Asymmetry in Okun's Law Revisited: New evidence on cyclical unemployment–cyclical output trade-off in the Free State Province using NARDL model.

Omoshoro-Jones, Oyeyinka Sunday

Free State Provincial Treasury, Bloemfontein, South Africa

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Oyeyinka S. Omoshoro-Jones

Abstract

Since 1994, the ineffectiveness of adopted pro-growth policies to reduce the persistently high unemployment rate in the Free State (FS) province has become a conundrum for policymakers, begging the questions: Is Okun's law that predicts an inverse unemployment-output relationship exists in the FS province? If so, what is the nature of Okun's relationship? This paper re-examines the asymmetric unemployment–output tradeoff employing the nonlinear autoregressive distributed lag (NARDL) modelling framework. Cyclical components of unemployment and output are generated from annual data covering the period 1994-2019, using Hodrick-Prescott and Corbae-Ouliaris filters. We controlled for structural breaks and performed a sensitivity analysis on the models estimated. Irrespective of the filtering method, our results confirm asymmetric Okun's relationship among variables in both the long-and-short run. The negative and statistically significant coefficient on changes of the positive cyclical output reveals that a 1% rise in cyclical output could lower cyclical unemployment between –0.87 to –0.70 percentage points, reliant on a sustained economic expansion. Estimated long-run coefficients of changes in the positive cyclical output show that an economic upswing between 1.88% and 2.03% would lower unemployment by 1%, in the FS province, consistent with the accepted 2:1 ratio for Okun's law in the empirical literature. We also find significant contemporaneous effects of changes in cyclical output on cyclical unemployment, where a 1% increase (decrease) in one-period lagged positive (negative) cyclical output reduces (increases) cyclical unemployment between –0.52 and –0.41 (+0.99 and +0.56) percentages points. The detection of these asymmetries explains the failure of the enacted policies to successfully reduce the prevalent high unemployment rate in the FS province. Based on these findings, some apt remedial actions are suggested to policymakers.

JEL Classification: C51, E24, E32, J64.

Keywords: Okun's Law, cyclical unemployment, cyclical output, NARDL, Free State province

1. Introduction

In the past two decades or so, the high unemployment rate remains a perennial issue in South Africa (Fig.8 and Table 7 in the Appendix), and the ineffectiveness of subsequently adopted national policies aimed at reducing the continued rise in the numbers of unemployed persons, that is, the active working-age population (15-64 years) creates a conundrum for policymakers at national and provincial levels government. While, the inability of improved economic condition during an economic expansion (Fig.8 and Table 7 in the Appendix) to considerably reduce the unemployment rate has attracted the attention of researchers on the 'jobless growth or recovery' phenomenon in the country (see, e.g., Marinkov and Geldenhuys, 2007; Leshoro, 2013; Phiri, 2014), and at the sectoral level (Gumata and Ndou, 2019). A consensus in these studies indicates that unemployment seemed to be less responsive to economic expansion in South Africa.

In South Africa, between 2014 and 2019, the number of unemployed people in the country grew by 1.5 million, increasing the unemployment rate by 3. 6 percentage points (from 25.1% in 2014) to 28.7% in 2019 (Statistics South Africa, 2019 hereafter StatsSA). Conversely, changes in South Africa's labour participation and labour absorption rates are trivial, suggestive of deep-seated structural problems in the labour market. For instance, the labour participation rate and absorption rate recorded in 2019 stands at 42.4% and 59.8%, respectively, compared to 42.6% and 56.8% documented in 2014 (StatsSA, 2020). On the other hand, the unemployment rate in South Africa (SA) has been exacerbated by the marked increase in youth unemployment that is progressively outpacing adult unemployment due to a much lower absorption rate and labour participation rate. Available labour data shows a sizeable increase in youth unemployment rate to 41% in 2019 (from 35.9% in 2014), while the adult unemployment rate rose to 19% in 2019 (from 15.7% in 2014) (Stats SA, 2019).

On the policy front, in the past two decades, the growing concerns of the South African government on the prevalent high unemployment and dampened economic growth have led to the implementation of notable policies with central objectives focusing on boosting economic growth, reduce the prevailing high unemployment rate and improve general welfare (i.e., eradicate income inequality and reduce poverty rates. In this context, since 2004 till date, the South African government have introduced six prominent policies which include: the Reconstruction and Development Program (RDP in 1994), the Growth, Employment and Redistribution strategy (GEAR in 1998), the Accelerated and Shared Growth Initiative for South Africa (AsgiSA in 2004), the Joint Initiative for Priority Skills Acquisition (JIPSA in 2007), the New Growth Path (NGP in 2010), and the National Development Plan (NDP in 2014) in 2014. Nevertheless, the available data on the South African economy and labour market shows that none of these nationally implemented policies has achieved the key objective of raising economic growth to the desired 6% as well as lower unemployment rate below 15% (Table 7 in the Appendix)[1].

However, the pervasiveness of the high unemployment rate and dampened economic growth becomes glaringly worrisome at the provincial level (Table 8 in the Appendix). Among the provinces, the Free State has the most consistent alarming unemployment rate and lacklustre economic growth, in comparison to large provinces such as the Gauteng, Kwa-Zulu Natal and Western Cape. The highest unemployment rate among the working-age population (15-64 years) were recorded in Free State (22.2%), Mpumalanga (21.5%) and Gauteng (21.9%) provinces, from the period 2014 to 2019, (StatsSA, 2019). Also, there is no palpable improvement in the labour participation rate and absorption rate among the working-age population in the Free State. For instance, the labour participation rate and absorption rate recorded for the FS stands at 61.1% and 41.5% in 2014 (with an unemployment rate of 32.7%), while labour participation rate of 63.6% and absorption rate of 41.1% are recorded in 2019 (with an unemployment rate of 34.7%) (StatsSA, 2020). Further, Free State province is among the provinces with the unemployment rate reaching the 30% threshold in 2019 (Table 8 in the Appendix).

In the light of the above, for effective policymaking, it is important to understand the correlation between the unemployment rate and economic growth in the Free State province, from both the theoretical and empirical perspectives. This exercise would provide a useful guide on the appropriate labour market reform policies, growth-enhancing strategies and the optimal (or desirable) growth rate, necessary to address the reduce the persistently high unemployment rate, at the same time, stimulate economic activity level. To justify this rationale, Fig. 1 shows the evolution between unemployment and economic growth in the Free State, since the advent of democracy in South Africa, spanning the period 1996 to 2019 (Table 8 in the Appendix).

An ocular inspection reveals three distinct phases of synchronized and decoupled movements between the unemployment rate and economic growth (measured as a gross domestic product, GDP), which includes (i) the pre-recession period (2002-2007), the immediate post-recession period (2010 to 2012), and (iii) the post-recession period (2014-2018). In the first phase (2002 to 2007), an inverse correlation between GDP and unemployment seem to exist, keeping with the empirical regularity hypothesised by Okun (1962). During this period, it appears the significant increase in GDP growth helped in lowering the unemployment rate, but in reality, the average growth rate of the GDP nearly 3.8%, and the unemployment rate is relatively high at 29%. The second phase (2010 to 2012) is characterised by a synchronized and positive correlation (co-movement) between the variables, but there is a sharp increase in the unemployment rate and marginal growth in GDP. At a face value, the GDP only grew by 0.5 percentage point to 3% in 2012 (from 2.5% in 2010), in contrast, unemployment rose sharply by 3 percentage points to peak at 31% in 2012 (from 28% in 2010). In the third phase

[1]For instance, GEAR was replaced with AsGISA to raise economic growth to 4.5% between 2005 and 2009, followed by a steady growth of 6% between 2010 and 2014, as well as halved unemployment rate to 15% by 2014 (from 30% in 2004)(The Presidency, 2006). After this, the NGP was introduced to drive job creation, stimulate economic growth, and reduce unemployment rate from 25% to 14% in 2020 (Economic Development, 2011. The recently introduced NDP aimed at reducing unemployment rate to 6% in 2030 from 6% in 2010 (NDP, 2030).

(2014-2018), both variables decoupled given the dramatic fall in GDP and unprecedented surge in the unemployment rate. In particular, in this phase, the average growth rate of the GDP and unemployment stands at 0.5% and 33.9%, respectively. This disparity becomes clear cut the divergent correlation among the two variables is analysed at a face value, which shows a sizeable GDP drop of about 2.2 percentage points to -0.2% in 2018, while unemployment increased by 5.3 percentage points to the highest record of 38% in the same period, compared to the growth rates of the GDP (2%) and unemployment (32.7%) recorded in 2014.

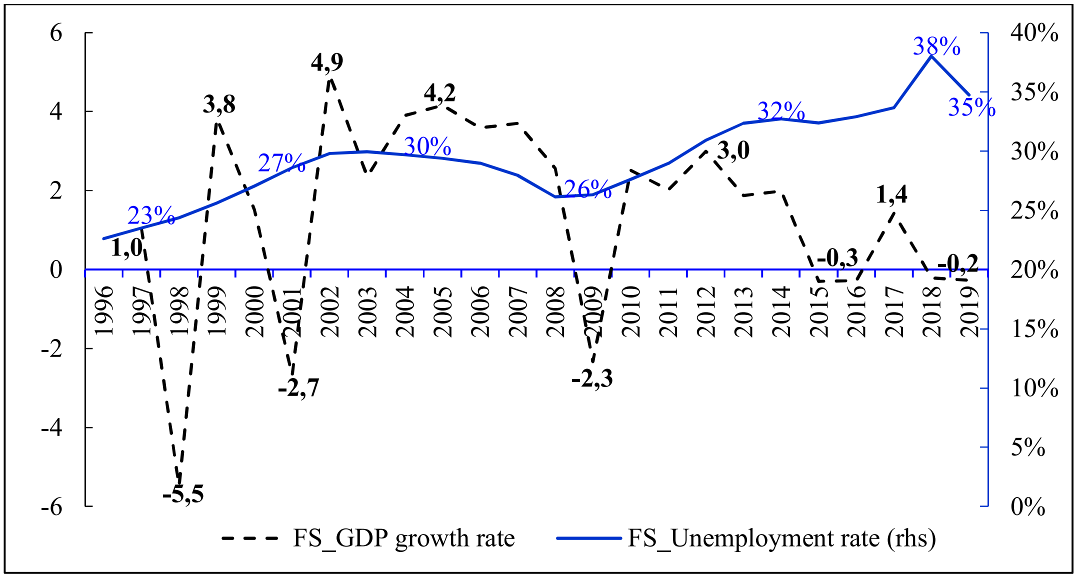

Fig.1. Historical evolution of real GDP and unemployment rate in Free State (1996-2019)
Data source: StatsSA, IHS Markit Regional eXplorer (ReX). Author's estimation.

Evidently, the disconnection between GDP growth and unemployment is a major concern for policymakers faced with the conundrum of how to formulate the appropriate pro-growth policy to tackle the acute unemployment rate in the Free State province. Empirically, the inability of an increase in GDP growth to have any dent on unemployment, while unemployment responds to fall in GDP begs the question of whether Okun's law applies to the Free State province (hereafter Free State). If so, what is the nature of the correlation between unemployment and output in the province?

Typically, the correlation between variations in unemployment and output over the business cycle relies on Okun's law. Okun (1962), noted an inverse relationship between the real gross national product (GNP) and unemployment gaps for the US, where a three percentage points increment in GNP reduces the unemployment rate by a percentage point (i.e., 3:1 trade-off ratio)[2]. In the past five decades, several studies have established theoretical foundations for the Okun's relationship, popularly referred to as Okun's law (see, e.g., Prachowny, 1993; Palley, 1993; Attfield and Silverstone, 1998; Gordon, 1984). In its simplest form, Okun's law links the activity in the goods market to activity in the labour market over the business cycle (Palley, 1993:148). In policy setting, changes in the unemployment rate and output, that is, Okun's relationship has important implications for macroeconomic policy, especially in determining the optimal or desirable growth rate (Sillvapulle et al. 2004). For instance, the combination of Okun's relationship with Phillip's curve yields the aggregate supply curve (Prachowny, 1993; Moosa, 1997), in effect, creating a link between inflation rate, unemployment rate and economic growth rate (Marinkov and Geldenhuys, 2007). Furthermore, the Okun's coefficient, measuring the responsiveness of unemployment to output growth can also reflect the cost of unemployment in terms of output (Sillvapulle et al. 2014), allowing policymakers to better understand how the labour market

[2]Studies by, for example, Freeman (2000), Mankiw (1994) and Gordon (1998) have found the estimate of Okun's relationship closer to 2 than 3.

adjusts (i.e., quick or slow) to changes in productivity, as well as knowing the appropriate policy-mix to adopt without adversely affecting the dynamics of the labour market and proper functioning of the domestic economy. As such, the Okun's coefficient is widely accepted as a useful 'rule of thumb' in policymaking and forecasting (Harris and Silverstone, 2001).

In the empirical literature, the Okun's coefficient is found to be relatively stable in some country-specific studies (see, e.g., Valadkhani and Symth, 2015; Sögner, 2001; Weber, 1995) and regional studies (see, e.g., Sögner and Stiassny, 2002; Freeman, 2000). Equally, changes in the Okun's coefficient has been well documented in many regional studies (see, e.g., Guisinger et al. 2018; Dixon et al. 2017; Durech et al. 2014; Villaverde and Maza, 2009) and cross-country studies (see, e.g., Jalles, 2019; Ball, 2017; Adanu, 2005; Moosa, 1997). While others have found the influence of structural changes, for example, associated with external shocks plays a notable role in the variability of Okun's coefficients across different regions or countries (Palley, 1993; Lee, 2000; Apergis and Rezitis, 2003; Owang and Sekhposyan, 2012). Despite the intense research interests and debates on the stability and/or usefulness of Okun's law (Knoteck, 2012), a common theme in the existing burgeoning literature relate to the Okun's relationship continues to hold in many countries/regions (Ball et al. 2017; Freeman, 2000; Sögner and Stiassny, 2002).

Most of the studies in the vast empirical literature have been devoted to the advanced countries (see, e.g., Jalles, 2019; Ball et al. 2017; Shin and Greenwood-Nimmo, 2014; Moosa, 1997), the US (see, e.g., Guisinger et al. 2018; Owang and Sekphosyan, 2012; Holmes and Silverstone 2006; Huang and Lin, 2006; Sillvapulle et al. 2004; Cuaresma, 2003; Altissimo and Violante, 2001; Harris and Silverstone, 2001) and the OECD countries/regions (see, e.g., Tang and Benthencourt, 2017; Huang and Yeh, 2013; Sögner and Stiassny, 2002; Virén, 2001; Harris and Silverstone, 2001; Lee, 2000).

Yet, despite the endemic problem of a persistently high unemployment rate and poor economic growth performance facing South Africa, only a handful of studies have investigated the asymmetry relationship between unemployment and output. Much of these studies focused on the South African economy (most notably, see, Marinkov and Geldenhuys, 2007; Phiri, 2014; Mazorodze and Siddiq, 2018; Sere et al. 2020), while the provincial economy has received no attention. To the best of our knowledge, only a recent study by Kavase and Phiri (2020) currently fill this research gap, but they found no evidence of asymmetric cyclical unemployment-cyclical output trade-off either in the long-run or short-run for the Free State province only, among the nine South African provinces.

Therefore, this study contributes to the literature by examining the presence of Okun's relationship in the Free State province over the period 1996 to 2019, employing the nonlinear autoregressive distributed lag (NARDL) model recently developed by Shin and Green-wood Nimmo (2014), which is simultaneously modelled long-run and short-run asymmetry in a statistically coherent manner. The NARDL model is suitable for our analysis because it also allows for an asymmetric bounds-testing cointegrated test (introduced by Pesaran et al. 2001) without the burden of pre-testing the data for the presence of unit roots, and produces asymmetric cumulative dynamic multipliers that are useful for tracing the nonlinear adjustment patterns after positive and negative shocks to the explanatory variables.

As it is common in the literature, we rely on the popular gap specification (i.e., gap model) originally proposed by Okun (1962), where cyclical components of unemployment and output growth rates are modelled symmetrically. To this end, we applied two different de-trending methods, namely Hodrick-Prescott (1997, hereafter HP filter) and Corbae Ouliaris (2006, hereafter CO filter) to isolate the cyclical components of the total unemployment and real gross domestic product (real GDP) for the Free State, over the period 1996 to 2019. Using the HP and CO filters allows us to ensure the robustness of drawn inferences and conclusion on the asymmetric Okun's relationship among the cyclical components (or gap variables). On top of that, for parsimony, the built dynamic NARDL models (based on the filtered data extracted using the HP and CO de-trending methods) were re-estimated using a cyclical component of real GDP per extracted capita based on the HP and CO filters, to conduct a sensitivity analysis. In this way, one would expect obtained results to considerably differ in magnitude, quantitatively and qualitatively, from those of the baseline models.

