Stochastic Choice: Rational or Erroneous?

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Abstract

Likelihood functions have been the central pieces of statistical inference. For discrete choice data, conventional likelihood functions are specified by random utility (RU) models, such as logit and tremble, which generate choice stochasticity through an ”error”, or, equivalently, random preference.

For risky discrete choice, this paper explores an alternative method to construct the likelihood function: Rational Expectation Stochastic Choice (RESC). In line with Machina (1985), the subject optimally and deterministically chooses a stochastic choice function among all possible stochastic choice functions; the choice stochasticity can be explained by risk aversion and the relaxation of the reduction of compound lottery. The model maximizes a simple two-layer expectation that disentangles risk and randomization, in the similar spirit of Klibanoff et al. (2005) where ambiguity and risk are disentangled.

The model is applied to an experiment, where we do not commit to a particular stochastic choice function but let the data speak. In RESC, well-developed decision analysis methods to measure risk attitude toward objective probability can also be applied to measure the attitude toward the implied choice probability. Stochastic choice functions are structurally estimated to estimate the stochastic choice functions, and use standard discrimination test to compare the goodness of fit of RESC and different RUs. The RUs are Expected Utility+logit and other leading contenders for describing decision under risk. The results suggest the statistical superiority of RESC over ”error” rules. With weakly fewer parameters, RESC outperforms different benchmark RU models for 30% – 89% of subjects. RU models outperform RESC for 0% – 2% of subjects. Similar statistical superiority is replicated in a second set of experimental data.

Key Words Experiment; Likelihood Function; Maximum Likelihood Identification; Risk Aversion Parameter; Clarke Test; Discrimination of Stochastic Choice Functions

1 Introduction

In 1912, a college student, Fisher fitted the probability mass function of a discrete data set through his maximum likelihood procedure, trying to challenge the ”error theory” and OLS popular at that time. He might not expect that his model later becomes the dominating method in modeling discrete choice data. In the setting of discrete choice, both theorists and experimentalists in economics confirmed that, even if ”the circumstances of choice seem in all relevant respects to be the same”, an individual does not always choose deterministically in the same choice tasks[1], especially when the alternatives are complicated prospects (Debreu 1959 page 3). The stochasticity in choice data makes it difficult to explain the data with deterministic preference over the primitive. Marschak, Debreu, Luce and others assumed that an individual always chooses a probability mass function (a.k.a. stochastic choice function) of the discrete alternatives, even if the individual is forced to choose only one alternative.

It is, therefore, a standard practice since Marschak and Luce to model the choice data with a stochastic choice function; the value of the stochastic choice function equals the value of the likelihood function, which can be estimated through a maximum likelihood procedure and discriminated by Likelihood Ratio Test (LRT) or its generalizations. The recent applications of aforementioned procedure include but do not limit to medical decision (de Bekker-Grob, Ryan and Gerard 2012), policy evaluation, school choice (Chen and Sonmez 2006), online rating system (Harper et al. 2005), job choice, food choice (Allen and Rehbeck 2019a), location choice, urban planning, informetrics (Liu et al. 2018), and decision under risk and uncertainty (Conte, Hey and and Moffatt 2011; Georgalos 2018). It is broadly applied everywhere.

To derive the stochastic choice function, Valavanis-Vail (1957), Block and Marschak (1957), and Marschak (1959) suggest ”stochastic utility”, ”random ordering”, or ”random utility” (RU) based on Thurstone (1927). The RU model is equivalent to the so-call ”discrete choice” (DC) model[2], that uses a *deterministic preference functionals* to predict *deterministic choices* and an extra error component to capture the discrepancy between the predicted choices and actual choices: the agent evaluates the utility of alternative f as $u(f)+\epsilon(f)$. The error component $\epsilon(f)$ is thus attributed to the randomness of preference, where the decision-makers are assumed to have randomly variating tastes or make errors. Two typical examples of this class are probit (Thurstone 1927), which is a DC model with normally distributed i.i.d. error, and logit (Luce 1958), a DC model with i.i.d. extreme-value type II error. RU has been one of the most widely adopted econometric model.

RU is a natural explanation of choice variation. For example, if a subject chose an alternative for a portion of time, it seems natural to assume that she prefers this alternative only in this portion of time (Machina 1985). However, evidence in experimental

[1] Psychologists accepted ”stochastic choice” a bit earlier (Thurstone 1927); Davidson and Marshack (1957) tried to advocate the stochasticity of choice to economists, who have been assuming that the preference is deterministic and transitive.

[2] For example, see Straklescki (2019).

studies suggest that fluctuating taste is not the sole source of stochasticity: when asked to repeatedly choose from the same menu of alternatives, subjects usually make different choices, even if the repetitive tasks are presented simultaneously[3]. Besides the taste shocks, RU and DC might originate from noise or "error-making" (tremble). In addition to the error term, RU and DC assume a deterministic ranking among the discrete alternatives in the menu. However, both recent theoretical and experimental works reaffirmed that in some decision tasks, there could be no alternative that is "clearly" better than the others (for example Fudenberg et al. 2015 and Agranov and Ortoleva 2017). Furthermore, in an experimental setting, the subjects' choices are usually inconsistent with the prediction of the deterministic utility for more than twenty percent of times (for example Marschak 1959). It makes sense to attribute the small discrepancy to the error-making, but does the consistently high "error rate" solely due to error?

In an RU, choosing the error distribution is important: any deterministic utility plus error can be represented with any other arbitrary deterministic utility plus some other error distribution[4]. A straightforward implication is that a utility function with a risk aversion parameter of 0.1 plus error is equivalent to the same utility function with a risk aversion parameter of 100 plus another error distribution. Thurstone himself admitted the arbitrariness in picking error distribution. This is unsatisfactory (Cosslett 1983; Manski 1975; Chiong and Shum 2018) because arbitrarily choosing the error distribution will lead to the misspecification of the choice probability model. Additionally, the analysts usually have no *a priori* knowledge about the distribution (Cosslet 1983; Chiong and Shum 2018). So one might ask: what distributions fit the best and why? Does "explaining" choice randomness with preference randomness really explains the ultimate source of randomness (Machina 1985)[5]?

In contrast to the fundamental assumptions of RU, most economists use a deterministic and transitive preference in their models despite of the stochasticity in the data.

[3] Tversky (1969), Camerer (1989), Starmer and Sugden (1989), Hey and Orme (1994), Carbone and Hey (1995), Ballinger and Wilcox (1997), Hey (2001), Dwenger, Kübler and Weizsäcker (2018), Agranov and Ortoleva (2017).

[4] This is a simple corollary to the fact that discrete choice rule is equivalent to a random utility model. For any utility functions u, v and error distribution ϵ, there exists an error distribution ϵ' such that $u + \epsilon = v + \epsilon'$. However, for any error distributions ϵ, ϵ', and any function u, there does not necessarily exist a utility function v such that the equity holds (for example, see Strzalecki 2019).

[5] RU also limits the dependence of error distribution on the menu, history, or choice frequency (Machina 1985; Brady and Rehbeck 2016; Manzini and Mariotti 2014). Other important limits of RU can be found, for example, in Bajari and Benkard (2003) and Echenique and Saito (2018).

This dilemma was resolved by Machina (1985), who first suggested an alternative approach, Stochastic Choice Generated by Deterministic Preference (SCGDP), in which the stochasticity originates from stable preference rather than unstable (i.e. random) preference over the lottery. Assume the choice functional assign the respective probabilities (ρ_f, ρ_g) to the alternatives $\{f, g\}$, then the individual must weakly prefer the generated lottery (with $\{f, g\}$ as outcomes and (ρ_f, ρ_g) as probability) over any other generated lotteries (stochastic choices) or deterministic choices. This approach is more satisfying than the random utility rules from a theoretical perspective (Carbone and Hey 1995; Cerreia-Vioglio et al 2019). The first experimental evidence could be Sopher and Narramore (2000).

There are several influential stories for SCGDP. Chew et al. (1991) propose a stochastic choice model generated by deterministic preference relaxing betweenness. It allows the discrimination of randomizing risk and objective risk. Fudenberg et al. (2015) first axiomatize the stochastic choice generated by additive perturbed utility (APU) using acyclicity; the stochasticity originates from a cost function. Another source could be regret-minimizing, which is first suggested by Dwenger, Kübler and Weizsäcker (2018) using field data of school choice. Cerreia-Vioglio et al (2019) propose "deliberately" stochastic choice, the SCGDP defining on lottery primitives and assuming the reduction of compound lottery. In general, the stochastic choice can originate from the deterministic *preferences for convexity* (Machina 1985). As an inspirational example (Machina 1989), suppose a mum is to give an indivisible treat to one of her two children; in this case, the mum prefers hedging, or "balancing", in the sense that the mum prefers to give some expected reward to both of the two "attributes" - her two children. Therefore, she strictly prefers a coin-flip over the decisive allocation to one child[6]. Several studies have used randomization devices to elicit similar "mental coins"[7].

