The Partition of Production between Households and Markets

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Abstract

The process of industrialization was accompanied by the switch from household production to firm production. The industrialization process was also a process of population growth, the appearance of general-purpose technologies, and the expansion of international trade. This paper studies the partition of production between households and firms in an analytically tractable general equilibrium model with a continuum of goods. We show that population growth, development of general-purpose technologies, and the opening of international trade increase the percentage of goods produced by firms. However, with the appearance of a technology biased toward home production, the percentage of goods produced by households can increase.

Keywords: Household production, economic development, structural change, technology choice, increasing returns

JEL Classification Numbers: L13, F12, O14

1. Introduction

The industrialization process was associated with the switch from household (family) production to market (firm) production. Compared with household production, factory production is relatively new. For families lived before the Industrial Revolution, it was not uncommon that clothes were family made, meat were supplied by livestock raised by the family, and houses were built by family members. Today, at least for families living in cities of developed countries, clothes, food, and houses are usually purchased from markets produced by firms. The industrialization process was also a process of population growth, the appearance and diffusion of general-purpose technologies, and the expansion of international trade.

There are two significant differences between household production and firm production. First, firm production is associated with the existence of significant levels of fixed costs.[1] Second, the level of output produced by a firm is usually much higher than that produced by a household. Those differences between household production and firm production are related because a high level of fixed cost needs to be recovered by producing a high level of output.

[1] One source of fixed costs comes from research and development expenditure. Another source of fixed costs is buildings and equipment. Chandler (1990) has a detailed discussion of the significance of fixed costs in modern production.

In this paper, we study the division of production between households and firms in a general equilibrium model in which households and firms choose production technologies optimally. We demonstrate that the division of production between households and firms can be addressed in an analytically tractable general equilibrium model. In this model, there is a continuum of products (Dornbusch, Fischer, and Samuelson, 1977; He and Yu, 2015; Chu and Ji, 2016; Ji and Seater, 2020). We assume that each good can be produced by either a constant returns to scale technology or an increasing returns to scale technology. In equilibrium, with relatively low levels of output, households choose constant returns to scale technologies; With relatively high levels of output, firms choose increasing returns to scale technologies.

In this model, firms producing the same good are assumed to engage in oligopolistic competition (Qiu and Zhou, 2007; Liu and Wang, 2010; Wen and Zhou, 2020). The relevance of oligopoly in developed countries such as USA is discussed in detail in Chandler (1990). In their textbook, Pindyck and Rubinfeld (2005, p. 441) write that "oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, aluminum, petrochemicals, electrical equipment, and computers."

The size of the market is measured by the size of the population. First, we show that an increase in population size increases the percentage of goods produced by firms. The reason is that a higher population leads to a higher quantity of demand for each good. Other things equal, the adoption of increasing returns to scale technologies becomes more profitable because the fixed costs can be spread over higher levels of output. This switch to firm production is beneficial to consumers. Second, if a general-purpose technology decreases fixed costs of production, then it helps the adoption of increasing returns to scale technologies. However, with a technology biased toward home production, the percentage of goods produced by firms will decrease. Third, as the opening of international trade increases the size of the market, it will increase the percentage of goods produced by firms.[2] In addition, the country with a smaller population gains more from the opening of international trade.

In the literature, Ng and Zhang (2007) have studied a model in which individuals allocate time among leisure, home production, and firm production. With the existence of fixed costs, both home and firm production have increasing returns. They show that it can be optimal for the

[2] If learning by doing increases an individual's supply of labor, the percentage of goods produced by firms will also increase over time.

government to tax home production and to subsidize firm production. There are some significant differences between their model and this one. First, in their model, firms engage in monopolistic competition. In this model, firms engage in oligopolistic competition. Second, the main questions addressed are different. They focus on studying optimal government policy while we are interested in addressing how the partition of production between households and firms is affected by factors such as population growth and the opening of international trade.

During a country's process of development, it is commonly observed that labor force moves out from the agricultural sector (traditional sector) to the manufacturing sector (modern sector) (Zhang, 1999; Weisdorf, 2006; Akbulut, 2011; Buera and Kaboski, 2012). If the family production is reinterpreted as the agricultural sector with a constant returns to scale production technology and firm production is reinterpreted as the manufacturing sector with increasing returns to scale production technologies, then the switch from family production to firm production is mapped into the relocation of labor force from the agricultural sector to the manufacturing sector. This switch of production is related to unified growth theory in terms of the question asked. For this line of literature trying to provide a unified framework to analyze growth over centuries, North (1981), Rosenberg and Birdzell (1986) and Acemoglu, Johnson and Robinson (2001, 2002) have discussed how good institutions (better protection of property rights) may stimulate greater specialization and make growth sustainable. Galor and Weil (2000) and Galor and Moav (2002) have demonstrated that a rising population leads to a rise in the rate of return to human capital investment, and a higher return induces a greater rate of human capital investment which leads to perpetual growth.