Most closely related to our paper is Kavase and Phiri (2020) who investigate the possibility of a nonlinear relationship between cyclical unemployment and cyclical output across the nine provinces in South Africa. Our paper differs from theirs in many dimensions. First, unlike these authors that relied on only one filtering method, we decompose the output and unemployment series into their trend and cyclical components using two different de-trending procedures to ensure robustness in the regressions analysis (Phiri, 2014). Second, our modelling approach is different from theirs. For instance, we account for structural breaks in our dynamic nonlinear models, which these authors ignored. Accounting for breaks in data is important to avoid spurious inferences on the presence of Okun's relationship and asymmetric long-run relationship between cyclical variables over the business cycle (Attfield and Silverstone, 1998; Lee, 2000; Harris and Silverstone, 2001). This particular omission could explain the rationale for the authors to erroneously conclude on the absence of an asymmetric relationship between cyclical variables being studied, in both the long-and-run for the Free State province.

The balance of this paper is structure as follows. Section 2 provides a brief survey of relevant studies testing asymmetry in Okun's law. Section 3 outlines the modelling strategy on the gap version of Okun's law within the NARDL modelling framework. Section 4 presents the annual data and descriptive statistics of the extracted cyclical components. The results of the estimated models and sensitivity analysis are discussed in Section 5, and Section 6 concludes with some policy recommendation.

2. Survey of relevant studies: Asymmetric unemployment-output trade-off

This sub-section provides a bird-eye view of the large body of studies in the empirical literature but focuses on the nascent strand of studies on asymmetric Okun's relationship in South Africa.

Broadly, the correlation between variations in unemployment and output over the business cycle relies on Okun's law. Okun (1962) noted an inverse relationship between the real gross national product (GNP) and unemployment gaps for the US, where a three percentage points increment in GNP reduces the unemployment rate by a percentage point, that is, a 3:1 trade-off ratio[3]. In the past five decades, subsequent empirical studies have established a theoretical foundation for the Okun's relationship– universally know as Okun's law (see, e.g., Prachowny, 1993; Palley, 1993; Attfield and Silverstone, 1998; Gordon, 1984).

In its simplest form, Okun's law links the activity in the goods market to activity in the labour market over the business cycle (Palley, 1993:148). In policy setting, changes in the unemployment rate and output, that is, Okun's relationship has important implications for macroeconomic policy, especially in determining the optimal or desirable growth rate (Sillvapulle et al. 2004). For instance, the combination of Okun's relationship with Phillip's curve produces the aggregate supply curve (Prachowny, 1993; Moosa, 1997), in effect, creating a link between inflation rate, unemployment rate and economic growth rate (Marinkov and Geldenhuys, 2007). Furthermore, the Okun's coefficient, measuring the responsiveness of unemployment to output growth can also reflect the cost of unemployment in terms of output (Sillvapulle et al. 2014), allowing policymakers to better understand how the labour market adjusts (i.e., quick or slow) to changes in productivity, as well as knowing the appropriate policy-mix to adopt without adversely affecting the dynamics of the labour market and proper functioning of the domestic economy. As such, the Okun's coefficient is widely accepted as a useful 'rule of thumb' in policymaking and forecasting (Harris and Silverstone, 2001).

In the voluminous literature, the Okun's coefficient is found to be relatively stable in some countries (Valadkhani and Symth, 2015; Sögner, 2001; Weber, 1995), states (Freeman, 2000), and region (Sögner and Stiassny, 2002). On the contrary, changes in the Okun's coefficient has been documented in some regional studies (see, e.g., Guisinger et al. 2018; Dixon et al. 2017; Durech et al. 2014; Villaverde and Maza, 2009) and cross-country studies (see, e.g., Jalles, 2019; Ball et al. 2017; Adanu, 2005; Moosa, 1997). While others have found the influence of structural changes, for example, associated with external shocks plays a notable role in the variability of Okun's coefficients across different regions or countries (Palley, 1993; Lee, 2000; Apergis and Rezitis, 2003; Owang and Sekhposyan, 2012). Despite the intense research interests and debates on the stability and/or usefulness of Okun's law (Gordon 2010; Knoteck, 2012), a common theme in the existing burgeoning literature relates to the Okun's relationship continues to hold in many countries/regions (Ball et al. 2017; Freeman, 2000; Sögner and Stiassny, 2002).

[3]Studies by, for example, Freeman (2000), Mankiw (1994) and Gordon (1998) have found the estimate of Okun's relationship closer to 2 than 3.

In the extant literature, most of the studies assume a linear (symmetric) relationship between changes in output and unemployment over the business cycle, implying that economic upturns and downturns have the same (absolute) effect on unemployment. On the contrary, unemployment can either be more responsive to changes in output during economic upswings or downswings. Theoretically, firms may become risk-averse during an economic recovery, if employers fired a given amount of labour after a negative output shock (i.e., during a recession), they might not recruit the same amount of labour after a positive shock (i.e., during recovery) of equal magnitude. This is situation is directly relatable to the labour market hysteresis phenomenon, where cyclical output shocks may have a permanent effect on structural unemployment (Blanchard and Summers, 1987), to the extent that the labour market does not tend to return to its initial state, after an economic growth recovery.

The importance of testing for an asymmetric relationship between unemployment and output has been stressed in recent studies, for instance, the knowledge on the extent of asymmetric unemployment-output trade-off can be a useful guide to policymakers on designing and/or adopting effective stabilisation policies and structural policies (e.g., labour market reforms), whereas if asymmetry is ignored when it is present may lead to forecasting errors and model misspecification (Harris and Silverstone, 2001). Similarly, ignoring the presence of asymmetry between unemployment and output when it exists can also lead to erroneous inference in hypothesis testing, by rejecting the null hypothesis of a long-run relationship among the variable, resulting in wrong policy prescriptions or inappropriate policy formulation (Sillvapulle et al. 2004). Equally, identifying the asymmetry relationship between unemployment and output can also improve the effectiveness of an unemployment policy (Virén, 2001). Consequently, studies have shifted their attention to exploring the possibility of an asymmetric inverse relationship between unemployment and output, in recent times.

However, the majority of the studies on the nonlinear relationship between unemployment and output largely focus on OECD countries, particularly the US, employing sophisticated econometric models. For instance, Altissimo and Violante (2001) find that propagated shocks during recession induces a nonlinear relationship between output and unemployment in the US due to the significantly large impact of the shock on unemployment than output, in a nonlinear vector autoregressive (VAR) model. Cuaresma (2003) constructed a regime-dependent specification of Okun's law to examine the asymmetric cyclical unemployment and cyclical output trade-off in the US, and find a significantly higher asymmetric contemporaneous effect of output on unemployment during economic recessions than in expansions, but shocks to unemployment seemed to be more persistent in the expansionary regime.

Using a developed dynamic model, Sillvapulle et al. (2004) confirms a negative nonlinear relationship between cyclical output and unemployment in the US and finds that the contemporaneous effects of positive cyclical output on cyclical unemployment quantitatively differ from those of negative ones. Holmes and Silverstone (2006) used a Markov regime-switching model that captures asymmetries within and across regimes, and find a significant inverse relationship between cyclical output and unemployment in the US during expansionary regimes.

Elsewhere, Lee (2000) studied the presence of an asymmetric Okun's relationship across 16 OECD countries utilizing a static model that allows changes (negative and positive) in unemployment to determine output growth rate, and find that a significantly higher Okun coefficient for decreases (than for increases) in the unemployment rate for Finland, Japan, and the United States, while the opposite holds for Canada, France, and the Netherlands. Viren (2001) introduced an error correction–asymmetric based model, in which changes in unemployment are determined by positive and negative changes in output, to assess the asymmetric relationship among the variables across 20 OECD countries; obtained results show that output growth exerts a stronger impact on unemployment when it is low and output is high, and vice versa.

Harris and Silverstone (2001) test for asymmetry in Okun's law in seven OECD countries using asymmetric error correction model (where Okun's coefficient either above or below long-run equilibrium), and finds that unemployment reacts asymmetrically to contemporaneous changes in output contingent on upswings or downswings in the business cycle; they found asymmetry for Australia, Japan, New Zealand, United Kingdom, United States, and Germany. While, Huang and Yeh (2013) find a highly significant unemployment-output trade-off for 53 OECD countries both in the short and long run at both the state and country level, using panel ARDL (with Pool Mean Group estimator) estimated for the period 1980 to 2005.

Shin and Greenwood-Nimmo (2014) find evidence of an asymmetric negative relationship between cyclical output and unemployment in the US, Canada and Japan, using the NARDL model with monthly data spanning 1982 to 2003. More recently, Tang and Bethencourt (2017) consider the asymmetric unemployment-output tradeoff across 17 countries in the Eurozone employing the nonlinear autoregressive distributed lag (NARDL), which showed the evidence of long-run and short-run asymmetries in most of the countries considered, while labour markets respond quickly to changes in cyclical outputs in a short period, but the adjustments towards new equilibrium become weak in the long run.

Despite the endemic problem of a persistently high unemployment rate and poor economic growth performance, only a handful of studies have investigated the asymmetry relationship between unemployment and output. Among these, Marinkov and Geldenhuys (2007) relied on an error correction-based asymmetric model and controlled for structural breaks in annual series for South Africa. They find evidence for both symmetric relationship (estimates range from -0.77 to -0.16) and asymmetric relationship (estimates range from -0.77 to -0.18) between cyclical output and cyclical unemployment, irrespective of the version of Okun's specifications and filtering procedure employed. Their results also show highly significant negative and nonlinear contemporaneous impacts of cyclical output on cyclical unemployment. Using a momentum threshold autoregressive model (MTAR), Phiri (2014) find support for the existence of a negative and nonlinear relationship between unemployment and output in South Africa from 2000 to 2013, regardless of the de-trending methods applied, while unemployment Granger-causes economic growth in the long run, suggestive of the probable cause of the jobless-growth phenomenon in South Africa.

Meanwhile, Mazorodze and Siddiq (2018) employed a NARDL using quarterly data (1994Q1–2017Q4) for South Africa and finds significant asymmetric negative effects of changes in cyclical output on unemployment in both the long-run and short-run, where a 1% increase in positive (negative) cyclical output decreases (increases) unemployment by 0.80 (+1.03) in the long-run. They also find a significant and negative contemporaneous relationship between the cyclical output and unemployment in the short-run, such that a 1% increase (decrease) in positive (negative) cyclical output is associated with a reduction (rise) of 0.14 (+0.18) in unemployment. In the same vein, Sere et al. (2020) consider the existence of an asymmetric unemployment–output trade-off in a multivariate set-up (i.e., add investment variable) employing a NARDL model estimated for South Africa over the 1994Q1 to 2019Q4, and find an insignificant positive correlation between changes in output (positive and negative) on unemployment in the long-run asymmetry, but a negative and significant Okun's relationships existing amongst the two variables in the short-run, as predicted by Okun's law.

Finally, Kavase and Phiri (2020) explored the presence of an asymmetric Okun's relationship across nine South African provinces utilizing a nonlinear autoregressive distributed lag model (NARDL) estimated for the period 1996 to 2016 and confirms the Okun's relationships in the majority of the provinces. They found a significant asymmetric inverse between the unemployment gap and output gap both in the long-and short-run, only in 2 provinces (i.e., Western Cape and KZN) and evidence of short-run asymmetry in others, except the Free State province, where no asymmetry was found, while the unemployment gap and the output gap are positively correlated in the long-run.

3. The Model: Non-linear Autoregressive Distributed Lag (NARDL)

This sub-section outlines the econometric approach to model the gap specification of Okun's law using the NARDL modelling framework recently developed by Shin and Greenwood-Nimmo (2014) as an augmented version of the ARDL model of Pesaran et al. (2001).

Therefore, to examine the presence of Okun's law in the Free State province, we prefer the theoretical version of Okun's law which relates the deviation of the unemployment rate from its natural rate to deviation of output growth from its potential, popularly referred to as the gap specification or gap model[4]. The gap model is a bivariate system that links activity in the goods market to activity in the

[4]In the literature, Okun's law has been widely studied with the help of, either the gap model or the first difference model (regress differenced unemployment rate series on output series). Originally, Okun (1962) inferred the inverse unemployment rate-output growth nexus based on the gap model (Attfield and Silverstone, 1998:625), which has the advantage over the first difference model since it captures the movements in unemployment around

labour market. In essence, the gap model captures the responsiveness of cyclical unemployment to the fluctuations in cyclical output over the business cycle[5].

For our application, the preferred gap model relies on the theoretical notion that the inverse relationship between unemployment and output generally expressed as:

$$u_t - u_t^* = \alpha_0 + \beta(y_t - y_t^*) + \varepsilon_t \qquad \beta < 0 \tag{1}$$

Expressed in a reduced-form:

$$u_t^g = \alpha_0 + \beta y_t^g + \varepsilon_t$$

where y_t and u_t are observed real output growth rate (actual output) and unemployment rate, respectively. Since y_t^* represents the potential or trend level of output and u_t^* is the natural rate of unemployment, thus y_t^g captures the cyclical output or output gap $(y_t - y_t^*)$, and u_t^g accounts for the cyclical unemployment or unemployment gap $(u_t - u_t^*)$. The parameter β is the Okun's coefficient, and ε_t is the stochastic error term, which accounts for factors that cause changes in the shift the unemployment rate–output relationship (for example, unusual fluctuation in productivity, changes in labour force participation, variability in the distribution of sectoral growth, and factor substitution (see, e.g., Silvapulle et al. 2004; Weber, 1995; Moosa, 1997).

But, computing the conventional linear gap model expressed in Eq.(1) arbitrarily can lead to spurious and/or inconclusive inferences, since it assumes a symmetric unemployment rate–output inverse relationship, but ignores the possibility of an asymmetric response of unemployment to economic upswings (i.e., positive output gap) and downswings (i.e., negative output gap) stressed in subsequent empirical studies (Neftçi,1984; Palley, 1993; Attfield and Silverstone, 1998; Silvapulle et al. 2004; Virén, 2001; Lee, 2000; Cuaresma, 2003). Further, the linear gap equation is susceptible to misspecification bias if cyclical variables are cointegrated (Attfield and Silverstone, 1998), as it represents a short-run dynamic version of Okun's law but lacks an error correction mechanism (Harris and Silverstone, 2001).

To circumvent these drawbacks and accounts for possible asymmetrically response of the unemployment rate to positive and negative growth shocks over the business cycle, we rely on the flexible and coherent NARDL modelling framework of Shin and Greenwood-Nimmo. Compared to other sophisticated econometric tools used in the strand of literature exploring the possibility of asymmetric Okun's law (e.g., Markov-switching regime-dependent or threshold models, MTAR, error correction-asymmetric based model, nonlinear-VAR, Harvey structural time-series model, and Hamilton model), the NARDL modelling frame work produces a dynamic nonlinear model that models both long-run and short-run asymmetries by decomposing the independent variable into positive and negative partial sum processes. Moreover, the model is flexible, requires no pre-testing for the presence of unit root in time series, and effectively captures both long-run and short-run asymmetries regardless of the integration order (or linear combination) of variables as $I(1)$ or $I(0)$ processes[6].

Furthermore, the most appealing feature of the NARDL modelling framework lies in its ability to produce asymmetric dynamic multipliers which allows the assessment of the cumulative effects of

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the natural rate of unemployment due to fluctuations in actual output around its trend growth. As such, corresponding cyclical components included in the gap model effectively captures the oscillations of output around the trend such that a percentage rise in output growth above its trend growth rate leads to a corresponding fall in unemployment below its natural rate of unemployment.

[5]The gap version of the Okun's enjoys great attention in the expansive empirical literature. To mention a few, see the studies by Weber (1995), Moosa (1997), Lee (2000), Virén (2001), Harris and Silverstone (2001), Cuaresma (2003), Silvapulle et al. (2004), Marinkov and Geldenhuys (2007), Owang and Sekphosyan (2012), Valadkhani and Symth (2015), Dixon et al. (2017), Ball et al. (2017), Jalles (2019), and Kavase and Phiri (2020).

[6]NARDL model is an augmented version of the symmetric ARDL model, and it has become a more appealing econometric tool in cross-country studies (see, e.g., Tang and Bethencourt, 2017; Shin and Greenwood-Nimmo, 2014) and country-specific studies (see, e.g., Kavase and Phiri, 2020; Sere et al. 2020; Mazorodze and Siddiq, 2018) assessing asymmetric unemployment rate–output trade-off, in recent times.

shocks owing to changes in the explanatory variables (in our case, the positive and negative partial sum processes of cyclical output) as well as the asymmetric adjustment paths and/or duration of the disequilibrium. Dynamic multipliers encapsulate the transition between the initial equilibrium, short-run disequilibrium following a shock, and the new long-run equilibrium (Tang and Bethencourt, 2017). Also, dynamic multipliers do not require forecasting of the cointegrating variables, thus it is less susceptible to the uncertainty associated with standard impulse response analyses. Besides, akin to the ARDL model, the NARDL model efficiently deals with endogeneity problems[7]and a suitable econometric tool when the sample size is small, as in this study.

