Licensing Cost-Reducing Innovations Under Supply Function Competition

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Abstract. In this paper, we study the problem of licensing cost-reducing innovations in a duopoly under supply function competition. We show that the innovator prefers fixed-fee licensing to no licensing if its cost advantage is not extremely large. Moreover, if its cost advantage is not extremely small, the innovator prefers fixed-fee licensing to royalty licensing, as well.

Keywords: Duopoly; licensing; supply function competition.

JEL Codes: D43; L13; O30

1 Introduction

In this paper, we study licensing of (cost-reducing) innovations in a duopoly under supply function competitions. Licensing of innovations has been extensively studied under quantity and price competitions both when the innovator is an outsider, an R&D organization that does not compete in the product market, and when it is an insider, one of the producers. In the first case, the licensor generally prefers fixed-fee licensing to royalty licensing (e.g., Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien et al.,

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1992), whereas in the second case the reverse is true: the licensor, as an insider, generally finds royalty licensing superior (e.g., Wang, 1998; Wang and Yang 1999; Kamien and Tauman, 2002; Filippini, 2005; and San Martin and Saracho, 2010). The second result was also checked, by a vast literature, to be robust to many modeling variations including asymmetric information, leadership structure, moral hazard, product differentiation, quality variation, risk aversion, and strategic delegation.[1] Yet, these variations leave out an important issue, namely licensing under supply function competition where firms non-cooperatively choose supply functions instead of fixed quantities or prices. This relatively new form of competition, formulated by Grossman (1981) and developed by Klemperer and Meyer (1989), has found many applications in the last three decades especially in electricity markets (Green and Newbery 1992, Rudkevich and Duckworth 1998, Day et al. 2002, Newbery and Greve 2017, Escrihuela-Villar et al. 2020) where it is also known that innovation is essential, as indicated by European Commission (2015), to maintain leadership in the industry as well as to boost jobs and economic growth.

We believe that our paper is the first attempt to integrate technology licensing with supply function competition. Formally, our model involves a duopolistic industry where one of the firms has an (unrivaled) cost-reducing innovation. We are not interested in the development of this innovation and also assume that its efficiency is common knowledge. We consider two licensing arrangements, one involving fixed-fees and the other involving ad valorem royalties (per revenue units).[2] Empirical studies on technology licensing report the frequent use of royalties, and especially ad valorem royalties. Ros-

[1]See Sen (2005) for a classified list of papers on these issues.

[2]There are also other types of licensing arrangements such as per unit royalty licensing and two-part tariff licensing that combines fixed fees with either per-unit or ad valorem royalties. which are beyond the scope of this paper.

toker (1984) reports using survey data from randomly selected 150 corporations in the United States that royalties and fees are separately used in 39% and 13% of all transactions respectively. Bousquet et al. (1998) show using French data that in a sample of 278 contracts, 225 (81%) includes royalties of which 216 (96%) are ad valorem. Vishwasrao (2007) shows that patent licensors are empirically more likely to ask for royalties when sales are relatively high and involatile but profits are low. Theoretically, it is well-known that under quantity or price competition the licensor has an additional incentive to use (per-unit or ad valorem) royalties since they raise the effective cost of the licensee (see, for example, Wang, 1998, Kamien and Tauman, 2002). We will investigate whether this incentive continues to exist under supply function competition with the help of a three-stage (non-cooperative) game played by the duopolists. In the first stage, the innovator (licensor) makes a "take-it-or-leave-it" offer to its competitor (licensee) for its cost-reducing technology, and in the second stage the licensee decides whether to accept or reject the offer (or equivalently whether to produce at equal or unequal costs). Given this decision and the implied cost structure, the two firms finally engage in supply function competition in the third stage. Calculating the subgame-perfect equilibrium of this game we show that the innovator prefers fixed-fee licensing to no licensing if and only if its cost advantage is not extremely high. Moreover, if this cost advantage is mild, it prefers fixed-fee licensing to royalty licensing, too. Thus, royalty licensing becomes a preferred arrangement of licensing for the innovator only if the size of its innovation and its effect on profits is very minor.

The rest of the paper is organized as follows: Section 2 involves some basic structures and Section 3 presents for each licensing arrangement a strategic game played by duopolists under supply function competition along with its equilibrium. In more detail, Section 3.1 deals with royalty licensing and Section 3.2 with fixed-fee licensing. Next, section 4 contains our welfare results and finally Section 5 concludes.

2 Basic Structures

We consider a duopolistic industry where a single homogeneous good is produced under cost asymmetry. The demand curve faced by the firms is given by

$$D(p) = a - p, \tag{1}$$

where $a > 0$ denotes the constant intercept and p denotes the product price. Firm $i = 1, 2$ faces the cost function

$$C_i(q_i) = \theta_i q_i^2/2, \tag{2}$$

where q_i is the quantity produced by firm i and θ_i is a non-negative constant that is equal to the marginal cost of the unit output. (Fixed costs are normalized to zero to simplify the analysis.) We assume that firm 1 (innovator) has a superior and unrivaled production technology that yields an initial cost advantage. Specifically, the cost parameters of firm 1 and firm 2 are $\theta_1 = \theta - x$ and $\theta_2 = \theta$ where $0 < \theta < a$ and $0 < x \leq \theta$. In this paper, we are not interested in the development of innovation (i.e., determination of the parameter x). The value of x, a, θ, θ_1, and θ_2 are exogenously given. We also assume that both firms are rational (profit maximizers) and the industry structure described above is common knowledge.

We consider two arrangements of licensing. One of them is fixed fee licensing in which firm 1 demands a fixed fee from firm 2. The other arrangement is ad valorem royalty licensing in which firm 1 demands from firm 2 a fraction of its revenues. Under both arrangements, once an agreement is reached firm 1 makes its superior technology be accessible to firm 2, enabling it to produce using the cost parameter $\theta - x$ instead of θ.

3 Licensing Game

We consider a strategic game consisting of three stages. In the first stage, the licensor (firm 1) makes a "take-it-or-leave-it" offer to the licensee (firm 2), and in the second stage the licensee decides whether to accept or reject the offer. We assume that the licensee always accepts an offer whenever it becomes indifferent to accept or reject. After the licensee makes its decision, production costs become finalized and common knowledge, and finally the firms engage in supply function competition in the third stage.

3.1 Royalty Licensing

We assume that firm 1 offers to firm 2 the license for its innovation in return for a royalty payment, defined as a share of the revenues of firm 2, i.e. $r\,p\,q_2$ where $r \in \mathbb{R}_+$ is called the royalty rate. (Note that no offer with $r > 1$ can be acceptable for firm 2. Thus, by allowing firm 1 to make an offer with $r > 1$, we actually allow for the possibility that firm 1 does not want to sell its innovation.) After learning the royalty rate demanded by firm 1, firm 2 decides whether or not to buy the license (to accept the offer of firm 1). Let $\gamma^R \in \{0, 1\}$ represent this decision, where $\gamma^R = 1$ and $\gamma^R = 0$ respectively denote 'to buy' and 'not to buy' the license sold by firm 1. If $\gamma^R = 1$, then the cost parameters of the two firms become equivalent at $\theta - x$. Otherwise, the cost parameters of firms 1 and 2 will be $\theta - x$ and θ respectively. After the decision and cost parameter of firm 2 have been realized and become common knowledge, firm 1 and firm 2 engage in supply competition. More formally, the two firms play the following non-cooperative game involving three consecutive stages:

Stage 1: Firm 1 decides on the value of r and offers to firm 2 that it will license its innovation in return for a royalty payment of amount rpq_2.

Stage 2: Firm 2 decides on the value of γ^R, i.e., whether or not it accepts the offer of firm 1, and announces it.

Stage 3: Firm 1 and Firm 2 engage in supply function competition with cost parameters $\theta - x$ and $\theta - \gamma^R x$ respectively and then a royalty fee of $\gamma^R rpq_2$ is paid by firm 2 to firm 1. (Note that the royalty fee becomes zero and firm 2 produces with parameter θ when $\gamma = 0$.)