In a country's development process, institutions impacting property rights and contracting may affect the level of transaction costs of using markets. Thus, the division of production between families and firms may be affected. Sun, Yang, and Zhou (2004) have studied the impact of transaction costs on the division of labor. In this model, the impact of transaction costs on the division of production between families and firms is not addressed explicitly. However, if we view that transaction costs lead to an increase in the marginal or fixed costs of increasing returns to scale technologies while do not affect the constant returns to scale technologies, transaction costs can be accommodated into the framework and it can be shown that an increase in transaction costs leads to a decrease in the percentage of goods produced by firms.

The plan of this paper is as follows. Section 2 sets up the model and establishes the existence of a unique equilibrium in a closed economy. Section 3 conducts comparative statics about the properties of the equilibrium. Section 4 addresses the impact of international trade on the division of production between households and firms. Section 5 identifies some possible extensions of the model and concludes.

2. Equilibrium in a closed economy

There are two countries: home and foreign. In this section, countries do not trade with each other. Without loss of generality, we focus on the home country because the analysis for the foreign country is similar.

Labor is the only factor of production. Each household consists of only one individual. Families are assumed to be homogeneous. The number of households in the home country is L, a positive real number. Each individual supplies one unit of labor inelastically. There is a continuum of goods indexed by a number $z \in [0,1]$. With $c(z)$ denoting a consumer's consumption of product z, this consumer's utility function is specified as $\int_0^1 lnc(z)dz$. Firms are owned by individuals and profits will be distributed to individuals. However, as firms earn a profit of zero in equilibrium, the only source of income is wage income. The wage rate is w. The price of product z is $p(z)$. A consumer's budget constraint states that the total spending on all products equals wage income: $\int_0^1 p(z)c(z)dz = w$. A consumer takes the wage rate and prices of goods as given and chooses the quantities of consumption to maximize utility. With the above specification of the utility function, a consumer's utility maximization leads to a fixed percentage of income spent on each product:[3]

$$p(z)c(z) = w. \quad (1)$$

In equilibrium, some goods are produced by households and the others are produced by firms. Regardless of whether goods are produced by households or firms, equation (1) applies: For a home produced good, the price is the opportunity cost of not supplying labor to the market. From

[3] Equation (1) here is similar to equation (8) in Dornbusch, Fischer, and Samuelson (1977). The derivation of equation (1) is as follows. A consumer tries to maximize $\int_0^1 lnc(z)dz + \lambda[w - \int_0^1 p(z)c(z)dz]$, where λ is a multiplier. The first order condition with respect to $c(z)$ is $\frac{1}{c(z)} - \lambda p(z) = 0$. That is, $p(z)c(z) = \frac{1}{\lambda}$. Integration over z yields $\int_0^1 p(z)c(z)dz = \int_0^1 \frac{1}{\lambda}dz = \frac{1}{\lambda}$. Thus, $w = \int_0^1 p(z)c(z)dz = \frac{1}{\lambda}$. Plugging $w = \frac{1}{\lambda}$ into $p(z)c(z) = \frac{1}{\lambda}$ yields equation (1).

equation (1), since the wage rate is w and unit labor requirement is specified later as h, the price of a home produced good is wh and the quantity of consumption is $1/h$.

Each good can be produced by either a constant returns to scale technology or an increasing returns to scale technology. If a good is produced by using the constant returns to scale technology, the constant marginal cost in terms of labor units is h, a positive number. It is assumed that the marginal cost is the same for all goods.

If a good is produced by using an increasing returns to scale technology, the marginal cost in terms of labor units is β. This marginal cost is assumed to be the same for all goods. We also assume that the marginal cost of increasing returns to scale technology is lower than that of the constant returns to scale technology: $\beta < h$.