Therefore, to implement the NARDL centred on the gap model describe in Eq. (1), consider a dynamic unrestricted ARDL (*p*,*q*)–error correction (ECM) based model in the form:

$$\Delta u_t^g = \alpha_0 + \beta_1 u_{t-1}^g + \beta_2 y_{t-1}^g + \lambda_1 \sum_{k=1}^{p} \Delta u_{t-k}^g + \lambda_2 \sum_{k=1}^{q} \Delta y_{t-k}^g + \delta ECT_{t-1} + \xi_t \tag{2}$$

where Δ is the first difference operator, λ_1 and λ_2 are the short-run coefficients with β_2 being the long-run coefficient (normalized on β_1), ξ_t is the normally distributed residual term, such that $\xi_t \sim IID(0,\sigma^2)$, and δ is the coefficient of the one-period lagged error correction term ECT_{t-1}, which measures the adjustment speed of the system to its long-run equilibrium following a shock. Usually, the coefficient of the ECT_{t-1} term is expected to be negative and statistically significant to confirm the existence of a long-run relationship among variables.

Since our main interest is to assess the non-linearity of the relationship between the unemployment rate and output, the model estimated in Eq.(2) is unsuitable to achieve this empirical goal, as it assumes that changes in the cyclical unemployment are linearly associated (one-for-one) with fluctuations in cyclical outputs over the business cycle. Also, it is plausible to reject the null hypothesis of no long-run relationship among variables (i.e., hidden cointegration) since the latent positive and negative components of the cyclical output are not separated (Granger and Yoon, 2002), masking the responsiveness of cyclical unemployment to changes in cyclical output during economic expansions (positive cyclical output) and recessions (negative cyclical output) over the business cycle.

Following Shin and Greenwod-Nimmo (2014), short-run and long-run asymmetries are incorporated into Eq. (2) by defining the potential output, y_t^g as a $k \times 1$ vector consisting of multiple regressors, such that $y_t^g = y_t^g + y_t^{g+} + y_t^{g-}$, obtaining an asymmetric long-run regression:

$$u_t^g = \beta^+ y_t^{g+} + \beta^- y_t^{g-} + \xi_t \qquad \xi_t \sim N(0,\sigma^2) \tag{3}$$

where β^+ and β^- are nonlinear error correction term associated with asymmetric long-run parameters, whereas y_t^{g+} and y_t^{g-} are the partial sum processes of positive and negative changes in cyclical output y_t^g, respectively defined as:

$$\begin{aligned} y_t^{g+} &= \sum_{k=1}^{t} \Delta y_k^{g+} = \sum_{k=1}^{t} \max(\Delta y_k^g, 0) \\ y_t^{g-} &= \sum_{k=1}^{t} \Delta y_k^{g-} = \sum_{k=1}^{t} \max(\Delta y_k^g, 0) \end{aligned} \tag{4}$$

The decomposition of cyclical output into associated positive and negative changes in Eq.(4) accounts for the differentiated effects of output shocks on the unemployment rate during economic upswings (expansion) and downswings (recession).

Substituting Eq. (4) into Eq. (2) yields a dynamic nonlinear ARDL (*p*,*q*) (NARDL) model under the error correction condition:

[7]The bounds-test procedure employed to establish if variables are cointegrated sufficiently deals with (weak) endogeneity issues (associated with non-stationary regressors) and serial correlation in residuals, by allowing the inclusion of optimum lags.

$$\Delta u_t^g = \alpha_0 + \sum_{k=1}^{p} \rho_i u_{t-k}^g + \theta_k^+ y_{t-k}^{g+} + \theta_k^- y_{t-k}^{g-} + \sum_{k=1}^{p-1} \varphi_i \Delta u_{t-k}^g + \sum_{k=0}^{q-1} (\pi_k^+ y_{t-k}^{g+} + \pi_k^- y_{t-k}^{g-}) + \mu ECT_{t-1} + \omega_t \quad \text{for} \quad k = 1,...,q \tag{5}$$

where θ_k^+ and θ_k^- are the asymmetric distributed-lagged parameters, φ is the autoregressive parameter, and ω_t is an IID process with zero mean and constant variance, σ^2. Parameters ρ, θ_k^+, and θ_k^- are long-run coefficients, while π_k^+ and π_k^- are short-run coefficients. Whereas $\beta^{g+} = -\theta^+ / \rho$ and $\beta^{g-} = -\theta^- / \rho$ are the asymmetric long-run coefficients. Note that reliable estimates of $\theta_k^+, \theta_k^-, \pi_k^+$ and, π_k^- are computed by standard OLS.

The dynamic NARDL model computed in Eq.(5) is used to carry out the bound-testing procedure proposed by Pesaran et al.(2001) to establish whether variables are cointegrated (i.e., exhibits a long-run relationship) based on modified *F*-test of the joint null, $\hat{\rho} = \theta_k^+ = \theta_k^- = 0$. Whereas, the standard Wald test is applied to uncover the existence of asymmetric relationship among variables in the long-run ($H_0 : \hat{\theta} = \theta_k^+ = \theta_k^-$), and in the short-run (either as $H_0 : \pi_k^+ = \pi_k^-$ for all $i = 0,...q-1$ or $H_0 : \Sigma_{i=0}^{q-1} \pi_k^+ = \Sigma_{i=0}^{q-1} \pi_k^-$)[8].

Finally, we assess the cumulative asymmetric effects associated with one unit change in y_t^{g+} and y_t^{g-}, respectively on u_t^g by deriving dynamic multipliers from Eq. (5) as:

$$m_h^{g+} = \sum_{j=0}^{h} \frac{\partial y_{t+j}^g}{\partial u^{g+}}, \; m_h^{g-} = \sum_{j=0}^{h} \frac{\partial y_{t+j}^g}{\partial u^{g-}}, \quad \text{for} \; h = 0,1,2... \tag{6}$$

Notice that $h \to \infty$, $m_h^{g+} \to \beta^{g+}$ and $m_h^{g-} \to \beta^{g-}$, where $\beta^{g+} = -\theta^+ / \rho$ and $\beta^{g-} = -\theta^- / \rho^-$ are the asymmetric long-run coefficients. In our case, estimated dynamic multipliers shed light on adjustment asymmetry by capturing the adjustment patterns from initial equilibrium to the new equilibrium following output shocks propagation in the domestic economy.

4. Data

Our model contains annual data on the gross domestic product (GDP) and the total unemployment rate for the Free State province, over the period 1999 to 2019. The nominal GDP (constant prices, 2010=100) is converted to real series (i.e., adjusted for inflation) using the Consumer Price Index (CPI)[9]. All data were sourced from Statics South Africa[10]. Seasonality is removed from the annual series using the Seasonal and Trend decomposition (STL) method introduced by Cleveland et al. (1990)[11].

Usually, the gap version of Okun's law described in Eq. (1) requires information about unemployment and output trends, which are directly unobservable in the gap model. So far, the empirical literature provides no definite guideline on the appropriate filtering techniques to generate

[8]Applying the Wald test on the joint null of $\pi_k^+ = \pi_k^-$ and additive symmetry $\Sigma_{i=0}^{q-1} \pi_k^+ = \Sigma_{i=0}^{q-1} \pi_k^-$ produces a strong– and weak–form of short-run symmetry, accordingly (Shin and Greenwood-Nimmo, 2014:293).

[9]The series on CPI (2010=100) is retrievable from http://www.statssa.gov.za/?page_id=1854&PPN=P0141

[10]Available at http://www.statssa.gov.za/. Provincial data on GDP (constant prices, 2010=100) and total unemployment (by official definition) are only available and published since the post-Apartheid period (i.e., 1994). The annual data sourced from Stats SA were benchmarked with those provided by IHS Markit ReX.

[11] The STL can decompose annual, quarterly and monthly data into seasonal and trend components, which makes it a versatile and superior decomposition method, compared to the popular ARIMA based decomposition methods (notably, TRAMO-SEATs and X-11). STL decomposition method generate robust seasonal series from annual, quarterly and monthly data using Loess estimation. The method also effectively deals with outliers, thus producing robust estimates of the trend-cycle and seasonal components from a time-series data with irregular patterns and/or missing values.

these trend series (Lee, 2000). Thus, to implement the gap model, the unobserved cyclical component (gaps) and trend component of the unemployment growth and real GDP growth rates were extracted from the observed series y_t and u_t by applying the Hodrick-Prescott (1997) and Corbae and Ouliaris (2006) filters[12]. In this context, similarity in the empirical results of the gap version of Okun's law using both the HP and CO filters would reinforce the robustness of the specified NARDL models.

It has been argued that the arbitrary use of HP filter tends to generate spurious cycles which do not exist in the original series–differing from the actual business cycle dynamics, which may lead to biased estimates of cyclical components and erroneous inferences (Harvey and Jaeger, 1993; Cogley and Nason, 1995; King and Robello, 1993). Also, HP-filtered data are perceived as being sensitive to the sample period and have poor end-of-sample properties (Hamilton, 2017; Phillips and Jin, 2015). Yet, several studies have reported unbiased estimates of Okun's coefficient using HP filter compared to alternative filtering techniques (see, e.g., Jalles, 2019; Ball et al. 2017; Huang and Lin, 2006; Marinkov and Geldenhuys, 2007)[13].

Nevertheless, we remedied these limitations in two ways: Firstly, we set the HP filter smoothing parameter (λ) to 100[14]. Secondly, employing the CO filter–a frequency-domain filter, which has been proven to be superior in extracting cyclical components without losing observations at the end-points of series, generate de-trended series with super-consistent finite sample properties that are statistically reliable and asymptotically converges to their true growth cycle. Compared to time-domain filtering techniques (such as HP, Band-pass and Baxter King filters), the CO filter is impervious to end-points problems. To apply the CO filter, the maximum (s) and minimum (e) values are respectively set to 2 and 8 (frequencies 0.25 and 1) to retain the oscillation periods in our annual data, congruent to the business cycle[15]. The summary statistics of the cyclical components of the unemployment and output growth rates based on the HP and CO filters are presented in Table 1, which shows the cyclical unemployment exhibiting greater variability (3.3%) than the cyclical output (2.8%) based on the reported standard deviations. The insignificant probability value of the Jarque-Bera statistics for both cyclical components supports the assumption of normally distributed residuals.

Fig. 2 and Fig. 3 plots the evolution of the cyclical components (gaps) of the unemployment and output series based on the HP and CO filters. In both filters, the amplitude and magnitude of the cyclical components of the unemployment and output growth rates appear to be similar. But, at the end sample, the estimate of the cyclical output is negative (positive) and the cyclical unemployment is positive (positive) in the HP filter (CO filter). This inference is plausible, as different de-trending techniques may generate varying cyclical components (or gap estimates) (King and Robelo, 1993).

On the other hand, the three notable sharp troughs (peaks) in the cyclical output (cyclical unemployment) coincide with major global events such as (i) the 1997 Asian currency crisis, (ii) the unwinding of the speculative tech bubble (i.e., dot.com bubble) in 2000/2001, and (iii) the synchronized fall in global economic growth during the 2007/08 recession. Furthermore, it is noticeable that the cyclical unemployment exhibit a much higher magnitude compared to the cyclical output, corroborating the persistently high unemployment rate in the Free State province during an economic expansion (Table 1). Apart from this, the inverse relationship between the cyclical output and cyclical unemployment is less clear cut during the 25 years being studied, as the cyclical variables often cross at values different from zero, implying that a positive output gap is uncorrelated to a negative unemployment gap (or vice versa).

Table 1. Summary statistics of unobserved components of u_t and y_t (1994-2019).

	HP de-trending method	CO de-trending method

[12] We refer readers to the Appendix for a brief exposition on the respective de-trending procedures used by the HP and CO filters.

[13] Refer to Appendix A for more details.

[14] The chosen smoothing parameter is equivalent to value of 6.25 prescribed by Ravn and Uhlig (2002) to improve generated filtered series from annual data when implementing HP filter.

[15] See, '*Apply Corbae-Ouliaris frequency domain filter to time series*' http://fmwww.bc.edu/repec/bocode/c/couliari.html (Accessed 20 October 2020)

Description	Cyclical unemployment (%)	Cyclical output (%)	Cyclical unemployment (%)	Cyclical output (%)
	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$
Mean	-1.66e-14	2.982e-14	1.13e-15	9.27e-16
Median	-0.281	0.351	-0.062	0.569
Maximum	6.599	5.259	6.496	5.555
Minimum	-6.929	-5.902	-6.258	-5.395
Std. dev.	3.397	2.865	3.306	2.888
Skewness	0.198	-0.330	0.056	-0.394
Kurtosis	2.526	2.944	2.513	2.768
Jaque-Bera statistics (*p*-value)	0.366 (0.832)	0.422(0.809)	0.238 (0.887)	0.648 (0.604)
Sum	-3.81e-13	6.88e-13	2.35e-14	2.09e-14
Sum Sq. Dev.	253.885	180.611	240.575	183.552

Note: *p*-values in ( ) parenthesis.

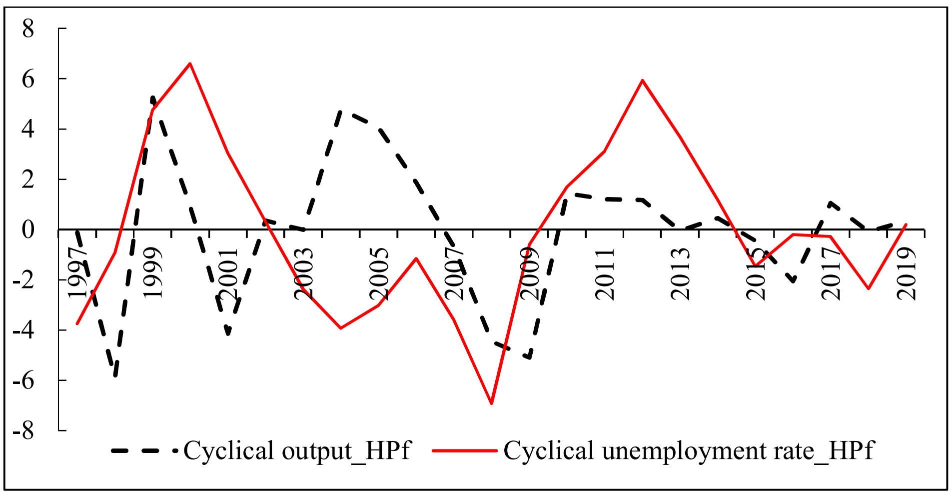

Fig.2. Estimated cyclical real GDP and cyclical unemployment using HP filter.

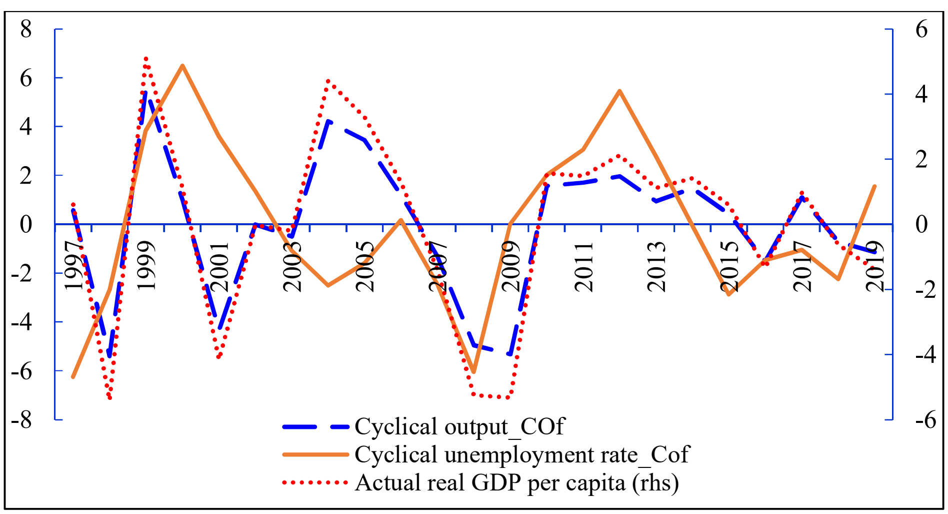

Fig.3 Estimated cyclical real GDP and cyclical unemployment using CO filter

Taken together, to a large extent, there is evidence of a dominant positive correlation between unemployment and output growth (most notably, during the periods: 1998 – 2000 and 2009–2013) in the Free State province, suggesting that the unemployment–output trade-off regularity postulated by

Okun's law may not holds in the province, and a rise in economic activity level during economic expansion will not translate into job opportunities in the labour market, consistent with the evidence of the 'jobless growth' phenomenon documented for South Africa during the past decades (see, Phiri, 2014; Marinkov and Geldenhuys, 2007).

