Budget Deficit to Achieve and Maintain Full-employment Under Growth by Technological Progress

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18 April 2021

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Abstract

The purpose of this paper is to show, using a simple two-periods overlapping generations (OLG) model in which goods are produced solely by labor in a monopolistically competitive industry, that a continuous budget deficit is necessary to maintain full-employment under economic growth driven by autonomous technological progress. Since the budget deficit must be continuous, it should be financed by seigniorage not by public debt. Also we will show that to achieve full employment in the presence of involuntary unemployment we need extra budget deficit. Budget deficit is necessary under growth because of deficiency of the savings of the older generation. These budget deficits are not debt and do not need to be redeemed. The money supply equals the savings. An increase in the money supply equals an increase in the savings. It equals the budget deficit. The rate of an increase in the savings equals the growth rate and therefore budget deficit does not cause inflation.

Keywords: overlapping generations model, budget deficit, full-employment, growth

JEL Classification No.: E12, E24.

1 Introduction

The purpose of this paper is to show, using a simple two-periods overlapping generations (OLG) model in which goods are produced solely by labor in a monopolistically competitive industry, that a continuous budget deficit is necessary to maintain full-employment under economic growth driven by autonomous technological progress[1]. Since the budget deficit must be continuous, it should be financed by seigniorage not by public debt. Also we will show that to achieve full employment in the presence of involuntary unemployment we need extra budget deficit. Budget deficit is necessary under growth because of deficiency of the savings of the older generation. These budget deficits are not debt and do not need to be redeemed. The money supply equals the savings. An increase in the money supply equals an increase in the savings. It equals the budget deficit. The rate of an increase in the savings equals the growth rate and therefore budget deficit does not cause inflation.

2 The model

We consider a two-periods (1: younger or working, and 2: older or retired) overlapping generations (OLG) model under monopolistic competition. Our model is according to Otaki (2007), Otaki (2009) and Otaki (2015), and a generalization of Tanaka (2020) in which perfect competition is assumed. The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0, 1]$. Good z is monopolistically produced by firm z with constant returns to scale technology. The technology progresses at the rate $\gamma - 1 > 0$.

2. During Period 1, each consumer supplies 1 unit of labor, consumes the goods and saves money for his consumption in Period 2. He is employed or not employed.

3. During Period 2, each consumer consumes the goods using his savings carried over from his Period 1 earnings

4. Each consumer determines his consumption in Periods 1 and 2 and the labor supply at the beginning of Period 1 depending on the situation that he is employed or not employed.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1, 2$.
C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1, 2$.
$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1, 2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i =$

[1]The rate of technological progress may be affected by investment by firms. However, it can be considered to be basically the result of scientific research.

1, 2.
P_i: the price of consumption basket in Period i, $i = 1, 2$.
$p_i(z)$: the price of good z in Period i, $i = 1, 2$.
$\rho = P_2/P_1$: (expected) inflation rate (plus one).
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
y: labor productivity, which increases by autonomous technological progress.

We assume that the population L_f is constant.

3 Behaviors of agents

3.1 Consumers' behavior

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods.

2. Then, they maximize their consumption baskets given the expenditure in each period.

The utility function of employed consumers of one generation over two periods is written as

$$u(C_1^e, C_2^e) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1, 2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, i = 1, 2.$$

σ is the elasticity of substitution among the goods. $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}, i = 1, 2.$$

The budget constraint for an employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi.$$

The budget constraint for an unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi.$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e}, 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}.$$

Since the utility functions $u(C_1^e, C_2^e)$ and $u(C_1^u, C_2^u)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

and

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi}{P_1},$$

$$C_2^e = (1 - \alpha) \frac{Wl + \Pi}{P_2},$$

and

$$C_1^u = \alpha \frac{\Pi}{P_1}, \ C_2^u = (1 - \alpha) \frac{\Pi}{P_2}.$$

Solving maximization problems in Step 2 (details of calculation are available upon request), the following demand functions of employed and unemployed consumers are derived .

