Value of Life and Annuity Demand

Pashchenko, Svetlana and Porapakkarm, Ponpoje

University of Georgia, National Graduate Institute for Policy Studies (GRIPS, Tokyo)

15 April 2021

Value of Life and Annuity Demand

Svetlana Pashchenko, and Ponpoje Porapakkarm*

April 15, 2021

Abstract

How does the value of life affect annuity demand? To address this question, we construct a portfolio choice problem with three key features: i) agents have access to life-contingent assets, ii) they always prefer living to dying, iii) agents have non-expected utility preferences. We show that as utility from being alive increases, annuity demand decreases (increases) if agents are more (less) averse to risk rather than to intertemporal fluctuations. Put differently, if people prefer early resolution of uncertainty, they are less interested in annuities when the value of life is high. Our findings have two important implications. First, we get a better understanding of the well-known annuity puzzle. Second, we argue that the observed low annuity demand provides evidence that people prefer early rather than late resolution of uncertainty.

Keywords: annuities, value of a statistical life, portfolio choice problem, life-contingent assets, longevity insurance

JEL Classification Codes: D91, G11, G22

*Pashchenko: University of Georgia, email: svetlanap.econ@gmail.com. Porapakkarm: National Graduate Institute for Policy Studies (GRIPS), email: p-porapakkarm@grips.ac.jp. Thor Watson provided excellent research assistance.

1 Introduction

How does the value of life affect investments in life-contingent assets? Standard portfolio choice theory with survival uncertainty, going back to Yaari (1965), usually sidesteps the issue of the value of life by not requiring the agent's utility when alive to be greater than when he is dead.[1] Empirical evidence shows, however, that people require substantial compensation for an increase in mortality risk, implying that the value of life is large (see Viscusi, 1993, for an extensive review). This, in turn, has dramatic effects on the welfare assessment of issues involving changes in health and longevity (Murphy and Topel, 2006; Hall and Jones, 2007).

Our goal in this paper is to analyze a portfolio choice problem in an environment where life is valuable but survival is uncertain. Specifically, we study how explicitly incorporating the value of life changes our understanding of consumers' decisions to invest in assets with survival-contingent payoffs such as annuities or life insurance.

The starting point of such an analysis is how to think about the utility of life. We take a stand that this utility is an additional component to overall utility that depends only on whether an individual is alive but not on his consumption, income, asset holding, etc. Put differently, intra-period utility from being alive represents a non-pecuniary element of an individuals' welfare not captured by other pecuniary factors.[2]

In a standard additive expected utility framework, this non-pecuniary felicity from being alive does not change consumption/savings decisions because it does not affect the marginal utilities of consumption or wealth. However, this is not necessarily the case with a more general preferences specification. In our analysis, we adopt a non-expected utility framework (Kreps and Porteus, 1978). The attractive feature of these preferences is that unlike the standard expected utility case, they allow to separately model aversion to risk

[1] In fact, in many parametrizations of such models, people have higher utility in the state of death. This happens, for example, if utility over consumption is of the constant relative risk aversion (CRRA) type with the coefficient of risk aversion set above one, and the utility in death is set equal to zero.

[2] In this approach, we follow Hall and Jones (2007). An alternative approach is to re-normalize disutility from being dead instead of assuming extra utility from being alive. Rosen (1988) shows that these two approaches are equivalent.

and aversion to intertemporal substitution. This feature adds an important dimension to the analysis: even though the utility from being alive does not affect the intra-period marginal utility, it does introduce additional fluctuations in utility both over time and over states of the world. This, in turn, matters for decisions of consumers who have different attitudes towards these two types of fluctuations.

In this framework, we study how consumers allocate their investments between survival-contingent assets. In the standard expected utility framework, the trade-off in this decision depends on the marginal benefits of having additional resources in each state: marginal utility of consumption if alive and marginal utility of bequests if dead. In the non-expected utility case, there is an additional consideration, which, following Weil (1990), we call the trade-off between safety and stability of utility. Importantly, each life-contingent asset can potentially affect this trade-off in a different way.

We proceed in several steps. First, we provide a general characterization of how changes in (intra-period) utility of being alive affect the relative benefits of allocating resources to states when alive versus when dead. We derive conditions that determine the sign of the corresponding change in annuity demand. We show that if agents are more (less) averse to risk than to intertemporal fluctuations, annuity demand decreases (increases) when intra-period utility of being alive increases. This happens because an increase in intra-period utility from being alive increases differences in utilities both across states of the world and over time. Annuity investments accentuate the former difference but can smooth the latter.

Second, in the same general framework, we turn to the concept of the value of a statistical life (VSL), which represents the willingness to pay to marginally reduce mortality risk and is commonly used in the health and longevity literature (e.g., Cordoba and Ripoll, 2017). We show that the relationship between intra-period utility of being alive and VSL is not necessarily positive. A negative relationship can arise when preferences are such that intertemporal fluctuations are disliked more than risk. As mentioned above, this also coincides with the situation where people invest more in annuities as intra-period utility of being alive increases. Thus, when agents prefer stability over safety, the following somewhat paradoxical result can arise: as felicity from being alive increases, people are willing to pay less to extend their life, but at the same time they reallocate their portfolios towards assets that pay off only when they are alive.

Third, we apply our analysis to three parametrizations of non-expected utility, namely, Epstein-Zin-Weil (Epstein and Zin, 1989; Weil, 1990), risk-sensitive (Hansen and Sargent, 1995) and disappointment aversion version of Chew-Deckel preferences (Chew, 1983; Dekel, 1986; Gul, 1991). The application of our approach to Epstein-Zin-Weil preferences allows for the most intuitive interpretation of our results: we show that annuity demand decreases (increases) with intra-period utility of being alive if the coefficient of relative risk aversion is higher (lower) than the inverse of the elasticity of intertemporal substitution (IES). This result is of a particular interest because the relationship between these two parameters determines agents' attitudes toward the timing of the resolution of uncertainty. Our result, thus, can be restated as follows: when agents prefer early (late) resolution of uncertainty, they are less (more) interested in annuities as intra-period utility of being alive increases.

In the final part of the paper, we provide a quantitative illustration of our theoretical findings using a retirement saving model where agents have access to the private annuity market. For this illustration, we choose Epstein-Zin-Weil preferences since it is most commonly used non-expected utility parametrizations in macroeconomics and finance. Using the distribution of retirees by wealth and pension income from the Health and Retirement Study (HRS), we simulate how annuity demand changes with the value of life. We show that if risk aversion is above the inverse of the IES, the percentage of people buying annuities quickly decreases as the VSL increases. Moreover, the demand for annuities is almost completely eliminated even for relatively low values of VSL. Put differently, when people prefer early resolution of uncertainty, the low demand for annuities can, to a significant extent, be accounted for by the fact that people derive utility from being alive, and to arrive to this result we do not need unrealistically high values of VSL.

The last result offers important insight into the long-standing annuity puzzle. The essence of this puzzle is that a standard life-cycle model predicts people should annuitize a substantial fraction (if not all) of their wealth (Yaari, 1965), while in reality, the demand for annuities is low. A number of explanations have been put forward to account for this discrepancy. The prominent explanations include, for example, bequest motives, market frictions, crowding out by Social Security, medical expenses, means-tested benefits, and high degree of impatience (Butler et al., 2017; Dushi and Webb, 2004; Inkman et al., 2011; Mitchell et al., 1999; Lockwood, 2012; Pashchenko, 2013; Pashchenko and Porapakkarm, 2019; Peijnenburg et al., 2017; Reichling and Smetters, 2015; Turra and Mitchell, 2008). Several studies have investigated the puzzle in the framework with non-standard preferences while allowing life to be valuable. Bommier and Le Grand (2012) use the expected utility framework with the concavification of the lifetime utility function to show that an increase in risk aversion leads to lower demand for annuities. Bommier et al. (2020) further show that this result holds in a framework with risk-sensitive preference when the value of life is positive. We contribute to this line of research by showing theoretically that demand for annuities is affected by the interplay between i) agents' attitudes toward uncertainty and intertemporal fluctuations; and ii) intra-period utility of being alive. Moreover, our simulations with Epstein-Zin-Weil preferences show that this mechanism is quantitatively important.

From another angle, we can also state that in light of our findings, the low demand for annuities can be considered as evidence that people prefer early resolution of uncertainty. The issue of whether empirical evidence supports preferences for early or late resolution of uncertainty is not entirely resolved. On the one hand, support for early resolution of uncertainty comes from three sources. First are the direct estimates of the risk aversion and the elasticity of intertemporal substitution from consumption data using an Euler equation. These estimates find that the former exceeds the inverse of the latter (Attanasio and Weber, 1989; Chen et al., 2013; Vissing-Jorgensen and Attanasio, 2003). Second are the results from controlled experiments where people choose between different lotteries and preferences are elicited from their choices. These experiments show that even though people have heterogeneous preferences, on average they prefer early resolution (Brown and Kim, 2014; Meissner and Pfeiffer, 2018). Finally, studies in macro-finance show that in order to account for a numbers of features of asset markets, such as the equity premium puzzle, people should prefer early resolution of uncertainty (Bansal and Yaron, 2004; Huang and Shaliastovich, 2013; Malloy et al., 2009; Yogo, 2006).

On the other hand, evidence that people may prefer late resolution of uncertainty comes from studies in health economics. A number of studies in this field show that people avoid learning about the true state of their health when it comes to serious illness (see, for example, Oster et al., 2013 for the case of genetic testing for Huntington disease or Kellerman et al., 2002 for HIV testing; see also Cordoba and Ripoll, 2017 for an excellent review of such evidence).

It is important to mention our relationship to several strands of literature not discussed above. Our paper belongs to a broad class of studies on saving and portfolio choice in the presence of survival uncertainty. These studies can be divided into four groups based on whether mortality is assumed to be exogenous or endogenous, and whether preferences are standard additive expected utility or of a more general type.

The literature in the first category (exogenous mortality and standard preferences) is very substantial and includes, among others, seminal work on saving behavior (Hubbard et al., 1994; De Nardi et al., 2010). As we mentioned earlier, these studies typically abstract from the value of life; i.e., they do not impose a constraint that individuals are better off being alive since, in the context of standard preferences and exogenous mortality, assuming life is valuable usually does not add any new insights. One exception is De Nardi et al. (2018) who explicitly incorporate the value of life in a structural consumption/saving model with exogenous mortality in order to understand the non-pecuniary implications of deteriorating health.

Among the literature in the second category (exogenous mortality and non-standard preferences) it is more common to encounter studies that incorporate the value of life. This happens because more general preferences oftentimes lead to non-trivial implications of treating life as valuable. For example, Bommier and Villeneuve (2012) show that in a model with standard preferences people are risk-neutral to mortality risk, and allowing for non-additive preferences can introduce mortality risk aversion which is important to take into account in many policy applications. Cordoba and Ripoll (2017) provide a detailed illustration of the advantages of using a non-expected utility approach when modeling the value of life.

In the last two categories of studies (with endogenous mortality) incorporating the value of life is crucial because otherwise agents will deliberately increase their mortality. A common approach in this literature, starting from the seminal work of Hall and Jones (2007), is to add a constant to an otherwise standard utility function to ensure life is preferred to death (e.g., Eslami and Karimi, 2019; Fonseca et al., 2020; Nygaard, 2019; Ozkan, 2017). Two alternative approaches is to introduce utility penalty from death (Hugonnier et al., 2013, 2020) or to assume that death happens when health declines below a certain level while assuming health enters the utility function as a necessary good (Yogo, 2016).

