An Economic Theory of Education Externalities: Effects of Education Capital

Harashima, Taiji

Kanazawa Seiryo University

7 May 2021

An Economic Theory of Education Externalities: Effects of Education Capital

HARASHIMA Taiji*

May 2021

Abstract

The source of education externalities has been mostly explained based on the concept of human capital, but this concept is very elastic, so the mechanism behind education externalities has not necessarily been sufficiently explained theoretically. In this paper, I construct a model of education externalities based on the concept of education capital, and show that it is education capital, not human capital, that generates education externalities. Unlike human capital, the effects of education capital have upper bounds because there is a division of labor (i.e., there are specialists). The uncovered mechanism of education externalities has the potential to provide many valuable insights for educational institutions and policy. For example, elementary schools should basically be compulsory, but whether education (not research) in universities should be subsidized by governments may depend on the degree of generosity of high-income people.

JEL Classification code: H52, I21, I22, I26, I28

Keywords: Education; Education Externality; Human capital

*Correspondence: HARASHIMA Taiji, Kanazawa Seiryo University, 10-1 Goshomachi-Ushi, Kanazawa, Ishikawa, 920-8620, Japan.
Email: harashim@seiryo-u.ac.jp or t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

In many countries, elementary schools are compulsory and subsidized by governments. The most important reason for compulsory elementary school education is that it is highly likely that education has positive externalities; that is, it provides positive effects not only to the people who receive the education but also to many third parties. Because of this nature, education has been regarded to be a driving force of endogenous economic growth.

The reason why education has positive externalities has usually been explained based on the concept of human capital, particularly in studies of endogenous growth (Romer, 1986, 1990a, 1990b; Lucas, 1988; Acemoglu, 1998; Aghion and Howitt, 1998). The importance of human capital has been stressed since Mincer (1958) and Becker (1962, 1964). If human capital is a factor input for production, and it is obtained by education and accumulated differently from other types of capital (e.g., physical capital), then the economy can grow endogenously along with the accumulation of human capital because of education's positive externalities. In the theory of endogenous growth, the origins of these externalities have been thought to be the exchange of ideas and information among educated persons and other related factors.

However, the concept of human capital is very elastic, and the mechanism explaining why human capital shows positive externalities is still not sufficiently clear. For example, it has not necessarily been clearly explained how human capital (or almost equivalently, the qualities of laborers) can increase constantly and indefinitely through education so as to be a driving force of endogenous growth. The results of many empirical studies are also mixed and inconclusive, particularly because it is difficult to appropriately measure human capital and the achievements of education (Rauch, 1993; Acemoglu and Angrist, 2000; Krueger and Lindahl, 2001; Pritchett, 2001; Moretti, 2004; Cohen and Soto, 2007; Breton, 2010).

In addition, it seems highly likely that education improves elements that are not directly related to economic outcomes and induces positive externalities indirectly through these elements (e.g., improved health, crime prevention, political participation, and civic engagement), but the mechanism behind these effects has also not been sufficiently clarified theoretically (Dee, 2004; Grossman, 2006; Groot and van den Brink, 2007; Lochner, 2011; Oreopoulos and Salvanes, 2011; Münich and Psacharopoulos, 2018).

In this paper, I examine the mechanism of education externalities from a different point of view by introducing the concept of "education capital" instead of human capital. Human and education capital are common in that both can be acquired through education, but they are different in the following aspects. First, human capital is regarded as a different kind of factor input from other kinds of capital (e.g., physical capital) in production processes. Hence, it can be accumulated and works in a different manner from the other kinds of capital. On the other hand, the education capital introduced in this paper is a part of the larger group of capital inputs used in production processes, and it plays the same role and works in the same manner as physical capital.

Second, although human capital is thought to be able to increase infinitely and thereby the effects of its externalities can also increase indefinitely, the effects of externalities generated by education capital have an upper bound (i.e., they do not increase infinitely). Therefore, although human capital can be a driving force of indefinite endogenous growth, education capital cannot. Education capital initially can enhance economic growth, particularly in a period when relatively few people receive an education, but the power to enhance economic growth diminishes as more people receive an education. Nevertheless, the effects of education externalities that have already been generated remain indefinitely even though they do not increase.

In this paper, I first show that the assumption behind the conventional production functions is inappropriate in the examination of education externalities. I then examine education externalities on the basis of an alternate production function presented in Harashima (2009[1], 2017[2]). I show that education enhances production not by improving labor productivity but by accumulating capital (i.e., education capital). I next show how education externalities are generated through the accumulation of education capital by constructing a model of education externalities. Because labor is fully divided (i.e., as specialized as possible), education capital cannot be accumulated infinitely, and as a result, education externalities have an upper bound. Finally, I examine the conditions for subsiding education (elementary schools and universities) and for compulsory education.

2 PRODUCTION FUNCTION

2.1 Problem with the conventional production function

2.1.1 Wages for high- and low-skilled workers

In many economic studies in which workers are assumed to be heterogeneous in their abilities, it is assumed explicitly or implicitly that the only difference between high- and low-ability workers is their skills, and skills can be acquired by any worker equally at the same costs. In particular, this assumption is commonly used in studies of the economic impacts of immigrants (Altonji and Card, 1991; Borjas, 1994, 1999, 2003; Friedberg and Hunt, 1995; Card, 2005, 2009; Bodvarsson and Van den Berg, 2009; Ottaviano and Peri, 2012).

---

1 Harashima (2009) is also available in Japanese as Harashima (2016).

2 Harashima (2017) is also available in Japanese as Harashima (2020a).

Suppose that there are two types of workers: high-skilled (HS) and low-skilled (LS) workers. Let L_{HS} and L_{LS} be the labor inputs of HS and LS workers, respectively. In this case, a CES (constant elasticity of substitution) production function can be described as

$$Y = A\left[(1-\lambda-\mu)K^q + \lambda L_{HS}^q + \mu L_{LS}^q\right]^{\frac{1}{q}} , \tag{1}$$

where Y is output, A is technology, K is capital inputs, and λ, μ, and q are parameters ($0 < \mu < \lambda < 1$, $\lambda + \mu < 1$ and $q \leq 1$). The skill difference is represented by the difference between the values of λ and μ. The value of λ is larger than that of μ because HS workers are assumed to have more skills than LS workers. If $q \to 0$, the production function degenerates to a Cobb–Douglas production function. Suppose that capital moves perfectly elastically. Let w_{HS} and w_{LS} be the wages for HS and LS workers, respectively. By equation (1),

$$w_{HS} = \frac{\partial Y}{\partial L_{HS}} = \frac{r\lambda}{1-\lambda-\mu}\left(\frac{K}{L_{HS}}\right)^{1-q} , \tag{2}$$

and

$$w_{LS} = \frac{\partial Y}{\partial L_{LS}} = \frac{r\mu}{1-\lambda-\mu}\left(\frac{K}{L_{LS}}\right)^{1-q} , \tag{3}$$

where

$$r = \frac{\partial Y}{\partial K} .$$

By equations (2) and (3), $w_{HS} > 0$, $w_{LS} > 0$, and

$$\frac{w_{HS}}{w_{LS}} = \frac{\lambda}{\mu}\left(\frac{L_{LS}}{L_{HS}}\right)^{1-q} .$$

If

$$\left(\frac{\lambda}{\mu}\right)^{\frac{1}{1-q}} < \frac{L_{HS}}{L_{LS}} ,$$

$w_{HS} < w_{LS}$; that is, the wage for HS workers is lower than the wage for LS workers. Because $0 < \mu < \lambda < 1$ and $q \leq 1$,

$$1 < \left(\frac{\lambda}{\mu}\right)^{\frac{1}{1-q}} ,$$

and thereby, if L_{HS} is sufficiently larger than L_{LS}, $w_{HS} < w_{LS}$. If λ and μ are nearly equal, then $w_{HS} < w_{LS}$, even if L_{HS} is only a little larger than L_{LS}. Hence, in these cases, HS workers will want to work as LS workers because $w_{HS} < w_{LS}$ (i.e., skills that HS workers obtain do not result in higher wages). By arbitrage, there will be an equilibrium ratio $\frac{L_{HS}}{L_{LS}}$ such that

$$\left(\frac{\lambda}{\mu}\right)^{\frac{1}{1-q}} = \frac{L_{HS}}{L_{LS}} ,$$

and at this equilibrium, $w_{HS} = w_{LS}$. If L_{HS} is sufficiently larger than L_{LS}, therefore, the wages of HS and LS workers are always equal. This is particularly true if λ and μ are not very different.

Nevertheless, acquiring skills entails costs. Suppose that a worker pays back a loan that was borrowed to cover the costs to acquire skills by c_S every year. The equilibrium ratio $\frac{L_{HS}}{L_{LS}}$ will be achieved when $w_{HS} - c_S = w_{LS}$. In this way, the net incomes (i.e., income minus the costs to acquire skills) of HS and LS workers are equal, and skill level is indifferent to net income.

2.1.2 Optimal choice to not receive education

In the model in Section 2.1.1, therefore, workers who do not want to receive education always exist as a result of their rational choices because there is an equilibrium between high and low skills acquired by education. As more workers receive education (i.e., more workers become HS workers), the value of HS workers decreases and eventually becomes lower than that of LS workers (i.e., $w_{HS} < w_{LS}$). Hence, if the number of workers who receive education exceeds a certain critical point (i.e., $w_{HS} = w_{LS}$), it is rational for the remaining workers not to receive education.

However, the possibility that $w_{HS} \leq w_{LS}$ is significantly at odds with reality. This implies that arguments in which (1) heterogeneous workers play an essential role, (2) conventional production functions are used, and (3) the only difference between high-and low-skill workers is assumed to be their skill levels will reach misleading conclusions. An alternative production function and different assumptions are needed.