In addition to marginal costs, there are fixed costs of production associated with increasing returns to scale technologies. There are various sources of fixed costs of production. One example of fixed costs is the research and development cost. To develop increasing returns technologies, different goods may require quite different levels of research and development expenditure. For example, developing machines to produce shoes may be much easier than developing machines for childbearing. To capture this difference, we assume that different goods have different fixed costs of production. For $z \in [0,1]$, let $f(z)$ denote the level of fixed cost of producing good z measured in labor units. We assume that f is differentiable. Like Dornbusch, Fischer, and Samuelson (1977), goods are arranged in such a way that the level of fixed cost increases with the index: $f'(z) > 0$. To ensure the existence of an equilibrium that both household and firm production exist, it is assumed that $f \geq \frac{1}{2}\left[L - \frac{2\beta}{h} - \sqrt{L\left(L - \frac{4\beta}{h}\right)}\right]$, and $\lim_{z \to 1} f(z) \to +\infty$.[4] The intuition for the need for this lower bound of fixed costs is as follows. Since the marginal cost of market production is assumed to be lower than that of household production, if fixed cost of market production goes to zero, all goods will be produced by firms. The existence of a lower bound of fixed costs of firm production ensures that in equilibrium some goods will be produced by households.

[4] The lower bound of the fixed cost is determined by the requirement that a firm's level of output is higher than that of a family. From equation (9), a firm's output is $x = (\sqrt{fL} - f)/\beta$. Equalizing this output with home output $1/h$ yields $f = \frac{1}{2}\left[L - \frac{2\beta}{h} - \sqrt{L\left(L - \frac{4\beta}{h}\right)}\right]$.

Initially, the market size might be too small, and all goods are produced by households. When population grows, market size becomes larger and firms will emerge. A firm hires labor from households and distributes profits to households. As will be verified later, as each household produces for its own consumption only, with its relatively low level of output, each household optimally chooses constant returns to scale technology. With relatively higher levels of output, firms optimally choose increasing returns to scale technologies. A firm producing good z with a level of output $x(z)$ has a total revenue of $p(z)x(z)$. Since its total cost is $(f+\beta x)w$, this firm's profit is $px-(f+\beta x)w$. The number of identical firms producing product z is $m(z)$. Firms producing the same good are assumed to engage in Cournot competition. For each firm, it takes the wage rate as given and chooses its level of output to maximize its profit. Combination of results from a consumer's utility maximization with a firm's optimal choice of output yields[5]

$$p(z)\left(1-\frac{1}{m(z)}\right)=\beta w. \qquad (2)$$

Equation (2) shows that a firm's price p is a markup over its marginal cost of production βw. The markup factor decreases with the number of firms producing the same good.

The number of firms producing a good is a real number rather than restricted to be an integer. The number of firms producing a good is determined by the zero-profit condition.[6] The zero-profit condition for a firm is given by

$$p(z)x(z)-[f(z)+\beta x(z)]w=0. \qquad (3)$$

In equilibrium, some goods will be produced by households while others will be produced by firms. Since the marginal cost for increasing returns technologies is the same for all goods, for the same level of output, average cost of producing a good increases with the level of fixed cost of producing this good. Thus, a good with a lower level of fixed cost is more likely to be produced by firms. Let z_c denote the cutoff level of product for which the price charged by firms is equal to

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[5] The derivation of equation (2) is as follows. A firm's optimal choice of output x yields $p\left(1+\frac{x}{p}\frac{\partial p}{\partial x}\right)=\beta w$. Let x_{-i} denote the sum of other firms' output. From the clearance of product market, total supply of a product equals total demand of this product: $x_{-i}+x_i=Lc$. Differentiation of this equation yields $\frac{\partial x_{-i}}{\partial p}+\frac{\partial x_i}{\partial p}=L\frac{\partial c}{\partial p}$. In a Cournot competition, a firm treats other firms' output as constant when it changes output. That is, a firm will treat x_{-i} as constant. Thus, $\frac{\partial x_i}{\partial p}=L\frac{\partial c}{\partial p}$. As a result, $\frac{\partial x_i}{\partial p}\frac{p}{c}=L\frac{\partial c}{\partial p}\frac{p}{c}$, or $\frac{\partial x_i}{\partial p}\frac{p}{c}\frac{c}{x}=L(\frac{\partial c}{\partial p}\frac{p}{c})\frac{c}{x}$. By using $\frac{\partial c(z)}{\partial p(z)}\frac{p(z)}{c(z)}=-1$ (that is, the absolute value of a consumer's demand for a good is one) from a consumer's utility maximization, $\frac{\partial x_i}{\partial p}\frac{p}{x}=\frac{\partial x_i}{\partial p}\frac{p}{c}\frac{c}{x}=-L\frac{c}{x}=-\frac{mx}{x}=-m$. Plugging this result into $p\left(1+\frac{x}{p}\frac{\partial p}{\partial x}\right)=\beta w$ yields equation (2).

