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# A Psychologically-Based Model of Voter Turnout\*

Ming Li and Dipjyoti Majumdar<sup>†</sup>

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## Abstract

We analyze a psychologically-based model of voter turnout. Potential voters experience regret if they fail to vote, which is the motivation for participation in voting. Regret from abstention is inversely related to the margin of victory. Voters on the winner's side experience less regret than those on the loser's side. We show that the unique equilibrium involves positive voter turnout. We show that the losing side has higher turnout. In addition, voter turnout is positively related to importance of the election and the competitiveness of the election. We also consider scenarios in which voters are uncertain

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about the composition of the electorate’s political preferences and show similar phenomena emerge.

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## 1 Introduction

After the 2000 US presidential election, CBS News / New York Times conducted a poll of 1720 Americans November 10-12, 2007 and found that 55% of those who did not vote regretted not voting.<sup>1</sup> After the surprising elimination of the socialist candidate Lionel Jospin at the hands of the far-right anti-immigration candidate Jean-Marie Le Pen in the first round of the 2002 French presidential election, many French voters regretted their decision not to vote.<sup>2</sup> Clearly, in these elections, each individual voter’s decision would not have affected the outcome of the election. Yet, voters who did not vote experienced regret afterwards. In this paper, we maintain that psychological payoffs are an important force in determining voting behavior. We posit that voters who do not vote in elections suffer regret and this gives them an incentive to vote in elections.

The question of why people vote in large elections is one that has occupied the minds of political scientists and economists alike. The initial idea of the “rational voter paradox” is in the celebrated work of Downs (1957). If voting is costly and the only motivation for people to vote is to affect the outcome of the election, then they should never vote. The reason is that in large elections the probability of any single voter to affect the outcome of the election is infinitesimal. However, observed turnout rates are quite high. Since the seminal

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<sup>1</sup>*CBS News*, November 13, 2000. Since it was a national poll, many of these voters reside in states where the election result was not in dispute.

<sup>2</sup>*Financial Times*, London, May 4, 2002, pg. 09.

work of Downs, there has been a lot of attempts to explain this paradox. In our opinion, a satisfactory theoretical model has to be able to achieve three goals. First, it must explain significantly positive turnouts as equilibrium behavior in large elections. Second, it must account for other frequently observed empirical facts, for example, the positive correlation between turnout and the perceived competitiveness of elections. Lastly, it must be parsimonious so that the model can be enriched to study other interesting issues in large elections, for example, candidates' political positioning.

In this paper, we explain voter turnout by introducing voter regret. We study an election with two candidates and two corresponding groups of voters. Voters who failed to vote in an election suffer regret. The magnitude of a voter's regret depends on whether or not his preferred candidate wins the election and on the margin of victory. The smaller the margin of victory, the higher is the regret suffered by any voter who did not vote. Moreover, for any fixed margin of victory, the regret of a voter on the losing side is higher than that on the winning side.<sup>3</sup> We call this assumption "winner regrets less." A potential voter votes if and only if his cost of voting is less than the expected regret he would suffer after the election if he did not vote.

Our model is a departure from the strictly "rational" model, where the incentive to vote depends solely on the voter's probability of affecting the outcome. In fact, since we have a continuum of voters in our model, the probability of any voter being pivotal is zero. A common approach to explaining voter turnout is assuming that voters derive utility from participation in elections, either because of self-expression or sense of civic duty. We neither dispute the plausibility of such concerns nor rule them out from our model. However,

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<sup>3</sup>We want to de-emphasize the importance of a voter's (nonzero) regret from abstention while his favorite candidate wins. As we show in Example 2 in Section 5, such regret is not necessary for our equilibrium construction when voters are uncertain about the composition of preferences of the electorate.

we enhance them with a psychological factor with an intuitive appeal.

Here, we give a short justification for our assumptions about voter regret, and we provide in Section 6 a detailed discussion of the concept of regret in our model. Voters feel regret if they do not vote because failure to vote is viewed negatively in a democratic society. It is conceivable that such negative (self-)perception is especially strong when the election is close or when a voter's favorite candidate lost the election. One possible interpretation is that ethical voters are empathetic towards those that belong to the same political group as theirs. Thus, an ethical voter thinks his action disappoints his peers or his favorite candidate if he does not vote.<sup>4</sup> Such regret becomes more poignant when the election result is close because the psychological effect of the letdown is stronger. This is also consistent with the anecdotal observation that voter apathy is contagious.

Assuming a decreasing density function of the distribution of the cost of voting, we show that there exists a unique equilibrium with a positive level of turnout. We account for the consistently observed empirical facts, namely, that turnout is lower for the winning side, that it is positively correlated with perceived closeness of the election, and that it is positively correlated with the importance of the election. We also allow voters to be uncertain about the composition of the electorate's political preferences, and derive similar results.

In our model, a voter votes because he anticipates that he will experience regret if he does not. The voting equilibrium in our model is a margin of victory and turnout levels for the two groups that are consistent with each other. First, an *anticipated* margin of victory determines a cost threshold for either group, such that voters with voting cost below those thresholds vote in the election, and thereby determines the turnouts for the two groups. Conversely, each pair of

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<sup>4</sup>An ethical voter may also feel regret about not voting because this contributes to making himself an apathetic person. This is reminiscent of arguments made by Frank (1988) (Ch. 11, "Human Decency.").

turnout levels determines a *resulting* margin of victory. Consistency requires the anticipated margin of victory and the resulting margin of victory to be the same. The first condition requires that turnouts decrease as the margin of victory increases, since regret is inversely related to margin of victory. On the other hand, the second condition causes margin of victory to increase as turnouts increase, given our “winner regret less” assumption and the decreasing pdf of the cost distribution. Therefore, there is a unique voting equilibrium.

Let us consider the intuition of the comparative statics results. First, lower turnout for the winning side is simply implied by the fact the winning voters regret less about not voting. Second, as the election is perceived to be closer, fixing turnout levels, the anticipated winning margin is going to be smaller. By our “winner regrets less” assumption and the decreasing pdf of the cost distribution function, an increase in turnout would cause the winning margin to increase. Thus, the equilibrium turnout must increase to restore consistency. Finally, as the election becomes more important, given the same winning margin, both turnouts will increase, causing the equilibrium turnout to increase.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model of voting with regrets. Section 4 characterizes the equilibrium and analyzes how voter turnout varies with various factors. Section 5 introduces an example in which there is uncertainty about the voter population. Section 6 discusses the concept of regret in our model and its relation to the existing literature. Section 7 concludes and discusses possible extensions.

## 2 Related Literature

In this section, we discuss the relationship of our paper to the literature on voter turnout. In Section 6, we discuss the concept of regret that

we use in our model.

Since the work of Downs, there has been a large literature that tries to explain voter turn-out in large elections. Most of the early works starting with Tullock (1968) were decision theoretic models. An important and controversial paper in this genre is by Ferejohn and Fiorina (1974). Following Savage, in their model voters make participation decisions not according to expected utility, but the *minimax regret criterion*. The regret attached to an act  $a_i$  if state  $S_j$  occurs is the difference between what the decision maker would have attained had he known the true state of nature and what he actually attains by choosing  $a_i$ .<sup>5</sup> They show that under some condition on the cost parameter, a voter's decision to vote for his most preferred candidate is the act that minimizes the maximum regret. Their approach has been subject to criticism in relation to the validity of minimax regret as a choice criterion. A rational decision maker would use the minimax regret criterion only if he is *uncertain* about the outcome, in the sense that he cannot estimate the probabilities of different outcomes at all. Many researchers argue that this assumption is implausible. In addition, the minimax regret criterion is also argued to lead to contradictions. For example, it is possible for a voter to have a traffic accident en route to the polls and get killed. Why would a minimax regretter not stay home instead?<sup>6</sup> In addition, their model is a decision theoretic model rather than game theoretic. In that sense, our model is not closely related to theirs.