The rest of the paper is organized as follows. Section 2 describes the environment and derives our main results. Section 3 applies our approach to several common parametrizations of non-expected utility preferences. Section 4 discusses the implication of our results for the debate about preferences for the timing of the resolution of uncertainty. Section 5 provides a quantitative illustration. Section 6 concludes.

2 Model

We start by considering the demand for annuities in an environment with a general preferences specification that allows us to separately characterize agents' attitudes towards fluctuations in utility across states of the world and over time. In the first step of our analysis, we do not impose parametric assumptions on preferences but rather show how demand for annuities depends on general properties of these preferences when individuals derive utility from being alive.

2.1 Preferences

Consider an environment where agents' preferences are characterized by a triple of functions (w, f, g), where $w(\cdot)$ is the intra-period utility function, $g(\cdot)$ determines the intertemporal aggregation rule and $f(\cdot)$ determines the uncertainty aggregation rule.

Denoting the value function of an agent at time t as V_t, we can write the preferences in recursive form as follows:

$$V_t = g^{-1}\Big[(1-\beta)g(w) + \beta g(z_{t+1})\Big],$$

where β is the discount factor and z_{t+1} is the certainty equivalent:

$$z_{t+1} = f^{-1}\Big[\sum_{i=1}^{I} p_i f(V_{t+1}^i)\Big]$$

Here, p_i is the probability of outcome i next period resulting in the value function V_{t+1}^i, $i = 1, ..., I$.

We assume that $w(\cdot)$ is continuous, and $f(\cdot)$ and $g(\cdot)$ are strictly increasing, concave and twice continuously differentiable.[3]

Before we proceed, it is important to note the followings regarding our preferences structure. First, when $f(\cdot) = g(\cdot)$, we are dealing with the standard expected utility case. Second, when $f(\cdot) \neq g(\cdot)$, we can disentangle agents' attitudes towards inter- and intra-temporal utility fluctuations. Specifically, an agent's attitude toward utility fluctuations over time is determined by the function $g(\cdot)$, while his attitude toward utility fluctuations across states of the world is determined by the function $f(\cdot)$.[4]

2.2 Portfolio choice problem

To illustrate how the value of life can affect demand for annuities, we incorporate the preferences described above into a portfolio choice problem. There is only one type of uncertainty, which is the uncertainty in survival: with probability s_t an agent is still alive in period $t+1$ (conditional on being alive in period t), and with probability $1-s_t$ he is dead in period $t+1$. Denote the corresponding value functions as V^a_{t+1} and V^d_{t+1}, respectively.

Assume that an agent can invest in two state-contingent assets. The first type of asset delivers gross return R^a_{t+1} next period if an agent is alive and nothing otherwise, while the second type of asset delivers gross return R^d_{t+1} only in the state when an agent is dead and nothing otherwise.[5] Denote the current holding of Type 1 assets as k^a_t and of Type 2 assets as k^d_t.

[3]As an example, consider two well-known parametrizations of the non-expected utility preferences commonly used in macroeconomics and finance. The first is Epztein-Zin-Weil parametrization, where both $g(\cdot)$ and $f(\cdot)$ are assumed to be constant elasticity of substitution functions. The second is risk-sensitive preferences (Hansen and Sargent, 1995), where $g(x) = x$ and $f(x) = -\frac{1}{k}exp(-kx)$. Both parametrizations are discussed in detail in Section 3.

[4]It is important to stress that $g(\cdot)$ determines an agent's attitude toward intertemporal *utility* fluctuations. His attitude toward intertemporal *consumption* fluctuations depends not only on $g(\cdot)$ but also on the intra-period utility function $w(\cdot)$.

[5]The Type 1 asset can be thought of as an annuity and the Type 2 asset as life insurance. A more common approach in the literature is to study the demand for annuities in the environment when people can invest in regular risk-free bonds as an alternative to annuity. Our results can easily be extended to that framework as well (as we will show later); however, our specification allows for a clearer illustration.

Note that if an agent does not place any value on having assets in the state when he is dead, the portfolio choice problem becomes trivial as he will allocate 100% of his resources to Type 1 assets. We assume that agents have a bequest motive; and we denote utility from leaving a bequest in the amount k_t^d as $\mho(k_t^d)$, where $\mho(\cdot)$ is increasing and continuously differentiable. We further assume that $\mho(0) = \underline{\mho} > -\infty$.[6]

Denoting an agent's time t consumption as c_t, we can summarize the environment as follows: if alive, an agent derives utility from consuming c_t; otherwise, he enjoys utility from leaving a bequest $k_t^d R_t^d$.

To ensure that a state when an agent is alive always brings higher utility than a state when he is dead, we assume that being alive brings additional utility.[7] We model this additional utility by introducing a constant b, such that when $b = 0$, we go back to the standard framework where life is not necessarily valuable, and $\partial V_t^a / \partial b > 0$, i.e., the higher is b, the more valuable it is to be alive.

We specify the intra-period utility as follows:

$$w(\cdot) = \begin{cases} \psi(c_t, b) & \text{if an agent is alive in period } t \\ \mho(k_t^d) & \text{otherwise }^{8} \end{cases}$$

Assumption 1 $\dfrac{\partial g(\psi(c_t, b))}{\partial c_t}$ does not depend on b, i.e., the g-transformed marginal utility of consumption does not depend on b.

[6] This assumption is common in the literature since otherwise even the poorest agents are compelled to leave a bequest (see De Nardi, 2004). In Appendix A we show that this assumption does not affect our results.

[7] Note that without this assumption, it is not guaranteed that an agent who is alive has higher utility than an agent who dies, i.e., life is not necessarily valuable. For example, assuming CRRA utility over consumption and setting $\underline{\mho} = 0$, we have a situation where an agent who left no bequest has utility equal to zero when he dies and negative utility when he is alive (assuming the coefficient of relative risk aversion is greater than one). We discuss in more details the interpretation of the parameter b in a context with a bequest motive in Appendix B.

[8] Note that the actual amount of bequeathed assets is $k_t^d R_t^d$. We omit R_t^d in the bequest function to make notation less cumbersome.

We will show below that this assumption ensures that in the standard expected utility framework, parameter b does not affect consumption/savings decisions.

Denoting per-period income of an agent as y_t, we can write the individual's optimization problem as follows:

$$V_t^a(k_t^a) = \max_{k_{t+1}^a, k_{t+1}^d} \left\{ g^{-1}\Big[(1-\beta)g(\psi(c_t,b)) + \beta g(z_{t+1})\Big] \right\}$$

$$z_{t+1} = f^{-1}\Big[s_t f(V_{t+1}^a(k_{t+1}^a)) + (1-s_t) f(V_{t+1}^d(k_{t+1}^d))\Big]$$

$$\text{s.t} \qquad c_t + k_{t+1}^a + k_{t+1}^d = k_t^a R_t^a + y_t$$

$$V_t^d(k_t^d) = g^{-1}\Big[(1-\beta)g(\mho(k_t^d))\Big] \; ^{9}$$

For the ease of exposition, we introduce the following notations:

$$\psi(c_t,b) \equiv \psi_t, \qquad \mho(k_t^d) \equiv \mho_t.$$

Using these notations, we can write the first-order conditions for the investments in k_{t+1}^a and k_{t+1}^d as follows:

$$(1-\beta)\frac{\partial g(\psi_t)}{\partial c_t} = \beta \frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}} \; s_t \frac{\partial f(V_{t+1}^a)}{\partial V_{t+1}^a} \; \frac{\partial V_{t+1}^a}{\partial k_{t+1}^a}, \tag{1}$$

$$(1-\beta)\frac{\partial g(\psi_t)}{\partial c_t} = \beta \frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}} \; (1-s_t) \frac{\partial f(V_{t+1}^d)}{\partial V_{t+1}^d} \; \frac{\partial V_{t+1}^d}{\partial k_{t+1}^d}. \tag{2}$$

We can use the envelop theorem to find $\partial V_{t+1}^a/\partial k_{t+1}^a$ and $\partial V_{t+1}^d/\partial k_{t+1}^d$:

$$\frac{\partial V_{t+1}^a}{\partial k_{t+1}^a} = \frac{(1-\beta)\;\partial g(\psi_{t+1})/\partial c_{t+1}\; R_{t+1}^a}{\partial g\left(V_{t+1}^a\right)/\partial V_{t+1}^a}, \tag{3}$$

$$\frac{\partial V_{t+1}^d}{\partial k_{t+1}^d} = \frac{(1-\beta)\;\partial g(\mho_{t+1})/\partial k_{t+1}^d\; R_{t+1}^d}{\partial g\left(V_{t+1}^d\right)/\partial V_{t+1}^d}. \tag{4}$$

[9] We can also set $V_t^d(k_t^d) = \mho(k_t^d)$, which will not change our results. We have chosen this formulation for the sake of symmetry.

Note that the left-hand sides of Equation (1) and Equation (2) are the marginal costs of investing in k^a_{t+1} and k^d_{t+1}, respectively, while the right-hand sides represent the corresponding marginal benefits. We denote these marginal benefits as MB^a_{t+1} and MB^d_{t+1} for investment in k^a_{t+1} and k^d_{t+1}, respectively.

The key object of interest for our analysis is the ratio MB^a_{t+1}/MB^d_{t+1} since it determines the relative demand for Type 1 and Type 2 assets. Using Equation (3) and Equation (4), this ratio can be represented as follows:

$$RMB_{t+1}(k^a_{t+1}, k^d_{t+1}) = \frac{MB^a_{t+1}}{MB^d_{t+1}} = \frac{s_t}{1-s_t} \frac{\dfrac{\partial f(V^a_{t+1})/\partial V^a_{t+1}}{\partial g(V^a_{t+1})/\partial V^a_{t+1}}}{\dfrac{\partial f(V^d_{t+1})/\partial V^d_{t+1}}{\partial g(V^d_{t+1})/\partial V^d_{t+1}}} \frac{\partial g(\psi_{t+1})/\partial c_{t+1}}{\partial g(\mho_{t+1})/\partial k^d_{t+1}} \frac{R^a_{t+1}}{R^d_{t+1}}. \tag{5}$$

The ratio of marginal benefits in Equation (5) is a function of k^a_{t+1} and k^d_{t+1}. Denote the optimal solution to the household's problem as $\overline{k}^a_{t+1}$ and $\overline{k}^d_{t+1}$. Note that the ratio $RMB_{t+1}(k^a_{t+1}, k^d_{t+1})$ evaluated at this optimal bundle is equal to one:

$$RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1}) = \left.\frac{MB^a_{t+1}}{MB^d_{t+1}}\right|_{\overline{k}^a_{t+1}, \overline{k}^d_{t+1}} = 1 \tag{6}$$

Consider an agent who optimally chooses his portfolio allocation $\overline{k}^a_{t+1}, \overline{k}^d_{t+1}$ given a particular value of per-period utility of being alive b. Next, suppose b marginally increases. If the ratio $RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})$ evaluated with the new value of b is no longer equal to one, the previous optimal decisions no longer maximize an agent's utility. The key question is which way an agent will shift his portfolio allocation. If the ratio $RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})$ increases, an agent will reallocate his portfolio towards k^a_{t+1}, while if it decreases, he reallocates his portfolio towards k^d_{t+1}.

