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The Reward and Contract Theories of Patents in a Model of Endogenous Growth

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Abstract: I develop a general equilibrium model of endogenous growth to jointly analyze two distinct theories of the patent system's social value: (1) that patents stimulate innovation by enhancing private incentives to invest in R&D (reward theory) and (2) that patents disseminate technical information into the public domain through disclosure requirements (contract theory). The model features endogenous innovator selection into patents versus secrecy based on heterogenous innovation size, the effective cost of disclosure, and expected licensing revenue from holding a patent. Innovation is cumulative, patent rights overlap across industries, and new innovator's pay mandatory licensing fees to a subset of previous innovators if those innovators hold a patent. The economy's endogenous patent propensity determines each new innovator's licensing burden, consistent with the concept of patent thickets. The model captures the inherent tension between the two objectives of the patent system and highlights novel, competing effects of patent policy on both economic growth and social welfare.

Keywords: Innovation; Patent policy; Patent thickets; Trade secrets; Endogenous growth

JEL Classification: O31, O34, O43

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1 Introduction

Within economics, the patent system is traditionally framed as a device to stimulate innovation. This "reward theory" view of the patent system maintains that patents grant temporary monopoly power to successful innovators in order to foster the ex ante private incentive to invest in R&D. In the judicial system however, patents are instead often conceptualized primarily as a means to disseminate technical information into the public domain. Section 112 of US patent law states that patents "must contain a written description ... as to enable any person skilled in the art ... to make and use the same." Under this "contract theory" view of patents, temporary monopoly is offered to innovators ex post in exchange for disclosing information that would otherwise remain secret indefinitely. The primacy of this reasoning within the court system is exemplified by a unanimous decision of the United States Supreme Court issued in 1989, which stated that "the ultimate goal of the patent system is to bring new designs and technologies into the public domain through disclosure."[1]

There is a clear tension between these "twin purposes" of the patent system. After all, firms choose to patent innovations only when doing so increases profitability relative to keeping them secret. As noted by Kultti et al. (2007), "if patenting enhances the incentive to innovate by improving appropriability, how can it simultaneously spread information and thereby the possibilities to imitate the patented innovation?" To the extent that disclosed information can be used by competitors to imitate innovations in ways that either do not explicitly infringe on a patent, or that are difficult for the patent owner to prevent, disclosure requirements can undermine the reward function of the patent system. Indeed, firms do routinely decide not to patent eligible innovations. Evidence from surveys of European and US firms estimate patent propensity, defined as the proportion of innovations for which a patent application is made, to be between 30-55% (Cohen et al., 2002; Hall et al., 2014). Across a wide range of industries, firms report secrecy to be a more effective appropriation mechanism than patents, and disclosure requirements are cited as a key reason firms choose not to patent (Arundel, 2001; Cohen et al., 2000, 2002; Hall et al., 2013).

On the other hand, if secrecy truly offers superior appropriability, one may wonder why firms ever choose to patent. In some cases, the answer may simply be that firms obtain a patent when it is difficult to keep the underlying innovation secret. If, for example, a particular innovation can be easily reverse engineered, technical information quickly leaks to competitors regardless of whether a patent is obtained.[2] Alternatively, Anton and Yao (2004) and Zaby (2010) argue that the relative value of obtaining a patent depends on the size of the competitive advantage that the innovation provides. Patents may effectively deter imitation of less substantial innovations because the costs to competitors associated with the risk of patent infringement outweigh the benefits of catching up to the innovator through imitation. However, the larger the lead of an innovator over its competitors,

[1] Decision in Bonito Boats, Inc. v. Thunder Craft Boats, Inc., 489 U.S. 141 (1989). See Roin (2005) for an extensive discussion on the emphasis placed on the contract theory of patents in the courts.

[2] This view is summarized in Boldrin and Levine (2013), "ideas will be patented when it seems likely that the secret would have emerged before the patent expired and not patented if the secret can be kept."

the greater the incentive for competitors to attempt imitation, and the greater the effective cost of disclosing technical information through a patent. In fact, this type of selection into patenting has been a long standing critique of the contract theory, for it implies that the information disclosed through patents is concentrated among a subset of innovations that are either relatively minor or would have entered the public domain even in the absence of a patent system.[3]

However, patents do offer a distinct advantage over secrecy by granting a degree of *forward* protection against competing innovation. That is, the effective breadth of patent protection increasingly includes the use of the original idea in future applications (Merges and Nelson, 1990; Jaffe, 2000; Gallini, 2002). Survey evidence shows that firms often choose to patent specifically in order to leverage a "blocking" effect on competitor innovation despite viewing secrecy as a superior option to protect against imitation.[4] Through the lens of the contract theory, this suggests a social benefit of forward protection since it encourages more innovators to bear the costs associated with a patent's information disclosure requirements. In terms of the reward theory however, there is growing concern that forward protection engenders rent seeking behavior that ultimately stifles innovation. In particular, since modern innovation necessitates building on many existing components and ideas, there is "a very real danger that a single product or service will infringe on many patents ... imposing an unnecessary drag on innovation by enabling multiple rights owners to 'tax' new products" (Shapiro, 2001). Commenting on the implications of this "patent thicket" effect, Boldrin and Levine (2013) argue that "the main dynamic general equilibrium effect of a patent system is to subject future inventions to a gigantic hold-up problem: with many licenses to be purchased and uncertainty about the ultimate value of the new innovation, each patent holder, in raising the price of his 'component,' imposes an externality on other patent holders."[5]

In this paper, I develop a general equilibrium model of endogenous growth to evaluate these aspects of patent policy in terms of both the reward and contract theory. In so doing, I attempt to merge two distinct lines of literature: the patent design literature that models firm choice of patents versus secrecy in a partial equilibrium setting (Denicolò and Franzoni, 2003; Anton and Yao, 2004; Kultti et al., 2007; Zaby, 2010; Kwon, 2012) and the endogenous growth literature that analyzes patent policy in terms of its general equilibrium effect on innovation (O'donoghue and Zweimüller, 2004; Chu, 2009; Chu et al., 2012; Acemoglu and Akcigit, 2012; Denicolò and Zanchettin, 2012; Cozzi and Galli, 2014; Yang, 2018). To my knowledge, only Suzuki (2015) and Klein (2020) have incorporated endogenous patenting and secrecy decisions in the context of general equilibrium growth. In both cases, these authors analyze how this decision impacts the reward effect of patent protection under the assumption that all innovators are homogenous and choose the same

[3]As noted in Denicolò and Franzoni (2003), this critique dates back at least as far as Rogers (1863); "what kind of contract is this, where the innovator keeps the best innovations for himself and gives the worse ... to the state?"

[4]For example, of the over 1,000 U.S. manufacturing firms surveyed in Cohen et al. (2000), 81.8% of firms include the blocking function of patents among the reasons that they ultimately chose to apply for a patent. In the same survey, 51% of firms consider secrecy as an effective appropriation mechanism for product innovations compared to 34.8% for patents. For process innovations, 50.6% view secrecy as effective against just 23.3% for patents. See Arundel (2001) and Hall et al. (2014) for additional evidence.

[5]See Akcigit and Ates (2019) for a recent discussion highlighting the role of this strategic use of patenting in declining business dynamism in the US.

mixture of patents and secrecy to protect their innovations. These analyses make an important contribution to the endogenous growth literature by demonstrating that stronger patent protection can fail to stimulate economic growth when innovators have the option to rely on secrecy. However, by construction, they do not consider why some firms choose secrecy while others patent, nor the consequences associated with this selection for the degree of overlapping rights creating patent thickets and the type of information disclosed through patents.

In contrast, I explicitly incorporate these considerations into a Schumpeterian quality ladder framework of endogenous cumulative innovation. Following Minniti et al. (2013) and Chu et al. (2017, 2019), the step size of each innovation's quality improvement is randomly drawn from a stationary Pareto distribution. After receiving their draw, each new industry leader chooses either to patent its innovation or keep it secret. Patents offer innovators a degree protection in two dimensions: backward protection against potential imitators and forward protection against subsequent innovation that displaces their leadership position. In the spirit of Kultti et al. (2007), Kwon (2012), and Klein (2020), backward protection is modeled as a probabilistic right to exclude competitors from the use of the information disclosed within the patent. Following O'donoghue and Zweimüller (2004) and Chu (2009), forward protection takes the form of a profit-sharing rule between current and former inventors through mandatory licensing agreements. Although secrecy does not provide forward protection, it provides superior backward protection since firms avoid information disclosure and prevent imitation as long as technical information does not "leak" to competitors. To capture the presence of patent thickets, I assume that each new innovation builds on a subset of current innovations across industries. Each new innovator must pay a licensing fee to each of the owners of these current innovations, *if they hold a patent*. This implies that the total licensing burden of new innovators depends on the endogenous patent propensity of firms throughout the economy.

I demonstrate that heterogeneity in industry leaders' quality advantage over competitors delivers an endogenous selection into patents versus secrecy that mirrors the findings of the partial equilibrium analyses of Anton and Yao (2004) and Zaby (2010). Firms with relatively small innovations choose to patent because their expected licensing revenue offsets the expected reduction in profits from the sale of their own innovation implied by patent disclosure. Firms with relatively large innovations choose secrecy in order to better preserve their large profit flow over their tenure as industry leader. I show that general equilibrium considerations have important implications for this equilibrium partition. Specifically, the greater the economy's rate of innovation, the shorter the expected period that firms remain industry leader, implying a greater incentive to patent to secure licensing revenue.

Within this general equilibrium framework, changes to patent policy gives rise to novel, competing effects on both economic growth and welfare. First, increasing backward protection strengthens the monopoly position of patented innovations by decreasing the ability of competitors to utilize disclosed information. This implies a standard reward theory trade-off; increased appropriability enhances R&D incentives but reduces the welfare benefit of each innovation by limiting competition. However, the direction of these effects can be *reversed* when one accounts for the endogenous patenting decision of innovators. Specifically, the increased appropriability of patents relative to secrecy generates an increase in equilibrium patent propensity. This increases the expected licensing burden of each innovator and reduces ex ante R&D incentives. I show that the net change to economic growth can be positive or negative, and depends on the relative size of this patent thicket effect and the traditional reward effect. In addition, the shift into patenting implies an increase in the proportion of innovations for which technical information is disclosed. This creates a welfare trade-off specific to the contract theory of patents; the net effect depends on the relative size of the increase in the volume of disclosed information against the increased limitations on its use.

Although strengthening forward protection also increases the relative attractiveness of patents over secrecy, I show that the corresponding increase in innovator's expected licensing burden always decreases R&D incentives and economic growth. This finding agrees with existing analyses of forward protection in endogenous growth models that assume all innovations are protected by patents, such as O'donoghue and Zweimüller (2004) and Yang (2018). Nevertheless, by incorporating the option to keep innovations secret, the model still highlights a novel welfare trade-off specific to the contract theory of patents. The shift into patenting caused by strengthened forward protection unambiguously increases the volume of information disclosure in the economy. The overall welfare impact of the policy change is determined by the relative importance of this pro-competitive increase in disclosure against the reduction in the rate of economic growth.

To better understand the relative magnitude of these competing effects, I calibrate the model to basic long-run features of the US economy and analyze the impact of patent policy numerically. The benchmark simulations show that strengthening forward or backward patent protection ultimately decreases private R&D incentives and economic growth. That is, I find that the positive reward effect of greater appropriability is dominated by the patent thicket effect created by the increased licensing burden of each new innovator as more innovators select into patenting. Indeed, this finding appears surprisingly general, and holds across most plausible cases.[6] I find that stronger backward protection increases economic growth only when the economy's patent propensity is quite high (close to 100%), or the distribution of innovation size is skewed heavily towards minor innovations. Second, I find that the overall welfare impact of patent policy is dictated by its effect on economic growth in most cases. In other words, although the information disclosure function of the patent system generates a positive welfare effect as emphasized by the contract theory of patents, it is usually dominated by reward theory considerations. In particular, I find that the contract theory of patents provides an independent justification for strengthening patent protection only when R&D investment exhibits severe diminishing marginal returns.