2.1.3 Problem with the conventional production function

The problem with the conventional production function shown in Section 2.1.1 is related to two fundamental questions. First, there is a question about the CES between HS and LS workers. Cobb–Douglas and CES production functions both have the nature of CES in production, and CES indicates that any factor input is indispensable for production. Therefore, as a factor input becomes scarcer, its value and price increase. This relationship is very reasonable between labor and capital inputs, but it may not be true between HS and LS workers because HS workers can be employed as LS workers if LS workers become scarcer. Note that labor cannot be employed as capital, but it can substitute for capital inputs. Therefore, it is doubtful that the same nature of CES exists between HS and LS workers as exists between labor and capital inputs.

Second, there is a question about the assumption that the only difference between HS and LS workers is skills that can be acquired by any worker equally at the same cost, that is, that a worker can freely choose whether to be high skilled or low skilled. With this assumption, the ratio $\frac{L_{HS}}{L_{LS}}$ is endogenous. The importance of this assumption is easily understood if we instead assume that any worker is born either as an HS worker or an LS worker and cannot change from one to the other; that is, the ratio $\frac{L_{HS}}{L_{LS}}$ is exogenously given and fixed. In this case, Cobb–Douglas and CES production functions will predict that the phenomenon $w_{HS} < w_{LS}$ would be widely and frequently observed across countries and time periods. However, the phenomenon $w_{HS} < w_{LS}$ is regarded to be very unnatural and has actually rarely (probably never) been observed in market-oriented economies. This result indicates that there are factors that intrinsically differentiate workers' abilities other than skills that are equally acquirable at the same cost. If such factors exist, it is problematic to use the conventional production function to examine heterogeneous workers.

2.2 An alternative production function

2.2.1 The assumption behind an alterative production function

An alternative production function that overcomes the abovementioned problems is presented by Harashima (2009, 2017). In this case, instead of HS and LS workers, HI and LI workers are assumed, where an HI worker is a worker with higher innovative intelligence and an LI worker is one with lower innovative intelligence. Here, let ω be the magnitude of a worker's innovative intelligence, and it is exogenously given and constant.

ω indicates the productivity of the workers. HI and LI workers are identical to each other except for their values of ω; and $\omega_{HI} > \omega_{LI}$ where ω_{HI} and ω_{LI} are ω of HI and LI workers respectively. Note that the reasons why workers have different values of ω and why ω is exogenously given and constant are beyond the scope of economics and are the subject of study in other fields.

Let L_{HI} and L_{LI} be the numbers of HI and LI workers, respectively. Let L_S be a unit of the size of the economy, and initially $L_S = L_{HI} + L_{LI}$. Let also $S_{HI} = \frac{L_{HI}}{L_S}$ and $S_{LI} = \frac{L_{LI}}{L_S}$; thus, initially $S_{HI} = \frac{L_{HI}}{L_{HI}+L_{LI}}$ and $S_{LI} = \frac{L_{LI}}{L_{HI}+L_{LI}}$. S_{HI} and S_{LI} can be interpreted as the sizes of the economies of HI and LI workers within a country, respectively. Capital inputs are also assumed to move perfectly elastically. Based on the model of Total factor productivity shown in Harashima (2017), the production function can be described as

$$Y = \bar{\sigma} A K^{1-\alpha} \left(\omega_{HI} L_{HI}^{\alpha} S_{HI}^{1-\alpha} + \omega_{LI} L_{LI}^{\alpha} S_{LI}^{1-\alpha} \right) , \tag{4}$$

where α (> 0) is a constant parameter and $\bar{\sigma}$ is a worker's accessibility limit to capital and assumed to be independent of ω and constant.

Let $\bar{L}_S$ be a unit of the size of the economy when the population density is optimal (thereby, $\bar{L}_S$ is constant). Hence, by equation (4), the production function in the long run is described by

$$Y = \bar{\sigma} A K^{1-\alpha} \bar{L}_S^{\alpha-1} \left(\omega_{HI} L_{HI} + \omega_{LI} L_{LI} \right) , \tag{5}$$

where the population density is initially assumed to be optimal; thereby, $\bar{L}_S = \bar{L}_{HI} + \bar{L}_{LI}$, where $\bar{L}_{HI}$ and $\bar{L}_{LI}$ are the initial values of L_{HI} and L_{LI}.

If workers are not heterogeneous (i.e., if $\omega_{HI} = \omega_{LI} = \bar{\omega}$), then equation (4) degenerates to

$$Y = \bar{\omega} \bar{\sigma} A K^{1-\alpha} L^{\alpha} , \tag{6}$$

which is a Cobb–Douglas production function. In this case, α indicates the labor share. This means that, in the alternative production function, the elasticity of substitution between labor and capital inputs is still constant, as it is in the Cobb–Douglas and CES production functions, but it is not constant between HI and LI workers.

Equation (6) is a Cobb–Douglas production function, and therefore it is also a conventional production function. The assumption behind equation (6), however, is completely different from the conventional one; that is, HI and LI workers are not changeable by education, whereas HS and LS workers are changeable by acquiring skills through education at the same costs for everybody.

2.2.2 Education can be always advantageous

Let w_{HI} and w_{LI} be the wages of HI and LI workers, respectively. Based on equation (5), Harashima (2017) showed that

$$\frac{w_{HI}}{w_{LI}} = \frac{\omega_{HI}}{\omega_{LI}} > 1$$

is always kept in the long run regardless of the numbers of HI and LI workers. There is no possibility of $w_{HI} \leq w_{LI}$ if $\omega_{HI} > \omega_{LI}$. Because wages are not determined by skills acquired by education but by the values of ω, it can be always advantageous to receive education in this alternative production function described by equation (5) or (6) regardless of the numbers of HI and LI workers if education has some positive effects.

To this point, I have not shown what kinds of effects education has on economic output (e.g., production) with this alternative production function and assumption. I discuss these effects in the following section.

3 EDUCATION AND THE PRODUCTION FUNCTION

3.1 Effects of education on production

What effects does education have on economic outputs, or more specifically, how does it generate positive externalities? The production function described by equation (5) or (6) implies three possible channels through which education can generate positive externalities: (1) education increases productivity (ω), (2) it increases capital (K), and (3) it increases a worker's accessibility limit to capital ($\bar{\sigma}$).

In this paper, I focus particularly on possibilities (1) and (2); I briefly discuss possibility (3) in Section 4.6.

3.1.1 Difference between capital and labor inputs

Before examining possibilities (1) and (2), I reexamine the difference in the nature of capital and labor inputs because ω is related to labor inputs and K is related to capital inputs. These two inputs are equally indispensable as factor inputs for production. If either of them is not available, production cannot be materialized except in extremely rare cases. In this respect, they have much in common.

However, there are also essential differences between them. For example, labor inputs consist of human beings and capital inputs consist of items such as machines, buildings, and software. In addition, Harashima (2009, 2017) showed that there is an important difference between labor and capital inputs in that the performance of a worker (labor) improves as the worker repeats the same assigned task, but the performance of machines and tools (capital) does not change as the same task is repeated. The reason for this difference is that humans (labor) have intelligence, particularly fluid intelligence, but machines and tools (capital) do not.

3.1.2 Outcomes of education as capital

According to this difference in performance in repeating the same task, we can distinguish the effects of education on ω and K and determine which of possibilities (1) and (2) is valid. That is, if the performance of a worker improves as the worker repeats the same assigned task only when the worker has received education, this improvement in performance is related to ω (i.e., education increases the value of ω), but if the performance improves regardless of education, education will be related to K (and/or $\bar{\sigma}$).

It seems clear that the education received by a worker does not change even if the worker repeats the same task; therefore, even if the performance of the worker improves with repetition, this increase in performance is realized thanks to the worker's experiences after receiving education, as indicated by the theory of the experience curve effect that dates back to Wright (1936), Hirsch (1952), Alchian (1963), and Rapping (1965). That is, regardless of whether a worker has received education, performances improve after repeating the same task. Furthermore, the mechanism of this improvement is already reflected in the production function described by equation (5) or (6) by ω. Taking the above arguments into consideration, possibility (1) does not seem to be acceptable.

Therefore, it is highly likely that the benefits acquired by education are not an increase in labor productivity, as possibility (1) indicates. Rather, they are the accumulation of a kind of capital, as possibility (2) indicates. Education increases production because it increases capital. I refer to this kind of capital "education capital," and I call other and usually assumed capital inputs (e.g., physical capital) "usual capital."

Because education capital is a type of capital, receiving education is an investment. Making investments requires money; therefore, receiving education also requires money (i.e., tuition). This means that investments in education compete with other types of capital (i.e., usual capital) in financial markets. For example, student loans compete with other types of loans in financial markets.

3.2 Positive externalities of education

3.2.1 Externalities from increases in labor productivity

Before examining externalities generated by education capital, I examine externalities with regard to ω. As shown in Section 3.1, education basically does not affect ω, but even if it could affect ω, it would not have externalities. As Harashima (2017) showed, regardless of the numbers of HI and LI workers,

$$\frac{w_{HI}}{w_{LI}} = \frac{\omega_{HI}}{\omega_{LI}} > 1$$

is always kept. That is, even if a worker could switch from being an LI worker to an HI worker by receiving education (i.e., the worker's productivity changed from ω_{LI} to ω_{HI} thanks to education), the wages of HI and LI workers would remain unchanged. A worker's change from LI to HI would increase the wage of the worker from w_{LI} to w_{HI}, but the other workers' wages would not be affected. The benefits from education are enjoyed only by the worker who received education. Therefore, there is no externality in education from the point of view of ω, even if education could affect ω.