Thus, our key point of interest is how the ratio MB^a_{t+1}/MB^d_{t+1} changes when being alive brings higher utility, *taking initial optimal allocation as a starting point.* [10] The direction of this change is stated in the Proposition 1 below. Before formulating the proposition, we introduce the following notations:

$$\frac{\partial f(V^a_{t+1})}{\partial V^a_{t+1}} \equiv f'_a, \qquad \frac{\partial^2 f(V^a_{t+1})}{\partial (V^a_{t+1})^2} \equiv f''_a,$$

$$\frac{\partial g(V^a_{t+1})}{\partial V^a_{t+1}} \equiv g'_a, \qquad \frac{\partial^2 g(V^a_{t+1})}{\partial (V^a_{t+1})^2} \equiv g''_a.$$

Proposition 1: Consider the ratio of marginal benefits of investing in two survival-contingent assets defined in Equation (5) evaluated at the optimal allocation $\overline{k}^a_{t+1}, \overline{k}^d_{t+1}$. The change in this ratio in response to the marginal increase in b, $\dfrac{\partial RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})}{\partial b}$ can be described as follows.

i) If $f(\cdot) = g(\cdot)$, then $\dfrac{\partial RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})}{\partial b} = 0$;

ii) If $f(\cdot) \neq g(\cdot)$ and $\left|\dfrac{f''_a}{f'_a}\right| > \left|\dfrac{g''_a}{g'_a}\right|$, then $\dfrac{\partial RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})}{\partial b} < 0$;

iii) If $f(\cdot) \neq g(\cdot)$ and $\left|\dfrac{f''_a}{f'_a}\right| < \left|\dfrac{g''_a}{g'_a}\right|$, then $\dfrac{\partial RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})}{\partial b} > 0$.

Proof Consider first the standard expected utility case where $f(\cdot) = g(\cdot)$. In this case, the ratio in Equation (5) reduces to:

$$\left.\frac{MB^a_{t+1}}{MB^d_{t+1}}\right|_{\overline{k}^a_{t+1}, \overline{k}^d_{t+1}} = \frac{s_t}{1-s_t}\left(\frac{\partial g(\psi_{t+1})/\partial c_{t+1}}{\partial g(\mho_{t+1})/\partial k^d_{t+1}}\right)\frac{R^a_{t+1}}{R^d_{t+1}} \tag{7}$$

which is just the ratio of marginal utilities of consumption and bequests multiplied by the ratio of expected returns of the two state-contingent assets.

[10]Note that if instead of two state-contingent assets, we consider portfolio allocation between annuities and regular bonds, we would consider the ratio: $MB^a_{t+1}/(MB^a_{t+1} + MB^d_{t+1})$, where the denominator represents the marginal benefits of investing in bonds since they pay out both in states when an individual is alive and not alive. Since this expression can be rewritten as $1/(1 + MB^d_{t+1}/MB^a_{t+1})$, the ratio MB^a_{t+1}/MB^d_{t+1} is still our main object of interest determining the relative weight of annuities in the optimal portfolio.

Note that given Assumption 1 (that $\dfrac{\partial g(\psi_{t+1})}{\partial c_{t+1}}$ does not depend on b), the ratio MB^a_{t+1}/MB^d_{t+1} does not depend on b as well, which proves part i) of Proposition 1.

Next, consider a case of non-expected utility, $f(\cdot) \neq g(\cdot)$. Before proceeding, we introduce the following notation:

$$\frac{\partial f(V^d_{t+1})}{\partial V^d_{t+1}} \equiv f'_d, \qquad \frac{\partial g(V^d_{t+1})}{\partial V^d_{t+1}} \equiv g'_d,$$

$$\frac{s_t}{1-s_t}\left(\frac{\partial g(\psi_{t+1})/\partial c_{t+1}}{\partial g(\mho_{t+1})/\partial k^d_{t+1}}\right)\frac{R^a_{t+1}}{R^d_{t+1}} \equiv D.$$

Note that D is positive and does not depend on b given fixed portfolio allocations. We can now express the derivative in question as follows:

$$\frac{\partial RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})}{\partial b} = \frac{\partial}{\partial b}\left(\left.\frac{MB^a_{t+1}}{MB^d_{t+1}}\right|_{\overline{k}^a_{t+1},\overline{k}^d_{t+1}}\right) = D\,\frac{g'_d}{f'_d}\,\frac{f'_a}{g'_a}\,\frac{1}{(g'_a)^2}\,\frac{\partial V^a_{t+1}}{\partial b}\left(\frac{f''_a}{f'_a} - \frac{g''_a}{g'_a}\right) \quad (8)$$

The whole expression on the right-hand side before the bracket is positive: $D > 0$; $\partial V^a_{t+1}/\partial b > 0$; in addition, the pairs (g'_a, g'_d) and (f'_a, f'_d) have the same sign because of monotonicity of $f(\cdot)$ and $g(\cdot)$. Thus, the sign of $\frac{\partial}{\partial b}RMB_{t+1}(\overline{k}^a_{t+1}, \overline{k}^d_{t+1})$ is determined by the expression in the bracket: $\dfrac{f''_a}{f'_a} - \dfrac{g''_a}{g'_a}$. Since both $f(\cdot)$ and $g(\cdot)$ are increasing and concave, both ratios $\dfrac{f''_a}{f'_a}$ and $\dfrac{g''_a}{g'_a}$ are negative, and the sign of the expression in the bracket depends on which ratio has higher absolute value. This finishes the proof of parts ii) and iii) of Proposition 1.

Part i) of Proposition 1 states that the demand for annuities and portfolio allocation is independent of b in the expected utility framework. Parts ii) and iii) state that the effect of per-period utility of being alive b on portfolio allocation depends on the relative properties of the functions $f(\cdot)$ and $g(\cdot)$. The ratios $\dfrac{f''_a}{f'_a}$ and $\dfrac{g''_a}{g'_a}$ can be thought of as measuring an agent's aversion to fluctuations in utility over states of the world and over time, respectively. When $\left|\dfrac{f''_a}{f'_a}\right| > \left|\dfrac{g''_a}{g'_a}\right|$, we can say $f(\cdot)$ is "more concave" than $g(\cdot)$. In this case, an increase in b leads to more investments in k^d_{t+1} and a decrease in demand for annuities.

In contrast, when $\left|\frac{f_a''}{f_a'}\right| < \left|\frac{g_a''}{g_a'}\right|$, $g(\cdot)$ is "more concave" than $f(\cdot)$ and an increase in b leads to higher demand for annuities since MB^a_{t+1} increases relative to MB^d_{t+1}.

Thus, the relative concavity of $f(\cdot)$ versus $g(\cdot)$ determines whether an increase in intra-period utility of being alive increases or decreases the demand for annuities. We discuss the intuition behind this result in the next section.

2.3 Why does the relative concavity of $f(\cdot)$ and $g(\cdot)$ matter?

To better understand the results of the previous section, here we further discuss the importance of the relative concavity of functions $f(\cdot)$ and $g(\cdot)$. Our focus is on how the properties of these functions affect an agent's decision of whether to allocate an extra dollar of investment in k^a_{t+1} (and thus increase V^a_{t+1}) or in k^d_{t+1} (and thus increase V^d_{t+1}).

To better illustrate the intuition, we consider two extreme cases which differ in whether $f(\cdot)$ or $g(\cdot)$ is more concave. In the first case, we assume that $g(\cdot)$ is linear while maintaining the assumption that $f(\cdot)$ is concave. In the second case, we assume that $f(\cdot)$ is linear and only $g(\cdot)$ is concave.

2.3.1 *Case 1: $g(\cdot)$ is linear and $f(\cdot)$ is concave*

We can rewrite the ratio MB^a_{t+1}/MB^d_{t+1} in Equation (5) as follows:

$$\frac{MB^a_{t+1}}{MB^d_{t+1}} = \frac{s_t}{1-s_t}\left(\frac{\partial f(V^a_{t+1})/\partial V^a_{t+1}}{\partial f(V^d_{t+1})/\partial V^d_{t+1}}\right)\left(\frac{\partial \psi_{t+1}/\partial c_{t+1}}{\partial \mho_{t+1}/\partial k^d_{t+1}}\right)\frac{R^a_{t+1}}{R^d_{t+1}}$$

Note that the only term in this expression that changes as b increases (fixing portfolio allocation at $\overline{k}^a_{t+1}, \overline{k}^d_{t+1}$) is $\partial f(V^a_{t+1})/\partial V^a_{t+1}$. Given the concavity of $f(\cdot)$, this term decreases as V^a_{t+1} increases.

Intuitively, the function $f(\cdot)$ determines an agent's attitude toward fluctuations in utility over states of the world. An increase in b widens the gap between V^a_{t+1} and V^d_{t+1}, so an agent tries to reverse this by increasing the value of being dead V^d_{t+1} through investments in k^d_{t+1}.

2.3.2 *Case 2:* $f(\cdot)$ *is linear and* $g(\cdot)$ *is concave*

We can rewrite the ratio in Equation (5) as follows:

$$\frac{MB^a_{t+1}}{MB^d_{t+1}} = \frac{s_t}{1-s_t}\left(\frac{\partial g(V^d_{t+1})/\partial V^d_{t+1}}{\partial g(V^a_{t+1})/\partial V^a_{t+1}}\right)\left(\frac{\partial g(\psi_{t+1})/\partial c_{t+1}}{\partial g(\mho_{t+1})/\partial k^d_{t+1}}\right)\frac{R^a_{t+1}}{R^d_{t+1}}$$

Note that the only term here that depends on b (fixing portfolio allocation at $\overline{k}^a_{t+1}, \overline{k}^d_{t+1}$) is $\partial g(V^a_{t+1})/\partial V^a_{t+1}$, which decreases as b marginally increases, thus making the ratio larger.

Intuitively, while risk aversion induces an agent to allocate resources to the worst state, his concern for intertemporal smoothing induces him to allocate resources to the best state. With linear $f(\cdot)$ the latter motive dominates: an increase in b makes the state of being alive better, increasing the efficiency of investing in it from the perspective of intertemporal smoothing.

2.4 The value of a statistical life

It is important to discuss how our analysis is related to the concept of the value of a statistical life (VSL). The VSL represents a monetary value of a reduction in mortality risk that would prevent one statistical death. More formally, it is the willingness to pay for a marginal reduction in mortality risk, or the marginal rate of substitution between wealth and survival probability (Andersson and Treich, 2011).

In our framework, it can be expressed as follows:

$$VSL = \frac{\partial V^a_t/\partial s_t}{\partial V^a_t/\partial k^a_t} \tag{9}$$

For the ease of exposition and comparison with other studies, we are going to rewrite the budget constraint in our optimization problem in terms of asset prices rather than asset returns:

$$c_t + p^a_{t+1}k^a_{t+1} + p^d_{t+1}k^d_{t+1} = y_t + k^a_t \tag{10}$$

Compared to the previous formulation, the return on assets is normalized to be equal to one (conditional on surviving), while the price of the unit of assets k^a_{t+1} (k^d_{t+1}) costs p^a_{t+1} (p^d_{t+1}).[11]

To derive $\partial V_t^a/\partial s_t$, we need to take into account that a change in s_t can potentially affect the price of survival-contingent assets, i.e., $\dfrac{\partial p_{t+1}^i}{\partial s_t}$ $(i = a, d)$ can be nonzero.

