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Abstract

This study explores the effects of monetary policy in a Schumpeterian growth model with endogenous quality increment and distinct cash-in-advance (CIA) constraints on consumption, manufacturing and R&D investment. Our results are summarized as follows. When the CIA constraint is solely on consumption expenditure, an increase in the nominal interest rate may stifle economic growth by lowering the arrival rate of innovation and stimulate it at the same time by raising the size of quality increment. An additional CIA constraint on manufacturing weakens the growth-retarding effect and enhances the growth-promoting effect, whereas an additional CIA constraint on R&D investment strengthens only the negative growth effect. The quantitative analysis finds that the relationship between inflation and growth can be either monotonically decreasing or hump-shaped, but the welfare effect of inflation is always negative.

JEL classification: O30; O40; E41
Keywords: Monetary Policy, Economic Growth, R&D, Endogenous Quality Increment

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1 Introduction

In this study, we develop a monetary Schumpeterian growth model to analyze the effects of monetary policy on the size of quality increment, economic growth, and social welfare. Distinct from previous studies relying on the assumption of an exogenous quality step size, this study extends the innovation-driven growth models and explores an endogenous quality increment channel through which monetary policy induces noticeable impact on the real variables. To incorporate money demand into this growth-theoretic framework, we impose various cash-in-advance (CIA) constraints; that is, a CIA constraint on consumption expenditure as in Lucas (1980) and Dotsey and Sarte (2000), a CIA constraint on manufacturing as in Arawatari *et al.* (2018), and a CIA constraint on R&D investment as in Chu and Cozzi (2014) and Chu *et al.* (2015).

In this monetary Schumpeterian growth model, we derive the following results. In the presence of a CIA constraint exclusively on consumption expenditure, an increase in the nominal interest rate raises real wage rate through reducing labor supply, which further generates two counteracting effects on economic growth. First, higher nominal interest rate discourages R&D incentives since entrepreneurs employing labor to produce inventions now face higher R&D costs. As a result, the arrival rate of innovation decreases, causing the economic growth rate to decline. Second, given that the price markup is increasing in the size of quality increment, a rising wage rate that dampens monopoly profit incentivizes entrepreneurs to pursue more radical innovations for a higher profit flow, which in turn boosts economic growth. Since the economic growth rate is jointly determined by the arrival rate of innovation and the size of quality increment, the overall effect of the nominal interest rate on economic growth depends on the balance between the above competing forces. By calibrating the model to the US economy, we find that the relationship between the nominal interest rate and economic growth is more likely to be monotonically decreasing. Conditional on the Fisher equation which predicts a positive *long-run* relationship between nominal interest rate and inflation rate (see Mishkin (1992) and Booth and Ciner (2001) for supportive empirical evidence), our model also implies a negative correlation between inflation and economic growth.

When CIA constraints on consumption and manufacturing are present, a rise in the nominal interest rate reinforces the aforementioned positive effect through causing a larger decline in the monopoly profit and weakens the negative effect through producing an additional reallocation effect that shifts labor employment from the manufacturing to R&D sector. In this case, the nexus between the inflation rate and the economic growth rate can be negative or hump-shaped, depending on the strength of the CIA constraint on manufacturing.[1]

Furthermore, when consumption expenditure and R&D investment are constrained by cash,

[1] Vaona (2012) and Barro (2013) find that the relationship between inflation and economic growth is monotonically decreasing. Nevertheless, a number of empirical studies, such as Khan and Senhadji (2001), Burdekin *et al.* (2004), and López-Villavicencio and Mignon (2011), have documented an inverted-U shaped relation.

a higher nominal interest rate weakens the positive effect on the quality step size and strengthens the negative effect on the innovation arrival rate. This is because a higher nominal interest rate now causes a larger increase in the R&D cost and therefore a larger decrease in the innovation arrival rate. In addition, the lowered R&D labor demand in turn mitigates the rise in the wage rate, which stems from a higher nominal interest rate under the CIA constraint on consumption. This then depresses the positive impact of the nominal interest rate on the size of quality increment, as the decline in the monopoly profit becomes smaller in this circumstance. Therefore, the economic growth rate is monotonically decreasing in the nominal interest rate. Moreover, in all above cases, the social welfare is always decreasing in nominal interest rate, implying that the Friedman rule is socially optimal.

This study closely relates to the literature on inflation and innovation. A noticeable representative along this line of effort is the pioneering work of Marquis and Reffett (1994), which explores the effects of inflation on growth in the framework of Romer (1990).[2] A great number of studies have analyzed the effects of inflation in a Schumpeterian quality-ladder model, such as Chu and Lai (2013), Chu and Cozzi (2014), Chu *et al.* (2015), Chu and Ji (2016), Huang *et al.* (2017), Oikawa and Ueda (2018), Zheng *et al.* (2019), Huang *et al.* (2019), and Gil and Iglésias (2020). These models, however, all feature an identical step size of quality improvement. One novel exception is Chu *et al.* (2017), who consider the heterogeneity of quality step sizes. However, they assume that the quality increment is drawn from an exogenously given distribution, instead of the endogenous choice by entrepreneurs. Accordingly, our study complements their interesting study and contributes to the literature by allowing the step size of quality increment to be endogenously chosen by profit-maximizing entrepreneurs. Combined with the conventional frequency-of-innovation channel, the novel feature of endogenous quality step size provides a new mechanism in explaining the (potentially) inverted-U relationship between inflation and economic growth, which helps to reconcile the discrepancies in the empirical literature.

In addition, the proposed model in this study implies a positive relationship between inflation and price markups, which is consistent with the result in Wu and Zhang (2001) within a growth framework,[3] but might seem inconsistent with the widely recognized implication of standard New Keynesian models featuring sticky prices. Due to mixed empirical evidence, however, the positive inflation-markup relationship is not necessarily implausible. Bils (1987), Rotemberg and Woodford (1991), Rotemberg and Woodford (1999), Martins and Scarpetta (2002), Gali *et al.* (2007) provide empirical evidence supportive of countercyclical markups; and Banerjee and Russell (2001), and Banerjee *et al.* (2001) identify a negative long-run relationship between inflation and markup in Australia and most of the G7 countries. In sharp contrast, exploiting the Solow residual to estimate the cyclical movements in markups, Haskel *et al.* (1995) explore a panel data set

[2] Hori (2017) and Arawatari *et al.* (2018) also consider monetary policy in the Romer variety-expanding model with heterogeneity in the productivity of R&D entrepreneurs.

[3] Wu and Zhang (2001) develop a neoclassical growth model with endogenous price markup, which is determined by firm number and firm size, and predict a positive linkage between inflation and markup.

of two-digit U.K. manufacturing industries, and find evidence for strongly procyclical markups. Using both aggregate and detailed manufacturing industry data, Nekarda and Ramey (2013) suggest that markups are procyclical unconditionally, and either mildly procyclical or acyclical conditional on demand shocks. Using detailed mirco data on local house prices, retail prices and households shopping intensity, Stroebel and Vavra (2019) show that rising house prices increase consumers' demand by reducing their sensitivity to price changes, and firms raise markups in response. Their novel evidence suggests a procyclical desired or natural markup, which responds to monetary policy endogenously.[4] In fact, recent empirical evidence has motivated macroeconomic theorists to reinvestigate existing general equilibrium models for a better understanding of the mechanism under which a positive relationship between inflation and price markups can be shaped.[5] This study exploits the Schumpeterian growth model and provides a discussion on an alternative possible channel inducing a positive inflation-markup relationship.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 and 4, respectively, analytically and numerically explore the effects of monetary policy on the quality increment, economic growth, and social welfare. The final section concludes.

2 Model

In this section, we present the monetary Schumpeterian growth model featuring quality increment that is endogenously chosen by optimizing entrepreneurs. The framework is based on the classical quality-ladder growth model in Grossman and Helpman (1991). We introduce money demand via CIA constraints on consumption as in Lucas (1980), on manufacturing as in Arawatari *et al.* (2018), and on R&D investment as in Chu and Cozzi (2014). The nominal interest rate serves as the monetary policy instrument and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on quality increment, innovation and economic growth, respectively.

