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Abstract

We will show, using a simple two-periods overlapping generations (OLG) model with bequest motive in which goods are produced solely by labor in a monopolistically competitive industry, that a continuous budget deficit is necessary to maintain full-employment under economic growth driven by technological progress. Since the budget deficit to maintain full-employment must be continuous, it should be financed by seigniorage not by public debt. Budget deficit is necessary under growth because of deficiency of the consumptions of the older generation. This budget deficit is not debt and does not need to be redeemed. If the budget deficit is excessive, inflation will be triggered. About this excessive budget deficit that has caused inflation, only the excess portion should be reduced, and there is no need to make up for past excesses by creating surpluses or reducing deficits. We also show that insufficient government expenditure causes involuntary unemployment.

Keywords: overlapping generations model, budget deficit, full-employment, growth, inflation

JEL Classification No.: E12, E24.

1 Introduction

In this paper we will show, using a simple two-periods overlapping generations (OLG) model with bequest motive in which goods are produced solely by labor in a monopolistically competitive industry, that a continuous budget deficit is necessary to maintain full-employment under economic growth driven by technological progress. Since the budget deficit to maintain full-employment must be continuous, it should be financed by seigniorage not by public debt. Budget deficit is necessary under growth because of deficiency of the consumptions of the older generation. This budget deficit is not debt and does not need to be redeemed.

If the budget deficit is excessive, inflation will be triggered. About this excessive budget deficit that has caused inflation, only the excess portion should be reduced. There is no need to make up for past excesses by creating surpluses or reducing deficits. We also show that insufficient government expenditure causes involuntary unemployment. This paper is one of the attempts to give a theoretical basis to the so-called functional finance theory by Lerner (1943) and Lerner (1944). This paper also present a theoretical basis to MMT (Modern monetary theory, Mitchell et al. (2019)). In particular, this paper provides a rationale for the following claims (Kelton (2020)). We refer to the summary of Kelton's book by Hogan (2021). In fact, Hogan argues that Kelton is wrong, but he summarizes Kelton's argument to the point.

1. The treasury creates new money.

   The money supply equals the savings. Thus, an increase in the money supply equals an increase in the savings. As expressed in the equation (5) in Section 4.2, an increase in the savings equals the budget deficit. The rate of an increase in the savings, which equals the rate of an increase in the money supply, equals the rate of economic growth, and therefore the budget deficit and an increase in the money supply does not cause inflation.

2. Inflation is caused by federal government deficit spending, not by Fed policy..

   As we will show in Section 4.3, if the actual budget deficit is larger than the budget deficit that is necessary and sufficient to maintain full-employment under economic growth, the prices of the goods will rise.

3. Federal government spending is not related to taxes or borrowing.

   As summarized above, sustained budget deficits are necessary to maintain full employment under economic growth, and these budget deficits make it possible to maintain full employment. It is impossible to maintain full employment in a growing economy with a balanced budget. Therefore, even if the budget deficit to maintain full employment is financed by the national debt, it does not need to be repaid or redeemed, and should not be repaid or redeemed. Future budget surpluses need not and should not make up the deficit for growth.

2 The model

We consider a two-periods (1: younger or working, and 2: older or retired) overlapping generations (OLG) model under monopolistic competition. Our model is according to Otaki (2007), Otaki (2009) and Otaki (2015), and a generalization of Tanaka (2020) in which perfect competition is assumed. The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0, 1]$. Good z is monopolistically produced by firm z with increasing or decreasing returns to scale technology. The technology progresses at the rate $\gamma - 1 > 0$.

2. During Period 1, each consumer supplies 1 unit of labor, consumes the goods and saves money for his consumption and bequest in Period 2. He is employed or not employed.

3. During Period 2, each consumer consumes the goods using his savings carried over from his Period 1 earnings, and he leaves a bequest to the next generation. We assume that the bequest is equally distributed to each consumer. A consumer of one generation receives the bequest at the end of his Period 1, and he leaves the bequest at the end of his Period 2.

