Estimation of a Supply Function

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CHAPTER 2: ESTIMATION OF A SUPPLY FUNCTION

In this chapter we provide a brief introduction of the production and supply function. The theoretical neoclassical model, has limited capacity to explain data. Recent structural econometric techniques provide better estimation results and are able to explain entry, exit and investment decisions of firms.

1.- Introduction

In a similar way as in demand estimation, where we maximized utility functions, in this chapter we estimate a production function, productivity measures and cost functions. Technological efficiency is represented in the production function while economic efficiency is represented in the cost function. In this chapter we will replicate the main studies on production and cost function using standard software packages such as Excel or R.

Productivity estimates together with demand elasticity are key components of market structure studies. Traditionally productivity was estimated empirically using industry panels using a theoretical production function equation. These estimates were not consistent with data and did not explain market dynamics.

This chapter acknowledges how researchers have refined productivity estimation with the development of a broader dynamical structural approach in a similar way as in demand estimation. Firms decisions on investment strategies will depend on future expected profits than invest in research and development will have a higher probability of staying in the market while others with lower investment and lower productivity will have higher probability of exit. Firms will invest if they have expectations on future profits based on expectations and past achievements.

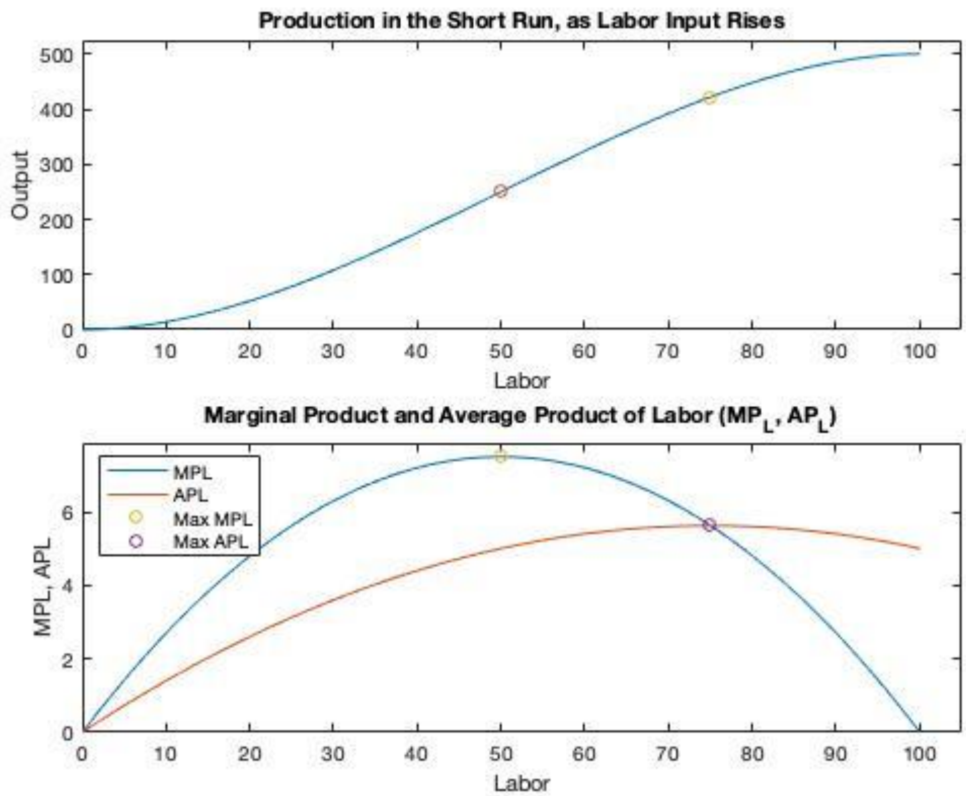
The chapter covers the dual approach of cost functions. Costs are a key component of the benefits and as such knowledge of the costs of a company or industry and the study of increasing returns of scale are fundamental to the analysis of competition. While the theoretical cost functions are generally known from the introductory economics courses, this chapter will also explain further research on increasing returns of scale and scope in industries where competition problems are common such as telecommunication and energy markets. The main concept here is economies of scale or the proportional increase in cost resulting from a small proportional increase in the level of output.

We begin with the traditional production function estimation, its endogeneity problems and further developments followed by cost function estimation developed by Viner (1932) and Sraffa (1926). Next scale economies and multiproduct scope economies are studied. Finally, the chapter explains other alternative methods that measure efficiency as a mere proportion of outputs and inputs while giving up production function determination. These methods are called frontier analysis approach and are divided into Stochastic Frontier Analysis (SFA) and Data Envelope Analysis (DEA).

