Bid Credits in Simultaneous Ascending Auctions

Rosa, Benjamin

Virginia Tech

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Benjamin V Rosa*

Virginia Tech

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Abstract

I study the impact of bid credits on simultaneous ascending auctions in a model where bidders potentially have complementary values. Although bid credits can lead to a more equitable distribution of items, I find an additional unintended consequence: bidders without credits are more exposed to winning a less desirable set of items and will drop out of the auction sooner when their competitors have credits. Calibrating the model to data from the Federal Communication Commission's sale of licenses in the 700 MHz guard bands, I find exposure reduced average non-credited dropout values by 5.7 percent but did not decrease revenues.

1 Introduction

Spectrum licenses, which grant owners the right to operate within a particular band of the spectrum, are valuable resources. In the US, sales from the Federal Communication Commission's (FCC's) first ten spectrum auctions generated over $23 billion in revenues for the US Treasury. A popular means of selling these licenses is through simultaneous ascending auction (SAA), where all licenses are up for sale simultaneously, and the auction ends when bidding stops. Additionally, the FCC, in compliance with their statutory obligation to ensure small businesses can compete in the auction process, will routinely offer bid credits to small firms—which lowers their final payments by a pre-established amount. For example, a small firm with a ten percent bid credit that wins a license for $100 would need to pay the FCC only $(1 - 0.1) \times 100 = \$90$ for that license. In this paper, I explore how credits impact spectrum auction outcomes both theoretically and through an illustrative calibration of the FCC's 25 percent bid credit for small businesses.

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Bid credits are typically used to increase the participation of a designated group of firms. Although bid credits are generally effective in that regard, they have received criticism for their potentially adverse effect on revenues. In a Congressional Budget Office report, Musick (2005) finds that small bidders paid an average of 20 percent less than the next highest bidder that did not qualify for bid credits in the FCC's Auction 35, which sold licenses for personal communications services (PCS). At first glance, one can imagine that having small bidders pay 75 percent of their bid would negatively affect revenues, but Ayres and Cramton (1996) and Cramton et al. (2011) show that this need not be the case; to compete with the small firms that have credits, large firms bid more aggressively, and if small firms create sufficient competition, then it is possible for revenues to increase. Ayres and Cramton (1996) then substantiate their theory with data from regional narrowband auctions, finding that the prices paid for regional licenses, which have credited bidders, were 6.2 percent higher than the national auction, which did not have credits.

What I demonstrate in this paper is that there is yet another, likely unintended consequence of using bid credits when licenses are awarded through SAA. Empirical studies show that bidders tend to have complementarities in their values for licenses, meaning that the marginal value of a license increases as bidders acquire more of them.[1] Since the SAA format requires firms to bid on individual licenses instead of license packages, firms interested in multiple licenses are exposed to the risk of winning a less desirable bundle at a price higher than their value. This phenomenon is known as the "exposure problem" and has been well documented in the theory literature.[2] I show in this paper that bid credits intensify the exposure problem for firms that value multiple licenses, causing them to drop out of bidding earlier than they would absent bid credits. This mechanism can potentially counteract the revenue increase from having more competitive small firms.

To investigate the possible magnitude of this channel and evaluate the potential effects of alternative bid credits, I calibrate a stylized model of equilibrium bidding to data from the FCC's auction of licenses in the A block of the upper 700 MHz guard bands, which was part of Auction 33.[3] In this auction, the FCC granted a 25 percent bidding credit to firms qualifying as (very) small businesses, and licenses were sold through SAA. My calibrated model indicates that increased exposure from bid credits reduced dropout values for the average non-credited bidder that bid on every license by 5.7 percent when multiple licenses had active regional bidders. However, heightened competition from small businesses combined with the relative strength of large businesses resulted in *increased* revenues for the FCC. Indeed, counterfactual simulations

[1] See Ausubel et al. (1997), Fox and Bajari (2013), Moreton and Spiller (1998) and Xiao and Yuan (2018).
[2] Meng and Gunay (2017), Goeree and Lien (2014), and Bulow et al. (2017) are a few examples.
[3] Auction 33 also had B block licenses available. I discuss this more in section 3.

reveal that the FCC could have used a 50 percent credit to increase the number of small business winners by 36.8 percent with a negligible effect on revenues.

I then use my calibrated model to compare the license allocation under SAAs with credits to the efficient allocation. I find that the early dropout induced by increased exposure tilted the distribution of licenses in favor of small regional firms. As a result, the FCC would have awarded more than the efficient number of licenses to small firms by using SAAs, and because exposure issues exist absent credits, this result would have held even if there were no credits.

Given my methods and application, this paper makes a couple of contributions to the existing literature. I have a theoretical contribution on how bid credits impact exposure, which I have not seen discussed in other papers in the literature. This channel can result in substantial decreases in equilibrium dropout values for non-credited firms, mainly when the credit and the complementarity between values are high. To my knowledge, this is also the first paper to attempt to quantify the potential effects of bid credits on FCC spectrum auctions through a calibrated model of equilibrium bidding. Other papers looking at bid credits tend to compare realized outcomes within and across auctions, as is the case for Ayres and Cramton (1996) and Musick (2005). Although stylized, my calibration allows me to use the economic theory to assess what would have happened under a variety of different bid credit configurations.

My data application relies on calibrated bidder value distributions, and there are a few other empirical papers that use FCC data to estimate bidder values. Hong and Shum (2003) model bidding as separate single-unit auctions, so they do not address the possibility of license complementarity. Fox and Bajari (2013) estimate bidder valuations using a pairwise stability condition on data from the 1995 - 1996 C block auction of the 1900 MHz PCS spectrum band. They find that combining licenses into four large regional licenses would result in a 48 percent increase in efficiency in the C block outcomes. More recently, Xiao and Yuan (2018) use an entry game to estimate bidder valuations and license complementarities on data from the 700 MHz radio frequency band. My paper differs from these in its focus on bid credits.

My theoretical analysis uses the framework of Goeree and Lien (2014). In their paper, they develop a tractable model for analyzing equilibrium behavior when an arbitrary number of objects are awarded through SAA. I extend their analysis by including bid credits and calibrating it to observed outcomes from the FCC. Meng and Gunay (2017) also develop an equilibrium model of bidding in a similar environment with two licenses, but they allow for heterogeneity in bidder license values. Although such heterogeneity is likely to occur in my setting, I use the more general model as a basis for my study and transform the data in the empirical analysis so that licenses in the calibrated model are roughly similar.

My paper also contributes to a rich literature on bid credits in other auction formats. In procurement auctions, these papers include simulation studies such as Hubbard and Paarsch (2009) and empirical studies such as Marion (2007), Krasnokutskaya and Seim (2011), and Rosa (2018). Athey et al. (2013) investigates bid subsidies in timber auctions. These papers are similar in their focus on single-unit auctions; my contribution to this strand of literature is to extend the analysis to the multi-unit simultaneous ascending auction, where exposure can become an issue if bidders have complementary values.

The remainder of the paper proceeds as follows. In section 2, I describe the bidding model and show how credits influence dropout values. Section 3 contains a description of the data and some institutional features of Auction 33. Section 4 outlines my calibration procedure and results, and section 5 shows predictions from the calibrated model on how outcomes would change with different credits. Section 6 discusses some additional considerations, and Section 7 concludes.

2 Model

In this section, I lay the theoretical foundation for my analysis of bid credits. I assume that two types of bidders bid for a total of n licenses labeled $1, 2, \ldots, n$. The first type of bidder is a global bidder, and they are interested in acquiring every license. The second type of bidder is a local bidder. I assume that each license has a local bidder that values it and that local bidders value only one license. Thus, every auction starts with a total of K global bidders that bid on each license and n local bidders that bid on individual licenses. This assumption on local bidding is, of course, an abstraction from reality since a single license may have multiple local bidders in real-life settings. I make this assumption to keep the model tractable, and one can interpret a local bidder on a license as an agent representing local bidding power.