The remainder of this paper is organized as follows: Section 2 develops the theoretical model. Section 3 examines patent policy analytically in the context of the reward and contract theory of patents. Numerical results are presented in Section 4. Section 5 concludes.

[6]This prediction of the model is supported by empirical evidence that the continued strengthening of the legal protection afforded to patent holders has failed to stimulate innovation, the so called "patent puzzle." See Klein (2020) for further discussion of this result in the context of endogenous growth models.

2 The Model

2.1 Patents Versus Secrecy

The economy consists of a unit continuum of structurally identical industries indexed by $\omega \in [0,1]$. In each industry ω and time t, there exists a single leading firm that has successfully innovated the industry's current state-of-the-art product. A unit mass of competitive R&D firms, or "followers," participate in R&D races to innovate the next quality improvement and supplant the current leader. The winner of the R&D race for the j^{th} quality vintage of industry ω's product at time t discovers an innovation that represents a $\lambda(j,\omega,t)$ size quality improvement over the previous vintage. That is,

$$\lambda(j,\omega,t) \equiv \frac{q(j,\omega,t)}{q(j-1,\omega,t)} > 1, \tag{2.1}$$

where $q(j,\omega,t)$ denotes the product quality associated with the specified vintage, industry, and time. Following Minniti et al. (2013) and Chu et al. (2017, 2019), the winner of each R&D race draws their innovation's quality improvement from a stationary Pareto distribution with probability density function,

$$f(\lambda) = \frac{1}{\kappa}\lambda^{-(1+\kappa)/\kappa}, \tag{2.2}$$

where $\kappa \in (0,1)$ determines the distribution's shape parameter, $1/\kappa$.[7]

As in the standard quality ladder framework, leaders and followers within industries compete in prices. Each new leader optimally exploits its innovation's quality advantage through limit pricing and captures its industry's entire market share. This implies that each $\lambda(j,\omega,t)$ quality advantage draw can be immediately translated into corresponding monopoly flow profits, $\pi(\lambda(j,\omega,t),t)$. However, leaders face two threats to their dominant market position: subsequent innovation and imitation by industry followers. Successful imitation of a leader's product enables a follower to copy the industry's state-of-the-art quality. That is, imitation implies full catch-up and eliminates the leader's quality advantage regardless of its initial size. As detailed further in Section 2.3, once a leader's product has been imitated, price competition drives the market price to marginal cost and the leader's flow profits to zero.

To protect their monopoly position, each new quality leader chooses either to patent their innovation or keep it secret as soon as its quality draw is realized. Both appropriation methods are imperfect. When a leader chooses to patent, there exists a probability $m_p \in (0,1)$ that a leader's patent will not effectively prevent follower imitation. As in Kultti et al. (2007) and Kwon (2012), this single probability of imitation is intended to distill all relevant aspects of the imperfect backward protection provided by patents. In particular, I interpret m_p to represent the aggregate threat of imitation due to information disclosure requirements and the ability of followers to utilize this disclosed information given limited patent length, breadth, and enforcement. I treat m_p as an exogenous policy parameter, where $m_p = 0$ represents perfect backward protection from patents and

[7] See Minniti et al. (2013) for evidence that the empirical distribution of innovation size is well approximated by a Pareto distribution.

$m_p = 1$ represents nonexistent backward protection. Although secrecy avoids formal information disclosure, followers are free to imitate the latest quality vintage if they can uncover its underlying technical information. This information leakage occurs with probability $m_s \in (0, 1)$.

Throughout the main analysis, I assume that $m_p > m_s$ so that secrecy provides superior protection from imitation in accordance with firm survey evidence. As we will see, this assumption will underpin the positive welfare effect of the information disclosure requirements of patents that are central to the contract theory of patents. In effect, $m_p > m_s$ ensures that more technical information enters into the public domain when a leader chooses to patent. Simplifying notation, the expected profit flows of a λ size quality leader in a typical industry under patent and secrecy respectively are

$$\pi_p(\lambda, t) = (1 - m_p)\pi(\lambda, t), \qquad \pi_s(\lambda, t) = (1 - m_s)\pi(\lambda, t). \tag{2.3}$$

Unlike secrecy however, patents offer a degree of forward protection. As in O'donoghue and Zweimüller (2004) and Chu (2009), forward protection takes the form of compulsory licensing agreements between the patent holder and subsequent innovators that build on the patented innovation. This existing literature assumes that this cumulative nature of quality improvement is restricted to new innovations within the same industry. That is, new quality vintages infringe only on patents covering previous iterations of the same product. In contrast, I assume that each new innovation builds on some potentially patent protected component of the current state-of-the-art product in a $\phi \in (0, 1)$ proportion of industries in the economy. New innovators must obtain a license from each associated industry leader, *if that incumbent leader holds a patent.* I treat ϕ as an exogenous policy parameter that represents the breadth of forward patent protection.[8] To maintain a symmetric equilibrium structure with a common rate of innovation in each industry, $I(\omega, t) = I(t)$, I assume that this forward protection breadth is common across industries. In this way, each successive quality vintage shares the same potential to infringe on multiple patents and each patent has the same potential to secure licensing agreements with multiple future innovators.[9]

In the spirit of O'donoghue and Zweimüller (2004) and Chu (2009), the licensing payment to the owner of each patented component is determined based on the expected value of a flow payment of $s \in (0, 1)$ share of the new innovator's monopoly profits $\pi(\lambda, t)$ over its tenure as industry leader. To keep the analysis tractable, I assume that innovators pay the present discounted value of the requisite licensing fee to all infringed patent holders as a lump sum as soon as innovation occurs. As we will see, this implies that the expected licensing revenue from owning a patent does not

[8]Empirical analyses of patent citations provide direct empirical support for the presence of such inter-industry patent overlap. Fung (2005) and Blazsek and Escribano (2010) document that newly granted patents routinely include citations of existing patents across different industries, and such inter-industry citations often comprise the majority of a patent's total citations. In addition, Niwa (2016, 2018) analyze licensing agreements across industries in the context of horizontal innovation in a variety expansion model of growth. In this work, each new variety is assumed to infringe on the patents of all existing incumbents. In the present model, I assume that the degree of such inter-industry patent overlap is determined by patent law.

[9]O'donoghue and Zweimüller (2004) and Chu (2009) do allow for new innovations to infringe on multiple patents covering several previous vintages of the industry's product. Instead, I follow Chu et al. (2012) and Yang (2018) and assume that infringement occurs only on the most recent vintage. However, since I extend this framework to incorporate infringement across industries, numerous innovations may still infringe on a single patent.

depend on the probability of imitation corresponding to the licensee's choice of patenting versus secrecy. Consequently, it ensures that the value of forward protection provided by patents depends only on its breadth ϕ and the size of each licensing deal determined by the policy parameter s, but is independent of the degree of backward protection m_p.[10]

Let $v_L(\lambda, t)$ denote the value of the lump sum licensing payment from a λ size quality leader to a single infringed patent holder. $v_L(\lambda, t)$ is calculated through a standard no-arbitrage condition that equates the expected return of the licensing deal over the licensee's duration as industry leader to the risk-free market rate $r(t)$. Over an interval of time dt, the licensee owes a $s\pi(\lambda, t)dt$ share of profits to the patent holder. With probability $I(t)dt$, the licensee is replaced as industry leader, terminating the agreement. If the licensee is not replaced, the value of the agreement changes by $\dot{v}_L(\lambda, t)dt$. The corresponding no-arbitrage condition is

$$r(t)v_L(\lambda, t)dt = s\pi(\lambda, t)dt - I(t)v_L(\lambda, t)dt + (1 - I(t)dt)\dot{v}_L(\lambda, t)dt. \tag{2.4}$$

Taking limits as $dt \to 0$ and collecting terms, we have

$$v_L(\lambda, t) = \frac{s\pi(\lambda, t)}{r(t) + I(t) - \frac{\dot{v}_L(\lambda, t)}{v_L(\lambda, t)}}. \tag{2.5}$$

The total licensing obligation of each new innovator depends on the patenting decision of industry leaders throughout the economy. Using the law of large numbers, the total expected licensing obligation of a λ size innovator, denoted $V_L(\lambda, t)$, is given by

$$V_L(\lambda, t) = \int_{\omega \in \phi} \mathbb{1}_p(\omega, t) v_L(\lambda, t) d\omega = \phi n_p(t) v_L(\lambda, t), \tag{2.6}$$

where forward protection breadth ϕ determines the proportion of industries for which a new innovation potentially infringes on existing patents, $\mathbb{1}_p(\omega, t)$ is an indicator function taking the value of one if the leader in industry ω owns a patent at time t, and $n_p(t)$ denotes the economy wide proportion of leaders that own a patent at time t.

Let $v_p(\lambda, t)$ and $v_s(\lambda, t)$ denote the expected value of a λ size innovation under patent or secrecy respectively, after paying the requisite upfront licensing fees of expected size $V_L(\lambda, t)$. When a leader chooses to patent, they earn an expected profit flow $\pi_p(\lambda, t)dt$ over an interval of time dt. Innovation occurs in each industry ω with probability $I(\omega, t)dt$, and each such innovation has a probability ϕ of resulting in a licensing deal for a patent holder. The expected number of licensing deals obtained during dt is

$$\int_0^1 \phi I(\omega, t) dt d\omega = \phi I(t) dt, \tag{2.7}$$

in a symmetric equilibrium with $I(\omega, t) = I(t)$. Each licensing deal results in an immediate payment

[10] Modeling licensing payments as a lump sum is also consistent with the fact that licensing fees are typically negotiated when the ultimate value of a new innovation remains uncertain (Boldrin and Levine, 2013).

that depends on the infringing party's λ draw according to (2.5), with expected size $\mathbb{E}_\lambda[v_L(\lambda,t)]$. There is a capital loss $v_p(\lambda,t)$ from replacement when the next innovation occurs in the patent holder's industry, with probability $I(t)dt$, and a change in valuation $\dot{v}_p(t)dt$ if the firm remains the leader with probability $(1-I(t)dt)$. Equating this overall expected return to the interest rate $r(t)$, taking limits as $dt \to 0$ and collecting terms, we have

$$v_p(\lambda,t) = \frac{(1-m_p)\pi(\lambda,t) + \phi I(t)\mathbb{E}_\lambda[v_L(\lambda,t)]}{r(t) + I(t) - \frac{\dot{v}_p(\lambda,t)}{v_p(\lambda,t)}}. \tag{2.8}$$

When a leader chooses secrecy, they forgo potential licensing revenue in exchange for a larger expected profit flow. The corresponding no-arbitrage condition is

$$v_s(\lambda,t) = \frac{(1-m_s)\pi(\lambda,t)}{r(t) + I(t) - \frac{\dot{v}_s(\lambda,t)}{v_s(\lambda,t)}}. \tag{2.9}$$

Since each innovator chooses either secrecy or a patent in order to maximize the expected present discounted value of their innovation, the total expected value of an innovation of size λ is given by

$$V(\lambda,t) = \max\{v_p(\lambda,t), v_s(\lambda,t)\} - V_L(\lambda,t). \tag{2.10}$$

2.2 Households

As in the traditional quality ladder framework, the economy is populated by a unit continuum of identical households. Each household is a dynastic family of infinitely lived members that begins with a single member at $t=0$ and grows at the common rate $n>0$. The population of the economy at time t equals is given by $N(t)=e^{nt}$. Each household maximizes discounted utility

$$U = \int_0^\infty e^{-(\rho-n)t}\; ln(u(t))dt, \tag{2.11}$$

where $\rho > n$ is the subjective discount rate. Per capita sub-utility at time t is defined as

$$ln(u(t)) = \int_0^1 ln\Big[\sum_j q(j,\omega,t)y(j,\omega,t)\Big]d\omega, \tag{2.12}$$

where $q(j,\omega,t)$ denotes the quality of the j^{th} vintage of industry ω's product at time t and $y(j,\omega,t)$ denotes the associated quantity consumed.