Based on these pieces of evidence, this paper proposes an easily estimatable model, namely Rational Expectation Stochastic Choice (RESC) model for decision under risk. RESC is a parameterizable special case of SCGDP. The stochastic choice function is

[6]This relaxes consequentialism. We confine our modest effort within the scope of individual decision making under risk, in-line with Machina (1989).

[7]Carbone and Hey (1995) was the first to discriminate SCGDP and RU. Sopher and Narramore (2000) first suggest the consistency of mixing across permutations as the empirical evidence favoring SCGDP. Dwenger, Kübler and Weizsäcker (2018) find extensive evidence in favor of SCGDP in both the field and the lab; Agranov and Ortoleva (2017) find that subjects would switch their choice when the same task is repeated and even pay for a physical randomizing device to randomize their choice.

distribution-free and the decision rule is "rational"[8]: the stochasticity in choices arises from maximizing a well-defined preference functional - a simple expectation. RESC postulates that an agent chooses probabilities to randomize her choice for lotteries. This results in a new (compound) lottery for which its payoff in each state is the expected payoff of those lotteries in that state, calculated based on the chosen probability for randomization. The agent's objective is to maximize the expected utility of the resulting lottery. We discuss this potential thinking process of the agent in Section 2.1. In short, the stochasticity in RESC arises from the hedging of negatively correlated acts and balancing the expected reward in each state:

$$\rho^* = \arg\max_{\rho} \mathbb{E}_{\mathbb{P}} u(\rho_f f + \rho_g g),$$

Where $\mathbb{P}$ is a probability measure of the states of nature. f and g are state-dependent lotteries that match each state of nature to a payoff.

A more general formulation allows the discrimination of the risk attitude towards objective probability and the attitude toward implied choice "probability" (**Appendix D**). Many seminal studies suggest that the relaxation of the reduction of compound lottery is both normatively and empirically appealing (For example, Segal 1990). In our analysis, there are two sources of uncertainty: the objective probability exogenously given by the experimenter, and the random choice "probability" endogenously "given" by the subject herself (flip a mental coin) or the nature (flip a physical coin). The choice "probability" needs not to be an exact probability in the sense of Chipman (1963). It is well-known that the attitudes towards risk and uncertainty is associated with pessimism, source, and trust[9]. So it is not too unnatural to hypothesize that the subject is pessimistic when facing the probability set by someone else, but not so pessimistic when dealing with the random choice probability "set" by the subject herself or the nature, because she trusts herself or the nature more than trusts others. In game theory, for instance, mixed strategies are based on the expected value of the payoff calculated based on the assigned probabilities: the agents are assumed to be risk-neutral when

[8] It is said to be "rational" in three senses. First, in the sense of classical economics, the preference is rational then it must be transitive and complete; a choice function c is rational if it can be written as $c \in \arg\max_c U(c)$ where U is a deterministic utility function (for example, see Mascolell et al. 1995; Firsch 1927). Second, the maximization of expectation is considered the "rational behavior" (von Neumann and Morgenstern 1944) in decision analysis, game theory, artificial intelligence, finance, cognitive science and behavioral economics (Tversky and Kahneman 1989). Third, the choice rule cannot be explained by "error".

[9] For example, see Abdellaoui et al (2011); Dimmock et al (2016). Bohnet et al (2008) find that people are more risk averse when dealing with other people than when dealing with nature.

evaluating the random choice ”probability”. Indeed, using the second-order model, the empirical finding in **Appendix D** suggests that most subjects are risk-averse if the probability is exogenous, but they are not risk-averse if it is endogenous. Suppose that the attitudes are the same, then the reduction axiom still holds, the indifference curves in the Marschak-Machina triangle become linear and RESC degenerates to the deterministic EU (Machina 1985). This might give a modest insight into the quest for the source of choice stochasticity.

RESC have several advantages, for example, the source of data stochasticity has an explanation, the analysts do not need to worry about choosing error distributions, traditional functional forms for EU can still be used, and the model can be estimated with the similar methods.

The purpose of constructing a likelihood function is to fit it with the data. This paper uses two sets of binary choice data to investigate the empirical performance of RESC and RUs through a maximum-likelihood procedure and discrimination tests. The experimental task (section 3) is completing several problems where each problem asks one to choose one lottery from a menu of two lotteries. The outcomes of the lotteries are monetary prizes and depend on the realization of equally-likely-occurring states. The subject is presented with a pair of alternatives and asked to choose one; she is assumed to choose the first alternative (f) with probability ρ_f and the other one with probability $\rho_g = 1 - \rho_f$ (Markschak 1957; Debreu 1959). One might still be in doubt that why do we use the stochastic choice function to model the deterministic choice data. In fact, this has long been the standard: although stochasticity was not introduced in the objects of choice, the subject can ”introduce it in the act of choice” (Debreu 1959)[10].

At individual choices level, we use *Maximum Likelihood* procedures to estimate RESC and RUs. The deterministic component for RU we have considered are Expected Utility(EU), Quiggin's (1982) Rank Dependent Expected Utility (RDEU), Regret Theory (RT)[11] and Salience Theory under Risk (ST) (Bordalo et al 2012).

Among known generalizations, RDEU and EU (both with RU) have long been the ”leading” benchmark RUs in measuring risk, not only because of their mathematical tractability, but also because of their statistical superiority and empirical robustness[12] (Conte, Hey and and Moffatt 2011). Despite shortfalls in explaining some named para-

[10]See also Markschak 1957. Harless and Cameron 1994 also argue that randomness occurs during the act of choice

[11]Bell (1980, 1982, 1983, 1985). Loomes and Sugden (1982)

[12]As Machina suggested, this is possibly due to many non-EUs relaxing independence are in fact locally EU

doxes, EU proves its statistical superiority over most of its generalization through standard discrimination tests (for example, see Hey and Orme, 1994).

The payoffs of the lotteries are state-dependent. Thus non-*Probability Sophistication (PS)*[13] models are also considered. PS models include EU, Subjective EU, Choquet EU, Prospect Theory, and RDEU. There are two notable non-PS models under risk: Regret Theory and Salience Theory[14]. Non-PS preference can be normative even if the lotteries are the vNM simple lotteries without correlation specified (Epstein and Halevy 2019).

Since Marschak and Luce, it has been the standard to structurally estimate the stochastic choice function by maximum likelihood procedure and compare the models by discrimination tests[15]. To our surprise, only one work has implemented this process to compare a non-RU SCGDP model with RU. Carbone and Hey (1995) might be the first to structurally estimate a non-RU SCGDP model – Chew *et al* (1991)'s Quadratic Utility – but it fits well for only one out of forty-four subjects. To our knowledge, all other important experimental works on differentiating SCGDP and RU did not implement likelihood discrimination; they were unable to quantitatively conclude on the goodness of fit of a stochastic choice function. Utilizing maximum likelihood estimation and sophisticated discrimination process, Section 4 manages to differentiate SCGDP and RU and suggests an alternative method of measuring risk under the classical setting of discrete choices. Section 5 repeats the analysis using data from another experiment.

2 The Model and Method

We layout the choice problem in a simple three-equally-likely states case. The model can be extended to the problem of different states with differing probabilities with trivial effort. A problem is to choose an alternative from a menu of two lotteries $\{f, g\}$. We use the notation f to indicate a typical alternative on the menu. The prizes (monetary payoffs) of the lotteries depend on three states of the world $s \in S = \{s_1, s_2, s_3\}$ eventuates.

Each lottery is a probability distribution that generates from the mapping from S to a prize space $X \subset R$, that is $f(s_i) = f_i, s_i \in S$ and $f_i \in X$. In the experimental setting of this paper, the states are equally likely to occur, therefore $p(s_1) = p(s_2) = p(s_3) = \frac{1}{3}$. Table 1

[13]PS means the preferences over lotteries depend exclusively on the independent probability distributions of the outcomes.

[14]Refer to **Appendix C** for details.

[15]Such as LRT for nested models and Vuong/Clarke test for non-nested models

illustrates the problem.

Table 1: Menu 1

	$s_1(p_1 = \frac{1}{3})$	$s_2(p_2 = \frac{1}{3})$	$s_3\ (p_3 = \frac{1}{3})$
f	f_1	f_2	f_3
g	g_1	g_2	g_3

2.1 Rational Expectation Stochastic Choice (RESC)

We emphasize that the optimizing object of RESC is a probability vector specifying how to randomize between two lotteries. The purpose of this *intentional* randomization[16] is to obtain a two-stage lottery that gives a higher expectation of utility than the two individual lotteries. The key idea of RESC is the *preference for convexity* (Marchina 1985): a probabilistic (hence convex) combination of f and g is preferred to the two individual alternatives [17].

Our model implies that the decision makers' evaluation of a simple lottery with objective probability can be different from the evaluation of a simple lottery with randomizing probability; this is supported by recent evidence that the subjects evaluate an objective coin differently in comparing to a mental coin (Agranov and Ortoleva 2017).