2.1 Household

Consider an economy with a representative household whose intertemporal preference is given by

$$U = \int_0^\infty e^{-\rho t}[\ln c_t + \theta \ln(1 - L_t)]dt, \tag{1}$$

where c_t is the consumption of final good and L_t is the supply of labor. The parameters $\rho > 0$ and $\theta \geq 0$ determine, respectively, the subjective discounting and leisure preference. We assume

[4] Desired or natural markup is defined as the markup under perfectly flexible prices. See Nekarda and Ramey (2013) for a detailed survey of the literature on the cyclicality of price markups

[5] For example, Phaneuf *et al.* (2018) propose a general equilibrium model with purely forward-looking price setters, and show that, in the existence of working capital financing, marginal cost can be directly affected by the nominal interest rate, the mechanism of which is able to induce procyclical movements in price markups.

that the size of household N_t does not grow over time and equals N_0 at time $t = 0$, which is normalized to unity.[6]

Suppose that the final good is chosen to be the numeraire. Thus, the household's budget constraint is given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t L_t - \pi_t m_t - c_t + \tau_t, \tag{2}$$

where a_t is the real value of assets and the return rate of assets is the real interest rate r_t. w_t is the real wage rate. m_t is the real money balance held by the household and π_t is the inflation rate determining the cost of money holding. The household also receives a lump-sum transfer τ_t from the government. We assume that real money balances are required prior to purchasing the consumption good. The CIA constraint on consumption is $\xi c_t \leq m_t$, where $\xi > 0$ measures the strength of the CIA constraint.

The household maximizes her utility subject to the budget constraint and the CIA constraint. From standard dynamic optimization, we derive the following no-arbitrage condition:

$$\frac{\zeta_t}{\eta_t} - \pi_t = r_t, \tag{3}$$

where η_t and ζ_t are the Hamiltonian co-state variables on the budget constraint and the CIA constraint, respectively. As addressed by Bond *et al.* (1996) and Chang *et al.* (2019), this no-arbitrage condition states that the real rate of return on money (i.e., $\zeta_t / \eta_t - \pi_t$) must equal to the real rate of return on asset (i.e., r_t). With this no-arbitrage condition, we can derive the familiar Euler equation such that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{4}$$

Moreover, we derive the optimality condition for labor supply such that

$$w_t(1 - L_t) = \theta c_t (1 + \xi i_t), \tag{5}$$

where $i_t = r_t + \pi_t$ is the nominal interest rate.

2.2 Production

There is a mass of competitive firms producing a unique final good by aggregating intermediate inputs according to the following Cobb-Douglas function:

$$y_t = \exp\left\{ \int_0^1 \ln x_t(j) dj \right\}, \tag{6}$$

[6]By this assumption, we sidestep the issue of scale effects for analytical tractability. Alternatively, Peretto (1998), Segerstrom (1998), and Howitt (1999a) provide important approaches of removing scale effects in the Schumpeterian growth model.

where $x_t(j)$ is the quantity of intermediate goods in industry $j \in [0,1]$. The final-good production function in (6) yields a unit-elastic demand with respect to each variety such that

$$x_t(j) = y_t / p_t(j), \tag{7}$$

where $p_t(j)$ denotes the price of $x_t(j)$.

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily occupied by an industry leader until the arrival of next innovation. We follow Peretto and Connolly (2007) and Arawatari, Hori and Mino (2018) to assume that a fixed operating cost is required in production. Accordingly, the production function for the leader in industry j is

$$x_t(j) = \lambda^{n_t(j)} \left[L_{x,t}(j) - \kappa \right], \tag{8}$$

where $\lambda > 1$ is the quality increment of an innovation, $n_t(j)$ is the number of innovations that have occurred in industry j up to time t, $L_{x,t}(j)$ is the production labor in industry j, and $\kappa > 0$ is the fixed operating cost. We assume that monopolists need to borrow cash to facilitate production. Therefore, given $\lambda^{n_t(j)}$, the marginal cost of production for the leader in industry j is $mc_t(j) = w_t(1 + \alpha i_t)/\lambda^{n_t(j)}$, where $(1 + \alpha i_t)$ represents the additional cost due to a CIA constraint on manufacturing and $\alpha \in [0,1]$ is the strength of the CIA constraint. Furthermore, we assume that the previous quality leader in industry j who owns the second-latest production technology is able to produce the same product $x_t(j)$ at a higher marginal cost of $(1 + \alpha i_t) w_t / \lambda^{n_t(j)-1}$. Bertrand competition implies that the profit-maximizing price $p_t(j)$ is

$$p_t(j) = \lambda mc_t(j),$$

which allows the current leader to exclude the competition of previous leader.[7] The monopoly profit in industry j is

$$\Pi_t(j) = p_t(j) x_t(j) - w_t L_{x,t}(j)(1 + \alpha i_t) = \left(\frac{\lambda - 1}{\lambda} \right) y_t - \kappa w_t (1 + \alpha i_t), \tag{9}$$

where we have applied (7) and (8). In addition, the demand function of manufacturing labor is

$$L_{x,t}(j) = \kappa + \frac{p_t(j) x_t(j) / [w_t(1 + \alpha i_t)]}{\lambda} = \kappa + \frac{y_t / w_t}{\lambda (1 + \alpha i_t)}, \tag{10}$$

where the second equality again applies (7). This equation implies that the demand of manufacturing labor is identical across industries.

[7] We assume that the previous leader is inactive when her profit is zero.

2.3 Innovation

Denote by $v_t(j, \lambda)$ the value of the monopolistic firm in industry j that attempts at creating an invention with a quality size of λ. Equation (9) implies that the profit flow of each monopolist across industries $j \in [0, 1]$ is identical such that $v_t(j, \lambda) = v_t(\lambda)$ in a symmetric equilibrium.[8] Then the no-arbitrage condition for v_t is

$$r_t v_t = \Pi_t + \dot{v}_t - \mu_t v_t, \tag{11}$$

where μ_t is the aggregate intensity of research targeting at a state-of-the-art product and also the arrival rate of next innovation. Intuitively, the value $r_t v_t$ is equal to the sum of the profit flow Π_t, the potential capital gain $\dot{v}_t$, and the expected loss $\mu_t v_t$ due to creative destruction.

There is a unit continuum of entrepreneurs who employ R&D labor for innovation. Suppose that an entrepreneur $\omega \in [0, 1]$ who undertakes at intensity $\mu_t(\omega)$ for a time interval of length dt achieves success with a probability of $\mu_t(\omega)dt$. We assume that the resource cost of research effort depends on the size of the innovation that the entrepreneur pursues. In particular, research at intensity $\mu_t(\omega)$ requires $\mu_t(\omega)f(\lambda)$ units of labor, where $f'(\lambda) > 0$ and $f''(\lambda) > 0$. The R&D cost is thus given by $\mu_t(\omega)f(\lambda)w_t$. The entrepreneur ω chooses λ and $\mu_t(\omega)$ at every moment to maximize her expected profit such that

$$\max_{\{\lambda, \mu_t(\omega)\}} \mu_t(\omega)v_t(\lambda)dt - \mu_t(\omega)f(\lambda)w_t dt.$$

The optimal choice of quality increment satisfies the following first-order condition:

$$v_t'(\lambda) = f'(\lambda)w_t. \tag{12}$$

which equates the marginal benefit of a larger innovation to the marginal cost of achieving it. The maximization of net benefits from R&D with respect to the choice of research intensity yields the zero-expected-profit condition such that

$$v_t(\lambda) = f(\lambda)w_t. \tag{13}$$

Moreover, in equilibrium, the unit measure of entrepreneurs implies that the aggregate research intensity (i.e., the innovation rate) is equal to the counterpart at the individual level, namely, $\mu_t \equiv \int_0^1 \mu_t(\omega)d\omega$.

[8] See, for example, Cozzi, Giordani and Zamparelli (2007) for a theoretical justification for the symmetric equilibrium in this strand of Schumpeterian growth model.

2.4 Monetary Authority

The monetary sector is formulated as in Arawatari *et al.* (2018). The monetary authority controls the nominal interest rate i, which is kept constant over time such that $i_t = i > 0$ for all time $t > 0$. The seigniorage revenue is rebated to households via lump-sum transfers. Denote by M_t the nominal money supply at time t. Thus, the budget constraint is given by $\tau_t = \dot{M}_t / P_t$, where P_t is the nominal price of the final good.