Our model is closer in spirits to that of Riker and Ordeshook (1968). They analyze a model of voter participation in which agents receive a duty payoff  $D > 0$  when they vote for their preferred candidate. In our model, one can reinterpret regret as a payoff (negative)

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<sup>5</sup>For a detailed description of this criterion, see Luce and Raiffa (1957), pp. 280-282.

<sup>6</sup>For a sample of such criticisms, see Beck (1975), Mayer and Good (1975), and Tullock (1975).

that the agents receive for not voting for their preferred candidate. However, in their model the payoff  $D$  is independent of the election outcome, which is fitting to the duty payoff interpretation. In our model, by contrast, for any fixed level of the margin of victory, the regret that a voter on the losing side incurs is higher than that incurred by one on the winning side. Moreover, regret is a decreasing function of the margin of victory. This, together with assumptions on the distribution of voting cost, allows our model to deliver comparative statics that is different from that of Riker and Ordeshook (1968). For example, in Riker and Ordeshook's model, turnout level is fixed and independent of the relative size of the minority. By contrast, our model predicts that turnout will be higher among the minority than among the majority and yet the majority's preferred candidate wins. Moreover, turnout progressively diminishes as the size of the minority goes down. In addition, in our model, turnout increases as the size of the minority approaches the size of the majority and as the importance of election increases. These are well in line with empirical evidence.<sup>7</sup>

Feddersen and Sandroni (2006c) (see also Feddersen and Sandroni (2006a) (2006b)) generate similar comparative statics using a more involved model than ours. In their model, there are both ethical and nonethical voters. The former are called "rule-utilitarian," who derive a positive payoff from taking the ethical action, namely the one that is called for by the minimization the social cost of voting. Therefore, voting is not necessarily the ethical action to take, which is determined endogenously in their model. Hence, voters have to know about the equilibrium aggregate behavior of all voters to decide what is the ethical thing to do. Also, in their model, there is the possibility that a voter abstains from an election and feels "righteous" about his choice not to vote. These two features are in stark contrast to Riker and

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<sup>7</sup>See, for example, Blais (2000) and Coate and Conlin (2004).



Ordeshook’s and our model, where voters’ ethical concerns are “private” in that they do not take into account other voters’ cost of voting and where voting is the always the ethical thing to do. Feddersen and Sandroni (2006c) impose a consistency condition on the behavior of the agents and look for consistent behavior profiles. They show that consistent behavior corresponds to a pure strategy equilibrium of a suitably defined two-player game. In order for the existence of the pure strategy equilibrium, they impose that the fractions of voters (both in the majority and in the minority group) who take the ethical action are random variables. In contrast, the voters in our model are all ethical and our equilibrium construction works regardless of whether voters are certain about the composition of voter preferences for candidates. Moreover, in their model the comparative statics results crucially depend on the fact that agents in both groups (the majority and the minority) care about aggregate voting costs rather than voting costs of the members of their own group. By contrast, in our model each voter only cares about his own cost. In short, our model can be viewed as a more parsimonious one with different theoretical justification.

Thus, our paper can be put in the broad category of research that uses “ethical-voter” models. The survey on political economy by Merlo (2006) contains a concise discussion of various voter turnout models.<sup>8</sup>

The first game-theoretical papers on voter turnout use the so-called pivotal-voter model. Ledyard (1982) is the first to analyze the problem of voting behavior in large elections using game theoretic models. The model also allows endogenous party-positions. The paper does not however characterize turnout levels though there are some examples. Palfrey and Rosenthal (1983) study a simplified version of the Ledyard model with complete information. They show that in the vot-

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<sup>8</sup>See also Aldrich (1993), Feddersen (2004) and Dhillon and Peralta (2002).

ing game with complete information there are multiple equilibria. The model shows that it is possible to have high turnout equilibrium even with large electorates. However, this large turnout equilibrium crucially depends on the assumption of complete information. Indeed, as shown by Palfrey and Rosenthal (1985), with incomplete information regarding the voting costs, there is essentially a unique low turnout equilibrium for large electorates. Myerson (1998) adds another source of uncertainty in elections by considering games with population uncertainty. In a setting where population uncertainty is modelled as a Poisson process, it is shown that the low turn-out equilibrium is the unique prediction.

According to the classification proposed by Merlo (2006),<sup>9</sup> in addition to the pivotal-voter and ethical-voter models, there is a third category: uncertain-voter model, where turnout is related to voters uncertainty about who is the best candidate. In all such models (e.g., those of Feddersen and Pesendorfer (1996), Matsusaka (1995), Degan (2006) and Degan and Merlo (2007)), the possibility of voting for the wrong candidate engenders a cost and therefore uncertain voters may prefer to abstain. While in Feddersen and Pesendorfer's (1996) model such a cost depends on pivotal considerations, in the models of Degan (2006) and Degan and Merlo (2007), as in ours, it is a psychological cost derived from doing the wrong thing, not because of the effect a voter's decision has on the outcome. In Matsusaka's (1995) model, a voter's utility from voting is explicitly defined as a function increasing in the quality of his information. Unlike in the other three papers and ours, a voter has no strategic considerations in Matsusaka's model.

There exists a literature that aims to explain voter turnout as results of learning or evolution. Diermeier and Mieghem (2000) study

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<sup>9</sup>Another branch of literature explains voter turnout as the result of mobilization by group leaders. See Uhlaner (1989), Morton (1991), and Shachar and Nalebuff (1999). They do not explicitly model why, however, leaders are able to influence voters' decisions.

a model of voter turnout where in each period a randomly selected voter receives information about turnout through polls and plays best response based on Bayesian updating. They show that large turnout is possible in large elections when voting factions are of similar size. There are also papers by Conley, Toossi, and Wooders (2001) and Sieg and Schulz (1995), who use evolutionary models and investigate conditions under which high turnout survives. Their research may also be interpreted as providing explanation to why voters may have ethical preferences.

We know of two other papers that assume voters' utility endogenously depends on the election outcome. Llavador (2006) introduces a model in which voters care about the margin of victory out of instrumental concerns, for example, power sharing between parties according to their received votes. In his model, each voter has an ideal margin of victory for a candidate. So, each voter decides whom to vote for based on both her preferences on the winning margin and whom the other voters are voting for. By contrast, a voter in our model only votes for his favorite candidate, and makes participation decisions based on whether the other voters are voting. Callander (2007, forthcoming) studies performance of the sequential voting systems when voters have a preference for voting for the eventual winning candidate. He shows that sequential voting aggregates information better in lopsided elections and simultaneous voting is better in tight elections.

Recently, there has been some interesting work that empirically or experimentally tests voting models. Coate and Conlin (2004) structurally estimate a modified version of the rule-utilitarian voter model of Feddersen and Sandroni (2006c) and find evidence supporting it over simple expressive voting. Coate, Conlin, and Moro (2004) test the pivotal-voter model with small elections, and find that it is outperformed by an expressive voting model. Degan and Merlo (2007) structurally estimate a model of turnout and voting in multiple elections, using data from the US presidential and congressional elections.

They find their structural model matches data quite well and utilize the model to perform counterfactual analysis of the effect of policies that increase voter information or sense of civic duty. Levine and Palfrey (2006, forthcoming) test the pivotal-voter model in small elections in the lab and argue that a combination of the pivotal-voter model and the logit equilibrium explains the experimental data well.

### 3 The Model

Two candidates,  $A$  and  $B$ , compete for one position in an election. Correspondingly, there are two types of voters. Let us call them  $A$ -voters and  $B$ -voters respectively. Each voter favors the candidate that bears the same label as his own. Voters are uniformly distributed on the interval  $[0, 1]$ , with  $A$ -voters occupying  $[0, \alpha]$  and  $B$ -voters occupying  $(\alpha, 1]$ . We assume

$$\alpha > \frac{1}{2}.$$

Let  $\delta$  and  $0$  be respectively the utility a voter gets when his favorite candidate gets elected and when the opposing candidate gets elected, due to the different policies they will implement. So,  $\delta$  can be interpreted as the importance of the election.