We now discuss the potential thinking process underpinning RESC. Figure 1(a)[18] illustrates what we usually assume for an agent ascribes to make the discrete choice. The agent is determined to choose either f and g, doing so she would end up with a lottery with three possible outcomes.

In RU, the observed stochasticity in choices is captured by error. However, classical RU might not be able to explain the subject when she relaxes consequentialism: the utility of choosing f might depend on the stochastic choice probability she has assigned to g (Machina 1989).

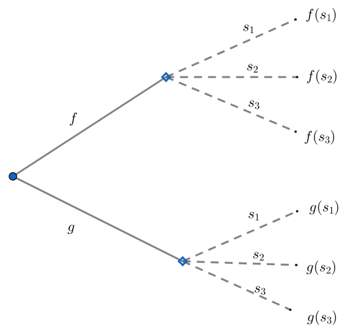
The alternative evaluation process can also be demonstrated in Figure 1(b), the agent

[16] The subject could be either conscious or subconscious.

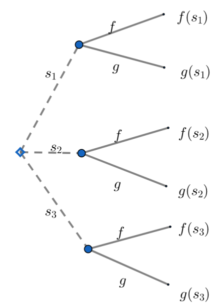
[17] The first experimental evidence can be found in Sopher and Narramore (2000). The experimental task here explicitly allows the subject to choose a convex combination over two lotteries. The authors find that preference for mixtures is common.

[18] We borrow these figures from Ke and Zhang (2019), where they use them for preferences over Anscombe-Aumann acts.

Figure 1: Two Ways of Perceiving Pairwise State-dependent Lottery Choice Problem



(a) Classical interpretation: The agent moves first to deterministically choose one lottery, then natures moves to decide the realization of the state. There is only one one-stage risk that represented by the tree branches originates from either f or g.



(b) RESC: Nature decides the state first, then the agent needs to consider how to randomize conditioned the occurrence of each state. The branch originates from the states represents *randomization* risk, which is determined by the agent herself.

would *start* from the point view of the states [19]. Even if the problem specifies the objective time as in Figure 1(a), the subjective timing and objective timing does not have to be the same (Ke and Zhang, 2019). The agent first conditions on the occurrence of a state, and then thinks of what is resulted from her action. For example, if state 1 were to happen, by randomization between f and g, the agent gets an expected value of $\rho_f f(s_1) + \rho_g g(s_1)$[20].

The agent goes through all three states and looks for an optimal ρ^* to obtain a new lottery of three outcomes that may produce a more desirable distribution of outcomes than f and g individually.

Table 2 provides an extreme example to show the potential advantage of randomiza-

[19]This idea was introduced by Saito (2015) to explain the preferences of convexity. Their primitives are Anscombe-Aumann acts.

[20]We also estimated the general version of RESC by replacing this expected value with EU so the risk attitude toward randomizing probability can be separately estimated. The subjects can be viewed as second-order expected utility maximizers (Segal 1990). Doing so would involve introducing an extra parameter in the utility function. We found this generalization extra statistical improvement for only 11 subjects. Most subjects are not risk-averse when dealing with the stochastic choice probability instead of facing the objective probability (**Appendix D**)

Table 2: An Example of Convex Combination

	$s_1(p_1 = \frac{1}{3})$	$s_2(p_2 = \frac{1}{3})$	$s_3\ (p_3 = \frac{1}{3})$
f	10	4	1
g	2	8	11
$k = 0.5f + 0.5g$	6	6	6

The last row shows the resulting lottery k if an agent chooses to randomize f and g by probability $(0.5, 0.5)$. The outcome of k in each state is the expected value (in terms of $(0.5, 0.5)$) of two outcomes of f and g in that state.

tion where an agent obtains a lottery with reduced risk generated from objective probability, if she ascribes to the thought process as indicated in Figure 1(a).

We now write RESC formally as follows:

$$(\rho_f^*, \rho_g^*) = \underset{\rho}{\operatorname{argmax}} \sum_{1}^{3} p(s_i)u(\rho_f f(s_i) + \rho_g g(s_i)), \tag{1}$$

where

$$\begin{cases} \rho_f + \rho_g = 1 \\ 0 \le \rho_f \le 1 \\ 0 \le \rho_g \le 1 \end{cases}$$

and $u(\cdot)$ is a utility function such as Constant Absolute Risk Aversion (CARA) or Constant Relative Risk Aversion (CRRA). These are the standard assumptions in the MLE of risk aversion parameter. **Appendix A** gives the mathematical derivation of how to calculate the optima in equation (1).

We argue that it is the innate intention of randomization makes the observed choice data appear to be stochastic. Note equation (1) gives a very neat way of constructing the likelihood function for the estimation. We assume the following CARA utility function form [21]

$$u = -\frac{exp(-rx)}{r}. \tag{2}$$

Suppose for problem n the actual choice $c_n = f(g)$, then the likelihood of observing this choice is assigned as $\rho_f(\rho_g)$. For N independent number of problems, the sum of log-

[21] We do not use the alternative CRRA utility function in this paper because the experimental parameters involve evaluating utility at 0, which can be difficult to deal with in power utility functions (Wakker 2008).

likelihood is

$$\mathcal{L}(r) = \sum_{i}^{N} log((\rho_f^*, \rho_g^*) \circ I) \tag{3}$$

where I is an identification vector, that is

$$I = \begin{cases} [1\ 0] & if \quad c_n = f \\ [0\ 1] & if \quad c_n = g, \end{cases}$$

and $\circ$ denotes operation for Hadmard product.

The advantage of RESC is that it deals with preferences and stochasticity simultaneously. RESC allows a more accurate way of predicting people's choices in the case when choices have to be made deterministically in a one-shot fashion, which is the nature of many real-life choices.

2.2 Model of Deterministic Choice plus Error

Under RU model, the utility of an alternative $\tilde{U}(f)$ consists of the evaluation of an alternative $v(f)$ that is determined by a preference functional and an error part $\tilde{\epsilon}(f)$:

$$\tilde{U}_\omega(f) = v(f) + \tilde{\epsilon}(f) \tag{4}$$

The error ϵ could possibly depend on the whole menu but it is assumed to be only dependent on f in the mainstream theoretical and empirical literature of random utility or discrete choice. Because it is well-known that the menu-dependent error model trivially explains everything. For empirical analysis, we choose the most popular and standard RU model, Logit (Luce 1958), which assumes ϵ to be *i.i.d.* extreme value. The Logit choice probability is as follows[22].

$$\rho_f = \frac{e^{v(f)}}{e^{v(f)} + e^{v(g)}}. \tag{5}$$

[22]Note this equation implies the variance of the ϵ is $\frac{\pi^2}{6}$. In general one can add an extra scale parameter σ such that the variance becomes $\sigma^2 \frac{\pi^2}{6}$. Then equation (5) becomes

$$\rho_f = \frac{e^{v(f)/\sigma}}{e^{v(f)/\sigma} + e^{v(g)/\sigma}}.$$

If $v(\cdot)$ takes a linear form then scale parameter does not affect the ratio of the parameters in $v(\cdot)$ (Chapter 3, Train 2009).

The interpretation is the likelihood of choosing an alternative is proportional to its evaluation compared to the sum of the two evaluations and the error is decreasing with the advantage in evaluation.

The sum log-likelihood for a dataset of N problems is

$$\mathcal{L}(\cdot) = \sum_{i}^{N} log((\rho_f, 1 - \rho_f) \circ I) \tag{6}$$

where the arguments of $\mathcal{L}$ are the parameters in the preference functionals. For example, in EU, the argument can be the risk aversion parameter in a CARA utility function.

We consider the following most relevant deterministic theories for our problem: Expected Utility (EU), Rank Dependent Expected Utility (Quiggin 1982), hereafter RDEU, and two non-PS models: Salience Theory under Risk (ST) and Regret Theory (RT). For all these theories we assume CARA. The preference functional $v(\cdot)$ for these deterministic theories are defined in **Appendix C**.

2.3 Comparison of RESC and RU: An Illustration

In this subsection, we point out the major difference between RU models and RESC.

Consider the choice task 1 and 2 with different menus: $\{f, g\}$ and $\{f, h\}$. There are two states of nature, s_1 and s_2. f pays zero dollar in s_1 and one dollar in s_2. g pays zero dollar in s_1 and 1.01 dollars in s_2. h pays 1.01 dollars in s_1 and zero dollar in s_2. The probability of s_1 and s_2 are equal. The setting is summarized in the following Table 3.

The choice probability predicted by i.i.d. RU (EU+logit) and Random Expected Utility (Gul and Pesendorfer 2006) is obvious. The error in an RU model only depends on the specific alternative. Therefore the choice probability in Tasks 1 and 2 would not change in an RU model. It is well known that if the error depends on the menu, then the RU model just trivially explains everything.

In reality, many subjects would not regard Tasks 1 and 2 as the same task. RESC is one approach to rectifies this limit of the RU model.