Households maximize (2.11) by allocating per capita consumption expenditure $c(t)$ given prices at time t. Since quality adjusted products within each industry are perfect substitutes, households purchase only the product with the lowest quality adjusted price. Products enter utility symmetrically, so households optimally spread expenditure evenly across each industry. Demand for the good with the lowest quality adjusted price in a typical industry ω is

$$y(\omega, t) = \frac{c(t)N(t)}{p(\omega, t)}, \tag{2.13}$$

where $p(\omega, t)$ is the market price of the associated good. Given (2.13), maximizing (2.11) subject to the standard intertemporal budget yields

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{2.14}$$

where $r(t)$ is the instantaneous market interest rate.

2.3 Production and R&D

In each industry, labor is used for R&D and the manufacture of final goods. Labor is the only factor of production and one unit of labor is required to produce one unit of output, regardless of the product's quality. All firms share a common marginal cost equal to the wage rate, which is normalized to unity and serves as the numéraire. In each industry, firms compete in prices under two possible cases depending on whether successful imitation of the industry's state-of-the-art product has occurred. While an industry leader maintains its quality advantage, it optimally charges a limit price of $p(\omega, t) = \lambda(\omega, t)$. Given equal costs of production, this limit price ensures that followers can do no better than break even and exit the market. Using (2.13), the instantaneous flow profits associated with a $\lambda(\omega, t)$ quality lead are

$$\pi(\lambda(\omega, t), t) = c(t)N(t)\frac{\lambda(\omega, t) - 1}{\lambda(\omega, t)}, \tag{2.15}$$

where $\lambda(\omega, t) - 1$ is the profit margin and $c(t)N(t)/\lambda(\omega, t)$ is total quantity sold. If the leader's product is imitated, competition drives the market price to marginal cost and flow profits to zero. Since this implies that both the leader and imitating follower exactly break even, I assume that the leader continues to serve the entire market, with total quantity sold at the competitive level of $c(t)N(t)$. Thus, leaders remain active in the market until they are displaced by subsequent innovation.

Followers in each industry participate in R&D races to innovate the next quality vintage and supplant the current leader. A follower i that employs $l_{R,i}(\omega, t)$ units of labor in R&D at time t successfully innovates with instantaneous probability

$$i_i(\omega, t) = \frac{l_{R,i}(\omega, t)}{\alpha N(t)} \cdot \left[\frac{L_R(\omega, t)}{N(t)}\right]^{-\beta}, \tag{2.16}$$

where $0 < \beta < 1$ and $L_R(\omega, t) = \sum_i l_{R,i}(\omega, t)$ is the total R&D labor used by firms in industry ω. Equation (2.16) combines two common features of R&D technology specifications in endogenous growth models. Following the approach pioneered by Dinopoulos and Segerstrom (1999) and

Dinopoulos and Thompson (2000), the first term in (2.16) implies that R&D difficulty is proportional to the size of the economy's population. This eliminates the counterfactual scale effects present in first-generation endogenous growth models, while maintaining a tractable analytical structure. The second term in (2.16) implies that each firm's instantaneous probability of successful innovation is a decreasing function of total R&D investment in the industry. This captures the well-established presence of duplicative R&D investment among firms competing in R&D races, and imposes decreasing returns to R&D at the industry level.[11] The industry wide innovation rate is obtained by summing across all followers,

$$I(\omega, t) = \frac{1}{\alpha}\Big[\frac{L_R(\omega, t)}{N(t)}\Big]^{1-\beta}. \tag{2.17}$$

Each follower chooses $l_{R,i}(\omega, t)$ in order to maximize its expected discounted profits. Free-entry into R&D implies that in every industry with positive research expenditure, the expected return to R&D must exactly offset its cost. Let $\mathbb{E}_\lambda[V(\lambda, t)]$ denote the ex ante expected value of a successful innovation, taking into account the innovator's optimal choice of patenting versus secrecy and expected licensing burden as in (2.10). In a symmetric equilibrium with $I(\omega, t) = I(t) > 0$, free-entry implies that followers in each industry choose R&D employment so that $l_{R,i}(t) = i_i(t)\mathbb{E}_\lambda[V(\lambda, t)]$. Using (2.16) and (2.17), the free-entry condition can be written as

$$\mathbb{E}_\lambda[V(\lambda, t)] = \hat{\alpha} N(t) I(t)^{\frac{\beta}{1-\beta}}, \tag{2.18}$$

where $\hat{\alpha} = \alpha^{1/1-\beta}$.

2.4 Equilibrium

I now solve the model for a steady state equilibrium in which $I(\omega, t) = I(t)$, $n_p(t)$, and $c(t)$ are constant, the labor market clears, the free-entry condition (2.18) holds, and each leader chooses either secrecy or a patent to maximize their value. It follows immediately from (2.5), (2.8), (2.9), (2.15), and (2.18) that the value of each innovation, whether patented or secret, grows at the rate of population growth n in equilibrium. That is, $\dot{v}_p(\lambda, t)/v_p(\lambda, t) = \dot{v}_s(\lambda, t)/v_s(\lambda, t) = n$. This is the standard implication of endogenous growth specifications in which R&D difficulty grows at rate n, as in (2.17). From the Euler equation (2.14), constant per capita consumption expenditure implies that $r(t) = \rho$. In the main text, I restrict attention to the case where neither appropriation method strictly dominates the other, so that both secrecy and patents are chosen by some firms in equilibrium and $0 < n_p(t) < 1$. Henceforth, I drop the time index for all variables that are constant in equilibrium.

[11] See Jones and Williams (2000) for a discussion of the importance of this R&D duplication or "stepping on toes effect" in endogenous growth models. Additional examples of this approach to decreasing returns to R&D at the industry level include Impullitti (2010), Chu et al. (2012), and Cozzi and Galli (2014).

2.4.1 Patents versus secrecy in equilibrium

Let $V_p(\lambda, t)$ and $V_s(\lambda, t)$ denote the total expected value of an innovation of size λ under patent and secrecy respectively, including licensing payments. That is, $V_p(\lambda, t) = v_p(\lambda, t) - V_L(\lambda, t)$ and $V_s(\lambda, t) = v_s(\lambda, t) - V_L(\lambda, t)$. Using (2.5), (2.8), and (2.9), we have

$$V_s(\lambda, t) = \frac{(1 - \phi s n_p - m_s)\pi(\lambda, t)}{\rho - n + I} \tag{2.19}$$

$$V_p(\lambda, t) = \frac{(1 - \phi s n_p - m_p)\pi(\lambda, t)}{\rho - n + I} + \frac{\phi s I \mathbb{E}_\lambda[\pi(\lambda, t)]}{(\rho - n + I)(\rho - n + I)} \tag{2.20}$$

As in Chu (2009) and Yang (2018), each innovator's licensing obligation acts as a reduction in their expected flow profits over their tenure as industry leader. In the present model however, this licensing obligation depends on the endogenous proportion of firms that choose to patent, n_p. In addition, observe that the impact of forward patent protection on both the licensing obligation of all new innovators and the expected licensing revenue of patent holders is determined by the product of forward protection breadth and licensing magnitude, $\phi s \in (0, 1)$. Without loss of generality, I henceforth treat ϕs as a single parameter that determines the overall strength of forward patent protection. In order to ensure that new innovators always enter the market after receiving their λ draw, I impose the following parameter restriction

Assumption 1. $\phi s < 1 - m_s$.

Under Assumption 1, each firm's licensing obligation is sufficiently small relative to its expected flow profits as industry leader so that $V(\lambda, t) = \max\{V_p(\lambda, t), V_s(\lambda, t)\} > 0$ for all $\lambda > 1$.

Although all firms that choose to patent share a common expected licensing revenue, the effective cost of patenting depends on the innovator's flow profits. This implies that an innovator's optimal choice of a patent versus secrecy depends on the size of its innovation. More formally, an innovator will choose to patent if and only if $V_s(\lambda, t) \leq V_p(\lambda, t)$. Using (2.19) and (2.20), this condition can be rewritten

$$(m_p - m_s)\pi(\lambda, t) \leq \frac{\phi s I \mathbb{E}_\lambda[\pi(\lambda, t)]}{(\rho - n + I)}. \tag{2.21}$$

The left hand side of (2.21) captures the cost of patenting in terms of reduced expected profit flows due to information disclosure and is strictly increasing in λ. The right hand side captures the benefit in terms of expected licensing revenue, which is constant in the innovator's λ. Therefore, we can characterize the patent, secrecy decision of all firms in the economy in terms of a threshold innovation size $\tilde{\lambda}$. When both patenting and secrecy occur in equilibrium, $\tilde{\lambda}$ is determined by the single crossing of the left and right hand side of (2.21) given by

$$(m_p - m_s)(1 - \tilde{\lambda}^{-1}) = \frac{\phi s I \Omega}{(\rho - n + I)}, \tag{2.22}$$

Where $0 < \Omega \equiv \kappa/(1 + \kappa) < 1$ denotes the portion of expected profit that depends on the draw of

λ. As shown in the Appendix, we have

$$\mathbb{E}_\lambda[\pi(\lambda,t)] = \int_1^\infty cN(t)(1-\lambda^{-1})f(\lambda)d\lambda = cN(t)\frac{\kappa}{1+\kappa} = cN(t)\Omega. \tag{2.23}$$

Rearranging gives the following equilibrium condition,

$$\tilde{\lambda} = \frac{(\rho-n+I)(m_p-m_s)}{(\rho-n+I)(m_p-m_s)-\phi sI\Omega}, \tag{2.24}$$

where all firms that receive an innovation draw of $\lambda \leq \tilde{\lambda}$ choose to patent and all firms that receive $\lambda > \tilde{\lambda}$ choose secrecy. Therefore, as in the partial equilibrium analyses of Anton and Yao (2004) and Zaby (2010), the model delivers an equilibrium partition into patenting and secrecy in which firms with relatively small innovations choose to patent, while firms with relatively large innovations choose secrecy. Using the law of large numbers, we can express the economy wide proportion of innovations under patent in terms of the probability of receiving a $\lambda \leq \tilde{\lambda}$ draw. That is,

$$n_p(\tilde{\lambda}) = F(\tilde{\lambda}) = 1-\tilde{\lambda}^{-1/\kappa} \tag{2.25}$$

When convenient, I refer to $n_p(\tilde{\lambda})$ as the economy wide "patent propensity" and to $\tilde{\lambda}$ as the equilibrium "patent threshold." Note that patent propensity is uniquely determined by and strictly increasing in the patent threshold.