Let  $c$  be the cost of voting, a random variable that are distributed on  $[0, +\infty)$  according to the same distribution function  $F$  and density function  $f$  for both  $A$ - and  $B$ -voters. The parameters  $\alpha$ ,  $\delta$ , and  $F$  are common knowledge among voters.

A significant part of the cost of voting is the opportunity cost. The opportunity cost of voting for a retiree is lower than that for an active worker. The opportunity cost of voting for a worker with a flexible schedule is lower than that for a worker with a rigid one.

A  $I$ -voter's expected "material" payoff from voting for candidate  $J$  is

$$-c,$$

as the probability of his vote affecting the election outcome is zero, given that we have a continuum of voters.

In addition to his material payoff, a voter incurs regret when the election is close and he fails to vote or votes for the opposing candidate. The level of regret is determined by the closeness of the election. The closer the election, the higher the regret. In addition, his regret from not voting is larger when his favorite candidate loses by a certain margin than when the candidate wins by the *same* margin.

Since a voter experiences regret if he votes for the wrong candidate, voting for the opposing candidate is dominated by voting for one's favorite candidate. Therefore, we will only consider the case where voters vote for their favorite candidate if they vote at all.

We define candidate  $I$ 's *margin of victory* to be equal to the difference between the number of voters voting for candidate  $I$  and the number of those voting for his opponent,  $-I$ . Let  $\tau_A$  and  $\tau_B$  be the turnouts for each type of voters. Thus, candidate  $A$ 's margin of victory is

$$m_A = \alpha\tau_A - (1 - \alpha)\tau_B.$$

We assume the voter's regret takes the form

$$R(m, \delta) = \delta r(m),$$

where  $m$  is the margin of victory (or, margin of loss if  $m < 0$ ) for the voter's favorite candidate. Hereafter, we will refer to the function  $R$  as the absolute regret function, and  $r$  as the relative regret function, or simply the regret function. Note that absolute regret is simply relative regret scaled up by  $\delta$ , which measures the importance of the election.

A voter's expected payoff from not voting is

$$-\delta E_m(r(m)).$$

The function  $r$  is nonnegative, continuously differentiable except possibly at  $m = 0$ , strictly increasing for  $m < 0$ , and strictly decreasing for  $m > 0$ .

For clarity of exposition, we separate the regret function into two parts:  $r_- : \mathbb{R}_- \rightarrow \mathbb{R}_+$  and  $r_+ : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ . They are defined by

$$\begin{aligned} r_-(m) &= r(m) \text{ for } m < 0; \\ r_+(m) &= r(m) \text{ for } m > 0; \\ r_-(0) &= \lim_{m \rightarrow 0^-} r(m); \\ r_+(0) &= \lim_{m \rightarrow 0^+} r(m). \end{aligned}$$

We assume that  $r$  has the following property.

**Assumption 1.** (*Winner regrets less.*) The regret function  $r$  satisfies:

- a.  $r_-(0) \geq r_+(0)$ ;
- b.  $\frac{r'_-(-m)}{-r'_+(m)} < 1$  for all  $m > 0$ .

The first part of the assumption says that when an abstaining voter's favorite candidate loses by an arbitrarily small margin, his regret is higher than that if his favorite candidate wins by an arbitrarily small margin. The second part can be interpreted as: "when the margin of victory increases by a small amount, the decrease in regret for an abstaining voter on the winner's side is more than that if he is on the loser's side." In other words, "a winner gets complacent more easily." These two parts imply

$$r_+(m) < r_-(-m)$$

for all  $m > 0$ , or "winner regrets less."

## 4 Equilibrium

We look for equilibria of the model defined as follows.

**Definition 1.** A *voting equilibrium* is a profile of strategy and beliefs that satisfies:

- a. each voter forms Bayesian beliefs about the result of the election;
- b. based on these beliefs, each voter chooses to vote or not based on the expected utility of these two alternatives;
- c. the candidate with a majority of the votes wins.

As we concluded in the previous section, the pivotal probability for any single voter is zero. Therefore, a voter chooses to vote if

$$-c < -\delta E_m(r(m)) \quad \text{or} \quad \delta E_m(r(m)) > c. \quad (1)$$

and not to vote if the inequality goes the other direction. If the two alternatives give him the same utility, he can choose either option.

We focus on the case  $\alpha > 1/2$ . That is, a majority of voters prefer candidate  $A$  to  $B$ . As we will demonstrate below, this will preclude the possibility of a tie in the election.

First, observe that if an  $I$ -voter ( $I = A, B$ ) of cost  $c$  finds it optimal to vote, then another  $I$ -voter of cost  $c' < c$  must also find it optimal to vote. Therefore, the equilibrium can be characterized by two cutoff cost values for each group of voters,  $c_A$  and  $c_B$ . Thus, the equilibrium is characterized by the following equations:

$$\tau_A = F(c_A), \quad (2)$$

$$\tau_B = F(c_B), \quad (3)$$

$$c_A = \delta r(\alpha \tau_A - (1 - \alpha) \tau_B), \quad (4)$$

$$c_B = \delta r((1 - \alpha) \tau_B - \alpha \tau_A). \quad (5)$$

The following lemma states that the candidate with a higher proportion of supporters,  $A$ , wins the election.

**Lemma 1.** *In equilibrium, voters on the winner's side must have a lower turnout. Hence,  $A$  must win the election in equilibrium.*

*Proof.* The first statement is a direct consequence of Assumption 1. A voter on the winner's side has lower regret. Therefore, the cutoff cost value for the winner's side must be lower, which implies lower turnout since both types have the same cost distribution,  $F$ .

The second statement is implied by the first. Since  $\alpha > \frac{1}{2}$ , in order for  $B$  to win,  $B$ -voters must have a higher turnout than  $A$ -voters, which contradicts the first statement.  $\square$

Note that the above discussion is conditional on the existence of an equilibrium, to which we turn now. As we know  $A$  wins the election in equilibrium, let  $m$  be the winning margin by  $A$ . From equations (4) and (5), we have

$$\tau_A = F(\delta r(m)), \quad (6)$$

$$\tau_B = F(\delta r(-m)), \quad (7)$$

$$m = \alpha\tau_A - (1 - \alpha)\tau_B. \quad (8)$$

A triple  $(\tau_A, \tau_B, m)$  constitutes a voting equilibrium outcome if and only if it is a solution to equations (6), (7), and (8). Together, they imply

$$m = \alpha F(\delta r(m)) - (1 - \alpha)F(\delta r(-m)). \quad (9)$$

An  $m$  that satisfies the above equation uniquely determines a voting equilibrium outcome.

Let us use  $\mu(\cdot)$  to denote the function defined by the right hand side of (9).<sup>10</sup>

The right hand side of Equation (9) can be interpreted as the *resulting* winning margin by candidate  $A$  given that  $A$ -voters with voting costs lower than or equal to  $\delta r(m)$  vote in the election and  $B$ -voters with voting costs lower than or equal to  $\delta r(-m)$  vote in the

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<sup>10</sup>Note that  $\mu$  is a function of  $\alpha$ ,  $\delta$ ,  $F(\cdot)$ , and  $m$ . However, whenever without confusion, we treat it as only a function of  $m$ . In this sense, an  $m$  satisfying (9) is a fixed point of  $\mu$ .



election. Thus, an equilibrium winning margin  $m$  is a fixed point of the function  $\mu(\cdot)$ .

We make the following assumption to guarantee the existence of an equilibrium.

**Assumption 2.**  $\alpha F(\delta r_+(0)) - (1 - \alpha)F(\delta r_-(0)) > 0$ .

Intuitively, the assumption requires that when the winning margin is arbitrarily close to zero, the voters on the majority side who prefer voting to abstention will outnumber such voters on the minority side. That is, the majority voters will not be so complacent as to make the election outcome to go the other way. Mathematically, taking  $\alpha$  and  $F$  as fixed, this assumption requires that the discontinuity of  $r$  at 0 cannot be too severe.<sup>11</sup> In particular, if  $r$  is continuous at 0, then this assumption is satisfied as  $\alpha > \frac{1}{2}$ .