Thus,

$$\frac{\partial V_t^a}{\partial s_t} = \beta \left(\frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}}\right) \left(\frac{f(V_{t+1}^a) - f(V_{t+1}^d)}{\partial g(V_t^a)/\partial V_t^a}\right) - (1-\beta)\frac{\frac{\partial g(\psi_t)}{\partial c_t}\left[k_{t+1}^a \frac{\partial p_{t+1}^a}{\partial s_t} + k_{t+1}^d \frac{\partial p_{t+1}^d}{\partial s_t}\right]}{\partial g(V_t^a)/\partial V_t^a} \tag{11}$$

Using the envelop condition, we can write the denominator of Equation (9) as follows:

$$\frac{\partial V_t^a}{\partial k_t^a} = (1-\beta)\,\frac{\partial g(\psi_t)/\partial c_t}{\partial g(V_t^a)/\partial V_t^a} \tag{12}$$

Thus,[12]

$$VSL = \frac{\beta}{1-\beta}\left(\frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}}\right)\left(\frac{f(V_{t+1}^a) - f(V_{t+1}^d)}{(\partial g(\psi_t)/\partial c_t)}\right) - k_{t+1}^a \frac{\partial p_{t+1}^a}{\partial s_t} - k_{t+1}^d \frac{\partial p_{t+1}^d}{\partial s_t} \tag{13}$$

Next, consider how VSL in our model changes with b, the additional utility of being alive. Taking the derivative of Equation (13) with respect to b, we get:[13]

$$\frac{\partial VSL}{\partial b} = \frac{\beta}{1-\beta}\,\frac{s_t}{\partial g(\psi_t)/\partial c_t}\left(\frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}}\right)\frac{\partial f(V_{t+1}^a)}{\partial V_{t+1}^a}\,\frac{\partial V_{t+1}^a}{\partial b}$$

$$\times\left\{\left(\frac{f(V_{t+1}^a) - f(V_{t+1}^d)}{\partial f(z_{t+1})/\partial z_{t+1}}\right)\left[\left(\frac{\partial^2 g(z_{t+1})/\partial z_{t+1}^2}{\partial g(z_{t+1})/\partial z_{t+1}}\right) - \left(\frac{\partial^2 f(z_{t+1})/\partial z_{t+1}^2}{\partial f(z_{t+1})/\partial z_{t+1}}\right)\right] + 1\right\} \tag{14}$$

[11] Assets prices and returns are linked as follows: $p_{t+1}^a = 1/R_{t+1}^a$ and $p_{t+1}^d = 1/R_{t+1}^d$.

[12] This expression is similar to the VSL derived in Cordoba and Ripoll (2017) (Equation (17) in their paper). In their framework, they assume $V_{t+1}^d = 0$, $f(x) = x^{1-\gamma}$, $g(x) = x^{1-\sigma}$, and $\partial p_{t+1}^a/\partial s_t = \delta/(1+r)$, where δ shows the degree of imperfection in the annuity market (with $\delta = 0$ meaning the annuity market does not exist) and $(1+r)$ is the gross return on a risk-free bond.

[13] Note that we consider the response of VSL to the marginal change in b evaluated at the optimal portfolio choice.

Similar to Equation (8) which determines whether the demand for annuities increases or decreases in b, the sign of the expression above depends on:[14]

$$\left[\frac{\partial^2 g(z_{t+1})/\partial z_{t+1}^2}{\partial g(z_{t+1})/\partial z_{t+1}} - \frac{\partial^2 f(z_{t+1})/\partial z_{t+1}^2}{\partial f(z_{t+1})/\partial z_{t+1}}\right],$$

i.e., the relative concavity of functions $g(\cdot)$ and $f(\cdot)$. When $f(\cdot)$ is more concave than $g(\cdot)$, the expression in the square bracket is positive (since both $\frac{\partial^2 g(z_{t+1})/\partial z_{t+1}^2}{\partial g(z_{t+1})/\partial z_{t+1}}$ and $\frac{\partial^2 f(z_{t+1})/\partial z_{t+1}^2}{\partial f(z_{t+1})/\partial z_{t+1}}$ are negative), thus $\partial VSL/\partial b > 0$. In other words, the willingness to pay to increase the survival probability increases as life brings more utility.

Note that this is not necessarily the case when $g(\cdot)$ is more concave than $f(\cdot)$. In this situation, the expression in the square bracket is negative which can result in $\partial VSL/\partial b$ being negative. In this case, as b increases, people may be willing to pay less to extend their life.

Importantly, a situation where $g(\cdot)$ is more concave than $f(\cdot)$ also corresponds to the situation when demand for annuities increases as life becomes more valuable (b increases) as was shown by our analysis above. Thus, in the situation where an increase in b makes an agent *less* willing to pay for mortality reduction, he also has *higher* demand for annuities.[15] In other words, when we disentangle an agent's attitude towards inter- and intra-temporal utility fluctuations, the following situation can arise: as life becomes more valuable, an agent buys more annuities which further increases his utility when alive, while at the same time, he is less willing to pay to increase his survival probability.

2.5 Extension: The case of irreversible annuity investment

In the portfolio choice problem we have considered so far, we assume that an annuity pays out for one period: an agent, who, in period t, purchases an annuity gets paid only in period $t+1$; to continue receiving an annuity payout in period $t+2$, he needs to purchase

[14]Note that $f(V_{t+1}^a) - f(V_{t+1}^d)$ is positive because living is preferred to dying and $f(\cdot)$ is an increasing function.

[15]Note, the opposite is not necessarily true. When $g(\cdot)$ is more concave than $f(\cdot)$, an agent's demand for annuities increases in b but his willingness to pay for mortality reduction can still be increasing in b. This can be seen from Equation (14), which shows that $\partial VSL/\partial b$ can be positive in this case.

an annuity again in period $t+1$. This is analogous to purchasing lifelong annuities, which can be freely adjusted both upwards and downwards every period. Annuities, however, can represent an irreversible investment: if an agent holds annuities in the amount k_t^a in period t, he can only increase it.

To illustrate how modeling annuities as irreversible investments can affect our findings, consider the following modified portfolio choice problem. We modify the budget constraint in Equation (10) as follows:

$$c_t + \tilde{p}_{t+1}^a (k_{t+1}^a - k_t^a) + p_{t+1}^d k_{t+1}^d = y_t + k_t^a,$$

where the annuity price $\tilde{p}_{t+1}^a$ takes into account the irreversibility, and is thus higher than in the case we considered earlier.[16] In addition, we must add another constraint to reflect this irreversibility:

$$k_{t+1}^a \geq k_t^a \tag{15}$$

We denote the Lagrange multiplier on this constraint as μ_t.

Now consider two situations. First, suppose the constraint in Equation (15) is not binding, then $\mu_t = 0$ and our expression for MB_{t+1}^a / MB_{t+1}^d is unchanged.[17] Thus, our results carry through and the relative concavity of functions $f(\cdot)$ and $g(\cdot)$ determines whether demand for annuities increases or decreases as life becomes more valuable.

Second, suppose the constraint in Equation (15) is binding, $\mu_t > 0$. In this case, in period t, an agent does not invest in annuities, thus at the optimum $MB_{t+1}^a < MB_{t+1}^d$. As we established earlier, when $g(\cdot)$ is more concave than $f(\cdot)$, an increase in b (extra utility from being alive) increases the demand for reversible (or liquid) annuities. This, however, is not necessarily the case now: even though an increase in b increases MB_{t+1}^a relative to MB_{t+1}^d (keeping portfolio choice fixed at the initial optimal level), this may not be enough to induce an agent to start investing in annuities, i.e., MB_{t+1}^a can still be less than MB_{t+1}^d.

To summarize, when $f(\cdot)$ is more concave than $g(\cdot)$, the illiquidity of annuities does not change the result that an increase in b decreases demand for annuities. When, however,

[16] Note that in this specification an agent who already has lifelong annuity income k_t^a, adds $k_{t+1}^a - k_t^a$ to it in period t.

[17] Since we have rewritten the budget constraint in terms of assets prices, the ratio R_{t+1}^a / R_{t+1}^d in Equation (5) is changed to p_{t+1}^d / p_{t+1}^a.

$g(\cdot)$ is more concave than $f(\cdot)$, an increase in b either increases the demand for annuities or has no effect on it.

3 Parametric illustrations

In this section, we provide an illustration of our results by making parametric assumptions on functions $f(\cdot)$ and $g(\cdot)$. We consider three parameterizations: Epstein-Zin-Weil (EZW), risk-sensitive preferences, and preferences with Chew-Dekel risk aggregator. Among these three, EZW is probably the most widely used in macroeconomics, asset-pricing, and household finance, far from complete list includes Guvenen, 2009; Inkman et al., 2011; Kaplan and Violante, 2014; Krueger and Ludwig, 2019; Love, 2017. Several notable applications of risk-sensitive preferences developed by Hansen and Sargent (1995) include Andersen (2005); O'Dea et al. (2020); and Tallarini (2000).[18] In Chew-Dekel class of preferences, the most well-known is those with disappointment aversion developed by Gul (1991). Applications of these preferences can be found in Campanele et al. (2010) and their generalization in Routledge and Zin (2010).

3.1 Epstein-Zin-Weil preferences

In EZW preferences both uncertainty and time aggregators are assumed to be constant elasticity of substitution (CES)-type functions. We assume that:

$$f(x) = \frac{x^{1-\sigma}}{1-\sigma}$$

and:

$$g(x) = \frac{x^{1-\frac{1}{\alpha}}}{1-\frac{1}{\alpha}}$$

[18]Two recent studies provide deep theoretical insight into the latter two parametrizations. Specifically, Bommier et al. (2017) show that risk-sensitive preferences satisfy the property of monotonicity, while this is not the case for EZW preferences. Dillenberger et al. (2020) show that both these parametrizations violate stochastic impatience property for a range of parameters values.

For the intra-period utility functions $\psi(\cdot)$ and $\mho(\cdot)$, we assume the following parametrization:

$$\psi(c_t, b) = \begin{cases} \left[\xi c_t^{\rho} + (1-\xi)b^{\rho}\right]^{1/\rho} & \text{if } b > 0 \\ c_t & \text{if } b = 0, \end{cases}$$

$$\mho(k_t^d) = \eta(\varphi + k_t^d R_t^d)$$

Note that when $b = 0$ and $\eta = 0$, we have standard EZW preferences. When $\varphi = 0$, bequests become a necessity.[19] To maintain Assumption 1 ($\frac{\partial g(\psi(c_t, b))}{\partial c_t}$ does not depend on b), we set $\rho = 1 - 1/\alpha$.[20]

In this parametrization, the concavity of $f(\cdot)$ is characterized by the parameter σ, which is also the coefficient of relative risk aversion, and the concavity of $g(\cdot)$ is characterized by the parameter $1/\alpha$, which is also the inverse of the elasticity of intertemporal substitution α. In light of our earlier results, the relationship between these two parameters is important for our subsequent analysis. Note that when $\sigma = 1/\alpha$ (and thus $f(\cdot) = g(\cdot)$), we are back to the standard expected utility case.

[19]In our parametrization of the bequest motive, we follow De Nardi (2004). One way to think about this bequest motive is that an agent derives utility from leaving wealth to his children and thus from increasing their consumption; and the bequest function captures the extra utility of children from this additional consumption. This represents a limited altruism in a sens that the entire lifetime utility of children does not become part of the utility of parents. This latter case would be an interesting but not altogether straightforward extension for future work: parents then would enjoy not only utility from being alive themselves but also from the fact that their children are alive.