The auctioneer sells all licenses through a simultaneous ascending auction. Each license with two or more active bidders has a price clock that continuously ticks upward at an equal pace. A clock pauses when only one bidder is willing to buy its license at the current price, and that bidder becomes the provisional winner. Firms can bid on any license while at least one price clock is still ticking, which enables bidders to bid on licenses at later times. In practice, the FCC limits this behavior by imposing activity rules; inspired by the FCC's rules, I limit late bidding in my model by using the following activity rule: a bidder cannot increase the total number of licenses on which they are bidding.[4] The auction concludes when all licenses have a provisional winner, and each winner pays their license's corresponding price or discounted price if they have

[4]The FCC's activity rule is similar. Prior to participating, bidders buy eligibility and are required to bid on a portion of their eligibility in each round. A bidder that does not meet this requirement must either use an eligibility waiver or forfeit their eligibility.

credits.

I denote local valuations for licenses by v_i, and I assume that local bidders draw their valuations independently and identically from the same distribution F_v with support $[0, \overline{v}]$. For global bidders, I assume that each potentially heterogeneous bidder has a total valuation for all n licenses of V^i. Important to a global bidder's bidding decision is the marginal value of an additional license, as it will dictate how long global bidders will continue bidding on a given license. Following Goeree and Lien (2014), I denote the marginal value for global bidder i on license $n-j+1$ as V_j^i, where $j = 1, \ldots, n$. Thus, V_1^i is the marginal value of the n^{th} license for global bidder i, while V_n^i is the marginal value of the first license. The value of all licenses is then $V^i = \sum\limits_{j=1}^{n} V_j^i$, and I denote the vector that collects the marginal values as $\boldsymbol{V}^i = (V_1^i, V_2^i, \ldots, V_n^i)$.

As has been shown in the empirical literature, there tends to be value complementarity between spectrum licenses. In other words, bidders have higher marginal values as they acquire more licenses. I allow for, but do not impose, complementarity by assuming that $V_1^i \geq V_2^i \geq \cdots \geq V_n^i$. Let x_j^i be independently distributed according to the CDF $F_{x_j^i}$ with support $\left[0, \overline{x}_j^i\right]$. To allow for complementarity, I assume that $V_n^i = x_n^i$, $V_{n-1}^i = x_{n-1}^i + x_n^i, \ldots, V_1^i = \sum\limits_{l=1}^{n} x_l^i$. Observe that this specification contains the no-complementarity case, which occurs when all distributions except $F_{x_n^i}$ are set to a degenerate distribution at zero. I make the additional assumption that the support of local values is large enough relative to the support of marginal global values to rule out extreme solutions.[5]

I incorporate a bid credit rule that mirrors the one used by the FCC in their licensing auctions. At the beginning of the auction, a known subset of bidders qualify for a credit of $\alpha \in [0, 100)$ percent; for simplicity, I assume that local bidders either all receive credits or not, but a subgroup of global bidders may receive credits. When determining the final payments, the bids of credited firms are reduced by α percentage points. Therefore, a bidder that would normally leave the auction at a price of p will now leave the auction at a price of ϕp, where $\phi = \frac{1}{1-\frac{\alpha}{100}}$. For example, a bidder that would drop out at a price of \$75 without credits would, with a 25 percent bid credit, be willing to stay in until the price is $75 \times \left(\frac{1}{1-\frac{25}{100}}\right) = \$100$ and would pay \$75.

As will be shown later, bid credits change how credited bidders behave in an auction by allowing them to bid above their value yet pay a lower price. The magnitude of this behavior is tied to the scaling factor, ϕ, which serves as measure of how high above their value a bidder is willing to bid. As such, I define a bidder's *credited value* as their value scaled up by ϕ, and I denote credited values and their associated distributions

[5]Specifically, let $\overline{V}_j^i$ be the highest possible marginal value for bidder i. I assume that if $\sum\limits_{j=1}^{n} \overline{V}_j^i = m$, then $\overline{v} \gg \frac{m}{n}$. This assumption prevents global bidders from attaining a value so high that they would always beat the local bidders. Otherwise, the optimal strategy for these high-value global bidders would be to drop out at a price of $\frac{V^i}{n}$ if they have no credits or $\frac{\widehat{V}^i}{n}$ if they have credits.

with hats. Global bidders with credits have a credited value of $\widehat{V}^i = \phi V^i$, which implies credited marginal values of $\widehat{V}_j^i = \phi V_j^i$. When they receive credits, local bidders have a credited value of $\hat{v}_i = \phi v_i$. The implied credited value distribution for local bidders is $\widehat{F}_v(x) = \Pr(\phi v < x) = F_v\left(\frac{x}{\phi}\right)$.

2.1 Equilibrium with One Global Bidder and One license

I begin my analysis of bid credits by exploring the simplest case: where one global and one local bidder compete for one license. For now, I will consider scenarios without global credits, and I will discuss cases with global credits after my general result.

Without credits, local bidders always have a weakly dominant strategy to stay in the auction until the price reaches their value. Crucial to a global bidder's strategy is their beliefs about their competitors' value, which is represented by a distribution of values given the current price, p. To that end, I construct the distribution $F_k(v \mid p) = 1 - \left[\frac{1-F_v(v)}{1-F_v(p)}\right]^k$. In words, this distribution is the probability that v is higher than the lowest value of the remaining k active local bidders, given that the local values are all higher than p.[6]

Define $\Pi_k^K\left(\boldsymbol{V}^i, p\right)$ as the expected profit of global bidder i when there are K active global bidders and k active local bidders (out of n). Additionally, let $D_k^K\left(\boldsymbol{V}^i\right)$ be the equilibrium dropout value with K active global bidders and k active local bidders. At this value, the global bidder drops out of all k licenses. In the single-license case, the global bidder's objective is to choose a dropout value that maximizes

$$\Pi_1^1\left(\boldsymbol{V}^i, p\right) = \int_p^{D_1^1\left(\boldsymbol{V}^i\right)} \left(V_1^i - v_1\right) dF_1\left(v_1 \mid p\right).$$

The dropout value that optimizes this expression is the one that equates the term inside the parentheses, $V_1^i - v_1$, to zero, which is $D_1^1\left(\boldsymbol{V}^i\right) = V_1^i$.[7] This solution implies that the global bidder, like the local bidder, drops out at their (marginal) value.

[6] To see how I derive this object, consider the distribution of the smallest order statistic, F_{min}, for an arbitrary CDF, F: $F_{min}(x) = 1-(1-F(x))^k$. The truncated distribution for a random variable between a and b is $F(x \mid a < x < b) = \frac{F(x)-F(a)}{F(b)-F(a)}$. If $\overline{x}$ is the upper bound of the support of F, then setting $a = p$ and $b = \overline{x}$ yields $F(x \mid p < x) = \frac{F(x)-F(p)}{1-F(p)}$. Applying the truncated distribution to the order statistic gives the result:

$$F_{min}(x \mid p < x) = 1 - \left(1 - \frac{F(x) - F(p)}{1 - F(p)}\right)^k$$

$$F_{min}(x \mid p < x) = 1 - \left(\frac{1 - F(p) - F(x) + F(p)}{1 - F(p)}\right)^k$$

$$F_{min}(x \mid p < x) = 1 - \left(\frac{1 - F(x)}{1 - F(p)}\right)^k.$$

[7] To see why, note that the global bidder would like to continue bidding as long as the expected payoff is positive and will drop as soon as the expected payoff becomes negative. In the single-license case, the payoff is positive when $p < V_1^i$ and negative when $p > V_1^i$; therefore, the optimal time to drop out is when $p = V_1^i$.