In a steady state equilibrium with constant I, $\tilde{\lambda}$ and n_p are both constant. However, the equilibrium value of $\tilde{\lambda}$ is strictly increasing in the equilibrium I. This is because a greater rate of innovation makes the forward protection from subsequent innovations offered by patents relatively more attractive. Indeed, as $I \to 0$, the benefit of patenting disappears, and $\tilde{\lambda} \to 1$ implying all firms choose secrecy. As $I \to \infty$,

$$\tilde{\lambda} \to \tilde{\lambda}_{max} \equiv \frac{m_p-m_s}{m_p-m_s-\phi s\Omega}. \tag{2.26}$$

To ensure that the patent threshold is well defined for any positive, finite rate of innovation, I impose the following parameter restriction

Assumption 2. $\phi s\Omega < m_p - m_s.$

Under Assumption 2, the effective cost of patenting is sufficiently high such that some firms always prefer secrecy in equilibrium and $0 < n_p < 1$. Finally, note that it is immediate from (2.24) that the patent threshold decreases in m_p and increases ϕs. That is, stronger backward or forward patent protection increases the attractiveness of patenting relative to secrecy, increasing the patent threshold and patent propensity. Summarizing, we have,

Lemma 1. *Under Assumptions 1 and 2, the equilibrium choice of patenting versus secrecy of all* *innovators in the economy is completely characterized by a unique patent threshold* $1 < \tilde{\lambda} < \infty$*. All innovations of size* $\lambda \leq \tilde{\lambda}$ *are protected by patent and all innovations of size* $\lambda > \tilde{\lambda}$ *are protected by secrecy. All else equal, the economy's patent threshold and corresponding patent propensity are strictly increasing in backward patent protection, forward patent protection, and the equilibrium rate of innovation.*

2.4.2 Labor market clearing and free-entry

Labor market clearing requires that the total labor employed in production and R&D equals the economy's population. Rearranging (2.17), total labor employed in R&D can be written in terms of the equilibrium innovation rate,

$$L_R(t) = \hat{\alpha} N(t) I^{\frac{1}{1-\beta}}. \tag{2.27}$$

Labor employed in manufacturing within each industry depends on the price with in that industry $l_y(\omega, t) = c(t)N(t)/p(\omega, t)$. Prices are either equal to $p(\omega, t) = \lambda(\omega, t)$ if the latest innovation has not been imitated or $p(\omega, t) = 1$ if it has. Using the equilibrium selection into patenting and secrecy from (2.24) and the law of large numbers, total labor employed in manufacturing is given by

$$L_y(t) = cN(t)\Big[\int_1^{\tilde{\lambda}} \Big[(1-m_p)\frac{1}{\lambda} + m_p\Big] f(\lambda) d\lambda + \int_{\tilde{\lambda}}^{\infty} \Big[(1-m_s)\frac{1}{\lambda} + m_s\Big] f(\lambda) d\lambda\Big]. \tag{2.28}$$

As shown in the Appendix,

$$\begin{aligned} L_y(t) &= cN(t)M(\tilde{\lambda}), \\ \text{where} \quad M(\tilde{\lambda}) &= n_p m_p + (1-n_p)m_s + \frac{1}{1+\kappa}\big[1 - m_p + (m_p - m_s)\tilde{\lambda}^{-\frac{1}{\Omega}}\big] \end{aligned} \tag{2.29}$$

captures the mean labor requirement per unit of consumption expenditure as a function of the patent threshold. Note that $M(\tilde{\lambda}) > 0$ and is strictly increasing in $\tilde{\lambda}$. This is an immediate consequence of the disclosure requirements of patenting delivering $m_p > m_s$. Greater patent propensity implies a greater proportion of industries produce under competitive conditions where each unit of consumption expenditure corresponds to a greater quantity of output.

Using (2.27) and (2.29), the labor market clearing condition of $N(t) = L_y(t) + L_R(t)$ becomes,

$$1 = cM(\tilde{\lambda}) + \hat{\alpha} I^{\frac{1}{1-\beta}}. \tag{2.30}$$

Thus, the usual resource allocation trade-off between consumption expenditure and innovation is present in the model. An increase in the economy's patent propensity effectively tightens the resource constraint since it increases the manufacturing labor required to maintain a constant level of per capita consumption expenditure.

The free-entry condition, equation (2.18), equates the expected value of developing an innovation to the associated R&D cost. Under the equilibrium selection into patenting and secrecy given by the patent threshold in equation (2.24), we can write the ex ante expected value of an innovation as

$$\mathbb{E}_{\lambda}[V(\lambda,t)] = \int_{1}^{\tilde{\lambda}} V_p(\lambda,t)f(\lambda)d\lambda + \int_{\tilde{\lambda}}^{\infty} V_s(\lambda,t)f(\lambda)d\lambda. \tag{2.31}$$

As shown in the Appendix, performing the required integration yields

$$\mathbb{E}_{\lambda}[V(\lambda,t)] = \frac{cN(t)}{\rho-n+I}\Big[\frac{\phi s\Omega n_p I}{\rho-n+I} - \phi s\Omega n_p + \Big(\Omega(1-m_p) + (m_p-m_s)\Big(\frac{\tilde{\lambda}-\frac{1}{1+\kappa}}{\tilde{\lambda}^{1/\Omega}}\Big)\Big)\Big]. \tag{2.32}$$

As usual, the ex ante expected value of an innovation depends on the stream of profits extracted from consumption expenditure in the industry discounted at an effective rate that includes the threat of replacement, $\rho - n + I$. The first term in brackets captures the contribution of expected licensing revenue. Since only innovators that choose to patent receive licensing revenue, this term is weighted by the probability of receiving a draw associated with patenting, $F(\tilde{\lambda}) = n_p$. The second term captures expected licensing payments. All firms pay the same proportion of profits in licensing fees and the magnitude of these fees depends on economy wide patent propensity. Together, these two terms capture the traditional effect of blocking patents in endogenous growth models such as O'donoghue and Zweimüller (2004), Chu (2009), and Yang (2018). Although ex ante expected licensing payments equal expected licensing revenue, innovators discount future licensing revenue relative to their immediate required payments. Consequently, the expected reward from innovation decreases in forward patent protection.

The final term captures expected profit flows, net of licensing payments. Note that if all firms chose to patent with $\tilde{\lambda} = \infty$, then $n_p = 1$ and this net expected profit flow becomes $\Omega(1 - m_p)$. If all firms chose secrecy with $\tilde{\lambda} = 1$, then $n_p = 0$ and the entire term in brackets collapses to a net expected profit flow of $\Omega(1 - m_s)$ since no licensing occurs. Furthermore, for any $\tilde{\lambda} > 1$, we have that $\mathbb{E}_{\lambda}[V(\lambda,t)]$ is strictly decreasing in m_p. That is, as long as some firms choose to patent, stronger backward protection from patents increases the expected value of an innovation. This reflects the traditional reward theory motivation for strengthening patent protection.

Finally, note that the expected value of an innovation is decreasing in the economy's patent threshold $\tilde{\lambda}$ for two reasons. First, a greater patent threshold implies that innovators are more likely to receive an innovation draw associated with patenting. This corresponds to lower expected profit flows. Second, due to the presence of overlapping innovations across industries, the increase in the economy wide patent propensity increases the volume of licensing payments between new and incumbent innovators. That is, in terms of the ex ante value of an innovation, an increase in $\tilde{\lambda}$ behaves as an increase in the forward protection of patents. Summarizing, we have the following,

Lemma 2. *Under Assumptions 1 and 2, the reward for successful innovation is strictly increasing in backward patent protection, strictly decreasing in forward patent protection, and strictly decreasing in the patent threshold and corresponding patent patent propensity. That is,* $\frac{\partial \mathbb{E}_{\lambda}[V(\lambda,t)]}{\partial m_p} < 0$,

$\frac{\partial \mathbb{E}_\lambda[V(\lambda,t)]}{\partial \phi s} < 0$, *and* $\frac{\partial \mathbb{E}_\lambda[V(\lambda,t)]}{\partial \tilde{\lambda}} < 0$.

Of course, Lemma 2 does not account for the general equilibrium effects of patent policy on the private incentive to invest in R&D. In particular, the reward to innovation depends on equilibrium consumption expenditure and the innovation rate, which in turn depend on the allocation of labor resources between R&D and production captured by the labor market clearing condition. To incorporate the general equilibrium determination of c and I, I combine the labor market clearing condition (2.30), the free-entry condition (2.18), and the expected value of an innovation (2.32),

$$1 = \frac{(1-\hat{\alpha} I^{\frac{1}{1-\beta}})\left[\Omega\left(\frac{\phi s n_p I}{\rho - n + I} + 1 - m_p - \phi s n_p\right) + (m_p - m_s)\left(\frac{\tilde{\lambda} - \frac{1}{1+\kappa}}{\tilde{\lambda}^{1/\Omega}}\right)\right]}{\hat{\alpha} I^{\frac{\beta}{1-\beta}}(\rho - n + I) M(\tilde{\lambda})}. \tag{2.33}$$

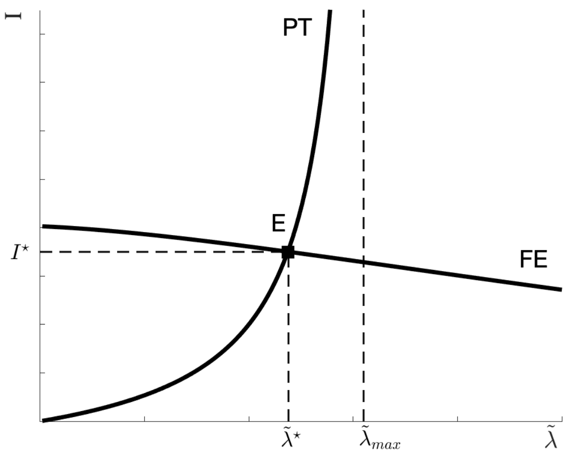
Equation (2.33) represents a single free-entry (FE) condition that captures the relationship between the private incentive to invest in R&D and the endogenous patent threshold, after incorporating the general equilibrium effect of the resource allocation trade-off between equilibrium consumption and innovation. Note that this general equilibrium effect compounds the negative relationship between I and $\tilde{\lambda}$. This is because an increase in patent propensity increases the effective resource cost of c through $M(\tilde{\lambda})$, leaving fewer resources for R&D. Combined with the negative effect of patent propensity on the reward to innovation at any constant level of c established by Lemma 2, equation (2.33) specifies a strictly downward sloping relationship between I and $\tilde{\lambda}$.

2.4.3 The steady state equilibrium

The model's equilibrium is determined by solving the patent threshold (PT) condition given by (2.24) and the free-entry (FE) condition given by (2.33) for the equilibrium values of I and $\tilde{\lambda}$. As shown in the Appendix, Assumptions 1 and 2 guarantee a unique steady state equilibrium exists in which $I > 0$ and $1 < \tilde{\lambda} < \infty$. Figure 1 illustrates the equilibrium by graphing the PT and FE conditions in $(\tilde{\lambda}, I)$ space.[12]

[12]Note that the economy immediately jumps to the steady state equilibrium at time zero, when the economy begins with an initial quality draw for each industry. As in Klein (2020), this is because R&D investment, per-capita consumption expenditure, and each innovator's patent versus secrecy decision are choice variables.

Figure 1: Equilibrium



2.5 Growth and Welfare

As is standard, the rate of economic growth is defined as the rate of growth of per capita sub-utility $u(t)$. Using (2.12) and (2.13), we have

$$ln(u(t)) = \int_0^1 ln\Big(\frac{c \cdot q(\omega, t)}{p(\omega, t)}\Big) d\omega = ln(c) - \int_0^1 ln(p(\omega, t)) d\omega + \int_0^1 ln(q(\omega, t)) d\omega \qquad (2.34)$$

In each industry, $p(\omega, t)$ equals $\lambda(\omega, t)$ if the leader's product has not been imitated and one if it has. Using the law of large numbers, we have

$$\int_0^1 ln(p(\omega, t)) d\omega = (1 - m_p) \int_1^{\tilde{\lambda}} ln(\lambda) f(\lambda) d\lambda + (1 - m_s) \int_{\tilde{\lambda}}^{\infty} ln(\lambda) f(\lambda) d\lambda \qquad (2.35)$$

As shown in the Appendix, we can express $\int_0^1 ln(p(\omega, t)) d\omega$ as a stationary price index as a function of the patent threshold, $P(\tilde{\lambda})$. That is,

$$P(\tilde{\lambda}) \equiv \int_0^1 ln(p(\omega, t)) d\omega = (1 - m_p)\kappa + (m_p - m_s)\tilde{\lambda}^{-\frac{1}{\kappa}} \Big[ln(\tilde{\lambda}) + \kappa \Big]. \qquad (2.36)$$

The price index is constant in equilibrium, and is strictly decreasing in $\tilde{\lambda}$ because $m_p > m_s$. Note that if all innovations were kept secret, $P(\tilde{\lambda} = 1) = (1 - m_s)\kappa$, and if all innovations were patented, $P(\tilde{\lambda} = \infty) = (1 - m_p)\kappa$.