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi)}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1 - \alpha)(Wl + \Pi)}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha \Pi}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1 - \alpha)\Pi}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha\frac{Wl+\Pi}{P_1}, (1-\alpha)\frac{Wl+\Pi}{P_2}\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi}{P_1}, (1-\alpha)\frac{\Pi}{P_2}\right).$$

Let

$$\omega = \frac{W}{P_1}.$$

This is the real wage rate. Then, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi}{P_1}, \rho\right) - \Gamma(l),$$

$$V^u = \varphi\left(\frac{\Pi}{P_1}, \rho\right),$$

Denote

$$I = \omega l + \frac{\Pi}{P_1}.$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{1}$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial C_1^e} + (1-\alpha)\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (1) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > (<)0$, the labor supply is increasing (decreasing) with respect to the real wage rate ω. But, we assume that the real wage rate does not have a significant effect on the individual labor supply. l may depend on employment L in some way. However, we assume that Ll is increasing with respect to L.

3.2 Firms' behavior

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(WLl + L_f\Pi)}{P_1} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(WLl + L_f\Pi)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(WLl + L_f\Pi)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(\overline{W}\ \overline{Ll} + L_f\overline{\Pi})}{P_2}.$$

where $\overline{W}$, $\overline{\Pi}$, $\overline{L}$ and $\overline{l}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, respectively, during the previous period.

Let

$$M = (1-\alpha)(\overline{W}\ \overline{Ll} + L_f\overline{\Pi}).$$

This is the total savings or the total consumption of the older generation consumers. It is the planned consumption that is determined in Period 1 of them. Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{M}{P_1}$.

Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{Y}{P_1}, \tag{2}$$

where Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi) + G + M.$$

G is the government expenditure. Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are

$$\int_0^1 Ldz = L,\ \int_0^1 Lldz = Ll.$$

The output of firm z is Lly. At the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial Ll} = y.$$

From (2)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial Ll} = -\frac{p_1(z)y}{\sigma d(z)} = -\frac{p_1(z)}{\sigma Ll}.$$

The profit of firm z is

$$\pi(z) = p_1(z)Lly - LlW.$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial Ll} = p_1(z)y - Lly\frac{p_1(z)}{\sigma Ll} - W = p_1(z)y - \frac{p_1(z)y}{\sigma} - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{1 - \frac{1}{\sigma}y}W.$$

Let $\mu = 1/\sigma$. Then,

$$p_1(z) = \frac{1}{(1-\mu)y}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)y}W.$$

4 Budget deficit to maintain full-employment under economic growth

4.1 Market equilibrium

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly.$$

The (nominal) aggregate demand is

$$\alpha(WL + L_f\Pi) + G + M = \alpha P_1Lly + G + M.$$

Since they are equal,

$$P_1Lly = \alpha P_1Lly + G + M. \tag{3}$$

In real terms

$$Lly = \frac{G+M}{(1-\alpha)P_1}.$$

The equilibrium value of Ll cannot be larger than $L_fl(L_f)$. $l(L_f)$ is the labor supply when full-employment is achieved. However, it may be strictly smaller than $L_fl(L_f)$. Then, we have $L < L_f$ and involuntary unemployment exists. If the government collects a tax T from the younger generation consumers, (3) is rewritten as

$$P_1Lly = \alpha(P_1Lly - T) + G + M.$$

4.2 Budget deficit to maintain full-employment

Suppose that up to Period t full-employment has been achieved. We write

$$P_1^t L_f l(L_f) y = \alpha(P_1^t L_f l(L_f) y - T^t) + G^t + M^t. \tag{4}$$

Superscript t represents the values in Period t. In such a steady state with full-employment we can assume that the prices of the goods are constant. Then, $\rho = 1$. The savings of the younger generation is

$$(1-\alpha)(P_1^t L_f l(L_f) y - T^t) = G^t - T^t + M^t.$$

In order to maintain full-employment under growth by autonomous technological progress this must be equal to γM^t. Therefore, we obtain

$$G^t - T^t = (\gamma - 1) M^t. \tag{5}$$

M^t is the savings of the younger generation in Period t. It is equal to their consumption in the next period. Since it is positive, $G^t > T^t$ when $\gamma > 1$. (5) means that the budget deficit equals an increase in the savings. Thus, the accumulated budget deficit equals the savings.