2.5 General equilibrium

Definition 1. *The general equilibrium consists of a sequence of prices $\{P_t, w_t, r_t, i_t, p_t(j), v_t\}_{t=0}^{\infty}$ and allocations $\{c_t, a_t, m_t, y_t, L_t, L_{x,t}, L_{r,t}\}_{t=0}^{\infty}$ such that the representative household maximizes utility taking $\{r_t, w_t\}$ as given; competitive final-good firms produce $\{y_t\}$ to maximize profits taking $\{p_t(j)\}$ as given; each differentiated intermediate-good producer j produces $x_t(j)$ and chooses $\{L_{x,t}(j), p_t(j)\}$ to maximize profits taking $\{w_t\}$ as given; entrepreneurs choose $\{\mu_t, \lambda\}$ to maximize expected profits taking $\{w_t\}$ as given; and all markets clear. That is, the final-good and asset markets clear such that $c_t = y_t$ and $a_t = v_t$, respectively, where v_t is the aggregate firm value. The labor-market-clearing condition is*

$$L_{x,t} + L_{r,t} = L_t, \tag{14}$$

where $L_{x,t} \equiv \int_0^1 L_{x,t}(j)dj$ and $L_{r,t} = \int_0^1 \mu_t(\omega) f(\lambda) d\omega = \mu_t f(\lambda)$ are the aggregate demand of manufacturing labor and R&D labor, respectively.

Then we obtain the following result.

Lemma 1. *Holding constant the nominal interest rate i, the economy immediately jumps to a unique and stable balanced growth path along which each variable grows at constant (possibly zero) rate.*

Proof. See Appendix A. □

On the steady state, the firm value v_t grows at the rate of consumption and final good, and labor allocations are stationary. By applying the Euler equation (4) and the no-arbitrage condition (11), we can obtain the steady-state value of innovation such that

$$v_t(\lambda) = \frac{\Pi_t}{\rho + \mu}. \tag{15}$$

Now $v_t'(\lambda)$ can be calculated by using (15). Substituting $v_t'(\lambda)$ and (15) into the two first-order conditions for each entrepreneur (i.e., (12) and (13)), we have

$$\frac{v_t'(\lambda)}{v_t(\lambda)} = \frac{f'(\lambda)}{f(\lambda)} \Leftrightarrow \frac{1}{(\lambda - 1) - \lambda \kappa w_t (1 + \alpha i)/y_t} = \frac{\lambda f'(\lambda)}{f(\lambda)} \equiv \epsilon \Leftrightarrow \lambda = \frac{1 + 1/\epsilon}{1 - \kappa w_t (1 + \alpha i)/y_t}, \tag{16}$$

where ϵ is defined as the elasticity of the resource requirement with respect to the size of the attempted innovation. Notice that each entrepreneur takes the aggregate research intensity μ as given.

By substituting (9) and (15) into (13), the steady-state ratio of output and wage is given by

$$\frac{y_t}{w_t} = \frac{(\rho+\mu)f(\lambda)+\kappa(1+\alpha i)}{(\lambda-1)/\lambda}. \tag{17}$$

Substituting (17) into (10), together with the fact that $L_{x,t} = L_{x,t}(j)$, yields the aggregate demand of manufacturing labor such that

$$L_x = \kappa + \frac{(\rho+\mu)f(\lambda)+\kappa(1+\alpha i)}{(\lambda-1)(1+\alpha i)}. \tag{18}$$

Moreover, using (5) and (17), we can rewrite the aggregate labor supply as

$$L = 1-\theta(1+\xi i)\frac{c_t}{w_t} = 1-\theta(1+\xi i)\frac{(\rho+\mu)f(\lambda)+\kappa(1+\alpha i)}{(\lambda-1)/\lambda}, \tag{19}$$

where the final-good resource condition has been applied. Next, substituting (18) and (19) into the labor-market-clearing condition (14) yields the following equation:

$$\begin{aligned} &\kappa+\mu f(\lambda)+\frac{f(\lambda)(\rho+\mu)+\kappa(1+\alpha i)}{(\lambda-1)(1+\alpha i)}+\theta(1+\xi i)\frac{(\rho+\mu)f(\lambda)+\kappa(1+\alpha i)}{(\lambda-1)/\lambda}=1 \\ \Leftrightarrow &\mu = \frac{\frac{(1-\kappa)(\lambda-1)(1+\alpha i)}{f(\lambda)} - \left[\frac{\kappa(1+\alpha i)}{f(\lambda)}+\rho\right]\left[1+\theta\lambda(1+\xi i)(1+\alpha i)\right]}{1+(\lambda-1)(1+\alpha i)+\lambda\theta(1+\xi i)(1+\alpha i)}, \end{aligned} \tag{20}$$

which contains two endogenous variables $\{\lambda, \mu\}$. The other equation for solving the model is obtained by inserting (17) into (16) such that

$$\lambda - \frac{\kappa(1+\alpha i)(\lambda-1)}{f(\lambda)(\rho+\mu)+\kappa(1+\alpha i)} = 1+1/\epsilon \Leftrightarrow \mu = \frac{\kappa(1+\alpha i)/\epsilon}{(\lambda-1-1/\epsilon)f(\lambda)} - \rho. \tag{21}$$

Given the equilibrium innovation arrival rate and size of quality increment, we derive the growth rate of output by substituting (8) into (6) to rewrite the production function of final good such that

$$y_t = Q_t L_x. \tag{22}$$

In this equation, Q_t is the aggregate technology level and defined as

$$Q_t = \exp\left(\int_0^1 n_t(j)dj \ln\lambda^*\right) = \exp\left(\int_0^t \mu_s ds \ln\lambda^*\right),$$

where the second equality applies the law of large number. Accordingly, the steady-state growth rate of final good and of technology is given by

$$g \equiv \frac{\dot{y}_t}{y_t} = \frac{\dot{Q}_t}{Q_t} = \mu^* \ln \lambda^*. \tag{23}$$

Before closing this section, we show that our analysis on how the nominal interest rate relates to quality increment, economic growth, and social welfare, also applies to the counterpart on how inflation relates to those variables, as justified in Chu and Cozzi (2014) and Chu *et al.* (2017). To see this, we combine the Fisher equation and the Euler equation to show that the inflation rate is given by $\pi = i - r = i - g(i) - \rho$. As long as $\partial g(i)/\partial i < 1$, we have $\partial \pi/\partial i = 1 - \partial g(i)/\partial i > 0$.[9] This positive *long-run* relationship between the inflation rate and the nominal interest rate is also supported by the empirical evidence in Mishkin (1992) and Booth and Ciner (2001).

3 Implications of monetary policy

In this section, we analyze the effects of monetary policy on the optimal size of quality increment, innovation, economic growth, and social welfare. In Subsection 3.1, we consider a special case in which the CIA constraint is only on consumption. In Subsection 3.2, we consider the general case of both CIA constraints on consumption and manufacturing. In the next section (i.e., Section 4), we numerically evaluate the impacts of monetary policy on the aforementioned variables along with social welfare.

3.1 Monetary effects under CIA constraint on consumption

To better understand how monetary policy affects the real aspect, we first consider the special case where CIA constraint is exclusively on consumption. When the manufacturing activities are no longer constrained by cash, which can be obtained by setting $\alpha = 0$, (20) and (21) are reduced to

$$\mu = \frac{\frac{(1-\kappa)(\lambda-1)}{f(\lambda)} - \left[\frac{\kappa}{f(\lambda)} + \rho\right]\left[1 + \theta\lambda(1+\xi i)\right]}{1 + (\lambda - 1) + \lambda\theta(1+\xi i)}, \tag{24}$$

and

$$\mu = \frac{\kappa/\epsilon}{(\lambda - 1 - 1/\epsilon) f(\lambda)} - \rho, \tag{25}$$

respectively. Equation (24) features a positive slope and a positive $\lambda-$intercept in the $\{\lambda, \mu\}$ space as shown in Figure 1; (20) is denoted as the "labor condition". In addition, equation (25) also contains two endogenous variables $\{\mu, \lambda\}$ but features a negative slope, with no intercepts, in the $\{\lambda, \mu\}$ space as shown in Figure 1; (21) is denoted as the "R&D condition". The intersection

[9] Under our calibrated parameter values, steady-state inflation is increasing in the nominal interest rate.

at point O in Figure 1 determines the unique steady-state values for μ and λ.[10]

Figure 1 shows that an increase in the nominal interest rate shifts down the "labor condition" curve and leaves the "R&D condition" curve unaffected, leading to a lower innovation rate accompanied by a larger size of quality increment. Intuitively, due to the CIA constraint on consumption, (5) shows that a higher nominal interest rate raises the opportunity cost of consumption, causing households to substitute for leisure. As a consequence, the decline in labor supply drives up the real wage rate, inducing two opposing effects on economic growth. On the one hand, the rise in the wage rate decreases the monopoly profit flow for a given size of quality increment, as shown in (9). This in turn induces entrepreneurs to pursue a more radical innovation with a higher innovating firm value. On the other hand, the rise in the wage rate increases the R&D cost, which discourages the R&D incentive and thus reduces the innovation rate. Moreover, an attempt of a more radical innovation is associated with more R&D labor demand and a larger R&D cost, which reinforces the negative impact of a rise in the nominal interest rate on the innovation arrival rate. The above results are summarized in the following Proposition 1.