Since c and $P(\tilde{\lambda})$ are constant in equilibrium, the equilibrium rate of growth $g \equiv \frac{\dot{u}(t)}{u(t)}$ is equal to the time derivative of $\int_0^1 ln(q(\omega, t)) d\omega$. Within each industry, product quality evolves according to $q(j, \omega, t) = \lambda(\omega, t) q(j-1, \omega, t)$ as new vintages are introduced at rate I. As shown in the Appendix, this implies that the rate of economic growth is proportional to the rate of innovation,

$$g = I\int_1^{\infty} ln(\lambda)f(\lambda)d\lambda = I\kappa. \tag{2.37}$$

Combining terms and using (2.11), we have the following expression for welfare discounted to time zero,

$$(\rho - n)U = \frac{\kappa}{\rho - n}I + ln(c) - P(\tilde{\lambda}) \tag{2.38}$$

To better understand the role of patent policy on equilibrium welfare, I consider a social planner who chooses the levels of c and I to maximize social welfare in (2.38), subject to the resource constraint (2.30). As in Klein (2020), I assume that the social planner cannot directly control innovators' patenting decisions, which implies that each innovator's patent versus secrecy choice remains determined by the PT in equation (2.24). The associated Lagrangian is

$$\mathcal{L}(c, I, \Gamma) = \frac{\kappa}{\rho - n}I + ln(c) - P(\tilde{\lambda}) + \Gamma\Big[1 - cM(\tilde{\lambda}) - \hat{\alpha}I^{\frac{1}{1-\beta}}\Big], \tag{2.39}$$

where Γ is the Lagrange multiplier. Optimization yields the following expression that equates the social cost and benefit of R&D

$$\hat{\alpha}I^{\frac{\beta}{1-\beta}} = (1-\beta)cM(\tilde{\lambda})\left(\frac{\kappa}{\rho - n} - \frac{\partial\tilde{\lambda}}{\partial I}\left[\frac{1}{M(\tilde{\lambda})}\frac{\partial M(\tilde{\lambda})}{\partial\tilde{\lambda}} + \frac{\partial P(\tilde{\lambda})}{\partial\tilde{\lambda}}\right]\right). \tag{2.40}$$

Using (2.18), the analogous expression for the market cost and return of R&D is

$$\hat{\alpha}I^{\frac{\beta}{1-\beta}} = \frac{1}{N(t)}\mathbb{E}_{\lambda}[V(\lambda, t)], \tag{2.41}$$

where $\mathbb{E}_{\lambda}[V(\lambda, t)]$ is given by (2.32).

Although (2.40) and (2.41) are too complex to directly compare analytically, they still illuminate several reasons that the market equilibrium may fail to deliver the socially optimal level of R&D. First, the social planner scales the entire benefit of R&D by $1 - \beta$, while β does not affect the market return to R&D. This reflects the negative externality associated with duplicative R&D investment embedded in the R&D technology of (2.17). As emphasized by Jones and Williams (2000), this implies that the value of β plays a crucial role in determining if the market equilibrium exhibits over or under investment in R&D. Second, the social planner internalizes the feedback effect of the innovation rate on the equilibrium selection into patenting. In other words, unlike private researchers, the social planner realizes that a consequence of greater R&D is greater patent propensity since $\partial\tilde{\lambda}/\partial I > 0$.

This feedback effect impacts the social value of R&D through the disclosure function of the patent system. Since $m_p > m_s$, required information disclosure coupled with imperfect backward patent protection implies that more, usable technical information enters the public domain when a leader chooses to patent. As a result, a greater proportion of innovations under patent translates to a greater proportion of industries that produce under competitive conditions. This increase in competition effects the social value of R&D through two related channels. (i) The first term in square brackets represents the effect of increased competition on the labor requirements of maintaining a constant level of consumption expenditure. That is, as in standard models, the social planner recognizes that increasing the labor resources devoted to R&D leaves fewer resources for the production of final goods. However, since $\partial M(\tilde{\lambda})/\partial\tilde{\lambda} > 0$, the endogenous increase in selection into patenting effectively tightens the resource constraint, and increases the scale of this resource trade-off at the margin. (ii) The second term in brackets captures the welfare benefit of increased competition through its effect on price index. Since $\partial P(\tilde{\lambda})/\partial\tilde{\lambda} < 0$, the shift into patenting implies a greater utility increase from each innovation. This positive effect of the information disclosure requirements of the patent system forms the basis for the model's representation of the contract theory of patents.

3 Patent Policy

In this section, I analyze the implications of strengthened backward and forward patent protection in terms of the reward and contract theory of patents. When considering the reward theory, I evaluate the implications of the policy change for private R&D incentives and the equilibrium rate of innovation. For the contract theory, I evaluate the policy change's impact on information disclosure and competition within industries through the price index.

3.1 Backward Protection

I begin by considering a change to patent disclosure policy that strengthens backward patent protection (decreases m_p). For example, this policy change could represent an explicit reduction in information disclosure requirements, or increased enforcement against the use of disclosed information by imitating followers. The direct effect of strengthened backward patent protection is an increase in the expected profit flows generated by each patented innovation. The corresponding increase in the ex ante expected value of an innovation stimulates private R&D investment, and captures the traditional motivation for strengthened patent protection under the reward theory of patents.

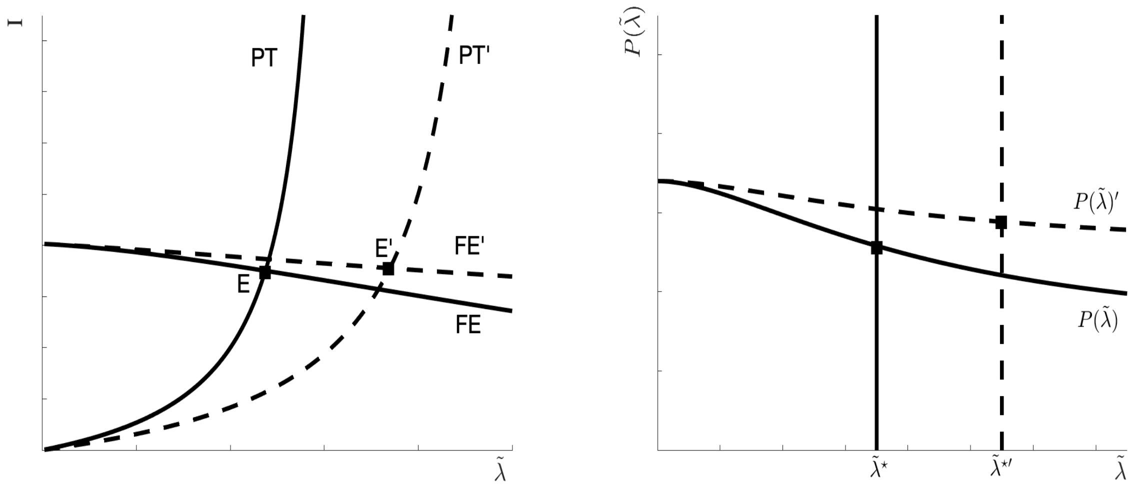
However, the model's treatment of endogenous firm selection into patents or secrecy augments the reward effect in two fundamental ways. First, since only patented innovations enjoy an expected profit increase, the strength of the reward effect depends on the economy's endogenous patent propensity.[13] In the extreme case where all firms choose to rely on secrecy ($\tilde{\lambda} \to 1$), the improved appropriability offered by patents has no impact on the reward for innovating successfully. In

[13]Suzuki (2015) and Klein (2020) emphasize a similar result in a framework in which homogenous innovators choose an appropriation strategy in the form of a patenting, secrecy mix. In these papers, the strength of the reward effect depends on the weight of patents in this mix. In the present paper, the strength of the reward effect depends on the economy wide reliance on patents because this is equivalent to the likelihood of receiving an innovation draw that will be protected by a patent.

contrast, existing analyses of patent policy where patents are the sole appropriation mechanism by default implicitly assume that the full reward effect is present. The dependency of the size of the reward effect from strengthening backward patent protection on endogenous patenting behavior is depicted in Figure 2 as a nonparallel rightward shift in the FE condition, where the size of the shift is greater for larger $\tilde{\lambda}$.

Figure 2: Strengthening backward protection

(a) Equilibrium (b) Price index



Second, strengthening backward patent protection influences equilibrium patent propensity, which impacts the reward from innovation through each firm's licensing obligation. Setting aside the effect of m_p on the PT condition, note that the rightward shift to the FE curve results in movement along the upward sloping PT curve. Intuitively, the increase in the innovation rate implied by the reward effect implies that the forward protection offered by patents is more valuable at any level of backward protection, enticing more innovators to select into patenting. Moreover, by directly reducing the probability that a patented innovation will be imitated, strengthening backward patent protection reduces the effective cost of patenting relative to secrecy. This further increases the relative attractiveness of choosing to patent over secrecy at at any level of $I > 0$, and shifts the PT condition rightward in Figure 2. Since both of these forces imply an increase in the equilibrium patent threshold, we conclude that strengthening backward patent protection unambiguously increases patent propensity. Due to the model's treatment of overlapping innovations across industries, the increase in patent propensity raises each firm's licensing obligation, which reduces the expected reward from innovation through the patent thicket effect. This is represented in Figure 2 as movement along the downward sloping FE curve. The overall change to the equilibrium innovation rate is determined by the relative magnitude of the traditional reward effect and the competing patent thicket effect, and is ambiguous in the general case.

However, even in cases where the reward theory motivation for strengthening backward patent protection fails, the policy may still be independently justified through the welfare benefits of information disclosure as emphasized by the contract theory of patents. This is illustrated in Panel (b) of Figure 2, which graphs the price index of equation (2.36) along with the equilibrium value of $\tilde{\lambda}^{\star}$ as determined in Panel (a). The movement along the $P(\tilde{\lambda})$ curve associated with the equilibrium increase in the patent threshold from stronger backward patent protection captures the positive welfare effect of increased competition due to information disclosure as more firms select into patenting. However, since the size of the increase in imitation associated with patenting is determined by $m_p - m_s > 0$, strengthening backward protection decreases this welfare benefit associated with each patent. This is captured by a rightward shift in in the price index in Panel (b). Therefore, just as the reward theory, the direction of the change to welfare associated with the contract theory of patents is determined by the magnitude of two competing effects: the increase in disclosed information from the endogenous shift into patenting and the decrease in pro-competitive effect of each individual patent's disclosure.