Below, we will solve for the equilibrium of the game described above. Using subgame-perfection we will start from the last stage.

Stage 3: Firms simultaneously choose their supply functions.

In stage 3, a strategy for firm $i \in \{1, 2\}$ is a linear function mapping prices into quantities, i.e., $S_i^R(p) = \nu_i^R p$ where $\nu_i^R \geq 0$. Given the strategies $S_1^R(p)$ and $S_2^R(p)$, the duopolistic product market clears if

$$D(p) = S_1^R(p) + S_2^R(p) \tag{3}$$

or

$$a - p = \nu_1^R p + \nu_2^R p, \tag{4}$$

implying an equilibrium price $p\left(\nu_1^R, \nu_2^R\right)$ that satisfies

$$p\left(\nu_1^R, \nu_2^R\right) = \frac{a}{1 + \nu_1^R + \nu_2^R}. \tag{5}$$

Then, for any $r \in \mathbb{R}_+$ and $\gamma \in \{0, 1\}$ the profits of firm 1 and 2 respectively become

$$\begin{aligned} \pi_1^R(v_1^R, v_2^R, r, \gamma^R) \quad &= \quad p(\nu_1^R, \nu_2^R) S_1(p(\nu_1^R, \nu_2^R)) - (\theta - x) S_1^R(p(\nu_1^R, \nu_2^R))^2/2 \\ &\quad + \gamma^R rp(\nu_1^R, \nu_2^R) S_2^R(p(\nu_1^R, \nu_2^R)) \end{aligned} \tag{6}$$

and

$$\pi_2^R(\upsilon_1^R, \upsilon_2^R, r, \gamma^R) \;=\; p(\nu_1^R,\nu_2^R)S_2^R(p(\nu_1^R,\nu_2^R)) - (\theta - \gamma^R x)S_2^R(p(\nu_1^R,\nu_2^R))^2/2$$
$$-\gamma^R r p(\nu_1^R,\nu_2^R)S_2^R(p(\nu_1^R,\nu_2^R)). \tag{7}$$

(The superscript R appearing in π_1^R and π_2^R, as well as in some variables, stands for *royalty licensing*.) Given any $r \in \mathbb{R}_+$ and $\gamma^R \in \{0,1\}$, the supply functions $S_1^{R*}(p) = \nu_1^{R*}(r,\gamma^R)p$ and $S_2^{R*}(p) = \nu_2^{R*}(r,\gamma^R)p$ form a Nash equilibrium if $\nu_i^{R*} \equiv \nu_i^{R*}(r,\gamma^R)$ solves

$$\max_{\nu_i \geq 0} \pi_i^R(\nu_i, \nu_j^{R*}, r, \gamma^R) \tag{8}$$

for each $i, j \in \{1,2\}$ with $i \neq j$.

Proposition 1. *For any $r \in \mathbb{R}_+$ and $\gamma \in \{0,1\}$, the supply function competition under royalty licensing has always a unique Nash equilibrium where firm 1 and firm 2 respectively compete with supply functions $\nu_1^{R*}(r,\gamma)p$ and $\nu_2^{R*}(r,\gamma)p$ satisfying*

$$\nu_1^{R*}(r,\gamma^R) \;=\; \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \tag{9}$$

$$\nu_2^{R*}(r,\gamma^R) \;=\; \frac{-b_2 + \sqrt{b_2^2 - 4a_2c_2}}{2a_2} \tag{10}$$

where

$$a_1 = \theta_1\theta_2^R + \theta_1 + \theta_2^R, \tag{11}$$

$$b_1 = \theta_1\theta_2^R - 2\gamma^R r\theta_1 + 2\theta_1, \tag{12}$$

$$c_1 = -(\theta_2^R - 4\gamma^R r + 2), \tag{13}$$

$$a_2 = \theta_1\theta_2^R + (1 - 2\gamma^R r)(\theta_1 + \theta_2^R), \tag{14}$$

$$b_2 = \theta_1\theta_2^R - 2\gamma^R r\theta_1 + 2\theta_2^R, \tag{15}$$

$$c_2 = -(\theta_1 + 2), \tag{16}$$

$$\theta_2^R = \theta - \gamma^R x = \theta_1 + (1 - \gamma^R)x. \tag{17}$$

Proof. Pick any $r \in \mathbb{R}_+$ and $\gamma^R \in \{0, 1\}$. Let $\theta_2^R = \theta - \gamma^R x = \theta_1 + (1 - \gamma^R)x$. If the pair of supply functions $S_1^R(p) = \nu_1^R(r, \gamma^R)p$ and $S_2^R(p) = \nu_2^R(r, \gamma^R)p$ form a Nash equilibrium, then the associated market clearing price must be a solution to

$$\max_{p \geq 0} p\left(a - p - S_2^R(p)\right) - \theta_1(a - p - S_2^R(p))^2/2 + \gamma^R rpS_2^R(p) \tag{18}$$

and

$$\max_{p \geq 0} p\left(a - p - S_1^R(p)\right) - \theta_2^R\left(a - p - S_1^R(p)\right)^2/2 - \gamma^R rpS_2^R(p). \tag{19}$$

The first-order necessary condition for the problem in (18) implies

$$\begin{aligned} 0 \quad &= \quad a - bp - S_2^R(p) + \gamma^R r\Big(S_2^R(p) + pS_2^{R'}(p)\Big) \\ &\quad\quad + \left(p - \theta_1\left[a - p - S_2^R(p)\right]\right)\Big(-1 - S_2^{R'}(p)\Big), \end{aligned} \tag{20}$$

or

$$\begin{aligned} 0 \quad &= \quad S_1^R(p) + 2\gamma^R rS_2^R(p) + (p - \theta_1 S_1^R(p))\left(-1 - \nu_2^R\right) \\ &= \quad (\nu_1^R + 2\gamma^R r\nu_2^R)p + (1 - \theta_1\nu_1^R)\left(-1 - \nu_2^R\right)p, \end{aligned} \tag{21}$$

further implying

$$\nu_1^R = \frac{1 + (1 - 2\gamma^R r)\nu_2^R}{1 + \theta_1(1 + \nu_2^R)}. \tag{22}$$

On the other hand, the first-order necessary condition for the problem in (19) implies

$$\begin{aligned} 0 &= a - bp - S_1^R(p) - \gamma^R r(S_2^R(p) + pS_2^{R'}(p)) \\ &\quad + \left(p - \theta_2^R\left[a - p - S_1^R(p)\right]\right)\left(-1 - S_1^{R'}(p)\right), \end{aligned} \tag{23}$$

or

$$\begin{aligned} 0 &= S_2^R(p) - 2\gamma^R r S_2^R(p) + (p - \theta_2^R S_2^R(p))\left(-1 - \nu_1^R\right) \\ &= \left(\nu_2^R - 2\gamma^R r \nu_2^R\right)p + \left(1 - \theta_2^R \nu_2^R\right)\left(-1 - \nu_1^R\right)p, \end{aligned} \tag{24}$$

further implying

$$\nu_2^R = \frac{1 + \nu_1^R}{1 - 2\gamma^R r + \theta_2^R(1 + \nu_1^R)} \tag{25}$$

or

$$\nu_1^R = \frac{(1 - 2\gamma^R r + \theta_2^R)\nu_2^R - 1}{1 - \theta_2^R \nu_2^R}. \tag{26}$$

Solving (22) and (26) together yields (9)-(17). To check the second-order sufficiency condition, we differentiate the right-hand side of (21) with respect to p to obtain

$$\left(\nu_1^R + 2\gamma^R r \nu_2^R\right) + \left(1 - \theta_1 \nu_1^R\right)\left(-1 - \nu_2^R\right), \tag{27}$$

which is always equal to zero (hence nonpositive) by (22). Similarly, we differentiate the right-hand side of (24) with respect to p to obtain

$$\left(\nu_2 - 2\gamma^R r \nu_2^R\right) + \left(1 - \theta_2^R \nu_2^R\right)\left(-1 - \nu_1^R\right), \tag{28}$$

which is always equal to zero (hence nonpositive) by (25). ■

Stage 2: Firm 2 chooses $\gamma^R \in \{0, 1\}$; i.e., whether or not to buy the royalty license.