[20]Note that when $\rho \neq 1 - 1/\alpha$, the change in b affects marginal utility of consumption even in the case of standard expected utility preferences.

We can now write the value functions as follows:

$$V_t^a(k_t^a) = \max_{k_{t+1}^a, k_{t+1}^d} \left\{ (1-\beta)\left(\xi c_t^{1-\frac{1}{\alpha}} + (1-\xi) b^{1-\frac{1}{\alpha}}\right) + \beta z_{t+1}^{1-\frac{1}{\alpha}} \right\}^{\frac{1}{1-\frac{1}{\alpha}}}$$

$$z_{t+1} = \left\{ s_t \Big[V_{t+1}^a(k_{t+1}^a) \Big]^{1-\sigma} + (1-s_t) \Big[V_{t+1}^d(k_{t+1}^d) \Big]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

$$V_t^d(k_t^d) = \left\{ (1-\beta) \Big[\eta(\varphi + k_t^d R_t^d) \Big]^{1-\frac{1}{\alpha}} \right\}^{\frac{1}{1-\frac{1}{\alpha}}} = \Big(1-\beta\Big)^{\frac{1}{1-\frac{1}{\alpha}}} \eta(\varphi + k_t^d R_t^d)$$

s.t. $\quad c_t + k_{t+1}^a + k_{t+1}^d = k_t^a R_t^a + y_t$

We can express the ratio of marginal benefits of investing in each life-contingent asset asset as follows:

$$RMB_{t+1}(k_{t+1}^a, k_{t+1}^d) = \frac{MB_{t+1}^a}{MB_{t+1}^d} = \frac{s_t}{1-s_t} \; \frac{\xi}{\eta^{1-\frac{1}{\alpha}}} \; \frac{c_{t+1}^{-\frac{1}{\alpha}}}{\left(\varphi + k_{t+1}^d R_{t+1}^d\right)^{-\frac{1}{\alpha}}} \; \frac{R_{t+1}^a}{R_{t+1}^d} \; \frac{\left(V_{t+1}^a\right)^{\frac{1}{\alpha}-\sigma}}{\left(V_{t+1}^d\right)^{\frac{1}{\alpha}-\sigma}} \tag{16}$$

Consider how this ratio, evaluated at the optimal bundle $(\overline{k}_{t+1}^a, \overline{k}_{t+1}^d)$ changes when b marginally increases. As before, $RMB_{t+1}(\overline{k}_{t+1}^a, \overline{k}_{t+1}^d) = 1$ and the direction of the change in this ratio when b marginally changes indicates which direction an agent will reallocate his portfolio (see also Proposition 1).

Starting with the expected utility case, note that when $\frac{1}{\alpha} = \sigma$, the last term in Equation (16) disappears and the relative benefits of investing in two state-contingent assets are determined by the ratio of the marginal utility of consumption to that of bequests. In this case, a change in b does not affect annuity demand.

Next, consider the case when $\frac{1}{\alpha} \neq \sigma$. When evaluated at the optimal portfolio choice, the only term in Equation (16) that depends on b is $\left(V_{t+1}^a\right)^{\frac{1}{\alpha}-\sigma}$. Taking the derivative of this term with respect to b while keeping portfolio choice fixed at $(\overline{k}_{t+1}^a, \overline{k}_{t+1}^d)$, we get:

$$\left(\frac{1}{\alpha} - \sigma\right) \left[\left(V_{t+1}^a\right)^{\frac{1}{\alpha}-\sigma-1} \right] \frac{\partial V_{t+1}^a}{\partial b}$$

The sign of this expression is determined by $\frac{1}{\alpha} - \sigma$. When $\frac{1}{\alpha} < \sigma$, in response to an increase in b, agents reallocate investments from k_{t+1}^a to k_{t+1}^d. This also corresponds to the case when $f(\cdot)$ is more concave than $g(\cdot)$ and agents dislike uncertainty more than they dislike intertemporal fluctuations. In contrast, when $\frac{1}{\alpha} > \sigma$ investments in k^a_{t+1} increase as b increases.

To summarize, when the coefficient of risk aversion is above (below) the inverse of the IES, an increase in intra-period utility of being alive, b, leads to lower (higher) demand for annuities.

3.2 Risk-sensitive preferences

The risk-sensitive preferences correspond to the following functional forms for $f(\cdot)$ and $g(\cdot)$:

$$f(x) = -\frac{1}{k}\exp(-kx)$$

$$g(x) = x$$

Here the parameter k can be thought of as risk aversion since it determines the aversion to utility fluctuations over states of the world.[21] The value function has the following representation:

$$V^a_t(k^a_t) = (1-\beta)\psi(c_t, b) - \frac{\beta}{k} ln[s_t \exp(-kV^a_{t+1}(k^a_{t+1})) + (1-s_t)\exp(-kV^d_{t+1}(k^d_{t+1}))]$$

Note that this also corresponds to the case of linear $g(\cdot)$ and concave $f(\cdot)$ discussed in Section 2.3. Using the results of that section, we can write the ratio of marginal benefits of investing in Type 1 and Type 2 assets as follows:

$$RMB_t(k^a_{t+1}, k^d_{t+1}) = \frac{MB^a_{t+1}}{MB^d_{t+1}} = \frac{s_t}{1-s_t}\frac{\exp(-kV^a_{t+1})}{\exp(-kV^d_{t+1})}\frac{\partial\psi_{t+1}/\partial c_{t+1}}{\partial\mho_{t+1}/\partial k^d_{t+1}}\frac{R^a_{t+1}}{R^d_{t+1}} \tag{17}$$

When evaluated at $(\overline{k}^a_{t+1}, \overline{k}^d_{t+})$, the only term in Equation (17) that responds to marginal change in b is $\exp(-kV^a_{t+1})$, moreover, this term decreases as b increases. This means

[21] Because of linearity of $g(\cdot)$ an agent is neutral to intertemporal *utility* fluctuations, but he may not be neutral to intertemporal *consumption* fluctuations. The latter also depends on function $\psi(\cdot)$.

that given the initial optimal allocation, the marginal increase in b will drive an agent to reallocate from Type 1 to Type 2 assets, i.e., to purchase less annuities. Put differently, for any value of k, which measures the degree of risk aversion, an increase in the value of life will lead an agent with risk-sensitive preferences to buy less annuities.[22]

3.3 Chew-Dekel risk aggregator

Chew (1983) and Dekel (1986) developed a class of preferences where a certainty equivalent is defined implicitly.[23] Denoting a risk aggregator as L, in our context, we can define the certainty equivalent z_{t+1} as follows:

$$z_{t+1} = s_t L(V^a_{t+1}, z_{t+1}) + (1 - s_t) L(V^d_{t+1}, z_{t+1}). \tag{18}$$

Among the tractable members of the Chew-Dekel class of preferences, we are going to focus on the disappointment aversion (Gul, 1991). Agents with these preferences are more sensitive to bad outcomes. In the original Gul's formulation bad outcomes are those that are below the certainty equivalent, while Routledge and Zin (2010) allow for a different definition of the cutoff at which an outcome is considered bad. In our case, there are only two outcomes, life and death, and arguably the second is a bad outcome. With disappointment aversion, the risk aggregator L takes the following form:

$$L(V^a_{t+1}, z_{t+1}) = \frac{(V^a_{t+1})^{1-\sigma} z^{\sigma}_{t+1}}{1-\sigma} + z_{t+1}\left(1 - \frac{1}{1-\sigma}\right) \tag{19}$$

$$L(V^d_{t+1}, z_{t+1}) = \frac{(V^d_{t+1})^{1-\sigma} z^{\sigma}_{t+1}}{1-\sigma} + z_{t+1}\left(1 - \frac{1}{1-\sigma}\right) + \delta \frac{((V^d_{t+1})^{1-\sigma} z^{\sigma}_{t+1} - z_{t+1})}{1-\sigma} \tag{20}$$

Here, $\delta \geq 0$, and when $\delta = 0$ this can be shown to correspond to the case when the uncertainty aggregation rule is determined by the CRRA function with parameter σ. When

[22] This result complements that of O'Dea et al. (2020) who show that an increase in risk aversion for agents with risk-sensitive preferences decreases annuity demand *given* fixed positive value of life. We show that *given* fixed risk aversion k, an increase in the utility of being alive decreases annuity demand. From Equation (17), one can see that this effect is larger for higher values of k.

[23] See Backus et al. (2004) for a more detailed discussion of these preferences.

δ is positive, an agent places additional weight on an outcome when he dies. This will become more transparent after we explicitly solve for z_{t+1}. To do this, we can combine Equation (18)-Equation (20):

$$z_{t+1} = \frac{z_{t+1}^{\sigma}}{1-\sigma}(s_t(V_{t+1}^a)^{1-\sigma} + (1-s_t)(V_{t+1}^d)^{1-\sigma}) + z_{t+1}(1-\frac{1}{1-\sigma}) + \delta(1-s_t)\frac{((V_{t+1}^d)^{1-\sigma} z_{t+1}^{\sigma} - z_{t+1})}{1-\sigma}$$

Transforming this further we get:

$$z_{t+1}^{1-\sigma} = \frac{s_t}{1+\delta(1-s_t)}(V_{t+1}^a)^{1-\sigma} + \frac{(1-s_t)(1+\delta)}{1+\delta(1-s_t)}(V_{t+1}^d)^{1-\sigma.}$$

We can now redefine the probabilities as follows:

$$\tilde{s}_t = \frac{s_t}{1+\delta(1-s_t)},$$

and

$$(1-\tilde{s}_t) = \frac{(1-s_t)(1+\delta)}{1+\delta(1-s_t)}.$$

Thus,

$$z_{t+1} = [\tilde{s}_t(V_{t+1}^a)^{1-\sigma} + (1-\tilde{s}_t)(V_{t+1}^d)^{1-\sigma}]^{\frac{1}{1-\sigma}}.$$

This now looks like an uncertainty aggregation rule where $f(x) = x^{1-\sigma}/(1-\sigma)$ and the probabilities of outcomes (life and death) get reweighted based on the disappointment aversion parameter δ. Note that the reweighting is such that $\tilde{s}_t < s_t$ and $1-\tilde{s}_t > 1-s_t$, i.e., the bad outcome (death) gets higher weight when $\delta > 0$. Assuming next that the time aggregation rule has the CRRA form, $g(x) = x^{1-\frac{1}{\alpha}}/(1-\frac{1}{\alpha})$, we can rewrite the ratio of marginal benefits of investing in Type 1 and Type 2 assets as follows:

$$RMB_t(k_{t+1}^a, k_{t+1}^d) = \frac{MB_{t+1}^a}{MB_{t+1}^d} = \frac{\tilde{s}_t}{1-\tilde{s}_t}\frac{(V_{t+1}^a)^{\frac{1}{\alpha}-\sigma}}{(V_{t+1}^d)^{\frac{1}{\alpha}-\sigma}}.\frac{\partial g(\psi_{t+1})/\partial c_{t+1}}{\partial g(\mho_{t+1})/\partial k_{t+1}^d}\frac{R_{t+1}^a}{R_{t+1}^d}$$

This is similar to the case with EZW preferences considered earlier, except for the term $\tilde{s}_t/(1-\tilde{s}_t)$. This difference, however, does not affect our earlier conclusions: evaluated at the optimal allocation, the only term in this expression that changes with marginal increase in b is $\left(V_{t+1}^a\right)^{\frac{1}{\alpha}-\sigma}$, and its response to b is determined by the relationship between risk aversion (σ) and the inverse of IES ($1/\alpha$).