3.2.2 Externalities from increases in capital

Next, I examine externalities with regard to capital. As indicated in Section 3.1.2, education capital competes with usual capital in financial markets, and therefore they are traded in the same manner in financial markets. Ramsey-type growth models indicate that, through arbitrage in markets, the return on investments in capital is kept identical and equal to the rate of time preference (RTP) at steady state such that

$$\frac{dY}{dK} = r = \theta \, , \tag{7}$$

where r is the real interest rate and θ is RTP. Because education capital and usual capital are indifferent with regard to the return on investments in capital in markets, the return on investments in education capital will also be r and equal to θ. Even if a worker acquires education capital, equation (7) is kept. That is, the benefits from education are only distributed to the worker who acquired education capital. This suggests that no education externalities exist from the point of view of capital.

However, in this paper, I show that, unlike the case with usual capital, there is a mechanism through which education capital generates positive externalities, and furthermore, this externality is consistent with equation (7). This mechanism is explained in detail in the following section.

4 EXTERNALITY OF EDUCATION

4.1 *A possible source of externality of education*

Production processes consist of not only processes in which machines or tools are used (e.g., in factories), but they also include many kinds of transactions, dealings, and coordination with parties inside and outside of companies. If a worker's counterparts have the same education capital as the worker has, the transactions between the worker and the counterparts will proceed more smoothly and efficiently than if not, because troubles will be avoided when both have the same knowledge (i.e., education capital).

For example, if one of the parties cannot read and write, transactions will become very inefficient, particularly when transactions are very complicated, as compared with the case where both parties can read and write. Reading, writing, and arithmetic are unquestionably the most fundamental types of education capital, and business among people who do not have these will be extremely inefficient in modern society.

This nature is important because if a worker newly acquires some type of education capital, that worker's performance and that of counterparts who already possess the same education capital will potentially improve at the same time. That is, the benefits from acquiring education capital are distributed not only to the worker but also to the worker's counterparts. They can enjoy benefits without any additional cost and effort because another worker additionally acquired the same education capital (i.e., externalities exist in education).

4.2 *The model*

4.2.1 The key mechanism

If a worker coincidentally transacts business with another worker who possesses the same education capital, the transaction between them will be implemented more smoothly, efficiently, and productively. Note that although it is implemented more "productively," this improvement is unrelated to labor productivity (ω), as indicated in the previous sections.

In the model in this paper, this improvement in efficiency or productivity is described as an increase in the value of education capital. That is, the value of the capital increases from its initial value by this improvement. It increases because a larger amount of production can potentially be generated using the same initial amount of education capital. When two workers who possess the same education capital coincidentally transact business with each other, an increase in the value of the education capital of the two workers is actually materialized. This is the key mechanism in the model of education externalities in this paper.

This mechanism indicates that, if a worker newly acquires education capital, the value of education capital that is already possessed by other workers increases. These workers can enjoy this increased value without any additional cost and effort. Furthermore, these workers can enjoy benefits without paying any compensation to the worker who newly acquired the education capital.

4.2.2 Environment

Suppose that there are many workers and they are uniformly distributed over a unit line segment [0, 1], and therefore the quantity (number) of all workers is unity. Each worker transacts business with each of the other workers once in a unit period. All workers are identical except for education capital (i.e., there is no difference between an LI and an HI worker). Therefore, the production function used in this model is the one described by equation (6).

Workers can possess different types of education capital. Let δ_i be the ratio of workers who have acquired and possess type i education capital to all workers, and $0 < \delta_i \leq 1$. Because each worker transacts business with each of the other workers once in a unit period, the ratio of transactions between that worker and the other workers who possess the same type i education capital to those between the worker and all other workers (regardless of whether they possess type i education capital) is δ_i^2. The performances of δ_i^2 transactions are therefore improved by the mechanism explained in Section 4.2.1, whereas the performances of the other $1 - \delta_i^2$ transactions are not.

4.2.3 Capital

Capital is composed of usual capital and education capital. The value of usual capital is equal to the costs that are required to produce or obtain it. The "initial" value of education capital is also equal to the costs required to acquire it. The word "initial" here means before the mechanism of externalities has worked or when the effects of externalities are ignored or removed. The value of type i education capital is determined not only by the initial value but also by the effects of externalities (i.e., the value of δ_i). Hence, the value of type i education capital is a function of δ_i. The value of all capital (K) in the production function is the sum of the values of usual and education capital.

Suppose that there are M (> 1) types of education capital. Let $K_{EDU,i}$ be the total (summed over all workers) value of type i education capital possessed by workers in the economy. Taking the effect of δ_i on education capital shown in Section 4.2.1 into consideration, it is assumed for simplicity that, for any i,

$$K_{EDU,i} = \widetilde{K}_{EDU,i}\left(1 + \frac{\delta_i^2}{\chi}\right) , \quad (8)$$

where $\chi(>0)$ is a parameter and common for any i, and $\widetilde{K}_{EDU,i}$ is the initial value of $K_{EDU,i}$ (i.e., it is $K_{EDU,i}$ when the effect of δ_i on education capital is ignored or removed); therefore, $\widetilde{K}_{EDU,i}$ indicates the total (or summed) costs that are needed to acquire type i education capital by workers in the economy. Parameter χ represents the strength of effect of δ_i, and as the value of χ decreases, the strength increases. The total value of education capital in the economy (K_{EDU}) is therefore

$$K_{EDU} = \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_i^2}{\chi} \right) \right] .$$

Hence, capital in the economy is

$$K = K_{USU} + K_{EDU} = K_{USU} + \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_i^2}{\chi} \right) \right] ,$$

where K_{USU} is the total value of usual capital in the economy.

4.2.4 The production function

By equation (6), the production function is described as

$$Y = \bar{\sigma}\bar{\omega} L^{\alpha} \left\{ K_{USU} + \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_i^2}{\chi} \right) \right] \right\}^{1-\alpha} . \quad (9)$$

4.3 Externalities

4.3.1 Effects of δ_i on *Y*

Equation (8) indicates that education has the nature of externalities, and in addition, the degree of externality increases as the value of δ_i increases.

If the effects of δ_i are ignored or removed and the value of δ_i is set to be zero for any i, the education capital values are equal to their initial values, and the production function described by equation (9) degenerates to

$$Y_{\delta=0} = \bar{\sigma}\bar{\omega} L^{\alpha} \left\{ K_{USU} + \sum_{i=1}^{M} \widetilde{K}_{EDU,i} \right\}^{1-\alpha} , \quad (10)$$

where $Y_{\delta=0}$ is Y in this case. By equations (9) and (10),

$$\frac{Y}{Y_{\delta=0}} = \left\{ \frac{K_{USU} + \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_i^2}{\chi} \right) \right]}{K_{USU} + \sum_{i=1}^{M} \widetilde{K}_{EDU,i}} \right\}^{1-\alpha} > 0 \tag{11}$$

because $0 < \delta_i \leq 1$ and $0 < \chi$. Inequality (11) indicates that production increases because of the effect of δ_i (i.e., by education externalities).

On the other hand,

$$\frac{dY}{d\delta_i} = \bar{\sigma}\bar{\omega}L^{\alpha}(1-\alpha)\left\{K_{USU} + \sum_{i=1}^{M}\left[\widetilde{K}_{EDU,i}\left(\frac{\chi + \delta_i^2}{\chi}\right)\right]\right\}^{-\alpha}\left\{\frac{d\widetilde{K}_{EDU,i}}{d\delta_i}\left(1 + \frac{\delta_i^2}{\chi}\right) + \widetilde{K}_{EDU,i}\frac{2\delta_i}{\chi}\right\} . \tag{12}$$

Because an increase in $\widetilde{K}_{EDU,i}$ indicates that the number of workers who acquire and possess type *i* education capital increases and thereby δ_i increases, then

$$\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}} > 0 \ . \tag{13}$$

Hence, by equation (12) and inequality (13),

$$\frac{dY}{d\delta_i} > 0 \ . \tag{14}$$

Inequalities (11) and (14) again indicate that the production function shows the existence of education externalities.

4.3.2 Increasing returns to scale

By equation (9), for some parameter $\gamma(> 1)$,

$$Y_{\gamma} = \bar{\sigma}\bar{\omega}(\gamma L)^{\alpha}\left\{\gamma K_{USU} + \sum_{i=1}^{M}\left[\gamma\widetilde{K}_{EDU,i}\left(1 + \frac{\delta_{i,\gamma}^2}{\chi}\right)\right]\right\}^{1-\alpha} , \tag{15}$$

where Y_{γ} and $\delta_{i,\gamma}$ are Y and δ_i after $\widetilde{K}_{EDU,i}$ increased to $\gamma\widetilde{K}_{EDU,i}$, respectively.

Because

$$\delta_{i,\gamma}^2 > \delta_i^2$$

by inequality (13) and $\gamma > 1$, then by equations (9) and (15),

$$\frac{Y_\gamma}{Y} = \gamma \left\{ \frac{K_{USU} + \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_{i,\gamma}^2}{\chi} \right) \right]}{K_{USU} + \sum_{i=1}^{M} \left[\widetilde{K}_{EDU,i} \left(1 + \frac{\delta_i^2}{\chi} \right) \right]} \right\}^{1-\alpha} > \gamma \, .$$

That is, the production function shows increasing returns to scale, which is another aspect of education externalities.