Summarizing these findings, we have the following

Proposition 1. *Strengthening backward patent protection (decreasing m_p) increases the equilibrium patent threshold $\tilde{\lambda}$ and associated patent propensity n_p, but has an ambiguous effect on the innovation rate, I, and the price index, $P(\tilde{\lambda})$.*

Before proceeding, note that these findings reflect the inherent tension between the reward and contract theory of patents. Both the positive welfare impact of increased disclosure and the negative impact on the innovation rate through the patent thicket effect are driven by the endogenous shift into patenting. The strength of the patent thicket effect depends on level of forward patent protection ϕs, which determines how each firm's licensing obligation scales with patent propensity n_p. However, the value of ϕs also determines each patent holder's expected licensing revenue, which is the key motivation for firm selection into patenting. In this way, the positive disclosure effect of the patent system will be large only when it is accompanied by a large patent thicket effect. The pro-innovation reward effect is similarly interrelated and opposed to the positive effect of disclosure on welfare. The larger the reduction in m_p, the larger the size of the reward effect, but the smaller the disclosure benefit of each patented innovation.[14]

3.2 Forward Protection

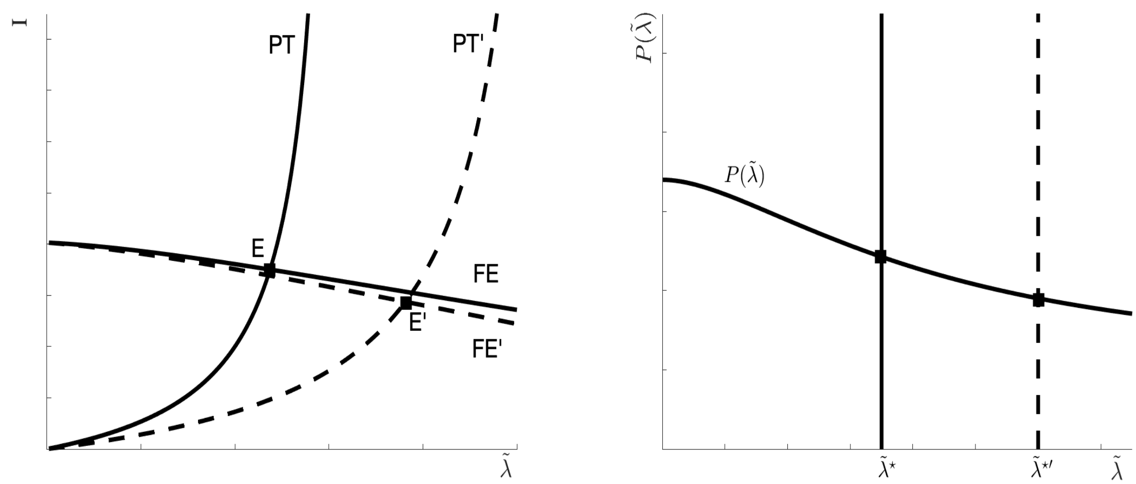
The direct effect of strengthening forward patent protection (increasing ϕs) is an increase in patent holder licensing revenue. This implies a greater licensing obligation for each new innovator at any $0 < n_p < 1$, reducing the expected value of an innovation as established in Lemma 2. This generates a leftward shift in the FE condition in Figure 3. Once again, the size of the shift is larger for greater $\tilde{\lambda}$, since the effective increase in licensing obligation scales with patent propensity. Note

[14] For comparison with existing analyses that assume innovations are solely protected by patents, I examine a special case of the model in the the Appendix where patents strictly dominate secrecy for all innovators. In this case, since policy changes do not impact selection into patenting, the model isolates the welfare trade-off central to the reward theory of patents, but omits welfare consideration underpinning the contract theory.

that this creates movement down the PT condition as the relative attractiveness of patents decreases as a result of the decrease in the innovation rate implied by the shift. However, strengthening forward protection also generates greater total expected licensing revenue from holding a patent at any $I > 0$, and a corresponding increase in the relative attractiveness of choosing to patent over secrecy. Since the expected number of licensing deals generated from a single patent depends positively on the the economy's rate of innovation, the size of this increase to the relative advantage of patenting increases in I. This is represented in Figure 3 by a nonparallel rightward shift in the PT condition.

Figure 3: Strengthening forward protection

(a) Equilibrium (b) Price index



Through the patent thicket effect, the shift of the PT condition further reduces the private incentive to invest in R&D, and the rate on innovation decreases unambiguously. This reflects the innovation stifling effect of forward patent protection emphasized by existing literature such as O'donoghue and Zweimüller (2004) and Chu (2009), and implies that strengthening forward patent protection is never justified through the reward theory of patents. The overall change to the patent threshold is ambiguous in general, as it depends on the relative magnitude of the downward pressure implied by the decrease in innovation and the upward pressure from the increase in forward patent protection. Since the price index does not directly depend on ϕs, the direction of change to welfare through the disclosure effect of patenting is solely determined by the change in $\tilde{\lambda}$. Summarizing, we have the following proposition,

Proposition 2. *Strengthening forward patent protection (increasing ϕs) decreases the equilibrium innovation rate, I, but has an ambiguous effect on the patent threshold, $\tilde{\lambda}$, patent propensity, n_p, and the price index, $P(\tilde{\lambda})$.*

4 Numerical Analysis

In this section, I provide a quantitative assessment of the effects of patent policy on the model's equilibrium. My first objective is to better understand the relative magnitudes of the competing effects described in the previous section. To this end, I calibrate the model to approximate basic long run features of the U.S. economy, and use this baseline to analyze the effects of policy changes that strengthen backward and forward protection. Then, I proceed to examine the robustness of these findings across a range of plausible parameter values. The goal of this exercise is to characterize the types of situations in which the patent system can be justified on either reward or contract theory grounds.

4.1 Baseline Calibration

I begin by pre-setting several parameters that are common in endogenous growth models. I set $\rho = 0.07$ and $n = 0.01$ to reflect a 7% long-run real return of the U.S. stock market and 1% average growth rate of the U.S. labor force. This implies an effective discount rate of $\rho - n = 0.06$. I follow Minniti et al. (2013) and Chu et al. (2017) and set the parameter controlling the Pareto distribution of innovation quality improvements to $\kappa = 0.21$. Since each innovator chooses a limit price equal to the size of its innovation, the value of κ also determines the distribution of innovator mark-up over cost. The choice of $\kappa = 0.21$ implies an average mark-up of about 1.266, which is in the range of empirical estimates commonly considered in endogenous growth models of 1.05 - 1.4. As discussed in Impullitti (2010), estimates suggest an empirically relevant range for the degree of diminishing returns to R&D of 0.4 - 0.9. I set a conservative value of $\beta = 0.4$, which is closest to the case of linear R&D technology within this range.

I choose the model's remaining parameters of $\{\alpha, \phi s, m_s, m_p\}$ in order to obtain realistic values of patent propensity, the scope of licensing agreements, and economic growth. As mentioned in the introduction, empirical estimates of patent propensity range from 30 to 55% (Cohen et al., 2002; Hall et al., 2014). In the baseline calibration, I target an intermediate value of $n_p = 0.45$. Following Minniti et al. (2013), I target a growth rate of $g = \kappa I = 2\%$ to reflect the long-run growth trend of the U.S. Finally, I rely on estimates provided in Chu (2009) and Yang (2018) to determine a target for the total expected share of new innovator profits that are transferred to previous innovators through licensing agreements. In existing models in which all innovators patent, this "backloading effect" of patents can be set directly. In the present model however, the corresponding total licensing obligation of a $\phi s n_p$ share of profits depends on the endogenous economy wide patent propensity. As in Yang (2018), I target a total backloading effect of patents equal to a 15% share of profits.[15] Given our target patent propensity of $n_p = 0.45$, this implies a value of $\phi s = 1/3$.

Since firm patenting decisions depend on the relative backward protection provided by secrecy

[15] As discussed in Chu (2009) and Yang (2018), empirical estimates suggest a range of $[0.15, 0.52]$ for the backloading effect of patents. Following Yang (2018), I choose a conservative value of 0.15 in the baseline calibration.

and patenting, m_s and m_p are not separately identified without an additional restriction. To account for this, I set $m_s = 0.49$ to reflect survey evidence reported in Cohen et al. (2002) that 51% of firms consider secrecy to be an effective appropriation mechanism for product innovations. This leaves two free parameters, backward patent protection m_p and the innovation difficulty parameter α, which I jointly calibrate to match the 2% growth rate and 45% patent propensity targets. This calibration yields values of $\alpha = 1.640$ and $m_p = 0.7909$. See Appendix C for a summary of baseline parameter values.

4.2 Baseline Results

I begin the numerical analysis by examining the impact of strengthening forward patent protection in the baseline economy. Column 2 of Table 1 displays results following a 25% increase in ϕs. In this case, the improved attractiveness of patents relative to secrecy generates an increase in the economy's patent propensity of almost 7%. As a result, the licensing burden of new innovators increases by an equivalent of 6.6% of their flow monopoly profits. In accordance with Proposition 2, this patent thicket effect reduces the private incentive to invest in R&D, and economic growth decreases as a consequence of stronger forward protection. On the other hand, the fall in R&D does free up labor resources for the production of final goods, and welfare from equilibrium consumption expenditure does increase modestly. In addition to this allocative trade-off effect that is standard in endogenous growth models, the shift into patenting generates a welfare benefit as more information is disclosed through patents. The associated increase in competition within industries drives the price index down by 2.8%. However, these positive effects are ultimately dominated by the cost of reduced economic growth, and total welfare falls as a result of the policy change.

Table 1: Baseline Results

	Baseline	$\phi s \uparrow$	$m_p \downarrow$
$g(\%)$	2.000	1.828	1.918
n_p	0.450	0.519	0.551
$\phi s n_p$	0.150	0.216	0.184
$L_{R\&D}(\%)$	4.532	3.900	4.226
$(\rho - n)U$	27.22	25.00	26.18
$\frac{g}{(\rho-n)}$	33.34	30.47	31.97
$ln(c)$	3.826	4.194	4.010
$-P(\tilde{\lambda})$	-9.944	-9.657	-9.803

Table 1 displays results from strengthening forward patent protection by increasing ϕs 25% and strengthening backward patent protection to generate a 25% decrease in $(m_p - m_s)$. The final six rows report the total backloading effect of patents ($\phi s n_p$), the share of labor employed in R&D, and total welfare as specified in (2.38) followed by total welfare's three component parts. All welfare measures have been multiplied by 100 for display purposes.

Next, I examine the effects of strengthening backward patent protection. In particular, I consider a decrease in m_p that generates a 25% decrease in the effective cost of patenting, $m_p - m_s$.[16] Recall from Proposition 1 that strengthening backward protection creates opposing effects on the private incentives to invest in R&D. On the one hand, the improved appropriability provided by patents increases the ex ante expected value of an innovation and encourages R&D, as emphasized by the reward theory. On the other hand, the corresponding shift into patenting increases each firm's licensing burden, which discourages R&D. In this case, the results in Table 1 show that the policy change induces a large increase in patent propensity of about 10%. Although there is no change to the size of each individual licensing deal, each firm's licensing burden still increases 3.4%. Overall, the reward effect only partially offsets this patent thicket effect, and economic growth falls as a result of the policy change.

Moreover, the policy change generates competing effects on the welfare benefit of the disclosure of information through patents. Specifically, the realized benefit of disclosure is the lower price of goods implied by the greater rate of imitation for innovations under patent relative to those that remain secret. Since this relative increase in imitation is determined by $m_p - m_s$, the policy change directly decreases the welfare benefit of each patented innovation. On the other hand, the associated shift into patenting implies that fewer innovations are kept secret. In this case, the increase in patent propensity is sufficiently large so that the price index falls. However, this positive welfare effect of the policy change is not enough to justify the policy change on contract theory grounds. Once again, the welfare cost of reduced economic growth dominates, and social welfare decreases when backward protection is strengthened from its baseline level.

4.3 Backward Patent Protection and the Reward Theory

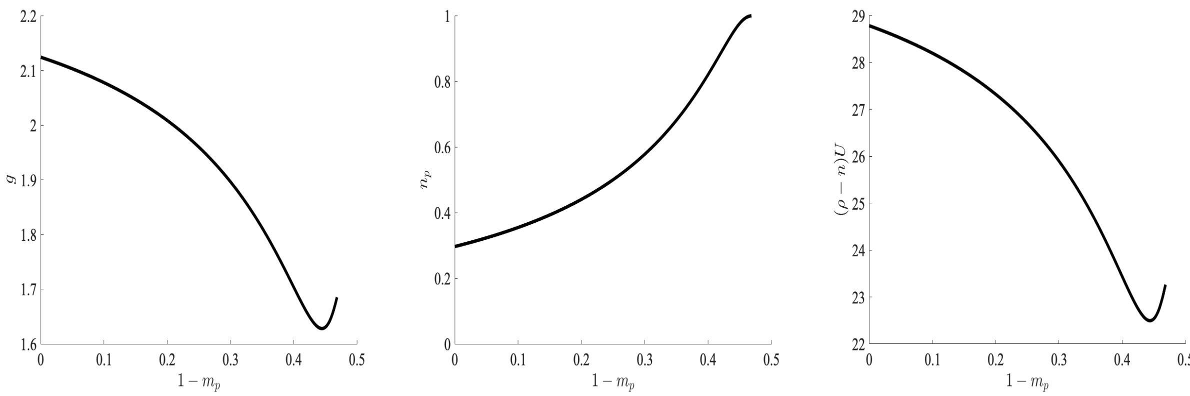
I now turn to a closer examination of the relationship between backward patent protection and economic growth. I begin by considering results across a wider range for backward patent protection, holding all other parameters to their baseline value. The results of this experiment are displayed in Figure 4, which plots economic growth, patent propensity and welfare against backward patent protection $(1-m_p)$. In each case, I vary $1-m_p$ from zero, representing nonexistent backward protection, to its upper bound implied by Assumption 2 so that patent propensity remains below one. With the baseline values of m_s, ϕs, and κ, this upper bound is about 0.45.