Proposition 1. *Under the endogenous quality step size λ^*, a higher nominal interest rate decreases the arrival rate of innovation but increases the size of quality increment.*

Proof. Proven in the text. □

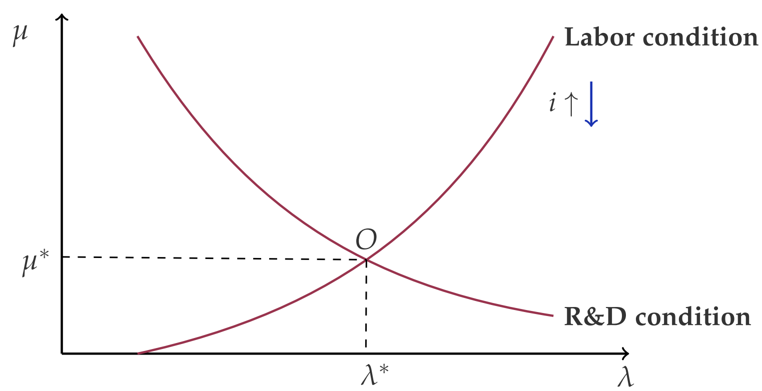

Figure 1: The steady-state equilibrium under a CIA constraint on consumption.

Differentiating (23) with respect to the nominal interest rate i yields

$$\frac{\partial g}{\partial i} = \underbrace{\frac{\partial \mu^*}{\partial i}}_{<0} \cdot \ln \lambda^* + \underbrace{\frac{\partial \lambda^*}{\partial i}}_{>0} \cdot \frac{\mu^*}{\lambda^*}.$$

In an economy in which the quality increment is exogenously given, the channel of changing the size of quality increment through which monetary policy affects economic growth is shut down,

[10]See Appendix A.2 for the details for which the intersection between the labor condition (20) and the R&D condition (21) is unique.

i.e., $\partial\lambda^*/\partial i = 0$. In this case, the economic growth rate g is a decreasing function of the nominal interest rate i, as in the existing studies such as Chu and Cozzi (2014). Nevertheless, in the economy in which the quality increment can be endogenously determined by the entrepreneurs, a change in the nominal interest rate can affect the economic growth rate through the size of quality increment in addition to the frequency of innovation. This is the novel mechanism in our model that could cause a non-monotonic effect of the nominal interest rate on the economic growth rate.

3.2 Monetary effects under CIA constraints on consumption and manufacturing

We now proceed to the general case with CIA constraints on consumption and manufacturing. Figure 2 describes the effects of a higher nominal interest rate on the quality step size and the innovation arrival rate. Comparing (20) and (21) to (24) and (25), it is obvious that the presence of an additional CIA constraint on manufacturing causes the "R&D condition" to rise, but leads to an ambiguous impact on the "Labor condition".

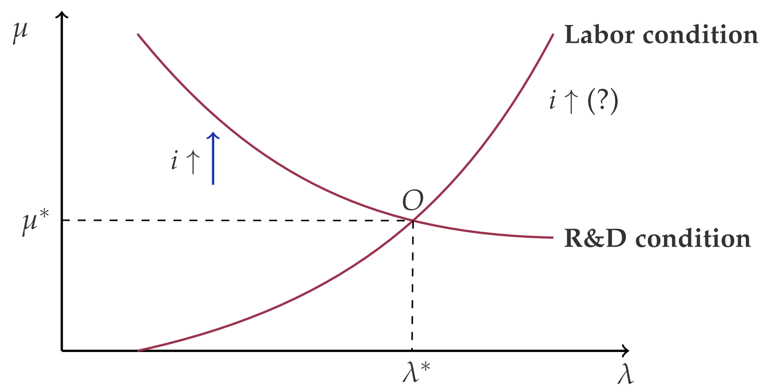

Figure 2: The steady-state equilibrium under CIA constraints on consumption and manufacturing.

In this case, the overall impact of an increase in the nominal interest rate on the quality increment and innovation becomes ambiguous. The intuition for this result is as follows. On the one hand, with higher nominal interest rate, imposing a CIA constraint on manufacturing further reduces monopoly profit, which reinforces the negative effect from rising real wage rate. Both these effects motivate entrepreneurs to pursue an even more radical innovation aiming to set a larger price markup and gain a higher profit flow. On the other hand, a CIA constraint on manufacturing creates incentives for labor reallocation from the manufacturing sector to the R&D sector, which mitigates the negative effect of inflation on R&D originating from the consumption-leisure decision channel. Whether a higher nominal interest rate increases or decreases the quality increment and innovation depends on the relative magnitude of the above effects. Given this ambiguity, we provide a discussion in the numerical analysis that follows.

4 Quantitative analysis

In this subsection, we calibrate the model to the US data and numerically evaluate the effects of nominal interest rate (and inflation rate) on quality increment, innovation, economic growth and social welfare, respectively. To facilitate the analysis, we assume the functional form $f(\lambda) = \beta\lambda^5$ as the benchmark and consider alternative functions in the sensitivity analysis.[11] To perform this quantitative analysis, we assign steady-state values to the structural parameters $\{\rho, \xi, \alpha, \theta, \kappa, \beta\}$. The discount rate ρ is set to a conventional value of 0.02. As for the strength of the CIA constraint on consumption (i.e., ξ), we follow Zheng *et al.* (2019) to set it to 0.17, for matching the ratio of M1-consumption in the US. As for the strength of the CIA constraint on manufacturing, we follow Arawatari *et al.* (2018) to set $\alpha = 1$ as the benchmark. To pin down the value of remaining parameters, we match the following long-run empirical moments. (a) The conventional value of the economic growth rate is 2%; (b) The long-run average inflation rate in the US is about $\pi = 2.5\%$. Thus, the nominal interest rate in the steady state is determined by the Fisher equation such that $i = r + \pi = \rho + g + \pi = 6.5\%$; (c) The standard time of employment to 1/3; (d) The arrival rate of innovation μ^* is set to 8% as the benchmark value.[12] Table 1 summarizes these moments and calibrated parameter values.

Table 1: Parameter values and targeted moments

Targeted moments		Parameters	
Innovation arrival rate	8%	ρ	0.02
M1-consumption ratio	0.17	ξ	0.17
Economic growth rate	2%	α	1
Time of employment	1/3	κ	0.0223
Average inflation rate	2.5%	θ	1.8146
		β	0.1622

4.1 Results

Given the benchmark estimated parameters, we now quantify the impacts of the nominal interest rate (and the inflation rate) on the quality increment, the innovation rate, the economic growth rate, and the social welfare, respectively. Figure 3a and 3b display that the size of quality

[11] We consider $f(\lambda) = \beta\lambda^5$ for the following reason. Assuming $f(\lambda) = \beta\lambda^5$ means the elasticity is $\epsilon = 5$. According to (21), $\lambda > 1 + 1/\epsilon = 1.2$ must hold. As shown below, given the conventional economic growth rate and arrival rate of innovation, the benchmark quality step size, namely the price markup, is 1.284. In general, the market value of price markup is lower than 1.4 (see, for example, Jones and Williams (2000)). Therefore, to correspond to the empirical evidence, we take $f(\lambda) = \beta\lambda^5$ as the benchmark. We also consider a sensitivity analysis on the function form of $f(\lambda)$ in Subsection 4.2.

[12] The existing literature has considered different values. For example, using a structural model to estimate, Caballero and Jaffe (2002) report an innovation arrival rate of 4%. Laitner and Stolyarov (2013) find the roughly same value (i.e., 3.5%), whereas Lanjouw (1998) shows that the probability of obsolescence is in the range of 7%-12%. We thus select an intermediate value in this exercise.

increment is increasing in the inflation rate, but the arrival rate of innovation is decreasing in it. When raising the inflation rate from -0.0400 (i.e., $i = 0$) to 0.1601 (i.e., $i = 0.2$), the quality step size rises from 1.2795 to 1.2937, whereas the arrival rate of innovation declines from 0.0810 to 0.0773. As a result, the growth rate of output becomes an inverted-U function of the inflation rate. Figure 4a shows that the growth-maximizing inflation rate is around 3.87%, which is consistent with the estimates in a number of empirical evidence such as Burdekin *et al.* (2004) and Kremer *et al.* (2013). This result indicates that the positive effect of inflation on the quality increment dominates the negative effect of inflation on the innovation arrival rate when the inflation rate is in a low level, and the positive effect is dominated by the negative one when the inflation rate becomes sufficiently high.