Next, consider the case where local bidders receive credits and the global bidder does not. With credits, local bidders have a new weakly dominant strategy of dropping out when the price reaches their credited value. In turn, global bidders have new beliefs about the local value distribution. Let $\widehat{F}_k(v \mid p)$ be the probability that v is higher than the lowest of the k active local bidders' *credited* values given the price. If $\tilde{\Pi}_k^K(\boldsymbol{V}^i, p)$ is a global bidder's expected profits with local credits and $\tilde{D}_k^K(\boldsymbol{V}^i)$ is the corresponding dropout value, then the global bidder maximizes

$$\tilde{\Pi}_1^1(\boldsymbol{V}^i, p) = \int_p^{\tilde{D}_1^1(\boldsymbol{V}^i)} (V_1^i - v_1)\, d\widehat{F}_1(v_1 \mid p).$$

Following the same logic as before, the equilibrium dropout value is $\tilde{D}_1^1(\boldsymbol{V}^i) = V_1^i$. Observe that global bidders are not exposed to the risk of acquiring a sub-optimal bundle of licenses in the one-license case and, therefore, do not face an exposure problem. As a result, the dropout values with and without local credits are the same.

2.2 Equilibrium with One Global Bidder and Two Licenses

Suppose now that the auctioneer wishes to sell two licenses and that one global bidder participates. The local bidders desire only one license and have the same weakly dominant strategy. The global bidder now needs to consider the possibility that they may not win both licenses, even though they may marginally value the second license more than the first.

The global bidder's expected profits are

$$\Pi_2^1(\boldsymbol{V}^i, p) = \int_p^{D_2^1(\boldsymbol{V}^i)} \left[\int_{v_2}^{V_1^i} (V_1^i - v_1)\, dF_1(v_1 \mid v_2) + (V_2^i - v_2)\right] dF_2(v_2 \mid p),$$

where v_1 is the highest local value and v_2 is the lowest local value.[8] Following Goeree and Lien (2014), the dropout value (for both licenses) that equates the term inside of the brackets to zero is

$$D_2^1(\boldsymbol{V}^i) = V_2^i + \int_{D_2^1(\boldsymbol{V}^i)}^{V_1^i} (V_1^i - v_1)\, dF_1\left(v_1 \mid D_2^1(\boldsymbol{V}^i)\right),$$

or

$$D_2^1(\boldsymbol{V}^i) = V_2^i + \Pi_1^1\left(\boldsymbol{V}^i, D_2^1(\boldsymbol{V}^i)\right).$$

[8]The upper bound on the inner integral is the dropout value if there is one license, which was solved previously.

From these expressions, it is evident how multiple licenses affect dropout. The global bidder is willing to stay in both markets at a price above the marginal value of one license, risking the possibility that they may win only one license at that higher price. This observation highlights the nature of the exposure problem faced by global bidders. When there is only one license remaining, the dropout value is the marginal value of the second license: $D_1^1\left(\boldsymbol{V}^i\right) = V_1^i$.

What remains is how credits affect the two-license case. Suppose that local bidders receive credits and that the global bidder does not. Following the derivation above, the new dropout value is

$$\tilde{D}_2^1\left(\boldsymbol{V}^i\right) = V_2^i + \tilde{\Pi}_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right),$$

where $\tilde{\Pi}_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right) = \int_{\tilde{D}_2^1\left(\boldsymbol{V}^i\right)}^{V_1^i}\left(V_1^i - v_1\right) d\widehat{F}_1\left(v_1 \mid \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right)$. To compare these dropout values, it is useful to note the following proposition on global beliefs; this proof and all other proofs are contained in the appendix.

Proposition 1. *If $\phi > 1$, then $\widehat{F}_k \leq F_k$ for all k, with a strict inequality on the interval $(p, \phi\overline{v})$.*

In words, Proposition 1 says that global bidders facing local bidders with credits are less likely to win. With this intuition in mind, I arrive at my first result: global dropout values are lower with local credits when both local bidders are active.

Proposition 2. *If the global bidder receives no credits, then $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) < D_2^1\left(\boldsymbol{V}^i\right)$.*

The intuition behind this result is that credits make local bidders drop out at higher values; therefore, the global bidder believes they have less of a chance to win if they stay in longer and will drop out earlier. This result is tied to the exposure problem, as credits essentially increase the chance that the global bidder fails to acquire both licenses.

2.3 Equilibrium with One Global Bidder and Multiple Licenses

Applying the insights from the two-license case recursively, one can extend the two-license results to an arbitrary number of licenses. Assume that there is still one global bidder and suppose that there are n

licenses for sale. The equilibrium dropout, which was derived in Goeree and Lien (2014), is

$$\begin{aligned}
D_k^1\left(\boldsymbol{V}^i\right) &= V_k^i + \Pi_{k-1}^1\left(\boldsymbol{V}^i, D_k^1\left(\boldsymbol{V}^i\right)\right) \\
\Pi_k^1\left(\boldsymbol{V}^i, p\right) &= \int_p^{D_k^1\left(\boldsymbol{V}^i\right)} \left[\Pi_{k-1}^1\left(\boldsymbol{V}^i, v_k\right) + \left(V_k^i - v_k\right)\right] dF_k\left(v_k \mid p\right) \\
\Pi_0^1\left(\boldsymbol{V}^i, p\right) &= 0.
\end{aligned}$$

As before, endowing local bidders with credits leads to global bidders dropping out at lower values so long as there are at least two active local bidders. This result extends to the n license case, with the strict inequality holding until there is one active local left.

Proposition 3. *If the global bidder receives no credits, then* $\tilde{D}_k^1\left(\boldsymbol{V}^i\right) \leq D_k^1\left(\boldsymbol{V}^i\right)$*, with a strict inequality whenever* $k \geq 2$.

I arrive at this result by using induction to extend the proof from the two-license case, and this result demonstrates the pervasiveness of the increased exposure problem generated by bid credits.

2.4 Equilibrium with Multiple Global Bidders and Multiple Licenses

Now consider the n license case with K global bidders. The critical insight here is that a global bidder that remains in the auction will no longer win a license when a local bidder drops out. Rather, the price will continue to rise on licenses without local bidders until the rest of the global bidders drop out. In addition, global bidders do not change their optimal dropout values if $K > 2$ because the only relevant information on final prices is revealed when the final competing global bidder drops out. As derived in Goeree and Lien (2014), the equilibrium dropout value is then characterized by the following set of equations:

$$\begin{aligned}
D_k^K\left(\boldsymbol{V}^i\right) &= D_k^2\left(\boldsymbol{V}^i\right) \quad \text{for} \quad K \geq 2 \\
0 &= \Pi_k^1\left(\boldsymbol{V}^i, D_k^2\left(\boldsymbol{V}^i\right)\right) + \sum_{l=k+1}^{n} \left(V_l^i - D_k^2\left(\boldsymbol{V}^i\right)\right) \\
0 &= \Pi_n^1\left(\boldsymbol{V}^i, D_n^2\left(\boldsymbol{V}^i\right)\right).
\end{aligned}$$

When local bidders receive credits, the additional consideration of rising prices on licenses with inactive local bidders does not change the ranking of dropout values with and without local credits. As such, I arrive at my general result.

Proposition 4. *If global bidder i receives no credits, then $\tilde{D}_k^K\left(\boldsymbol{V}^i\right) \leq D_k^K\left(\boldsymbol{V}^i\right)$, with a strict inequality whenever $k \geq 2$ or when $K \geq 2$, $n > k$, and $k = 1$.*

Note that this and my previous results also apply to global bidders that receive credits. A global bidder receiving credits with marginal values $\boldsymbol{V}^i$ behaves as a global bidder without credits but with marginal values $\phi\boldsymbol{V}^i$ because credits allow them to buy licenses at a price of ϕ times their value. Thus, the increased exposure problem for global bidders is universal and is generated by local credits in this model.