The prediction of RESC in Task 1 and Task 2' is obvious. In Task 2', the subject chooses to equally split her choice probability not because of ”error”, but because of a desire of balancing the expected payoff in two ”attributes”: the two states[23]. If we further assume ”continuity” which is assumed in most of the mainstream stochastic choice

[23] This is suggested in the spirit of Machina (1989)'s mum example of normative decision under risk.

Table 3: Choice Probability under Different Theories

				Choice Probability		
				RU (iid)	REU	RESC
		s_1	s_2			
Task 1	Lottery f	0	1	~40%	0%	0%
	Lottery g	0	1.01	~60%	100%	100%
Task 2	Lottery f	0	1	~40%	0%	~40%
	Lottery h	1.01	0	~60%	100%	~60%
Task 2'	Lottery f	0	1	50%	50%	50%
	Lottery h'	1	0	50%	50%	50%

theories, then the stochastic choice ρ is a continuous function (or correspondence). In task 2, intuitively, where h' is altered a little bit to become h, the choice probability should also be only altered a little bit. Therefore, in task 2, the subject would still assign some probability to lottery f, otherwise continuity is violated. The experimental setting is similar to but more complicated than this example.

3 Experiment and Data

The data used in this study was obtained in a research university's decision lab from 89 subjects drawn from both graduate and undergraduate students. There were 28 pair-wise choice tasks over two state-dependent lotteries as in Table 1. **Appendix B** presents a screenshot of the sample choice task and the list of all tasks. The subjects were told at the beginning that the computer would randomly draw a choice task (Random-Lottery Incentive System) and draw a state of that choice task to decide their payments after they finish all tasks. The sequence of the tasks was randomized. Each subject's choice of the lotteries (lottery 1 or lottery 2) in each task was recorded. Therefore, the dataset is a matrix of size 28 by 89 with the element of the binary number 1 or 2. Average reward is ~ 13 dollars. Completing time was about 30 min. Four sessions were run. A subject can only participate in one session. No data were excluded from the analysis.

All subjects signed relevant consent forms and all experimenters took relevant training. The experiment was approved by the ethical approval committee.

4 Estimation Strategy and Discrimination Results

This paper uses *Maximum Likelihood* procedures to estimate the parameters in stochastic choice functions (RESC and RU) at an individual subject level[24]. The sum of log-likelihood for RESC and RU models are defined in equation (3) and equation (6) respectively. In terms of RU models, we stick with the standard logit form (5) in Luce's original work[25].

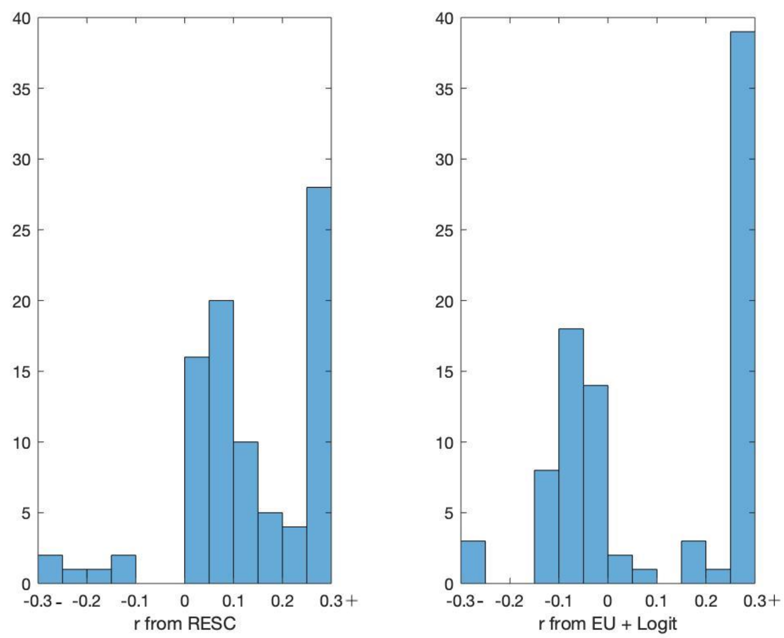
Figure 2 shows the histogram of the estimated risk aversion parameter from RESC and EU. In the estimation we constraint r between -0.3 and 0.3. Values outside these bounds imply extremely risk-seeking or risk aversion and they are unidentifiable given the experimental parameters. To set a constraint for r is standard in structural estimation literature [26]

[24] There are no analytical solutions for the optimizing of the choice problem, therefore there are no explicit solutions to maximize the sum of log-likelihood functions either. We write the algorithms for optimization in MatLab.

[25] The main result remains similar when using other RUs such as more general forms of logit, probit, and tremble.

[26] For example, see Georgalos (2019). Some intuition of these numbers: under $r = -0.3$, the Certainty Equivalent of the lottery $(22, 0.5; 0, 0.5)$ is 21; under $r = 0.3$ it is 2. A detailed description of the intuition of the boundaries of risk aversion parameters and the unidentifiability beyond the bound can be found in Holt and Laury (2002). The main result remains robust when the boundary is shifted.

Figure 2: Estimated Risk Aversion Parameter r



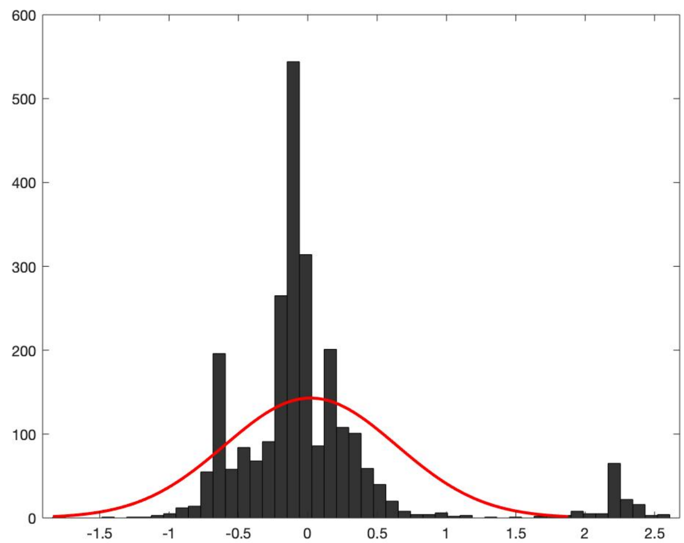
y axis is the number of subjects. x axis is the ML estimated risk aversion parameter r as in equation (2). "0.3+" means 0.3 or higher. "-0.3-" means -0.3 or lower. A higher r indicates higher risk aversion. Values less (more) than -0.3 (0.3) suggest extreme risk seeking (aversion) and are unidentifiable in the dataset. Consistent with previous experimental studies, the majority of the subjects are risk averse. On average, r is higher in RESC than in EU + Logit.

We can see under RESC risk aversion is higher than EU. This is probably due to the choice of a riskier lottery is considered to be hedging in RESC but is considered to be an error in EU.

RESC does not nest any of the RU models; therefore it is standard to use the Clarke test (2003) to compare the goodness of fit of two non-nested competing likelihood functions (Imai and Tingley 2012). Vuong (1989) is the other canonical test to compare non-nested models. However, Clarke (2003, 2007) demonstrate the former test is more suitable with small sample size and likelihood ratios exhibiting sharp peaks than a normal distribution, which is our case as shown in Figure 3.

In recent literature, Clarke test is generally considered appropriate and sufficient to discriminate stochastic choice rules, especially under risk and uncertainty (Among many others, see Hey and Panaccione 2011; Panagiotelis and Czado and Joe 2012; Hey and Pace 2014; Wilcox 2015); no more robustness check is usually performed.

Figure 3: Histogram of the Log of the Likelihood Ratios



x axis the individual likelihood ratio of EU+logit to RESC under each problem. It is calculated based on the estimated parameter from MLE. y axis is the number of problems. All subjects' data in all problems were pooled to get this histogram. The red line was the fitted normal distribution curve. It is clear the distribution has a higher peak than the normal distribution.

In the Clarke test, the null hypothesis is that the two competing models are equally good, hence on a particular problem the probability of the log-likelihood for one model being larger than the probability of the other model is $\frac{1}{2}$.