First, note that even when patents do not provide any backward protection $(1 - m_p = 0)$, a sizable proportion of about 30% of innovators still select into patenting. This is because some innovators, necessarily those with sufficiently low monopoly profits associated with small innovations, are willing to sacrifice their entire profit stream in exchange for the expected licensing revenue offered by forward patent protection. Second, note that economic growth is non-monotonic in backward patent protection. This reflects the changing relative magnitude of the competing reward and patent thicket effects as patent propensity increases along with backward patent protection. That is, the greater the proportion of innovations that are protected by patents, the greater the impact

[16] Given the baseline values of $m_p = 0.7909$ and $m_s = 0.4900$, this policy change corresponds to a 9.51% decrease in m_p to 0.7157.

Figure 4: Baseline: Backward Patent Protection and Growth

(a) Growth (b) Patent Propensity (c) Welfare



of strengthening backward protection has on the ex ante expected value of an innovation. However, in the baseline case, we see that patent propensity must be close to one (about 0.96) before the reward effect begins to dominate the patent thicket effect from further strengthening protection. Clearly, the economy achieves its highest growth rate, and social welfare, when backward protection is set to zero.

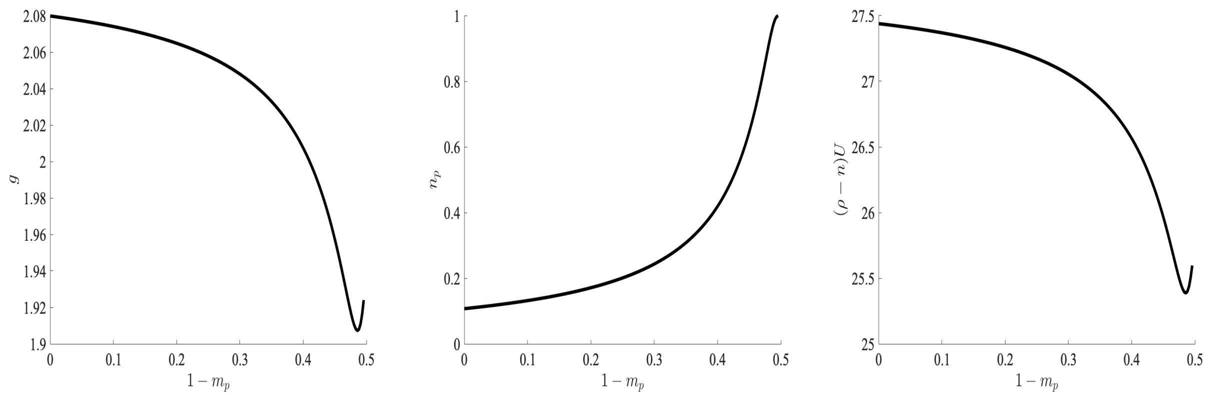
Next, I turn to examining the relationship between backward patent protection and economic growth under alternate parameters. First, I consider weaker forward protection, which implies a smaller patent thicket effect for any corresponding increase in patent propensity. I set $\phi s = 0.111$ so that that each innovator's expected licensing obligation, $\phi s n_p$, is 5% of monopoly profits when patent propensity is 45%, instead of the 15% of profits assumed in the baseline equilibrium. Second, I consider a distribution of innovation quality increments with a smaller right tail by setting $\kappa = 0.0475$. This implies that the average mark-up in the economy is 1.05, instead of the 1.266 assumed in the baseline. In both cases, I recalibrate the value of α so that the growth rate remains 2% at the starting point of 45% patent propensity. See Appendix C for associated parameter values.

Interestingly, Figure 5 illustrates that weaker forward protection does little to alter the relationship between backward patent protection and economic growth. Although a lower ϕs implies a smaller patent thicket effect at each level of patent propensity, it also implies that the option to patent is less attractive at each level of backward protection. Since fewer firms patent, strengthening backward protection generates a smaller increase in the ex ante expected value of an innovation. In other words, since firms select into patents specifically to gain expected licensing revenue, the size of the reward effect is directly linked to the size of the patent thicket effect. As a result, the reduction in forward protection decreases the magnitude of both effects. This implies a smaller absolute impact of backward protection on economic growth, but growth continues to decrease in backward protection until patent propensity approaches one.

In contrast, Figure 6 shows that the distribution of innovation size has a substantial impact of the relative importance of the patent thicket and reward effects. A smaller right tail of the distribution reduces the weight of very large innovation outliers on the ex ante expected value of an

Figure 5: Low ϕs: Backward Patent Protection and Growth

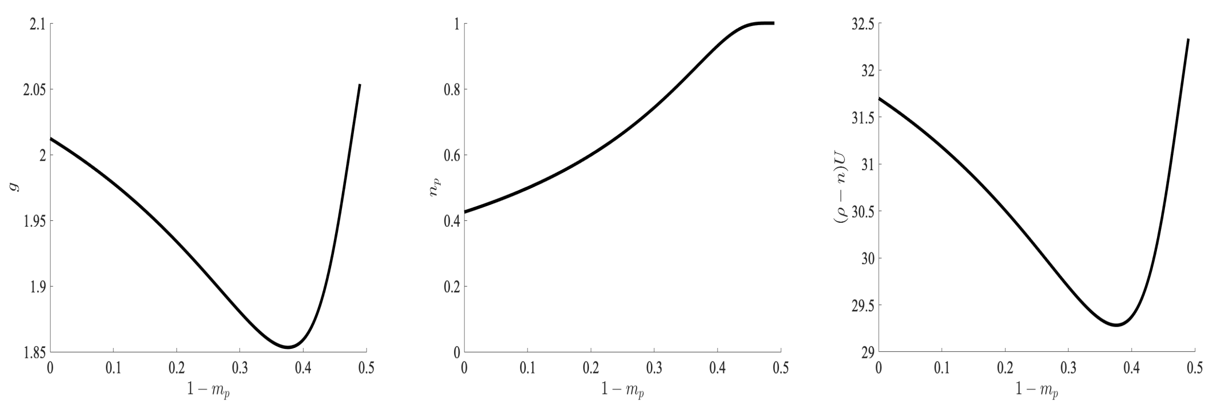
(a) Growth (b) Patent Propensity (c) Welfare



innovation. Since these outliers are always protected by secrecy, reducing their prevalence increases the importance of patent protection on the expected value of an innovation, and leads to a larger reward effect. In this case, the reward effect begins to dominate the patent thicket effect when patent propensity reaches about 88.5%. Moreover, we see that backward patent protection can have an overall positive impact on growth and welfare. Compared to the case of zero protection, strengthening backward protection to the point where patent propensity approaches one increases economic growth, and social welfare.

Figure 6: Low κ: Backward Patent Protection and Growth

(a) Growth (b) Patent Propensity (c) Welfare



4.4 Forward Patent Protection and the Contract Theory

In the cases examined so far, the welfare effect of patent policy has largely been determined by the policy's impact on economic growth. Any change to welfare through the information disclosure function of the patent system as emphasized by the contract theory has been secondary to reward theory considerations. In this section, I explore whether plausible cases exist in which strengthening patent protection can be justified through the contract theory. In particular, I repeat the policy experiment of strengthening forward protection through a 25% increase in ϕs across different levels of decreasing returns to R&D with β ranging from $0.4-0.9$. As before, I recalibrate α in each case so that the growth rate remains 2% in the pre-policy change equilibrium with patent propensity at 45%. The calibrated values of α can be found in Appendix C.

In Table 2, I report all results in terms of the change from the corresponding initial equilibrium to the new equilibrium after forward protection has been strengthened. In accordance with Proposition 2, strengthening forward protection is always growth reducing in the model. However, the larger the value of β, corresponding to a greater degree of diminishing returns to R&D, the smaller the associated decrease in growth. Indeed, this result holds despite a greater increase in patent propensity and the backloading effect of patents as β increases. This is because the reduction in the labor employed in R&D implies a larger increase in R&D productivity when the duplication of R&D efforts within industries is more severe.

In terms of welfare, this implies that the negative impact of growth reduction shrinks, while the positive disclosure effect grows as β increases. At the highest degree of diminishing returns to R&D with $\beta = 0.9$, strengthening forward patent protection becomes welfare improving. Importantly, note that the increase in consumption expenditure alone is not sufficient to overcome the welfare cost of reduced growth. Thus, the positive disclosure effect of the patent system does play a crucial role in the welfare effect of the policy change. On the other hand, although this result demonstrates that the contract theory *can* justify stronger forward protection, it seems that it is best interpreted as an illustrative special case. Across the clear majority of plausible cases, the overall welfare effects of patent policy are drive by the policy's impact on economic growth.

Table 2: Strengthening Forward Patent Protection

	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$
$\Delta g(\%)$	-0.172	-0.139	-0.107	-0.078	-0.051	-0.025
$\Delta n_p(\%)$	6.854	7.135	7.391	7.626	7.841	8.040
$\Delta \phi s n_p(\%)$	6.607	6.724	6.831	6.928	7.079	7.101
$\Delta L_{R\&D}(\%)$	-0.632	-0.607	-0.584	-0.563	-0.544	-0.526
$\Delta(\rho - n)U$	-2.218	-1.684	-1.185	-0.717	-0.278	0.136
$\Delta \frac{g}{(\rho-n)}$	-2.874	-2.313	-1.791	-1.301	-0.841	-0.408
$\Delta ln(c)$	0.369	0.330	0.294	0.261	0.230	0.202
$-\Delta P(\tilde{\lambda})$	0.287	0.300	0.312	0.323	0.333	0.343

Table 2 displays results from strengthening forward patent protection by increasing ϕs 25% across different values of β. Results in the first column, with $\beta = 0.4$, correspond to the baseline case as reported in Table 1. All results are reported in terms of the change from the initial equilibrium to the new equilibrium after forward protection has been strengthened.

5 Conclusion

The objectives of the patent system are characterized by two distinct theories of its social value. According to the reward theory, patents stimulate innovation by enhancing the ex ante incentive to invest in R&D. According to the contract theory, patents encourage successful innovators to disclose technical information that would otherwise remain secret. Despite the clear interdependence of these twin purposes of the patent system, the extant literature has thus far either analyzed them separately, or restricted attention to partial equilibrium settings.

In this paper, I contribute to the literature by developing a novel general equilibrium model of endogenous growth that incorporates both potential justifications of the patent system. The model features profit maximizing innovators that select into either patents or secrecy based on their innovation's size, the effective cost of information disclosure, and the expected licensing revenue from holding a patent. Firms with relatively large innovations select into secrecy to avoid disclosure requirements and maintain their large competitive advantage over competitors. Firms with relatively small innovations choose to patent because the effective cost of disclosure is low relative to expected licensing revenue extracted from subsequent innovators. The combination of forward patent protection in the form of mandatory licensing agreements and voluntary selection into patenting gives rise to the endogenous determination of each new innovator's licensing burden, capturing the presence of patent thickets.

In this context, the model illustrates that endogenous innovator selection into patents and secrecy is central to understanding the tension between the patent system's objectives of stimulating innovation and disseminating technical information. On the one hand, strengthening backward patent protection implies a classic reward theory trade-off; improved appropriability enhances R&D incentives at the expense of limiting competition. On the other hand, the increased relative attractiveness of patenting generates an increase in patent propensity. This shift into patenting expands the proportion of innovations for which technical information is disclosed, while increasing the licensing burden of innovator's through the patent thicket effect. I show that these effects stemming from changes to innovator selection into patenting can be sufficiently strong such that the qualitative predictions of patent policy analyzed through a traditional reward theory lens can be reversed.