2.5 Comparative Statics

For my next result, I consider how a change in local credits impacts global dropout values. Suppose that credits were initially set at $\alpha \geq 0$ but then increase to $\breve{\alpha}$. Let $\tilde{D}_k^K\left(\boldsymbol{V}^i\right)$ be the equilibrium dropout value for a global bidder when credits are α, and let $\breve{D}_k^K\left(\boldsymbol{V}^i\right)$ be the equilibrium dropout when credits are $\breve{\alpha}$. The following corollary to Proposition 4 relates $\tilde{D}_k^K\left(\boldsymbol{V}^i\right)$ to $\breve{D}_k^K\left(\boldsymbol{V}^i\right)$.

Corollary 1. *If $\breve{\alpha} > \alpha$ and global bidder i receives no credits, then $\breve{D}_k^K\left(\boldsymbol{V}^i\right) \leq \tilde{D}_k^K\left(\boldsymbol{V}^i\right)$, with a strict inequality whenever $k \geq 2$ or when $K \geq 2$, $n > k$, and $k = 1$.*

In words, Corollary 1 says that increasing local credits weakly decreases equilibrium dropout values, with the intuition being that increased credits lead to an increased exposure problem. Noting that higher credits tend to increase the number of credited winners, this result highlights a potential revenue trade-off when using higher credits to allocate more licenses to credited bidders.

Although my theoretical results give insights on the sign of global dropout changes when adding credits, a question of practical importance is its magnitude and likely effect on revenues in reality. The answer to these questions depends on the distribution of valuations and the size of the credit; if the credit is not large enough for credited bidders to compete, then, because all credited bidders would drop out early, this exposure issue would be irrelevant, and the final outcome would be driven by non-credited bidders. To answer these more practical questions as well as what would happen with different credits, I turn to an illustrative calibration to FCC data.

3 Data

In this section, I describe the FCC's auctioning process and data. I examine data from the FCC's sale of licenses in the 700 MHz guard bands, which concluded on September 21, 2000 and is also known as Auction

33. In this auction, the FCC offered 104 licenses using a simultaneous ascending auction format, of which nine winning bidders won 96 licenses and generated a total of $520 million in revenues for the US Treasury. The FCC separated licenses into an A block and a B block, where a license in the B block offered 2 MHz more bandwidth than an A block license in the same area. Since blocks are dissimilar and there is insufficient variation in the B block outcomes to identify the model's primitives, I focus my analysis on the A block.

Licenses covered the US, its territories, and the Gulf of Mexico. In setting the boundaries for each license, the FCC utilized major economic areas (MEAs), which is an aggregation of the US and the Gulf of Mexico into 52 distinct regions. Because licenses in disconnected regions may not be subject to the same exposure issues, I consider only licenses for the 46 MEAs spanning the contiguous 48 states.

To encourage small business participation, the FCC granted bid credits to businesses qualifying as either small or very small. In Auction 33, the credit's size was 15 percent for small businesses and 25 percent for very small businesses—where the small and very small business requirement is a three-year average revenue of less than 40 million and 15 million, respectively. Every business that qualified for credits qualified as a very small business in the data, so to keep the exposition concise, I will continue to refer to very small businesses as small businesses.

Mirroring the bidder classification used in the model, I divide bidders into two separate groups: global bidders and local bidders. I define a global bidder as any bidder that bids on all licenses, while a local bidder is a bidder that does not bid on every license. Note that local bidders under this classification could have submitted bids on multiple licenses in Auction 33, whereas local bidders in the model can bid on only one license. In the calibration, I will treat bids from these local bidders as coming from agents representing local bidding power; these agents will bid on individual licenses. In the data, nine out of the 14 bidders are local, and the remaining five are global. Local bidders submit bids on an average of 14.9 licenses.

Table 1 summarizes key statistics from the data. Because large and small bidders may have different underlying values, I separate the global bidders into large global bidders and small global bidders. As is expected from this classification, large global bidders tend to have higher dropout values than small global bidders—where I define the dropout values as the highest bid submitted on a license, excluding the winning bidder and net of any bidding credit. Average local dropout values are higher because more local bidders bid on licenses with broader population coverage.

In determining a license's coverage, which is likely to be tied to the value of that license, the FCC used population estimates from the 1990 Census. The middle panel in table 1 shows statistics on those estimates. Each license served an average of 5.37 million people, with a median of 3.97 million. These statistics suggest

Table 1: Summary Statistics

	Mean	Stdev	25th %ile	Median	75th %ile	Obs
Bid level						
Dropout value ($ millions)	1.39	2.32	0.16	0.50	1.68	319.00
Small global dropout ($ millions)	1.13	1.20	0.36	0.81	1.55	75.00
Large global dropout ($ millions)	1.18	1.97	0.09	0.27	1.41	112.00
Local dropout ($ millions)	1.72	2.96	0.19	0.58	2.18	132.00
License level						
Population (millions)	5.37	5.07	2.34	3.97	5.90	46.00
Bidder level						
Upfront payment ($ millions)	4.31	2.97	3.62	3.62	5.43	14.00
Small global upfront payment ($ millions)	4.52	1.28	4.07	4.52	4.98	2.00
Large global upfront payment ($ millions)	5.43	0.00	5.43	5.43	5.43	3.00
Local upfront payment ($ millions)	3.88	3.66	0.55	3.62	3.62	9.00

Note: Summary statistics for global and local bidders at the bid, license, and bidder level. Dropout values are the highest bids each bidder submits on a license, not including the winning bidder. Dropout values do not include any bidding credits. The population variable is in millions and comes from the 1990 U.S. Census. The upfront payment is a refundable payment to the FCC for the rights to bid on a license or group of licenses. The size of the upfront payment is determined by how many licenses a bidder wishes to buy.

that the distribution of population estimates is skewed toward several high-density areas, such as the New York MEA that had a population of near 30 million.

The FCC also required firms to make a payment upfront in proportion to the number of licenses on which they wish to bid. The bottom panel of table 1 contains statistics on upfront payments tabulated by bidder type. Given that global bidders have higher average upfront payments than local ones, my bidder classification scheme appears to be within reason.

4 Calibration

I seek to calibrate bidder value distributions and use them to illustrate the magnitude of how credit-induced increases to exposure can impact spectrum auctions. I do so by calibrating features of the A block data to equivalent outcomes determined by the model. This section contains a description of my calibration procedure. I begin by outlining my parametric assumptions on the model's primitives and then move to the variation in the data that allows me to identify those primitives.

Before diving into the calibration details, however, I remark here that the model is a stylized representation of the more complex rules and features present in actual FCC spectrum auctions. My intent for the model is that it be used as an illustrative example of how credits can affect dropout rather than a full-on empirical estimation of all elements of bidder values that accounts for all aspects of FCC spectrum auctions. My calibration strategy thus revolves around generating model primitives within the context of a simplified model that produce outcomes matching similar data outcomes.

In an ideal world, I would have multiple observations of the FCC's auction for this block of the spectrum, each containing the same number of bidders and licenses for sale. I could then calibrate the general model. However, my single observation of this auction requires that I use within-auction variation in bidding instead of variation across auctions. As such, I calibrate the two-license model using moments calculated at the license level. Although this choice is inherently an abstraction, the two-license model is still able to capture most if not all of the insights from the general model and, as will be discussed later, generates value structures similar to ones found in other empirical papers with more detailed models.

To minimize the effect of heterogeneity across licenses arising from population size, I normalize all bids by the population in the calibration so that values are in per capita terms. In a more flexible two-license model, Meng and Gunay (2017) allow for heterogeneity in how bidders value licenses. A limitation of using that model, however, is that it requires a shared ranking of licenses, in the sense that all bidders prefer one license over the other. In practice, there are likely to be differences in how bidders rank licenses, so my calibration employs the simpler model with normalized values. My results should thus be interpreted as a measure of average per capita license value, with the caveat that values can vary across regions in reality.