The choice of firm 2 on γ^R depends on the comparison of its stage-3 profits obtained under the supply function competition when it decides to buy the license ($\gamma^R = 1$) and when it decides not to buy it ($\gamma^R = 0$). Proposition 1 implies that if $\gamma^R = 0$, the firms' profits will be $\pi_1^R(\nu_1^{R*}(r,0), \nu_2^{R*}(r,0),\ r,0)$ and $\pi_2^R(\nu_1^{R*}(r,0), \nu_2^{R*}(r,0), r, 0)$ for any $r \in \mathbb{R}_+$. On the other hand, if $\gamma^R = 1$, the firms' profits will be $\pi_1^R(\nu_1^{R*}(r,1),\ \nu_2^{R*}(r,1), r, 1)$ and $\pi_2^{R*}(\nu_1^{R*}(r,1),$ $\nu_2^{R*}(r,1), r, 1)$. For any $i \in \{1,2\}$ and $r \in \mathbb{R}_+$, let $\Delta\pi_i^{R*}(r)$ denote the difference between the profits of firm i obtained under $\gamma^R = 1$ and $\gamma^R = 0$:

$$\Delta\pi_i^{R*}(r) = \pi_i^R(\nu_1^{R*}(r,1), \nu_2^{R*}(r,1), r, 1) - \pi_i^R(\nu_1^{R*}(r,0), \nu_2^{R*}(r,0), r, 0). \tag{29}$$

Also, let $\gamma^{R*}(r)$ denote the optimal choice made by firm 2 at $r \in \mathbb{R}_+$. Since firm 2 is rational, we must have

$$\gamma^{R*}(r) = \begin{cases} 1 & \text{if } \ \Delta\pi_2^{R*}(r) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Note from (7) and (29) that $\Delta\pi_2^{R*}(r)$ is decreasing for all $r \in \mathbb{R}_+$, as we have

$$\partial\Delta\pi_2^{R*}(r)/\partial r = -p(\nu_1^{R*}(r,1), \nu_2^{R*}(r,1))^2\, \nu_2^{R*}(r,1)) < 0. \tag{31}$$

We will use this observation to prove the following result.

Proposition 2. *There exists* $\bar{r} \in \mathbb{R}_+$ *such that* $\Delta\pi_2^{R*}(\bar{r}) = 0$. *Moreover,* $\bar{r}$ *is unique and lies in* $(0,1)$.

Proof. Note from (7) that $\pi_2^R(\nu_1^{R*}(r,\gamma^R), \nu_2^{R*}(r,\gamma^R), r, \gamma^R)$ is decreasing in $\theta_2^R = \theta - \gamma^R x$, while θ_2^R is decreasing in γ^R. So, when the license of firm 1 is free, i.e., $r = 0$, π_2^R is increasing in γ^R, implying $\Delta\pi_2^{R*}(0) > 0$. On the other hand, when the license of firm 1 is too costly for firm 2, i.e. $r \geq 1$, (7) implies that the profit of firm 2 is negative when it buys the license (i.e., $\gamma^R = 1$). In contrast, when firm 2 decides not to buy the license (i.e., $\gamma^R = 0$), its profit is always positive. Therefore, $\Delta\pi_2^{R*}(r) < 0$ for all $r \geq 1$. Since $\Delta\pi_2^{R*}(r)$ is continuous in r, there exists $\bar{r} \in (0,1)$ such that $\Delta\pi_2^{R*}(\bar{r}) = 0$. Moreover, $\bar{r}$ is unique since (31) implies that $\Delta\pi_2^{R*}(r)$ is decreasing for all $r \in (0,1)$. ■

Proposition 2 and (31) together imply that equation (30) can be rewritten as

$$\gamma^{R*}(r) = \begin{cases} 1 & \text{if } \; r \leq \bar{r}, \\ 0 & \text{if } \; r > \bar{r}, \end{cases} \tag{32}$$

where $\bar{r}$ is the unique solution to $\Delta\pi_2^{R*}(r) = 0$.

Stage 1: Firm 1 chooses the royalty rate $r \in \mathbb{R}_+$.

The profit of firm 1 in (6) is increasing in the royalty rate r by the envelope function theorem if $\gamma^R = 1$ and independent of r if $\gamma^R = 0$. On the other hand, equation (32) shows that firm 2 accepts to pay a royalty at rate r if and only if $r \leq \bar{r}$. Therefore, if firm 1 decides to sell the license for its innovation, then it must set the royalty rate at $\bar{r}$. Recall that firm 2 does not buy the license when $r \geq 1$. So, for any $r \geq 1$ the profit of firm 1 becomes equivalent to its profit at $r = 1$. So, firm 1 decides not to sell the license if and only if it obtains a lower profit at the royalty rate $r = \bar{r}$ (implying $\gamma^{R*}(r) = 1$) than at the rate $r = 1$ (implying $\gamma^{R*}(r) = 0$). Thus, the optimal royalty rate r^* for firm 1 must satisfy

$$r^* \in \begin{cases} \{\bar{r}\} & \text{if } \; \Delta\pi_1^{R*}(\bar{r}) \geq 0, \\ [1, \infty) & \text{otherwise.} \end{cases} \tag{33}$$

Note that r^* is not unique when $\Delta\pi_1^{R*}(\bar{r}) < 0$, however each value of r^* in $[1, \infty)$ leads to the same response by firm 2, i.e. $\gamma^{R*}(r^*) = 0$ for all $r^* \geq 1$. In other words, r^* is essentially unique. Considering the equilibrium strategies played in all three stages together, we observe the following.

Proposition 3. *The rate of royalty r^* chosen by firm 1, the decision plan $\gamma^{R*}(r)$ of firm 2 for each $r \in \mathbb{R}_+$, and the contingent supply functions $\nu_1^{R*}(r, \gamma^R)p$ and $\nu_2^{R*}(r, \gamma^R)p$ of firms 1 and 2 for each $r \in \mathbb{R}_+$ and $\gamma^R \in \{0, 1\}$ altogether form the subgame-perfect Nash equilibrium of the three-stage game under royalty licensing.*

Proof. Follows from (9)-(17), (32), and (33). ∎

3.2 Fixed-Fee Licensing

Here, we assume that firm 1 offers to firm 2 the license for its innovation in return for a fixed fee $F \in \mathbb{R}_+$. After learning the value of F asked by firm 1, firm 2 decides whether to buy the license or not. Let $\gamma^F \in \{0, 1\}$ represent this decision of firm 2. (We add the superscript F to variables in this subsection to denote that they are associated with fixed fee licensing.) Specifically, $\gamma^F = 1$ and $\gamma^F = 0$ respectively denote the decisions 'to buy' and 'not to buy' the fixed fee license sold by firm 1. If $\gamma^F = 1$, then the cost parameters of the two firms become equivalent at $\theta - x$. Otherwise, the cost parameters of firms 1 and 2 will be $\theta - x$ and θ respectively. After the decision and cost parameter of firm 2 have been realized and become common knowledge, firm 1 and firm 2 engage in supply competition. More formally, the two firms play the following non-cooperative game involving three consecutive stages:

Stage 1: Firm 1 decides on the value of F (in the units of profits) and announces that it will sell to firm 2 the license for its innovation in return for a payment of amount F.

Stage 2: Firm 2 decides on the value of γ^F, i.e., whether or not it accepts the offer of firm 1, and announces it.

Stage 3: Firm 1 and Firm 2 engage in supply function competition with cost parameters $\theta_1 = \theta - x$ and $\theta_2^F = \theta - \gamma^F x$ respectively and choose their equilibrium supply functions. After the productions are realized, the licensing fee of amount $\gamma^F F$ is paid by firm 2 to firm 1.

Below, we will solve for the equilibrium of the above game. Using subgame-perfection we will start from the last stage.

Stage 3: Firms simultaneously choose their supply functions.