Next, we determine the degree of integration of the cyclical variables. Even though it is straightforward to utilize the NARDL 'gap' model specified in Eq. (5) without pre-testing for the stationarity properties of the cyclical unemployment and cyclical gap variables, nonetheless, the estimated bivariate asymmetric gap model becomes invalid when the integration order of the variables is higher than 2. In this context, a cursory inspection of Fig.2 and Fig.3 indicates that the two cyclical variables may contain structural breaks, partly attributed to the impact of adverse global shocks, which is usually undetectable by conventional Augmented Dickey-Fuller (ADF) test (Barnejee et al. 1992; Perron and Vogeslang, 1992). And, the failure to account for existing endogenous structural breaks in the cyclical variables can lead to spurious inferences that are biased towards a false null hypothesis when data are trend stationary (Perron, 1989).

Hence, the integration order of the cyclical unemployment and cyclical output series was established using the augmented Dickey-Fuller GLS (Elliot et al. 1996) and Phillips-Perron (Phillips and Perron, 1988, hereafter PP) unit root tests. On top of that, we applied the Breakpoint (BP) unit root test (with DF min-*t*) that allows for a structural break in the deterministic trend to be endogenously determined at unknown dates (or time)[16]. The results of the DF-GLS, PP and BP (for both innovation outliers and additive outliers break test) unit root tests are reported in Table 2.

We also account for the effects of structural changes on the cyclicality of the output growth and unemployment growth rates owing to significant global and/or domestic shocks[17] by incorporating a dummy variable into the specified dynamic gap model in Eq. (5), which is re-estimated as:

$$\Delta u_t^g = \sum_{k=1}^{p} \rho_i u_{t-k}^g + \theta_k^+ y_{t-k}^{g+} + \theta_k^- y_{t-k}^{g-} + \gamma_1 D_{t-k} + \sum_{k=1}^{p-1} \varphi_i \Delta u_{t-k}^g + \sum_{k=0}^{q-1} (\pi_k^+ y_{t-k}^{g+} + \pi_k^- y_{t-k}^{g-}) + \gamma_2 \Delta D_{t-k} \\ + \mu ECT_{t-1} + \omega_t \qquad \omega_t \sim N(0, \sigma^2) \tag{7}$$

where γ_1 and γ_2 are the coefficients of the dummy variable accounting for structural changes. In our case, D_t encapsulates all the break dates (*BD*) identified by the Breakpoint unit root test, and specified such that $D_t = 1$ for $t \geq BD$ and 0 otherwise.

5. Empirical Results and Discussion

This section discusses the results of the deployed (i) unit root tests to determine the order of integration of the unemployment and output gap variables, (ii) asymmetric bound-test to establish long-run relationship among gap variables, and (iii) dynamic NARDL models (with gap variables extracted using the HP and CO filters) to examine whether the cyclical unemployment and cyclical output exhibit an asymmetric and negative relationship (as postulated by Okun's law), in both the long and short-run.

We also check the robustness of the baseline dynamic NARDL-gap models, as well as the reliability of the estimation results by conducting a sensitivity analysis, in which the cyclical component of real GDP (measuring cyclical output) in the baseline model, is substituted with that of real GDP per capita (constant prices, 2010=100), generated using the HP and CO filters.

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[16]The Breakpoint unit root test is developed based on theoretical contributions of, notably Perron (1989), Vogelsang and Perron (1998), Zivot and Andrews (1992), and Banerjee et al. (1992).

[17]In the empirical literature, it has become a common practice to include dummy variables into asymmetric and symmetric models (especially, the gap model) assessing Okun's law, to capture the effects of structural changes over the business cycle (see, Gordon, 1984; Harris and Silverstone, 2001; Sögner and Stiassny, 2002; Apergis and Rezitis, 2003; Palley, 1993; Huang and Yeh, 2013; Tang and Bethencourt, 2017, among others).

5.1. Unit root tests

We begin our analysis by considering the results of the unit root tests carried out presented in Table 2, which shows both the unemployment gap and output gap variables are generally stationary in levels, $I(0)$, without exceeding the integration order of one. Overall, the results of the DF-GLS, PP, and Breakpoint unit-root tests confirm that both the HP and CO filters are successful in extracting a potential unit root and endogenous structural breaks existing in the gap series (or cyclical components), since the generated cyclical components of unemployment and output are stationary (with or without trend). As anticipated, the Breakpoint unit root tests reveal structural breaks in the cyclical variables being considered, coinciding with prominent global events. This result justifies the inclusion of a dummy variable (created for the periods 2001 to 2010) in the estimated dynamic NARDL models, accounting for the impacts of structural changes in the cyclical unemployment and cyclical output, over the business cycle.

5.2. Bounds-Testing for Asymmetric cointegration (nonlinear long-run relationship)

Having established the stationarity properties of the cyclical variables, we investigate whether the cyclical components of the unemployment and output growth rates are cointegrated, that is, exhibits a long-run relationship, as stressed in the empirical literature (see, e.g., Harris and Silverstone, 2001; Lee, 2000; Attfield and Silverstone, 1998). After estimating the NARDL model in Eq. (7) with optimal lags of 2 based on the Akaike information criterion (AIC), the bound testing procedure is applied to test the joint *F*-test for the null of, $\hat{\rho} = \hat{\theta}_k^+ = \hat{\theta}_k^+ = 0.$ The results of the bound-tests for the estimated NARDL models reported in Table 3 shows that, in the baseline models, the values of the joint *F*-statistics exceed the upper bounds *I* (1) at all significance levels, indicating that cyclical unemployment and cyclical output (based on the HP and CO filters) have at least one long-run relationship, that is, the cyclical variables are cointegrated in the long-run. Besides, the negative and statistically significant coefficients of the one-period lagged error correction terms, ECT_{t-1} in the nonlinear dynamic models reported in Table 4 further reinforce the evidence of a long-run relationship (i.e., co-movement) between cyclical variables being studied. Also, the significant negative ECT_{t-1} affirm the reversion of the bi-variate system (including the independent variables) to a long-run equilibrium (or steady-state) following an economic perturbation or external shocks.

5.3. Does asymmetric Unemployment-Output trade-off exist in the Free State province?

Given the clear evidence of a nonlinear long-run relationship existing between cyclical unemployment and output, the next step is to examine whether cyclical unemployment reacts asymmetrically (or symmetrically) to the 'positive' and 'negative' changes in cyclical output, in times of economic expansion (upswing) and recession (downswing) in the Free State province. To this end, we employ the unrestricted dynamic NARDL specified in Eq. (7) representing the dynamic 'gap' version of Okun's law.

For robustness, separate dynamic nonlinear models were estimated as the baseline models, with one comprising of HP-filtered cyclical variables, and the second one contains the CO-filtered cyclical variables. The estimation results are reported in Table 4[18]. As can be seen, the results of the two NARDL-gap models estimated based using different de-trending techniques (i.e., HP and CO filters) are qualitatively quite similar, considering the magnitude coefficients of the positive and negative cyclical outputs, while the point estimates of the contemporaneous effects of cyclical outputs on unemployment do not differ considerably.

Table 2. Results of the unit root test on the estimated y_t^g and u_t^g using HP and CO filters.

	HP filter	CO filter

[18]We utilize the general-to-specific procedure (starting with 4 lags) to obtain the parsimonious NARDL models, and the AIC to select optimal lags (starting with 4 lags). These procedures mostly favoured the NARDL (2, 2, 1, 1) model, in all cases.

	Intercept	Intercept & trend	Intercept	Intercept & trend
DF-GLS				
y_t^g	-4.218*	-4.241*	-4.124*	-4.102*
u_t^g	-2,160**	-2.257	-2.367**	-2.658
Δu_t^g		-3.432**	-3.072*	-3.303**
PP				
y_t^g	-4,078*	-3.958**	-4.059*	-2.994***
u_t^g	-2.489	-2.444	-3.940**	-2.874
Δu_t^g	-3.282**	-6.986*		-7.195*
Breakpoint				
Innovative outlier: (DF min-*t*, *F*-stat.)				
y_t^g	-4.340***	-6.507*	-4.224***	4.631
Break date	2004	2007	2004	2007
Δy_t^g	-8.397*		8.259*	7.612*
Break date	2001		2001	2002
u_t^g	-5.829*	-4.991***	-5.397*	-5.325**
Break date	2008	2008	2008	2008
Δu_t^g		-5.804*		
Break date		2008		
Additive outlier: (DF min-*t*, F-stat.)				
y_t^g	-4.629**	5.094***	-4.861*	-4.851
Break date	2003	2006	2009	2010
Δy_t^g		-7.677*		-7.674*
Break date	2003	2001		2010
u_t^g	-5.261*	-4.500	-5.628*	-5.506**
Break date	2006	2005	2005	2005
Δu_t^g		-7.728*		
Break date		2001		

Note: *, **, and *** denotes significance of associated *p*-values at 1%, 5%, and 10% levels respectively. Breaks are chosen based on Dickey-Fuller *min-t* with optimal lag length (up to 4) being selected using *F*-statistics. Parameters y_t^g and u_t^g refers to cyclical output and cyclical unemployment.

Table 3. Bounds F-test for nonlinear cointegration (baseline models)

Dependent variable: ΔU_t^g		Function: $\hat{u}_t^g = f(\hat{y}_t^{g+}, \hat{y}_t^{g-})$			
De-trending method	Dynamic specification	F-statistic (F_{PSS})		Conclusion	
HP filter	NARDL (2,2,1,1)	20.712		Cointegration – asymmetric relationship exists	
CO filter	NARDL (2,2,1,1)	7.6087		Cointegration – asymmetric relationship exists	
Asymptotic CVs (k = 3)[1]: HP and CO gap variables					
1%		5%		10%	
I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
5.333	7.063	3.71	5.018	3.008	4.15

Note: I(0) and I(1) denotes the upper bound and lower bound levels, respectively. F_{PSS} refers to the F-test of the joint null, $\hat{\rho} = \hat{\theta}_k^+ = \hat{\theta}_k^+ = 0$, as in Pesaran et al. (2001). Asymptotic CVs for the two NARDL–gap models with HP and CO filtered data are tested based on Case III (unrestricted constant and restricted).

Next, we consider first the existence of asymmetric cyclical unemployment–cyclical output trade-off in the long run. Clearly, in the nonlinear dynamic models (with HP and CO de-trended series), only the 'positive' changes in cyclical outputs are found to be negative and statistically significant, with values of -0.87 (at 1% level) and -0.72 (at 10% level), implying that unemployment only responds to sustained and favourable output fluctuations (i.e., during economic expansion), over the business cycle. By implication, in the long run, a 1% increase in the 'positive' cyclical output will reduce cyclical unemployment between 0.87 and 0.72 percentage points, during a sustained economic upswing, in the Free State. This would indicate that the Okun's relationship (i.e., cyclical unemployment-output trade-off) is only present during an expansion in the Free State province. This inference aligns with Marinkov and Geldenhuys (2007) who find a 1% increase in cyclical output associated with a decrease in cyclical unemployment by 0.77 percentage points for South Africa, using an asymmetric-based error correction model estimated for the period 1970 to 2005.

On the contrary, unemployment appears to be irresponsive to output fluctuation during economic recessions, suggesting a breakdown in the inverse relationship between unemployment and output as predicted by Okun's law (Gordon, 2010; Palley, 1993). Specifically, the estimation results of the dynamic models unanimously show a positive relationship (or co-movement) between the 'negative' changes in cyclical output and cyclical unemployment, indicating that a 1% decrease in cyclical unemployment would reduce unemployment between 0.41 and 0.31 percentage points, during economic downswings (i.e., recession). This result agrees with, for example, Kavase and Phiri (2020), who find that a percentage decrease in cyclical output would reduce the cyclical unemployment gap by 1.66 for the Free State province, as well as Sere et al. (2020), who reported an insignificant positive relationship between unemployment and economic growth in South Africa. Likewise, Marinkov and Geldenhuys (2007) documented a significant positive relationship between cyclical output and cyclical unemployment for South Africa.

It is important to point out that, it is unsurprising to find a positive correlation between cyclical unemployment and output in the Free State province, as the provincial government is the largest employer of labour[19], in effect crowding out private sector employment growth. On one hand, the moral responsibility to reduce the pervasively high unemployment rate, rigid labour union negotiations and employment protection laws inhibit the provincial government from firing public workers during --

[19]See, '*Four facts about our provincial economies*', at http://www.statssa.gov.za/?p=12056 (Accessed on 18 December 2020)

Table 4. Dynamic asymmetric estimation of cyclical unemployment-cyclical output estimation (baseline models)

Dependent variable: ΔU_t^g							
		HP filter				CO filter	
Variable	Coefficient	Standard error	*t*-ratio (*p*-values)	Variable	Coefficient	Standard error	*t*-ratio (*p*-values)
u_{t-1}^g	-1.520	0.186	-8.133 (0.000)	u_{t-1}^g	-1.357	0.253	-5.357 (0.000)
y_{t-1}^{g+}	-0.876	0.245	-3.566 (0.000)	y_{t-1}^{g+}	-0.720	0.365	-1.971 (0.080)
y_{t-1}^{g-}	0.408	0.240	1.679 (0.123)	y_{t-1}^{g-}	0.314	0.335	0.936 (0.373)
Δu_{t-1}^g	0.921	0.142	6.447 (0.000)	Δu_{t-1}^g	0.934	0.208	4.487 (0.001)
Δy_t^{g+}	-1.636	0.294	-5.548 (0.000)	Δy_t^{g+}	-1.554	0.457	-3.395 (0.007)
Δy_t^{g-}	1.862	0.316	5.892 (0.000)	Δy_t^{g-}	1.857	0.449	4.131 (0.002)
Δy_{t-1}^{g+}	-0.529	0.172	-3.076 (0.013)	Δy_{t-1}^{g+}	-0.418	0.256	-1.629 (0.137)
Δy_{t-1}^{g-}	-0.831	0.350	-2.371 (0.041)	Δy_{t-1}^{g-}	-0.563	0.468	-1.201 (0.260)
$\gamma_1 D_{t-1}$	-15.341	2.758	-5.561 (0.000)	$\gamma_1 D_{t-1}$	10.395	3.172	-3.277 (0.009)
$\gamma_2 \Delta D_t$	-9.699	2.871	-3.377 (0.041)	$\gamma_2 \Delta D_t$	-7.577	1.938	-3.908 (0.003)
ECT_{t-1}	-1.520	0.144	10.510 (0.000)	ECT_{t-1}	-1.357	0.193	-7.022 (0.000)
Constant	39.626	6.713	5.902 (0.000)	*Constant*	32.079	9.125	3.515 (0.006)
L_y^+	-0.576	0.156	-3.680 (0.005)	L_y^+	-0.530	0.268	-1.978 (0.079)
L_y^-	0.2686	0.144	1.862 (0.095)	L_y^-	0.231	0.227	1.015 (0.336)
Diagnostic statistics							
R^2	0.951	$\bar{R}^2$	0.896	R^2	0.876	$\bar{R}^2$	0.740
F-statistic	17.474 (0.000)	*DW*	2.04	*F*-statistic	6.407 (0.005)	*DW*	2.07
χ_{SC}^2	1.306 (0.520)	χ_{NOR}^2	3.676 (0.159)	χ_{SC}^2	0.803 (0.669)	χ_{NOR}^2	2.961 (0.227)
$\chi_{HET-ARCH}^2$	0.763 (0.382)	χ_{FF}^2	2.620 (0.144)	$\chi_{HET-ARCH}^2$	1.309 (0.252)	χ_{FF}^2	2.124 (0.183)
W_{LR}	32.567(0.000)	W_{SR}	5.297 (0.021)	W_{LR}	11.607 (0.000)	W_{SR}	8.800 (0.003)

Note: Subscripts “+” and “−” denotes the positive and negative partial sum processes, respectively. L_y^+ (i.e., $\hat{\beta}^{g+} = -\hat{\theta}_k^+ / \hat{\rho}$) and L_y^- (i.e., $\hat{\beta}^{g+} = -\hat{\theta}_k^- / \hat{\rho}$) are the long-run coefficients associated with positive and negative changes of cyclical output (y^{g+}), respectively. W_{LR} and W_{SR} refers to the Wald test for long-run symmetry (i.e., $L_y^+ = L_y^+$) and the additive short-run symmetry condition (i.e., $\Sigma_{i=0}^{q-1}\hat{\pi}_k^+ = \Sigma_{i=0}^{q-1}\hat{\pi}_k^-$). χ_{SC}^2, χ_{H-ARCH}^2, χ_{NOR}^2, χ_N^2 and χ_{FF}^2 are the LM tests for serial correlation, heteroscedasticity, normality, and function form (Ramsey’s RESET), accordingly. Corresponding *p*-values in ( ) parenthesis

--economic downturns[20]. On the other hand, our results of a positive co-movement (rather than the predicted inverse relationship) between unemployment and output can be explained in the context of labour hoarding theory (hysteresis effect), as firms have the incentive to retain skilled and/or trained employees (for example, reducing work-hours per week) during economic downturns (see, e.g., Blanchard and Summers, 1986; Blanchard, 1999).