4.1 Parametric Assumptions

I take a parametric approach in calibration, which requires parametric assumptions on the model's primitives. To this end, I assume that the (per capita) values and their components follow a Weibull distribution. Let $\mathcal{W}(\lambda, k)$ denote a Weibull distribution with scale parameter λ and shape parameter k. For local values, I assume that $v_i \sim \mathcal{W}(\lambda_l, k_l)$. The credit for small bidders is $\alpha = 25$ percent, which implies $\phi = \frac{1}{1-\frac{25}{100}} = \frac{4}{3}$ for any bidder that receives bid credits.

I follow the parameterization used by Krishna and Rosenthal (1996) for auctions with synergies in my parameterization of global values. In particular, I assume that global bidder i has a standalone value of x_i and a complementarity value of θ_i, which is added to the standalone value if the bidder wins both licenses. Under this parameterization, the marginal value of the first license is $V_2^i = x_i$, while the marginal value of the next license is $V_1^i = \theta_i + x_i$. The total value for bidder i is then $V^i = V_1^i + V_2^i = 2x_i + \theta_i$.

The summary statistics suggest that there may be heterogeneity between groups of global bidders. To accommodate this possibility, I allow for between-group heterogeneity in the global value distributions. Let $g(i) \in \{S, L\}$ denote the group affiliation of global bidder i, with the possible groups being either small (S) or large (L). I assume that $x_i \sim \mathcal{W}\left(\lambda_x^{g(i)}, k_x^{g(i)}\right)$ and that $\theta_i \sim \mathcal{W}\left(\lambda_\theta^{g(i)}, k_\theta^{g(i)}\right)$. An attractive property of the Weibull distribution is that it can be readily adapted to allow for bid credits. Specifically, a small global bidder has a credited standalone distribution of $\phi x_i \sim \mathcal{W}\left(\phi\lambda_x^S, k_x^S\right)$ and a credited complementarity distribution of $\phi\theta_i \sim \mathcal{W}\left(\phi\lambda_\theta^S, k_\theta^S\right)$. A local bidder has a credited value distribution of $\phi v_i \sim \mathcal{W}\left(\phi\lambda_l, k_l\right)$.

To maintain the consistency between the theory and empirical model, I must truncate all value distributions. I select my truncation values so that they are high enough to prevent any substantial changes to the calibrated outcomes. For global bidders, I set a truncation value for the standalone and complementarity distribution of 2 (or \$2 per capita), which is above the highest observed per capita global bid in the data. For local bidders, I truncate the value distribution at 10 (or \$10 per capita) to bypass situations where global bidders draw values that guarantee a win over all local bidders. I find that these truncation values are far enough away from the observed bids that changing them has no meaningful effect on my results.

4.2 Moments

Given these parametric assumptions, I can simulate draws from the value distributions and use them to compute outcomes such as a bidder's equilibrium dropout value. To calibrate the model to the data, I must match moments from these simulated outcomes to moments arising in the data. Thus, my next step is to select the targeted data moments.

One outcome of the model is the per capita price at which global and local bidders drop out of bidding given the group affiliation of their competition. Thus, my first set of data moments are the mean and standard deviations of each type of bidder's dropout values, which are the last bid each bidder makes in the data excluding the winning bidder.

When local bidders are active, inter-license complementarities generate an exposure problem for all global bidders in the model. To extract this information from the data, I require variation in dropout prices engendered by a local bid presence that is tied to how global firms value multiple licenses. As has been documented by Ausubel et al. (1997) and Moreton and Spiller (1998) in the first two broadband PCS auctions, bidders tend to prefer contiguous licenses over non-contiguous ones. I exploit this preference to identify the complementarity parameter by separating the global dropout moments by whether there is a local bidder active on the license or on a contiguous license. In effect, I am assuming that the exposure problem, if present, is generated by local bidders on nearby licenses.

Another model outcome that would ideally match the data is the bid that ultimately wins a license. The continuous nature of the model, coupled with the discrete structure of the FCC's auction format, results in discrepancies between model winning bids and data winning bids, unfortunately. Hence, I do not use the winning bid amounts in the calibration. I do capitalize on some information contained in the winning bids, though, by matching the proportion of observed winners of each type to the proportions predicted by the model.

Not all local bidders receive credits in the data. Because the model does not allow for heterogeneity in the local-bidder dimension, I account for this issue by applying weights to the moments. Specifically, I solve the model twice: once with local credits and once without them. The final moments are thus a weighted average of the two solutions, with the weights determined by the percent of local bids in the data that have credits applied to them. Approximately 52 percent of all local dropout bids have credits, so I use a weight of 52 percent for the solution with local credits and 48 percent for the solution without them.

In sum, I have 13 total data moments, which are all outlined in Table 3. The first eight moments are the means and standard deviations of the global dropout values tabulated by whether there is a local bidder in a contiguous market and whether the bidder is small or large. The next two moments are the mean and standard deviation of the observed local dropout values, and the last three moments are the proportions of each type of bidder (large global, small global, and local) that win a license.

4.3 Calibration Methodology

My calibrated parameter values are the Weibull parameters that generate weighted outcomes most similar to the ones observed in the data. I obtain these parameters by minimizing the distance between the model and data moments. Let $\check{m}$ denote the weighted data moments, and let $\mathbf{\Psi}$ be a vector collecting every Weibull parameter. If $m(\mathbf{\Psi})$ are the weighted moments generated from simulating the model with Weibull parameters $\mathbf{\Psi}$, then the minimum-distance criterion function is

$$Q(\mathbf{\Psi}) = \left[\check{m} - m(\mathbf{\Psi})\right]' \left[\check{m} - m(\mathbf{\Psi})\right].$$

The calibrated parameters, $\hat{\mathbf{\Psi}}$, are the ones that minimize this criterion function.

To produce weighted moments from the model, I simulate a total of 200 auctions: 100 with local credits and 100 without local credits. In each simulated auction, I draw standalone and complementarity values for the global bidders from their type-specific value distributions, as well as values for the local bidders on each of the two licenses. I use these draws in conjunction with the model to compute equilibrium dropout values for each type of bidder, which will depend on the type and number of active bidders for global bidders. I then record the prices at which each bidder drops out, whether there was an active local bidder at the time of dropout, and the type of the winning bidder (or bidders). I calculate the model moments from these records.

4.4 Identification

There are a total of ten Weibull distribution parameters I aim to recover from the data. These parameters include shape and scale parameters of each global bidder's standalone value distributions (λ_x^g,k_x^g), the shape and scale parameters of each global bidder's complementarity distribution (λ_θ^g,k_θ^g), and the shape and scale parameters of the local value distribution (λ_l,k_l).

The local dropout moments identify the local value parameters since local bidders drop out when the price reaches their value or credited value. Separating the dropout moments for global bidders by whether there is a local bidder active in a contiguous market identifies the parameters of each component of the global bidder's value distribution. Observe that global bidders who drop out when a local bidder is active faced an exposure problem in the model, and the magnitude of the exposure problem is governed by the complementarity parameter. By contrasting dropout behavior with and without exposure, I can disentangle the part of global bidders' values that are standalone from the part that is complementarity. The proportion of winning bidders from each group acts as an additional aid in identifying the value distributions because it contains information from the winning bids.

4.5 Results

Table 2 lists the parameters obtained from the calibration. The parameters imply that a large global bidder that draws the average standalone and complementarity has 58.0 percent of their value attributed to complementarity. For small global bidders, a similar calculation reveals that 80.7 percent of their average value is due to complementarity. The average global bidder value due to complementarity is, therefore, 67.1 percent, which is close to the findings from Xiao and Yuan (2018) in their study on the 700 MHz radio frequency band.