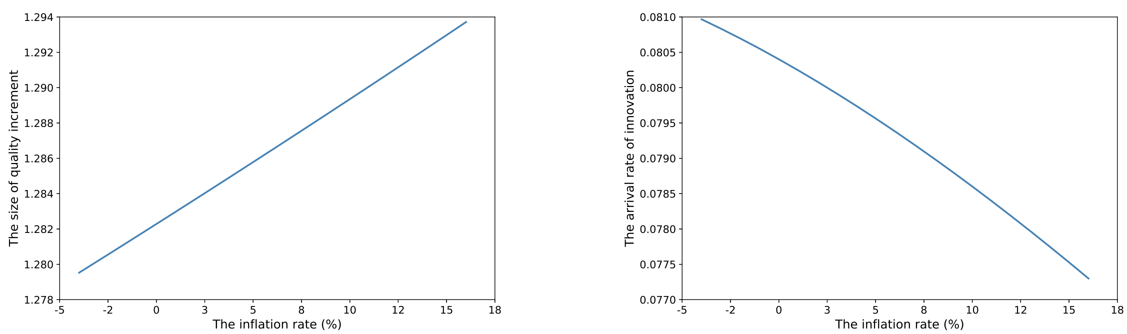

Figure 3: (a) Inflation and size of quality increment; (b) Inflation and arrival rate of innovation.

To explore the welfare effect of inflation, we derive the steady-state welfare function. This is obtained by imposing balanced growth on (1), which yields

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) = \frac{1}{\rho}\left(\ln Q_0 + \ln L_x + \frac{g}{\rho}\right), \tag{26}$$

where Q_0 is normalized to unity, and L_x and $g = \mu \ln \lambda$ are given by in (18) and (23), respectively. Figure 4b shows that the social welfare level is decreasing in the inflation rate. For example, raising the inflation rate from -0.0400 to 0.1601 causes the social welfare U to decline from -9.7924 to -17.8742. This result implies that the optimality of the Friedman rule holds in this case.

4.2 Robustness analysis

In this subsection, we conduct two experiments: one is to reduce the strength of the CIA constraint on manufacturing to zero, and the other is to examine the extent to which the quantitative results would change under an alternative function of $f(\lambda) = \beta\lambda^3$.

We first consider the case of the CIA constraint only on consumption (i.e., $\alpha = 0$). By keeping

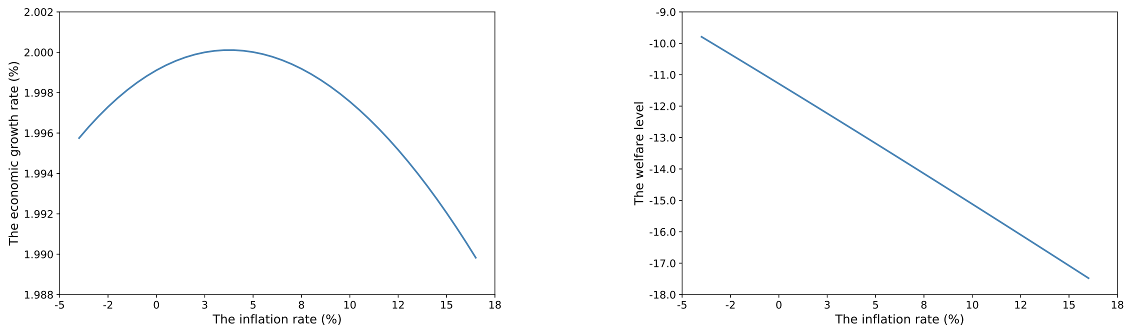

Figure 4: (a) Inflation and economic growth; (b) Inflation and social welfare.

other parameter values unchanged as in the benchmark, we evaluate the impacts of inflation on the interested variables. Figure 5a, 5b and 6b show that, similar to the previous benchmark case, the size of quality step is increasing in the inflation rate and the innovation arrival rate is decreasing in it; these results are consistent with the implications of Proposition 1. However, the growth rate of output is now a monotonically decreasing function of the inflation rate as described in Figure 6a. Recalling the analysis in Subsection 3.2, when the CIA constraint on manufacturing is present, the growth-promoting effect of higher inflation is two-fold as follows: (a) higher inflation reduces the monopoly profit, which tends to induce entrepreneurs to pursue a more radical innovation; (b) this more radical innovation reallocates labor from the intermediate-good sector to the R&D sector, which tends to raise the innovation arrival rate. When the CIA constraint on manufacturing is absent, these two layers of the positive growth force are significantly weakened, leading to a monotonically decreasing effect of inflation on economic growth in the dominant position.

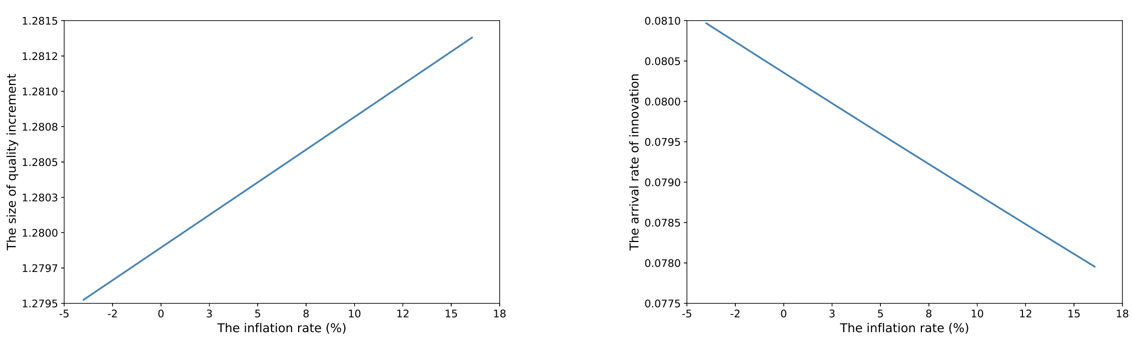

Figure 5: (a) Inflation and size of quality increment ($\alpha = 0$); (b) Inflation and arrival rate of innovation ($\alpha = 0$).

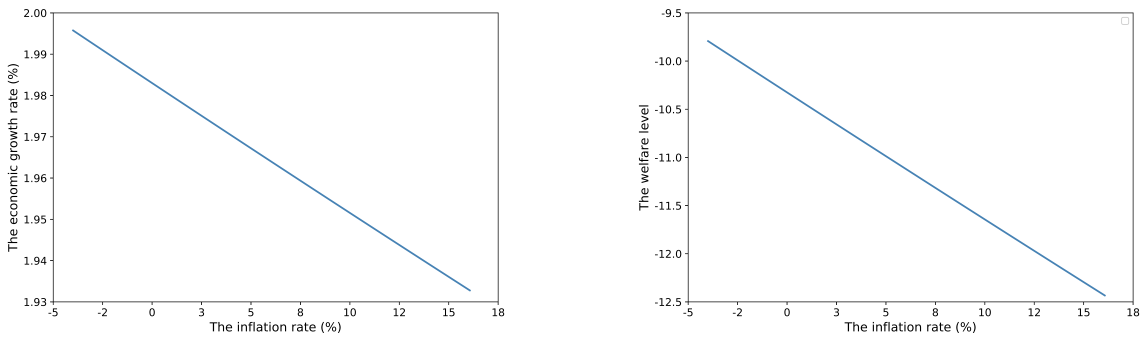

Figure 6: (a) Inflation and economic growth ($\alpha = 0$); (b) Inflation and social welfare ($\alpha = 0$).

Next, we examine the robustness of quantitative results under $f = \lambda^3$, while keeping other parameter values unchanged as in the benchmark. The results regarding the impacts of inflation on the size of quality increment, the arrival rate of innovation, the economic growth rate and the social welfare are reported in Figure 7a, 7b, 8a and 8b, respectively. It is shown that our model results are robust to this functional change. For example, raising the inflation rate still increases the quality step size and decreases the innovation arrival rate and welfare level. Moreover, despite of a larger threshold value of inflation rate (i.e., 10.3%), the growth rate of output continues to be a hump-shaped function of the inflation rate.