Theoretically, firms may hoard labour, rather than firing workers during an economic recession, due to higher transaction associated with, for example, payment of severance packages to retrenched (or fired) employees, as well as hiring (and training) new employees when the economy recovers. During a transitory economic recession, firms become risk-averse and tend to hoard or spread (for instance, reducing work-hours per week or cut overtime) labour. Consequently, labour hoarding makes the employment of workers less sensitive to changes in output growth rate (Palley, 1993:149). Aside from this, a decline in output growth decreases the rate of job creation, which in turn reduced labour force entry and/or cause exit from the labour force by marginal and discouraged workers. In addition, some studies have found that countries with highly protected labour market are faced with a persistently high unemployment rate, due to low reaction of employment to output fluctuations as a result of labour hoarding (Sögner and Stiassny, 2002). These rationales convincingly justify the evidence of a positive correlation between cyclical unemployment and output observed in the Free State province.

Based on the foregoing analysis, it is instructive to focus on the estimates of the long-run coefficients $\hat{L}_y^+$, which are negative and statistically significant, at 1% and 10% levels, in the baseline model with HP and CO filtered data, respectively. In this case, $\hat{L}_y^+$ are -0.57 and -0.53 for the Free State province, thus we can infer that an economic upturn of 1.73% and 1.88% is required to reduce unemployment by 1%. Interpretively, a real GDP growth rate of about 1.7 and 1.8% would lower unemployment by 1%. Thus, we can conclude that the real GDP must grow at approximately 1.8 percentage points faster than the rate of unemployment, in the Free State province. Our result on the ratio of Okun's relationship for the province is consistent with recent estimates of Okun's relationship being closer to 2 (Freeman, 2000; Gordon, 1998; Mankiw, 1994), as stated earlier.

So far, the evidence of a negative and statistically significant coefficient of the 'positive' cyclical output and cyclical unemployment, is consistent with Okun's law, but it also reveals the existence of a '*partial*' asymmetric cyclical unemployment-output trade-off, over the business cycle. Interpretively, the unemployment gap will only respond to a sustained economic expansion (i.e., positive output gap), in the Free State province.

Turning to the nature of the relationship between cyclical unemployment and output, in the short run. The estimation results of the baseline NARDL-gap models reveals a highly significant contemporaneous and inverse relationship between cyclical unemployment and output. Specifically, the coefficients of both the lagged and one period-lagged 'positive' and 'negative' cyclical outputs are negative and statistically significant at a 1% level, in all cases. This particular results on short-run asymmetry indicate that changes in cyclical outputs have a considerable contemporaneous effect on cyclical unemployment, during both economic upswings and downswings (over the business cycle), in the Free State province.

Collectively, the results of both nonlinear models suggest that a 1% increase in the (one-period lagged) 'positive' cyclical output is contemporaneously associated with a decrease in cyclical unemployment between 1.63 and 1.55 (-0.52 and -0.41) percentage points during an economic expansion, while a 1% decline in the one-period lagged 'negative' cyclical output will increase cyclical unemployment between 0.83 and 0.56 percentage points, during recessions. This evidence of contemporaneous cyclical unemployment-output trade-off in the Free State province implies that Okun's relationship between the gap variables is asymmetric, and present during transitory economic upswings and downswings, over the business cycle. Similar findings have been documented in recent studies, for example, see Kavase and Phiri (2020) for Kwazulu-Natal province in South Africa, and

[20]See, e.g., Moosa (2008:21) reached similar conclusion, arguing that the lack of correlation between unemployment and output reflected as insignificant Okun's coefficient between unemployment and output for 4 Arab countries being studied can be attributed to labour market rigidity as a result of the dominant role of the government as the prime source of demand for labour, as well as the tendency to protect the jobs of workers in the public sector.

both Marinkov and Geldenhuys (2007) and Mazorodze and Siddiq (2018) for South Africa[21]. Elsewhere, evidence on the asymmetric short-run effects of positive output on cyclical unemployment, which quantitatively differs from the negative ones, as found by, for example, Silvapulle et al. (2004) and Cuaresma (2003) for the US.

On the short-term dynamics of the nonlinear NARDL-gap models, as expected, the coefficients $(\hat{\mu}_s)$ of the ECT_{t-1} terms are negative and statistically significant, indicating the reversion of the systems (including the cyclical components of unemployment and output growth rates) to a long-equilibrium (i.e., steady-state) in the short-run, following an external shock. Particularly, the speed of adjustments of -1.52 and -1.35[22] observed in the dynamic nonlinear models, suggests rapid convergence of the bivariate systems to their long-run equilibrium, after an external shock, entails an oscillatory adjustment process, as opposed to the gradual monotonic adjustment process, if the lagged error correction term coefficients are slightly less than 1. By implication, the estimated baseline models are restored to their steady-state equilibrium takes place in less than a year, after a shock to the systems. Analogous to the short-run dynamics reported here, Kavase and Phiri (2020) reported a statistically significant and negative ECT_{t-1} with coefficients greater than 1, for some South African provinces that exhibits asymmetric unemployment-output gap trade-off in both the short-run and long-run (Western Cape, with ECT_{t-1} term coefficient of 1.29) or only in the short-run (for example, Limpopo and Eastern cape with ECT_{t-1} term coefficients of -1 .70 and -1.2, respectively).

Next, we checked the reliability of the inferences on both the long-run and short-run asymmetries that underlies the negative cyclical unemployment–cyclical output relationships in the Free State. The *F*-statistics of the implemented Wald test is presented in Table 4 (lower panel), which unequivocally rejects the null hypothesis of a symmetric relationship between the cyclical variables in both the long-run and short-run, at a statistically acceptable significance level. As can be seen, the *F*-statistics of the Wald test for the 'partial' long-run symmetry W_{LR} (defined as $\theta_k^+ = \theta_k^-$) and the additive sum of short-run symmetry W_{SR} (defined as $\Sigma_{i=0}^{q-1}\hat{\pi}_k^+ = \Sigma_{i=0}^{q-1}\hat{\pi}_k^-$) are either statistically significant at 1% or 5% level, in the nonlinear models (with HP-and CO-filtered data). More specifically, the values of the statistically significant joint *F*-statistics for W_{LR} are 32.5 and 11.6 in the nonlinear model with HP-and CO-filtered gap variables, respectively. In the same vein, the significant joint *F*-statistics for W_{SR} are 5.29 and 8.80 in the nonlinear model with HP-and CO-filtered gap variables, respectively. Overall, the results of the Wald tests concretely affirm the existence of both long-run and short-run asymmetric negative relationships between cyclical unemployment and cyclical output, in the province being studied, as deduced in the preceding analyses.

As regards the impacts of structural changes on the pattern of cyclical unemployment and output, the qualitative variables D_t have statistically significant and negative coefficients, in most cases. This inference accentuates the exposure of the provincial economy to external and/or endogenous shocks, with the spillover effects of these shocks indirectly affect the domestic labour market through fluctuations in cyclical output (negative and positive) output on cyclical unemployment. Our results also stressed the importance of accounting for structural breaks when testing the different specification of Okun's law, especially at the provincial (states) or regional level.

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[21] These studies employs the same NARDL model and de-trending methods (based on the HP and CO filters), as in this paper.

[22] Although the coefficient of the lagged error correction terms, which also measures the adjustment speed may be greater than 1, in our dynamic nonlinear models; even so, they still fall within the acceptable range of -1 and -2 to ensure convergence (i.e., stability) of the bi-variate systems consisting of cointegrated gap variables, following economic perturbations or external shocks, in the short-run (Johansen, 1995). In fact, in a stable error correction model, the coefficient of error correction term must be strictly inferior to zero and superior to -2. This means that the error correction process fluctuates around the long-run value in a dampening manner, instead of converging directly to the equilibrium path in a monotonic process, but once this process is complete, convergence to the equilibrium path becomes rapid (Narayan and Smyth, 2006:239).

Finally, the computed dynamic NARDL-gap models were subjected to a wide array of diagnostic tests to validate the robustness and reliability of inferred results and conclusion. The results of the diagnostic tests reported in the lower panel of Table 4, shows that the baseline models are parsimonious and good of a fit, given the relatively high adjusted R-squared statistics of 89% and 74% in the model with extracted gap variables based on the HP and CO filters, respectively. In all cases, the Durbin-Watson (DW) statistics of 2 indicates no evidence of serial correlation (i.e., first-order correlation) and the Breusch-Godfrey (BG) serial correlation LM statistics failed to reject the null hypothesis of serial correlation in the residuals (up to 2 lags). Whereas, the Jarque-Bera (JB) statistics with insignificant p-values indicates normally distributed residuals, and the Autoregressive Conditional Heteroscedasticity (ARCH) tests accept the null hypothesis of no ARCH effects in the residuals. The insignificant Q-statistics (up to 12 lags) of the estimated autocorrelation (AC) and partial autocorrelation (PAC) coefficients of the correlogram of standardised residuals rejects evidence of higher-order serial correlation in residuals, and the correlogram of standardised residuals squared refutes the presence of any ARCH/GARCH effects (Table10 in the Appendix). Also, Ramsey's regression equation specification error test (RESET) test using the square of the fitted values of the cyclical unemployment, do not support model misspecification due to incorrect functional form. Lastly, the cumulative sum of recursive residuals (CUSUM) and the cumulative sum of squares of recursive residuals (CUSUMSQ) assessing the structural stability of the dynamic asymmetric models plotted in Fig. 4 and Fig. 5 affirms the constancy of estimated parameters (or coefficients) and the stability of the models, since the CUSUMs and CUSUMSQs lie within the 5% level of significance (i.e., 95% confidence interval).

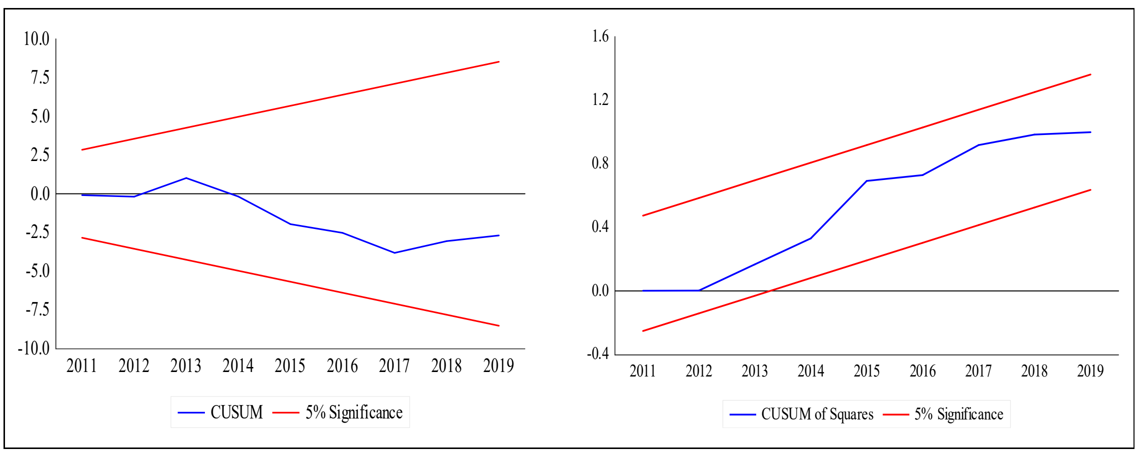

Fig.4 CUSUM and CUSUMSQ (Baseline model with HP filtered cyclical variables)

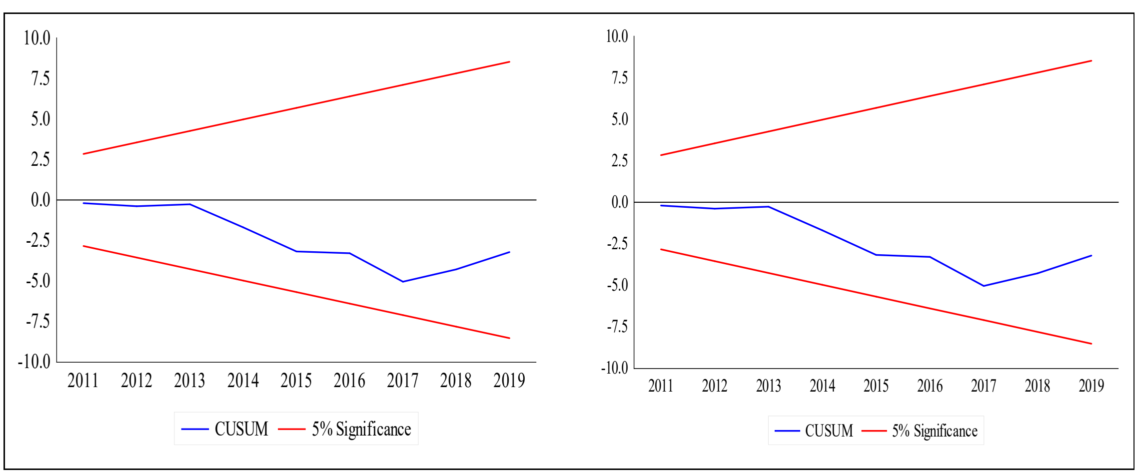

Fig.5 CUSUM and CUSUMSQ (Baseline model with CO filtered cyclical variables)

5.4. Sensitivity analysis: How reliable is the Asymmetric Okun's Law using provincial data?

In the final stage of our analysis, we perform a sensitivity analysis to substantiate the robustness of the drawn deductions and conclusion in the preceding analyses investigating the asymmetric inverse relationship between cyclical unemployment and cyclical output, in the Free State province. For this purpose, the bivariate NARDL-gap equation in (7) is re-estimated using the cyclical growth rates of real GDP per capita (yc_t^g) (as an alternative proxy of cyclical output)[23] and total unemployment. Intuitively, one would expect that using a different proxy as a measure of real GDP would produce different results on nature (i.e., symmetric or asymmetric)-, the stability- and the presence- of Okun's relationship over a business cycle. On this premise, the estimation results of the re-estimated nonlinear gap models will most likely differ from those obtained in the baseline models, either quantitatively or qualitatively or both.

We follow the same estimation and analytical strategy as in the previous sub-section. As before, the growth rate of the real GDP per capita was de-trended using the HP and CO filtering procedures, and the summary statistics of the extracted cyclical output is presented in Table 9 in the Appendix[24]. The DF-GLS, PP and Breakpoint unit root tests are applied to the extracted cyclical output (yc_t^g) using the HP and CO filters, and the results presented in Table 11 (in the Appendix) shows that the time series is either $I(0)$ or $I(1)$ stationary (with or without trend breaks).

In what follows, the bounds-testing procedure is implemented, and the results of the cointegration analyses provided in Table 5 confirm the existence of a long-run relationship between cyclical unemployment and cyclical output. In this case, the joint F-statistics exceeds the asymptotic critical values for the upper bound (I (1)) levels, in the re-estimated NARDL-gap models, irrespective of the filtering method used.

Table 5. Bounds F-test for nonlinear cointegration (sensitivity analysis models)

Dependent variable: ΔU_t^g		Function: $\hat{u}_t^g = f(\hat{y}c_t^{g+}, \hat{y}c_t^{g-})$	
De-trending method	Dynamic specification	F-statistic (F_{PSS})	Conclusion
HP filter	NARDL (2,2,1,1)	14.17898	Cointegration – asymmetric relationship exists
CO filter	NARDL (2,2,2,0)	6.516974	Cointegration – asymmetric relationship exists

Asymptotic CVs (k = 3) for the estimated models with HP and CO filtered series

1%		5%		10%	
$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
5.333	7.063	3.71	5.018	3.008	4.15

Note: See, Table 3.

Having established that the cyclical unemployment and output are cointegrated, we compute two dynamic NARDL gap model (with each comprising of HP-and CO-filtered data) to capture unobserved long- and short-run asymmetries among the cyclical variables. Table 6 presents the estimation results of the dynamic models, which are quite similar to those obtained in the baseline models (consisting of

[23]Annual data on nominal GDP per capita (constant prices, 2010=100) is sourced from IHS Markit ReX database (benchmarked with the same series from StatsSA). Real GDP per capita is generated using CPI (2010=100).
[24]Fig.7 and Fig.8 (in the Appendix) plots the evolution of the cyclical output, actual real output, and cyclical unemployment using the HP and CO filters, respectively.

cyclical components of real GDP (y_t^g) using the HP and CO filters). In general, the estimation results of the dynamic models further reinforces the evidence of a *partial impact* of changes in cyclical output on cyclical unemployment, in the long run. Precisely, we note that cyclical unemployment tends to be more responsive to the positive change in cyclical output (economic upswings), but generally irresponsive to the negative change in cyclical output (downswings) over the business cycle, in the Free State province.

On long-run asymmetry, the coefficients of the 'positive' changes in cyclical output are negative and statistically significant at a 5% level, only in the dynamic nonlinear model with HP-filtered data, indicating that a 1% increase in cyclical output reduces cyclical unemployment by 0.70 percentage points, in the long-run, in times of economic upswings. These results support the existence of a nonlinear Okun's relationship in the Free State province, as expected. However, the coefficient of the positive changes in cyclical output is insignificant, in the NARDL model consisting of CO filtered data, affirming the evidence of a *partial* asymmetric cyclical unemployment–cyclical output trade-off, as previously noted.