Given any $F \in \mathbb{R}_+$ and $\gamma^F \in \{0,1\}$, let $v_1^{F*}(F,\gamma^F)p$ and $v_2^{F*}(F,\gamma^F)$ respectively denote the equilibrium supply functions of firm 1 and firm 2 under fixed fee licensing. Then, the profits of firm 1 and 2 can respectively be written as

$$\begin{aligned}\pi_1^F(v_1^{F*}(F,\gamma^F), v_2^{F*}(F,\gamma^F), F, \gamma^F) &= \gamma^F F + v_1^{F*}(F,\gamma^F)p \\ &\quad -(\theta - x)v_1^{F*}(F,\gamma^F)^2 p^2/2 \qquad (34)\end{aligned}$$

and

$$\begin{aligned}\pi_2^F(v_1^{F*}(F,\gamma^F), v_2^{F*}(F,\gamma^F), F, \gamma^F) &= -\gamma^F F + v_2^{F*}(F,\gamma^F)p \\ &\quad -(\theta - \gamma^F x)v_2^{F*}(F,\gamma^F)^2 p^2/2 \qquad (35)\end{aligned}$$

for any $F \in \mathbb{R}_+$ and $\gamma^F \in \{0,1\}$. Recalling that under royalty licensing the equilibrium supply functions of firm 1 and firm 2 are respectively $\nu_1^{R*}(r,\gamma^R)p$ and $\nu_2^{R*}(r,\gamma^R)p$ which are implied by (9)-(17) for any $r \in \mathbb{R}_+$ and $\gamma^R \in \{0,1\}$, and also noting that the term $\gamma^F F$ in (34) and (35) is constant with respect to the output choices of the firms, we trivially obtain the slopes of the equilibrium supply functions under fixed fee licensing by the equations

$$\nu_1^{F*}(F,\gamma^F) = \nu_1^{R*}(0,\gamma^F) \qquad (36)$$

and

$$\nu_2^{F*}(F,\gamma^F) = \nu_2^{R*}(0,\gamma^F) \qquad (37)$$

for any $F \in \mathbb{R}_+$ and $\gamma^F \in \{0, 1\}$. That is, by inserting $r = 0$ and $\gamma^R = \gamma^F$ into (9)-(17), we can obtain $\nu_1^{F*}(F, \gamma^F)$ and $\nu_2^{F*}(F, \gamma^F)$ for any $F \in \mathbb{R}_+$.

Stage 2: Firm 2 chooses $\gamma^F \in \{0, 1\}$; i.e., whether or not to buy the fixed fee license.

The choice of firm 2 on γ^F depends on the comparison of its stage-3 profits obtained under the supply function competition when it decides to buy the license ($\gamma^F = 1$) and when it decides not to buy it ($\gamma^F = 0$). For any $F \in \mathbb{R}_+$, equations (34) and (35) imply that if $\gamma^F = 0$, the profit of firm 2 will become $\pi_2^F(\nu_1^{F*}(0, 0),\ \nu_2^{F*}(0, 0), F, 0)$. On the other hand, if $\gamma^F = 1$, firm 2 will earn $\pi_2^F(\nu_1^{F*}(0, 1),\ \nu_2^{F*}(0, 1), F, 1)$. For any $i \in \{1, 2\}$, let $\Delta\pi_i^{F*}(F)$ denote the difference between the profits of firm i obtained under $\gamma^F = 1$ and $\gamma^F = 0$; i.e.,

$$\Delta\pi_i^{F*}(F) = \pi_i^F(\nu_1^{F*}(0, 1), \nu_2^{F*}(0, 1), F, 1) - \pi_i^F(\nu_1^{F*}(0, 0), \nu_2^{F*}(0, 0), F, 0). \quad (38)$$

Also, let $\gamma^{F*}(F)$ denote the optimal decision of firm 2 when firm 1 demands a fee of $F \in \mathbb{R}_+$. As firm 2 is rational, we have

$$\gamma^{F*}(F) = \begin{cases} 1 & \text{if} \quad \Delta\pi_2^{F*}(F) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (39)$$

Note that using (38) for $i = 2$ and (7), we obtain

$$\partial\Delta\pi_2^{F*}(F)/\partial F = -1, \quad (40)$$

implying that $\Delta\pi_2^{F*}(F)$ is decreasing for all $F \in \mathbb{R}_+$. We use this observation to prove the following result.

Proposition 4. *There exists $\bar{F} \in \mathbb{R}_+$ such that $\Delta\pi_2^{F*}(\bar{F}) = 0$. Moreover, $\bar{F}$ is unique and equal to $\Delta\pi_2^{F*}(0)$.*

Proof. Using (38) for $i = 2$ along with (37) we obtain $\Delta\pi_2^{F*}(F) = \Delta\pi_2^{F*}(0) - F$, implying that $\Delta\pi_2^{F*}(\bar{F}) = 0$ if $\bar{F} = \Delta\pi_2^{F*}(0)$. Also, we know from (40) that $\partial\Delta\pi_2^{F*}(F)/\partial F < 0$ for all $F \in \mathbb{R}_+$. Therefore, $\bar{F}$ must be unique. ■

Proposition 4 and equation (40) together imply that equation (39) can be rewritten as

$$\gamma^{F*}(F) = \begin{cases} 1 & \text{if} \quad F \leq \bar{F}, \\ 0 & \text{if} \quad F > \bar{F}. \end{cases} \tag{41}$$

Stage 1: Firm 1 chooses the fixed fee $F \in \mathbb{R}_+$.

Equation (34) shows that the profit of firm 1 under fixed fee licensing is increasing in the fixed fee F if $\gamma^F = 1$ and independent of F if $\gamma^F = 0$. On the other hand, equation (41) shows that firm 2 accepts to buy the license with fixed fee F if and only if $F \leq \bar{F}$. Therefore, if firm 1 decides to sell the license for its innovation, then it must set the fixed fee at $\bar{F}$. Also, recall that firm 1 cannot sell its license to firm 2 ($\gamma^{F*}(F) = 0$) when $F > \bar{F}$. So, firm 1 decides not to sell the license if and only if it obtains a lower profit at the fixed fee $F = \bar{F}$ (implying $\gamma^{F*}(F) = 1$) than at any $F > \bar{F}$ (implying $\gamma^{F*}(F) = 0$). Thus, the optimal level of the fixed fee F^* for firm 1 must satisfy

$$F^* \in \begin{cases} \{\bar{F}\} & \text{if} \quad \Delta\pi_1^{F*}(\bar{F}) \geq 0, \\ (\bar{F}, \infty) & \text{otherwise.} \end{cases} \tag{42}$$

Note that F^* is not unique when $\Delta\pi_1^{F*}(\bar{F}) < 0$; however each value of F^* in $(\bar{F}, \infty)$ induces the same response by firm 2, i.e. $\gamma^{F*}(F^*) = 0$ for all $F^* > \bar{F}$. Thus, the equilibria we have characterized above is essentially unique. Considering the equilibrium strategies in all three stages together, we have the following result:

Proposition 5. *The fixed fee F^* chosen by firm 1, the decision plan $\gamma^{F*}(F)$ of firm 2 for each $F \in \mathbb{R}_+$, and the contingent supply functions $\nu_1^{F*}(F, \gamma^F)p$ and $\nu_2^{F*}(F, \gamma^F)p$ of firms 1 and 2 for each $F \in \mathbb{R}_+$ and $\gamma^F \in \{0, 1\}$ altogether form the subgame-perfect Nash equilibrium of the three-stage game under fixed fee licensing.*

Proof. Follows from (36), (37), (41), and (42). ■

4 Welfare Analysis

In this section, we ask the following questions: Is fixed fee licensing or royalty-licensing always beneficial for consumers? Does the innovator always prefer to sell the license for its superior technology under any arrangement of licensing or could it be better off keeping its innovation to itself? If licensing ever becomes optimal for firm 1, which arrangement would it prefer? To answer the first of these questions, we have to find how consumers' surplus is affected by licensing. As the two forms of licensing affect the supply functions differently, let us first consider the easier case of fixed fee licensing. As we have seen in equations (36) and (37), the supply function equilibrium under fixed fee licensing can be obtained from the equilibrium under royalty licensing by setting the royalty rate to zero. Correspondingly, the reaction functions of firm 1 and firm 2 under fixed fee licensing can be obtained, under the setting $r = 0$, from (22) and (25) as

$$\nu_1^F(\nu_2^F) = \frac{1 + \nu_2^F}{1 + \theta_1(1 + \nu_2^F)}. \tag{43}$$

and

$$\nu_2^F(\nu_1^F) = \frac{1 + \nu_1^F}{1 + \theta_2^F(1 + \nu_1^F)} \tag{44}$$

respectively. Figure 1 plots these reaction functions and the equilibrium slopes ν_1^{F*} and ν_2^{F*} obtained at their intersection.