[6] The number of firms is determined by the zero-profit condition. See Liu and Wang (2010) and Zhou (2004, 2009, 2013, 2014a, 2014b) for models that firms engaging in Cournot competition earn zero profits.

the cost of household production. Specifically, in equilibrium goods in the range $[0, z_c]$ are produced by firms and goods in the range $(z_c, 1]$ are produced by households. For goods produced by firms, the price is equal to the average cost because firms earn a profit of zero. Since the average cost of a firm is $\frac{w(f+\beta x)}{x}$ and the cost of household production is wh, the cutoff level good z_c is defined by equalizing the average cost of a firm with the cost of household production:

$$\frac{f(z_c)+\beta x(z_c)}{x(z_c)} = h. \quad (4)$$

For each good in the range $[0, z_c]$, each of the L consumers demands c units of output and the total demand for this good is Lc. Each of the m firms supplies x units of output and the total supply of this good is mx. The clearance of product market requires that demand equals supply:

$$Lc(z) = m(z)x(z). \quad (5)$$

For the labor market, the demand for labor is the sum of demand from firms and demand from households. The demand for labor from firms is $\int_0^{z_c} m(f + \beta x)dz$. Since each consumer spends a given percentage of income on each good, a consumer's quantity of consumption of each household-produced good is $\frac{1}{h}$. As a result, the total demand for labor used in household production is $hL\int_{z_c}^1 cdz$, or $L(1 - z_c)$. Each of the L individuals supplies one unit of labor and the total supply of labor is equal to L. The clearance of the labor market equilibrium requires

$$\int_0^{z_c} m(f + \beta x)dz + hL\int_{z_c}^1 cdz = L. \quad (6)$$

For goods produced by firms, the price level $p(z)$, output level $x(z)$, and consumption $c(z)$ are indexed by z. In the following, if there is no confusion, this dependence may not be explicitly stated. In equilibrium, each household produces goods in the range $(z_c, 1]$, and supplies z_c units of labor to the market and uses the resulting income to purchase goods produced by firms in the range $[0, z_c]$. An individual is indifferent between allocating labor for home production and supplying labor to firms. Equations (1)-(6) form a system of six equations defining six variables $p, w,$ $x,$ $m,$ $c,$ and z_c as functions of exogenous parameters. An equilibrium is a tuple (p, w, x, m, c, z_c) such that

(i) Given the wage rate w and the price levels p, a consumer chooses consumption c to maximize utility;

(ii) Given the wage rate w, a firm chooses output x to maximize profit;

(iii) The same wage rate w applies for household production and firm production;

(iv) Labor market and markets for products clear;

(v) Adjustment through entry and exit of firms is over: firms earn a profit of zero.

For the rest of this section, we establish the existence of a unique equilibrium. The wage rate is normalized to one: $w \equiv 1$.

From equations (1) and (5), for goods in the range $[0, z_c]$, the number of firms producing a good is given by

$$m = \frac{L}{px}. \tag{7}$$

Plugging the value of x from (3) and the value of m from (7) into equation (2), the price of a firm produced good is[7]

$$p(z) = \frac{\beta\sqrt{L}}{\sqrt{L}-\sqrt{f(z)}}. \tag{8}$$

From equation (8), the price of a good increases with the level of fixed costs.

Plugging the value of p from equation (8) into equation (3), a firm's level of output is

$$x = \frac{\sqrt{fL}-f}{\beta}. \tag{9}$$

From equation (9), a firm's scale of production increases with the size of the population. A firm's scale of production also increases with the level of fixed costs. The reason is that a higher level of fixed costs requires a higher level of output so that a firm can break even.

Plugging the value of p from equation (8) and the value of x from equation (9) into equation (7), the number of firms producing good z is given by

$$m(z) = \sqrt{\frac{L}{f(z)}}. \tag{10}$$

Equation (10) shows that the number of firms producing the same good increases with the size of the population and decreases with the level of fixed costs of producing this good.

Plugging the value of x from (9) into equation (4) leads to the following equation defining the cutoff level of good z_c as a function of exogenous parameters:[8]

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[7] The derivation of equation (8) is as follows. From (3), $x = \frac{f}{p-\beta}$. Plugging this result into (7) yields $m = \frac{L}{px} = \frac{L(p-\beta)}{pf}$. Plugging this result into (2) yields $p\left(1 - \frac{pf}{L(p-\beta)}\right) = \beta$. Solving this equation yields $\frac{p-\beta}{p} = \pm\sqrt{\frac{f}{L}}$. The root $\frac{p-\beta}{p} = -\sqrt{\frac{f}{L}}$ is discarded because this root means that a firm charges a price lower than its marginal cost. The remaining root yields equation (8).