4. Each consumer determines his consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1 depending on the situation that he is employed or not employed.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1, 2$.
B^e: Bequest by an employed consumer.
C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1, 2$.
B^u: Bequest by an unemployed consumer.
$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1, 2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1, 2$.
P_i: the price of consumption basket in Period i, $i = 1, 2$.
$p_i(z)$: the price of good z in Period i, $i = 1, 2$.
$\rho = P_2/P_1$: (expected) inflation rate (plus one).
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
y: labor productivity, which increases by technological progress, also it is increasing or decreasing with respect to the total employment, Ll.

We assume that the population L_f is constant.

We denote a bequest to each consumer by consumers of the previous generation by $\tilde{B}$. We have

$$\tilde{B} = \frac{1}{L_f}\left[L\tilde{B}^e + (L_f - L)\tilde{B}^u\right],\ L_f\tilde{B} = L\tilde{B}^e + (L_f - L)\tilde{B}^u.$$

$\tilde{B}^e$ and $\tilde{B}^u$ are, respectively, bequests by an employed consumer and an unemployed consumer of the previous generation.

3 Behaviors of agents

3.1 Consumers' behavior

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods.

2. Then, they maximize their consumption baskets given the expenditure in each period.

The utility function of employed consumers of one generation over two periods is written as

$$u(C_1^e, C_2^e, \frac{B^e}{P_2}) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, \frac{B^u}{P_2}).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1, 2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1, 2.$$

σ is the elasticity of substitution among the goods. $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}}, i = 1, 2.$$

The budget constraint for an employed consumer is

$$P_1C_1^e + P_2C_2^e + B^e = Wl + \Pi + \tilde{B}.$$

The budget constraint for an unemployed consumer is

$$P_1C_1^u + P_2C_2^u + B^u = \Pi + \tilde{B}.$$

Let

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + P_2C_2^e + B^e}, \beta = \frac{P_2C_2^e}{P_1C_1^e + P_2C_2^e + B^e}, \ 1 - \alpha - \beta = \frac{B^e}{P_1C_1^e + P_2C_2^e + B^e}.$$

$0 < \alpha < 1,\ 0 < \beta < 1$. Since the utility functions $u(C_1^e, C_2^e, \frac{B^e}{P_2})$ and $u(C_1^u, C_2^u, \frac{B^u}{P_2})$are homothetic, α and β are determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + P_2C_2^e + B^e} = \frac{P_1C_1^u}{P_1C_1^u + P_2C_2^u + B^u},$$

and

$$\beta = \frac{P_2C_2^e}{P_1C_1^e + P_2C_2^e + B^e} = \frac{P_2C_2^u}{P_1C_1^u + P_2C_2^u + B^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets and bequests.

$$C_1^e = \alpha\frac{Wl + \Pi + \tilde{B}}{P_1},\ C_2^e = \beta\frac{Wl + \Pi + \tilde{B}}{P_2},\ \frac{B_2^e}{P_2} = (1 - \alpha - \beta)\frac{Wl + \Pi + \tilde{B}}{P_2},$$

$$C_1^u = \alpha\frac{\Pi + \tilde{B}}{P_1},\ C_2^u = \beta\frac{\Pi + \tilde{B}}{P_2},\ \frac{B^u}{P_2} = (1 - \alpha - \beta)\frac{\Pi + \tilde{B}}{P_2}.$$

Solving maximization problems in Step 2 (details of calculation are available upon request), the following demand functions of employed and unemployed consumers are derived .

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi + \tilde{B})}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(Wl + \Pi + \tilde{B})}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(\Pi + \tilde{B})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(\Pi + \tilde{B})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha\frac{Wl + \Pi + \tilde{B}}{P_1}, \beta\frac{Wl + \Pi + \tilde{B}}{P_2}, (1 - \alpha - \beta)\frac{Wl + \Pi + \tilde{B}}{P_2}\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi+\tilde{B}}{P_1}, \beta\frac{\Pi+\tilde{B}}{P_2}, (1-\alpha-\beta)\frac{Wl+\Pi+\tilde{B}}{P_2}\right).$$

Let

$$\omega = \frac{W}{P_1}.$$

This is the real wage rate. Then, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi+\tilde{B}}{P_1}, \rho\right) - \Gamma(l),$$

$$V^u = \varphi\left(\frac{\Pi+\tilde{B}}{P_1}, \rho\right),$$

Denote

$$I = \omega l + \frac{\Pi+\tilde{B}}{P_1}.$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{1}$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial C_1^e} + \beta\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > (<) 0$, the labor supply is increasing (decreasing) with respect to the real wage rate ω. But, we assume that the real wage rate does not have a significant effect on the individual labor supply. l may depend on employment L in some way. However, we assume that Ll is increasing with respect to L.