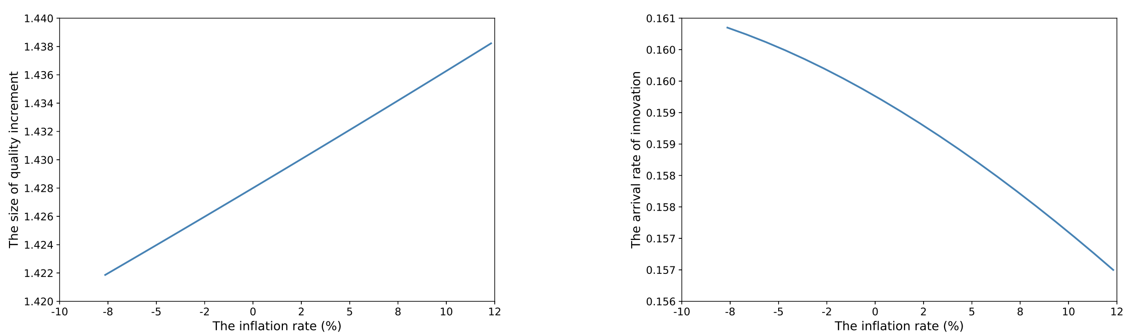

Figure 7: (a) Inflation and size of quality increment ($f(\lambda) = \lambda^3$); (b) Inflation and arrival rate of innovation ($f(\lambda) = \lambda^3$).

4.3 An extension of a CIA constraint on R&D

When entrepreneurs' R&D activities are constrained by cash, they make borrowing from households to facilitate the wage payment for R&D labor and make returns based on the nominal interest rate i. In this case, the R&D cost for a typical firm $\omega \in [0, 1]$ is given by $\mu_t(\omega) f(\lambda) w_t(1 +$

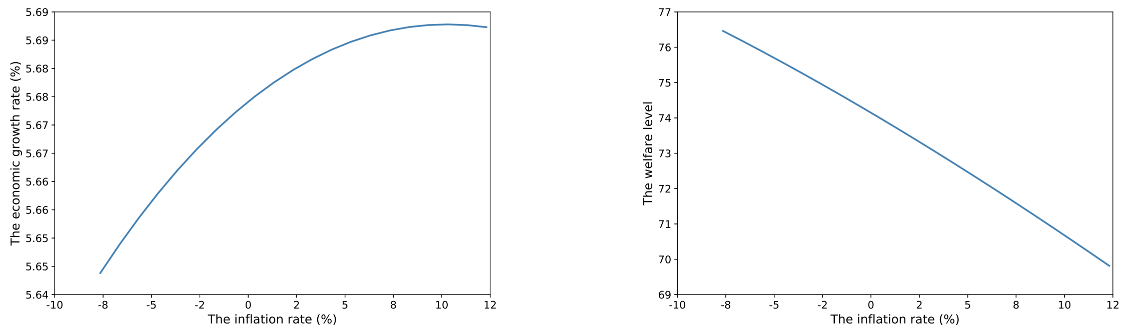

Figure 8: (a) Inflation and economic growth ($f(\lambda) = \lambda^3$); (b) Inflation and social welfare ($f(\lambda) = \lambda^3$).

ηi), where $0 \leq \eta \leq 1$ represents the degree of the CIA constraint on R&D. Accordingly, the two first-order conditions in (12) and (13) now are given by

$$v_t'(\lambda) = f'(\lambda) w_t (1 + \eta i), \tag{27}$$

$$v_t(\lambda) = f(\lambda) w_t (1 + \eta i). \tag{28}$$

After some manipulations, we can derive the output to wage ratio, the aggregate demand for manufacturing labor, and the aggregate labor supply such that[13]

$$\frac{y_t}{w_t} = \frac{(\rho + \mu) f(\lambda)(1 + \eta i) + \kappa}{(\lambda - 1)/\lambda}, \tag{29}$$

$$L_x = \kappa + \frac{(\rho + \mu)(1 + \eta i) f(\lambda) + \kappa}{(\lambda - 1)}, \tag{30}$$

$$L = 1 - \theta(1 + \xi i) \frac{(\rho + \mu)(1 + \eta i) f(\lambda) + \kappa}{(\lambda - 1)/\lambda}. \tag{31}$$

Solving the model yields the two steady-state conditions for λ and μ such that

$$\mu = \frac{\frac{(1-\kappa)(\lambda-1)}{f(\lambda)} - \left[\frac{\kappa}{f(\lambda)} + \rho(1 + \eta i)\right][1 + \theta\lambda(1 + \xi i)]}{\eta i + \lambda + \theta\lambda(1 + \xi i)(1 + \eta i)}, \tag{32}$$

$$\mu = \frac{\kappa/\epsilon}{(\lambda - 1 - 1/\epsilon) f(\lambda)(1 + \eta i)} - \rho. \tag{33}$$

As shown in Figure 9, a higher nominal interest rate shifts down both the "Labor condition" and "R&D condition" curves. Therefore, the innovation arrival rate is lowered unambiguously.

[13] To focus on how the incorporation of CIA constraint on R&D affects the model results, we do not consider the CIA constraint on manufacturing in this extension for simplicity.

However, the impact on the size of quality increment can be either positive or negative.

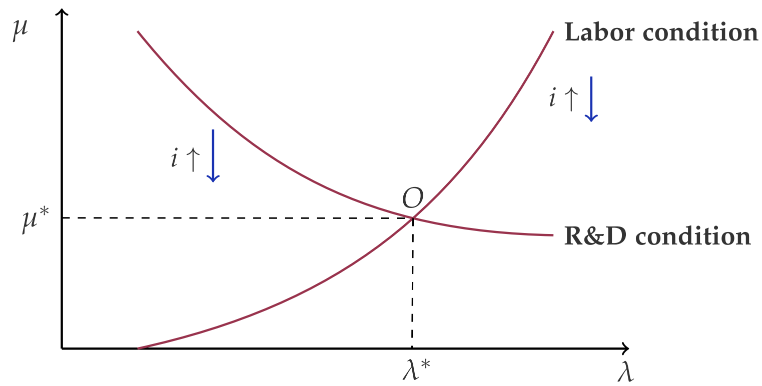

Figure 9: The steady-state equilibrium under CIA constraints on consumption and R&D.

Similar to the previous exercises, we resort to a quantitative analysis to evaluates the effect of inflation on the quality step size, the innovation arrival rate, economic growth, and social welfare, respectively, in this extension. We recalibrate this extended model to pin down the value of the parameter η. In addition to the moments used in the benchmark, we use the R&D labor share in the US for calibration. Specifically, we use the ratio of scientists and engineers engaged in R&D over the manufacturing labor force, which is around 4.2%.[14] The calibrated parameter values are reported in Table 2.

Table 2: Parameter values and targeted moments

ρ	ξ	κ	θ	β	η
0.02	0.17	0.0226	1.9108	0.1504	0.4526

Given the above recalibrated parameters, we quantify the effects of inflation on the aggregate variables. In the presence of CIA constraints on both consumption expenditure and innovative activities, Figure 10a shows that the size of quality increment is still increasing in inflation. In addition, the innovation arrival rate remains as a decreasing function of inflation, as described in Figure 10b. Intuitively, when the CIA constraint on R&D is present, a higher nominal interest rate (and the inflation rate) generates an additional negative impact on the innovation arrival rate, since the increase in the R&D cost discourages R&D incentives. Moreover, the lowered R&D labor demand mitigates the rise in the real wage rate and weakens the impact of the nominal interest rate on the monopoly profit, which in turn lessens the positive growth effect due to a large quality increment. Therefore, a higher inflation rate continues to result in a lower economic growth rate, as in the benchmark case. Figure 11a shows that raising the nominal interest rate from 0 to 20

[14]The number of scientists and engineers engaged in R&D is obtained from Science and Engineering Indicators 2000 (Appendix Tables 3-25) published by the National Science Foundation. The data on manufacturing employees are obtained from the Bureau of Labor Statistics.

percentage point causes a decline in the economic growth rate by 8.521% (percentage), and this magnitude is larger than the one in the benchmark case (i.e., 3.246%). As for the welfare effect of inflation, Figure 11b indicates that the Friedman rule still can lead to a socially optimal outcome.

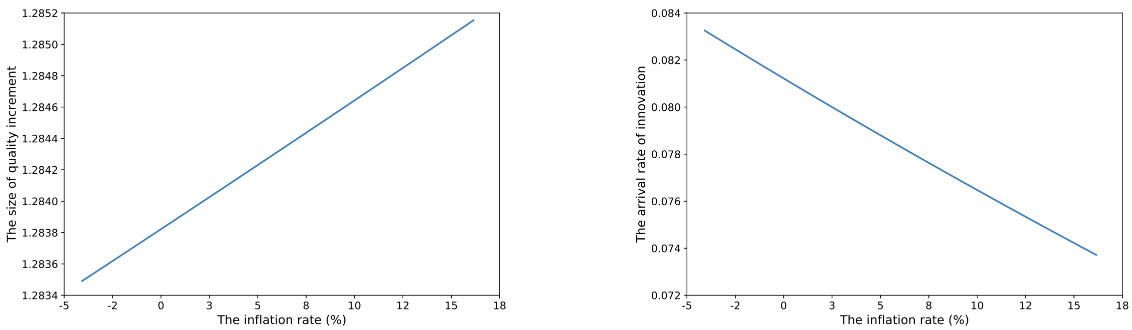

Figure 10: (a) Inflation and size of quality increment ($\eta = 0.4526$); (b) Inflation and arrival rate of innovation ($\eta = 0.4526$).