In addition, by equation (9),

$$\frac{dY}{d\widetilde{K}_{EDU,i}} =$$

$$\bar{\sigma}\bar{\omega}L^{\alpha}(1-\alpha) \left\{ K_{USU} + \sum_{j=1}^{M} \left[\widetilde{K}_{EDU,j} \left(1 + \frac{\delta_j^2}{\chi} \right) \right] \right\}^{-\alpha} \left[1 + \frac{\delta_i^2 + \widetilde{K}_{EDU,i} \frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi} \right] . \qquad (16)$$

On the other hand,

$$\frac{dY}{dK_{USU}} = \bar{\sigma}\bar{\omega}L^{\alpha}(1-\alpha) \left\{ K_{USU} + \sum_{j=1}^{M} \left[\widetilde{K}_{EDU,j} \left(1 + \frac{\delta_j^2}{\chi} \right) \right] \right\}^{-\alpha} . \qquad (17)$$

Hence, by equations (16) and (17) and inequality (13),

$$\frac{\frac{dY}{d\widetilde{K}_{EDU,i}}}{\frac{dY}{dK_{USU}}} = 1 + \frac{\delta_i^2 + \widetilde{K}_{EDU,i} \frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi} > 1 \, . \qquad (18)$$

That is, the return on investments in education capital is larger than that on usual capital. The extra return on investments in education is

$$\frac{dY}{d\widetilde{K}_{EDU,i}} - \frac{dY}{dK_{USU}} > 0 \,. \tag{19}$$

Inequalities (18) and (19) imply that investments in education capital increase more rapidly and eventually become far larger than those of usual capital because of the higher returns. However, there is a mechanism that acts to dampen investments in education capital. This mechanism is discussed in Section 4.5.

4.3.3 Distribution of increased incomes

If the value of type *i* education capital increases because a worker newly acquired it, the total production and income in the economy increase because the increase in the value of type *i* education capital indicates that the total capital inputs for production in the economy increase.

To whom is this increase in incomes distributed? Can only the worker who newly acquired type *i* education capital obtain all the increase in incomes? Because of the existence of externalities, the increase in incomes is distributed equally to all workers who possess type *i* education capital, not only to the worker who newly acquired it.

4.4 Tree diagram of education capital

Basically, knowledge and technology increase incrementally and additively. New knowledge and technologies are added to the foundation of previous knowledge and technologies. Of course, great leaps in knowledge and technologies may have occasionally occurred (i.e., new discoveries), but they could not be discovered without the foundation of accumulated past knowledge and technologies.

Therefore, various pieces of knowledge and technologies can be described by a tree diagram. Because education capital depends crucially on accumulated knowledge, various types of education capital can be also described by a tree diagram. A tree has the base, branches, and many twigs, which means that a tree consists of many parts with different features. Similarly, knowledge and education capital are composed of many elements that have different features.

The base of a knowledge tree is the knowledge that is the foundation of all the other pieces of knowledge. On the other hand, each knowledge twig deals with very limited pieces of knowledge. Branches that are located closer to the base will correspond to more elementary pieces of knowledge, and those located farther from the base will correspond to more specialized and sophisticated pieces of knowledge.

In this analogy, the base of the education capital tree corresponds to reading, writing, and arithmetic, which are usually taught at elementary schools, whereas the twigs correspond to the educational content studied mostly at universities and graduate schools.

It seems highly likely that, if type *i* education capital is an elementary piece of education, the value of δ_i is large (e.g., almost unity), but on the other hand, if it is a very specialized piece of knowledge, the value of δ_i is very small (e.g., almost zero).

4.5 Limit of education externalities

Because education capital has the tree-like structure described above, the value of δ_i for most types of education capital is highly likely to have an upper bound that is far lower than unity because of specialization. Specialization means that most other workers need not acquire the education capital that a small number of specialists already possess.

4.5.1 Division of labor

Workers are specialized as much as possible within an economy; that is, there is a division of labor. The number of specialists for each specialized skill is constrained by the structure of the division of labor within the economy, and the number of specialists for each specialized skill is very small compared with the number of all workers. Given this division of labor, the demands of firms (or employers) for workers who are specialists (i.e., those who possess twigs of education capital) have an upper bound.

Even if the wage of specialists with regard to a particular type of education capital decreases significantly, specialists of this type will never be demanded and hired by firms beyond its upper bound because the upper bound is determined by the structure of division of labor in the economy, not by wages. That is, the demand for a worker for a particular specialized type of education capital will be very inelastic and its demand curve will be almost vertical. These specialists are recruited by firms up to the upper bound, but no more are demanded past that point.

4.5.2 The upper bound of δ_i

Even if inequality (18) always holds (i.e., if the return on investments in education capital is always higher than that in usual capital), the number of workers who possess type *i* education capital does not exceed a certain critical number.

Let $\bar{\delta}_i$ be the upper bound of type *i* education capital, and $\delta_i \leq \bar{\delta}_i \leq 1$ for any *i*. Because of inequality (18), any type of education capital will be accumulated until δ_i reaches $\bar{\delta}_i$. The existence of the upper bound of δ_i means that the effect of education externalities has a limit.

Note, however, that knowledge itself will increase indefinitely, and the degree of division of labor (i.e., specialization) will also increase. That is, increases in knowledge are almost canceled out by increases in the degree of specialization; therefore,

$$\sum_{j=1}^{N} \bar{\delta}_j < 1$$

will be kept in an economy. Of course, there may be workers who are specialists in multiple fields and possess diplomas in different fields, but specialists usually operate in a single field.

4.5.3 Education externalities and equilibrium investments

If δ_i reaches its upper bound for any *i* (i.e., if $\delta_i = \bar{\delta}_i$), investments in education capital stop being made. After that, investments are made only in usual capital; therefore, through arbitrage in markets,

$$\frac{dY}{dK_{USU}} = r = \theta \tag{20}$$

is always kept. Because $r = \theta$ is kept, equation (20) is consistent with equation (7). Because equation (20) is consistent with education externalities in the sense that equation (20) and education externalities coexist without any problems, education externalities are also consistent with equation (7). Hence, by equation (20) and inequality (18),

$$\frac{dY}{dK_{USU}} = r = \theta < \frac{dY}{d\widetilde{K}_{EDU,i}}$$

is kept consistently with education externalities and equation (7).

In actuality, of course, investments in education capital never stop being made because some workers retire and their replacements are needed in every period. For simplicity, I ignore this replacement in this paper.

4.6 Other possible sources of externalities in education

As mentioned in Section 3.1, possibility (3) also exists as an effect of education. As Harashima (2009) showed, $\bar{\sigma}$ is a worker's accessibility limit to capital and is determined not only by the availability of physical transportation facilities but also by efficiencies of law enforcement, regulations, the financial system, and other related factors. It seems likely that, if the average education level in a country increases, these efficiencies will improve and thereby the value of $\bar{\sigma}$ will increase. The production function described by equation (5) indicates that an increase in $\bar{\sigma}$ increases production using the same amounts of inputs and benefits everyone. Therefore, if an increase in $\bar{\sigma}$

is generated by education, education benefits everyone (i.e., it has externalities through the effect on $\bar{\sigma}$).

In addition, Harashima (2020b) showed that many people are often intentionally misled by other parties when making decisions in dealings in business, and as a result, they unconsciously give away economic rents to the other parties. It seems likely that the probability of a person being misled by other people decreases if the person is more educated. A decrease in this probability may also increase the value of $\bar{\sigma}$ because it may cause the number of attempts to mislead people to decrease because of the lower level of success in doing so. Education therefore may also have externalities through this channel.

5 SUBSIDIZED OR COMPULSORY EDUCATION

5.1 Elementary schools

It is highly likely that if the knowledge corresponding to education capital i is closer to the base of the knowledge tree, the value of $\bar{\delta}_i$ is higher. As indicated in Section 4.4, the materials taught at elementary schools (i.e., reading, writing, and arithmetic) correspond to the base of the knowledge tree. Hence, the value of $\bar{\delta}_i$ of education capital acquired in elementary schools will be near unity, and almost all employers will require their employees to possess this type of education capital.

Note that the costs to attend elementary schools are almost always paid by parents (through taxes in many cases), not the children. However, in this paper, it is assumed for simplicity that these costs are paid by the children themselves (i.e., the future workers) on the assumption that the costs are paid by them later when they become adults. Hence, in this paper, it is "workers" who study at elementary schools and at the same time pay their costs.

5.1.1 Costs for education externalities

Suppose for simplicity that the cost of acquiring education capital is covered only by loans. Let the payment of loans for the cost of acquiring a unit of type i education capital ($\widetilde{K}_{EDU,i}$) in each period be c_i. Suppose also that the value of c_i is common for all types of education capital (i.e., $c_i = \bar{c}(> 0)$) for any i. Through arbitrage in markets, c_i is kept equal to the real interest rate such that

$$\bar{c} = \frac{dY}{dK_{USU}} = r \ . \tag{21}$$

5.1.2 Benefits from education externalities

By equation (16), if

$$\frac{dY}{d\widetilde{K}_{EDU,i}} =$$

$$\bar{\sigma}\varpi L^{\alpha}(1-\alpha)\left\{K_{USU} + \sum_{j=1}^{M}\left[\widetilde{K}_{EDU,j}\left(1+\frac{\delta_j^2}{\chi}\right)\right]\right\}^{-\alpha}\left(1+\frac{\delta_i^2 + \widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi}\right) > \bar{c}\ ;$$

therefore, by equations (17) and (20), if

$$\frac{dY}{d\widetilde{K}_{EDU,i}} = \frac{dY}{dK_{USU}}\left(1+\frac{\delta_i^2 + \widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi}\right)$$

$$= r\left(1+\frac{\delta_i^2 + \widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi}\right) > \bar{c}\ , \tag{22}$$

workers will continue to acquire type *i* education capital up to the point $\delta_i = \bar{\delta}_i$. Because $\bar{c} = r$ as indicated by equation (21), by inequality (22), if

$$\frac{\delta_i^2 + \widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi} > 0\ , \tag{23}$$

type *i* education capital will continue to be acquired up to the point $\delta_i = \bar{\delta}_i$. By inequality (13), $\delta_i > 0$, $\widetilde{K}_{EDU,i} > 0$, and $\chi > 0$, inequality (23) is always held; therefore, $\delta_i = \bar{\delta}_i$ will be always eventually be achieved and maintained.