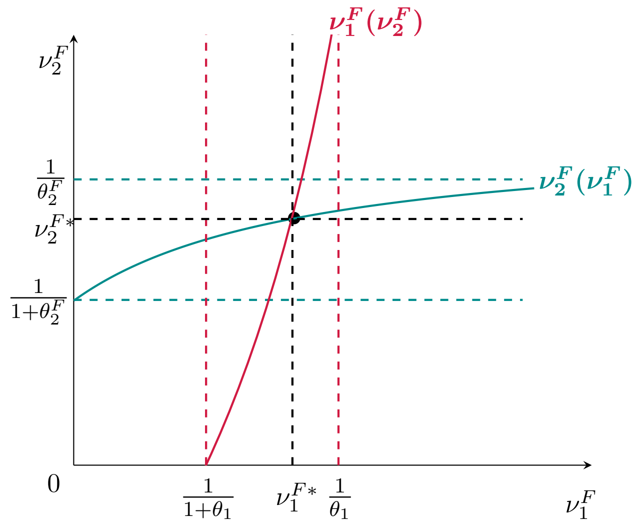

Figure 1. The Slopes of Equilibrium Supply Functions Under Fixed-Fee Licensing

Recall that the cost parameter of firm 2 is $\theta_2^F = \theta - \gamma^F x$, which is increasing in $\gamma^F \in \{0, 1\}$. So, the horizontal lines at $1/(1+\theta_2^F)$ and $1/\theta_2^F$ would each be drawn at a higher level if $\gamma^F = 1$ than if $\gamma^F = 0$, and consequently the blue-colored reaction curve of firm 2, $\nu_2^F(\nu_1^F)$, would be at a higher position if $\gamma^F = 1$. This implies the following remark.

Remark 1. The slopes ν_1^{F*} and ν_2^{F*} always attain a higher level when firm 2 buys the license sold by firm 1 at any fixed fee than when firm 2 does not buy the license.

Remark 1 has an implication on the equilibrium industry output and consumers' surplus under fixed fee licensing.

Proposition 6. *The equilibrium industry output and consumers' surplus are always higher under fixed fee licensing than under no licensing.*

Proof. Note that price p^* clears the product market under fixed fee licensing if $D(p^*) = S_1^F(p^*) + S_2^F(p^*)$ or $a - p^* = \nu_1^F p^* + \nu_2^F p^*$, implying $p^* = a/(1 + \nu_1^F + \nu_1^F)$. From Remark 1, we also know that the slopes of the equilibrium supply functions, $\nu_1^{F*}(F, \gamma^F)$ and $\nu_2^{F*}(F, \gamma^F)$, are higher if $\gamma^F = 1$ than if $\gamma^F = 0$, irrespective of the value of F. So, for all F it is true that the equilibrium price $p^* = 1/(a + \nu_1^{F*}(F, \gamma^F) + \nu_1^{F*}(F, \gamma^F))$ will be lower if $\gamma^F = 1$ than if $\gamma^F = 0$. Consequently, the equilibrium demand $D(p^*)$ and the equilibrium industry supply $Q^* = S_1^F(p^*) + S_2^F(p^*)$, which must be equal to each other, will be higher if $\gamma^F = 1$. Since the resulting consumers' surplus, $(Q^*)^2/2$, is increasing in Q^* and thereby increasing in γ^F, it follows that consumers' surplus is always higher under fixed fee licensing, irrespective of the fee, than under no licensing. ■

By Remark 1, the equilibrium supply functions of both firms have always higher slopes under fixed fee licensing than under no licensing. Given a fixed demand curve, the positive effect of fixed fee licensing on supply functions results in smaller equilibrium prices, and consequently in higher industry output and consumers' surplus.

Whether consumers also (always) prefer royalty licensing to no licensing is not easy to answer. The reason is that the royalty rate affects the equilibrium industry output in an ambiguous way. However, we can obtain an immediate result using Proposition 6. Since the equilibrium supply function chosen by any firm under fixed fee licensing is the same as it would be chosen under royalty licensing with $r = 0$, Proposition 6, along with the continuity of supply functions and profit functions, implies a welfare comparison at sufficiently small rates of royalty.

Let $\hat{r}$ denote a maximal threshold for r below which the industry supply curve is always steeper under royalty licensing than under fixed fee licensing; i.e.,

$$\hat{r} = \max\{r \in [0,1] : \sum_{i=1}^{2} \nu_i^{R*}(r',1) \geq \sum_{i=1}^{2} \nu_i^{F*}(0,1) \text{ for all } r' \leq r\}. \quad (45)$$

Note that $\hat{r} > 0$ since $\sum_{i=1}^{2} \nu_i^{R*}(r,1)$ is continuous in $r \in [0,1]$ and $\nu_i^{R*}(0,1) = \nu_i^{F*}(F,1) > \nu_i^{F*}(F,0) = \nu_i^{F*}(0,0)$ for all $F \in \mathbb{R}_+$ and $i \in \{1,2\}$. This leads to the following result.

Corollary 1. *If the equilibrium royalty rate r^* is such that firm 2 buys the license and $r^* \leq \hat{r}$, then the equilibrium industry output and consumers' surplus are always higher under royalty licensing than they would be under no licensing.*

According to (33), royalty licensing is observed if and only if $r^* = \bar{r}$ and $\Delta\pi_1^{R*}(r^*) \geq 0$, i.e., firm 1 prefers royalty licensing to the option of no licensing. If firm 2 buys the license, then r^* must be $\bar{r}$ ensuring zero profit to firm 2, i.e., $\Delta\pi_2^{R*}(\bar{r}) = 0$. So, Corollary 1 says that if the optimal royalty rate r^* chosen by firm 1 is such that $\Delta\pi_2^{R*}(r^*) = 0$, $\Delta\pi_1^{R*}(r^*) \geq 0$, and $r^* \leq \hat{r}$, then the equilibrium of the royalty licensing game leads to higher industry output and consumers' surplus than the supply function equilibrium with no licensing.

In the absence of any fee or royalty, firm 1 would become worse off under licensing, i.e., $\Delta\pi_1^{F*}(0) < 0$, as it would lose its cost advantage over firm 2. Oppositely, firm 2 would become better off, i.e., $\Delta\pi_2^{F*}(0) > 0$, if it had free access to the superior technology invented by firm 1. In order for any arrangement of licensing to take place, the necessary condition is that licensing must be bilaterally beneficial when it is free; i.e., its benefit to firm 2 in terms of additional profits due to lower costs must exceed its cost to firm 1 in terms of lost profits due to sharing its invention. Below, we will see that this industry-wide net benefit of licensing depends on how large the cost advantage x of firm 1 is with respect to the common cost parameter θ. First let us write $\Delta\pi_i^{F*}(F)$ as $\Delta\pi_i^{F*}(F, \theta, x)$ for any $i = 1, 2$, $F \in \mathbb{R}_+$, $\theta \in (0, a)$, and $x \in [0, \theta]$.

Assumption 1. Given the cost parameter θ, the cost advantage x of firm 1 is such that $\Delta\pi_1^{F*}(0, \theta, x) + \Delta\pi_2^{F*}(0, \theta, x) \geq 0$.

The above assumption ensures that if the license is free, the industry profits are higher when firm 1 shares its invention with firm 2 than when firm 1 keeps it to itself. If this assumption holds, firm 1 and firm 2 can always reach a bilaterally beneficial allocation under fixed fee licensing while the amount of fixed fee would affect the welfares of the two firms but not their production levels.