Next, I turn to the model's fit, which is reported in Table 3. The dropout statistics are normalized so that the reported dropout values are per capita, and "local active" refers to whether a local bidder is active on that license or a contiguous license. Overall, the model seems to fit the dropout moments well despite being stylized. The model overestimates the number of large global winners and underestimates the number of small global bidders, though, which is likely due to a particularly strong small global bidder—Access Spectrum. Access Spectrum is the only small bidder that wins any licenses. Because I do not use the winning bids in the dropout moments, their bids are less likely to appear in the small global bidder distribution, which causes

Table 2: Calibrated Per Capita Value Parameters

	Value
Small global	
Standalone scale parameter (λ_x^S)	0.04
Standalone shape parameter (k_x^S)	1.57
Complementarity scale parameter (λ_θ^S)	0.30
Complementarity shape parameter (k_θ^S)	0.83
Large global	
Standalone scale parameter (λ_x^L)	0.10
Standalone shape parameter (k_x^L)	0.31
Complementarity scale parameter (λ_θ^L)	2.71
Complementarity shape parameter (k_θ^L)	0.47
Local	
Scale parameter (λ_l)	0.28
Shape parameter (k_l)	1.16

Note: Calibrated Weibull parameters for the per capita value distributions.

the calibration to underestimate the number of small global winners.

5 Counterfactual Analysis

With the results from the calibration established, I shift focus to my analysis of counterfactual credits. I use the calibrated value distributions to investigate how auction outcomes would change had the FCC granted small businesses a different credit. I begin my analysis with an exploration of the increased exposure problem that arises from local bidders receiving credits by contrasting equilibrium dropout with different credits to the no-credit alternative. I then simulate the dropout values for all bidders over a range of bid credit values and employ those results to infer counterfactual auction outcomes.

5.1 Exposure Analysis

The model suggests local bid credits heighten exposure issues for global bidders, causing them to drop out of bidding earlier. To explore the magnitude of this effect, I solve the equilibrium dropout values for global bidders under multiple bid credits, ranging from no credits to a sizable bid credit of 75 percent. Because the exposure problem originates from local bidding, my analysis focuses on dropout values when both local bidders are active.

Figure 1 illustrates the changes in equilibrium dropout at the per capita level. Equilibrium dropout depends on the standalone and complementarity component in global values. To construct a two-dimensional figure, I fix the standalone value at its mean of $0.18 per capita and allow the complementarity parameter to

Table 3: Model Fit

	Data	Model
Avg. local dropout	0.23	0.26
Std. local dropout	0.24	0.23
Local Active		
Avg. large global dropout	0.12	0.12
Avg. small global dropout	0.18	0.13
Std. large global dropout	0.16	0.15
Std. small global dropout	0.11	0.12
Local Inactive		
Avg. large global dropout	0.77	0.60
Avg. small global dropout	0.32	0.38
Std. large global dropout	0.27	0.25
Std. small global dropout	0.20	0.19
Winners		
Prop. large global winners	0.58	0.68
Prop local winners	0.04	0.09
Prop. small global winners	0.38	0.23

Note: Comparison of the moments in the data with the moments generated by the model at the calibrated parameters. All moments are in terms of dollars per capita. Local Active refers to whether a local bidder is still bidding in the license or a contiguous license.

vary. As is evident by that figure, bid credits can lead to considerable reductions in global dropout values, particularly at the 75 percent level. Fixing the complementarity at its mean value of \$0.50 per capita, I find that, relative to no local credits, large global dropout values decrease by 5.7 percent with a 25 percent credit, 13.8 percent with a 50 percent credit, and 26.6 percent with a 75 percent credit.

5.2 Counterfactual Credits

Although local credits can lead to marked decreases in large global dropout values, the effect on revenue hinges on the relative strength of large firms. If large firms have sufficiently high values, then bid credits can increase revenues by making small firms more competitive. I investigate the possible impact of various bid credits on revenues as well as the proportion of small winners in Table 4, which contains statistics on outcomes simulated at multiple credit levels. Since licenses in the data serve populations of different sizes, and my model assumes symmetry in licenses—I scale up the per capita results by the average population in the data, which is about 5.4 million. Thus, the bid and revenue results in Table 4 are per license. I also limit the highest credit to 50 percent in these simulations so that I do not overextend my analysis too far beyond the observed data.

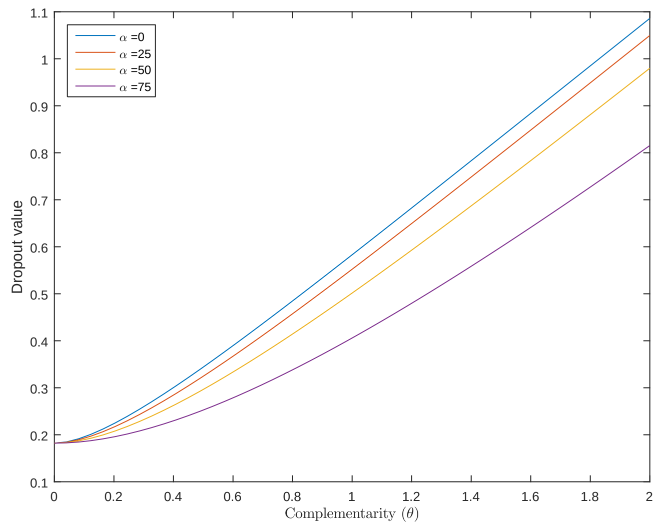

Figure 1: Equilibrium Dropout Values for Large Global Bidders

Note: Dropout values are in dallors per capita and are evaluated holding x_i fixed at its mean value of 0.18 . Figure assumes that both local bidders are active.

To contrast these outcomes against the efficient one, I use the simulated values to run the Vickrey–Clarke–Groves (VCG) mechanism. This well-known allocation and payment rule requires that each firm pay their social cost of participation in exchange for an efficient outcome, even when there are complementarities. Because bidders report their values to the mechanism rather than bidding, there are no average bid outcomes.

Table 4: Counterfactual License-Level Outcomes

	VCG	$\alpha = 0$	$\alpha = 12.5$	$\alpha = 25$	$\alpha = 37.5$	$\alpha = 50$
Avg. large global bid (in millions)		1.63	1.65	1.65	1.67	1.71
Avg. small global bid (in millions)		0.88	1.00	1.15	1.36	1.67
Avg. local bid (in millions)		1.22	1.31	1.43	1.58	1.79
Avg. winning bid (in millions)		2.60	2.70	2.82	3.00	3.29
Avg. revenue (in millions)	2.61	2.60	2.62	2.62	2.68	2.62
Prop. preferred winners	0.21	0.22	0.26	0.29	0.30	0.40

Note: Table shows average outcomes for the VCG mechanism and different credits. Outcomes are averaged at the license level.

As a general trend, increasing the credit leads to higher average bids for each type of bidder, with the most significant increase coming from small global bidders. The pattern of growing average bids for large global bidders, despite them not receiving any credits, suggests that large global bidders are strong enough to withstand the exposure effect outlined previously. Small global bidders have the largest increase because they always receive credits, whereas only a fraction of local bidders receive credits. Average winning bids follow a pattern similar to that of the average bids in that they increase with the size of the credit. Average revenues, which adjusts the winning bids for credits where applicable, remain stable at around 2.6 million per license, with a small increase at a 37.5 percent credit. When contrasted against the increasing proportion of small winning bidders, the revenue statistic suggests that revenues are mostly unresponsive to the credit: revenue losses from small bidders that win because of the credit are offset by the large bidders that the credit forces to bid higher.