[8] Plugging the value of m from (7), the value of p from (8), and the value of x from (9) into equation (6), it can be shown that equation (6) is always valid. This redundancy of equation (6) is consistent with Walras's law.

$$V \equiv \sqrt{f(z_c)} - \left(\frac{h-\beta}{h}\right)\sqrt{L} = 0. \quad (11)$$

The following assumption about parameter values is made:

Assumption 1: $L > \frac{h}{h-\beta}$.

A sufficient condition for Assumption 1 to be satisfied is that the size of the population L is sufficiently large. Also, Assumption 1 will be valid when the marginal cost of the constant returns to scale technology h is relatively high or the marginal cost of increasing returns to scale technologies β is relatively low. Since the size of the population is positively related to the size of the market, an interpretation of Assumption 1 is that it puts a lower bound on the size of the market for increasing returns to scale technologies to be adopted.

The following proposition studies the existence and uniqueness of an equilibrium in a closed economy.

Proposition 1: With Assumption 1, an equilibrium exists. If an equilibrium exists, it is unique.

Proof: Existence: From (11), for $z_c = 0$, $V = \frac{1}{2}\left(\sqrt{L} - \sqrt{L - \frac{4\beta}{h}}\right) - \left(\frac{h-\beta}{h}\right)\sqrt{L}$. Thus, $V < 0$ if $L > \frac{h}{h-\beta}$. That is, under Assumption 1, $V < 0$. For $z_c = 1$, $V > 0$. Since V is a continuous function of z_c, there exists at least one value of $z_c \in [0, 1]$ such that $V = 0$.

Uniqueness: From (11), $\frac{dV}{dz_c} > 0$. Thus V is a monotonic function of z_c. ■

In this equilibrium, for goods produced by firms, the price of a good decreases with its fixed cost. All goods produced by households have the same price which is equal to wh, or h.

We now verify that for goods in the range $z \in [0, z_c]$, firms optimally choose increasing returns technologies. This requires that the average cost with increasing returns technology is lower than that with constant returns technology:

$$\frac{f + \beta\frac{\sqrt{fL} - f}{\beta}}{\frac{\sqrt{fL} - f}{\beta}} < h.$$

We also need to verify that for goods in the range $z \in (z_c, 1]$, a household optimally chooses constant returns to scale technology. This requires that the average costs for those goods with increasing returns technology are higher than that with constant returns technology: $\frac{f+\beta\frac{1}{h}}{\frac{1}{h}} > h$. It can be checked that both inequalities are valid.

3. Comparative statics

In this section, we study how the division of production between households and firms is affected by the size of the population, the appearance of general-purpose technologies, and other relevant factors.

For economies with different levels of population, is there any systematic relationship between the percentage of goods produced by firms and the size of the population? The following proposition shows that an economy with a larger population has a higher percentage of goods produced by firms.

Proposition 2: An increase in population size increases the percentage of goods produced by firms.

Proof: From equation (11), the relationship between the cutoff level of good z_c and the size of the population L is given by $\frac{dz_c}{dL} = -\frac{\partial V/\partial L}{\partial V/\partial z_c}$. Partial differentiation of equation (11) yields $\frac{\partial V}{\partial L} < 0$ and $\frac{\partial V}{\partial z_c} > 0$. As a result, $\frac{dz_c}{dL} > 0$. ∎

The intuition behind Proposition 2 is as follows. Other things equal, an increase in the size of the population increases the demand for each good (Zhou, 2004, 2009, 2019). First, for goods produced by firms before the population growth, population growth makes the adoption of increasing returns technologies more profitable. Thus, these goods will remain to be produced by firms. For these goods, the number of firms producing the same product may increase. This increased degree of competition means that firms receive lower prices for their goods. Second, for some goods initially produced by households close to the cutoff level, the increase in demand makes the adoption of increasing returns to scale technologies profitable. Those goods will experience a switch from household production to firm production. Thus, the percentage of goods produced by firms increases with the population size.

As the wage rate is normalized to one, a lower price means a higher level of consumer welfare. An increase in population size is beneficial to a consumer for two reasons.[9] First, for those goods already produced by firms before the increase in population, their prices are lower. Second, some goods originally produced by households are now produced by firms and their prices become lower. Both effects are beneficial to a consumer.

Similarly, from (11), it can be shown that a decrease in marginal cost of increasing returns technologies increases the percentage of goods produced by firms. A decrease in marginal cost of increasing returns technologies is also beneficial to a consumer.