3.2 Firms' behavior

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(WLl + L_f\Pi + L_f\tilde{B})}{P_1} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(WLl + L_f\Pi + L_f\tilde{B})}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{\beta(WLl + L_f\Pi + L_f\tilde{B})}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{\beta(\overline{W}\ \overline{Ll} + L_f\overline{\Pi} + L_f\overline{\tilde{B}})}{P_2}.$$

where $\overline{W}$, $\overline{\Pi}$, $\overline{L}$, $\overline{l}$ and $\overline{\tilde{B}}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the bequest from the previous generation, respectively, during the previous period.

Let

$$M = \beta\left(\overline{W}\ \overline{Ll} + L_f\overline{\Pi} + L_f\overline{\tilde{B}}\right).$$

This is the total consumption of the older generation consumers. It is the planned consumption that is determined in their Period 1. Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. The total savings is

$$(1-\alpha)\left(\overline{W}\ \overline{Ll} + L_f\overline{\Pi} + L_f\overline{\tilde{B}}\right).$$

Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{Y}{P_1}, \tag{2}$$

where Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi + L_f\tilde{B}) + G + M.$$

G is the government expenditure. The government determines its demand for good z, $g(z)$, to maximize the following index.

$$\left(\int_0^1 g(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}},$$

subject to the constraint:

$$\int_0^1 p_1(z)g(z)dz = G.$$

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are

$$\int_0^1 Ldz = L,\ \int_0^1 Lldz = Ll.$$

The output of firm z is Lly. By increasing or decreasing returns to scale y is a function of Ll. In the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial Ll} = y + Lly'(Ll).$$

In a case of constant returns to scale

$$\frac{\partial d(z)}{\partial Ll} = y.$$

From (2)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)[y + Lly'(Ll)]}{\sigma d(z)} = -\frac{p_1(z)}{\sigma Ll}\left[1 + \frac{Lly'(Ll)}{y}\right].$$

Define the elasticity of the labor productivity by

$$\zeta = \frac{Lly'(Ll)}{y}.$$

Then,

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)(1+\zeta)}{\sigma Ll}$$

We assume that ζ is constant and $1 + \zeta > 0$. For increasing (or decreasing) returns to scale technology $\zeta > 0$ (or $(\zeta < 0)$).

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial Ll} = \left[p_1(z)y - Lly\frac{p_1(z)}{\sigma Ll}\right](1+\zeta) - W = \left[p_1(z)y - \frac{p_1(z)y}{\sigma}\right](1+\zeta) - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{\left(1 - \frac{1}{\sigma}\right)(1+\zeta)y}W.$$

Let $\mu = 1/\sigma$. Then,

$$p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)(1+\zeta)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y}W.$$

4 Budget deficit to maintain full-employment under economic growth and inflation by excessive budget deficit

4.1 Market equilibrium

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1 Lly.$$

The (nominal) aggregate demand is

$$\alpha(WL + L_f\Pi + L_f\tilde{B}) + G + M = \alpha P_1 Lly + \alpha L_f\tilde{B} + G + M.$$

Since they are equal,

$$P_1 Lly = \alpha P_1 Lly + \alpha L_f\tilde{B} + G + M. \tag{3}$$

In real terms

$$Lly = \frac{G + M + \alpha L_f\tilde{B}}{(1-\alpha)P_1}.$$

The equilibrium value of Ll cannot be larger than $L_f l(L_f)$. $l(L_f)$ is the labor supply when full-employment is achieved. However, it may be strictly smaller than $L_f l(L_f)$. Then, we have $L < L_f$ and involuntary unemployment exists. If the government collects a tax T from the younger generation consumers, (3) is rewritten as

$$P_1 Lly = \alpha(P_1 Lly + L_f\tilde{B} - T) + G + M.$$

4.2 Budget deficit to maintain full-employment

Suppose that up to Period t full-employment has been achieved under constant prices. Then, the following equation holds.

$$P_1^t L_f l(L_f)y = \alpha(P_1^t L_f l(L_f)y + L_f\tilde{B}^t - T^t) + G^t + M^t. \tag{4}$$