**Theorem 1.** *Given Assumptions 1 and 2, there exists at least one voting equilibrium.*

*Proof.* We show this by applying the intermediate value theorem to (9).

First, by our assumptions, both sides of (9) are continuous functions of  $m$ .

Second, when  $m = 0$ , the right hand side of (9),  $\mu(m)$ , is positive by Assumption 2. Therefore,  $m < \mu(m)$  at  $m = 0$ .

Third, let  $m = \alpha$ . Then,

$$\mu(m) \leq (2\alpha - 1)F(\delta r(m)) \leq 2\alpha - 1 < \alpha,$$

where the first inequality is implied by Assumption 1 and the third inequality by  $\alpha > \frac{1}{2}$ . Therefore,  $m > \mu(m)$  at  $m = \alpha$ .

Using the Intermediate Value Theorem, we conclude there exists  $m \in (0, \alpha)$  such that  $\mu(m) = m$ .  $\square$

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<sup>11</sup>This assumption is necessary here since we have an essentially deterministic model. We show in Section 5, however, this is not the case when there is uncertainty about  $\alpha$ .

The uniqueness of equilibrium is not guaranteed with the given set of assumptions. The following is a sufficient condition.

**Assumption 3.** The density function of cost distribution,  $f$ , is non-increasing.

A rough interpretation of the assumption is that voters with low costs are more populous than those with high costs.

**Theorem 2.** *Given Assumptions 1, 2, and 3, the equilibrium is unique.*

*Proof.* Consider equation (9). It suffices to show its right hand side,  $\mu(m)$ , is non-increasing in  $m$ .

To see this, observe that

$$\mu'(m) = \alpha f(\delta r(m)) \delta r'_+(m) + (1 - \alpha) f(\delta r(-m)) \delta r'_-(-m), \quad (10)$$

which we conclude to be negative by Assumptions 1 and 3 and  $\alpha > \frac{1}{2}$ .

Thus, the solution to (9) is unique, which implies uniqueness of equilibrium.  $\square$

To understand the argument for uniqueness, note that in equilibrium candidate  $A$  wins the election and that the winning margin  $m$  uniquely determines the equilibrium. Let us consider an alternative equilibrium with a lower winning margin for  $A$ . Thus, the turnouts of  $A$ -voters and  $B$ -voters should both increase. However, since the corresponding cutoff cost level for  $B$ -voters is higher than that for  $A$ -voters (Assumption 1) and the density of high-cost voters is lower (Assumption 3), since the cutoff level for  $B$ -voters increases at a slower rate than that for  $A$ -voters (Assumption 1), and since  $B$ -voters are less populous, this will cause  $A$ 's winning margin to go up, contradicting our stipulation that the winning margin goes down. We can derive a similar contradiction if we try to find an equilibrium with a higher winning margin. Thus, there is only winning margin that ensures the resulting turnouts produce exactly the same winning

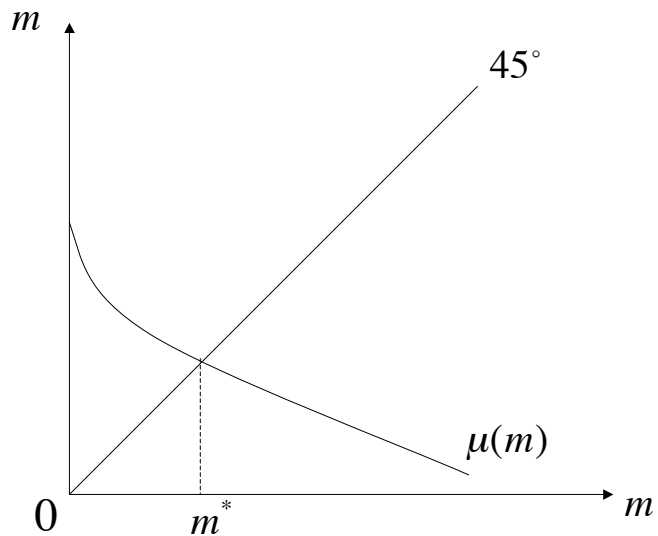


Figure 1: The determination of the equilibrium winning margin by  $A$ . The function  $\mu$  maps from an anticipated margin to a resulting margin.

margin, or the turnouts and winning margin are “mutually consistent.”

Figure 1 demonstrates the determination of the equilibrium winning margin when Assumptions 1, 2, and 3 are satisfied.

The following lemma makes explicit the relationship between turnout and margin of victory in equilibrium as an implication of properties of the regret function.

**Lemma 2.** *Fix the importance of the election,  $\delta$ , the regret function,  $r$ , and the cost distribution,  $F$ . In equilibrium, turnouts for  $A$ - and  $B$ -voters,  $\tau_A$  and  $\tau_B$ , are inversely related to the winning margin,  $m$ .*

*Proof.* This is simply a consequence of the equilibrium equations, and the properties of the regret function,  $r$ .  $\square$

Note that the turnouts in the above lemma is the within-group turnout, not the total turnout of all voters. The lemma states that

if a change in parameter values causes the turnout in  $A$ -voters (or  $B$ -voters) to rise, it must also cause the winning margin to drop.

We now turn to comparative statics of turnouts with respect to various aspects of the election. First, turnouts are inversely related to the dominance of the majority, or positively related to the competitiveness of the election.<sup>12</sup>

**Theorem 3.** *Turnouts for  $A$ - and  $B$ -voters,  $\tau_A$  and  $\tau_B$ , are decreasing in  $\alpha$ . Furthermore, the total turnout,  $\tau = \alpha\tau_A + (1 - \alpha)\tau_B$ , is also decreasing in  $\alpha$ .*

*Proof.* We show that the equilibrium turnout,  $m$ , increases as  $\alpha$  increases. Then, by Lemma 2, both  $\tau_A$  and  $\tau_B$  are decreasing in  $\alpha$ .

As we have seen in the proof of Theorem 2, the right hand side of (9),  $\mu(\cdot)$ , is strictly decreasing in  $m$ . Note also the left hand side of (9) does not depend on  $\alpha$ . In addition,

$$\frac{\partial \mu}{\partial \alpha} = \tau_A + \tau_B > 0.$$

A direct application of the implicit function theorem gives us the desired conclusion. In fact,

$$\frac{dm}{d\alpha} = -\frac{\tau_A + \tau_B}{\mu'(m) - 1},$$

where the denominator is negative and the numerator positive. The negative sign in front of the expression implies the whole expression is positive.

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<sup>12</sup>One need use caution in interpreting this result. As Lemma 2 states, the *equilibrium* turnout is inversely related to the *equilibrium* winning margin, or the *realized* lopsidedness of the election. However, it is not immediately clear that the *ex ante* lopsidedness of the election will translate into *realized* lopsidedness. Theorem 3 shows that such translation does occur. Thus, it is appropriate to interpret the theorem as saying turnouts increase with the *perceived* lopsidedness of the election. See also Corollary 6.3 in Section 5.

We now turn to how the total turnout,  $\tau = \alpha\tau_A + (1-\alpha)\tau_B$ , varies with  $\alpha$ . We have

$$\frac{d\tau}{d\alpha} = \tau_A - \tau_B + \alpha \frac{d\tau_A}{d\alpha} + (1-\alpha) \frac{d\tau_B}{d\alpha}.$$

First, observe  $\tau_A < \tau_B$  in equilibrium by Lemma 1. Second, both  $d\tau_A/d\alpha$  and  $d\tau_B/d\alpha$  are negative. Thus, we may conclude

$$\frac{d\tau}{d\alpha} < 0. \quad \square$$

This theorem states that within-group turnouts must decrease as the winning side becomes more dominant, or the level of disagreement among all voters becomes lower. As the dominance of majority increases, the resulting winning margin at a given level of turnout for  $A$ -voters is higher. Thus, the equilibrium turnout has to decrease to restore the consistency between turnout and winning margin. In the meantime, the total turnout also decreases, as the turnouts for both groups are lower and more weight is put on the low-turnout group, the  $A$ -voters.

Our next result concerns the relationship between turnouts and the importance of the election.