4 Implications for the debate about early versus late resolution of uncertainty

The application of our approach to EZW parametrization of non-expected utility described in Section 3.1 allows us to link annuity demand and preferences for the timing of the resolution of uncertainty. In this section, we discuss this connection in some details.

When people have standard additive expected utility they demonstrate indifference to the timing of the resolution of uncertainty. In a simple two-period example this means they are indifferent between two lotteries, A and B, that can be described as follows. Let both lotteries have the same expected payoff. In lottery A, in the first period the outcomes for both the current and future period are revealed, i.e., all uncertainty is resolved. In lottery B, the outcome for the second period is not known until the second period arrives.[24]

In the case of non-expected utility, people are no longer indifferent between these two lotteries. Specifically, in the case of EZW preferences, when risk aversion exceeds the inverse of the IES, people are said to demonstrate preference for early resolution of uncertainty, i.e., in our example, lottery A brings higher ex-ante utility than lottery B. In contrast, when risk aversion is below the inverse of the IES, late resolution of uncertainty is preferred, i.e., lottery B brings higher expected utility.

In this light, we can restate our finding as follows: when people prefer early resolution of uncertainty, the annuity puzzle can to a significant degree be explained by the fact that people value life. We argue that one reason why theoretical models starting with Yaari (1965) consistently over-predict annuity demand is that in these models people are typically indifferent to the timing of uncertainty resolution and do not necessarily prefer living to dying.

A number of studies show that people are not indifferent to the timing of uncertainty resolution, but the question of whether early or late resolution is preferred is not entirely resolved. Three branches of literature discussed in the introduction provide evidence that early resolution is preferred; specifically, studies that estimate an Euler equation using

[24]The comparison between the two lotteries is made under the assumption that people cannot do anything, whether they know the outcomes early or not, i.e., they cannot re-optimize.

consumption data, experimental studies and macro finance literature (e.g., Bansal and Yaron, 2004; Brown and Kim, 2014; Vissing-Jorgensen and Attanasio, 2003). However, there are some studies pointing in the other direction. In particular, several studies in health economics suggest that people's attitude towards testing for serious diseases may signal preferences for late resolution (e.g., Oster et al., 2013).

This suggests another angle in which our results can be viewed. Specifically, we can contribute to the debate on whether early or late resolution of uncertainty is preferred. We show that one well-documented empirical fact (the VSL is high) can explain another well-known empirical regularity (low demand for annuities) in a framework where early resolution of uncertainty is preferred. Therefore we suggest that the annuity puzzle in combination with the high VSL estimates can be considered as additional evidence that people prefer early resolution of uncertainty.

5 Quantitative illustration: annuitization at retirement

In this section, we quantitatively solve a retirement saving model where retirees have access to a private annuity market. For this exercise, we use EZW preferences described in section 3.1 above. The main purpose of this exercise is to show that the theoretical mechanisms described above are quantitatively important.

5.1 Setup

A retiree enters the model at time $t = 1$ with initial wealth k_1 and pension income n_1, representing pre-existing annuity income provided by Social Security. At the start of retirement, he chooses whether to acquire additional annuities through the private market, i.e., whether to annuitize a part of his wealth k_1. Starting from period $t = 2$ he only solves a consumption/saving problem.[25] We denote the annuity income of a retiree starting from

[25]Pashchenko (2013) proves that in a retirement saving model with no uncertainty (except for survival uncertainty) an agent always chooses to annuitize only once in the first period.

period $t = 2$ as n, where $n = n_1 + \Delta$, and Δ is newly acquired annuity income in period $t = 1$. Note that annuity investments are irreversible, i.e., retirees receive Δ every period as long as they are alive. The price of annuity $\tilde{p}^a$ is determined as follows:

$$\tilde{p}^a = \gamma \sum_{t=1}^{T-1} \frac{S_{t+1|1}}{(1+r)^t}. \tag{21}$$

Here $S_{t+1|1}$ is the probability an agent survives to age $t+1$ ($t = 1, ..., T-1$) conditional on being alive in period 1. It has the following relationship with per-period survival probabilities s_t: $S_{j|1} = s_2 s_3 ... s_j$.

We model two types of frictions in the private annuity market. First, there is a load denoted as γ in Equation (21). It represents the discrepancy between the actual and actuarially fair annuity prices for an individual with average mortality. It arises because of administrative costs and adverse selection.

Second, there is a minimum purchase requirement, i.e., a retiree cannot buy an arbitrarily small annuity income flow. This reflects an important feature of the market: insurance companies usually put a restriction on minimum premiums for a life annuity. We denote the minimum purchase requirement as $\overline{n}$, thus $\Delta \geq \overline{n}$, and the minimum premium is $\overline{n}\tilde{p}^a$.

5.2 Calibration

Retirees enter the model at the age of 65 (corresponding to $t = 1$) and can live at most to age 95 (i.e., the maximum lifespan is $T = 30$). We use the Social Security life tables to construct survival probabilities s_t, $t = 1, ..., 30$.

We take the initial distribution of retirees by total wealth and annuity income from the Health and Retirement Study dataset (HRS). The HRS is a nationally representative sample of individuals over the age of 50. We use the RAND Version P of this dataset. To create the initial distribution we use retirees aged 64-66 in this dataset to increase the number of observations. Initial wealth (k_1) includes the value of housing and real estate, vehicles, value of business, IRAs, Keoghs, stocks, bonds, checking, saving and money market accounts, minus mortgages and other debts. Preexisting annuity income (n_1) corresponds to income from a Social Security pension.

We use the EZW parametrization of preferences described in Section 3.1. We assume the following parameter values. To set risk aversion, IES, discount factor and bequest parameters, we use parametrization from Pashchenko and Porapakkarm (2019), who adjust these parameters to match labor supply and saving behavior over the life-cycle. Specifically, we set the discount factor β to 0.96, the risk aversion parameter σ to 4, and the IES parameter α to $2/3$. Note that in their estimation, risk aversion exceeds the inverse of the IES. We also consider how our quantitative results change when the opposite is true.

We adjust bequest parameters so that the marginal propensity to bequeath (MPB) and the bequest threshold in our model are equal to 0.97 and $3,600, respectively (values estimated by Pashchenko and Porapakkarm, 2019). The threshold and the MPB can be expressed as functions of parameters η and ϕ in a simple two-period consumption-savings model (see De Nardi et al. (2010) and Pashchenko (2013) for more details). They have the following interpretation: only people whose wealth is above the threshold will leave a bequest (i.e., have an operational bequest motive) and 97 cents of every dollar above the threshold will be considered as potential bequests.[26] We set the weight of consumption in the intra-period utility function ξ to 0.5.[27]

We set the load in annuity price γ to 1.1 based on the estimates of Mitchell et al. (1999). Following Pashchenko (2013) we set the minimum purchase requirement $\overline{n}$ to $2,500. She shows that this number produces a minimum premium consistent with that set by large insurance companies.

5.3 Results

Figure 1 displays the results of our simulations. The top panel shows the percentage of individuals who purchase annuities at the beginning of retirement as a function of intra-period utility of being alive (b), while the bottom panel shows the corresponding change in the value of a statistical life (VSL).

When b is close to zero, almost 60% of retirees annuitize at least some part of their

[26]The corresponding values of η and ϕ are 10^{-4} and $120,000$, respectively.

[27]Our results are robust to alternative values of this parameter. Changing the value of this parameter, while keeping everything else the same, changes the MPB and bequest threshold. However, once other parameters are adjusted to reset the MPB and threshold to the targeted values, the effect of the change in ξ becomes insubstantial.

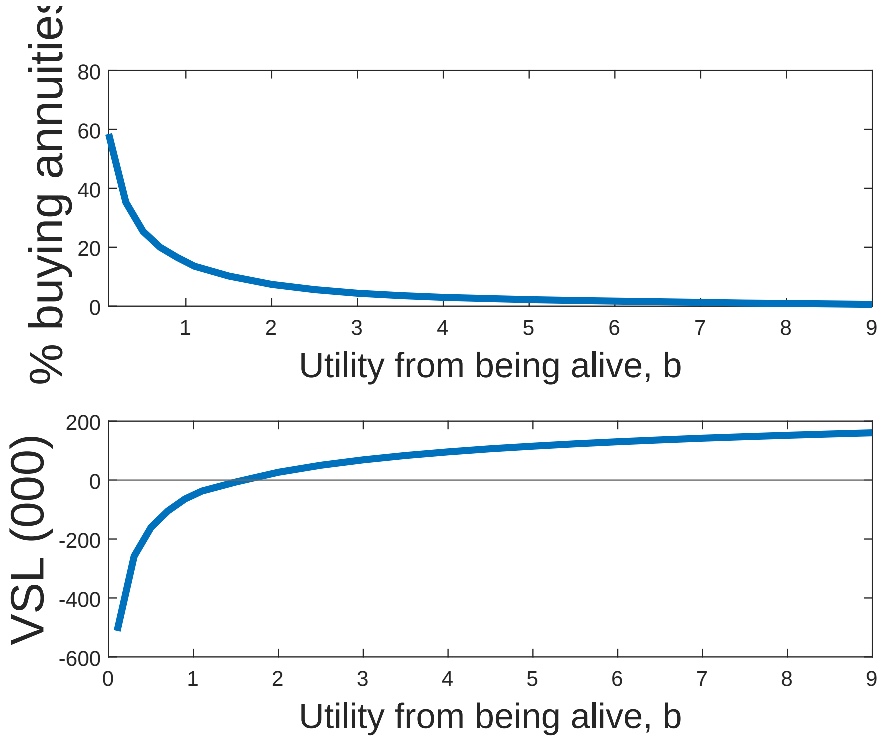

Figure 1: Annuity demand and the value of life when risk aversion is above 1/IES. Top panel: the percentage of retirees buying annuities. Bottom panel: VSL in thousand of dollars.

wealth.[28] This situation also corresponds to a negative VSL, i.e., the state of being alive is valued less than the state of being dead. As b increases, the VSL increases, while at the same time, demand for annuities goes down. The VSL becomes positive once b is close to 2, and in this situation only around 10% of people buy annuities. Increasing b to around 6 almost entirely eliminates the demand for annuities. Note that the corresponding VSL is less than $200K.

We consider next a situation when risk aversion is below the inverse of the IES. We decrease the coefficient of risk aversion to 0.5 (compared to the benchmark value of 4), which is now below 1/IES (equal to 0.67), while keeping all other parameters unchanged.

Figure 2 displays the results when using this new parametrization. The top panel looks very different from the previous case: now when b is zero, no retirees buy annuities.

[28]In the canonical life-cycle model this number would be 100%. Our model, however, features several impediments to annuitization; specifically, preannuitized wealth, market frictions, and bequest motives. Note that despite all these impediments, more than half of retirees choose to annuitize.