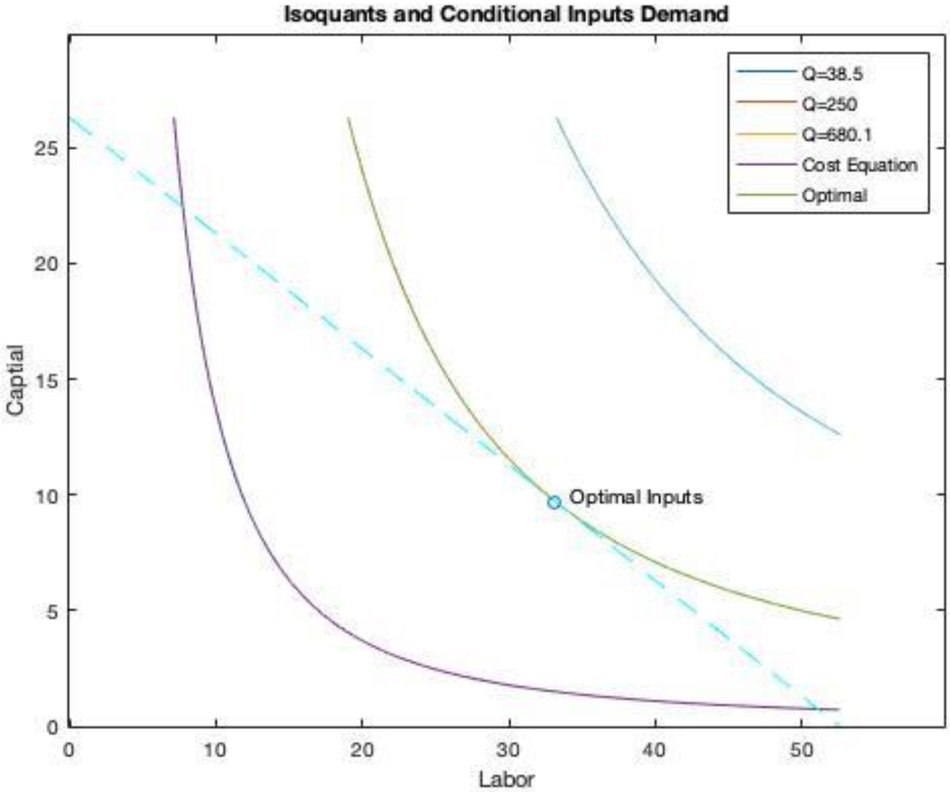
2.- Production function estimation

A production function is a mathematical function that relates the maximum quantity of output that can be obtained with inputs for example capital and labor (2007). Productivity is a measure of efficiency, the contribution of a particular input to the production. Syverson (2011) documented massive differences in total factor productivity (TFP) between industries in the US and stated that productivity is a matter of survival for business in a market. Duration, entry and exit in a particular market will most likely be related to a measure of efficiency or productivity of a firm. This is relevant when discussing market structure.

There are several economic properties of a production functions, namely nonnegativity, non-decreasing in inputs and concavity, that are generally expected and tested (for a complete description of assumptions of production functions in Microeconomics see Mas-Colell et al. (1995)). There are several mathematical functions that fit these assumptions, but Cobb-Douglas, Translog and Quadratic or Cubic production functions have been widely used in productivity analysis as we will see in this Chapter.
Classical graphs of the production function and average and marginal productivity are shown below assuming that only one input remains variable (e.g. Labor), while the rest are fixed. The graph represent a production function that only fulfills the general assumptions explained above in the segment of 50 and 100 of labor input. To the left of 50 labor units, the function is convex and to the right of 100, it is decreasing in labor, so the economically feasible region is only between 50 and 100. In the figure below average productivity AP (or output per unit of input) and marginal productivity MP (increase in output when input increases in one unit) are represented:



If we take into account two inputs the production function is generally represented by a family of isoquants which are combinations of two inputs (capital and labor) that are capable of producing an output level. There is one for each level of output and show higher level of output the farther they are from the origin. The expected shape of the production function implies convex isoquants with the below shape that can be easily minimized. The cost function will be the pairs of minimal cost for each unit of production. Unfortunately the theoretical model is too general and does not explain well specific production and cost data. Authors have dealt with different econometrical problems that surge when fitting easy functional forms to data.



As we mentioned, several mathematical functions that comply with minimal demanded assumptions have been fitted to production data and tested to comply with the abovementioned minimal assumptions.
A common function used in economics is Cobb-Douglas production function (see Ackerberg et al. (2007)):

$$Y = AL^{\beta_L}K^{\beta_K}$$

Where Y is output, K and L are capital and labor inputs, β are parameters and A is total factor productivity[1]. It is usually expressed as a linear equation in natural logarithms:

$$y = \beta_0 + \beta_k k + \beta_l l + \epsilon$$

β_0 is the mean efficiency level while ϵ are unobserved sources of differences such as managerial ability or technology differences between firms. Again, the problem if endogeneity appears when estimating this equation using simple OLS regression (see Econometric Appendix), as acknowledged by Marschak et al. (1944) . The problem is that inputs K and L are not independent variables but are correlated with the unobserved error term ϵ. We face a problem of endogeneity if, for example, higher productivity companies, i.e. those with the highest unobserved productivity also demand a large number of inputs, see Davis and Garcés (2009), and a selected sample as only the most efficient firms would appear in the panel data, as the least efficient would have exited the market.

[1] TFP - is the variation in output not explained by inputs depending on the functional approach, in this particular case the residuals or TFP would equal $\beta_0 + \epsilon$

If we do not take into account the problem of endogeneity, our estimated coefficients on endogenous inputs would be overestimated. One possible solution is to use proxy instrumental variables, or variables correlated with the company's demand for input but not correlated with firm production.

Olley and Pakes (OP) (1996) suggested using investment as a proxy for productivity to control for endogeneity. We will see that this approach takes into account the decision process of a firm that will also be covered in chapter three, Market Structure and Dynamics. Furthermore, Levinsohn and Petrin (2003) refined the OP work.