**Theorem 4.** *Turnouts for  $A$ - and  $B$ -voters,  $\tau_A$  and  $\tau_B$ , are increasing in the importance of the election,  $\delta$ . In addition, the total turnout,  $\tau$ , is also increasing in  $\delta$ .*

*Proof.* We discuss  $\tau_A$  only, as the argument for  $\tau_B$  is similar. From (6), we have

$$\frac{d\tau_A}{d\delta} = f(\delta r(m)) \left( r(m) + \delta r'_+(m) \frac{dm}{d\delta} \right).$$

Consider again Equation (9). Its left hand side does not depend on  $\delta$ . But derivative of the right hand side with respect to  $\delta$  can be written

$$\frac{\partial \mu}{\partial \delta} = \alpha f(\delta r(m)) r(m) - (1-\alpha) f(\delta r(-m)) r(-m).$$

Applying the implicit function theorem to (9), we have

$$\frac{dm}{d\delta} = -\frac{\partial\mu/\partial\delta}{\mu'(m) - 1}.$$

Using (10), we have

$$\frac{d\tau_A}{d\delta} = \frac{\delta(1 - \alpha)f(\delta r(-m))[r(m)r'_-(-m) + r(-m)r'_+(m)] - r(m)}{\mu'(m) - 1}.$$

Assumption 1 implies the first term in the numerator is negative. Since  $\mu'(m) < 0$  as we have shown in the proof of Theorem 2, we conclude  $d\tau_A/d\delta > 0$ . Similarly,  $d\tau_B/d\delta > 0$ . Combining them, we have  $\tau = \alpha\tau_A + (1 - \alpha)\tau_B$  also increases in  $\delta$ .  $\square$

Note that the relationship between the equilibrium winning margin and the importance of the election is ambiguous. If a higher  $\delta$  always caused  $\mu(\cdot)$ , the “resulting winning margin,” to rise, then equilibrium winning margin would go up, and it would go down if a higher  $\delta$  had the opposite effect. Fixing a winning margin, an increase in the importance of election causes turnouts of both groups to increase. The effect on  $A$ -voters, the winning group, is weaker than on  $B$ -voters by the assumption of “winner regrets less.” However, since the pdf of the cost distribution is non-increasing, and since there are more  $A$ -voters than  $B$ -voters, the marginal effect of an increase in the importance of the election on the resulting margin is ambiguous.

Despite such ambiguity in the relationship between the equilibrium margin and the importance of the election, the previous theorem states that turnouts always increase with the importance of the election. The reason is that even if an increase in the importance of the election causes the equilibrium winning margin to go up, thereby having the *indirect* effect of causing turnouts to fall, such effect is offset by the *direct* turnout-increasing effect of an increase in the importance of the election.

We now turn to the investigation of the relationship between voting cost and equilibrium voter turnout. Here, our results are more

limited in scope. Intuitively, since voters base their decisions on the comparison between their voting cost and expected regret, if the cost of voting becomes higher, turnout should become lower. However, since lower turnouts may result in a closer election, which increases voters' anticipated regret, the eventual effect of an increase in the cost of voting is not clear-cut. Below, we provide sufficient conditions for turnouts to decrease as voting becomes more costly.

**Theorem 5.** *Suppose  $F$  and  $G$  are two distributions of the voting cost that satisfy Assumption 3, where  $F$  first order stochastically dominates  $G$ . If, in addition, either of the following conditions is satisfied, then turnouts for  $A$ -voters and  $B$ -voters are both lower under  $F$  than under  $G$ .*

1. *The margin of victory under  $F$  is higher than that under  $G$ .*
2. *The distribution function  $F$  is a rightward shift of  $G$ , that is,  $F(c) = G(c - \varepsilon)$  for all some  $\varepsilon > 0$  and all  $c$  in the support of  $F$ .*

*Proof.* 1. This is a direct implication of the property that the regret function is inversely related to the margin of victory and the assumption that  $F$  first order stochastically dominates  $G$ .

2. Denote the equilibrium margins of victory for candidate  $A$  under  $F$  and  $G$  respectively by  $m^*$  and  $m'$ , and similarly the equilibrium turnouts for  $J$ -voters ( $J = A, B$ ) by  $\tau_J^*$  and  $\tau_J'$ .

First, we argue that  $m^* < m'$ . By assumption, for all  $m$ ,

$$\alpha G(\delta r(m)) - (1 - \alpha)G(\delta r(-m)) > \alpha G(\delta r(m) - \varepsilon) - (1 - \alpha)G(\delta r(-m) - \varepsilon),$$

because (i)  $r(m) < r(-m)$  by Assumption 1; (ii)  $G$  satisfies Assumption 3; (iii)  $\alpha > \frac{1}{2}$ . But, the right hand side of the inequality is equal to  $\alpha F(\delta r(m)) - (1 - \alpha)F(\delta r(-m))$ . This

means that compared to  $G$ ,  $F$  shifts  $\mu$  downwards in Figure 1. As a result, the equilibrium winning margin must be lower under  $F$  than under  $G$ .

Second, we argue that  $\tau_A^* < \tau_A'$  and  $\tau_B^* < \tau_B'$ . Suppose instead either  $\tau_A^* \geq \tau_A'$  or  $\tau_B^* \geq \tau_B'$ . Note

$$\begin{aligned}\tau_A^* &= F(\delta r(m^*)) = G(\delta r(m^*) - \varepsilon), & \tau_A' &= G(\delta r(m')); \\ \tau_B^* &= F(\delta r(-m^*)) = G(\delta r(-m^*) - \varepsilon), & \tau_B' &= G(\delta r(-m')).\end{aligned}$$

Thus, either  $\delta r(m^*) - \varepsilon \geq \delta r(m')$  or  $\delta r(-m^*) - \varepsilon \geq \delta r(-m')$ . But given  $m^* < m'$ ,

$$(\delta r(m^*) - \varepsilon - \delta r(m')) - (\delta r(-m^*) - \varepsilon - \delta r(-m')) = \delta \int_{m'}^{m^*} r'(m) + r'(-m) dm > 0,$$

by the second part of Assumption 1. Thus,

$$\delta r(m^*) - \varepsilon - \delta r(m') \geq \max\{\delta r(-m^*) - \varepsilon - \delta r(-m'), 0\}.$$

Since  $r(-m') > r(m')$ ,  $\alpha > 1/2$ , and  $G$  satisfies Assumption 3, we have

$$\alpha G(\delta r(m^*) - \varepsilon) - (1 - \alpha)G(\delta r(-m^*) - \varepsilon) > \alpha G(\delta r(m')) - (1 - \alpha)G(\delta r(-m')),$$

or  $m^* > m'$ , contradicting our conclusion above. Thus,  $\tau_J^* < \tau_J'$  for  $J = A, B$ .  $\square$

We now present an example that illustrates the voting equilibrium in our model.

**Example 1.** Suppose the regret function takes the following form:

$$r(m) = \begin{cases} 1 - 2m, & \text{if } m \in [0, \frac{1}{2}], \\ 1 + m, & \text{if } m \in [-1, 0], \\ 0, & \text{otherwise.} \end{cases}$$

In addition, assume that voting cost is uniformly distributed on  $[0, C]$  with  $C \geq \delta$  (this is to ensure no corner solutions exist in which all



B-voters vote). Substituting these into the equilibrium equations, we obtain

$$\begin{aligned} m &= \frac{(2\alpha - 1)\delta}{C + (3\alpha - 1)\delta}, \\ \tau_A &= \delta \frac{1 + (1 - \alpha)\delta/C}{C + (3\alpha - 1)\delta}, \\ \tau_B &= \delta \frac{1 + \alpha\delta/C}{C + (3\alpha - 1)\delta}. \end{aligned}$$

The equilibrium margin of victory is decreasing in  $C$  and increasing in  $\alpha$ . Both turnouts increase with  $\delta$ , and increase as  $\alpha$  approaches  $\frac{1}{2}$ , which confirms Theorems 3 and 4. Note distributions with higher  $C$  FOSD distributions with lower  $C$ . The turnouts are decreasing with  $C$ , which means higher voting costs induces lower turnouts.