Let

$$TB_i = \widetilde{K}_{EDU,i}\left(\frac{dY}{d\widetilde{K}_{EDU,i}} - \bar{c}\right)$$

for $\delta_i = \bar{\delta}_i$. TB_i indicates the total (or summed) benefits generated by the externality of type *i* education capital (i.e., the extra increases in production incurred due to the externalities) within the economy. By equations (21) and (22),

$$TB_i = \widetilde{K}_{EDU,i}\left(\frac{dY}{d\widetilde{K}_{EDU,i}} - \bar{c}\right) = r\widetilde{K}_{EDU,i}\frac{\delta_i^2 + \widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}}}{\chi} \ .$$

Here, suppose for simplicity that

$$\widetilde{K}_{EDU,i} = \delta_i \overline{K}_{EDU} \ ,$$

where $\overline{K}_{EDU}$ is a constant and common for any *i*. Hence, because

$$\widetilde{K}_{EDU,i}\frac{d\delta_i^2}{d\widetilde{K}_{EDU,i}} = \delta_i\overline{K}_{EDU}\frac{d\delta_i^2}{d(\delta_i\overline{K}_{EDU})} = 2\delta_i^2 \ , \tag{24}$$

then

$$TB_i = \frac{r\delta_i\overline{K}_{EDU}}{\chi}\left[\delta_i^2 + \delta_i\overline{K}_{EDU}\frac{d\delta_i^2}{d(\delta_i\overline{K}_{EDU})}\right] = \frac{3r\delta_i^3}{\chi}\overline{K}_{EDU} \ . \tag{25}$$

5.1.3 Benefits from externalities of education when $\delta_i = 1$

Consider the case where $\bar{\delta}_i = 1$. Let *i* in this case be *E*; therefore, $\bar{\delta}_E = 1$. Type *E* education capital is, for example, education that is taught in elementary schools. Because $\bar{\delta}_E = 1$, any worker can acquire type *E* education capital if they want. By equations (16) and (24),

$$\frac{dY}{d\widetilde{K}_{EDU,E}} = \bar{\sigma}\varpi L^{\alpha}(1-\alpha)\left\{K_{USU} + \sum_{j=1}^{M}\left[\widetilde{K}_{EDU,j}\left(1 + \frac{\delta_j^2}{\chi}\right)\right]\right\}^{-\alpha}\left(1 + \frac{3\delta_E^3}{\chi}\right) \ .$$

If $\delta_E = \bar{\delta}_E = 1$,

$$\frac{dY}{d\widetilde{K}_{EDU,E}} = \bar{\sigma}\varpi L^{\alpha}(1-\alpha)\left\{K_{USU} + \sum_{j=1}^{M}\left[\widetilde{K}_{EDU,j}\left(1 + \frac{\delta_j^2}{\chi}\right)\right]\right\}^{-\alpha}\left(1 + \frac{3}{\chi}\right) \ ,$$

and by equations (17) and (20),

$$\frac{dY}{d\widetilde{K}_{EDU,E}} = \frac{dY}{dK_{USU}}\left(1+\frac{3}{\chi}\right)$$
$$= r\left(1+\frac{3}{\chi}\right) .$$

Hence, the benefits ($\frac{dY}{d\widetilde{K}_{EDU,E}}$) at $\delta_E = \bar{\delta}_E = 1$ exceed the costs ($\bar{c}$). That is, if

$$\frac{dY}{d\widetilde{K}_{EDU,E}} = r\left(1+\frac{3}{\chi}\right) > \bar{c} ,$$

and therefore, because $r = \bar{c}$, if

$$\frac{3}{\chi} > 0 , \tag{26}$$

then type *E* education capital will be demanded and acquired by all workers. Because $\chi > 0$ and thereby inequality (26) is always satisfied, all workers will eventually acquire type *E* education capital; that is, the state $\delta_E = \bar{\delta}_E = 1$ will be naturally realized.

In addition, by equation (25),

$$TB_E = \frac{3r}{\chi}\overline{K}_{EDU} , \tag{27}$$

and

$$\frac{TB_E}{TB_i} = \frac{\frac{3r}{\chi}\overline{K}_{EDU}}{\frac{3r\delta_i^3}{\chi}\overline{K}_{EDU}} = \frac{1}{\delta_i^3}$$

for any *i* with $\delta_i < 1$; therefore,

$$\frac{TB_E}{TB_i} > 1 .$$

Particularly, if the value of δ_i is far smaller than unity, the difference between TB_E and TB_i will be very large because of the factor of δ_i in TB_i.

5.1.4 Benefits from externalities of education when $\delta_i < 1$

5.1.4.1 Decrease in benefits

As shown in Section 5.1.3, the state $\delta_E = \bar{\delta}_E = 1$ will be naturally realized, which means, in theory, that all workers will attend elementary schools. However, some workers may not enter elementary schools because of poverty, borrowing constraints, or other reasons. Particularly, poverty with borrowing constraints may prevent some workers from attending school.

Suppose that type E education capital is that taught at elementary school in a country and $\bar{\delta}_E = 1$, but elementary schools in the country are not compulsory. Therefore, $1 - \eta_E \left(0 < 1 - \eta_E < \bar{\delta}_E = 1\right)$ of the workers do not attend elementary schools because of poverty with borrowing constraints. That is, η_E workers acquire type E education capital, but $1 - \eta_E$ of them do not.

Let $TB_{\eta,E}$ be TB_E in this case. Therefore, by equations (25) and (27),

$$TB_{\eta,E} = \bar{K}_{EDU} \frac{3r}{\chi} \eta_E^3 .$$

Hence,

$$TB_E - TB_{\eta,E} = \frac{3r}{\chi} \bar{K}_{EDU} - \frac{3r\eta_E^3}{\chi} \bar{K}_{EDU} = \bar{K}_{EDU} (1 - \eta_E^3) \frac{3r}{\chi} ,$$

and because $\eta_E < 1$,

$$TB_E - TB_{\eta,E} > 0 .$$

That is, $TB_{\eta,E}$ is smaller than TB_E.

In this case, the benefits per unit of education capital (units of $TB_{\eta,E}$ for a unit of $\tilde{K}_{EDU,E} = \eta_E \bar{K}_{EDU}$) enjoyed by each of the η_E workers are

$$\frac{TB_{\eta,E}}{\tilde{K}_{EDU,E}} = \frac{TB_{\eta,E}}{\eta_E \bar{K}_{EDU}} = \frac{\bar{K}_{EDU} \frac{3r}{\chi} \eta_E^3}{\eta_E \bar{K}_{EDU}} = \frac{3r}{\chi} \eta_E^2 . \tag{28}$$

For comparison, consider the case where $\eta_E = 1$. In this case, the benefits per unit of education capital enjoyed by each of the η_E workers are

$$\frac{TB_{\eta,E}}{\widetilde{K}_{EDU,E}} = \frac{TB_{\eta,E}}{\eta_E \overline{K}_{EDU}} = \frac{TB_E}{\overline{K}_{EDU}} = \frac{3r}{\chi} \ . \tag{29}$$

Equations (28) and (29) indicate that because of the existence of the $1-\eta_E$ workers, the benefits per unit of education capital enjoyed by each of the η_E workers are reduced by

$$Rd_1 = \frac{3r}{\chi}(1-\eta_E^2) \ . \tag{30}$$

That is, the η_E workers who attend elementary schools cannot fully enjoy the benefits of externalities by Rd_1 because of the existence of the $1-\eta_E$ workers who do not attend them.

5.1.4.2 Condition for compulsory elementary education

To fully enjoy the benefits of externalities, the η_E workers may choose to pay the schooling costs of the $1-\eta_E$ workers and make them attend elementary schools, e.g., compulsorily. As shown in Section 5.1.1 and by equation (21), the cost per unit of education capital ($\bar{c} = r$) is constant regardless of η_E. Therefore, the burden for each η_E worker to pay the costs for the $1-\eta_E$ workers, each of whom acquires a unit of education capital, is

$$Ac_1 = \frac{\bar{c}(1-\eta_E)}{\eta_E} = \frac{r(1-\eta_E)}{\eta_E} \ . \tag{31}$$

Equation (31) means that the total (summed) costs for the $1-\eta_E$ workers are shared equally by all of the η_E workers.

If the benefits per unit of education capital enjoyed by each η_E worker exceed the burdens for each η_E worker to pay the costs of the $1-\eta_E$ workers (i.e., if $Rd_1 > Ac_1$), the η_E workers may choose to support the $1-\eta_E$ workers by making them compulsorily attend elementary schools at the expense of the η_E workers. That is, by equations (30) and (31), if

$$r\frac{3}{\chi}(1-\eta_E^2) > \frac{\bar{c}(1-\eta_E)}{\eta_E} \ ,$$

and therefore, if

$$\eta_E^2 + \eta_E - \frac{\chi}{3} > 0 \ , \tag{32}$$

then it may be better for them to pay the expense. Here, if

$$1 \geq \eta_E > \frac{\sqrt{9+12\chi}}{6} - \frac{1}{2}\ , \tag{33}$$

inequality (32) is satisfied. For example, when $\eta_E = 0.5$, if

$$\frac{25}{12} \geq \chi > 0\ , \tag{34}$$

inequality (32) is always satisfied.