Let L_1 and L_2 denote the individual log-likelihoods of the 28 problems, which are calculated using the estimated parameters. When the two models have the same number of parameter, the test statistic is

$$T = \sum_{1}^{28} I(L_2 - L_1)$$

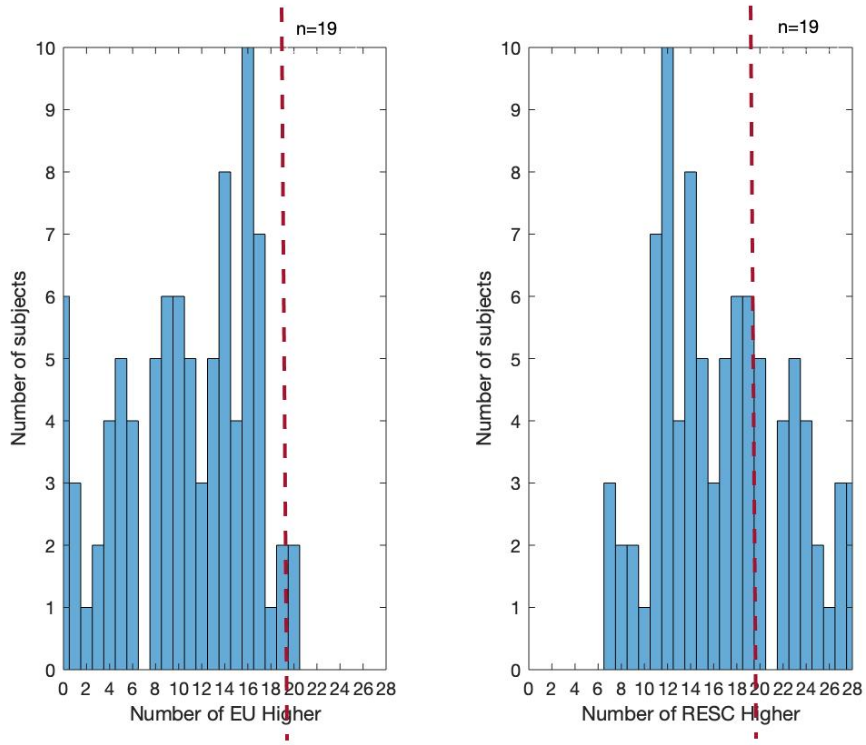
where

$$I(L_2 - L_1) = \begin{cases} 1 \quad if \quad L_2 > L_1 \\ 0 \quad if \quad L_2 \leq L_1 \end{cases}$$

Under the null hypothesis T has a binomial distribution with parameters $n = 28$

and $p = 0.5$. Thus an observation greater than 19 rejects the null hypothesis at the 5% significance level.

Figure 4: Clarke Test Between EU and RESC



On the left panel, x axis is the number of problems that EU+logit has a higher likelihood than RESC for each subject. If the number was more than 19, then we rejected the null that the two models perform the same at 5% and concluded that EU+Logit was better for this subject. There was only 2 out of 89 subjects that EU+logit was significantly better. The right panel reversed the comparison by counting the numbers that RESC has a higher likelihood than the EU. It shows 27/89 subjects' data fitted better in RESC.

Both RSEC and EU have one parameter to estimate, therefore individual likelihoods can be directly compared. In general, the individual likelihood of models with more parameters is penalized by $log(N)/N$, where N is the number of the extra parameters. Therefore, when we compare RESC with ST, the individual likelihoods of ST is subtracted by $log(28)/28$ since ST has one more parameter. The results of all comparisons between models using the Clarke test are summarized in the following Table 4. It shows the advantage of RESC, compared to both PS theories (EU and RDEU) and non-PS theories (ST and RT).

Table 4: Clarke Test Results

(a) RESC *vs* EU+error

EU better	RESC better	neither better
2%	30%	68%

(b) RESC *vs* RDEU+error

RDEU better	RESC better	neither better
0%	89%	11%

(c) RESC *vs* ST+error

ST better	RESC better	neither better
0	55%	45%

(d) RESC *vs* RT+error

RT better	RESC better	neither better
0%	57%	43%

5 Replication

As a robustness check, we replicate the analysis using another set of data first analyzed in Agranov and Ortoleva (2017). This experiment consisted of four parts of different settings. We use the data from the part that the experimental task was making a pairwise state-dependent lottery choice. The randomization device was the same. There were four possible states for the lottery and subjects were told the realization of the state would be determined by a computerized four-sided dice. There were 7 choice problems and each problem was repeated three times in a row, which the subjects were informed. The figure in **Appendix B** shows the 7 problems. Random lottery incentive system was also adopted. The subjects were 80 university students. Therefore the dataset is a matrix of row 21 by 80 with binary number 1 or 2.

The original analysis follows a simple descriptive fashion: if a subject did not switch preference in all repetitions, then it is concluded that the individual makes deterministic choice. If the subject did switch, then it is concluded that the individual makes stochastic choice. However, since deterministic choice is a special case of stochastic choice, even if the subject makes deterministic choice in all three repetitions, we cannot make sure that the subject will always choose deterministically if the task is changed or more repetition is administrated. This paper discriminates the deterministic plus error model and SCGDP using sophisticated discrimination tests.

We fit the data into the same RESC and EU+logit as defined in Section 2. For both, the same CARA utility function, as in equation (2), is adopted. The same standard Clarke test was adopted: since there are 21 problems, a model giving more than 14 higher likelihood fits better than the other at a significance of 5% level. Figure 5 indicates that

RESC strikingly outperforms EU+logit. The null hypotheses that RESC is not superior than EU+logit are rejected for **all** subjects. The null hypotheses that EU+logit is not better than RESC cannot be rejected for any of the subjects. This is consistent with the message from the original paper - a large majority of subjects exhibit ”stochastic choice” and this choice inconsistency in repetitions is difficult to be explained by deterministic preference theories and RUs.

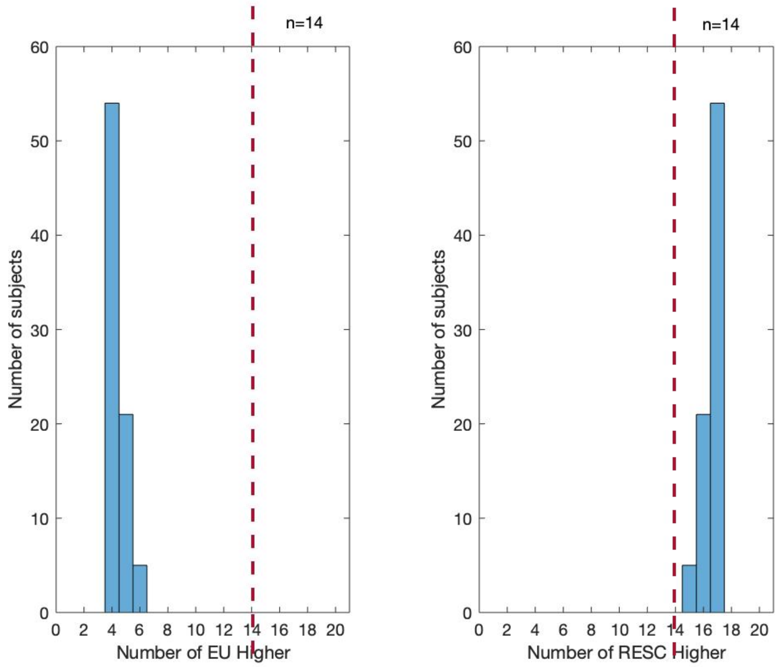
Many have been questioning the original 2017 experimental results. For example, some of them believe the subjects are *”framed”* to randomize, because of the repetitive setting of the experiment.

However, it is worth noting that, even if the subject does not seem to ”choose stochastically” in the repetitions[27], her preference is sometimes still incompatible with deterministic EU or EU+error. The reasoning is as follows. A subject’s choice over two different lotteries gives us information about her risk attitudes under EU (curvature of a utility function). If we fix a particular form of the utility function, then we can use that choice to calculate a one-sided range of risk parameter in the utility function[28]. For example, suppose a subject consistently chooses one lottery in one set of the three, which implies risk-averse. Meanwhile, the same subject’s consistent choices in another set can indicate risk-seeking. One can always trivially argue this is due to ”error”, but it is difficult to imagine a subject would make the same ”error” three times in a row. We have identified 25 subjects who ”did not make stochastic choices” based on simple descriptive statistics; however, among them, there are 10 subjects are of the type of that cannot be explained by EU well while RESC predicts their choice much more nicely. The other subjects might be explained by EU, but RESC still fits significantly better.

[27] i.e. they always choose the same object in repetition. This is determined by using simple descriptive statistics.

[28] For example, consider the choice task of Hard3 as the figure in **Appendix B**. If a subject had chosen the lottery in the second row, then we know her risk aversion $r > 0.0042$ as in the CARA utility function (2).

Figure 5: Clarke Test Between EU+error and RESC



The x-axis is the total number that one model has a higher likelihood than the other among one subject's all 21 problems. By Clarke test of non-nested models, a number of more than 14 indicates the former model fits better at 5% confidence interval. There were 80 subjects in total. RESC fits significantly better than EU+error for all subjects. EU+error fits significantly better than EU+error for no subjects.

6 Discussion

The results seem surprising are not surprising because it is known that explaining randomness in data with randomness of preference still does not explain the source of randomness (Machina 1985). Estimating utility function from binary choice data can often be not very successful due to the randomness. When the utility function suggests the subject prefers one alternative to the other, it might make sense to attribute the small discrepancy in choice probability to errors. However, suppose one subject consistently chooses an alternative for 55% of the time, can we claim that she strictly prefers one alternative to the other? When the deterministic utility function suggests she should be choosing it for sure, can we claim this choice probability of 55% instead of 100% is due to error? How could we refute the hypothesis that she chooses the randomizing probability by an instinctive optimization process? Of course, future theoretical investigation and empirical evidence is needed to thoroughly understand the rationale behind.