## 5 Uncertainty about the Electorate's Political Preferences

In the previous sections, we have introduced an essentially deterministic model. In equilibrium, a voter correctly anticipates the outcome of the election and the exact value of his regret if he fails to vote. We believe the model captures the idea that voter participation is driven by anticipated regret from not voting. In reality, of course, voters are uncertain about the election outcome. To encompass this in our model, we may allow  $\alpha$  to be uncertain to each voter, while maintaining our assumptions about the regret function.

To be specific, let  $G$  be the distribution of  $\alpha$  and  $g$  its corresponding density function.

**Assumption 4.** The density function  $g$  is symmetric around  $\frac{1}{2}$ , non-decreasing in  $[0, \frac{1}{2}]$ , and nonincreasing in  $[\frac{1}{2}, 1]$ .

Note that this implies that  $G$  second order stochastically dominates the uniform distribution ( $G$  can be the uniform distribution

itself). We may write the equilibrium equations as

$$\begin{aligned}\tau_A &= F(c_A), \\ \tau_B &= F(c_B), \\ c_A &= \delta \int_0^1 r(\alpha\tau_A - (1-\alpha)\tau_B)G(d\alpha), \\ c_B &= \delta \int_0^1 r((1-\alpha)\tau_B - \alpha\tau_A)G(d\alpha).\end{aligned}$$

Our next theorem demonstrates that a unique positive-turnout equilibrium exists in the voting game with uncertainty about  $\alpha$ .

**Theorem 6.** *Suppose Assumptions 1 and 4 hold. Then, in the voting game with uncertainty, there exists a unique equilibrium with positive turnout, in which  $\tau_A = \tau_b = \tau^*$ , where  $\tau^*$  satisfies*

$$F^{-1}(\tau) = \delta \int_0^1 r((1-2\alpha)\tau)G(d\alpha). \quad (11)$$

*Proof.* First, we show that in equilibrium,  $\tau_A = \tau_B$ . Suppose instead  $\tau_A > \tau_B$ . With a change of variables in the equilibrium characterization above, we have

$$\begin{aligned}c_A - c_B &= \frac{\delta}{\tau_A + \tau_B} \int_{-\tau_B}^{\tau_A} [r(m) - r(-m)] g\left(\frac{\tau_B + m}{\tau_A + \tau_B}\right) dm \\ &= \frac{\delta}{\tau_A + \tau_B} \int_{\tau_B}^{\tau_A} [r(m) - r(-m)] g\left(\frac{\tau_B + m}{\tau_A + \tau_B}\right) dm \\ &\quad + \frac{\delta}{\tau_A + \tau_B} \int_{-\tau_B}^{\tau_B} [r(m) - r(-m)] g\left(\frac{\tau_B + m}{\tau_A + \tau_B}\right) dm.\end{aligned}$$

The first term on the right hand side is negative by our “winner regrets less” assumption (Assumption 1). The second term can be rewritten as

$$\frac{\delta}{\tau_A + \tau_B} \int_0^{\tau_B} [r(m) - r(-m)] \left[ g\left(\frac{\tau_B + m}{\tau_A + \tau_B}\right) - g\left(\frac{\tau_B - m}{\tau_A + \tau_B}\right) \right] dm.$$

By Assumption 4 and the assumption  $\tau_A > \tau_B$ , we have  $g\left(\frac{\tau_B + m}{\tau_A + \tau_B}\right) - g\left(\frac{\tau_B - m}{\tau_A + \tau_B}\right) > 0$  for all  $m \in (0, \tau_B]$ . Therefore, the second term is also

negative by Assumption 1. Hence  $c_A < c_B$ , which contradicts our assumption that  $\tau_A > \tau_B$ . Thus, in equilibrium,  $\tau_A = \tau_B$ .

Now, we demonstrate that a unique equilibrium exists. Let  $\tau$  denote the turnout level for both groups. The equilibrium condition becomes (11). The left hand side is strictly increasing and the right hand side strictly decreasing in  $\tau$ . Furthermore, when  $\tau = 0$ ,  $F^{-1}(\tau) = 0$ , while the right hand side is strictly positive; when  $\tau \rightarrow 1$ ,  $F^{-1}(\tau)$  approaches  $\infty$  if the voting cost  $c$  has an unbounded support,<sup>13</sup> while the right hand remains bounded. Therefore, there exists a unique  $\tau^*$  that satisfies the equilibrium condition.  $\square$

The following two corollaries consider how voting cost and the importance of the election affect turnout. The higher the voting cost, the lower the turnout; the more important the election is, the higher the turnout. The former is different from our result in the deterministic case, mainly because here the two groups are ex ante symmetric while it is not the case in the deterministic case. The latter is similar to that in the deterministic case.

**Corollary 6.1.** *Let  $\tilde{F}$  and  $F$  be two distributions of voting cost,  $c$ , such that  $\tilde{F}$  first order stochastically dominates  $F$ . Then, the equilibrium voter turnout is lower under  $\tilde{F}$  than that under  $F$ .*

**Corollary 6.2.** *The equilibrium voter turnout is increasing in  $\delta$ , the importance of the election.*

The proofs are straightforward and omitted here. They use similar arguments to those in the proof of Corollary 6.3.

Our next result concerns how voter turnouts are affected by voters' uncertainty about  $\alpha$ .

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<sup>13</sup>If  $c$  has a bounded support, we need to make the assumption that the upper bound is large enough.

**Corollary 6.3.** *Let  $G$  and  $\tilde{G}$  be two distributions of  $\alpha$  that satisfy Assumption 4 and let  $\tilde{G}$  second order stochastically dominate  $G$ . Then, the equilibrium voter turnout under  $\tilde{G}$  is higher than that under  $G$ .*

*Proof.* Note that the equilibrium turnout is the  $\tau$  at which the left hand and right hand sides of (11) intersect. Also, the left hand side is increasing while the right hand side is decreasing in  $\tau$ . Thus, it suffices to show that under  $\tilde{G}$ , the right hand side of (11) is shifted up from that under  $G$ . Using integration by parts, we have

$$\begin{aligned} & \int_0^1 r((1-2\alpha)\tau)\tilde{G}(d\alpha) - \int_0^1 r((1-2\alpha)\tau)G(d\alpha) \\ &= r((2\alpha-1)\tau)[\tilde{G}(\alpha) - G(\alpha)] \Big|_{\alpha=0}^1 - \int_0^1 [\tilde{G}(\alpha) - G(\alpha)] dr((2\alpha-1)\tau)d\alpha. \end{aligned}$$

The first term is equal to zero. The second term can be rewritten

$$- \int_0^{\frac{1}{2}} [\tilde{G}(\alpha) - G(\alpha)] 2\tau r'((2\alpha-1)\tau) d\alpha - \int_{\frac{1}{2}}^1 [\tilde{G}(\alpha) - G(\alpha)] 2\tau r'((2\alpha-1)\tau) d\alpha.$$

Using Assumption 4 and the assumption that  $\tilde{G}$  second order stochastically dominates  $G$ , we have

$$\begin{aligned} \tilde{G}(\alpha) - G(\alpha) &< 0 \text{ when } \alpha \in (0, \frac{1}{2}); \\ \tilde{G}(\alpha) - G(\alpha) &> 0 \text{ when } \alpha \in (\frac{1}{2}, 1). \end{aligned}$$

Combining these with the assumption that  $r'(m)$  is positive for  $m < 0$  and negative for  $m > 0$ , we conclude

$$\int_0^1 r((1-2\alpha)\tau)\tilde{G}(d\alpha) - \int_0^1 r((1-2\alpha)\tau)G(d\alpha) > 0.$$

Hence, the equilibrium turnout under  $\tilde{G}$  is higher than that under  $G$ .  $\square$

The above theorem shows that if the election is anticipated to be close, then voter turnout is higher, again echoing our result in the

deterministic case. The reason is that as elections become close, a voter is more likely to experience a high regret if he does not vote.<sup>14</sup>

The assumption that a voter experiences regret even if his favorite candidate wins may appear counterintuitive. Our answer to this critique is two-fold. First, as mentioned in Section 3, although we use regret as justification for voter participation, we do not exclude the possibility of “warm glow” utility from voting, which is independent of the (anticipated) election outcome. Second, as the following numerical example demonstrates, when we allow uncertainty about  $\alpha$ , it is not necessary to require voters on the winning side to have positive regret.