OP designed an algorithm to simulate the decision process of an incumbent firm. Firms at the beginning of each period decide whether to exit or continue in the market. If they decide to exit they will receive a liquidation value of Φ, if they continue in the market they will choose inputs (labor, materials and energy) and a level of investment I_{it}. The sequence of decisions of a firm maximizing the expected discounted value of net future profits is shown in the following Bellman equation[2] :

$$V(k_{jt}, a_{jt}, w_{jt}, \Delta_t\ )=\max\{\phi(k_{jt}, a_{jt}, w_{jt}, \Delta_t), max\{\pi(k_{jt}, a_{jt}, w_{jt}, \Delta_t) - c(i_{jt}\ , \Delta_t) + \delta E[V(k_{jt+1}, a_{jt+1}, w_{jt+1}, \Delta_{t+1}) \vdots k_{jt}, a_{jt}, w_{jt}, \Delta_t, i_{jt}]\ \}$$

This equation describes the dynamic decision process of a firm. First, the firm decides to exit a market if its liquidation value, $\phi(\ )$, exceeds the expected discounted value of net future profits. Second, it decides on investing or not i_{jt}, which is the solution to the second maximization bracket, where π is the profit function and C is the cost of investment, δ is the discount function and *E ( )* is the firm expectation conditional on information at *t*. Firms with positive productivity shocks in *t* will invest more in that period *t*. OP derive the unobserved productivity shock as:

$$\Omega_{it} = h(i_{it}, k_{it}, a_{it})$$

Firms will invest in the future if there is an increase in current productivity. OP derive the following Cobb-Douglas production function:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \beta_a a_{it} + u_{it}$$

whereby,

$$u_{it} = \Omega_{it} + \eta_{it}$$

Substituting Ω_{it} and u_{it} in the production function, we obtain:

$$y_{it} = \beta_l l_{it} + \beta_m m_{it} + \beta_a e_{it} + (\beta_0 + \beta_k k_{it} + \beta_a a_{it} + h(i_{it}, k_{it}, a_{it})) + \eta_{it}$$

Where *y* is output, *k* and *l* are capital and labor inputs, β are parameters, *a* is age of the firm, all in logs, Ω_{it} is a productivity shock observed by the firm and η_{it} unobserved shocks. This specification takes into account the relation between inputs and Ω_{it} and controls for unobserved productivity, while traditional models based on OLS estimates will be biased upwards, this specification can be estimated with OLS without bias. With this development estimation of parameters is more accurate and a structural explanation of entry and exit of firms in a market.

[2] See appendix for an explanation of the Bellman equation.

The tables below show a dataset of 2544 Chilean firms consisting of value added, capital, labor, electricity, water, investment between 1986 and 1996 (2018). We will estimate parameters of the production function using the OP estimation technique on that dataset[3]:

Chilean firm-level production data 1986-1996

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Y	2,544	13.003	1.463	8.770	11.965	13.884	18.580
Capital	2,544	11.668	2.157	2.708	10.498	13.049	18.217
Skilled Labor	2,544	1.770	1.358	0	0.7	2.7	7
Un. Labor	2,544	1.680	1.448	0	0	2.7	6
Elec	2,544	4.109	1.546	0.000	3.045	4.997	9.944
Water	2,544	11.137	2.333	1.072	9.855	12.631	18.088
Inv	2,544	11.137	2.333	1.072	9.855	12.631	18.088
idvar	2,544	15,770.490	7,355.940	10,007	11,497	16,337	40,475
timevar	2,544	2,001.042	3.223	1,996	1,998	2,004	2,006

The resulting parameters as expected are biased upward when using OLS without controlling for simultaneity and selection bias as shown in the table below (columns OLS and OP). Levinsohn and Petrin (2003) suggest an alternative approach to control for missing data (column LEVPET) Both Olley – Pakes and Levinson and Petrin show lower value of parameters. These parameter values show the marginal variation of output with an increase in one unit of input, e.g. an increase in one unit of capital increases output in 0.485 units. Recall that the negative value of water parameter does not comply with theoretical assumptions described above.

OLS, OP and LP methods: Chilean dataset

	OLS	OP	LEVPET
K	0.485***	0.168	0.162***
	(0.050)	(0.029)	(3.95)
Skilled	0.453***	0.314	0.319***
	(0.014)	(0.03)	(8.78)
Unskilled	0.362***	0.256	0.258***
	(0.013)	(0.027)	(9.79)
Water	-0.159***	0.311	
	(0.046)	(0.208)	
_cons	7.625***		
	(0.109)		
N	2544	2544	2544

SE in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Residual standard error: 0.7773 on 2539 degrees of freedom
Multiple R-squared: 0.7181, Adjusted R-squared: 0.7177
F-statistic: 1617 on 4 and 2539 DF, p-value: < 2.2e-16

In conclusion, the naïve traditional method to estimate production functions was incorrect, several authors designed new methodologies that provide consistent estimations of firm productivity.