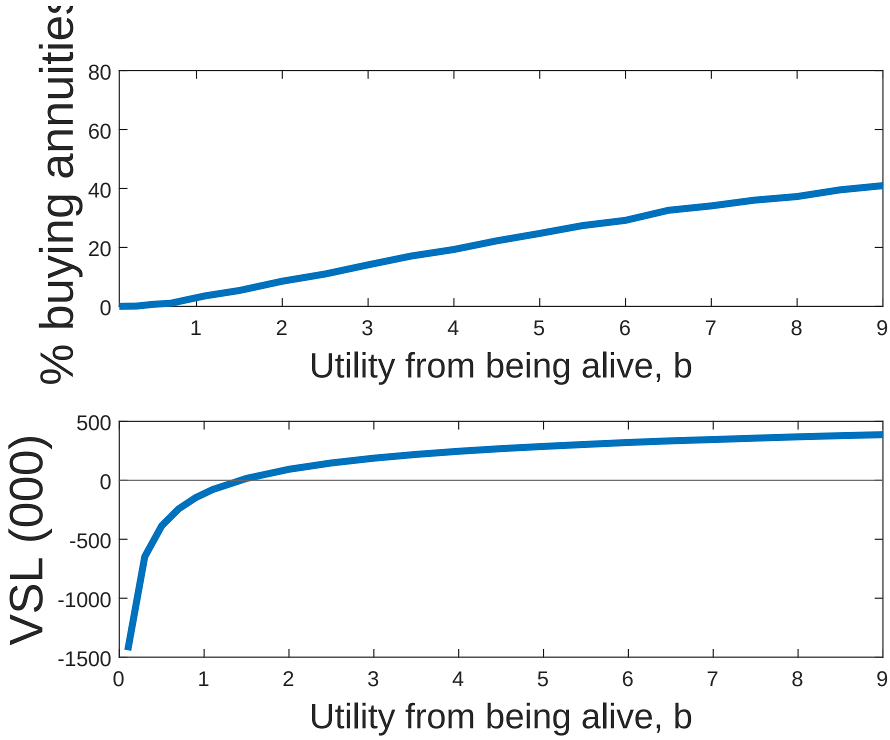

Figure 2: Annuity demand and the value of life when risk aversion is below 1/IES. Top panel: the percentage of retirees buying annuities. Bottom panel: VSL in thousands of dollars.

However, as b increases, more and more people start annuitizing. For example, when b is close to 9, almost 40% of retirees purchase private annuities. This is in sharp contrast with the previous case: when risk aversion was above 1/IES, a value of b close to 9 resulted in zero annuity demand.[29]

To summarize, the results of this section reinforce our earlier conclusion: when we disentangle an agent's attitude toward risk and intertemporal fluctuations, utility of being alive affects the demand annuities. Moreover, this mechanism is quantitatively important: using the EZW parametrization with risk aversion exceeding the inverse of the IES, we show that the demand for annuities is substantially lower when the VSL is positive compared to the situation with the negative VSL.[30]

[29]Note that VSL in the bottom panel of Figure 2 increases in b. This does not necessarily have to be the case, as we demonstrated theoretically in Section 2.

6 Conclusion

In this paper, we study the relationship between the value of life and the demand for assets with survival-contingent payoffs. Two key features of our approach compared to a standard portfolio choice problem with survival uncertainty is that i) we enforce the restriction that living is preferred to dying by allowing for non-pecuniary utility of being alive, ii) we allow for a more general preference specification where attitudes towards risk and intertemporal fluctuations can be separated.

We show theoretically that increasing non-pecuniary benefits from being alive can increase or decrease the demand for annuities depending on whether people are more averse to risk or to intertemporal fluctuations. When safety is of greater concern than intertemporal stability, people buy less annuities when utility of being alive increases.

We apply our approach to the three common parametrizations of non-expected utility. The most intuitive interpretation of our results comes from the application of our approach to Epstein-Zin-Weil preferences: we show that annuity demand decreases (increases) with intra-period utility of being alive if the coefficient of relative risk aversion is higher (lower) than the inverse of the elasticity of intertemporal substitution (IES).

To illustrate our findings quantitatively, we use Epstein-Zin-Weil parametrization of preferences and simulate annuity demand in a retirement saving models using the data from the HRS. We show that when risk aversion exceeds the inverse of the IES, the fraction of retirees buying annuities quickly decreases as life becomes more valuable. Moreover, when other common impediments to annuitization are present, the demand for annuities is nearly eliminated for values of VSL which are positive but not necessarily very large.

We can rephrase our findings in two ways. First, the well-known annuity puzzle can be at least partially explained by a combination of two factors: i) the value of life is positive

[30] It is important to mention that empirical estimates of the VSL are typically above the value that produces almost zero annuity demand in our simulations (which is less than \$200K). Viscusi (1993) provides an extensive review documenting that the estimates vary from \$1 million to \$16 million (in 1990 dollars). US government agencies (Department of Transportation, Food and Drug Administration, Environmental Protection Agency) use a VSL between \$1-10 million in their analyses involving mortality risk (Robinson, 2007).

and sufficiently large, ii) people are more averse to risk than to intertemporal fluctuations.

Second, the annuity puzzle provides evidence that people prefer early resolution of uncertainty, i.e., that risk aversion is above the inverse of the IES. This is because when life is valuable, early resolution of uncertainty must be preferred in order to account for this puzzle.

References

[1] Ameriks, J., Briggs, J., Caplin, A., Shapiro, M., Tonetti, C., 2019. Long-Term Care Utility and Late in Life Saving. Journal of Political Economy, forthcoming.

[2] Anderson, E., 2005. The dynamics of risk-sensitive allocations. Journal of Economic Theory, Volume 125(2), pp 93-150

[3] Attanasio, O., Weber, G., 1989. ”Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption”. Economic Journal, 99, 59-73

[4] Andersson, H., Treich, N., 2011, “The Value of a Statistical Life” in “Handbook in Transport Economics”, Edward Elgar, Cheltenham, UK.

[5] Backus, D., Routledge, B., Zin, S., 2004. Exotic preferences for macroeconomists, NBER Macroeconomics Annual 2004, Volume 19

[6] Bansal, R., Yaron, A., 2004. ”Risks for the Long-Run: A Potential Resolution of Asset Pricing Puzzles.” Journal of Finance, 54, 1481-1509

[7] Bommier, A., Harenberg, D.,LeGrand, F., O’Dea, C., 2020. Recursive Preferences, the Value of Life, and Household Finance. Cowles Foundation Discussion Paper N 2231

[8] Bommier, A., Le Grand, F., 2014. Too Risk Averse to Purchase Insurance? A Theoretical Glance at the Annuity Puzzle. Journal of Risk and Uncertainty, 48, pp. 135-166

[9] Bommier, A., Kochov, A., Le Grand, F., 2017. On Monotone Recursive Preferences. Econometrica, 85, pp 1433–1466.

[10] Bommier, A., Villeneuve, B., 2012. Risk Aversion and the Value of Risk to Life. Journal of Risk and Insurance, Vol. 79, No. 1, pp. 77-103

[11] Brown, A., Kim, H., 2014 ”Do Individuals Have Preferences Used in Macro-Finance Models? An Experimental Investigation”, Management Science, 60, 939-958

[12] Butler, M., Peijnenburg, K., Staubli, S., 2016. How much do means-tested benefits reduce the demand for annuities? Journal of Pension Economics and Finance, 1(04), pp 1-31

[13] Campanale, C., Castro, R., Clementi, G., 2010. Asset Pricing in a Production Economy With Chew-Dekel Preferences. Review of Economic Dynamics, 13(2), pp 379-402

[14] Chen, X., Favilukis, J., Ludvigson, S., 2013."An Estimation of Economic Models with Recursive Preferences". Quantitative Economics, 4, 39-83

[15] Chew, S,H., 1983. A generalization of the quasi-linear mean with applications to the measurement of inequality and decision theory resolving the Allais paradox. Econometrica, 51, pp 1065-1092.

[16] Cordoba, J.C., Ripoll, M., 2017, "Risk Aversion and the Value of Life", Review of Economic Studies, vol.84(4), pp. 1472–1509.

[17] Dekel, E., 1986. An Axiomatic Characterization of Preferences under Uncertainty: Weakening the independence axiom. Journal of Economic Theory, 40, pp 304-331

[18] De Nardi, M., 2004. "Wealth Inequality and Intergenerational Links". Review of Economic Studies, 71(7), 743-768

[19] De Nardi, M., French, E., Jones, J., 2010., Why Do the Elderly Save? Journal of Political Economy, 118(1), pp.39-75.

[20] De Nardi, M., French, E., Jones, J., 2016, Medicaid Insurance in Old Age. American Economic Review, 106(11), pp.3480-3520

[21] De Nardi, M., Pashchenko, S., Porapakkarm, P., 2018. The Lifetime Cots of Bad Health. NBER Working Paper No. 23963

[22] Dillenberger, D., Gottlieb, D.,Ortoleva, P., 2020. Stochastic Impatience and the Separation of Time and Risk Preferences. Mimeo, Princeton University

[23] Dushi, I., Webb, A., 2004. Household Annuitization Decisions: Simulations and Empirical Analysis. Journal of Pension Economics and Finance 3(2), 109-143

[24] Epstein, L., Zin, S, 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, Econometrica, 57(4), pp 937-969

[25] Eslami K., Karimi, S., 2019. Health Spending: Luxury or Necessity? Mimeo, Ryerson University

[26] Fonseca, R., Michaud, P-C., Kapteyn, A., Galama, T., 2020. Accounting for the Rise in Health Spending and Longevity. Journal of the European Economic Association, forthcoming

[27] Gomes, F., Michaelides, A., 2008. Asset Pricing With Limited Risk Sharing and Heterogeneous Agents. Review of Financial Studies, 21, pp 415–448

[28] Guvenen, F., 2009. A Parsimonious Macroeconomic Model for Asset Pricing. Econometrica, Vol. 77, N6, pp 1711-1750

[29] Hall, R., Jones, C., 2007, "The Value of Life and the Rise in Health Spending", Quarterly Journal of Economics, vol.122(1) , pp. 39-72.

[30] Hansen, L., Sargent, T., 1995. Discounted Linear Exponential Quadratic Gaussian Control. IEEE Transactions on Automatic Control, 40(5), pp 968–971.

[31] Huang, D., Shaliastovich, I., 2013. "Risk Adjustment and the Temporal Resolution of Uncertainty: Evidence from Options Markets". Mimeo, University of Pennsylvania

[32] Hubbard, G., Skinner, J., Zeldes, S., 1994. The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving. Carnegie-Rochester Conference Series on Public Policy, 40(1), pp. 59

[33] Hugonnier, J.,Pelgrin, F.,St-Amour, P., 2013. Health and (Other) Asset Holdings. Review of Economic Studies, Vol. 80, No. 2 (283), pp. 663-710

[34] Hugonnier, J.,Pelgrin, F.,St-Amour, P., 2020. Closing Down the Shop: Optimal Health and Wealth Dynamics Near the End of Life. Health Economics, 29(2), pp 138-153

[35] Inkman J., Lopes, P., Michaelides, A., 2011. How Deep is the Annuity Market Participation Puzzle? Review of Financial Studies, 24(1), pp. 279-319.

[36] Kaplan, G.,Violante, G. 2014. A Model of the Consumption Response to Fiscal Stimulus Payments. Econometrica, 82(4), pp 1199-1239.