**Example 2.** Consider the regret function

$$r(m) = \begin{cases} 1 + m, & \text{if } m \in [-1, 0], \\ 0, & \text{otherwise.} \end{cases}$$

Again, the cost distribution is uniform on  $[0, C]$ , where  $C \geq \frac{\delta}{2}$ . In addition, we assume that it is common knowledge that  $\alpha$  is uniformly distributed on  $[0, 1]$ . Thus,

$$\begin{aligned} \tau_A &= \frac{c_A}{C}, \\ \tau_B &= \frac{c_B}{C}, \\ c_A &= \delta \int_{\alpha\tau_A - (1-\alpha)\tau_B \leq 0} 1 + (\alpha\tau_A - (1-\alpha)\tau_B) d\alpha, \\ c_B &= \delta \int_{\alpha\tau_A - (1-\alpha)\tau_B \geq 0} 1 + ((1-\alpha)\tau_B - \alpha\tau_A) d\alpha. \end{aligned}$$

Again, we can show that in equilibrium  $\tau_A = \tau_B$  by deriving a contradiction from supposing they are not equal. Solving the equations

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<sup>14</sup>A similar result is obtained by Taylor and Yildirim (2005). They show that when voters are strategic, giving them more information about the composition of the voting population increases turnout. But, it also increases the possibility that the election turns out in favor of the minority candidate.

gives us

$$\tau_A = \tau_B = \frac{2\delta}{4C + \delta}.$$

Clearly, the turnout is increasing in the importance of the election  $\delta$ . In addition, it is decreasing in the upper bound of the cost,  $C$ , which implies that in this case higher voting cost causes lower turnout.

## 6 The Concept of Regret

In this section, we elaborate on the concept of regret used in our model, and compare our use with its use in the literature.

Regret is a widely observed psychological phenomenon. According to Landman (1993),

Regret is a more or less painful cognitive and emotional state of feeling sorry for misfortunes, limitations, losses, transgressions, shortcomings, or mistakes. It is an experience of felt-reason or reasoned- emotion. The regretted matters may be sins of commission as well as sins of omission; they may range from the voluntary to the uncontrollable and accidental; they may be actually executed deeds or entirely mental ones committed by oneself or by another person or group; they may be moral or legal transgressions or morally and legally neutral. . . .(p. 36)

It is commonly known that regret tends to have an unpleasant emotional effect on an individual's mind. Therefore, it is natural to expect that while making decisions, people take the anticipated regret into account, at least, in situations where they are frequently involved.

Some economic and decision theorists have emphasized the role of anticipated regret in decision making. Loomes and Sugden (1982) and Bell (1982) propose decision regret as explanations of paradoxical

behavior that cannot be accommodated by expected utility theory. The concept of regret in these models is very narrowly defined. For example, Bell (1982) defines it as “the difference in value between the actual assets received and the highest level of assets produced by other alternatives.” As a result, the application of their theory to economic models is quite limited.

In psychological research, by contrast, regret has increasingly become recognized as an important factor in decision making.<sup>15</sup> One of the first such studies is by Kahneman and Tversky (1982). Subsequent studies have revealed many interesting characteristics of regret. For example, people regret less about choices with justifications (Inman and Zeelenberg (2002)); they regret more about behavior inconsistent with their intentions (Pieters and Zeelenberg (2005)); they regret more about inactions than actions in the long run and vice versa in the short run (Gilovich and Medvec (1995)); and so on. Connolly and Zeelenberg (2002) propose Decision Justification Theory to explain many empirical phenomena related to regret and decision making. They break down regret into bad-outcome regret and self-blame regret. A person suffers the former if as a result of his action or inaction a bad outcome occurs, and suffers the latter if his action was unjustified or unwise regardless of what outcome occurs.

In our model, we introduce regret as an enhancement of Riker and Ordeshook’s civic-duty model. We believe that the “warm glow” utility received by a voter as a result of fulfilling his civic duty should be independent of the election outcome. In addition to this type of utility, we believe voters’ psychological involvement in the success or failure of their favorite candidates or alternatives is a significant factor

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<sup>15</sup>The psychologist Neal Roese at University of Illinois maintains a web site “Counterfactual Research News,” where an extended list of references can be found. The URL is <http://www.psych.uiuc.edu/~roese/cf/cfbib.htm>. In his book, Roese (2005) describes the relation between regret and decision making to the general public.

in driving voter participation decisions. Abstaining voters suffer regret because not voting is deemed ethically wrong in democratic societies and disappoints peers from the same partisan group.<sup>16</sup> We make the assumption that an abstaining voter’s regret on the winning side is less than that on the losing side. In addition, such regret increases as the election result becomes close. We believe these assumptions are plausible and intuitively appealing. Though empirical evidence that supports them is not widely available, they can be readily tested with surveys of potential voters. As we discuss below, the experimental study conducted by Pieters and Zeelenberg (2005) does provide evidence in support of the existence of voter regret.

In the following discussion, we want to address the concern that voters in our model experience regret even though they know their votes do not affect the outcome. It is our contention that people frequently regret their decisions because they deem them ethically and morally questionable and not because outcomes would have been different if they had acted differently. First, this fits into the broad definition by Landman (1993) above. In particular, regret can be caused by feeling sorry for “transgressions, shortcomings, or mistakes,” none of which automatically implies real consequences. Second, examples of people feeling regret abound where their actions are not decisive in the outcomes. Many lawmakers (for example, Senator Robert Byrd, Congressmen Bob Barr and Don Young) said they regretted their votes for the Patriot Act even though the vote of each of them would not have altered the outcome of the vote since the results were so lopsided. According to an ABC News report (January 5, 2007), of the 77 senators who voted to give President George W. Bush war powers, 34 (including John Kerry, John Edwards, Gordon Smith) expressed regret for voting that way. As widely reported in the media, many voters say they regret their vote or abstention in the 2000 election, even though

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<sup>16</sup>Surveys consistently show that roughly 90 percent of Americans believe they should vote even if their preferred candidate is certain to lose (Brody (1978)).



each of them would not have altered the outcome of the election.<sup>17</sup> Third, Pieters and Zeelenberg (2005) provide some further evidence with an experiment in a real election. They conducted a study with the Dutch national elections in which they asked voters to express their regret for their various actions (voting, abstention, voting for a particular party, etc.). Again, in such elections, each individual's vote does not affect the outcome of the elections. Nevertheless, their results showed that voters regretted their intention-action inconsistencies, like intending to vote but failing to do so, or intending to vote for  $x$  but voting for  $y$  instead. Though intention-action inconsistencies were their focus, their results also showed that voters who did not vote experience higher regret than those who did: those who intended to vote but failed to do so experienced the strongest regret among all groups; those who intended not to vote and did not vote (therefore does not commit intention-action inconsistency) nevertheless experienced higher regret than those who intended to vote for  $x$  and voted for  $x$  in the election (see Table 1, p. 22 of their paper).

## 7 Concluding Remarks and Further Research

In this paper, we have introduced a two-candidate model of electoral competition in which potential voters are ethical in that they experience regret after the election if they did not vote. The regret is inversely related to the margin of victory. Furthermore, voters on the winning side experience less regret than those on the losing side. We show that there is a unique equilibrium with strictly positive turnout. Furthermore, voter turnout is positively related to the importance of the election, and negatively related to the perceived lopsidedness of

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<sup>17</sup>See, in addition to the poll we cited at the beginning of our paper, the book by Patterson (2002) (p. 3).

the competition. These are true both when voters are sure about the proportion of voters that favor each candidate and when they are not.