We have shown the following proposition.

Proposition 1. *1. We need continuous budget deficit to maintain full-employment when the economy grows at the positive rate by technological progress.*

2. The accumulated budget deficit equals the savings.

Since the budget deficit must be continuous, it should be financed by seigniorage not by public debt.

Some notes

1. In Period $t+1$ $M^{t+1} = \gamma M^t$, and we can assume $G^{t+1} = \gamma G^t$ and $T^{t+1} = \gamma T^t$. Thus, with $P^{t+1} = P^t$ we obtain

   $$P_1^t L_f l(L_f) \gamma y = \alpha(P_1^t L_f l(L_f) \gamma y - \gamma T^t) + \gamma G^t + \gamma M^t.$$

   This is equivalent to (4).

2. If the prices of the goods are constant when full-employment is achieved, both the nominal and the real wage rates rise by technological progress.

3. Our conclusion implies that without government expenditure nor tax, full employment can not be maintained under growth. If $G^t = T^t = 0$, (4) is rewritten as

   $$P_1^t L_f l(L_f) y = \alpha P_1^t L_f l(L_f) y + M^t.$$

The savings of the younger generation is

$$(1-\alpha)P_1^t L_f l(L_f)y = M^t.$$

This cannot be equal to γM^t with $\gamma > 1$ unless $\alpha = 1$ and $M^t = 0$. By the same logic full employment can not be maintained with positive growth rate ($\gamma > 1$) under balanced budget. Conversely, without growth ($\gamma = 1$) we need balanced budget. Budget deficit is necessary under growth because of deficiency of the savings of the older generation.

4. The money supply equals the savings. An increase in the money supply equals an increase in the savings. (5) means that it equals the budget deficit. The rate of an increase in the savings equals the growth rate and therefore the budget deficit does not cause inflation.

5 Achievement of full-employment in the presence of involuntary unemployment

Suppose that there exists involuntary unemployment in Period $t-1$ due to insufficient demand, and full-employment is achieved in the next period, Period t. Let $\hat{G}^t$ and $\hat{M}^t$ be the real values of the government expenditure and the savings of the younger generation. They may be different from their steady state values, G^t and M^t. Then, we have

$$P_1^t L_f l(L_f)y = \alpha(P_1^t L_f l(L_f)y - T^t) + \hat{G}^t + \hat{M}^t. \tag{6}$$

Comparing (4) and (6), we get

$$\hat{G}^t - G^t = M^t - \hat{M}^t.$$

Since there exists involuntary unemployment in Period $t-1$, $\hat{M}^t$ is smaller than M^t which is the steady state value of the savings of the younger generation in Period $t-1$. Thus, we obtain

$$\hat{G}^t - G^t > 0.$$

This means that we need extra budget deficit to achieved full employment in the presence of involuntary unemployment. This extra budget deficit should also be financed by seigniorage not by public debt because we need continuous budget deficit after achievement of full employment. We have shown the following proposition.

Proposition 2. *To achieve full employment in the presence of involuntary unemployment we need extra budget deficit over continuous budget deficit for maintaining full-employment.*

6 Concluding Remark

We have shown the following results.

1. We need continuous budget deficit to maintain full-employment when the economy grows at the positive rate by autonomous technological progress. It should be financed by seigniorage not by public debt.

2. To achieve full employment in the presence of involuntary unemployment we need extra budget deficit over continuous budget deficit. This extra budget deficit should also be financed by seigniorage not by public debt because we need continuous budget deficit after achievement of full employment.

These budget deficits are not debt and do not need to be redeemed.

We want to extend discussions of this paper to a more general three-periods OLG model with childhood period used in Tanaka (2020) . In such a model consumers have debts as ell as savings.

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