Based on the inferences above, we only focus on the estimate of the long-run coefficient L_y^+ is -0.49, which is negative and statistically significant at a 5% level, only in the dynamic nonlinear model (with HP-filtered gap series). In this case, we can deduce that an economic upswing of 2.03% is necessary to reduce unemployment by 1% in the Free State province. Interpretively, a growth rate of real GDP by 2% will lower unemployment by 1%, implying that real GDP must grow at approximately 2 percentage points faster than the rate of unemployment, in the Free State province. This particular finding is noteworthy, as it aligns with the lower 2:1 ratio (instead of the original 3:1 trade-off ratio predicted in Okun's law) underpinning the unemployment-output growth relationship in the empirical literature, as mentioned earlier (see, e.g., Sögner, 2001; Attfield and Silverstone, 1998; Moosa, 1997; Gordon, 1984).

Akin to the estimation results of the coefficients associated with changes in negative cyclical outputs (denoting economic recession) in the baseline models (Table 4), we also find the coefficients of the negative cyclical output to be positive and insignificant in the specified dynamic nonlinear models, indicating that a positive correlation between cyclical unemployment and cyclical output, such that, in the long-run, a 1% change in negative cyclical output causes cyclical unemployment to fall between 0.41 and 0.51 percentage points in the long run, but these impacts are statistically insignificant. As explained earlier, this finding is plausible, partly due to severely low private-sector employment growth in the Free State province, as well as the inability of the provincial government (public sector)– the largest employer in the domestic labour market to lay off workers, owing to factors such as stringent labour protection laws, rigid labour unions and labour hoarding practices by firms, during an economic recession.

Next, we check for a possible short asymmetry between cyclical unemployment and cyclical output. Remarkably, the estimation results mirror the inferences observed in the baseline models, showing a relatively strong contemporaneous effect of changes in cyclical output on cyclical unemployment, in the short run.

As expected, the coefficients of the partial sums of the positive and negative processes (i.e., changes) of cyclical output are negative and highly significant (mostly at a 1% significant level), in both models. For example, in the re-estimated nonlinear model with HP-filtered gap variables, a 1% increase in the lagged (one-period lagged) positive cyclical output is associated with a decrease of 1.49 (-0.46) percentage point in cyclical unemployment, whereas a 1% decrease in the one-period lagged negative cyclical output raises cyclical unemployment by 0.99 percentage points, in the short run. Alike, in the dynamic nonlinear model with CO-filtered cyclical variables, a 1% increase in the lagged positive cyclical output is associated with a decrease of 1.06 percentage point in cyclical unemployment, while a 1% drop in the one-period lagged negative cyclical output increases cyclical unemployment by 0.94 percentage point, in the short run.

Altogether, these findings affirm the presence of a nonlinear Okun's law relationship between cyclical unemployment and cyclical output in the short-run, that is, during transitory economic upswings and downswings, with cyclical outputs exerting a relatively strong contemporaneous effect on cyclical unemployment in an asymmetric manner.

Table 6. Dynamic asymmetric estimation of unemployment-output estimation (sensitivity analysis models)

Dependent variable: ΔU_t^g

HP filter				CO filter			
Variable	Coefficient	Standard error	t-ratio (p-values)	Variable	Coefficient	Standard error	t-ratio (p-values)
u_{t-1}^g	-1.423	0.210	-6.754 (0.000)	u_{t-1}^g	-1.146	0.244	-4.697 (0.000)
yc_{t-1}^{g+}	-0.702	0.276	-2.539 (0.031)	yc_{t-1}^{g+}	-0.319	0.350	-0.912 (0.381)
yc_{t-1}^{g-}	0.506	0.278	1.815 (0.102)	yc_{t-1}^{g-}	0.409	0.350	-0.912 (0.234)
Δu_{t-1}^g	0.871	0.161	5.386 (0.000)	Δu_{t-1}^g	0.727	0.163	4.445 (0.001)
Δyc_t^{g+}	-1.498	0.347	-4.309 (0.002)	Δyc_t^{g+}	-1.069	0.254	-4.208 (0.001)
Δyc_t^{g-}	1.718	0.351	4.886 (0.000)	Δyc_t^{g-}	1.425	0.322	4.415 (0.001)
Δyc_{t-1}^{g+}	-0.466	0.149	-3.116 (0.012)	Δyc_{t-1}^{g+}			
Δyc_{t-1}^{g-}	-0.993	0.220	-4.495 (0.001)	Δyc_{t-1}^{g-}	-0.942	0.284	-3.316 (0.006)
$\gamma_1 D_{t-1}$	-1.498	0.347	-4.309 (0.002)	$\gamma_1 D_{t-1}$	-7.996	3.118	-2.564 (0.026)
$\gamma_2 \Delta D_t$	-10.414	3.434	-3.031 (0.014)	$\gamma_2 \Delta D_t$			
ECT_{t-1}	-1.423	0.1637	-8.696 (0.000)	ECT_{t-1}	-1.146	0.198	-5.759 (0.000)
Constant	36.030	7.634	4.719 (0.001)	*Constant*	21.452	7.359	2.915 (0.014)
L_y^+	-0.493	0.186	-2.638 (0.027)	L_y^+	-0.278	0.308	-0.905 (0.384)
L_y^-	0.355	0.173	2.045 (0.071)	L_y^-	0.357	0.252	1.416 (0.184)
Diagnostic statistics							
R^2	0.898	$\bar{R}^2$	0.785	R^2	0.811	$\bar{R}^2$	0.674
F-statistic	7.971 (0.002)	DW	1.714	F-statistic	5.915 (0.004)	DW	2.108
χ_{SC}^2	2.645 (0.266)	χ_{NOR}^2	0.904 (0.636)	χ_{SC}^2	2.738 (0.254)	χ_{NOR}^2	0.274 (0.881)
$\chi_{HET-ARCH}^2$	1.722 (0.189)	χ_{FF}^2	4.160 (0.075)	$\chi_{HET-ARCH}^2$	1.349 (0.245)	χ_{FF}^2	0.266 (0.617)
W_{LR}	20.912 (0.000)	W_{SR}	10.151 (0.001)	W_{LR}	7.757 (0.005)	W_{SR}	4.221 (0.039)

Note: See, Table 4. L_y^+ and L_y^- represent the long-run coefficients associated with positive and negative changes of cyclical output (yc^{g+}) computed based on real per capita GDP), respectively.

Considering the short-run dynamics of the models, the one-period lagged error correction ECT_{t-1} term is negative and highly significant (at 1% level), as anticipated. Notice that, while the correctly signed and significant coefficients of the result ECT_{t-1} terms provides concrete support for the evidence on the long-run relationship between cyclical unemployment and cyclical output, as previously inferred, they also indicate that the bi-variate system (including the independent gap variables) will revert to its long-run equilibrium, after a disturbance in the domestic economy and/or exogenous shock to the system, in the re-estimated dynamic nonlinear models (with the HP-and CO-filtered gap variables). The highly significant values for the speed of adjustment range between -1.14 and 1.42 in the models, suggesting a rapid and dampened oscillatory adjustments of the error correction processes in the bi-variate systems back to their long-run equilibrium (or steady-state path) would be less than a year, in the aftermath of economic perturbations or external shocks to the system.

Turning now to the results of the Wald tests for long-run symmetry W_{LR} and the additive sum of short-run symmetry condition W_{SR} reported in Table 6 (lower panel), shows statistically significant F-statistics at the acceptable p-values (i.e., 1% and 5% significant levels), refuting the null hypothesis of a symmetric relationship between cyclical unemployment and cyclical output, in both the long and short-run. Precisely, the corresponding values of highly significant joint F-statistics for W_{LR} are 32.5 and 11.6 in the nonlinear model with HP-and CO-filtered gap variables, respectively. Also, the significant joint F-statistics for W_{SR} are 10.15 and 4.22 in the nonlinear model with HP-and CO-filtered gap variables, respectively. Altogether, the Wald test results strongly support the evidence on the existence of an asymmetric cyclical unemployment–cyclical output trade-off, in the Free State province, as previously noted.

As regards the impacts of structural changes on the presence of an asymmetric negative Okun's relationships in both the long-and-short-run, the estimated parameters γ_1 and γ_2 for the qualitative dummy variables are negative and statistically significant (mostly at a 1% level), reinforcing the evidence of a nonlinear cyclical unemployment–cyclical output trade-off, in our focus province. Furthermore, the highly significant qualitative dummy variables accentuate the considerable influence of global and/or domestic shocks on the fluctuations in cyclical output and responsiveness of cyclical unemployment.

To ensure model parsimony and robustness of results, we carry out a battery of diagnostic tests on the re-estimated dynamic model. The results are provided in Table 6. (lower panel), which clearly shows that the re-specified NARDL-gap models are of a good fit, given that the adjusted R-squared is above 60%, in all cases. The DW statistics range from 1.7 to 2 indicating no evidence of first-order serial correction. The test for model stability and parameter constancy is plotted in Fig. 4 and Fig.5 (in the Appendix), which shows both the CUSUM and CUSUMSQ lies within the 5% significant line (i.e., 95% confidence interval), suggesting that the estimated coefficients and constructed NARDL models are stable. Whereas, the results of the JB statistics, BG serial correlation tests, ARCH tests and the Ramsey RESET tests provide irrefutable evidence that re-specified dynamic nonlinear models are devoid of normal errors, serial correlation, heteroscedasticity and incorrect functional form. Last but not the least, the insignificant Q-statistics (up to 12 lags) of the estimated autocorrelation (AC) and partial autocorrelation (PAC) coefficients of the correlogram of standardised residuals rejects evidence of higher-order serial correlation in residuals, and the correlogram of standardised residuals squared refutes the presence of any ARCH/GARCH effects (Table12 in the Appendix).

Hitherto, the findings of our empirical inquiry have been thorough, and firmly supports the notion that Okun's law is applicable in South Africa, particularly in the Free State province, and the inverse relationship between cyclical unemployment and cyclical output is asymmetry, indicating that cyclical upturns and downturns do not have symmetrical effects on unemployment. Particularly, in the case of the Free State province, the asymmetric Okun's relationship implies that unemployment is more responsive to only changes in output during upswings, with output fluctuations exerting significant contemporaneous effects on unemployment during economic upswings and downswings.

In general, our results contradicts the findings of Kavase and Phiri (2020), who failed to find an asymmetric trade-off between cyclical unemployment and output in the Free State province, despite employing a similar CO filter to extract cyclical components from annual data, econometric model, and specification of Okun's law. Nonetheless, the conclusion of this study on the inexistence of Okun's relationship in the Free State province can be considered unreliable since the authors failed to account for structural breaks in data used, which can lead to erroneous inferences and conclusion. Albeit, accounting for structural breaks in data has become an essential condition to appropriately test for asymmetric or symmetric unemployment-output trade-off, given that the reality that global spillovers and structural endogenous shocks may affect the stability of the inverse unemployment-output trade-off as well as the Okun's coefficient (Gordon, 1984), especially at the state (or provincial) and regional levels (see, e.g., Palley, 1993; Harris and Silverstone, 2001; Sögner and Stiassny, 2002; Apergis and Rezitis, 2003; Huang and Yeh, 2013).

All in all, our findings on the asymmetric cyclical unemployment-cyclical output trade-off align with Phiri (2014) and Marinkov and Geldehuys (2007), who documented evidence of a nonlinear Okun's law in South Africa, irrespective of the model specification and de-trending method used. Elsewhere, for instance, similar results on asymmetric cyclical unemployment and output trade-off have been reported by Tang and Benthencourt (2017) for some countries in the Eurozone, Shin and Greenwood-Nimmo (2014) for advanced countries (which includes the US, Canada and Japan) and Huang and Yeh (2013) for a large group of OECD countries.

Our analytical exercise thus far elucidates that the longstanding empirical regularity postulated by Okun's law exists in an asymmetric manner, in the Free State province, given the evidence of the nonlinear inverse relationship between cyclical unemployment and cyclical output. Furthermore, the evidence of a partial asymmetry cyclical unemployment-cyclical output trade-off driven by a sustained economic upturn, and asymmetry contemporaneous cyclical unemployment-cyclical output trade-off shed more light on how Okun's law operating in the province. This type of knowledge becomes a useful macro-economic foundation for policymaking on how to simultaneously develop effective pro-growth and unemployment-reducing policies necessary to alleviate the undesirable twin problems of a persistently high unemployment rate and severely weak economic growth in the Free State province, to improve the general welfare of the citizenry as well as boost economic activity across different sectors.

6. Conclusion and some policy recommendation.

Among the nine South African provinces, the unemployment rate is relatively high, and persistent in the Free State province since 1994, even during periods of economic expansion. The failure of adopted pro-growth policies to reduce the persistently high unemployment rate in the Free State province has become a conundrum for policymakers, and begs the questions: Is the long-standing Okun's law that predicts an inverse unemployment-output relationship applies in the FS province? If so, what is the nature of this Okun's relationship?

This study re-examines the asymmetric relationship between cyclical unemployment and cyclical output for the Free State province, over the period 1994 to 2019. We adopt the NARDL modelling approach which allows for asymmetric cointegration tests without pre-testing the time-series, and simultaneously modelled asymmetries among variables in the long-and short-run, in a coherent manner. Cyclical components of the total unemployment and output growth (real GDP) rates were separated from associated trends components employing the Hodrick-Prescott and Corbae-Ouliaris de-trending procedures. We also checked for the robustness of inferred inferences by performing a sensitivity analysis on the baseline models using a different measure of cyclical output (real GDP per capita). In all estimates, and irrespective of the filtering technique used, we find strong evidence supporting the presence of Okun's relationship, indicating a statistically significant asymmetric cyclical unemployment–cyclical output trade-off, in both long- and short-run. Our results refute a recent finding on the absence of asymmetric Okun's relationship in the Free State province. This conclusion can be attributed to co-breaking (structural breaks) in the data, not being accounted for, as in this present study.

Our findings are summarized as follows. Firstly, only the coefficient changes in 'positive' cyclical output exhibit a statistically significant and negative relationship with cyclical unemployment, suggesting a '*partial*' long-run asymmetric trade-off, implying that a 1% increase in positive changes in cyclical output increases cyclical unemployment by 0.87 to 0.70 percentage points, conditioned on a sustained economic upswing. Secondly, the estimated long-run coefficients show that an economic upswing between 1.88% and 2.03% is required to reduce unemployment by 1% in the province being studied, keeping in line with the documented lower 2:1 ratio for Okun's relationship in the extant empirical literature. Interpretively, real GDP growth of 2% would lower unemployment by 1%, indicating that real GDP must grow at nearly 2 percentage points faster than the rate of unemployment, in the Free State province. Thirdly, changes in cyclical output exert a strong contemporaneous effect on cyclical unemployment, indicating short-run asymmetry among variables. We find the estimates of contemporaneous (i.e., lagged and one-period lagged) coefficients of both the positive and negative cyclical outputs are found to be negative and statistically significant at 1% level, indicating that a 1% increase in positive cyclical output is associated with a decrease in cyclical unemployment between 1.63 and 1.06 percentage points, in the Free State province. While, a 1% increase (decrease) in positive (negative) cyclical output reduces (increases) cyclical unemployment between 0.52 and 0.41 (+0.99 and +0.56) percentages points. Overall, our empirical results align with those documented in the international literature, as well as country-specific studies on South Africa, exploring the same line of empirical inquiry.

Our findings have important policy implications. It is worth noting that the detection of an asymmetric negative relationship between cyclical unemployment and cyclical output sheds more light on the ineffectiveness of the various economic growth-inducing policies and unemployment-reducing strategies implemented–at the national and provincial level– up till now, has failed to curb the prevalent high unemployment rates and dampened economic growth experienced in the province. Similarly, the presence of long-and short-run asymmetries suggests that some important supply- and demand-side factors are inadvertently overlooked during the policymaking process. Then again, if the 2:1 ratio found for the existing Okun's relationship in the Free State province becomes relatively stable over an extended period, thus it implies that economic policy would not be able to decoupled unemployment from economic growth (Sogner, 2001:562), meaning that problem with both variables has to be effectively addressed simultaneously (i.e., not in isolation) by implementing an integrated policy.

It is widely argued that the sole responsibility of policymakers in a domestic economy, is to create a conducive environment to facilitate private sector participation, implementing effective policies to support and stimulate sustained economic growth. But, the concrete evidence on asymmetric cyclical output–cyclical unemployment trade-off in the Free State province, imply that the mammoth task to reduce the prevalent high unemployment rates in the provincial economy, cannot be myopically limited to boosting economic growth alone. But, it requires policymakers to design and implement a well-integrated policy that encapsulates the enacted macroeconomic-, fiscal-, labour-, and trade strategies, to tackle the influence of structural shocks (or structural changes), putting a wedge between the domestic economy and the labour market.