Relative to the efficient VCG outcome, auctions allocate a higher proportion of licenses to small firms at every credit, even when there are no credits. This result comes from exposure favoring small local firms, with allocative differences becoming more pronounced as credits increase the exposure problem. Average VCG revenues are higher than average revenues from the no-credit auction, consistent with Example 1 from Goeree and Lien (2014). With high enough credits, however, that result flips because of the relative strength of large firms.

Table 5: Change in Counterfactual Outcomes

	VCG	$\alpha = 0$	$\alpha = 12.5$	$\alpha = 37.5$	$\alpha = 50$
%Δ large global bid		-1.39	-0.46	1.23	3.27
%Δ small global bid		-23.49	-13.16	18.00	44.92
%Δ local bid		-14.38	-8.33	10.85	25.63
%Δ winning bid		-7.84	-4.27	6.50	16.83
%Δ revenue	-0.52	-1.03	-0.17	2.15	-0.05
%Δ prop. preferred win	-28.42	-25.00	-9.45	3.34	36.82

Note: Table shows changes in outcomes when the FCC uses the VCG mechanism and different credits. The results are in percentages and are relative to the 25 percent baseline.

To delineate how different credits alter the status quo, I compare percent changes in outcomes from the observed 25 percent credit level. Table 5 summarizes those results. The table highlights the trends discussed previously, with the policy-relevant results in the bottom two lines. An increase in the credit from 25 percent to 37.5 percent results in a modest increase in *both* revenues (2.15 percent) and the proportion of winning bidders (3.34 percent). With a 50 percent credit, the FCC could have attained a 36.8 percent increase in the proportion of small winners with a negligible effect on revenue. My results indicate that the FCC may have been able to use the credit to increase the representation of small winners without adversely affecting revenues.

6 Discussion

My analysis is based on a stylized model that captures how equilibrium bidding changes with credits in a tractable way. There are other rules and features of FCC auctions that may also be of interest in other applications. In this section, I outline those additional considerations and discuss how they might influence my analysis.

To conclude their auctions in a reasonable time frame, the FCC used a round-based format, where a bidder would decide whether to submit a bid on each license and choose their bid from a menu of pre-determined amounts in each round. Because bidding took place in discrete rounds instead of continuously, the model is inherently different than what occurred in practice. In a discrete model, the optimal dropout price would be the bid increment before expected profits turn negative given the number of active competitors and is likely to be lower than the ones implied by the continuous model.

There is an additional possibility that bidder values exhibit some form of substitutes. Since each MEA had an A and B block license for sale in Auction 33, firms in the B block might have considered bidding on a similar A block license when the B block price became too high (and vice versa). In general, allowing for substitutes results in a more pronounced exposure problem. Goeree and Lien (2014) investigate a case with one global bidder where local bidders can substitute between licenses. Substitution lets local bidders drive up the price on all licenses uniformly until they drop out, so to the extent that substitution occurs, dropout values in the data are higher than what one would expect if substitution was not allowed. Nevertheless, exposure issues are still present in these models and are made worse through substitution, so exposure problems from credits may be even more severe than predicted.

A more recent addition to FCC spectrum auctions that does not affect the data from Auction 33 is anonymous bidding. Under anonymous bidding, bidders know neither the identity nor characteristics of their competitors until the auction concludes. Because global bidders would be unaware of whether local bidders have credits in that environment, the analysis of their dropout strategies becomes considerably more complex. However, if credit information is revealed before the auction, my analysis would remain the same: local bidders have the same weakly dominant strategy of bidding up to their value, and global bidders can use the number of dropped licenses to infer whether a dropping bidder is local or global. Thus, the ranking derived in my main results should still apply.

7 Conclusion

This paper presents a new channel through which bid credits can affect bidding and revenues in simultaneous ascending auctions. When bidders interested in multiple licenses, or global bidders, have complementarities in their values, they face a known exposure problem: bidders can potentially win a bundle of licenses at a price higher than their value. I show in this paper that bid credits magnify these exposure issues for global bidders, causing them to drop out of bidding earlier and potentially decreasing revenues.

The strength and relevance of this channel depend crucially on the complementarities between bidder valuations and the level of the credit. Since these objects will vary with the context in which they arise, I use a calibrated model to explore its likely impact on A block bidding in FCC Auction 33. In this auction, small businesses received a 25 percent bidding credit, while large businesses received no credits. With few exceptions, the calibrated model fits the data well and suggests the average large global bidder drops out of bidding at a price 5.7 percent lower than they would in an auction without credits when multiple licenses have active local bidders. I go a step further by using the model to explore how outcomes—such as bidding, revenues, and the proportion of winners that are small businesses—change when the FCC adjusts the credit. My calibration indicates that large firms have high enough values that the FCC could have used a 50 percent bidding credit with a minimal effect on revenues. A credit of that magnitude would have increased the proportion of small winners by 36.8 percent. These results suggest that the FCC may have had considerable flexibility in changing the credit without adversely affecting revenues.

There are several areas open to future research. Due to the added complexity of allowing firms to substitute between licenses, I do not allow for license substitution in my analysis. I also simplify the FCC's format by using continuous prices when their actual format is round based. I leave the investigation of these points to future research.

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A Proofs

Proof of Proposition 1. When $v \leq p$, $F_k = \widehat{F}_k = 0$, and when $v \geq \phi\overline{v}$, $F_k = \widehat{F}_k = 1$. Suppose now that v is in the interval $(p, \phi\overline{v})$. Consider the truncated distributions for F_v and $\widehat{F}_v$ given $v > p$, and call them $\overline{F}$ and $\widehat{\overline{F}}$ respectively. Then,

$$\widehat{\overline{F}}\,(x) = \overline{F}\left(\frac{x}{\phi}\right) < \overline{F}\,(x)\,.$$

Therefore, the minimum order statistics adhere to the following inequality: $\widehat{F}_k = 1 - \left(1 - \widehat{\overline{F}}\right)^k < 1 - \left(1 - \overline{F}\right)^k = F_k$. □

Proof of Proposition 2. I prove this proposition through contradiction. Suppose that either $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) = D_2^1\left(\boldsymbol{V}^i\right)$ or $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) > D_2^1\left(\boldsymbol{V}^i\right)$.

Case 1. If $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) = D_2^1\left(\boldsymbol{V}^i\right)$, then given that $D_2^1\left(\boldsymbol{V}^i\right) = V_2^i + \Pi_1^1\left(\boldsymbol{V}^i, D_2^1\left(\boldsymbol{V}^i\right)\right)$ and $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) = V_2^i + \tilde{\Pi}_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right)$, it must be the case that $\tilde{\Pi}_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right) = \Pi_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right)$, which is a contradiction given Proposition 1.

Case 2. If $\tilde{D}_2^1\left(\boldsymbol{V}^i\right) > D_2^1\left(\boldsymbol{V}^i\right)$, then

$$\tilde{\Pi}_1^1\left(\boldsymbol{V}^i, \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right) = \int_{\tilde{D}_2^1(\boldsymbol{V}^i)}^{V_1^i} \left(V_1^i - v_1\right) d\widehat{F}_1\left(v_1 \mid \tilde{D}_2^1\left(\boldsymbol{V}^i\right)\right) < \int_{D_2^1(\boldsymbol{V}^i)}^{V_1^i} \left(V_1^i - v_1\right) d\widehat{F}_1\left(v_1 \mid D_2^1\left(\boldsymbol{V}^i\right)\right)$$

since expected profits decrease as the price increases. However,

$$\int_{D_2^1(\boldsymbol{V}^i)}^{V_1^i} \left(V_1^i - v_1\right) d\widehat{F}_1\left(v_1 \mid D_2^1\left(\boldsymbol{V}^i\right)\right) < \int_{D_2^1(\boldsymbol{V}^i)}^{V_1^i} \left(V_1^i - v_1\right) dF_1\left(v_1 \mid D_2^1\left(\boldsymbol{V}^i\right)\right) = \Pi_1^1\left(\boldsymbol{V}^i, D_2^1\left(\boldsymbol{V}^i\right)\right),$$

which is a contradiction. Observe that the above inequality is a property of first-order stochastic dominance.