Superscript t represents the values in Period t. The savings of the younger generation consumers are

$$M^{t+1} + L_f\tilde{B}^{t+1} = (1-\alpha)(P_1^t L_f l(L_f)y + L_f\tilde{B}^t - T^t) = G^t + L_f\tilde{B}^t - T^t + M^t. \tag{5}$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$M^{t+1} = \beta(P_1^t L_f l(L_f)y + L_f\tilde{B}^t - T^t) = \frac{\beta}{1-\alpha}[G^t + L_f\tilde{B}^t - T^t + M^t],$$

and

$$L_f\tilde{B}^{t+1} = (1-\alpha-\beta)(P_1^t L_f l(L_f)y + L_f\tilde{B}^t - T^t) = \frac{1-\alpha-\beta}{1-\alpha}[G^t + L_f\tilde{B}^t - T^t + M^t].$$

In order to maintain full-employment under growth by technological progress (5) must be equal to $\gamma(M^t + L_f\tilde{B}^t)$. Therefore, we obtain

$$G^t - T^t = (\gamma - 1)(M^t + L_f\tilde{B}). \tag{6}$$

Since $M^t + L_f\tilde{B}$ is positive, $G^t > T^t$ when $\gamma > 1$. In Period $t+1$ $M^{t+1} = \gamma M^t$, $\tilde{B}^{t+1} = \gamma\tilde{B}^t$, and we can assume $G^{t+1} = \gamma G^t$ and $T^{t+1} = \gamma T^t$. Thus, with $P_1^{t+1} = P_1^t$ we obtain

$$P_1^t L_f l(L_f)\gamma y = \alpha(P_1^t L_f l(L_f)\gamma y - \gamma T^t + \gamma L_f\tilde{B}^t) + \gamma G^t + \gamma M^t.$$

This is equivalent to (4), and full-employment is maintained by $G^{t+1} = \gamma G^t$ and $T^{t+1} = \gamma T^t$.

Since the budget deficit to maintain full-employment must be continuous, it should be financed by seigniorage not by public debt. Budget deficit is necessary under growth because of deficiency of the savings of the older generation. This budget deficit is not debt and does not need to be redeemed.

The money supply equals the savings. An increase in the money supply equals an increase in the savings. (6) means that it equals the budget deficit. The rate of an increase in the savings equals the growth rate and therefore the budget deficit in this case does not cause inflation.

Summarizing the results.

Proposition 1. *We need continuous budget deficit to maintain full-employment when the economy grows at the positive rate by technological progress under constant prices.*

4.3 Excessive budget deficit and inflation

Suppose that up to Period $t-1$ full-employment has been achieved under constant prices, but the government expenditure and/or the tax in Period t may be different from their steady state values. The steady state is a state of full-employment is continuously maintained under constant prices. Denote them by $\hat{G}^t$ and $\hat{T}^t$. We denote also the actual price by $\hat{P}_1^t$. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f)y = \alpha(\hat{P}_1^t L_f l(L_f)y + L_f\tilde{B}^t - \hat{T}^t) + \hat{G}^t + M^t. \tag{7}$$

The savings of the younger generation is

$$M^{t+1} + L_f\tilde{B}^{t+1} = (1-\alpha)(\hat{P}_1^t L_f l(L_f)y + L_f\tilde{B}^t - \hat{T}^t) = \hat{G}^t + L_f\tilde{B}^t - \hat{T}^t + M^t.$$

Their consumptions in their Period 2 and the bequests are, respectively,

$$M^{t+1} = \beta(\hat{P}_1^t L_f l(L_f)y + L_f\tilde{B}^t - \hat{T}^t) = \frac{\beta}{1-\alpha}[\hat{G}^t + L_f\tilde{B}^t - \hat{T}^t + M^t],$$

and

$$L_f\tilde{B}^{t+1} = (1-\alpha-\beta)(\hat{P}_1^t L_f l(L_f)y + L_f\tilde{B}^t - \hat{T}^t) = \frac{1-\alpha-\beta}{1-\alpha}[\hat{G}^t + L_f\tilde{B}^t - \hat{T}^t + M^t].$$