Based on our findings, we put forward some supply-and demand-side interventions for consideration by the provincial governments and policymakers. On the supply side, it is vital to reduce the increasing role of the provincial government as the largest employer in the province, as job opportunities in the public sector continued to crowd out private sector employment required to stimulate economic growth. In this context, the provincial government needs to provide financially incentivize private firms to offer internship opportunities, train low or unskilled workers, and hire new workers. This strategy would directly raise private-sector employment growth rate, reduce labour market rigidities (e.g., wage rigidity) as well as improves labour participation and labour absorption rates in the medium-term. Indirectly, the cumulative (multiplier) effects of a rise in private sector employment growth would increase total productivity and economic activity, across all sectors.

On one hand, it has become increasingly important for the policymakers in the provincial government to address the perennial issue of structural unemployment due to skill mismatch, as the domestic labour market has become skill-intensive, creating job opportunities for most highly skilled workers. In this instance, the provincial government can encourage self-employment and entrepreneurial spirit, especially among the youth population, by financing (partially or wholly) business development plans, and proposals of prospective entrepreneurs, as well as the establishment of privately-owned business, such as small, medium and micro-enterprises (SMMEs).

On the other hand, given the increasing role of the informal sector in the economy: the provincial government can capitalize on this indirect source of economic growth by addressing market rigidities due to, for example, obstructive bureaucratic red tapes on acquiring trading (or business) permits, high company registration fees, turn-around period for company/business registration, and stringent labour laws.

On the demand-side interventions, the presence of a short-run asymmetry observed on the cyclicality between unemployment and output growth in the Free State province requires the provincial government to increase the public investment spending on infrastructure, concentrating on preventative maintenance of public buildings, road networks and highways. This initiative creates transitory job opportunities for low-skilled, unskilled and discouraged workers, in the short term, while lowering the unemployment rate as the labour absorption rate increases. Aside from this, the provincial government needs to intensify its efforts in implementing flagship public employment programmes (PEPs), such as Expanded Public Work Programme and Community Work Programmes across the metropole, municipalities and districts in the Free State province. Even so, the provincial government should actively and properly monitor the implementation of the PEPs in terms of their rotational structure (i.e., a stipulation of 2 years employment period) to eradicate the problem of an extended participation period. Failure to effectively manage these flagship PEPs, with the capacity to absorb a large proportion of low-skilled or unskilled workers, could perpetuate the experience of high unemployment and poverty rates, in the province.

Another way to increase the labour absorption rate in the province is by adopting proactive strategies geared towards revitalizing the severely weakened productive sectors (particularly, the mining, agricultural, and manufacturing sectors), with a significant propensity to absorb a large proportion of low-skilled and unskilled workers. In the same vein, the provincial fiscal framework should be refined for optimal allocation of the limited fiscal resources, improve 'own' revenue generation, and efficiently manage the budgeting process (at department levels) to curb wasteful expenditures, as well as provide quality service delivery, at both the provincial and municipal levels. Moreover, policymakers should strengthen the role of the provincial government in the public-private-partnership (PPP), to deepen the impacts of favourable technological spillovers and gain the technical know-how (expertise) crucial for economic growth sustainability, in the domestic economy.

Finally, future research can build on our work by extending the bivariate nonlinear system employed here, to a multivariate model, incorporating some notable drivers of asymmetric Okun's relationship such as labour participation rate, labour productivity, employment growth rate, gender, and demographics[25]. This type of research will allow policymakers and researchers to better understand the unemployment rate-output growth nexus, and how to improve the efficacy of government policies to create productive economic growth (i.e., an employment-reducing output growth), in the Free State province.

[25]Refer to studies by Ball et al. (2013); Knotek (2012), Moosa (1997), Lee (2000), Palley (1993), and Prachowny (1993), among others.

Appendix A: Filtering Techniques

A1: Hodrick-Prescott time-domain filter

As common in the literature, the cyclical components of the growth rates of real GDP, real GDP per capita and unemployment time series used in our analysis were estimated using the Hodrick-Prescott filter, which is quite popular and extensively used in the strand of literature assessing Okun's law and related issues.

By the way of exposition, the HP filter is a two-sided linear that computes the smoothed series $y*$ of y by minimising the variance of y around $y*$ subject to a penalty function that contains the change in the trend growth of $y*$:

$$\Theta = \sum_{t=1}^{T}(y_t - y_t^*)^2 + \lambda\sum_{t=2}^{T-1}((y_{t+1}^* - y_t^*) - (y_t^* - y_{t+1}^*))^2 \tag{8}$$

where the penalty parameter λ controls the smoothness of the series $(y*)$, the residual $(y_t - y*)$ indicates the deviation from the trend and commonly referred to as the business cycle component.

Despite the critique on the shortcomings of the HP filter, a number of studies have reported unbiased estimates of Okun's coefficient using HP filter compared to alternative filtering techniques. For instance, Jalles (2019) exploited the time-varying properties of Okun's coefficients across 20 advanced economies, and obtained results reveal no qualitative difference from results of the country-specific gap models using HP filter and Hamilton (2017) filtering approach. Also, Ball et al. (2017) consider both the difference and gap versions of Okun's law and finds no qualitative difference in the results of both models using HP filter to generate cyclical series. Marinkov and Geldenhuys (2007) applied different filtering techniques (which includes HP, Beveridge Nelson decomposition, Band-Pass filter), and do not find any qualitative difference in the results of the first difference, gap and production versions of Okun's law tested for South Africa. While Huang and Lin (2006) found similar results using both the structural time series approach and HP filter to generate cyclical components from quarterly data when assessing the asymmetric Okun's law in the US.

A2: Courbae-Oularis frequency-domain filter.

To resolve the end-point problems common to time-frequency filters such as BK and HP, Corbae and Ouliaris (2006) developed a frequency-domain filter (referred to as CO filter), which is superior in extracting latent cyclical components of a time-series (such as HP and BK), without losing observations at the end-points of series (i.e., produce unreliability of end-of-sample estimates) and generating different cycles which are not in the original series. Using Monte Carlo simulations, Corbae and Ouliaris (2006) show that the CO filter has superior finite sample properties, and de-trended series are statistically reliable (e.g., much lower mean-squared error than time-domain filters such as HP and BK), as asymptotically converges to their true growth cycle. Moreover, the CO filter does not suffer from endpoints problems commonly found in trend filters.

Consider a non-stationary series x_t (in our case, real GDP) consisting of a deterministic component, z_t and an unobserved stochastic component $\tilde{x}_t$ defined as:

$$x_t = \Pi_2' z_i + \tilde{x}_t, \tag{9}$$

where z_t is a $(p+1)$ dimensional deterministic sequence and $\tilde{x}_t$ is a zero-mean time series.

Given that $\tilde{x}_t$ $I(1)$ is a process with the first difference series $\Delta x_t = v_t$, then v_t has a Wold representation:

$$v_t = \Sigma_{j=0}^{\infty} cj\xi_{t-j} \tag{10}$$

where the spectral density of v_t is $f_{vv}(\lambda) > 0$ for all λ, while coefficients $c_j = \Sigma_{j=0}^{\infty} j^{\frac{1}{2}} \left| c_j \right| < \infty$, and $\xi = iid(0, \sigma^2)$ has finite fourth moments.

Conversely, since $\tilde{x}_t$ is a $I(1)$ process, then the discrete Fourier transform of $\tilde{x}_t$ for $\lambda_s \neq 0$ can be expressed as:

$$w_{\tilde{x}}(\lambda_s) = \frac{1}{1-e^{i\lambda_s}} w_v(\lambda_s) - \frac{e^{i\lambda_s}}{1-e^{i\lambda_s}} \frac{[\tilde{x}_n - \tilde{x}_0]}{n^{1/2}} \tag{11}$$

where $\lambda_s = \frac{2\pi s}{n} (s = 0,1,...,n-1)$ are the fundamental frequencies of a sample n.

However, it is worth noting that the Fourier transform of a $I(1)$ process are not asymptotically independent across fundamental frequencies, because it is a deterministic trend in the frequency domain with a random coefficient $\frac{[\tilde{x}_n - \tilde{x}_0]}{n^{1/2}}$, instead they are frequency-wise dependent by the virtue of the component $n^{1/2}\tilde{x}_n$, which produces a common leakage to all frequencies, $\lambda_t \neq 0$, and in the limit as $n \to \infty$. As such, the frequency domain estimates of a cyclical component of a GDP series would be inconsistent. Also, as shown in Corbae et al. (2002), filtered series still has a leakage problem even when the original series is first de-trended in the time domain, while generated cyclical GDP series would be badly distorted, at any frequency domain.

However, Corbae and Ouliaris (2006) solved this leakage problem that yields biased filter estimates by de-trending the second term in Eq. (6) using a frequency domain regression to obtain an unbiased estimate of $w_{\tilde{x}}(\lambda_s)$ written as:

$$w_{(1/n)}(\lambda_s) = \frac{-1}{\sqrt{n}} \left(\frac{e^{i\lambda_s}}{1-e^{i\lambda_s}} \right) \tag{12}$$

Evidently, the leakage from the low frequency (expressed in Eq.6) can be removed by de-trending in the frequency domain, leaving an asymptotically unbiased of $\frac{1}{1-e^{i\lambda_s}} w_v(\lambda_s)$ estimate, over the nonzero frequencies. Also, this estimate is $\sqrt{n}$ -consistent and has a good finite samples series.

Appendix B: Figures and Tables

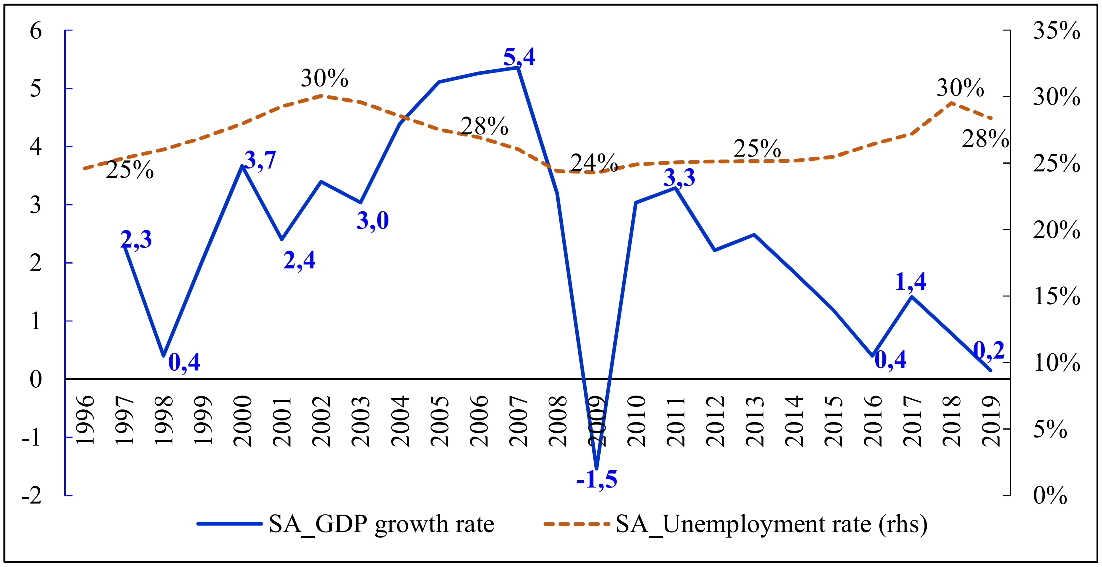

Fig.6. Historical evolution of real GDP and unemployment rate in South Africa (1996-2019)
Data source: StatsSA, IHS Markit ReX. Author's estimation.

Table 7. Historical growth rates of gross domestic product (year-on-year, %)

	SA	EC	FS	GP	KZN	LIM	MP	NC	NW	WC
1997	2.31	1.53	1.04	2.15	1.53	8.51	1.19	4.05	0.68	2.49
1998	0.40	-0.38	-5.48	1.00	0.73	3.71	0.49	1.92	0.21	-0.42
1999	2.06	2.49	3.83	2.06	0.10	1.26	2.79	2.34	1.27	3.73
2000	3.67	4.26	1.51	5.09	4.31	0.09	2.72	1.88	1.17	4.20
2001	2.40	1.59	-2.68	1.85	4.47	6.81	1.07	-2.22	2.23	3.51
2002	3.39	1.62	4.93	4.24	2.15	4.31	1.66	1.08	3.20	4.15
2003	3.04	2.75	2.36	2.87	2.82	2.14	2.78	3.65	5.59	3.62
2004	4.39	3.64	3.90	4.88	4.44	3.02	3.95	2.37	3.44	5.60
2005	5.11	5.03	4.16	4.98	5.64	3.88	4.31	3.16	6.97	5.97
2006	5.26	5.26	3.59	5.87	5.41	4.42	4.23	3.84	4.78	5.77
2007	5.36	5.32	3.71	5.78	6.10	4.27	4.03	3.30	4.52	6.25
2008	3.19	3.16	2.56	3.72	3.85	1.71	1.26	1.66	2.33	3.90
2009	-1.54	-0.98	-2.32	-1.52	-1.36	-1.59	-1.40	-2.26	-2.28	-1.39
2010	3.04	2.35	2.52	3.33	3.57	2.61	2.59	2.20	3.91	2.53
2011	3.28	3.72	2.02	3.56	3.68	2.41	2.22	1.97	3.00	3.77
2012	2.21	2.03	3.00	2.53	2.62	1.02	2.10	3.23	-1.29	2.81
2013	2.49	1.37	1.88	2.73	2.52	2.66	2.01	2.42	3.18	2.57
2014	1.85	1.30	1.99	2.27	2.45	1.10	2.91	2.95	-3.49	2.24
2015	1.19	0.81	-0.29	1.15	0.93	1.81	-0.19	1.10	4.67	1.43
2016	0.40	0.68	-0.27	1.14	0.55	-0.47	0.10	-1.21	-3.62	0.97
2017	1.41	0.59	1.43	1.04	1.81	2.09	1.89	2.84	2.19	1.25
2018	0.79	0.58	-0.21	1.11	0.72	0.57	0.60	0.50	0.73	0.82
2019	0.15	0.29	-0.27	0.53	-0.06	-0.34	-0.02	0.02	-0.94	0.31

Data sources: StatsSA, IHS Markit ReX. Author's estimation

Note: SA=South Africa, EC=Eastern Cape, FS=Free State, GP=Gauteng, KZN=Kwazulu-Natal, LP=Limpopo, MP=Mpumalanga, NC=North West, WC=Western Cape.

Table 8. Historical trend of unemployment rate in South Africa and across provinces (%), 1996-2019

	SA	EC	FS	GP	KZN	LP	MP	NC	NW	WC
1996	24.6	34.8	22.6	22.1	28.6	30.5	26.8	26.0	23.2	13.3
1997	25.4	35.2	23.5	23.1	29.3	31.2	27.8	26.3	23.9	13.9
1998	26.0	34.4	24.4	24.3	29.7	31.3	28.6	26.6	24.6	14.7
1999	26.9	33.0	25.6	25.8	30.6	31.3	29.5	27.5	25.4	15.8
2000	28.0	32.0	27.0	27.5	31.8	31.6	30.5	28.8	26.4	17.1
2001	29.3	32.2	28.6	29.1	33.5	32.4	31.5	30.6	27.5	18.3
2002	30.1	32.3	29.8	29.9	34.8	32.9	32.1	32.1	28.2	19.1
2003	29.6	31.5	30.0	29.2	34.5	32.3	31.3	32.4	27.8	19.1
2004	28.6	30.6	29.7	27.5	33.6	31.4	30.4	32.4	27.3	18.6
2005	27.5	29.6	29.4	25.9	32.5	30.8	29.8	32.3	26.8	18.1
2006	26.9	29.1	29.0	25.0	31.3	31.2	29.5	31.9	26.8	18.0
2007	26.1	28.6	28.0	24.1	28.8	31.1	28.3	30.3	26.8	18.6
2008	24.4	27.4	26.1	23.3	24.4	29.3	26.4	27.5	26.3	18.8
2009	24.3	27.5	26.3	24.3	22.0	27.0	26.7	27.1	26.8	20.0
2010	24.9	27.9	27.6	25.9	20.9	24.6	28.2	27.7	27.2	21.4
2011	25.1	28.2	29.0	26.1	20.7	22.3	29.2	28.1	26.6	22.2
2012	25.1	28.9	30.9	25.6	21.1	20.8	29.4	28.5	26.3	22.7
2013	25.2	29.6	32.3	25.3	21.6	19.3	29.0	29.1	26.6	22.7
2014	25.2	29.4	32.7	25.8	21.8	18.2	28.4	29.7	26.8	22.1
2015	25.5	29.2	32.4	27.1	22.0	18.6	28.3	30.1	26.9	21.3
2016	26.4	30.5	32.9	28.6	23.0	19.7	29.5	30.1	27.3	21.1
2017	27.2	33.0	33.7	29.4	23.8	20.0	31.2	29.5	27.5	20.8
2018	29.5	32.4	38.0	32.2	24.2	22.0	36.5	31.6	33.7	22.5
2019	28.4	36.9	34.7	30.2	25.1	20.4	33.5	28.1	29.5	20.5

Source: StatsSA

Note: SA=South Africa, EC=Eastern Cape, FS=Free State, GP=Gauteng, KZN=Kwazulu-Natal, LP=Limpopo, MP=Mpumalanga, NC=North West, WC=Western Cape.