Similar evidence can be found in the psychology and evolutionary dynamics literature. For example, the well-documented phenomena of ”probability weighting” is widely observed in human and animals.

	$s_1(p_1 = \frac{1}{3})$	$s_2(p_2 = \frac{1}{3})$	$s_3\ (p_3 = \frac{1}{3})$
f	10	4	1
g	2	8	11
$k = 0.5f + 0.5g$	6	6	6

The last row shows the resulting lottery k if an agent chooses to randomize f and g by probability $(0.5, 0.5)$. The outcome of k in each state is the expected value (in terms of $(0.5, 0.5)$) of two outcomes of f and g in that state.

Expected utility maximizer will always not to perform probability weighting.

The theory can be applied in building more efficient machine intelligence (AI). There are two primary goals of building AI: first, build AI that makes better decisions; second, build the AI that makes decisions just like a human. All decisions in real world are to some extend risky. For an abstract example $(1.1, 0)$ and $(0, 1)$, with the probability $\mathbb{P}(s_1) = s_2$ There are three way of designing AI: 1) Each AI program optimize a deterministic expected utility. 2) Each AI optimize a deterministic expected utility plus some error. 3)

More recently, Zhang et al (2014 PNAS) have proposed a binary-choice model that provides an evolutionary framework for generating a variety of behaviors that are considered anomalous from the perspective of traditional economic models (i.e., loss aversion, probability matching, and bounded rationality). In this framework, natural selection yields standard risk-neutral optimizing economic behavior when reproductive risk is idiosyncratic (i.e., uncorrelated across individuals within a given generation). However, when reproductive risk is systematic (i.e., correlated among individuals within a given generation), some seemingly irrational behaviors, such as probability matching and loss aversion, become evolutionarily dominant.

2019 (PNAS)

The example of AI is more suitable for a ”Rational” Stochastic Choice rather than a RU-based stochastic choice because unlike human, AI does not necessarily need to have variating taste or ”make errors”.

In sum, the EU maximizer appears to be a RESC decision maker in a single choice if one of the following conditions hold: 1) the agents are sufficiently altruistic: they maximize the EU of the whole group; 2) the agent is an individual EU maximizer and is expected to make the same or similar decisions for multiple time in its whole lifespan.

Future Perspectives

Future improvements include replicating with health discrete choice data, applying the same technique into multi-attribute binary choices, applying the model into decision under ambiguity, further theoretically axiomatization of this model, and comparing this model with other nascent stochastic choice models such as REU. Additionally, this paper primarily focuses on lotteries with determined correlations. To apply the model to simple lotteries with non-deterministic but ambiguous correlation, one might adopt the thoughts in Epstein and Halevy (2019): the subjects might have a subjective belief of the unknown correlation.

7 Concluding Remarks

This paper introduces a distribution-free approach to modeling discrete choice and measuring risk. Without assuming any error distribution, the functional form of RESC is even simpler than many of the conventional tractable models such as the EU+error models. Many commonly used features and methods in the EU, such as estimating the risk-averse parameter with CRRA or CARA utility function, are applicable in the same way in RESC. In this case, RESC's statistical superiority might because the subjects strategically choose a probability mass function to optimize their expectation, rather than the randomization due to an "error". The second reason for statistical superiority might be directly fitting the stochastic choice with one parameter. This was done in spirit of Alpha Go Zero' improvement, in which the policy function and value function are combined to one, contrary to the separately estimated functions in Alpha Go (Silver et al. 2017 Nature). In classic RU, people often fit the deterministic component with many parameters, when fitting the stochastic component - error distribution - with zero or one parameter. As discussed in the introduction, in an RU model, the error distribution could be more important than the deterministic component. Therefore, the classic application of RU models might involve an over-fitting and over-modeling of the deterministic component, while under-fitting and under-modeling the stochastic component.

In contrast, RESC models both components with one variable, reaching a natural balance. RESC ranks the stochastic choice functions but does not rank the primitives – everything has its place; no one is superior to the others.

Appendix - Literature Relations on Stochastic Choice Generated by Deterministic Preferences

The strain of literature initiated by Machina (1985) does provide richer information about the origin of choice stochasticity. The most influential examples are Fudenberg et al's (2015) Additive Perturbed Utility (APU), Ortoleva et al's (2017, 2019) "deliberate stochastic" (DS) and cautious stochastic choice (CSC), and Dwenger, Kübler and Weizsäcker (2018)'s regret minimization. The first one generates stochastic choice because of a cost function, which could be meaningfully interpreted as the cost of learning or the cost of attention. The second one introduces a max-min structure, which can be interpreted as being cautious or pessimistic. The third one suggests that deterministic choice generates more aftermath regret. RESC generates stochastic choice by balancing the expected payoff between different states.

The intersect between SCGDP and RU model is not an empty-set. APU is neither a special case nor a generalization RU model, though both logit and probit, due to their stringent functional form, are special cases of both RU and APU. There are several different characteristics of SCGDP and RU. As the first effort to axiomatize stochastic choice function with acyclicity, APU assumes an elegant and estimable functional form. APU inherited both the advantages of SCGDP - generated by deterministic preference - and the advantages of RU - the separation of the deterministic component and the stochastic component. An important generalization of APU into multi-attribute alternatives can be found in Allen and Rehbeck (2019ab). RESC cannot be expressed as APU or RU because it can violate regularity. Additionally, APU and RU usually consider the discrete set as the menu, while RESC considers the acts or state-dependent lottery as primitive. Both APU and RESC can violate stochastic transitivity. Both models cannot be represented with a utility plus error. Both models have well-defined parameters ready to be estimated. RESC is similar to DS and CSC, in the sense that both relax regularity and both may be considered the relaxing of consequentialism. RESC is different from RU and CSC as RESC is a simple maximization of expectation, considers the space of acts, relaxes probability sophistication, and most importantly, relaxes the reduction of compound lottery to allow the discrimination of the objective probability and stochastic choice probability.

There are several other lines of literature try to rationalize the stochastic component in choices. Motivated by neurophysiological evidence, one possible source of stochastic choice is considered to be the noise in signal and signal-processing (Woodford 2014); in this line of research the preference is not necessarily fluctuating but the cognitive processes can. Bounded rationality is sufficient to generate the stochastic choice in a deterministic setting, even if the indifference curves in the Marschak-Machina triangle are linear (Stoye 2015). Another approach is based on the bounded rationality of decision-maker who optimally acquires costly information (among others, see Sims 2003; Caplin, Dean, and Leahy 2019). The rational inattention models maximize a generalized expectation minus attention cost. It explains better than random utility models in decision with time variation (Webb 2019) and decisions with information acquisition. The earliest dynamic stochastic choice model generated by optimizing long-term rewards might be credited to Thompson (1933); it is plausible that stochastic choice is a result of sampling and learning (Natenzon 2017). Our experimental set-ups control the uncertainty-learning factor of randomizing choices, because only the single-period choice under risk is considered: there is no ambiguity involved.

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Appendix A - Rational Expectation Stochastic Choice (RESC) Optima

A problem is a choice from a menu of two state-dependent lotteries $\{f, g\}$. The prizes (monetary payoffs) of the lottery depend on three states of the world $s \in S\{s_1, s_2, s_3\}$ eventuates. We assume that $p(s_1) = p(s_2) = p(s_3) = \frac{1}{3}$. Table 5 illus the problem.

Table 5: Menu 1

	$s_1(p_1 = \frac{1}{3})$	$s_2(p_2 = \frac{1}{3})$	$s_3\ (p_3 = \frac{1}{3})$
f	f_1	f_2	f_3
g	g_1	g_2	g_3

The RESC theory postulates the subject randomizes her choice over f and g. The objective of choice is the probability of choosing f, denoted by ρ (the probability of choosing g then is 1ρ.) If state i occurred, the expected (expectation over the randomization between f and g) payoff in state i then is

$$x_i =_i +(1-\rho)g_i = \rho(f_i - g_i) + g_i$$

RESC theory further postulates the subject wants to maximize the following expected utility :

$$U(\rho) = u(x_1) + u(x_{2+}u(x_3)$$

where

$$x_i = \rho f_i + (1-\rho)g_i, \quad , i = 1, 2, 3.$$

The first order condition is

$$U'(\rho) = u'(x_1)(f_1 - g_1) + u'(x_2)(f_2 - g_2) + u'(x_3)(f_3 - g_3) \tag{7}$$