χ is a parameter in equation (8) that represents the strength of the education externality. As the value of χ decreases, the strength of the externality increases. Therefore, inequalities (33) and (34) indicate that, if the strength of externality is sufficiently strong, the probability that the η_E workers choose to support the $1-\eta_E$ workers and as a result elementary schools become compulsory is high.

5.1.4.3 Necessity of other reasons for compulsory elementary education

Inequality (33) indicates that, if

$$1-\eta_E \geq \frac{3}{2} - \frac{\sqrt{9+12\chi}}{6}\ ,$$

then inequality (32) is not satisfied. That is, if a sufficiently large number of workers initially do not attend elementary schools, the cost of compulsory education for the η_E workers exceeds their benefits. As a result, η_E workers may reject the idea of compulsory elementary school.

In this case, another rationale for compulsory elementary school attendance is required. For example, a government may force its people to attend elementary schools to strengthen national security.

5.1.5 Private or public schools?

The costs of compulsory education are usually paid by taxpayers. This means that these costs are most often paid by high-income workers because of progressive income taxes. However, in many countries, public and private schools exist. Moreover, high-income workers more often attend private schools. These high-income workers therefore pay double costs—they pay the costs for public schools through taxes and they pay for tuition at private schools. Why do they choose to attend private schools?

One possibility is that schools provide not only education capital but also other items or experiences of value (e.g., friends, connections, tradition, status, and security), and different things are provided in different schools. Another possibility is that teachers' skills vary across schools. If a private school provides something of more value as compared with public schools or if its teachers' skills are higher than those of public school teachers, some workers may choose private schools.

Generally, students in public elementary schools are not selected by the schools but are mechanically allocated by some pre-existing set of rules. Therefore, workers (children) with various backgrounds study at the same public school. On the other hand, private schools can select students, and these students may be far more homogeneous. Education in a homogeneous environment for selected students may provide additional benefits to the students.

5.2 Universities

As indicated in Section 4.4, the contents of education at universities and graduate schools correspond to the twigs of the knowledge tree. Therefore, the value of $\bar{\delta}_i$ of the education capital acquired at universities and graduate schools will be very small. Equation (25) indicates that, if the value of $\bar{\delta}_i$ is very small, TB_i is also very small.

It is assumed that the costs to acquire education capital at universities and graduate schools are $c_i = \bar{c}(> 0)$ for any *i*, and that equation (21) always holds.

5.2.1 Benefits from externalities of education in the case of a small $\bar{\delta}_i$

Suppose that *N* among *M* types of education capital correspond to education at universities (0 < *N* < *M*), and in addition, types 1, 2, 3, … , *N* of education capital are types of education capital that correspond to education only obtained at universities ("university education capital"). It is also assumed for simplicity that each worker acquires only one type of university education capital. Hence,

$$\sum_{j=1}^{N} \delta_j < 1 \, .$$

Let $TB_{i,\bar{\delta}}$ be TB_i when $\delta_i = \bar{\delta}_i$. Hence, by equation (25),

$$TB_{i,\bar{\delta}} = \frac{r\bar{\delta}_i \bar{K}_{EDU}}{\chi} \left[\bar{\delta}_i^2 + \bar{\delta}_i \bar{K}_{EDU} \frac{d\bar{\delta}_i^2}{d\left(\bar{\delta}_i \bar{K}_{EDU}\right)} \right] = \bar{K}_{EDU} \frac{3r\bar{\delta}_i^3}{\chi} \, , \qquad (35)$$

and thereby,

$$\frac{TB_{i,\bar{\delta}}}{TB_i} = \frac{\overline{K}_{EDU}\dfrac{3r\bar{\delta}_i^3}{\chi}}{\overline{K}_{EDU}\dfrac{3r\delta_i^3}{\chi}} = \left(\frac{\bar{\delta}_i}{\delta_i}\right)^3 \ .$$

For any $\delta_i < \bar{\delta}_i(\leq 1)$, therefore,

$$\frac{TB_{i,\bar{\delta}}}{TB_i} > 1 \ .$$

By equations (25) and (35),

$$TB_{i,\bar{\delta}} - TB_i \ = \overline{K}_{EDU}\frac{3r\bar{\delta}_i^3}{\chi} - \overline{K}_{EDU}\frac{3r\delta_i^3}{\chi} = \overline{K}_{EDU}\frac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\chi} > 0 \ ,$$

and thereby,

$$TB_i \ = TB_{i,\bar{\delta}} - \overline{K}_{EDU}\frac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\chi} \ .$$

Hence, the benefits per unit of education capital enjoyed by each of the δ_i workers are

$$\begin{aligned}\frac{TB_i}{\delta_i\overline{K}_{EDU}} &= \frac{TB_{i,\bar{\delta}} - \overline{K}_{EDU}\dfrac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\chi}}{\delta_i\overline{K}_{EDU}} \\ &= \frac{TB_{i,\bar{\delta}}}{\delta_i\overline{K}_{EDU}} - \frac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\delta_i\chi} \ . \qquad (36)\end{aligned}$$

On the other hand, if $\delta_i = \bar{\delta}_i$,

$$\frac{TB_{i,\bar{\delta}}}{\delta_i\overline{K}_{EDU}} = \frac{TB_{i,\bar{\delta}}}{\bar{\delta}_i\overline{K}_{EDU}} \ . \qquad (37)$$

Equations (36) and (37) indicate that, because of the existence of the $\bar{\delta}_i - \delta_i$ workers who do not acquire university education capital, the benefits per unit of education capital enjoyed by each of the δ_i workers is reduced by

$$Rd_2 = \frac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\delta_i \chi} \ . \tag{38}$$

5.2.2 $\boldsymbol{\delta_i}$ workers support $\boldsymbol{\bar{\delta}_i - \delta_i}$ workers

If some workers cannot pay the costs and enter universities for some reason, particularly because of poverty with borrowing constraints, and thereby $\bar{\delta}_i = \delta_i$ cannot be achieved, how should the δ_i workers respond? If the benefits gained by achieving $\bar{\delta}_i = \delta_i$ are larger than the cost to achieve it by paying the costs for the $\bar{\delta}_i - \delta_i$ workers, the δ_i workers may choose to pay the costs, even if they have to pay the costs for both themselves and the other workers.

If the δ_i workers pay the costs for the $\bar{\delta}_i - \delta_i$ workers, the burden for each δ_i worker is

$$Ac_2 = \frac{\bar{c}\left(\bar{\delta}_i - \delta_i\right)}{\delta_i} \ . \tag{39}$$

Equation (39) means that the total (summed) costs for the $\bar{\delta}_i - \delta_i$ workers (i.e., $\bar{c}\left(\bar{\delta}_i - \delta_i\right)$) are shared equally by all of the δ_i workers. Therefore, if $Rd_2 > Ac_2$, the δ_i workers may choose to pay the costs for the $\bar{\delta}_i - \delta_i$ workers. That is, by equations (38) and (39), if

$$\frac{3r\left(\bar{\delta}_i^3 - \delta_i^3\right)}{\delta_i \chi} > \bar{c}\frac{\bar{\delta}_i - \delta_i}{\delta_i} \ ,$$

and thereby if

$$3\left(\bar{\delta}_i^2 + \bar{\delta}_i \delta_i + \delta_i^2\right) > \chi \ , \tag{40}$$

then the δ_i workers may choose this option.

Inequality (40) indicates that as the value of δ_i increases (i.e., as a larger number of workers already have acquired type i education capital), the probability that the δ_i workers support the $\bar{\delta}_i - \delta_i$ workers increases. In addition, as the value of χ decreases (i.e., the strength of externality is stronger), the probability increases.

5.2.3 All $\boldsymbol{\delta_i}$ workers support all $\boldsymbol{\bar{\delta}_i - \delta_i}$ workers

Next, consider the case that all δ_i workers (i.e., the sum of all δ_i workers for all $i(\leq N)$) in a country collectively pay the costs for all $\bar{\delta}_i - \delta_i$ workers (i.e., the sum of the $\bar{\delta}_i - \delta_i$ workers for all $i(\leq N)$) in the country.

In this case, by equation (38), because of the existence of $\bar{\delta}_i - \delta_i$ workers, the benefits per unit of education capital enjoyed by each δ_i worker are reduced on average by

$$Rd_3 = \frac{\sum_{j=1}^{N}\left[\left(\bar{\delta}_j^3 - \delta_j^3\right)\frac{3r}{\delta_j \chi}\right]}{N} \ . \tag{41}$$

The term "on average" is added because the values of δ_i and $\bar{\delta}_i$ vary among the types of education capital. In addition, the per capita burdens for all δ_i workers are

$$Ac_3 = \frac{\bar{c}\sum_{j=1}^{N}\left(\bar{\delta}_j - \delta_j\right)}{\sum_{j=1}^{N}\delta_j} \ . \tag{42}$$

Equation (42) means that the total (summed) costs for all $\bar{\delta}_i - \delta_i$ workers (i.e., $\bar{c}\sum_{j=1}^{N}\left(\bar{\delta}_i - \delta_i\right)$) are shared equally by all δ_i workers (i.e., $\sum_{j=1}^{N}\delta_j$). Therefore, if $Rd_3 > Ac_3$, many δ_i workers may choose to support all of the $\bar{\delta}_i - \delta_i$ workers; that is, by equations (41) and (42), if

$$\frac{\sum_{j=1}^{N}\left[\left(\bar{\delta}_j^3 - \delta_j^3\right)\frac{3}{\delta_j \chi}\right]}{N} - \frac{\sum_{j=1}^{N}\left(\bar{\delta}_j - \delta_j\right)}{\sum_{j=1}^{N}\delta_j} > 0 \ ,$$

then the δ_i workers may make this choice.