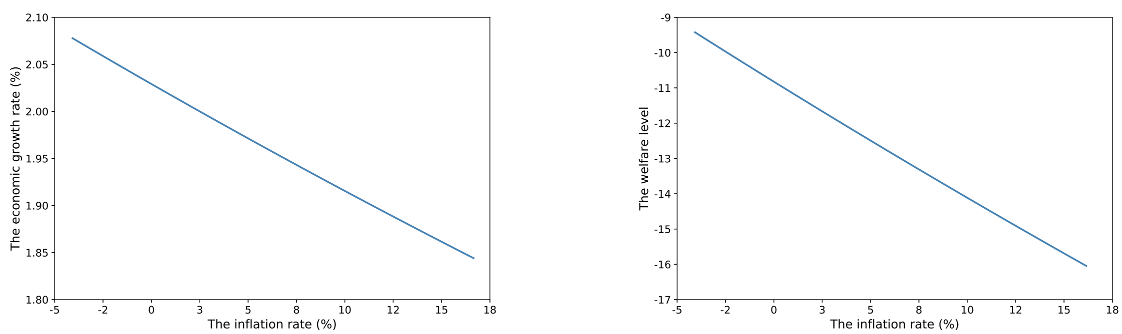

Figure 11: (a) Inflation and economic growth ($\eta = 0.4526$); (b) Inflation and social welfare ($\eta = 0.4526$).

5 Conclusion

In this study, we analyze the effects of monetary policy on quality increment, innovation, economic growth and social welfare, respectively. In the model with only a CIA constraint on consumption, we find that a higher nominal interest rate induces R&D firms to pursue a larger quality step size, which would stimulate economic growth. Nevertheless, a higher nominal interest rate raises the R&D cost and tends to depress innovation and economic growth. The CIA constraint on manufacturing reinforces the positive growth effect and weakens the negative effect. In contrast, the CIA constraint on R&D strengths exclusively the positive growth effect.

By calibrating our model to the US economy, we find that the economic growth rate can be either a monotonically decreasing or hump-shaped function of the inflation rate, whereas the social welfare is always decreasing in inflation.

This study can be extended in two directions. First, by normalizing the population size to unity, this study sterilizes the strong scale-effect problem present in the first-generation endogenous growth model such as in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Alternatively, one may remove the scale effects in the Schumpeterian growth model by considering the semi-endogenous-growth approach as in Kortum (1997) and Segerstrom (1998) or the second-generation approach as in Peretto (1998) and Howitt (1999b). Second, monetary policy in this study is introduced by imposing CIA constraints in different sectors. One may revisit how the impacts of inflation on nominal macroeconomic variables would change in a Schumpeterian growth model with endogenous quality increment if other formulations that incorporate monetary policy, such as money-in-utility function in Chu and Lai (2013) and price rigidity (via menu costs) in Oikawa and Ueda (2018), are considered. Due to its complexity, we leave these potentially interesting extensions to future research.

Appendix A

A.1 Proof of Lemma 1

Suppose that a time path of $[i_t]_{t=0}^{\infty}$ is stationary such that $i_t = i$ for all t. Define a transformed variable by $\Phi_t \equiv y_t / v_t$. Therefore, its law of motion is given by

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{y}_t}{y_t} - \frac{\dot{v}_t}{v_t}. \tag{A.1}$$

Using the final-good resource condition $c_t = y_t$ and the Euler equation in (4), the law of motion for y_t is

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{A.2}$$

From (11), the law of motion for v_t is

$$\frac{\dot{v}_t}{v_t} = r_t + \mu_t - \frac{\Pi_t}{v_t}, \tag{A.3}$$

where $\mu_t = L_{r,t}/f(\lambda)$ and Π_t stems from (9). Substituting (A.2) and (A.3) into (A.1) yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \left(\frac{\lambda - 1}{\lambda}\right) \Phi_t - \frac{\kappa}{f(\lambda)} - \frac{L_{r,t}}{f(\lambda)} - \rho, \tag{A.4}$$

where $v_t = f(\lambda)w_t$ in (13) has been applied. To derive a relationship between $L_{r,t}$ and Φ_t, we first use (18) and (13) to derive

$$L_{x,t} = \kappa + \frac{(y_t/v_t)(v_t/w_t)}{\lambda} = \kappa + \frac{\Phi_t f(\lambda)}{\lambda}. \tag{A.5}$$

In addition, substituting $c_t = y_t$ and (13) into (5) yields

$$L_t = 1 - \theta(1+\xi i)\frac{c_t}{w_t} = 1 - \theta(1+\xi i)\Phi_t f(\lambda). \tag{A.6}$$

Then, substituting (A.5) and (A.6) into the labor-market-clearing condition yields

$$L_{r,t} = L_t - L_{x,t} = 1 - \kappa - f(\lambda)\Phi_t\left[\theta(1+\xi i) + \frac{1}{\lambda}\right]. \tag{A.7}$$

Substituting (A.7) into (A.4) yields an autonomous dynamical equation of Φ_t such that

$$\frac{\dot{\Phi}_t}{\Phi_t} = [1+\theta(1+\xi i)]\Phi_t - \left[\frac{1}{f(\lambda)} + \rho\right]. \tag{A.8}$$

Given that λ is stationary over time and Φ_t is a control variable, the coefficient associated with Φ_t being positive implies that the dynamics of Φ_t is characterized by saddle-point stability such that Φ_t jumps immediately to its steady-state value given by

$$\Phi = \frac{1/f(\lambda) + \rho}{1+\theta(1+\xi i)}. \tag{A.9}$$

Equations (A.5), (A.6), and (A.7) imply that if Φ is stationary, then L_x, L_r, and L must all be stationary as well.

A.2 Uniqueness of the steady-state equilibrium

For any given i, differentiating (24) with respect to λ yields

$$\begin{aligned}
&\frac{\partial \mu}{\partial \lambda} \gtreqless 0 \\
\Leftrightarrow &\frac{(1-\kappa)\left\{\lambda f(\lambda) - (\lambda-1)[\lambda f'(\lambda) + f(\lambda)]\right\}}{[\lambda f(\lambda)]^2} + \frac{\rho}{\lambda^2} - \frac{\kappa\theta\lambda f(\lambda)(1+\xi i) - \kappa[1+\theta\lambda(1+\xi i)][\lambda f'(\lambda) + f(\lambda)]}{[\lambda f(\lambda)]^2} \gtreqless 0 \\
\Leftrightarrow &\frac{(1-\kappa)\left[\lambda - (\lambda-1)(1+\epsilon)\right]}{f(\lambda)} + \rho - \frac{\kappa\theta\lambda(1+\xi i) - \kappa[1+\theta\lambda(1+\xi i)](1+\epsilon)}{f(\lambda)} \gtreqless 0 \\
\Leftrightarrow &(1-\kappa)\left[\lambda - (\lambda-1)(1+\epsilon)\right] + \rho f(\lambda) + \kappa[1+\epsilon+\theta\lambda\epsilon(1+\xi i)] \gtreqless 0 \\
\Leftrightarrow &(1-\kappa)(1+\epsilon-\lambda\epsilon) + \rho f(\lambda) + \kappa[1+\epsilon+\theta\lambda\epsilon(1+\xi i)] \gtreqless 0 \\
\Leftrightarrow &1+\epsilon+\rho f(\lambda) + \lambda\epsilon[\kappa - 1 + \kappa\theta(1+\xi i)] \gtreqless 0.
\end{aligned} \tag{A.10}$$

Apparently, the left-hand side of the last inequality is an increasing function of κ. As $\kappa \to 1$, the last inequality is reduced to $1 + \epsilon + \rho f(\lambda) + \lambda\epsilon\theta(1 + \xi i) > 0$. As $\kappa \to 0$, the last inequality is reduced to $1 + \epsilon + \rho f(\lambda) - \lambda\epsilon > 0$ if $\lambda < 2$, which holds since the value of λ in empirical studies is generally smaller than 2. Therefore, we obtain $\partial\mu/\partial\lambda > 0$. This implies that μ is a monotonically increasing function of λ and features a positive slope and a positive λ-intercept in the $\{\mu, \lambda\}$ space as shown in Figure 1. Moreover, it is straightforward to verify that (21) implies that μ is a monotonically decreasing function of λ and features a negative slope, with no intercepts,[15] in the $\{\mu, \lambda\}$ space in Figure 1. Therefore, there must exist a unique equilibrium in which λ and μ are solely determined.