CARA utility function

Suppose $u(x)$ takes the following CARA utility function, that is

$$u(x) = -\frac{e^{-rx}}{r}.$$

Note that $u'(x) = e^{-rx}$. The first order condition (7)becomes

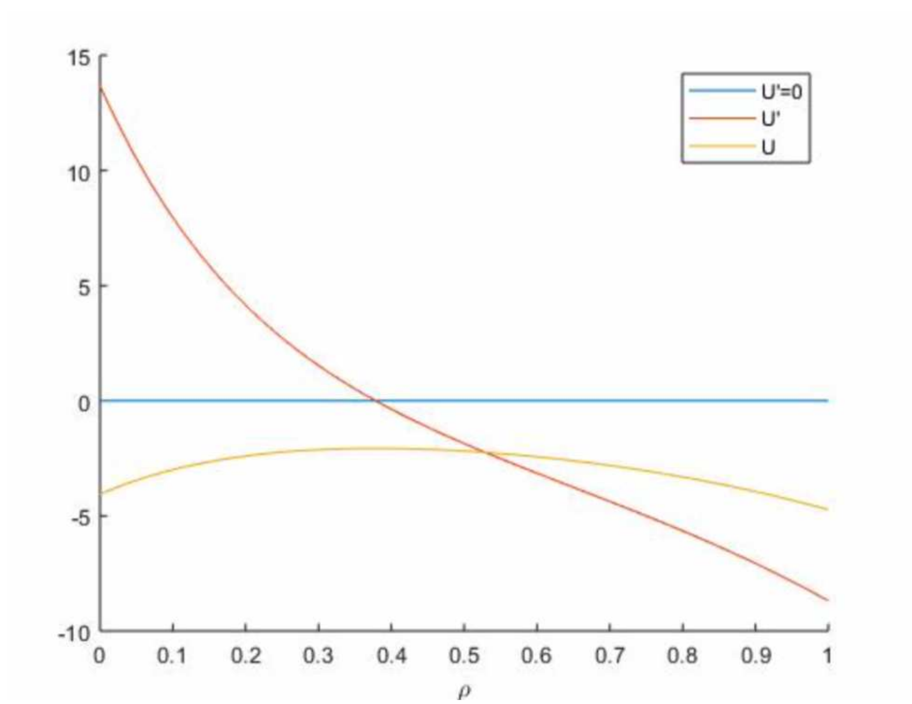
$$e^{-x_1}(f_1 - g_1) + e^{-x_2}(f_2 - g_2) + e^{-x_3}(f_3 - g_3) = 0.$$

which has the following explicit form

$$e^{-(\rho f_1+(1-\rho)g_1)}(f_1 - g_1) + e^{-(\rho f_2+(1-\rho)g_2)}(f_2 - g_2) + e^{-(\rho f_3+(1-\rho)g_3)}(f_3 - g_3).$$

There is no explicit solution for the equation above. Figure 6 gives the plot of $U(\rho)$ and its first derivative $U'(\rho) = \frac{\partial U}{\partial \rho}$ against ρ. It shows $U(\rho)$ is nicely concave and $U'(\rho)$ is monotone with one unique root. The parameters used for the plot are $f_1 = 15, f_2 = 0, f_3 = 3, g_1 = 0, g_2 = 6, g_3 = 10, r = 0.3.$

Figure 6: RESC Objective Function and its First Derivative



CRRA utility function

Suppose that $u(x)$ takes the following CRRA utility function form

$$u(x) = \begin{cases} \frac{x^(1-r)}{1-r} & r \neq 1 \\ log(x) & r = 1 \end{cases}$$

Noting $u'(x) = x^{-r}$, then first order condition (7) becomes

$$x_1^{-r}(f_1 - g_1) + x_2^{-r}(f_2 - g_2) + x_3^{-r}(f_3 - g_3) = 0,$$

which also does not have an explicit solution and will need to be solved numerically.

Appendix B - Experiment

Figure 7: Screenshot of the Experiment Interface

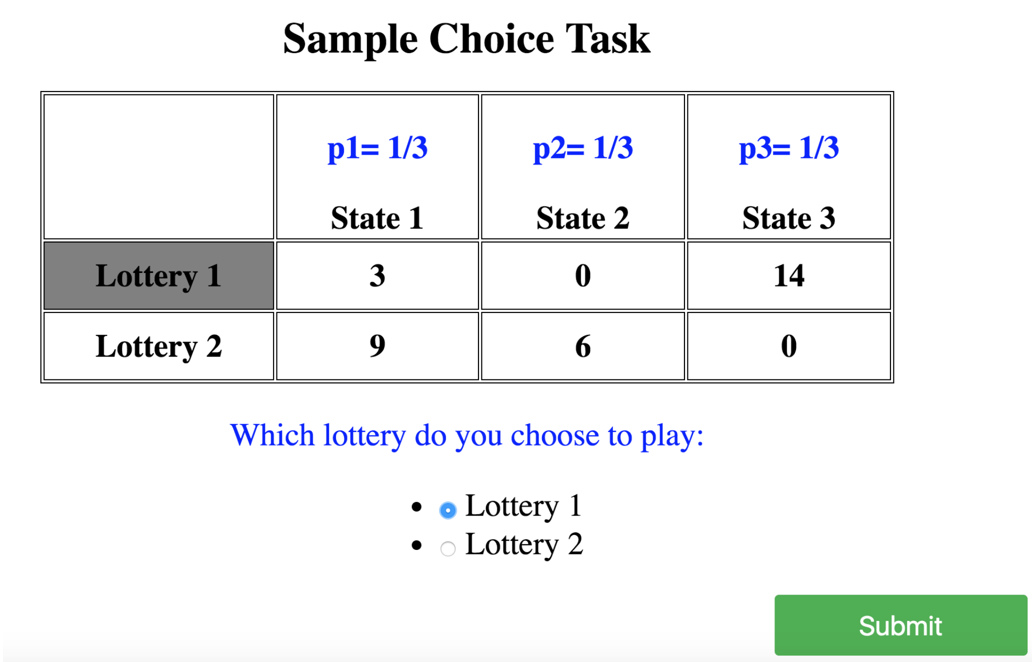


Figure 8: The 28 Experiment Tasks

15	0	3	6	10	0
0	6	10	0	4	15
3	15	0	6	0	10
10	6	0	15	0	4
14	0	3	5	0	3
0	6	9	1	8	0
3	14	0	0	5	3
9	6	0	0	1	8
16	0	3	10	0	8
0	6	10	4	17	0
3	16	0	0	10	8
10	6	0	0	4	17
8	0	1	10	0	6
0	3	4	2	17	0
1	8	0	0	10	6
4	3	0	0	2	17
0	3	10	12	0	6
6	6	0	4	15	0
10	0	3	0	6	12
6	0	6	0	4	15
0	3	18	4	5	0
6	12	0	0	0	23
18	0	3	6	7	0
6	0	12	0	0	22
0	3	16	5	4	0
6	9	0	0	0	20
16	0	3	2	3	0
6	0	9	0	0	10

Figure 9: The Seven Problems in Agranov and Ortoleva (2017

	State A	State B	State C	State D
FOSD1	98	98	98	98
	103	103	103	103
EASY1	23	23	30	30
	5	5	5	31
EASY2	12	14	16	96
	85	85	85	85
HARD1	38	38	38	77
	16	16	94	94
HARD2	10	10	90	90
	32	45	45	56
HARD3	6	84	105	200
	54	60	117	135
HARD4	13	30	51	81
	19	32	38	86

In the experiment, the two lotteries were not presented in this tabular format. However the implementation of the experiment implicitly set the outcomes of lotteries to be state-dependent. Interested readers can find the procedural details of the experiment in the paper and its online supplemental material.

Appendix C- Deterministic Choice Rules

Expected Utility(EU)

Under EU, an individual behaves as if compares the two expected utilities $EU(f) = \sum_1^3 p_i f(s_i)$ and $EU(g) = \sum_1^3 p_i f(s_i)$ and choose the act that has a higher expected utility.

Rank Dependent Expected Utility (RDEU)

RDEU differs from EU only in that an individual does not use the given objective probabilities to calculate the expected utility. Instead, probabilities are distorted according to the rank of the three payoffs. Let $f_1 \le f_2 \le f_3$ such the three payoffs that is ranked in an ascending order and denote the corresponding states s'_1, s'_2, s'_3. Then the RESC of act f is calculated as

$$u(f_1)(1 - w(p(s'_2 \cap s'_3)) + u(f_2)(w(p(s'_2 \cap s'_3) - w(p(s'_3)) + u(f_3)w(p(s_3)) \tag{8}$$

where $w(.)$ is a probability weighting function weakly increasing in $[0,1]$ and $w(0) = 0, w(1) = 1$. Note that if $w(.)$ is linear RDEU reduces to EU. A convex $w(.)$ leads to overweight of bad outcomes and therefore represents a pessimistic attitude. Similarly a concave $w(.)$ represents an optimistic attitude.