Here, suppose for simplicity that $\delta_i = \psi\bar{\delta}_i$ and $\bar{\delta}_i = \bar{\delta}$ for any $i(\leq N)$ where $\bar{\delta}$ and ψ are constants, and $0 < \psi < 1$ and $1 > \bar{\delta} > 0$. That is, the value of δ_i is identical for any type $i(\leq N)$. Hence, if

$$3(1 + \psi + \psi^2)\bar{\delta}^2 > \chi \ , \tag{43}$$

then many δ_i workers may choose to support all of the $\bar{\delta}_i - \delta_i$ workers.

Inequality (43) indicates that if ψ is sufficiently high (i.e., if enough workers have already acquired university education capital), the probability that all of the δ_i workers will support all of the $\bar{\delta}_i - \delta_i$ workers may be high. In addition, equality (43) also indicates that, as the value of $\bar{\delta}$ increases and as the value of χ decreases (i.e., the strength of the externality increases), this probability increases.

5.2.4 Only high-income workers are burdened

If university education is subsidized by the government, the subsidies are paid by taxpayers, particularly by high-income workers because of progressive income taxes. I now examine the case where only high-income workers pay the costs for all $\bar{\delta}_i - \delta_i$ workers. It is assumed for simplicity that

$$\sum_{j=1}^{N} \bar{\delta}_j = 1 \; ;$$

that is, any worker can acquire any type of university education capital, but they may only acquire one type. Therefore, $\sum_{j=1}^{N} \bar{\delta}_j = 1$ means that the quantity of all workers in the country is unity, and they are uniformly distributed over a unit line segment [0, 1].

Let $\rho (0 < \rho < 1)$ be the ratio of high-income workers to all workers in a country. Because the quantity of all workers is unity (i.e., $\sum_{j=1}^{N} \bar{\delta}_j = 1$), ρ indicates not only the ratio but the quantity of high-income workers. In this case, by equation (42), the per capita burden for the ρ high-income workers is

$$Ac_5 = \frac{\bar{c} \sum_{j=1}^{N} \left(\bar{\delta}_j - \delta_j\right)}{\rho \sum_{j=1}^{N} \bar{\delta}_j} = \frac{\bar{c} \sum_{j=1}^{N} \left(\bar{\delta}_j - \delta_j\right)}{\rho} \tag{44}$$

because $\sum_{j=1}^{N} \bar{\delta}_j = 1$ indicates the quantity of all workers. Hence, if $Rd_3 > Ac_5$, most high-income workers may choose to support all $\bar{\delta}_i - \delta_i$ workers; that is, by equations (41) and (44), if

$$\frac{\sum_{j=1}^{N} \left[\left(\bar{\delta}_j^3 - \delta_j^3\right) \frac{3}{\bar{\delta}_j \chi} \right]}{N} - \frac{\sum_{j=1}^{N} \left(\bar{\delta}_j - \delta_j\right)}{\rho} > 0 \, , \tag{45}$$

and thereby, if

$$\rho 3 (1 + \psi + \psi^2) \bar{\delta}^2 > \chi \, , \tag{46}$$

then most high-income workers may make that choice. Inequalities (43) and (46) indicate that the hurdle to support all $\bar{\delta}_i - \delta_i$ workers is higher in this case than it is in the case that all δ_i workers support $\bar{\delta}_i - \delta_i$ workers because $0 < \rho < 1$.

In this case, although all low-income δ_i workers can receive support from the high-income (ρ) workers, some ρ workers who belong to the $\bar{\delta}_i - \delta_i$ workers also can receive the support from other ρ workers, even if they are high-income workers. In this sense, some people may feel a sense of injustice or unfairness. Therefore, inequality (46) will not be the only determining factor as to whether this kind of scheme is accepted by people in a country.

5.2.5 Only low-income $\bar{\delta}_i - \delta_i$ workers are supported by high-income workers

A more realistic scheme will be one in which only high-income (ρ) workers are burdened and only low-income $\bar{\delta}_i - \delta_i$ workers are supported. Scholarships for low-income workers are one possible type of support scheme. In this case, however, the motivation for ρ workers to accept this scheme may not be its economic benefits; rather, it may be because of their generosity or spirit of social service.

Suppose for simplicity that $\tau_i(\bar{\delta}_i - \delta_i)$ in $\bar{\delta}_i - \delta_i$ workers are low-income workers (the $\tau_i(\bar{\delta}_i - \delta_i)$ workers) where $0 < \tau_i < 1$. In addition, let

$$\delta_{\tau,i} = \tau_i(\bar{\delta}_i - \delta_i) + \delta_i \ ;$$

thereby,

$$\delta_{\tau,i} < \bar{\delta}_i \ , \tag{47}$$

and

$$\delta_{\tau,i} = \tau_i \bar{\delta}_i + (1 - \tau_i)\delta_i > \delta_i \ .$$

$\delta_{\tau,i}$ is the δ_i increased by the support from ρ workers. As inequality (47) indicates, $\delta_{\tau,i}$ never increases up to $\bar{\delta}_i$.

In this case, per capita burdens for the ρ high-income workers are

$$Ac_6 = \frac{\bar{c}\sum_{j=1}^{N}\tau_j(\bar{\delta}_j - \delta_j)}{\rho\sum_{j=1}^{N}\bar{\delta}_j} = \frac{\bar{c}\sum_{j=1}^{N}\tau_j(\bar{\delta}_j - \delta_j)}{\rho} \ . \tag{48}$$

On the other hand, benefits per unit of education capital enjoyed by each δ_i worker are reduced owing to the existence of the $\tau_i(\bar{\delta}_i - \delta_i)$ workers (or equivalently, they are increased if they support all $\tau_i(\bar{\delta}_i - \delta_i)$ workers) not by Rd_3 but by

$$Rd_4 = Rd_3 - \frac{\sum_{j=1}^{N} \langle \left\{ \bar{\delta}_j^3 - \left[\tau_j \bar{\delta}_j + (1 - \tau_j) \delta_j \right]^3 \right\} \frac{3r}{\delta_j \chi} \rangle}{N}$$
$$= \left\{ \sum_{j=1}^{N} \frac{\left[\tau_j \bar{\delta}_j + (1 - \tau_j) \delta_j \right]^3}{\delta_j} - \sum_{j=1}^{N} \delta_j^2 \right\} \frac{3r}{N\chi} \ . \qquad (49)$$

In equation (49), the term

$$\frac{\sum_{j=1}^{N} \langle \left\{ \bar{\delta}_j^3 - \left[\tau_j \bar{\delta}_j + (1 - \tau_j) \delta_j \right]^3 \right\} \frac{3r}{\delta_j \chi} \rangle}{N}$$

is the reduced benefits owing to the existence of the $\bar{\delta}_i - \delta_{\tau,i}$ workers. Because

$$\bar{\delta}_i - \delta_i = (1 - \tau_i)(\bar{\delta}_i - \delta_i) + \tau_i(\bar{\delta}_i - \delta_i)$$
$$= \bar{\delta}_i - \delta_{\tau,i} + \tau_i(\bar{\delta}_i - \delta_i) \ ,$$

and thereby,

$$\tau_i(\bar{\delta}_i - \delta_i) = (\bar{\delta}_i - \delta_i) - (\bar{\delta}_i - \delta_{\tau,i}) \ ,$$

Rd_4 indicates only a part of the reduced benefits resulting from all $\bar{\delta}_i - \delta_i$ workers, that is, the part resulting from $\tau_i(\bar{\delta}_i - \delta_i)$ workers.

If $Rd_4 > Ac_6$, ρ workers may choose to support $\tau_i(\bar{\delta}_i - \delta_i)$ workers. That is, by equations (48) and (49), if

$$\left\{ \sum_{j=1}^{N} \frac{\left[\tau_j \bar{\delta}_j + (1 - \tau_j) \delta_j \right]^3}{\delta_j} - \sum_{j=1}^{N} \delta_j^2 \right\} \frac{3r}{N\chi} - \frac{\bar{c} \sum_{j=1}^{N} \tau_j (\bar{\delta}_j - \delta_j)}{\rho} > 0 \ , \qquad (50)$$

then they may make that choice. Suppose for simplicity that $\tau_i = \bar{\tau}$ for any $i (\leq N)$, where $\bar{\tau}$ is a constant and $0 < \bar{\tau} < 1$. Hence, by inequality (50), if

$$\left\{ \sum_{j=1}^{N} \frac{\left[\bar{\tau} \bar{\delta} + (1 - \bar{\tau}) \psi \bar{\delta} \right]^3}{\psi \bar{\delta}} - \sum_{j=1}^{N} \psi^2 \bar{\delta}^2 \right\} \frac{3}{N\chi} - \frac{\sum_{j=1}^{N} \bar{\tau} (\bar{\delta} - \psi \bar{\delta})}{\rho \sum_{j=1}^{N} \psi \bar{\delta}} > 0 \ ,$$

and thereby if

$$3\rho \frac{\left[\bar{\tau}\bar{\delta} + (1-\bar{\tau})\psi\bar{\delta}\right]^3 - \psi^3\bar{\delta}^3}{\bar{\tau}(1-\psi)\bar{\delta}} > \chi \ , \tag{51}$$

they may choose to support the $\tau_i\left(\bar{\delta}_i - \delta_i\right)$ workers. In any case, however, a comparison between inequalities (45) and (50) or (51) indicates that it is unclear whether the hurdle in this case is lower than that of the case in Section 5.2.5.