Table 9. Summary statistics of unobserved components of u_t and yc_t (1994-2019).

Description	HP de-trending method		CO filter de-trending method	
	Cyclical unemployment (%)	Cyclical output (%)	Cyclical unemployment (%)	Cyclical output (%)
	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$	$u_t^g = u_t - u_t^*$	$y_t^g = y_t - y_t^*$
Mean	-1.66e-14	2.24e-14	1.13e-15	7.63e-16
Median	-0.281	0.299	-0.062	0.606
Maximum	6.599	5.040	6.496	5.098
Minimum	-6.929	-5.963	-6.258	-5.402
Std. dev.	3.397	2.849	3.306	2.872
Skewness	0.198	-0.400	0.056	-0.496
Kurtosis	2.526	2.971	2.513	2.743
Jaque-Bera statistics (*p*-value)	0.366 (0.832)	0.616 (0.734)	0.238 (0.887)	1.006(0.604)
Sum	-3.81e-13	5.14e-13	2.35e-14	1.73e-14
Sum Sq. Dev.	253.885	178.606	240.575	181.542

Note: Note: *p*-values in ( ) parenthesis.

Table 10. Diagnostic tests on the residuals of estimated dynamic NARDL model (baseline model)

HP filtered data					CO filtered data				
Correlogram of standardised residuals					*Correlogram of standardised residuals*				
Lags	AC	PAC	Q-Stat	*p*-values	Lags	AC	PAC	Q-Stat	*p*-values
1	-0,030	-0,030	0,021	0,884	1	-0,085	-0,085	0,168	0,682
2	-0,165	-0,166	0,683	0,711	2	-0,023	-0,031	0,181	0,913
3	-0,232	-0,250	2,073	0,557	3	-0,202	-0,208	1,235	0,745
4	0,101	0,051	2,352	0,671	4	-0,021	-0,062	1,248	0,870
5	-0,099	-0,186	2,638	0,756	5	-0,348	-0,392	4,796	0,441
6	-0,271	-0,363	4,943	0,551	6	-0,054	-0,233	4,889	0,558
7	-0,017	-0,111	4,953	0,666	7	0,113	-0,023	5,325	0,620
8	0,151	-0,079	5,784	0,671	8	0,134	-0,063	5,983	0,649
9	0,026	-0,210	5,811	0,759	9	-0,144	-0,303	6,808	0,657
10	0,159	0,160	6,922	0,733	10	0,255	0,07	9,658	0,471
11	-0,176	-0,310	8,435	0,674	11	-0,126	-0,261	10,436	0,492
12	0,026	-0,168	8,473	0,747	12	-0,002	-0,139	10,437	0,578
Correlogram of standardised residual squared					*Correlogram of standardised residual squared*				
Lags	AC	PAC	Q-Stat	*p*-values	Lags	AC	PAC	Q-Stat	*p*-values
1	-0,195	-0,195	0,884	0,347	1	-0,258	-0,258	1,542	0,214
2	-0,038	-0,079	0,919	0,631	2	0,180	0,121	2,332	0,312
3	-0,197	-0,23	1,920	0,589	3	-0,084	-0,013	2,516	0,472
4	0,074	-0,022	2,073	0,722	4	-0,026	-0,076	2,534	0,638
5	-0,096	-0,130	2,342	0,800	5	0,224	0,235	4,009	0,548
6	0,000	-0,098	2,342	0,886	6	-0,174	-0,074	4,959	0,549
7	-0,308	-0,387	5,545	0,594	7	-0,175	-0,348	5,990	0,541
8	0,206	-0,033	7,096	0,526	8	0,068	0,052	6,162	0,629
9	-0,049	-0,159	7,191	0,617	9	-0,065	0,064	6,328	0,707
10	0,313	0,175	11,490	0,321	10	0,210	0,093	8,268	0,603
11	-0,077	0,043	11,776	0,381	11	-0,113	0,027	8,897	0,631
12	-0,092	-0,151	12,240	0,427	12	-0,012	0,008	8,905	0,711

Note: AC and PAC are the estimated Autocorrelation and Partial Correlation coefficients, respectively.

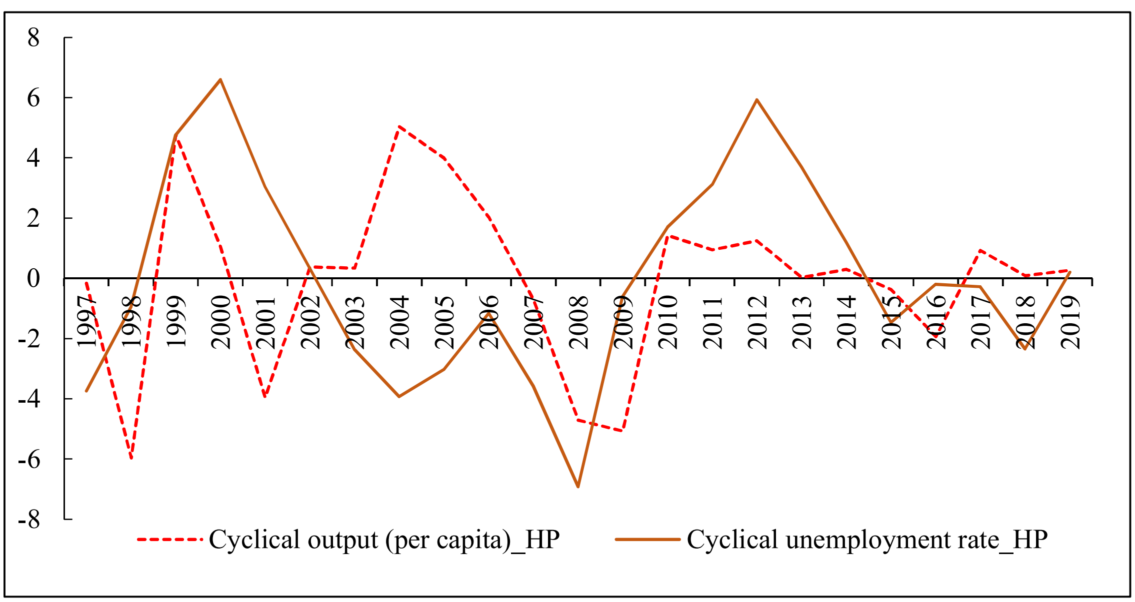

Fig.7. Estimated cyclical real GDP per capita and cyclical unemployment using HP filter

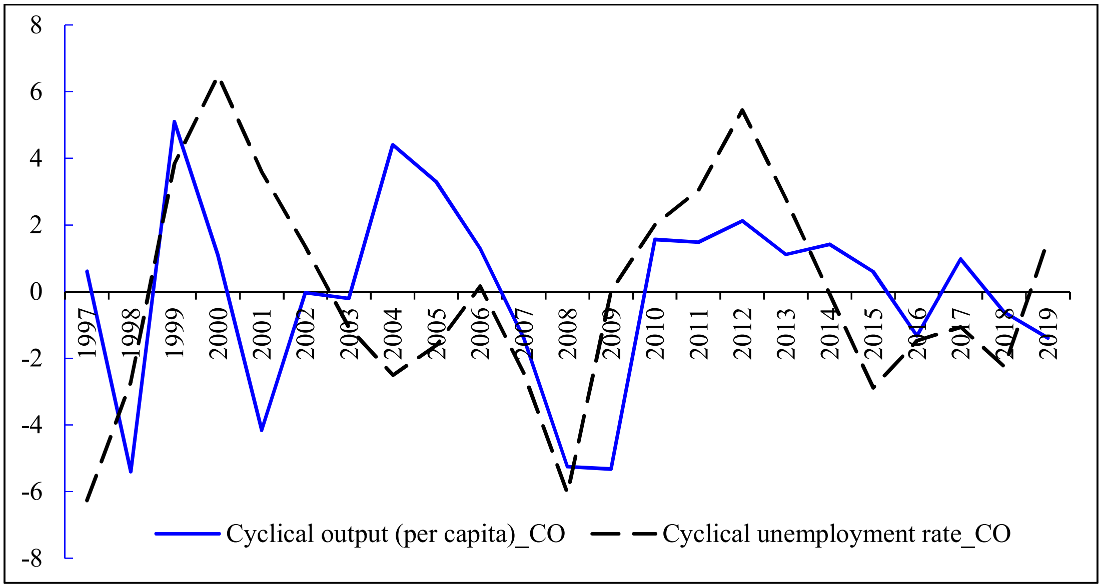

Fig. 8. Estimated cyclical real GDP per capita and cyclical unemployment using CO filter

Table 11. Results of the unit root test on the estimated yc_t^g using HP and CO filters.

	HP filter		CO filter	
	Intercept	Intercept & trend	Intercept	Intercept & trend
DF-GLS				
y_t^g	-4.049*	-4.048*	-3.941*	-3.890*
Phillips-Perron				
y_t^g	-3.894*	-3.781**	-3.882*	-3.770**
Breakpoint				
Innovative outlier: DF min-*t*, *F*-stat.				
y_t^g	-4.241***	-6.462*	-4.123	-4.041
Break date	2004	2007	2004	2001
Δy_t^g	-8.151*		-7.999*	-7.219*
Break date	2001		2001	2002
Additive outlier: DF min-*t*, *F*-stat.				
y_t^g	-4.617**	-5.039***	-4.668**	-4.633
Break date	2003	2006	2009	2009
Δy_t^g		-7.386*		-7.347*
Break date		2001		2001

Note: *, **, and *** denotes significance of associated *p*-values at 1%, 5%, and 10% levels respectively. Breaks are chosen based on Dickey-Fuller *min-t* with optimal lag length (up to 4) being selected using *F*-statistics. Parameters y_t^g and u_t^g refers to cyclical output and cyclical unemployment.

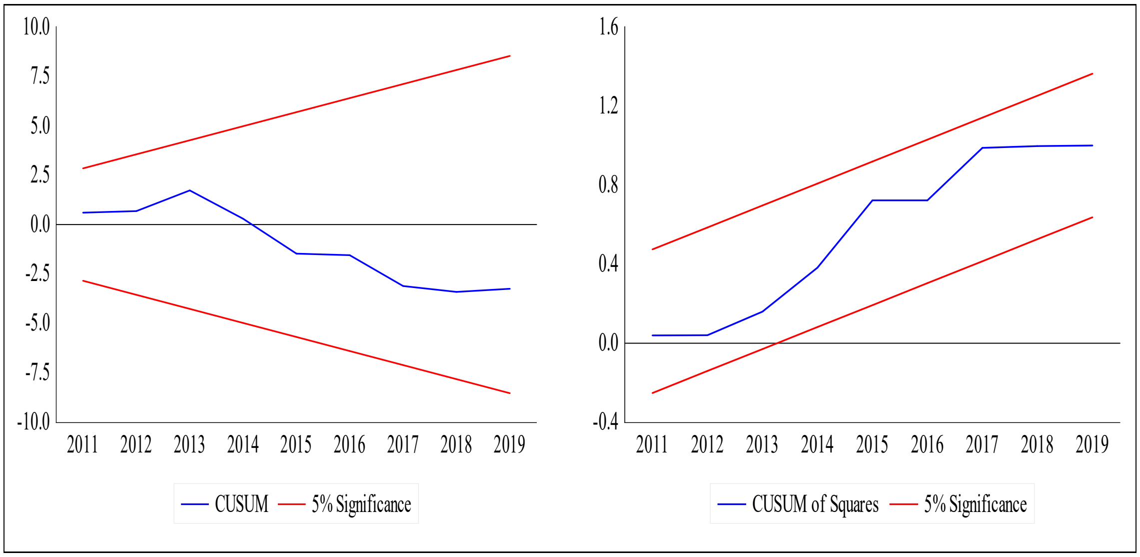

Fig.9. CUSUM and CUSUMSQ (sensitivity analysis model with HP filtered cyclical variables)

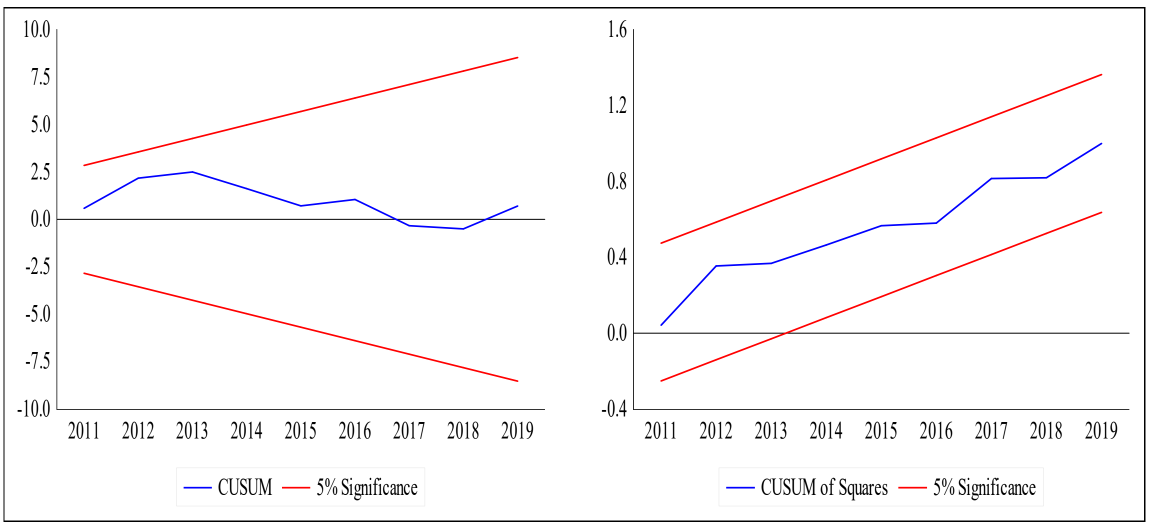

Fig.10 CUSUM and CUSUMSQ (sensitivity analysis model with CO filtered cyclical variables)

Table 12. Diagnostic tests on the residuals of estimated dynamic NARDL model (sensitivity)

HP filtered data					CO filtered data				
Correlogram of standardised residuals					*Correlogram of standardised residuals*				
Lags	AC	PAC	Q-Stat	*p*-values	Lags	AC	PAC	Q-Stat	*p*-values
1	0,142	0,142	0,464	0,496	1	-0,125	-0,125	0,3601	0,548
2	-0,217	-0,242	1,616	0,446	2	-0,22	-0,239	1,5452	0,462
3	-0,033	0,043	1,6445	0,649	3	0,013	-0,055	1,5497	0,671
4	0,094	0,043	1,8863	0,757	4	0,024	-0,039	1,565	0,815
5	-0,207	-0,251	3,1405	0,678	5	-0,364	-0,405	5,4429	0,364
6	-0,418	-0,351	8,6362	0,195	6	-0,051	-0,237	5,5231	0,479
7	0,015	0,043	8,6437	0,279	7	0,179	-0,094	6,607	0,471
8	0,165	-0,018	9,6436	0,291	8	0,09	-0,018	6,9022	0,547
9	-0,064	-0,103	9,8055	0,366	9	-0,142	-0,199	7,7073	0,564
10	-0,036	0,019	9,8624	0,453	10	0,118	-0,114	8,3162	0,598
11	-0,009	-0,239	9,8662	0,542	11	-0,063	-0,304	8,5097	0,667
12	0,058	-0,104	10,051	0,612	12	0,06	-0,019	8,7057	0,728
Correlogram of standardised residual squared					*Correlogram of standardised residual squared*				
Lags	AC	PAC	Q-Stat	*p*-values	Lags	AC	PAC	Q-Stat	*p*-values
1	-0,287	-0,287	1,9013	0,168	1	-0,255	-0,255	1,5045	0,22
2	0,047	-0,038	1,9557	0,376	2	0,052	-0,014	1,5708	0,456
3	-0,242	-0,261	3,4772	0,324	3	0,027	0,04	1,5903	0,662
4	0,156	0,014	4,1467	0,387	4	-0,14	-0,131	2,1261	0,713
5	0,049	0,099	4,2167	0,519	5	0,401	0,362	6,8425	0,233
6	-0,004	-0,01	4,2171	0,647	6	-0,169	0,009	7,7374	0,258
7	-0,413	-0,436	9,9983	0,189	7	0,026	-0,027	7,7602	0,354
8	0,266	0,081	12,597	0,127	8	0,105	0,107	8,1615	0,418
9	0,015	0,107	12,606	0,181	9	-0,036	0,092	8,2127	0,513
10	0,249	0,15	15,34	0,12	10	-0,034	-0,233	8,2641	0,603
11	-0,18	0,063	16,928	0,11	11	-0,163	-0,202	9,5576	0,571
12	-0,126	-0,138	17,801	0,122	12	-0,102	-0,2	10,127	0,605

Note: AC and PAC are the estimated Autocorrelation and Partial Correlation coefficients, respectively.

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