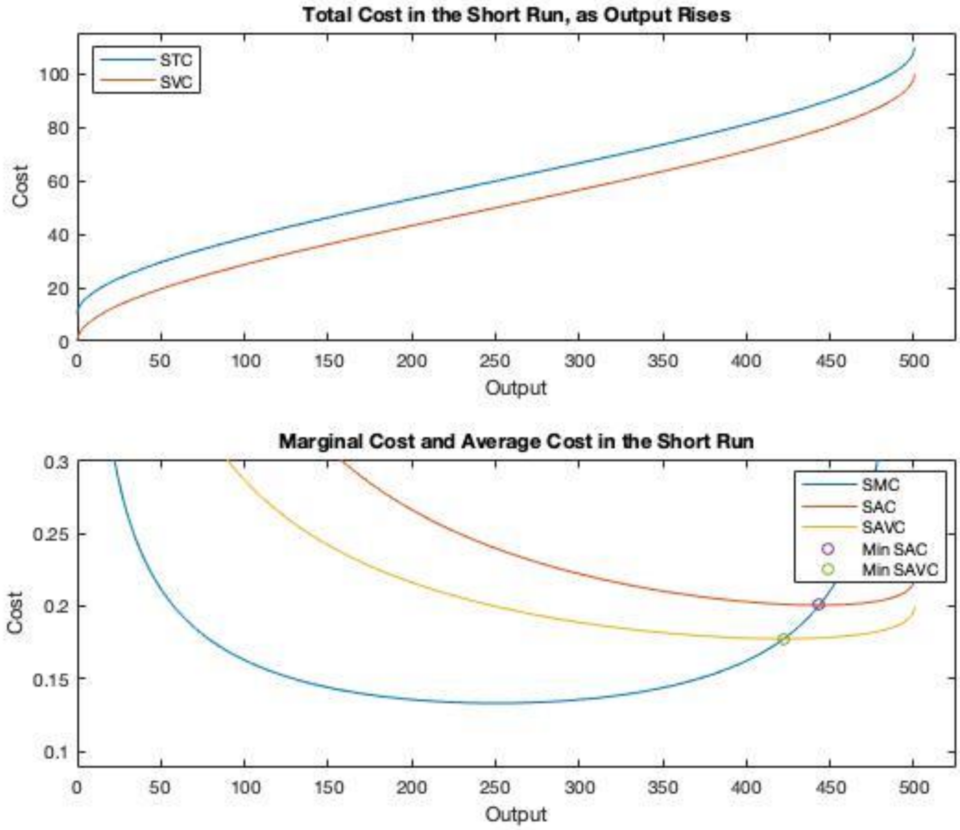
[3]See Rovigatti (2017).

3.- Cost function estimation

If we take factor prices as given and multiplying then by the quantities of production factors a cost equation is obtained. The cost function gives the minimal cost foreach level of production. The cost function has also general properties as the production function such as nonnegativity, nondecreasing in factor prices, non-decreasing in output, homogeneous and concave in input prices. Recall that through Shepard´s lemma (1953) deriving cost function with respect to price of factors one can obtain input demand functions and with them obtain the production function.

The graphic representation of cost functions was developed by Viner (1932). A cost function that complies with the minimum properties is represented below (STC are short term total cost and SVC short term variable cost). In the second graph below average and marginal cost are represented. Short term average cost function (SAC) (and equivalently variable cost curve SAVC) are unitary costs or total cost divided by total output and has a U-shape following production function curvature and fixed factor prices. Average cost decreases as output increases until the average cost is at the minimum. After this the law of diminishing returns makes average and marginal cost to rise. Marginal cost (SMC) is the derivative of the production function with respect to output or the infinitesimal change in cost when production increases in one unit. When prices are given in perfect competition, the part of marginal cost curve that is positively sloped (above min SAC) represents the short term supply of the individual firm.

Short term means a period of time when some inputs remain fixed. Economies of scale surge when all factors are variable and an equivalent increase in all factors generates a more than proportionate increase in production. In the long term all factors are variable, technological change and entry and exit can affect cost functions. The minimum average cost is known as Minimum Efficient Scale (MES) is the optimal level of output to produce (Greer, 2010). We will see below a classic example of economies of scale known as natural monopoly, where average costs are always declining so efficient production is better if it is concentrated in a single firm.



Economies of scale

The classic work on scale economies is Nerlove (1961), that studies power generation in the US using data of 145 firms in 1955. The sample collected consisted of cost data, fuel prices, labor and capital prices and total production of electricity for each electricity generation plant. The summary statistics of Nerlove´s data is[4] shown below:

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
cost	145	12.976	19.795	0.082	2.382	14.132	139.422
output	145	2,133.083	2,931.942	2	279	2,507	16,719
labor	145	1.972	0.237	1.450	1.760	2.190	2.320
laborshare	145	0.107	0.046	0.039	0.076	0.130	0.316
capital	145	174.497	18.209	138	162	183	233
capitalshare	145	0.427	0.115	0.090	0.351	0.486	0.845
fuel	145	26.177	7.876	10.300	21.300	32.200	42.800
fuelshare	145	0.467	0.123	0.070	0.394	0.544	0.759

Nerlove fitted the data to a Cobb-Douglas cost function derived from the following production function with three inputs: capital (K), labor (L) and fuel (F):

$$Y = AL^{\beta_L}K^{\beta_K}F^{\beta_F}u$$

[4]Table built using Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables. R package version 5.2.2. https://CRAN.R-project.org/package=stargazer

From this production function he obtained a cost function in logs in terms of units of fuel as follows:

$$Ln\left(\frac{C}{P_F}\right) = \beta_0 + \beta_y LnY + \beta_L Ln\left(\frac{P_L}{P_F}\right) + \beta_K Ln\left(\frac{P_K}{P_F}\right) + \varepsilon$$

Where Y measures billion Kwatt hour of electricity produced, P_K price of capital, P_L price of labor (dollars per hour) and P_F is the fuel price in cents of dollar per million BTUs.