References

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, **60** (2), 323–51.

Arawatari, R., Hori, T. and Mino, K. (2018). On the nonlinear relationship between inflation and growth: A theoretical exposition. *Journal of Monetary Economics*.

Banerjee, A., Cockerell, L. and Russell, B. (2001). An i (2) analysis of inflation and the markup. *Journal of Applied Econometrics*, **16** (3), 221–240.

— and Russell, B. (2001). The relationship between the markup and inflation in the g7 economies and australia. *Review of Economics and Statistics*, **83** (2), 377–384.

Barro, R. J. (2013). Inflation and economic growth. *Annals of Economics & Finance*, **14** (1).

Bils, M. (1987). The cyclical behavior of marginal cost and price. *American Economic Review*, pp. 838–855.

Bond, E. W., Wang, P. and Yip, C. K. (1996). A general two-sector model of endogenous growth with human and physical capital: balanced growth and transitional dynamics. *Journal of Economic Theory*, **68** (1), 149–173.

Booth, G. G. and Ciner, C. (2001). The relationship between nominal interest rates and inflation: international evidence. *Journal of Multinational Financial Management*, **11** (3), 269–280.

Burdekin, R. C., Denzau, A. T., Keil, M. W., Sitthiyot, T. and Willett, T. D. (2004). When does inflation hurt economic growth? different nonlinearities for different economies. *Journal of Macroeconomics*, **26** (3), 519–532.

Caballero, R. J. and Jaffe, A. B. (2002). *How High Are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth, In: A. Jaffe & M. Trajtenberg (Eds.), Patents, Citations, and Innovations: A Window on the Knowledge Economy*. Cambridge, MA: The MIT Press.

[15](21) shows that as λ approaches $1 + 1/\epsilon$, μ goes to infinity.

Chang, J.-j., Chang, W.-y., Tsai, H.-f. and Wang, P. (2019). Inflation targeting, pattern of trade, and economic dynamics. *Macroeconomic Dynamics*, **23** (7), 2748–2786.

Chu, A. C. and Cozzi, G. (2014). R&d and economic growth in a cash-in-advance economy. *International Economic Review*, **55** (2), 507–524.

—, —, Furukawa, Y. and Liao, C.-H. (2017). Inflation and economic growth in a schumpeterian model with endogenous entry of heterogeneous firms. *European Economic Review*, **98**, 392–409.

—, —, Lai, C.-C. and Liao, C.-H. (2015). Inflation, r&d and growth in an open economy. *Journal of International Economics*, **96** (2), 360–374.

— and Ji, L. (2016). Monetary policy and endogenous market structure in a schumpeterian economy. *Macroeconomic Dynamics*, **20** (5), 1127–1145.

— and Lai, C.-C. (2013). Money and the welfare cost of inflation in an r&d growth model. *Journal of Money, Credit and Banking*, **45** (1), 233–249.

Cozzi, G., Giordani, P. E. and Zamparelli, L. (2007). The refoundation of the symmetric equilibrium in schumpeterian growth models. *Journal of Economic Theory*, **136** (1), 788–797.

Dotsey, M. and Sarte, P. D. (2000). Inflation uncertainty and growth in a cash-in-advance economy. *Journal of Monetary Economics*, **45** (3), 631–655.

Gali, J., Gertler, M. and Lopez-Salido, J. D. (2007). Markups, gaps, and the welfare costs of business fluctuations. *Review of Economics and Statistics*, **89** (1), 44–59.

Gil, P. M. and Iglésias, G. (2020). Endogenous growth and real effects of monetary policy: R&d and physical capital complementarities. *Journal of Money, Credit and Banking*, **52** (5), 1147–1197.

Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *Review of Economic Studies*, **58** (1), 43–61.

Haskel, J., Martin, C., Small, I. *et al.* (1995). Price, marginal cost and the business cycle. *Oxford Bulletin of Economics and Statistics*, **57** (1), 25–41.

Hori, T. (2017). Monetary policy, financial frictions, and heterogeneous r&d firms in an endogenous growth model. *Scandinavian Journal of Economics*.

Howitt, P. (1999a). Steady endogenous growth with population and r&d inputs growing. *Journal of Political Economy*, **107** (4), 715–730.

— (1999b). Steady endogenous growth with population and r&d inputs growing. *Journal of Political Economy*, **107** (4), 715–730.

Huang, C.-Y., Yang, Y. and Cheng, C.-C. (2017). The growth and welfare analysis of patent and monetary policies in a Schumpeterian economy. *International Review of Economics & Finance*, **52**, 409–426.

—, — and Zheng, Z. (2019). *Monetary policy in a Schumpeterian growth model with Two R&D sectors*. Working paper, Department of Economics, University of Macau.

Jones, C. I. and Williams, J. C. (2000). Too much of a good thing? the economics of investment in r&d. *Journal of Economic Growth*, **5** (1), 65–85.

Khan, M. S. and Senhadji, A. S. (2001). Threshold effects in the relationship between inflation and growth. *IMF Staff papers*, **48** (1), 1–21.

Kortum, S. S. (1997). Research, patenting, and technological change. *Econometrica*, **65** (6), 1389–1420.

Kremer, S., Bick, A. and Nautz, D. (2013). Inflation and growth: new evidence from a dynamic panel threshold analysis. *Empirical Economics*, pp. 1–18.

Laitner, J. and Stolyarov, D. (2013). Derivative ideas and the value of intangible assets. *International Economic Review*, **54** (1), 59–95.

Lanjouw, J. O. (1998). Patent protection in the shadow of infringement: Simulation estimations of patent value. *Review of Economic Studies*, **65** (4), 671–710.

López-Villavicencio, A. and Mignon, V. (2011). On the impact of inflation on output growth: Does the level of inflation matter? *Journal of Macroeconomics*, **33** (3), 455–464.

Lucas, R. E. (1980). Equilibrium in a pure currency economy. *Economic Inquiry*, **18** (2), 203.

Marquis, M. H. and Reffett, K. L. (1994). New technology spillovers into the payment system. *Economic Journal*, pp. 1123–1138.

Martins, J. O. and Scarpetta, S. (2002). Estimation of the cyclical behaviour of mark-ups: a technical note. *OECD Economic studies*, **2002** (1), 173–188.

Mishkin, F. S. (1992). Is the fisher effect for real? : A reexamination of the relationship between inflation and interest rates. *Journal of Monetary Economics*, **30** (2), 195–215.

Nekarda, C. J. and Ramey, V. A. (2013). *The cyclical behavior of the price-cost markup*. Tech. rep., National Bureau of Economic Research.

Oikawa, K. and Ueda, K. (2018). The optimal inflation rate under schumpeterian growth. *Journal of Monetary Economics*, **100**, 114–125.

Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, **3** (4), 283–311.

— and Connolly, M. (2007). The manhattan metaphor. *Journal of Economic Growth*, **12** (4), 329–350.

Phaneuf, L., Sims, E. and Victor, J. G. (2018). Inflation, output and markup dynamics with purely forward-looking wage and price setters. *European Economic Review*, **105**, 115–134.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, **98** (5, Part 2), S71–S102.

Rotemberg, J. J. and Woodford, M. (1991). Markups and the business cycle. *NBER Macroeconomics Annual*, **6**, 63–129.

— and — (1999). The cyclical behavior of prices and costs. *Handbook of Macroeconomics*, **1**, 1051–1135.

Segerstrom, P. S. (1998). Endogenous growth without scale effects. *American Economic Review*, **88** (5), 1290–1310.

Stroebel, J. and Vavra, J. (2019). House prices, local demand, and retail prices. *Journal of Political Economy*, **127** (3), 1391–1436.

Vaona, A. (2012). Inflation and growth in the long run: A new keynesian theory and further semiparametric evidence. *Macroeconomic Dynamics*, **16** (1), 94–132.

Wu, Y. and Zhang, J. (2001). The effects of inflation on the number of firms and firm size. *Journal of Money, Credit and Banking*, pp. 251–271.

Zheng, Z., Huang, C.-Y. and Yang, Y. (2019). Inflation and growth: A non-monotonic relationship in an innovation-driven economy. *Macroeconomic Dynamics*, pp. 1–28.