Industrialization process is associated with the appearance of general-purpose technologies. Examples of general-purpose technologies include the factory system and information technology. What is the effect of the appearance of a general-purpose technology on the division of production between households and firms? There are various ways to incorporate the impact of the appearance of a general-purpose technology. First, both the marginal cost of constant returns technologies and increasing returns technologies can be reduced by the appearance of a general-purpose technology. Second, a general-purpose technology may just reduce the fixed costs of increasing returns to scale technologies. One justification of this is that technologies are embodied in the fixed costs of production as machines and thus only increasing returns technologies will be affected. If a general-purpose technology decreases fixed costs only, for tractability, it is assumed that a general-purpose technology reduces the fixed costs of all increasing returns to scale technologies by the same proportion. The following proposition studies the impact of the appearance of a general-purpose technology.

Proposition 3: If a general-purpose technology decreases the marginal cost of increasing returns to scale technologies by a higher proportion, the appearance of a general-purpose technology increases the percentage of goods produced by firms. If a general-purpose technology

[9] Here an increase in the size of the market is beneficial to everyone. This aspect is like Zhou (2004) which provides a general equilibrium model on the mutual dependence between the division of labor and the extent of the market. This paper departs from Zhou (2004) in two aspects. First, in this paper, in terms of their fixed costs of production, different sectors are heterogeneous rather than homogenous. Second, the division of production between families and firms and the impact of trade are not addressed in Zhou (2004).

decreases only the fixed costs of increasing returns technologies, the appearance of a general-purpose technology increases the percentage of goods produced by firms.

Proof: Equation (11) can be enlarged as $\sqrt{\varepsilon_1 f(z_c)} - \left(1 - \varepsilon_2 \frac{\beta}{h}\right)\sqrt{L} = 0$. In this enlarged equation, ε_1 and ε_2 are positive constants. Here, ε_1 captures the impact of a general-purpose technology on the fixed costs of increasing returns technologies. A decrease in this parameter means that fixed costs decrease. Similarly, ε_2 captures the impact that a general-purpose technology decreases the marginal costs of increasing returns technologies by a higher proportion. A decrease in this parameter means that marginal cost for increasing returns technologies decrease more. From this equation, it can be demonstrated that $\frac{dz_c}{d\varepsilon_1} < 0$ and $\frac{dz_c}{d\varepsilon_2} < 0$. ■

Proposition 3 shows that if a general-purpose technology decreases fixed costs of production, it helps the adoption of increasing returns to scale technologies. Rosenberg (1982) provides historical studies of the impact of some general-purpose technologies. Jovanovic and Rousseau (2005) provide a survey of empirical research of the impact of electricity and information technologies as general-purpose technologies on a society.

Consider the case of the internet providing streaming videos suggesting home “fix-up” solutions which entice a homeowner to try to repair something themselves (household production) instead of hiring a repairperson (firm production). In this case, the household production technological change can be greater than the technological change that reduces the marginal cost of firms with increasing returns technologies. We can capture this “home production augmenting” technology by an increase in ε_2. As shown in Proposition 3, with this kind of technological change, we have a “switch back,” or a reversal in the path of z_c. That is, the percentage of goods produced by households increases!

As the growth process is associated with a switch from household production to firm production, the relative size of the household production sector in a developing country is likely to be larger than that in a developed country. Since only market produced goods are included in GDP, international comparison of GDP may exaggerate the per capita real GDP differences between developing and developed countries. Also, during a country’s takeoff process, as the percentage of goods produced by firms increases, a country’s measured growth rate may change dramatically.

4. Impact of international trade on the division of production

In this section, we study the impact of the opening of international trade between the home country and the foreign country on the division of production between households and firms. The analysis can be easily extended to the case of multiple countries. Variables associated with the foreign country are denoted by asterisk marks. For example, population size in the foreign country is denoted by L^*.

We assume that the foreign country has the same technologies as the home country. Also, foreign consumers have the same preferences as domestic consumers. The only difference between the two countries is that the population in the foreign country may be different from that of the home country. There is no transportation cost between countries. Thus, for each good produced by firms, trade leads to equal price in the two countries. That is, markets in the two countries are integrated.

With trade, the equilibrium cutoff level of product between household and firm production in the two countries should be the same. This can be demonstrated as follows. Suppose that the two countries have different cutoff levels and the cutoff level of product in the home country is z_{th} and that in the foreign country is z_{tf}. Without loss of generality, assume that $z_{th} > z_{tf}$. Goods in the range (z_{tf}, z_{th}) in the foreign country are produced by households and their prices are equal to h. Goods in the range (z_{tf}, z_{th}) in the home country are produced by firms. Since equation (8) shows that the price of a product increases monotonically with the level of fixed cost, in the home country, prices of goods in the range (z_{tf}, z_{th}) will be lower than those in the foreign country. This contradicts the requirement that international trade leads to price equalization of the same product in the two countries.