Let us define a threshold value $\bar{x}(\theta)$ such that Assumption 1 holds if and only if $x \leq \bar{x}(\theta)$; i.e.,

$$\bar{x}(\theta) = \inf \left\{ x \in (0, \theta] : \sum_{i=1}^{2} \Delta\pi_i^{F*}(0, \theta, x') < 0 \text{ if and only if } x' > x \right\}. \quad (46)$$

Unfortunately, we cannot explicitly calculate $\bar{x}(\theta)$ analytically because of the complexity of our problem. However, computer calculations illustrated in Figure 2 reveal that $\bar{x}(\theta)/\theta$ has a unique graph (independent of the demand size parameter a) and moreover it is increasing in θ and it never exceeds θ. With these observations Figure 2 shows that there are situations in which Assumption 1 holds as well as situations in which it does not.

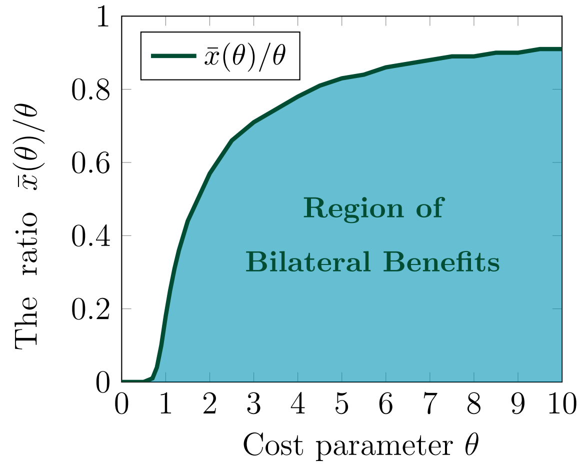

Figure 2. Region of Bilateral Benefits.

In our setup, firm 1 is assumed to have the ability to make a take-it-or-leave-it offer to firm 2, allowing firm 1 to fully exploit all the benefits of licensing. Under fixed fee licensing, firm 1 achieves this by demanding from firm 2 the highest agreeable fee of $F^* = \Delta\pi_2^{F*}(0, \theta, x)$. Therefore Assumption 1 is also sufficient for the license to be sold under fixed fee licensing. But this is not true in the case of royalty licensing, since the equilibrium royalty rate r^* is always positive as shown by (33). Thus, two firms engage in royalty licensing with the equilibrium royalty rate r^* if and only if the induced industry-wide net benefit is positive; i.e., $\Delta\pi_1^{R*}(r^*, \theta, x) + \Delta\pi_2^{R*}(r^*, \theta, x) \geq 0$. In order to answer whether or not this condition holds, we have to find how the function $\Delta\pi_1^{R*}(r, \theta, x) + \Delta\pi_2^{R*}(r, \theta, x)$ behaves with respect to r.

Proposition 7. *A change in the royalty rate r affects the supply functions of firms in opposite directions: The slope $\nu_1^{R*}(r, 1)$ is decreasing, while $\nu_2^{R*}(r, 1)$ is increasing, in r.*

Proof. Recall that when $\gamma^R = 1$, $\theta_2^R = \theta_1 = \theta - x$. Differentiating (9) with respect to r we obtain

$$\frac{\partial \nu_1^{R*}(r,1)}{\partial r} = \frac{1}{2a_1}\left(\frac{-(\partial b_1/\partial r)\left(\sqrt{b_1^2 - 4a_1c_1} - b_1\right) - 8a_1}{\sqrt{b_1^2 - 4a_1c_1}}\right).$$

We observe that the sign of the above derivative is equal to the sign of $\Gamma = -(\partial b_1/\partial r)\left(\sqrt{b_1^2 - 4a_1c_1} - b_1\right) - 8a_1 = 2\theta_1\left(\sqrt{b_1^2 - 4a_1c_1} - b_1\right) - 8\theta_1(2+\theta_1)$. Note that $\Gamma < 0$ if and only if $\sqrt{b_1^2 - 4a_1c_1} < b_1 + 4(2+\theta_1)$ or equivalently if and only if $\psi \equiv a_1c_1 + 2(2+\theta_1)b_1 + 4(2+\theta_1)^2$ is positive. Inserting a_1, b_1, and c_1 from (11), (12), and (13) into the above equation we get $\psi = -(4\theta_1^2 - 4r\theta_1 + 4\theta_1 + \theta_1^3 - 4r\theta_1^2) + 4{\theta_1}^2 - 8r\theta_1 + 8\theta_1 + 2\theta_1^3 - 4r{\theta_1}^2 + 4\theta_1^2 + 16 + 16\theta_1 + 4\theta_1^2$, which simplifies to $\psi = 4(1-r)\theta_1 + \theta_1^3 + 16 + 16\theta_1 + 8\theta_1^2$, which is always positive since $r \in [0,1]$. Therefore, $\partial\, \nu_1^{R*}(r,1)/\partial r < 0$.

Now, we will calculate the sign of $\partial\, \nu_2^{R*}(r,1)/\partial r$. Let $\nu_2^{R*} = \nu_2^{R*}(r,1)$. Then, equation (10) implies

$$\nu_2^{R*} = \left(-b_2 + \sqrt{b_2^2 - 4a_2c_2}\right)/2a_2.$$

It follows that

$$b_2 + 2a_2\nu_2^{R*} = \sqrt{b_2^2 - 4a_2c_2}.$$

Taking the square of both sides of the above equality and rearranging yields

$$b_2\nu_2^{R*} + a_2(\nu_2^{R*})^2 = -c_2.$$

The total differential of the above equation implies

$$\frac{\partial b_2}{\partial r}\nu_2^{R*} + b_2\left(\frac{\partial \nu_2^{R*}}{\partial r}\right) + 2a_2\nu_2^{R*}\left(\frac{\partial \nu_2^{R*}}{\partial r}\right) + \frac{\partial a_2}{\partial r}\left(\nu_2^{R*}\right)^2 = -\frac{\partial c_2}{\partial r}$$

or

$$\begin{aligned}\left(b_2 + 2a_2\nu_2^{R*}\right)\frac{\partial \nu_2^{R*}}{\partial r} &= -\frac{\partial c_2}{\partial r} - \frac{\partial b_2}{\partial r}\nu_2^{R*} - \frac{\partial a_2}{\partial r}(\nu_2^{R*})^2 \\ &= 2\theta_1\nu_2^{R*} + 4\theta_1(\nu_2^{R*})^2,\end{aligned}$$

further implying

$$\frac{\partial \nu_2^{R*}}{\partial r} = \left(\frac{1 + 2\nu_2^{R*}}{b_2 + 2a_2\nu_2^{R*}} \right) 2\theta_1 \nu_2^{R*}.$$

Since ν_2^{R*}, b_2, and a_2 are all positive, $\partial\, \nu_2^*(r,1)/\partial r > 0$. ■

Note that under royalty licensing the equilibrium price in equation (5) can be rewritten as

$$p^{R*}(r, \gamma^R) = \frac{a}{1 + \nu_1^*(r, \gamma^R) + \nu_2^*(r, \gamma^R)}. \tag{47}$$

To find the effect of r on $p^{R*}(r,1)$ we need to find its effect on $\nu_1^{R*}(r,1) + \nu_2^{R*}(r,1)$. This second effect cannot be obtained with the help of Proposition 7 where we show that $\nu_1^{R*}(r,1)$ and $\nu_2^{R*}(r,1)$ move in opposite directions as r changes. However, we can characterize a necessary and sufficient condition under which the effect of r on $\nu_1^{R*}(r,1)$ dominates the effect on $\nu_2^{R*}(r,1)$.