However, in this case, even if

$$\left\{\sum_{j=1}^{N} \frac{\left[\tau_j\bar{\delta}_j + \left(1-\tau_j\right)\delta_j\right]^3}{\delta_j} - \sum_{j=1}^{N} \frac{\delta_j^3}{\delta_j}\right\} \frac{3}{N\chi} - \frac{\sum_{j=1}^{N} \tau_j\left(\bar{\delta}_j - \delta_j\right)}{\rho} < 0 \ ,$$

or if

$$3\rho \frac{\left[\bar{\tau}\bar{\delta} + (1-\bar{\tau})\psi\bar{\delta}\right]^3 - \psi^3\bar{\delta}^3}{\bar{\tau}(1-\psi)\bar{\delta}} < \chi \ ,$$

ρ workers may choose to support the $\tau_i\left(\bar{\delta}_i - \delta_i\right)$ workers as long as

$$S + \left\{\sum_{j=1}^{N} \frac{\left[\tau_j\bar{\delta}_j + \left(1-\tau_j\right)\delta_j\right]^3}{\delta_j} - \sum_{j=1}^{N} \frac{\delta_j^3}{\delta_j}\right\} \frac{3}{N\chi} - \frac{\sum_{j=1}^{N} \tau_j\left(\bar{\delta}_j - \delta_j\right)}{\rho} > 0$$

is satisfied for $S > 0$. S represents the degree of generosity or intention to improve social equity and may represent, for example, donations from high-income people. Clearly, as S increases, this scheme will be more strongly supported by ρ workers.

CONCLUDING REMARKS

The reason why education shows positive externalities is usually explained based on the concept of human capital. However, this concept is very elastic, and it is still unclear how human capital generates externalities. More broadly, the mechanism behind education externalities has not necessarily been sufficiently explained theoretically.

In this paper, I examined the mechanism of education externalities from a different point of view by introducing the concept of education capital based on an alternative production function presented by Harashima (2009, 2017) and the assumptions behind this production function. Education capital and human capital are different in that education capital works in the same manner as the other kinds of capital (e.g., physical capital), but human capital does not. In addition, although it is generally thought that human capital can increase constantly and indefinitely and the effects of externalities can increase infinitely, the effects of externalities generated by education capital have an upper bound. Therefore, although human capital can be a driving force of indefinite endogenous growth, education capital cannot.

I showed that education does not improve labor productivities. Rather, it accumulates what I call education capital. Education capital has the unique nature that, if workers who possess the same type of education capital transact business with each other, their performances equally increase. Based on this nature, I constructed a model of externalities in education capital and showed how education externalities are generated by education capital. Because of the division of labor, education capital cannot be accumulated infinitely (i.e., the effects of have an upper bound). Hence, education capital cannot be a driving force of indefinite endogenous economic growth.

The uncovered mechanism of education externalities in this paper can provide many valuable insights for educational institutions and policies. I showed that elementary schools should basically be compulsory, as it is in many countries. Whether education (not research) in universities should be subsidized by governments, however, will vary across countries and conditions and may depend on the degree of generosity of high-income people.

References

Acemoglu, Daron (1998) "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *The Quarterly Journal of Economics*, Vol. 113, No. 4, pp. 1055-1089.

Acemoglu, Daron and Joshua Angrist (2000) "How Large Are Human-Capital Externalities? Evidence from Compulsory Schooling Laws," *NBER Macroeconomics Annual*, Vol. 15, pp. 9-59.

Aghion, Philippe and Peter W. Howitt (1998) *Endogenous Growth Theory*, Cambridge, MA, MIT Press.

Alchian, Armen A. (1963) "Reliability of Progress Curves in Airframe Production," *Econometrica*, Vol. 31, No. 4, pp. 679–693.

Altonji, Joseph G. and David Card (1991) "The Effects of Immigration on the Labor Market Outcomes of Less-skilled Natives," in John M. Abowd and Richard B. Freeman, eds., *Immigration, Trade and the Labor Market*, University of Chicago Press, Chicago.

Becker, Gary S. (1962) "Investment in Human Capital: A Theoretical Analysis," *The Journal of Political Economy*, Vol. 70, No. 5, pp. 9-49.

Becker, Gary S. (1964) *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*, Columbia University Press, New York.

Bodvarsson, Örn B. and Hendrik Van den Berg (2009) *The Economics of Immigration: Theory and Policy*, Springer Verlag, Berlin.

Borjas, George J. (1994) "The Economics of Immigration," *Journal of Economic Literature*, Vol. 32, No. 4, pp. 1667-1717.

Borjas, George J. (1999) "The Economic Analysis of Immigration," in *Handbook of Labor Economics*, Edition 1, Vol. 3, No. 3. (Ashenfelter, O. and D. Card, ed.), Elsevier, Amsterdam.

Borjas, George J. (2003) "The Labor Demand Curve Is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market," *Quarterly Journal of Economics*, Vol. 118, No. 1135-74.

Breton, Theodore R. (2010) "Schooling and National Income: How Large Are the Externalities?" *Education Economics*, Vol. 18, pp. 67-92.

Card, David (2005) "Is the New Immigration Really So Bad?" *Economic Journal*, Vol. 115, pp.300-323.

Card, David (2009) "Immigration and Inequality," *American Economic Review*, Vol. 99, No. 2, pp. 1–21.

Cohen, Daniel and Marcelo Soto (2007) "Growth and Human Capital: Good Data, Good Results," *Journal of Economic Growth*, Vol. 12, pp. 51–76.

Dee, Thomas S. (2004) "Are There Civic Returns to Education?" *Journal of Public Economics*, Vol. 88, No. 9–10, pp. 1697-1720.
Friedberg, Rachel and Jennifer Hunt (1995) "The Impact of Immigration on Host County Wages, Employment and Growth," *Journal of Economic Perspectives*, Vol. 9, No. 2, pp. 23-44.
Grossman, Michael (2006) "Education and Nonmarket Outcomes," in *Handbook of the Economics of Education*, Vol. 1, pp. 577-633, Elsevier.
Groot, Wim and Henriëtte Maassen van den Brink (2007) "The Health Effects of Education," *Economics of Education Review*, Vol. 26, No. 2, pp. 186-200.
Harashima, Taiji (2009) "A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 15508.
Harashima, Taiji (2016) "A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element," in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 50, No. 1, pp. 55 –80.（「全要素生産性の理論と収斂仮説：根源的要素としての一般労働者のイノベーション」『金沢星稜大学論集』第 50 巻第 1 号 55～80 頁）
Harashima, Taiji (2017) "A Theory on the Economic Impacts of Immigration," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 78821.
Harashima, Taiji (2020a) "A Theory on the Economic Impacts of Immigration," in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 54, No. 1, pp. 43 – 70.（「移民の経済的影響に関する理論的考察」『金沢星稜大学論集』第 54 巻第 1 号 43～70 頁）
Harashima (2020b) "Economic Inequality Exacerbated by Economic Rents from Exploitative Contracting," *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 104178.
Hirsch, Werner Z. (1956) "Firm Progress Ratios," *Econometrica*, Vol. 24, No. 2, pp. 136-143.
Krueger, Alan B. and Mikael Lindahl (2001) "Education for Growth: Why and for Whom?" *Journal of Economic Literature*, Vol. 39, No. 4, pp. 1101-1136.
Lochner, Lance (2011) "Nonproduction Benefits of Education: Crime, Health, and Good Citizenship," in *Handbook of the Economics of Education*, Vol. 4, pp. 183-282, Elsevier.
Lucas, Robert E. Jr. (1988) "On the Mechanics of Economic Development," *Journal of Monetary Economics*, Vol. 22, No. 1, pp. 3-42.
Mincer, Jacob. (1958) "Investment in Human Capital and Personal Income Distribution," *The Journal of Political Economy*, Vol. 6, No. 4, pp. 281-302.
Moretti, Enrico (2004) "Estimating the Social Return to Higher Education: Evidence from

Longitudinal and Repeated Cross-sectional Data," *Journal of Econometrics*, Vol. 121, No. 1–2, pp. 175-212.

Münich, Daniel and George Psacharopoulos (2018) "Education Externalities: What They Are and What We Know," prepared for the European Commission, *European Expert Network on Economics of Education (EENEE)Analytical Report*, No. 34,

Oreopoulos, Philip and Kjell G. Salvanes (2011) "Priceless: The Nonpecuniary Benefits of Schooling," *Journal of Economic Perspectives*, Vol. 25, No. 1, pp. 159-184.

Ottaviano, G., and G. Peri (2012) "Rethinking the Effects of Immigration on Wages," *Journal of the European Economic Association*, Vo. 10, No. 1, pp. 152–197.

Pritchett, Lant (2001) " Where Has All the Education Gone?" *The World Bank Economic Review*, Vol. 15, No. 3, pp. 367-391.

Rapping, Leonard (1965) "Learning and World War II Production Functions," *The Review of Economic Statistics*, Vol. 47, No. 1, pp. 81–86.

Rauch, James E. (1993) "Productivity Gains from Geographic Concentration of Human Capital: Evidence from the Cities," *Journal of Urban Economics*, Vol. 34, No. 3, pp. 380-400.

Romer, Paul M. (1986) "Increasing Returns and Long-Run Growth," Journal of Political Economy, Vol. 94, No. 5, pp. 1002-1037.

Romer, Paul M. (1990a) "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, No. 5, Part 2, pp. S71-S102.

Romer, Paul M. (1990b) "Human Capital and Growth: Theory and Evidence," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 32, pp. 251-286.

Wright, Theodore Paul (1936) "Factors Affecting the Cost of Airplanes," *Journal of the Aeronautical Sciences*, Vol. 3, No. 4, pp. 122-128.