Let z_t denote the cutoff level of good with international trade. In each country, goods in the range $[0, z_t]$ are produced by firms and goods in the range $(z_t, 1]$ are produced by households.[10] The cutoff level of product may be produced in either country. If the cutoff level of good is produced by the home country, it is defined by

[10] In this case, like models of international trade based on increasing returns to scale, for products produced by firms, only the total number of firms producing each product in the world and total world output can be determined. How the total output is allocated between the two countries is undetermined. For a product produced by firms, it can be produced either by firms located at only one country or by firms located at both countries. If a market provided product

$$\frac{f(z_t)+\beta x(z_t)}{x(z_t)} = h. \tag{12a}$$

If the cutoff level of good is produced by the foreign country, it is defined by

$$\frac{f(z_t)+\beta x^*(z_t)}{x^*(z_t)} = h. \tag{12b}$$

For a good produced by firms, total demand is the sum of domestic demand Lc and foreign demand L^*c^*. Total supply of this good is the sum of domestic supply mx and foreign supply m^*x^*. Goods market equilibrium requires that total demand equals total supply:

$$Lc + L^*c^* = mx + m^*x^*. \tag{13}$$

As firms engage in Cournot competition, a firm takes the output produced by other domestic firms and foreign firms producing the same good as given and chooses its output to maximize its profit. A domestic firm's optimal output choice yields

$$p\left(1 - \frac{x}{mx+m^*x^*}\right) = \beta w. \tag{14a}$$

Similarly, a foreign firm's optimal output choice yields

$$p\left(1 - \frac{x^*}{mx+m^*x^*}\right) = \beta w^*. \tag{14b}$$

Equations (14a) and (14b) show that a firm's price is a markup over its marginal cost of production. This markup factor increases with a firm's market share. When the number of foreign firms m^* equals zero, equation (14a) degenerates to equation (2).

The zero-profit condition for a domestic firm is

$$px - (f + \beta x)w = 0. \tag{15a}$$

Similarly, the zero-profit condition for a foreign firm is

$$px^* - (f + \beta x^*)w^* = 0. \tag{15b}$$

Labor market equilibrium in the home country requires that

$$\int_0^{z_t} m(f + \beta x)dz + hL\int_{z_t}^1 c(z)dz = L. \tag{16a}$$

Similarly, labor market equilibrium in the foreign country requires that

$$\int_0^{z_t} m^*(f + \beta x^*)dz + hL^*\int_{z_t}^1 c^*(z)dz = L^*. \tag{16b}$$

We now demonstrate that with international trade wage rates in the two countries should be equal in equilibrium: $w = w^*$. In terms of the production pattern of firm produced goods, there

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is not produced by the home country, $m(z) = 0$. Similarly, if a product is not produced by the foreign country, $m^*(z) = 0$.

are two cases. In the first case, at least one good is produced by both countries. Plugging the value of $mx + m^*x^*$ from equation (13) into equations (14a) and (14b), by employing equations (15a) and (15b) to eliminate x and x^*, it can be shown that the price of this good if produced at the home country is

$$p(z) = \frac{\beta\sqrt{wL+\quad {}^*L^*}}{\sqrt{wL+w^*L^*}-\sqrt{f(z)w}}. \tag{17}$$

The price of this good if produced in the foreign country is

$$p(z) = \frac{\beta w^*\sqrt{wL+w^*L^*}}{\sqrt{wL+w^*L^*}-\sqrt{f(z)w^*}}. \tag{18}$$

From equations (17) and (18), the wage rates in the two countries should be equal so that the same product has the same price regardless of the country it is produced in.

In the second case, the two countries produce distinct sets of market products. Without loss of generality, suppose that goods in the range $[0, z_h]$ are produced in the home country and goods in the range $(z_h, z_t]$ are produced in the foreign country. For a good produced in the foreign country z, as z approaches z_h in the limit, the price will approach that in equation (18). The price of z_h in the home country is given by equation (17). Thus, wage rates in the two countries should also be equal in this case.