Nerlove estimated the model using traditional OLS regression using cost and input prices for 145 companies. Results are shown in the table below were Nerlove work is replicated using his original data:

Table: Regression results Nerlove (1963)

	Dependent variable:
	log(cost/fuel)
log(output)	0.721***
	(0.017)
log(labor/fuel)	0.594***
	(0.205)
log(capital/fuel)	−0.008
	(0.191)
Constant	−4.686***
	(0.885)
Observations	145
R^2	0.932
Adjusted R^2	0.930
Residual Std. Error	0.392 (df = 141)
F Statistic	640.096*** (df = 3; 141)
Note:	*p<0.1; **p<0.05; ***p<0.01

From the above theoretical curves it is expected that the parameter of output should be positive as well as input price parameters (an increase in output or input price parameters should always increase cost). Indeed, β_y, β_L parameters are positive and significant, the former means that an increase in 1% in output yields a 0.72% increase in total cost, considering all other variables to remain fixed (2010).

Nerlove rejects the hypothesis of constant returns of scale as the inverse of the log Y parameter[5] $(0.72)^{-1} = 1.39$ is greater than one. This proves that power generating plants have increasing returns of scale (positive economies of scale). The scale parameter is also the ratio of marginal to average cost:

$$\frac{\partial lnC}{\partial lnQ} = \frac{Q}{C}\frac{\partial C}{\partial Q} = \frac{MC}{AC}$$

[5] Because $r = \sum_i \beta_i$, see (2010)

This ratio is positive/negative for increasing/decreasing returns to scale. Nerlove also divided the data in 5 groups according to the size of the firms. In the five regressions returns to scale were lower as the size of the firm increased.
The following figure shows the Nerlove data (in log), the estimated costs based on the output level and below the estimated residuals as a difference between the estimated and the actual values. For the estimate to be consistent under OLS, the residuals need to have no dependence on explanatory variables. On the contrary, the graph shows that they depend on the output level which violates the requirement for consistent estimates. At low and high levels of production, residuals are positive so the true cost is higher than the estimated values. On the other hand, for output intermediate values, the true value of the costs is better than the estimate. The graph of the residuals reflects a U-shape, see (2009) and (2018).