[37] Kellerman, S., Lehman,J., Lansky, A., Stevens, M., Hecht, F., Bindman, A., Wortley, P., 2002. ”HIV testing within at-risk populations in the United States and the reasons for seeking or avoiding HIV testing”. Journal of Acquired Immune Deficiency Syndromes, 31, 202-210

[38] Kreps, D., Porteus, E., 1978. ”Temporal Resolution of Uncertainty and Dynamic Choice Theory”. Econometrica, 46, 185-200

[39] Krueger, D., Ludwig, A., 2019. Optimal Taxes on Capital in the OLG Model with Uninsurable Idiosyncratic Income Risk. Mimeo, University of Pennsylvania

[40] Lockwood, L., 2012. Bequest Motives and the Annuity Puzzle. Review of Economic Dynamics, 15(2), pp. 226-243

[41] Lockwood, L., 2018. Incidental Bequests and the Choice to Self-Insure Late Life Risks. American Economic Review, 108(9), pp 2513-2550

[42] Love. D., 2017. Countercyclical Retirement Accounts. European Economic Review, Vol 98, pp 38-42

[43] Malloy, C., Moskowitz, T., Vissing-Jorgensen, A., 2009. ”Long-Run Stockholder Consumption Risk and Asset Returns”. Journal of Finance, LXIV, N 6

[44] Meissner, T., Pfeiffer, P., 2018. ”Measuring Preferences Over the Temporal Resolution of Consumption Uncertainty”. Unpublished manuscript

[45] Murphy, K., Topel, R., 2006, “The Value of Health and Longevity”, Journal of Political Economy, vol.114, N51, pp. 871-904

[46] Mitchell, O. S., Poterba, J.M., Warshawsky, M.J., Brown, J.R., 1999. New Evidence on the Money’s Worth of Individual Annuities. American Economic Review 89(5), pp.1299-1318

[47] Nygaard, N., 2019. Causes and Consequences of Life Expectancy Inequality. Mimeo, University of Houston

[48] Ozkan, S., 2017. Preventive vs. Curative Medicine: A Macroeconomic Analysis of Health Care over the Life Cycle. Mimeo, University of Toronto.

[49] Oster, E., Shoulson, D., Dorsey, R., 2013. ”Optimal Expectations and Limited Medical Testing: Evidence from Huntington Disease”, American Economic Review, 103, 804-830

[50] Pashchenko, S., 2013. Accounting for Non-Annuitization. Journal of Public Economics, 98, pp.53-67

[51] Pashchenko, S., Porapakkarm, P., 2019. Accounting for Social Security Claiming Behavior. Mimeo, University of Georgia

[52] Peijnenburg, K., Nijman, T., Werker, B., 2017. Health Cost Risk: A Potential Solution to the Annuity Puzzle. The Economic Journal, 127, pp 1598-1625

[53] Reichling, F., Smetters, K., 2015. Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs. American Economic Review, 105 (11), pp 3273-3320

[54] Robinson, L., 2007. How US Government Agencies Value Mortality Risk Reductions, Review of Environmental Economics and Policy, 1(2), pp.283-299.

[55] Rosen, S., 1988, “The Value of Changes in Life Expectancy”, Journal of Risk and Uncertainty, vol.I, pp.285-304.

[56] Routledge, B.,Zin, S., 2010. Generalized disappointment aversion and asset prices. Journal of Finance, 65(4), pp 1303–1332

[57] Tallarini, T., 2000. Risk-Sensitive Real Business Cycles. Journal of Monetary Economics, 45(3), pp 507-532.

[58] Traeger, C., 2014, “Capturing Intrinsic Risk Attitude”, mimeo, University of California Berkely

[59] Turra, C., Mitchell, O., 2008. The Impact of Health Status and Out-of-Pocket Medical Expenditures on Annuity Valuation. Ameriks, J., and Mitchell, O., editors, Recalibrating Retirement Spending and Saving, pp. 227-250. Oxford University Press.

[60] Viscusi, K., 1993, “The Value of Risks to Life and Health”, Journal of Economic Literature, vol.31(4), pp.1912-1946.

[61] Vissing-Jorgensen, A., Attanasio, O., 2003. “Stock Market Participation, Intertemporal Substitution and Risk Aversion”, American Economic Review Papers and Proceedings, 93, 383-391

[62] Weil, P., 1990, “Non-expected Utility in Macroeconomic?”, Quarterly Journal of Economics, vol.105, pp.29-42.

[63] Yaari, M., 1965. ”Uncertain Lifetime, Life insurance , and the Theory of the Consumer”. Review of Economic Studies, 32, 137-150

[64] Yogo, M., 2006, “A Consumption-based Explanation of Expected Stock Returns”, Journal of Finance, vol.LXI, N2, pp.539-580.

[65] Yogo, M., 2016. Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets. Journal of Monetary Economics, Vol 80, pp 17-34.

Appendix

A The role of bequest motive specification

In our specification we assume that the utility from leaving bequests satisfy the following property: $\mho(0) = \underline{\mho} > -\infty$, i.e., there is no infinite disutility of leaving no bequests. In this we follow a growing literature that allow bequests not to be a necessity and convincingly show that this assumption is consistent with the data (e.g., Ameriks et al., 2020, De Nardi, 2004, De Nardi et al, 2010 and 2016, Lockwood, 2018). In our framework, this assumption also means that there is a low bound on the utility of being dead.

One implication of treating bequests this way is that an individual whose wealth is relatively low, may choose not to leave a bequest. In this section, we first characterize the condition under which this happens, and then show that our main results (Proposition 1) still hold in this case.

Consider a portfolio choice problem described in Section 2.2. Suppose an individual is restricted not to invest in Type 2 assets, i.e., $k^d_{t+1} = 0$. Denote an optimal choice of Type 1 asset in this case as $\bar{\bar{k}}^a_{t+1}$. The question is whether the constraint $k^d_{t+1} = 0$ is binding, i.e., whether an agent starts investing in Type 2 assets if this constraint is removed. The following proposition provides an answer.

Proposition A1 It is optimal for an agent not to invest in Type 2 assets (set $k^d_{t+1} = 0$) if the following condition is true:

$$\left[\frac{\partial g(\psi_t)}{\partial c_t} - \beta(1-s_t)\,\frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}}\,\frac{\partial f(V^d_{t+1})/\partial V^d_{t+1}}{\partial g(V^d_{t+1})/\partial V^d_{t+1}}\,\frac{\partial g(\mho_{t+1})}{\partial k^d_{t+1}} \times R^d_{t+1}\right]\Bigg|_{k^a_{t+1} = \bar{\bar{k}}^a_{t+1},\, k^d_{t+1} = 0} \geqslant 0 \tag{22}$$

where $\bar{\bar{k}}^a_{t+1}$ is an optimal investment in Type 1 asset when Type 2 investments are set to zero.

Proof If it is optimal for an agent not to invest in Type 1 asset, than the first-order condition described in Equation (2) in the main text should not hold with equality at portfolio choice $(\bar{\bar{k}}^a_{t+1}, 0)$ and the following should be true:

$$\left[(1-\beta)\,\frac{\partial g(\psi_t)}{\partial c_t} - \beta\,\frac{\partial g(z_{t+1})/\partial z_{t+1}}{\partial f(z_{t+1})/\partial z_{t+1}}\,(1-s_t)\,\frac{\partial f(V^d_{t+1})}{\partial V^d_{t+1}}\,\frac{\partial V^d_{t+1}}{\partial k^d_{t+1}}\right]\Bigg|_{\bar{\bar{k}}^a_{t+1},0} \geq 0$$

Using the envelop condition in Equation (4), we arrive to the expression in Equation (22) which finishes the proof.

Consider a situation when the condition described in Equation (22) holds so an agent optimally chooses not to invest in Type 2 assets. Consider the change in his portfolio choice in response to the marginal change in b. Note that the ratio of marginal benefits of investing in Type 1 and Type 2 assets defined in Equation (5) is now greater or equal to one:

$$RMB_{t+1}(\bar{\bar{k}}^a_{t+1},0) = \frac{MB^a_{t+1}}{MB^d_{t+1}}\Bigg|_{\bar{\bar{k}}^a_{t+1},0} \geq 1$$

The direction of change in $RMB_{t+1}(\bar{\bar{k}}^a_{t+1},0)$ in response to the marginal change in b is the same as summarized in Proposition 1. However, the implications can be somewhat different. If $f(\cdot)$ is less concave than $g(\cdot)$, this ratio increases and then agents choose to buy more Type 1 assets while they still do not invest in Type 2 asset. When $f(\cdot)$ is more concave than $g(\cdot)$, the ratio in question decreases, and an agent decreases his investments in Type 1 asset. At the same time, he may or may not start investing in Type 2 asset. Thus, our main argument goes through with the slight modification that when $f(\cdot)$ is less concave than $g(\cdot)$, a marginal increase in b does not necessarily lead to more investments in Type 2 assets.

B How to think about per-period utility of being alive when there is a bequest motive?

In a standard expected utility case with no bequest motive and when utility of being dead is normalized to zero, constant b that is added to utility in order to ensure life is preferred to death has a straightforward interpretation. To see this, consider a simple static model with the CRRA utility. An individual is indifferent between life and death when utility when alive is equal to utility when dead (zero):

$$\frac{c^{1-\sigma}}{1-\sigma} + b = 0$$

From here we have

$$c = \left[b(\sigma - 1)\right]^{\frac{1}{1-\sigma}}$$

Thus, b determines the threshold consumption level so that if an individual consumes less than the threshold, he would rather be dead. In our case with bequest motive, even though the constant b plays a similar role by ensuring life is better than death, it can no longer be interpreted as consumption threshold. To see this, consider a version of the model described in Section 2.2 where an individual faces constant survival probability and receives the same income every period, i.e, $s_t = s$ and $y_t = y$ for all t. Consider a situation when an individual is indifferent between life and death, i.e., $V^a(k^a) = V^d(k^d)$. Using the definition of $V^a(k^a)$ we can write:

$$g(V^a(k^a)) = g(V^d(k^d)) = (1-\beta)g(\psi(c,b)) + \beta g(z),$$

and

$$f(z) = sf(V^a(k^a)) + (1-s)f(V^d(k^d)) = f(V^a(k^a)) = f(V^d(k^d))$$

Thus,

$$z = V^a(k^a) = V^d(k^d),$$

and

$$g(V^d(k^d)) = (1-\beta)g(\psi(c,b)) + \beta g(V^d(k^d)).$$

Rearranging and using the fact that $V^d(k^d) = g^{-1}\left[(1-\beta)g(\mho(k^d))\right]$ we get

$$g(\psi(c,b)) = (1-\beta)g(\mho(k^d)) \tag{23}$$

An important difference from the standard case without bequest motive is that an individual can change the utility in the death state. To make this more transparent, let us assume an agent does not invest in annuities ($k^a = 0$), so every period he receives income y and divide it between consumption c and bequests k^d. Equation (23) can be rewritten as:

$$g(\psi(y - k^d, b)) = (1-\beta)g(\mho(k^d))$$

Note that by increasing k^d an agent decreases his utility when alive and increases his utility when dead. His optimal choice of k^d is determined by equating marginal disutility from consuming less today to the marginal utility from leaving larger bequests. However, in this choice, nothing prevents him from choosing k^d such that utility of death will exceed utility of life since such a restriction does not enter in the optimal choice of k^d. Having constant b in per-period utility allows to avoid this problem. Put differently, extra utility from being alive b ensures an individual will not make utility of death exceed utility when alive by his choice of bequests.

To get the interpretation of b closest to the standard case, consider an individual who sets consumption equal to his income every period ($c = y$) and thus both k^a and k^d are zero. This individual is indifferent between life and death if

$$g\left(\psi(c, b)\right) = (1 - \beta) g(\underline{\mho})$$

Thus, the consumption threshold such that below it an individual would prefer to die is a function not only of b but also of $\underline{\mho}$, a low bound on utility of death.