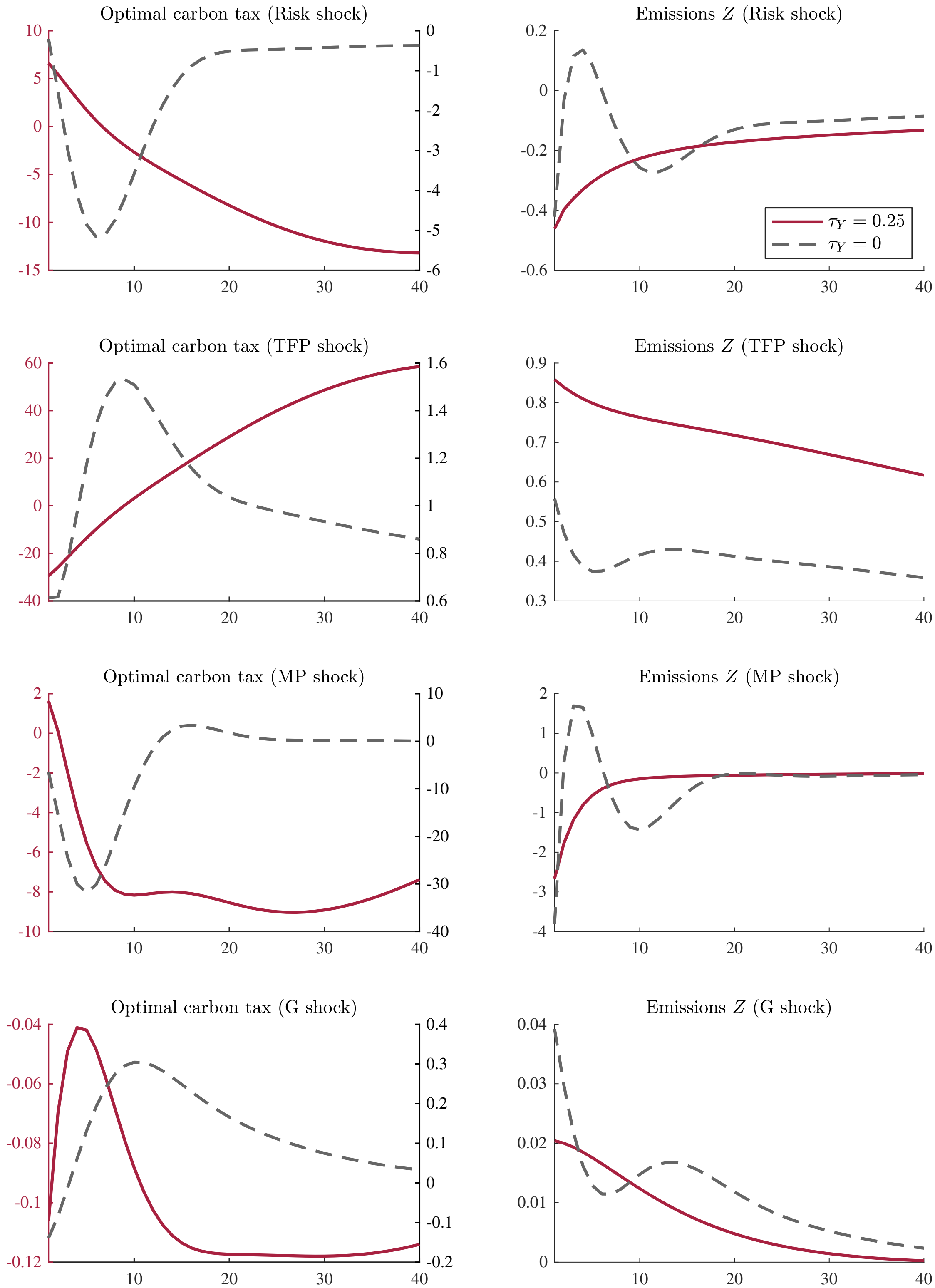


Fig. 14. IRFs of the optimal carbon tax rate and emissions to a one-standard-deviation of the four shocks in the model with different values of ι_Y. *Notes*: The 'MP shock' stands for the monetary policy shock. The 'G shock' stands for the government expenditure shock. The value of the IRFs are scaled up by 100 times.