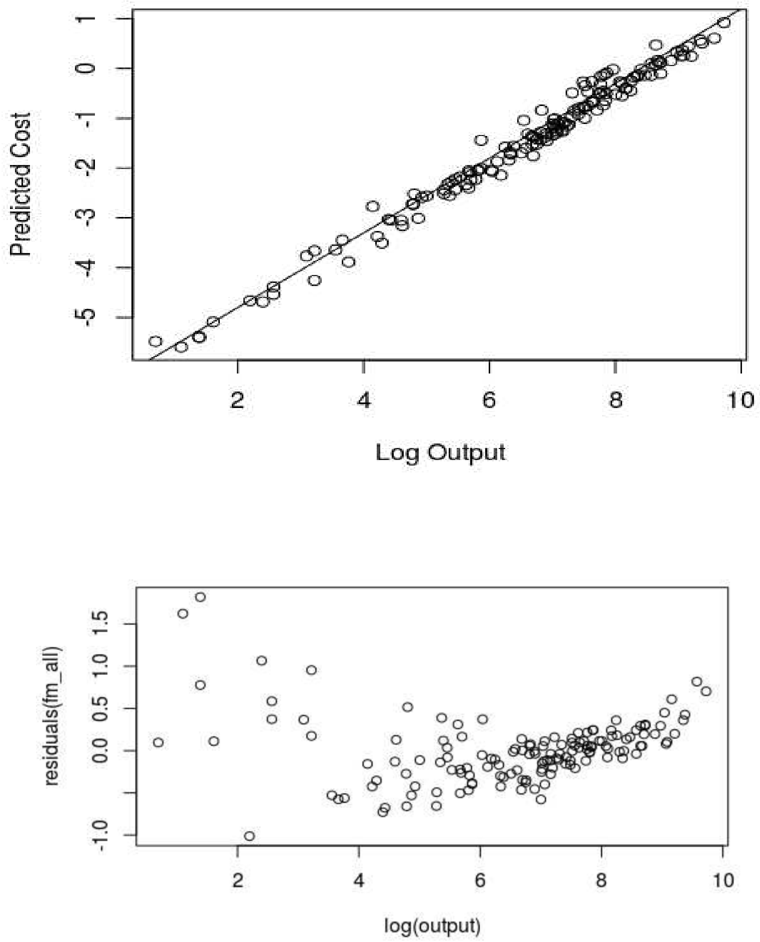


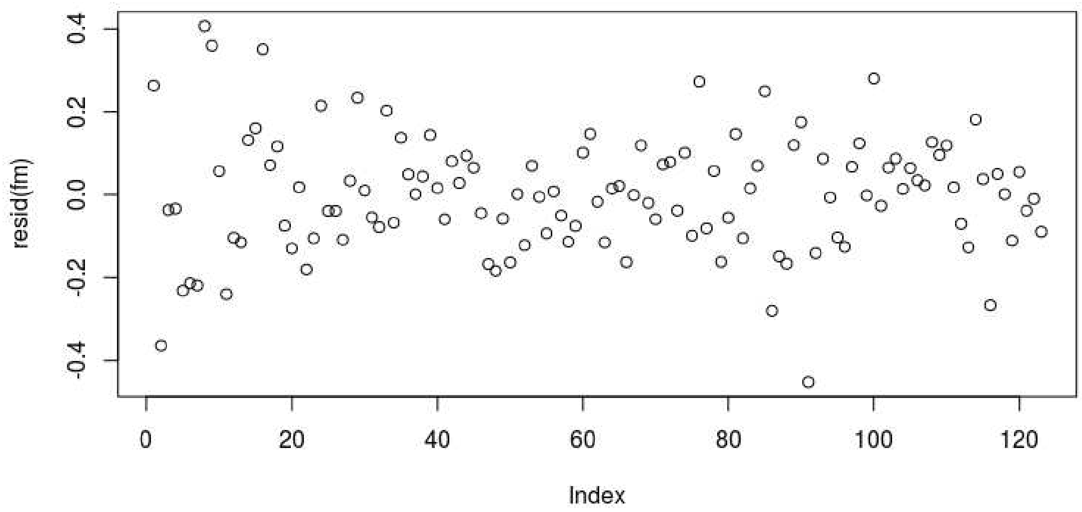
Figure: Nerlove graphs on fitted Log Cost and regression residuals

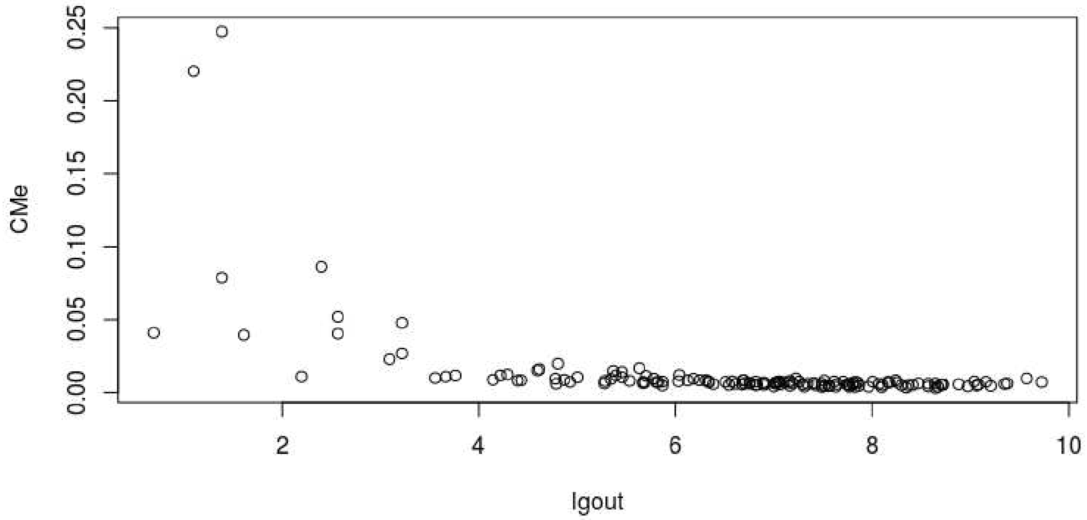
This diagnosis suggests that the shape of the cost function is incorrect, high residuals at low and high levels of output reveal an incorrect specification of the production function which should have a U type of cost function. The plot and data prove that there are increasing scale economies that are exhausted at a certain output level from which declining scale economies begin. Nerlove suggests that the specification can be corrected by expanding the function with a quadratic term. This generates a more flexible cost function that will allow cost to vary with the output level in a way that can generate economies of scale followed by diseconomies of scale as the level of production increases.

The table and figure below shows the Nerlove diagnostic test which consists of opposing the residues against the explanatory variable on the same chart. In this case using a Translog cost function, and unlike the previous specification, the graph shows that the expected value of the residuals in this regression is independent of the output level and remains around 0 as required by the MCO method. The table shows significant coefficients for output and labor.

Nerlove Translog Cost and regression residuals

	Dependent variable:
	log(cost/fuel)
log(output)	0.153**
	(0.062)
I(log(output)^2)	0.051***
	(0.005)
log(labor/fuel)	0.481***
	(0.161)
log(capital/fuel)	0.074
	(0.150)
Constant	−3.764***
	(0.702)
Observations	145
R^2	0.958
Adjusted R^2	0.957
Residual Std. Error	0.308 (df = 140)
F Statistic	800.667*** (df = 4; 140)
Note:	*p<0.1; **p<0.05; ***p<0.01





Christensen and Greene (1976) estimated the same cost function adjusted to the 1955 data by adding the 1970 data, using several models. Their model shows that the majority of firms were producing well beyond the efficient scale or with diseconomies of scale. These authors provided several models for data between 1955 and 1970 demonstrated the impact of technological progress over time moving the average cost function downwards and reducing de average cost of producing electricity.

Nerlove used a limited panel data and computing power available at that time and was strongly based on factor price taker assumption. Furthermore, other elements including long term contracts on fuel and trade unions regarding labor should be included in the study.