Lemma 1. *The slope of the industry supply curve, $\nu_1^{R*}(r,1) + \nu_2^{R*}(r,1)$, is increasing in r if and only if this slope is less than $(\nu_2^{R*}(r,1))^2 - 1$.*

Proof. Note that when $\gamma^R = 1$, $\theta_1 = \theta_2 = \theta - x$. Let $\nu_1^{R*} = \nu_1^{R*}(r,1)$ and $\nu_2^{R*} = \nu_2^{R*}(r,1)$. Taking the logarithm of equation (25) when $\gamma^R = 1$, we obtain

$$\ln(\nu_2^{R*}) = \ln\left(1 + \nu_1^{R*}\right) - \ln\left(1 - 2r + (\theta - x)(1 + \nu_1^{R*})\right).$$

The total differential with respect to r yields

$$\frac{1}{\nu_2^{R*}} \frac{\partial\, \nu_2^{R*}}{\partial\, r} = \frac{1}{1 + \nu_1^{R*}} \frac{\partial\, \nu_1^{R*}}{\partial\, r} - \frac{1}{1 - 2r + (\theta - x)(1 + \nu_1^{R*})} \frac{\partial\, \nu_1^{R*}}{\partial\, r}.$$

It follows that

$$\frac{\partial\,\nu_2^{R*}}{\partial\,r} = \frac{\nu_2^{R*}}{1+\nu_1^{R*}}\frac{\partial\,\nu_1^{R*}}{\partial\,r} - \frac{(\nu_2^{R*})^2}{1+\nu_1^{R*}}\frac{\partial\,\nu_1^{R*}}{\partial\,r}$$

and

$$\frac{\partial\,(\nu_1^{R*}+\nu_2^{R*})}{\partial\,r} = \left(1+\frac{\nu_2^{R*}}{1+\nu_1^{R*}} - \frac{(\nu_2^{R*})^2}{1+\nu_1^{R*}}\right)\frac{\partial\,\nu_1^{R*}}{\partial\,r}$$

or

$$\frac{\partial\,(\nu_1^{R*}+\nu_2^{R*})}{\partial\,r} = \left(\frac{1+\nu_1^{R*}+\nu_2^{R*}-(\nu_2^{R*})^2}{1+\nu_1^{R*}}\right)\frac{\partial\,\nu_1^{R*}}{\partial\,r}.$$

Since $\partial\,\nu_1^{R*}/\partial\,r < 0$ for all $r \in [0,1]$, we have $\partial\,(\nu_1^{R*}+\nu_2^{R*})/\partial\,r > 0$ if and only if $\nu_1^{R*}+\nu_2^{R*} < (\nu_2^{R*})^2 - 1$. ■

Now, define for each θ and x the largest threshold $\hat{r}(\theta, x) \in [0,1]$ under which the slope of the industry supply is increasing in r for all $r \leq \hat{r}(\theta, x)$; i.e.,

$$\hat{r}(\theta, x) = \begin{cases} \max\{r : r \in \hat{R}(\theta, x)\} & \text{if } \hat{R}(\theta, x) \neq \emptyset \\ 0 & \text{otherwise,} \end{cases} \tag{48}$$

where

$$\hat{R}(\theta, x) = \{r \in [0,1] : \nu_1^{R*}(r') + \nu_2^{R*}(r') < (\nu_2^{R*}(r'))^2 - 1 \text{ for all } r' \leq r\}. \tag{49}$$

Using the threshold $\hat{r}(\theta, x)$ we can propose the following sufficiency result.

Proposition 8. *The output of firm 2 and the industry output as well as consumers' surplus are higher, whereas the output of firm 1 is lower, under royalty licensing than under fixed fee licensing if the equilibrium royalty rate r^* is less than $\hat{r}(\theta, x)$.*

Proof. Assume $r^* < \hat{r}(\theta, x)$. Then by equation (33), we must have $r^* = \bar{r}$. Therefore, $\bar{r} < \hat{r}(\theta, x)$. By Lemma 1 and equations (48)-(49), we have

$\partial(\nu_1^{R*}(r,1)+\nu_2^{R*}(r,1))/\partial r > 0$ for all $r < \hat{r}(\theta,x)$. So, $\nu_1^{R*}(\bar{r},1)+\nu_2^{R*}(\bar{r},1) > \nu_1^{R*}(0,1)+\nu_2^{R*}(0,1)$. Equation (5) implies that $p^{R*}(\bar{r}) < p^{R*}(\bar{0})$. Therefore, $D(p^{R*}(\bar{r})) > D(p^{R*}(\bar{0}))$, since the function D is downward sloping by assumption. By market clearing conditions we can replace the demand at each side of the previous inequality by the industry supply, yielding $Q^{R*} = S_1^R(p^{R*}(\bar{r})) + S_2^R(p^{R*}(\bar{r})) > S_1^R(p^{R*}(0)) + S_2^R(p^{R*}(0)) = Q^{F*}$. Then, it follows that $CS^{R*} = (Q^{R*})^2/2 > (Q^{F*})^2/2 = CS^{F*}$. Thus, the equilibrium industry output and consumers' surplus are both higher under royalty licensing than under fixed fee licensing. Also, note that Proposition 7 implies that $\nu_1^{R*}(\bar{r},1) < \nu_1^{R*}(0,1)$ and $\nu_2^{R*}(\bar{r},1) > \nu_1^{R*}(0,1)$. Since we already found that $p^{R*}(\bar{r}) < p^{R*}(\bar{0})$, it must be true that $Q_1^{R*} = p^{R*}(\bar{r})\nu_1^{R*}(\bar{r},1) < p^{R*}(0)\nu_1^{R*}(0) = Q_1^{F*}$. As we earlier found that $Q^{R*} = Q_1^{R*} + Q_2^{R*} > Q_1^{F*} + Q_2^{F*} = Q^{F*}$, we must have $Q_2^{R*} > Q_2^{F*}$. Thus, we have established that the output of firm 1 is lower and the output of firm 2 is higher under royalty licensing than under fixed fee licensing. ■

Note from equation (33) that the equilibrium royalty rate r^* can be less than $\hat{r}(\theta,x)$ only if $r^* = \bar{r}$ and that is the case if $\Delta\pi_1^{R*}(\bar{r}) \geq 0$. So, the sufficiency condition in Proposition 8 holds only if $\bar{r} < \hat{r}(\theta,x)$ and $\Delta\pi_1^{R*}(\bar{r}) \geq 0$. We should also note that Propositions 6 and 8 together enable consumers to fully rank royalty licensing, fixed fee-licensing, and no licensing. No licensing is always inferior to fixed fee licensing, whereas fixed fee licensing is inferior to royalty licensing if the royalty rate is sufficiently small; i.e., $r^* < \hat{r}(\theta,x)$.

Our results have also implications on the equilibrium profit of firm 1. As the licensing contract is offered by firm 1, it ensures maximizing its own profit by setting a royalty rate under which the equilibrium profit of firm 2 is the same as it would obtain under the equilibrium of fixed fee licensing. Therefore, firm 2 is indifferent between the two forms of license that can be offered by firm 1. Moreover, in the region of bilateral benefits (defined by

Assumption 1) the industry profits are higher under fixed fee licensing than under no licensing. Given the aforementioned indifference of firm 2, it is clear that firm 1 prefers fixed fee licensing to no licensing when the size of its cost advantage, x, is sufficiently low with respect to θ. It should also be clear that firm 1 can never be worse under royalty licensing than under no licensing since it has always the option of not selling its license by demanding an unacceptable rate of royalty.

What our previous results are missing is a ranking on the equilibrium profits of firm 1 under royalty licensing and fixed fee licensing. To identify this ranking, let us define a threshold $\hat{x}(\theta)$ below which firm 1 always prefers royalty licensing to fixed fee licensing, i.e.,

$$\hat{x}(\theta) = \max\{x \in (0, \theta] : \pi_1^{R*}(r^*, \theta, x') \geq \pi_1^{F*}(F^*, \theta, x') \text{ for all } x' \leq x\}. \quad (50)$$

Using computer calculations, we illustrate in Figure 3 the graph of $\hat{x}(\theta)/\theta$ for values of $\theta \in [0, 10]$.