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Appendix A

A.1 Calculation of $\mathbb{E}_\lambda[\pi(\lambda,t)]$

Using (2.2) and (2.15),

$$\mathbb{E}_\lambda[\pi(\lambda,t)] = \int_1^\infty c(t)N(t)(1-\lambda^{-1})f(\lambda)d\lambda = c(t)N(t)\frac{1}{\kappa}\int_1^\infty (1-\lambda^{-1})\lambda^{-(1+\kappa)/\kappa}d\lambda \tag{A.1}$$

$$= \frac{c(t)N(t)}{\kappa}\int_1^\infty \Big(\lambda^{-(1+\kappa)/\kappa} - \lambda^{-(1+2\kappa)/\kappa}\Big)d\lambda = \frac{c(t)N(t)}{\kappa}\Big[-\kappa\lambda^{-1/\kappa} + \frac{\kappa}{1+\kappa}\lambda^{-(1+\kappa)/\kappa}\Big]_1^\infty \tag{A.2}$$

$$= c(t)N(t)\Big[\frac{1}{1+\kappa}\lambda^{-(1+\kappa)/\kappa} - \lambda^{-1/\kappa}\Big]_1^\infty = c(t)N(t)\big[1-\frac{1}{1+\kappa}\big] = c(t)N(t)\frac{\kappa}{1+\kappa} = cN\Omega. \tag{A.3}$$

A.2 Calculation of $\mathbb{E}_\lambda[V(\lambda,t)]$

From equation (2.31),

$$\mathbb{E}_\lambda[V(\lambda,t)] = \int_1^{\tilde{\lambda}} V_p(\lambda,t)f(\lambda)d\lambda + \int_{\tilde{\lambda}}^\infty V_s(\lambda,t)f(\lambda)d\lambda. \tag{A.4}$$

Using (2.19),

$$\int_{\tilde{\lambda}}^\infty V_s(\lambda,t)f(\lambda)d\lambda = \frac{cN(t)(1-\phi s n_p - m_s)}{\rho - n + I}\int_{\tilde{\lambda}}^\infty (1-\lambda^{-1})f(\lambda)d\lambda. \tag{A.5}$$

Using (A.1)-(A.3),

$$\int_{\tilde{\lambda}}^\infty V_s(\lambda,t)f(\lambda)d\lambda = \frac{cN(t)(1-\phi s n_p - m_s)}{\rho - n + I}\Big[\frac{-(\frac{1}{1+\kappa}-\tilde{\lambda})}{\tilde{\lambda}^{\frac{1}{\Omega}}}\Big]. \tag{A.6}$$

Similarly using (2.20),

$$\int_1^{\tilde{\lambda}} V_p(\lambda,t)f(\lambda)d\lambda = \frac{cN(t)}{\rho-n+I}\Big[(1-\phi s n_p - m_p)\int_1^{\tilde{\lambda}}(1-\lambda^{-1})f(\lambda)d\lambda + \frac{\phi s I\Omega}{\rho-n+I}\int_1^{\tilde{\lambda}} f(\lambda)d\lambda\Big] \tag{A.7}$$

$$= \frac{cN(t)}{\rho-n+I}\Big[(1-\phi s n_p - m_p)\Big(\frac{(\frac{1}{1+\kappa}-\tilde{\lambda})}{\tilde{\lambda}^{\frac{1}{\Omega}}} + \Omega\Big) + \frac{\phi s n_p I\Omega}{\rho-n+I}\Big]. \tag{A.8}$$

Combining (A.6) and (A.8), yields (2.32) in the main text.

A.3 Production labor

From equation (2.29) in the main text

$$L_y(t) = c(t)N(t)\Big[\int_1^{\tilde{\lambda}} \Big[(1-m_p)\frac{1}{\lambda} + m_p\Big] f(\lambda)d\lambda + \int_{\tilde{\lambda}}^{\infty} \Big[(1-m_s)\frac{1}{\lambda} + m_s\Big] f(\lambda)d\lambda \tag{A.9}$$

$$= c(t)N(t)\Big[n_p m_p + (1-n_p)m_s + (1-m_p)\int_1^{\tilde{\lambda}} \frac{1}{\lambda} f(\lambda)d\lambda + (1-m_s)\int_{\tilde{\lambda}}^{\infty} \frac{1}{\lambda} f(\lambda)d\lambda\Big]. \tag{A.10}$$

Since

$$\int_a^b \frac{1}{\lambda} f(\lambda)d\lambda = \Big[-\frac{1}{1+\kappa}\lambda^{-\frac{1+\kappa}{\kappa}}\Big]_a^b, \tag{A.11}$$

Solving the integration gives

$$n_y(t) = c(t)N(t)\Big[n_p m_p + (1-n_p)m_s + \frac{1}{1+\kappa}\big[1 - m_p + (m_p - m_s)\tilde{\lambda}^{-\frac{1}{\Omega}}\big]\Big]. \tag{A.12}$$

A.4 Calculation for welfare and growth

The calculation of the price index $P(\tilde{\lambda})$ and the growth rate g require solving an integral of the form

$$\int_a^b ln(\lambda)f(\lambda)d\lambda \tag{A.13}$$

Using integration by parts ($U = ln(\lambda)$, and $dV = f(\lambda)$) we have,

$$\int_a^b ln(\lambda)f(\lambda)d\lambda = \Big[-ln(\lambda)\lambda^{-1/\kappa} - \kappa\lambda^{-1/\kappa}\Big]_a^b. \tag{A.14}$$

Plugging this result into equations (2.35) and (2.37) in the main text give the stated results.

A.5 Existence and uniqueness of equilibrium

Under Assumptions 1 and 2, the patent threshold (PT) condition given by (2.24) and the free-entry (FE) condition given by (2.33) provide two equations in two unknown equilibrium values of I and $\tilde{\lambda}$. I demonstrate the existence and uniqueness of the model's steady state equilibrium by establishing the existence of a single crossing of the PT and FE conditions in ($\tilde{\lambda}$, I) space, with $1 < \tilde{\lambda} < \infty$ and $0 < I < \infty$.

As argued in Section 2.4.1, the PT condition is strictly upward sloping in ($\tilde{\lambda}$, I) space, with $I \to 0$ as $\tilde{\lambda} \to 1$ and $I \to \infty$ as $\tilde{\lambda} \to \tilde{\lambda}_{max}$, where $1 < \tilde{\lambda}_{max} < \infty$ as given by (2.26). From Section 2.4.2, the FE condition is strictly downward sloping in in ($\tilde{\lambda}$, I) space. Therefore, single crossing obtains if the innovation rate implied by the FE condition at $\tilde{\lambda} = 1$ is positive (the vertical intercept of the FE condition in Figure 1). Plugging $\tilde{\lambda} = 1$ into equation (2.33) implicitly defines this I,

$$\hat{\alpha} I^{\frac{\beta}{1-\beta}} (\rho - n + I)(1 + \kappa m_s) = (1 - \hat{\alpha} I^{\frac{1}{1-\beta}})(\kappa(1 - m_s)), \tag{A.15}$$

which has a unique solution with $0 < I < \infty$.

Appendix B

B.1 Patent dominant equilibrium

In the paper's main analysis, Assumption 2 ensures that the effective cost of the disclosure requirements of patenting are sufficiently high such that some firms always prefer secrecy in equilibrium ($0 < n_p < 1$). In this section, I briefly examine the model's equilibrium when patents strictly dominate secrecy for each possible innovation size $\lambda > 1$ and $n_p = 1$. Specifically, in place of Assumption 2, let us instead assume that $m_p < m_s$ and $\phi s > 0$.

In this case, patenting offers superior protection from imitation than secrecy in addition to forward protection, so it is clear that patenting is optimal for each innovator. Note that these parameters imply that the inequality in (2.21) is always strict, with $V_s(\lambda, t) < V_p(\lambda, t)$ for all $\lambda < 1$, so the PT equilibrium condition of equation (2.24) no longer applies, $\tilde{\lambda} = \infty$, and $n_p = 1$. To illustrate the similarity between traditional treatments and the model's patent dominant equilibrium, we can characterize the equilibrium through two conditions in two unknowns c and I. That is, given $\tilde{\lambda} = \infty$ and $n_p = 1$, we can write the labor market clearing condition (2.30) and the free-entry condition (2.18) as

$$1 = c\Big[\frac{1 + m_p\kappa}{1+\kappa}\Big] + \hat{\alpha} I^{\frac{1}{1-\beta}} \tag{B.1}$$

$$\hat{\alpha} I^{\frac{\beta}{1-\beta}} = \frac{c}{\rho - n + I}\Big[\frac{\kappa}{1+\kappa}\Big(1 - m_p - \frac{(\rho - n)\phi s}{\rho - n + I}\Big)\Big] \tag{B.2}$$

Observe that (B.1) specifies the traditional downward sloping resource constraint in (c, I) space and (B.2) specifies the traditional upward sloping free-entry condition. Finally, note from (2.36) that the price index is now completely given by parameter values with $P(\tilde{\lambda} = \infty) = (1 - m_p)\kappa$.

Given Assumption 1, it is straightforward to verify that a unique equilibrium exists. Figure B1 illustrates the equilibrium determination of I by graphing (B.1) and (B.2) in (c, I) space.[17] Repeating the shift analysis of Section 3 immediately gives the following

Proposition B1. *When patents strictly dominate secrecy* ($n_p = 1$),

- *Strengthening backward patent protection increases the equilibrium innovation rate and increases the price index.*

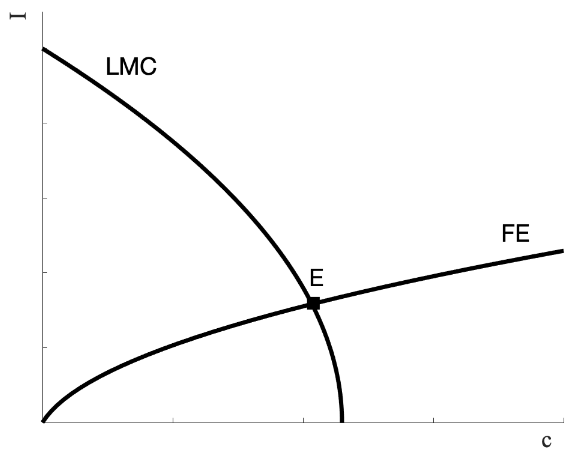
- *Strengthening forward patent protection decreases the equilibrium innovation rate and has no effect on the price index.*

Thus, strengthening backward patent protection unambiguously increases the economy's innovation rate when all innovations are protected by patent. This is because the full reward effect from the policy change is present, while there is no increase to the patent thicket effect since there is no change to innovators' selection into patents versus secrecy. On the other hand, the price index

[17] Since the FE condition of (2.33) incorporates both the free-entry condition and the labor market clearing constraint, one can also visualize the equilibrium determination of I in the patent dominant equilibrium as the asymptote of the FE condition as $\tilde{\lambda} \to \infty$ in Figure 1.

increases unambiguously since fewer innovations are imitated and there is no offsetting increase to the amount of information disclosure. In this way, the patent dominant equilibrium isolates the welfare trade-off central to traditional treatments of the reward theory of patents, but omits welfare considerations underpinning the contact theory.

Figure B1: Patent dominant equilibrium



Appendix C

C.1 Calibration Parameters

Table C3: Baseline Calibration Summary

Pre-set	Description	Value
ρ	Discount factor/ interest rate	0.07
n	Population growth	0.01
κ	Innovation dist. param.	0.21
β	Dim. returns to R&D	0.4
m_s	Secrecy imitation prob.	0.49
Calibrated	Description	Value
ϕs	Forward protection	0.333
α	Innovation difficulty	1.640
m_p	Patent imitation prob.	0.7909

Table C4: Recalibrated α for robustness checks

	$\phi s = 0.111$	$\kappa = 0.475$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$
$\alpha =$	1.7935	0.2074	2.235	3.045	4.150	5.655	7.705