□

Proof of Proposition 3. The equality case occurs when $k = 1$, as $\tilde{D}_1^1\left(\boldsymbol{V}^i\right) = D_1^1\left(\boldsymbol{V}^i\right) = V_1^i$. I prove the inequality case through induction. The base case, when $k = 2$, is shown above. Now suppose that $\tilde{D}_{k-j}^1\left(\boldsymbol{V}^i\right) < D_{k-j}^1\left(\boldsymbol{V}^i\right)$ for $j = 1, 2, \ldots, k-2$ and $k > 2$. From here, apply a contradiction argument by assuming $\tilde{D}_k^1\left(\boldsymbol{V}^i\right) \geq D_k^1\left(\boldsymbol{V}^i\right)$. If that assumption is true, then $D_k^1\left(\boldsymbol{V}^i\right) = V_k^i + \Pi_{k-1}^1\left(\boldsymbol{V}^i, D_k^1\left(\boldsymbol{V}^i\right)\right)$ and $\tilde{D}_k^1\left(\boldsymbol{V}^i\right) = V_k^i + \tilde{\Pi}_{k-1}^1\left(\boldsymbol{V}^i, \tilde{D}_k^1\left(\boldsymbol{V}^i\right)\right)$ imply $\tilde{\Pi}_{k-1}^1\left(\boldsymbol{V}^i, \tilde{D}_k^1\left(\boldsymbol{V}^i\right)\right) \geq \Pi_{k-1}^1\left(\boldsymbol{V}^i, D_k^1\left(\boldsymbol{V}^i\right)\right)$. But $\tilde{D}_{k-j}^1\left(\boldsymbol{V}^i\right) < D_{k-j}^1\left(\boldsymbol{V}^i\right)$ and Proposition 1 imply $\tilde{\Pi}_{k-2}^1\left(\boldsymbol{V}^i, p\right) < \Pi_{k-2}^1\left(\boldsymbol{V}^i, p\right)$. Thus,

$$\begin{aligned}
\tilde{\Pi}_{k-1}^1\left(\boldsymbol{V}^i, \tilde{D}_k^1\left(\boldsymbol{V}^i\right)\right) &= \int_{\tilde{D}_k^1\left(\boldsymbol{V}^i\right)}^{\tilde{D}_{k-1}^1\left(\boldsymbol{V}^i\right)} \left[\tilde{\Pi}_{k-2}^1\left(\boldsymbol{V}^i, v_{k-1}\right) + \left(V_{k-1}^i - v_{k-1}\right)\right] d\widehat{F}_{k-1}\left(v_{k-1} \mid \tilde{D}_k^1\left(\boldsymbol{V}^i\right)\right) \\
&< \int_{\tilde{D}_k^1\left(\boldsymbol{V}^i\right)}^{\tilde{D}_{k-1}^1\left(\boldsymbol{V}^i\right)} \left[\Pi_{k-2}^1\left(\boldsymbol{V}^i, v_{k-1}\right) + \left(V_{k-1}^i - v_{k-1}\right)\right] dF_{k-1}\left(v_{k-1} \mid D_k^1\left(\boldsymbol{V}^i\right)\right) \\
&< \int_{D_k^1\left(\boldsymbol{V}^i\right)}^{D_{k-1}^1\left(\boldsymbol{V}^i\right)} \left[\Pi_{k-2}^1\left(\boldsymbol{V}^i, v_{k-1}\right) + \left(V_{k-1}^i - v_{k-1}\right)\right] dF_{k-1}\left(v_{k-1} \mid D_k^1\left(\boldsymbol{V}^i\right)\right) \\
&= \Pi_{k-1}^1\left(\boldsymbol{V}^i, D_k^1\left(\boldsymbol{V}^i\right)\right),
\end{aligned}$$

which is a contradiction. Observe that since $\tilde{D}_k^1\left(\boldsymbol{V}^i\right) < D_k^1\left(\boldsymbol{V}^i\right)$, it must be the case that $\tilde{\Pi}_{k-1}^1\left(\boldsymbol{V}^i, \tilde{D}_k^1\left(\boldsymbol{V}^i\right)\right) < \Pi_{k-1}^1\left(\boldsymbol{V}^i, D_k^1\left(\boldsymbol{V}^i\right)\right)$. □

Proof of Proposition 4. There are two equality cases.

Case 1. If $k = 0$, then

$$\tilde{D}_0^2\left(\boldsymbol{V}^i\right) = \frac{\sum_{l=1}^n V_l^i}{n} = D_0^2\left(\boldsymbol{V}^i\right).$$

Case 2. If $K \geq 2$ and $n = k = 1$, then

$$\tilde{D}_1^2\left(\boldsymbol{V}^i\right) = \tilde{D}_0^2\left(\boldsymbol{V}^i\right) = V_1^i = D_1^2\left(\boldsymbol{V}^i\right) = D_0^2\left(\boldsymbol{V}^i\right).$$

Now, suppose neither of those cases hold. When $n = k$, $D_n^2\left(\boldsymbol{V}^i\right) = D_{n-1}^1\left(\boldsymbol{V}^i\right)$, and the inequality case follows from Proposition 3. When $n > k$, rewrite the equilibrium dropout values as

$$D_k^2\left(\boldsymbol{V}^i\right) = \frac{\Pi_k^1\left(\boldsymbol{V}^i, D_k^2\left(\boldsymbol{V}^i\right)\right) + \sum_{l=k+1}^n V_l^i}{n-k}$$

and

$$\tilde{D}_k^2\left(\boldsymbol{V}^i\right) = \frac{\tilde{\Pi}_k^1\left(\boldsymbol{V}^i, \tilde{D}_k^2\left(\boldsymbol{V}^i\right)\right) + \sum_{l=k+1}^n V_l^i}{n-k}.$$

To show that this proposition is true, it suffices to show that $\tilde{\Pi}_k^1\left(\boldsymbol{V}^i, \tilde{D}_k^2\left(\boldsymbol{V}^i\right)\right) < \Pi_k^1\left(\boldsymbol{V}^i, D_k^2\left(\boldsymbol{V}^i\right)\right)$. This inequality follows immediately from the proof of Proposition 3 by replacing $D_k^1\left(\boldsymbol{V}^i\right)$, $\tilde{D}_k^1\left(\boldsymbol{V}^i\right)$, $D_{k-j}^1\left(\boldsymbol{V}^i\right)$ and $\tilde{D}_{k-j}^1\left(\boldsymbol{V}^i\right)$ with $D_k^2\left(\boldsymbol{V}^i\right)$, $\tilde{D}_k^2\left(\boldsymbol{V}^i\right)$, $D_{k-j}^2\left(\boldsymbol{V}^i\right)$ and $\tilde{D}_{k-j}^2\left(\boldsymbol{V}^i\right)$, respectively. □

Proof of Corollary 1. Let $\breve{\phi} = \frac{1}{1-\frac{\breve{\alpha}}{100}}$. Because $\phi = \frac{1}{1-\frac{\alpha}{100}}$ is increasing in α, $\breve{\alpha} > \alpha$ implies $\breve{\phi} > \phi$. Let $\breve{F}_k$ be the probability of drawing a v that is higher than the lowest of the k active local bidders' credited values when the credits are $\breve{\alpha}$. Following the proof of Proposition 1, $\breve{F}_k < \widehat{F}_k$ on the interval $\left(p, \breve{\phi}\overline{v}\right)$ because $\overline{F}\left(\frac{x}{\breve{\phi}}\right) < \overline{F}\left(\frac{x}{\phi}\right)$. From there, Corollary 1 follows from the proofs of Propositions 2, 3, and 4. □