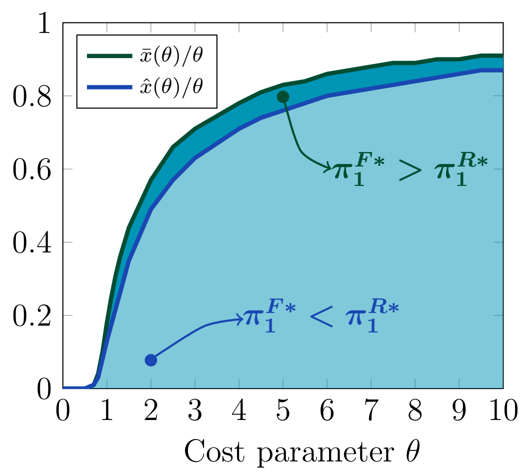

Figure 3. Preferences of Firm 1 on Licensing Arrangements.

Note that the dark blue curve, $\bar{x}(\theta)/\theta$, in Figure 3 draws the boundary of the region in which Assumption 1 holds and firm 1 strictly prefers fixed fee licensing to no licensing (as already illustrated in Figure 2). Moreover, under the dark-blue curve, fixed fee licensing always yields higher consumers' surplus and social welfare than royalty licensing. That is, for any (θ, x) such that $x < \bar{x}(\theta)$ we have $\Delta\pi_1^{F*}(0, \theta, x) > 0$, $\Delta\pi_2^{F*}(0, \theta, x) = 0$, $CS^{F*}(0, \theta, x) > CS^{R*}(0, \theta, x)$, and $SW^{F*}(0, \theta, x) > SW^{R*}(0, \theta, x)$. We also find that the dark-blue and light-blue curves can together characterize the preference relation of firm 1 on two licensing forms. In the dark-blue-shaded region, fixed fee licensing is more profitable for firm 1 whereas in the light-blue-shaded region, royalty licensing becomes more profitable.

5 Conclusion

In this paper we have dealt with the problem of licensing in a duopoly under supply function competition and studied the welfare effects of fixed fee licensing and royalty licensing in comparison with each other and with the case of no licensing. We have shown that the innovator firm may find it profitable to sell the license for its cost-reducing technology (by a take-it-or-leave-it contract) only if the size of its cost advantage is sufficiently low. Moreover, if the innovator finds fixed fee licensing more profitable than no licensing and sells its technology accordingly, then the equilibrium outputs of both firms and consequently the equilibrium industry output (which are all independent of the license fee) become higher than what could be attained in the absence of licensing. This also implies that consumers always prefer fixed fee licensing to the option of no licensing.

For royalty licensing, our results are more involving. The reason is that the equilibrium supply functions, hence the equilibrium output, of firms are not independent of the royalty rate like they are from the fixed fee. However, since in the extreme case where the royalty rate is zero, royalty licensing induces the same output allocations as in case of fixed fee licensing, we know that the welfare results under both arrangements of licensing must be the same in the limit. That is, if the equilibrium royalty rate is extremely small then the equilibrium industry output and consumers' surplus are always higher under royalty licensing (too) than they would be under no licensing. However, if the equilibrium royalty rate is not sufficiently low, then welfare analyses under two arrangements of licensing become independent. In this case, we can show that the royalty rate affects the equilibrium supply functions and outputs of the two firms in opposite directions. As a matter of fact, we find the effects to be negative for the innovator and positive for its rival. Given this conflict, the effect of the royalty rate on the equilibrium industry supply function, or output, becomes in general ambiguous. However, this ambiguity disappears if the equilibrium royalty rate is not very high.

Finally, we show that we can Pareto rank the two arrangements of licensing by the help of two distinct thresholds, $\hat{x}$ and $\bar{x}$ with $\hat{x} < \bar{x}$, that we put on the cost advantage of the inventor. Since in our model, the license contracts are always made by the innovator, independent of the licensed cost advantage the licensee is enforced under any arrangement of licensing to enjoy the same profit it would obtain under no licensing. Therefore, the licensee is indifferent between any arrangement of licensing and no licensing at all. On the other hand, the profit comparisons of the licensor is sensitive to the size of its cost advantage. The licensor prefers any arrangement of licensing to no licensing if and only if its cost advantage is smaller than the highest threshold value $\bar{x}$. Moreover, it prefers fixed fee licensing to royalty licensing if its cost advantage is mild; i.e., it falls in between the two thresholds $\hat{x}$ and $\bar{x}$. Finally, the licensor prefers royalty licensing to fixed fee licensing if its cost advantage is below the lowest of the thresholds; i.e., $\hat{x}$.

We should note that our results are different from the earlier findings in the literature where royalty licensing is preferred to fixed fee licensing by the innovator producer under Cournot competition (Wang 1998), Bertrand competition (Wang and Yang 1999), and Stackelberg competition (Filippini 2005). In contrast, we find that neither royalty nor fixed fee licensing is always superior for the innovator, whose preference on these arrangements depends on the size of its innovated cost advantage relative to the cost of its rival.

Although we have restricted ourselves to the case where licensing contracts are offered by the innovator entitling it to the whole producer benefit from licensing, one can also consider other forms of contracts by introducing a (non-dictatorial) cooperative or non-cooperative bargaining process between the duopolists. Future research may also study whether our results are robust with respect to the presence of asymmetric information, moral hazard problem, product differentiation, risk aversion, and strategic delegation among many other directions.

References

Bousquet, A., Cremer, H., Ivaldi, M., Wolkowicz, M., 1998. Risk sharing in licensing. International Journal of Industrial Organization, 16, 535-554.

Day, C.J., Hobbs, B.F., Pang, J.-S., 2002. Oligopolistic competition in power networks: a conjectured supply function approach. IEEE Transactions on Power Systems, 17:3, 597-607.

Escrihuela-Villar, M., Gutiérrez-Hita, C., and Vicente-Pérez J., 2020. Supply function competition in a mixed electric power market. Journal of Public Economic Theory, 22, 1151-1175.

European Commission, 2015. Energy union package: a framework strategy for a resilient energy union with a forward-looking climate change policy.

Filippini, L., 2005. Licensing contract in a Stackelberg model. The Manchester School, 73:5, 582-598.

Green, R., Newbery, D., 1992. Competition in the British electricity spot market. Journal of Political Economy, 100, 929-953.

Grossman, S., 1981. Nash equilibrium and the industrial organization of markets with large fixed costs. Econometrica, 49, 1149-1172.

Kamien, M.I., Tauman, Y., 1986. Fees versus royalties and the private value of a patent. Quarterly Journal of Economics, 101, 471-491.

Kamien, M.I., Oren, S.S., Tauman, Y., 1992. Optimal licensing of cost-reducing innovation. Journal of Mathematical Economics, 21, 483-508.

Kamien, M.I., Tauman, Y., 2002. Patent licensing: the inside story. Manchester School, 70, 7-15.

Katz, M.L., Shapiro, C., 1986. How to license intangible property. Quarterly Journal of Economics, 101, 567-589.

Klemperer, P.D., Meyer, M.A., 1989. Supply function equilibria in oligopoly under uncertainty. Econometrica, 57, 1243-1277.

Newbery, D.M., Greve, T., 2017. The strategic robustness of oligopoly electricity market models. Energy Economics, 68, 124-132.

Rostoker, M., 1984. A survey of corporate licensing. IDEA: Journal of Law and Technology, 24, 59-92.

Rudkevich, A., Duckworth, M., 1998. Strategic bidding in a deregulated generation market: implications for electricity prices, asset valuation and regulatory response. The Electricity Journal, 11, 73-78.

San Martín, M., Saracho, A.I., 2010. Royalty licensing. Economics Letters, 107, 284-287.

Sen, D., 2005. Fee versus royalty reconsidered. Games and Economic Behavior, 53:1, 141-7.

Vishwasrao, S., 2007. Royalties vs. fees: how do firms pay for foreign technology? International Journal of Industrial Organization, 25, 741-759.

Wang, X.H., 1998. Fee versus royalty licensing in a Cournot duopoly model. Economics Letters, 60, 55-62.

Wang, X. H., Yang, B.Z., 1999. On licensing under Bertrand competition. Australian Economic Papers, 38, 106-119.