Multi-product firms and economies of scale

(1984) conducted an empirical estimate of AT&T's cost function and economies of scale and scope in local and long-distance calls. this study was relevant for the decision of the U.S. government in 1982 to break AT&T into several local firms while leaving AT&T in the long-distance call market (2009). The allegation of AT&T against it was the efficiencies gained from managing all telecommunications services in a single company would be lost if it was divided by region and activities. Evans and Heckman proved empirically that they were wrong. They used a "subaditivity" test for the cost function, a property that implies that the cost is lower when performed by a firm that by several firms. The failure of this test would mean that a single firm is more inefficient that several firms.
To do this they defined a cost equation for two products:

$$C = C(q_L, q_T, r, m, w, t),$$

Where q_L the output level of local *L* calls is, and q_T is the output level of long distance T calls. Cost functions depend on the price of production factors: *r* is the return rate of capital, w is the wage rate, and *m* is the price of the materials, *t* is a variable that captures technical change.

These authors used a translog function with two products and three production factors:

$$\log C = \alpha_0 + \sum_i \alpha_i \, logp_i + \sum_i \beta_i \, logq_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} logp_i logp_j + \frac{1}{2}\sum_k \sum_j \delta_{kj} log\; q_k \log q_j + \sum_i \sum_k \rho_{ik} log\; q_k \log\; p_{\,i} + \sum_i \lambda_i \log t \log p_i + \sum_k \theta_k \log t + \tau \log(t)^2 + \mu \log t$$

This cost function is much more general than that used at first by Nerlove based on a Cobb-Douglas function. It is more flexible and can approach any cost function. Let's define it for two inputs and two outputs.
The Translog cost function is presented in an unrestricted way but in the estimate a number of restrictions on the cost functions suggested by the theory are imposed. Authors impose homogeneity in input prices and symmetry on input prices. Using Shepard´s Lemma[6], one can obtain the three inputs (i=1,2,3) participation equations:

$$s_i = \alpha_i + \sum_j \gamma_{ij} logp_j \;\; + \sum_k \; \rho_{ik} log\; q_k \;\; \sum_i \lambda_i \log t$$

[6] The derivatives of the cost function with respect to input prices are the input demand functions. $l_j = \frac{\P C(q_1, q_2, p_1, p_2, p_3, t)}{\P p_j}$

So the participation of input j in total costs would then be:

$$s_j = \frac{p_j l_j}{C} = \frac{p_j}{C}\frac{\P \ln C}{\P \ln p_j} = \frac{\P \ln C}{\P \ln p_j}$$

By applying Shepard's lemma to the cost function we get the participation equations.

The estimate parameters of these equations are shown in the table below. A SURE estimator (seemingly unrelated regressions) was used (2015).

Selected estimated results Translog function with 31 observations

	Estimate	*Std. Error*	*t value*	*Pr(>|t|)*	
Constant	*9.0542*	*0.0044524*	*20.335.750*	*< 2.2e-16*	***
Capital	*0.654*	*0.1555707*	*42.075*	*0.0018071*	**
Labour	*0.354*	*0.1414821*	*51.771*	*0.0004148*	***
Local (output)	*0.226*	*0.2221301*	*10.197*	*0.3319212*	
Toll (Output)	*0.504*	*0.5275837*	*-0.4318*	*0.6750634*	
Technology	*-0.201*	*0.0780344*	*-0.1255*	*0.9025945*	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
R-squared: 0.9999439   Adjusted R-squared: 0.9998318

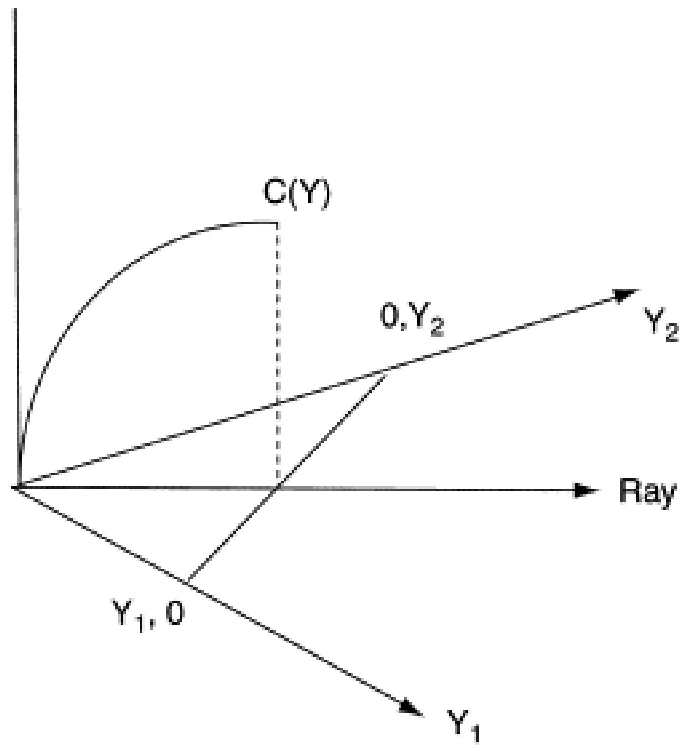
(Wales, 1987) proved that the function used was concave in input prices and monotonic but fails to comply non decreasing in output.