Let

$$\lambda = \frac{\hat{P}_1^t}{P_1^t} > 1.$$

If in Period $t+1$ full-employment is maintained with $P_1^{t+1} = \hat{P}_1^t > P_1^t$, we need

$$\hat{G}^t + L_f\tilde{B}^t - \hat{T}^t + M^t = \gamma\lambda(M^t + L_f\tilde{B}^t).$$

Thus, from (6)

$$\hat{G}^t - \hat{T}^t = (\gamma\lambda - 1)(M^t + L_f\tilde{B}^t) > (\gamma - 1)(M^t + L_f\tilde{B}^t) = G^t - T^t.$$

This means

$$(\lambda - 1)\gamma(M^t + L_f\tilde{B}^t) = (\hat{G}^t - \hat{T}^t) - (G^t - T^t),$$

or

$$\lambda - 1 = \frac{(\hat{G}^t - \hat{T}^t) - (G^t - T^t)}{\gamma(M^t + L_f\tilde{B}^t)} > 0.$$

Therefore, excessive budget deficit, $(\hat{G}^t - \hat{T}^t) - (G^t - T^t)$, causes inflation at the rate of $\lambda = \frac{\hat{P}_1^t}{P_1^t}$.

In Period $t+1$, $M^{t+1} = \gamma\lambda M^t$, $\tilde{B}^{t+1} = \gamma\lambda\tilde{B}^t$, and we can assume $G^{t+1} = \gamma\lambda G^t$ and $T^{t+1} = \gamma\lambda T^t$. Thus, with $P_1^{t+1} = \hat{P}_1^t$ we obtain

$$\hat{P}_1^t L_f l(L_f)\gamma y = \alpha(\hat{P}_1^t L_f l(L_f)\gamma y + \gamma\lambda L_f\tilde{B}^t - \gamma\lambda T^t) + \gamma\lambda G^t + \gamma\lambda M^t.$$

Since $\hat{P}_1^t = \lambda P_1^t$, this is equivalent to (4), and full-employment is maintained by $G^{t+1} = \gamma\lambda G^t$ and $T^{t+1} = \gamma\lambda T^t$. After one-period inflation full-employment can be maintained by continuous budget deficit under constant prices. About the excessive budget deficit that has caused inflation, only the excess portion should be reduced, and there is no need to make up for past excesses by creating surpluses or reducing deficits.

Summarizing the results.

Proposition 2. 1. *If the budget deficit is larger than the level which is sufficient to maintain full-employment, inflation is triggered.*

2. *About the excessive budget deficit that has caused inflation, only the excess portion should be reduced, and there is no need to make up for past excesses by creating surpluses or reducing deficits.*

Suppose that in Period $t+1$ $P_1^{t+1} = \lambda\hat{P}_1^t$, that is, inflation continues. Then, the following equation holds.

$$\hat{P}_1^t L_f l(L_f)\gamma\lambda y = \alpha(\hat{P}_1^t L_f l(L_f)\gamma\lambda y + \gamma\lambda L_f\tilde{B}^t - \gamma\lambda\hat{T}^t) + \gamma\lambda\hat{G}^t + \gamma\lambda M^t.$$

This is equivalent to (7).

4.4 Insufficient government expenditure and involuntary unemployment

Suppose that in (7) Ll may be different from $L_f l(L_f)$, the government expenditure may be different from G^t, and the prices of the goods are constant. Denote the actual value of the government expenditure by $\hat{G}^t$. The tax equals T^t. Then, we have

$$P_1^t Ll(L)y = \alpha(P_1^t Ll(L)y + L_f\tilde{B}^t - T^t) + \hat{G}^t + M^t. \tag{8}$$

Comparing (8) with (4), we obtain

$$Ll(L) - L_f l(L_f) = \frac{\hat{G}^t - G^t}{(1-\alpha)P_1^t y}.$$

If $\hat{G}^t < G^t$, we have $L < L_f$. It means that insufficient government expenditure causes involuntary unemployment.

As proved in Proposition 1, since a budget deficit is necessary to maintain full employment under growth, there will be involuntary unemployment under a balanced budget.