Like Section 2, we can normalize this wage rate to one. Like the analysis in Section 2, it can be shown that the price of a firm produced good is

$$p = \frac{\beta\sqrt{L+L^*}}{\sqrt{L+L^*}-\sqrt{f}}. \tag{19}$$

Plugging the value of p from equation (19) into equation (15a), a domestic firm's level of output is

$$x = \frac{\sqrt{f(L+L^*)}-f}{\beta}. \tag{20}$$

For each product, if this product is produced in both countries, the equilibrium scale of production in the two countries is the same. The number of firms in each country adjusts to clear the labor market.[11]

[11] By plugging the value of p from equation (19) and the value of x from (20) into equation (16a) the number of domestic firms can be determined. Similarly, the number of foreign firms can be determined by using (16b).

Plugging the value of x from (20) into equation (12a), the cutoff level of good with international trade is defined by[12]

$$\sqrt{f(z_t)} - \left(\frac{h-\beta}{h}\right)\sqrt{L+L^*} = 0. \tag{21}$$

The following proposition studies the impact of international trade on the division of production between families and firms.

Proposition 4: The opening of international trade increases the range of goods produced by firms.

Proof: In equation (21), the opening of trade is captured by an increase of L^* from zero to a positive number. From (21), it can be shown that $\frac{dz_t}{dL^*} > 0$. ■

Even without labor mobility across countries, a comparison of equations (11) and (21) shows that the impact of the opening of international trade is like that of an increase in domestic population. The reason is that an increase in domestic population or the opening up of trade leads to a decrease in a firm's monopoly power. The combination of zero transportation cost, a single factor of production, and that countries have access to the same production technologies leads to this result.

The opening of international trade is beneficial to each consumer in the two countries. In addition, the country with a lower endowment of labor benefits more from the opening of international trade because the increase of per capita utility in the smaller country is larger than that in the larger country. The reasoning is as follows. Before the opening of international trade, with the existence of increasing returns to scale, a consumer's utility in the smaller country is lower than that for a consumer in the larger country. With the opening of international trade, a consumer in each country now enjoys the same level of utility which is higher than that in each country before trade.

5. Conclusion

[12] If any equilibrium with the same level of total world output is viewed as one equilibrium, following the proof of Proposition 1, it can be shown that there exists a unique equilibrium with international trade.

The process of industrialization was accompanied by the switch from household production to firm production. The industrialization process was also a process of population growth, the appearance and diffusion of general-purpose technologies, and the expansion of international trade. In this paper, we have studied the partition of production between households and markets in an analytically tractable general equilibrium model in which families and firms choose production technologies optimally. With their relatively low levels of output, households choose constant returns to scale technologies. With their relatively high levels of output, firms choose increasing returns to scale technologies. First, we show that an increase in the size of the population increases the range of goods produced by firms. Second, if a general-purpose technology decreases fixed costs of production, it helps the adoption of increasing returns to scale technologies. However, if a technology is biased toward home production, then the percentage of goods produced by firms will decrease. Third, the opening of international trade increases the percentage of goods produced by firms.

There are some interesting extensions and generalizations of this model. First, this model abstracts from friction in the transition from household production to firm production. These frictions such as costs of moving to cities, capital constraint, and job market friction are important. In this paper, potential quality differences between household production and firm production are not considered. For goods produced within a family, it may be easier to assess the qualities of those goods. For some products provided by the market, such as car repair, medical service, it may be much more difficult to assess the qualities of those products. Other things equal, incorporating market frictions into the model is likely to increase the percentage of goods produced by families.

Second, in this paper, the size of a family is normalized to one. The size of a family can be modeled as a choice variable. In a country with weak institutions, costs of market transactions can be high and benefits of having a large family increase since a large family can also adopt some increasing returns to scale technologies and can avoid market transaction costs. With the development of better institutions and reductions in transaction costs for using markets, optimal family size is likely to decrease.

Third, in this model, families are homogeneous. It is commonly observed that individuals may have different levels of income and may spend quite different percentages of time on family production. This aspect can be captured in a model in which individuals differ in their endowments of labor or acquisition of human capital. An individual with a higher level of supply of effective labor has a higher level of income and may spend a lower percentage of time on family production.

Fourth, this paper does not address the impact of governmental behavior. Market transactions are usually taxed by governments. Tax may be incorporated into the model to study how a family's supply of labor to the market is affected by taxes. It is likely that a higher tax rate increases the percentage of goods produced by families.

Finally, in this model firms and households are distinguished only by their different cost structure. The emergence of social production through firms is related to the rise of the nation-state and the establishment of more efficient financial institutions. How factors such as the rise of nation-state and the level of efficiency in the financial sector affects the partition of production between households and firms will be an interesting avenue for future research.

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