Subaditivity Test

Heckman and Evans provided a test to prove if Bell was a natural monopoly (1984). The traditional approach to natural monopoly was to evaluate if the company was at a level of output where average cost was decreasing. In case of multiproduct companies like a telephone firm (at that moment the two products were local and long call), Baumol (1977) considered that the necessary and sufficient condition in order to identify a industry as a natural monopoly was that the cost function must be subadditive. This test compares the cost of producing several products separately from the cost of producing them together. There is subbaditivity if it is cheaper to concentrate production in a firm than dividing it in several firms, so the relevant question is whether it costs more to produce the total output with multiple firms compared to the case where a single firm produces everything. If it costs more to distribute production among several companies, then we have that the firm (in this case AT&T) is a natural monopoly even though it produces several products (1977). The test is summarized by Baumol as:

$$C\left(\sum_i q_i\right) < \sum_i C(q_i)$$

Graphically, scale economies in multiproduct firms are represented graphically, when outputs increase proportionately as the gradient of the the perpendicular ray from the origin. The degree of scale economies may be interpreted as a measure of the percentage rate of decline or increase in ray average cost with respect to output (Baumol et al., 1982).



Evans and Heckman used a translog production function with limited dataset to test for local subadditivity. Other authors such as Roller (1990b) contradict this Evans and Heckman and consider that Bell was a natural monopoly before splitting, and argued that the production function did not comply with general properties such as positive marginal productivity of production factors and was not suitable to explain technological progress.

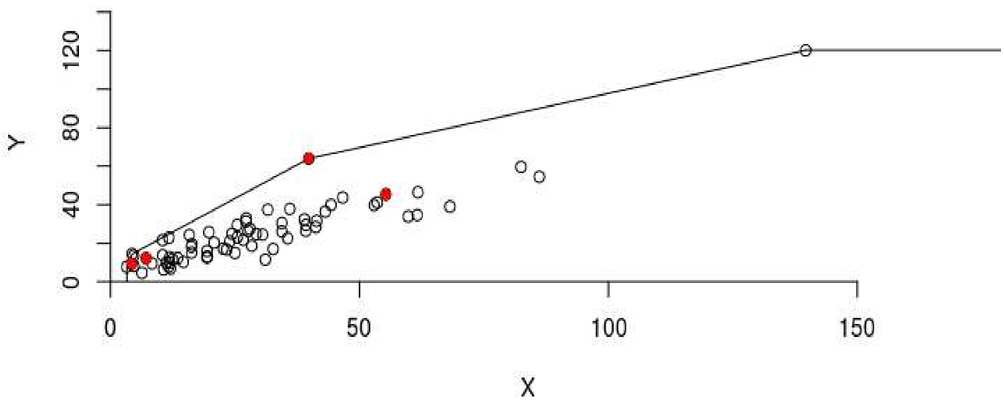
Frontier Analysis

Another approach to production function approach is frontier analysis, which estimates a measure of efficiency and productivity of outcomes and has been extensively used to assess outcomes in utilities, banks, hospitals, etc. (2012). These methods consider the cost and production functions as "ideal" or "frontier" to be estimated.
Instead of using a mathematical function, this analysis only requires input and output data and delivers efficiency ratios for each firm in the data, so that one can compare branches, production units or firms. The most efficient units will form a frontier line below which less efficient units will appear. Efficiency is measured as the ratio of the sum of outputs produced by each unit divided by the sum of inputs spent in the production process.

Charnes et. al. (1978) estimated for the first time a production frontier technology analysis based on a previous paper of Farrel (1957). The data consists of 70 school sites with the following variables[7]:

Firm - school site number
Inputs
x1 - education level of the mother
x2- highest occupation of a family member
x3- parental visits to school
x4- time spent with children in school-related topics
x5 - the number of teachers at the site
Outputs
y1- reading score
y2- math score
y3- self–esteem score
pft =1 if in program (program follow through) and =0 if not in program
name- Site name

The basic model of one input and one output is shown below and considers the maximum or border output that can be produced for each input level.



[7] See Benckmarking Package of Rstudio

Each point represents a school and the border has been drawn to find a frontier that encompasses all the data. There are some technically efficient schools at the border, and others below the border that can improve their efficiency. Efficiency is measured through a ratio, OY/OX, so that a call center operating at the frontier had a technical efficiency of 100% while those within the border operate at a level below the efficiency level. A firm is defined as combinations of inputs and outputs weighted by their v_i weight. The optimization program then states that we must increase the output level of the firm k as much as possible subject to the requirement that we can find the smallest firm that could have currently produced that higher level of production given the current combinations of inputs and outputs observed in the data.

The advantages of the DEA is that no functional form of the cost or production frontier should be assumed, while its critics consider that it relies too much on data and its sensitivity to outliers can be troubling. On the other hand, this method avoids imposing a specific parametric function on the production and cost function[8].

Conclusion

This chapter has reviewed production function analysis and cost functions, efficiency and productivity. Authors searched for a functional form that explained production data and complied with the minimum economic assumptions such as monotonicity, homogeneity and positive marginal productivity of factors. Newer techniques namely frontier analysis, have opened up possibilities for use in cases which have been resistant to other approaches due to complex (often unknown) nature of the relations between the multiple metrics labeled as inputs and outputs.

[8] **Stochastic Frontier Models: SFA** are parametric models (1992) where econometric theory is used to estimate pre-specified functional form and inefficiency is modeled as an additional stochastic term.(see Methodology and Applications of Stochastic Frontier Analysis, Andrea Furková)

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Annex: Alternative Fitting of Evans Heckman data in a Cubic Production Function