The savings of the younger generation consumers in Period t is

$$\hat{M}^{t+1} + L_f\hat{\tilde{B}}^{t+1} = (1-\alpha)(P_1^t Ll(L)y + L_f\tilde{B}^t - T^t) = \hat{G}^t + L_f\tilde{B}^t - T^t + M^t. \tag{9}$$

$\hat{M}^{t+1}$ and $\hat{\tilde{B}}^{t+1}$ are the actual values of the consumptions and the bequests of the younger generation consumers in their Period 2. Their consumptions in their Period 2 and the bequests are, respectively,

$$\hat{M}^{t+1} = \beta(P_1^t Ll(L)y + L_f\tilde{B}^t - T^t) = \frac{\beta}{1-\alpha}(\hat{G}^t + L_f\tilde{B}^t - T^t + M^t),$$

and

$$L_f\hat{\tilde{B}}^{t+1} = (1-\alpha-\beta)(P_1^t Ll(L)y + L_f\tilde{B} - T^t) = \frac{1-\alpha-\beta}{1-\alpha}(\hat{G}^t + L_f\tilde{B}^t - T^t + M^t).$$

Suppose that in Period $t+1$ full-employment is achieved with $P_1^{t+1} = P_1^t$ and $T^{t+1} = \gamma T^t$. Then, the following equation holds

$$P_1^t L_f l(L_f)\gamma y = \alpha(P_1^t L_f l(L_f)\gamma y + L_f\hat{\tilde{B}}^{t+1} - \gamma T^t) + \hat{\hat{G}}^{t+1} + \hat{M}^{t+1}.$$

$\hat{\hat{G}}^{t+1}$ is the actual value of the government expenditure in Period $t+1$. By (9) the savings of the younger generation consumers in Period $t+1$ is

$$\begin{aligned}&(1-\alpha)(P_1^t L_f l(L_f)\gamma y + L_f\hat{\tilde{B}}^{t+1} - \gamma T^t) = \hat{\hat{G}}^{t+1} + L_f\hat{\tilde{B}}^{t+1} - \gamma T^t + \hat{M}^{t+1}\\ =&\hat{\hat{G}}^{t+1} - \gamma T^t + \hat{G}^t + L_f\tilde{B}^t - T^t + M^t.\end{aligned}$$

To maintain full-employment this must equal $\gamma^2(M^t + L_f\tilde{B}^t)$. $M^t + L_f\tilde{B}$ is the steady state value of the savings of the younger generation consumers in Period t. Therefore,

$$\hat{\hat{G}}^{t+1} - \gamma T^t + \hat{G}^t - T^t = (\gamma^2 - 1)(M^t + Lf\tilde{B}) = \gamma(\gamma - 1)(M^t + Lf\tilde{B}) + (\gamma - 1)(M^t + Lf\tilde{B}).$$

In the steady state

$$\hat{\hat{G}}^{t+1} = \gamma G^t,\ \hat{G}^t = G^t,$$

$$\hat{G}^t - T^t = (\gamma - 1)(M^t + Lf\tilde{B}),$$

and

$$\hat{\hat{G}}^{t+1} - \gamma T^t = \gamma(\gamma - 1)(M^t + Lf\tilde{B}).$$

If $\hat{G}^t < G^t$, we have

$$\hat{G}^t - T^t < (\gamma - 1)(M^t + Lf\tilde{B}),$$

and then

$$\hat{\hat{G}}^{t+1} - \gamma T^t > \gamma(\gamma - 1)(M^t + Lf\tilde{B}).$$

To recover full-employment in Period $t + 1$ we need extra government expenditure over its steady state value.

Summarizing the results.

Proposition 3. *Insufficient government expenditure in Period t causes involuntary unemployment, and to recover full-employment in Period $t + 1$ we need extra government expenditure over its steady state value.*

In this subsection we have assumed constant prices. If the prices fall due to a decline in the nominal wage rate caused by the existence of involuntary unemployment, the real balance effect may work to increase consumption. However, it would be faster to restore full employment through fiscal policy.

5 Concluding Remark

We have shown the following results.

1. We need continuous budget deficit to maintain full-employment when the economy grows at the positive rate by technological progress.

2. If the budget deficit is larger than the level which is sufficient to maintain full-employment, inflation is triggered.

3. Insufficient government expenditure causes involuntary unemployment.

We want to extend discussions of this paper to a more general three-periods OLG model with childhood period used in Tanaka (2020) . In such a model consumers have debts as well as savings.

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