We view our current research as laying a tractable framework in which many interesting issues can be studied. In our model, the positions of candidates and voters are exogenously given, so are the proportions of voters who would vote for either candidate. Therefore, we see many directions in which this model can be extended. First, candidates need not have a predetermined position. They want to position themselves to appeal to a larger proportion of potential voters. Also, the potential voter population need not be fixed. Election campaigns dissipate information to more voters, and much effort is devoted to attracting one's own voters or discouraging opponents'. Recent presidential and congressional campaigns in the United States have seen the rise of negative tactics. An extended version of our model may be suitable to analyze the effect of such tactics on election outcomes. To be sure, many of the same issues have been studied using the pivotal-voter model, and many insights have been derived from these studies. However, as the pivotal-voter model fails to account for significantly positive turnout in large elections, arguably a crucial feature of democratic elections, a model that does capture this phenomenon is valuable at least as a device with which to verify the validity of those theoretical findings.

## References

- ALDRICH, J. H. (1993): “Rational Choice and Turnout,” *American Journal of Political Science*, 37, 246–278.
- BECK, N. L. (1975): “The Paradox of Minimax Regret,” *American Political Science Review*, 69, 918.
- BELL, D. E. (1982): “Regret in Decision Making under Uncertainty,” *Operations Research*, 30(5), 961–981.
- BLAIS, A. (2000): *To Vote or Not to Vote: The Merits and Limits of Rational Choice Theory*. University of Pittsburgh Press.
- BRODY, R. (1978): “The puzzle of political participation in America,” in *The new American political system*, ed. by A. King. Washington DC: American Enterprise Institute.
- CALLANDER, S. (2007, forthcoming): “Bandwagons and Momentum in Sequential Voting,” *Review of Economic Studies*.
- COATE, S., AND M. CONLIN (2004): “A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence,” *American Economic Review*, 94(5), 1476–1504.
- COATE, S., M. CONLIN, AND A. MORO (2004): “The Performance of the Pivotal-Voter Model in Small-Scale Elections: Evidence from Texas Liquor Referenda,” *mimeo*.
- CONLEY, J. P., A. TOOSI, AND M. H. WOODERS (2001): “Evolution and Voting: How Nature Makes Us Public Spirited,” *mimeo*.
- CONNOLLY, T., AND M. ZEELLENBERG (2002): “Regret in Decision Making,” *Current Directions in Psychological Science*, 11(6), 212–216.

- DEGAN, A. (2006): “Policy Positions, Information Acquisition, and Turnout,” *Scandinavian Journal of Economics*, 108(4), 669–682.
- DEGAN, A., AND A. MERLO (2007): “A Structural Model of Turnout and Voting in Multiple Elections,” *mimeo*.
- DHILLON, A., AND S. PERALTA (2002): “Economic Theories of Voter Turnout,” *Economic Journal*, 112, F332–352.
- DIERMEIER, D., AND J. A. V. MIEGHEM (2000): “Coordination and Turnout in Large Elections,” *mimeo*.
- DOWNES, A. (1957): *An Economic Theory of Democracy*. Harper, New York.
- FEDDERSEN, T. J. (2004): “Rational Choice Theory and the Paradox of Not Voting,” *Journal of Economic Perspectives*, 18(1), 99–112.
- FEDDERSEN, T. J., AND W. PESENDORFER (1996): “The Swing Voter’s Curse,” *American Economic Review*, 86(3), 408–424.
- FEDDERSEN, T. J., AND A. SANDRONI (2006a): “The Calculus of Ethical Voting,” *International Journal of Game Theory*, 35(1), 1–25.
- (2006b): “Ethical Voters and Costly Information Acquisition,” *Quarterly Journal of Political Science*, (1), 287–311.
- (2006c): “A Theory of Participation in Elections,” *American Economic Review*, 96(4), 1271–1282.
- FEREJOHN, J., AND M. P. FIORINA (1974): “The Paradox of Not Voting: A Decision-Theoretic Analysis,” *American Political Science Review*, 68, 525–536.
- FRANK, R. H. (1988): *Passions within Reason*. W. W. Norton and Company.

- GILOVICH, T. D., AND V. H. MEDVEC (1995): “The Experience of Regret: What, When, and Why,” *Psychological Review*, 102, 379–395.
- INMAN, J. J., AND M. ZEELLENBERG (2002): “Regretting Repeat versus Switch Decisions: The Attenuation Role of Decision Justifiability,” *Journal of Consumer Research*, 29, 116–128.
- KAHNEMAN, D., AND A. TVERSKY (1982): “The Simulation Heuristic,” in *Judgment under Uncertainty: Heuristics and Biases*, ed. by D. Kahneman, P. Slovic, and A. Tversky, pp. 201–208. Cambridge University Press.
- LANDMAN, J. (1993): *Regret: Persistence of the possible*. New York: Oxford University Press.
- LEDYARD, J. O. (1982): “The Paradox of Voting and Candidate Competition: a General Equilibrium Analysis,” in *Essays in Contemporary Fields of Economics*, ed. by G. Horwich, and J. Quirck. Purdue University Press: West Lafayette.
- LEVINE, D. K., AND T. R. PALFREY (2006, forthcoming): “The Paradox of Voter Participation: a Laboratory Study,” *American Political Science Review*.
- LLAVADOR, H. (2006): “Voting with Preferences over Margins of Victory,” *mimeo*, *Universitat Pompeu Fabra*.
- LOOMES, G., AND R. SUGDEN (1982): “Regret Theory: an Alternative Theory of Rational Choice under Uncertainty,” *The Economic Journal*, 92(368), 805–824.
- LUCE, R. D., AND H. RAIFFA (1957): *Games and Decisions*. Wiley, New York.
- MATSUSAKA, J. G. (1995): “Explaining voter turnout patterns: An information theory,” *Public Choice*, 84, 91–117.

- MAYER, L. S., AND I. J. GOOD (1975): "Is Minimax Regret Applicable to Voting Decisions?," *American Political Science Review*, 69, 916–917.
- MERLO, A. (2006): "Whither Political Economy? Theories, Facts, and Issues," in *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. K. Newey, and T. Persson. Cambridge: Cambridge University Press.
- MORTON, R. B. (1991): "Groups in Rational Turnout Models," *American Journal of Political Sciences*, 35(3), 758–776.
- MYERSON, R. (1998): "Population Uncertainty and Poisson Games," *International Journal of Game Theory*, 27, 375–392.
- PALFREY, T. R., AND H. ROSENTHAL (1983): "A Strategic Calculus of Voting," *Public Choice*, 41, 7–53.
- (1985): "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 79, 62–78.
- PATTERSON, T. E. (2002): *The Vanishing Voter: Public Involvement in an Age of Uncertainty*. Knopf.
- PIETERS, R., AND M. ZEELLENBERG (2005): "On Bad Decisions and Deciding Badly: When Intention-Behavior Inconsistency Is Regrettable," *Organizational Behavior and Human Decision Processes*, 97, 18–30.
- RIKER, W. H., AND P. C. ORDESHOOK (1968): "A Theory of the Calculus of Voting," *American Political Science Review*, 62, 25–42.
- ROESE, N. (2005): *If Only: How to Turn Regret Into Opportunity*. Broadway.

- SHACHAR, R., AND B. NALEBUFF (1999): “Follow the Leader: Theory and Evidence on Political Participation,” *American Economic Review*, 89(3), 525–547.
- SIEG, G., AND C. SCHULZ (1995): “Evolutionary Dynamics in the Voting Game,” *Public Choice*, 85, 157–172.
- TAYLOR, C. R., AND H. YILDIRIM (2005): “Public Information and Electoral Bias,” *mimeo*, *Duke University*.
- TULLOCK, G. (1968): *Toward a Mathematics of Politics*. Ann Arbor: University of Michigan Press.
- (1975): “The Paradox of Not Voting for Oneself,” *American Political Science Review*, 69, 919.
- UHLANER, C. J. (1989): “Rational Turnout: The Neglected Role of Groups,” *American Journal of Political Science*, 33(2), 390–422.