In our problem there are three states and the probabilities are kept as the same across all problems. Therefore, for each subject we only need to estimate two distorted probabilities, which are assumed to be used for all problems. We write the evaluation of the two alternatives as follows.

$$RDEU(f) = \tilde{p}_1 u(f_1) + \tilde{p}_2 u(f_2) + (1 - \tilde{p}_1 - \tilde{p}_2)u(f_3)$$
$$RDEU(g) = \tilde{p}_1 u(g_1) + \tilde{p}_2 u(g_2) + (1 - \tilde{p}_1 - \tilde{p}_2)u(g_3)$$

where we have either $\tilde{p_1} \le \tilde{p_2} \le 1 - \tilde{p}_1 - \tilde{p}_2$ or $\tilde{p}_1 \ge \tilde{p}_2 \ge 1 - \tilde{p_1} - \tilde{p_2}$. In the special case when $\tilde{p_1} = \tilde{p_2} = \frac{1}{3}$, RDEU reduces to EU.

Salience Theory under Risk (ST)

ST postulates that probability distortion is state-dependent. In RDEU, the evaluation of a lottery would not depend on how the state-contingent payoffs of one alternative correlate with of the other lottery. But in ST, how probabilities are distorted exactly by contrasting the the two payoffs of the two lotteries in each state. It postulates that individuals over weights the probability of the the most salient states. The salience of state i is defined as

$$\sigma_i = \begin{cases} \frac{|f(s_i)-g(s_i)|}{|f(s_i)|+|g(s_i)|} & if\ |f(s_i)|+|g(s_i)| \neq 0 \\ 0 & if\ |f(s_i)|+|g(s_i)| = 0. \end{cases}$$

Without loss of generality, let $\sigma(s_1) > \sigma(s_2) > \sigma(s_3)$. Then the probabilities are distorted as follows

$$\frac{\hat{p}_2}{\hat{p}_1} = \delta\frac{p_2}{p_1}, \ \frac{\hat{p}_3}{\hat{p}_2} = \delta\frac{p_3}{p_2}, \ \hat{p}_1 + \hat{p}_2 + \hat{p}_2 = 1$$

where $0 < \delta \leq 1$. When $\delta = 1$, ST reduces to EU. Sine we use equal probabilities for all problems, there is

$$\begin{cases} \hat{p}_1 = & \frac{1}{1+\delta+\delta^2} \\ \hat{p}_2 = & \frac{\delta}{1+\delta+\delta^2} \\ \hat{p}_3 = & \frac{\delta^2}{1+\delta+\delta^2} \end{cases} \tag{9}$$

.

Therefore, the Salience adjusted Expected Utility is calculated as follows.

$$ST(f) \quad = \frac{1}{1+\delta+\delta^2}u(f_1) + \frac{\delta}{1+\delta+\delta^2}u(f_2) + \frac{\delta^2}{1+\delta+\delta^2}u(f_3) \tag{10}$$

$$ST(g) \quad = \frac{1}{1+\delta+\delta^2}u(g_1) + \frac{\delta}{1+\delta+\delta^2}u(f_2)u(g_2) + \frac{\delta^2}{1+\delta+\delta^2}u(g_3). \tag{11}$$

Regret Theory (RT)

RT postulates that when an individual evaluates an alternative, she would take into account what she can obtain if she had chosen otherwise. Different functions forms had been given by Fishburn (1982), Bell (1982) and Loomes and Sugden (1982), which is the one this paper adopt.

Denote a preference relation as $\succeq$. We use the tractable Loomes and Sugden (1982), RT preference is written in the following way.

$$f \succeq g \Leftrightarrow \sum_{i}^{3} p_i Q(u(f_{s_i}) - u(g_{s_i})) \geq 0. \tag{12}$$

where $Q(.)$ is a strictly increasing, skew-symmetric and convex function. Here we further assume it has the function form that $Q(x) = x^{\alpha}$. When $\alpha = 1$ RT reduces to EU. $\alpha > 1$ repRESCnts regret aversion.

Based on equation (12), we can write the evaluation of the two acts as follows

$$\begin{aligned} RTEU(f) &= \textstyle\sum_i^3 I_i p_i Q(u(f_{s_i}) - u(g_{s_i})) & (13) \\ RTEU(g) &= \textstyle\sum_i^3 I_i p_i Q(u(f_{s_i}) - u(g_{s_i})) & (14) \end{aligned}$$

where

$$I_i = \begin{cases} 1, & if \quad f_{s_i} \geq g_{s_i} \\ 0, & if \quad f_{s_i} < g_{s_i} \end{cases} .$$

Appendix D - Empirical Performance of a General Version of RESC

We also estimated the following general version of RESC by replacing this expected value with expected utility. Therefore we assume an agent choose a stochastic choice function that maximizes the following objective function

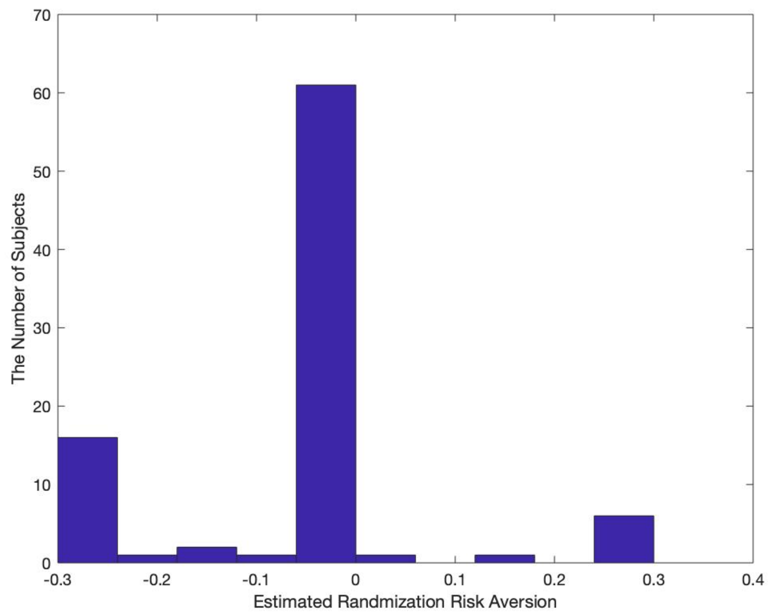
$$U(\rho) = u(x_1) + u(x_2) + u(x_3)$$

where

$$x_i = \rho v(f_i) + (1-\rho)v(g_i), \quad , i = 1, 2, 3 \tag{15}$$

where $v(x) = -\frac{exp(-rx)}{r}$, which is a CARA utility function. The risk attitude toward randomizing probability can be separately estimated as the r parameter in $v(\cdot)$. The subjects can be viewed as second-order expected utility maximizer (Segal 1990). Doing so would involve introducing an extra parameter. By LRT, the generalization of RESC generates extra statistical improvement for only 11 subjects comparing to the the RESC studied in section 4 and 5. Additionally, most subjects are not risk averse when dealing with the stochastic choice probability (Fig 10).

Figure 10: Estimated Randomization Risk Aversion



We fit each subject's data into the general version of RSEC to obtain the Maximum Likelihood estimates for r in equation 15t). It represents an subject's aversion to randomization risk. Most of the values are closes to 0. This indicates the majority of subjects are neutral to this type of risk.

Appendix E- Stochastic Choice Generated by Deterministic Preferences

The strain of literature initiated by Machina (1985) does provide richer information about the origin of choice stochasticity. The two most influential examples are Fudenberg et al's (2015) Additive Perturbed Utility (APU) and Ortoleva et al's (2017, 2019) "deliberate stochastic" (DS) and cautious stochastic choice (CSC). The former generates stochastic choice because of a cost function, which could be meaningfully interpreted as the cost of learning or the cost of attention. The latter introduces a max-min structure, which can be interpreted as being cautious or pessimistic. RESC generates stochastic choice by balancing the expected payoff between different states.

The intersect between SCGDP and RU model is not an empty-set. APU is neither a special case nor a generalization RU model, though both logit and probit, due to their stringent functional form, are special cases of both RU and APU. There are several different characteristics of SCGDP and RU. As the first effort to axiomatize stochastic choice function with acyclicity, APU assumes an elegant and estimable functional form. APU inherited both the advantages of SCGDP - generated by deterministic preference - and the advantages of RU - the separation of the deterministic component and the stochastic component. An important generalization of APU into multi-attribute alternatives can be found in Allen and Rehbeck (2019ab). RESC cannot be expressed as APU or RU because it can violate regularity. Additionally, APU and RU usually consider the discrete set as the menu, while RESC considers the acts or state-dependent lottery as primitive. Both APU and RESC can violate stochastic transitivity. Both models cannot be represented with a utility plus error. Both models have well-defined parameters ready to be estimated. RESC is similar to DS and CSC, in the sense that both relax regularity and both may be considered the relaxing of consequentialism. RESC is different from DS and CSC as RESC is a simple maximization of expectation, considers the space of acts, relaxes probability sophistication, and most importantly, relaxes the reduction of compound lottery to enable the discrimination of risk probability and stochastic choice probability.

There are several other lines of literature try to rationalize the stochastic component in choices. Motivated by neurophysiological evidence, one possible source of stochastic choice is considered to be the noise in signal and signal-processing (Woodford 2014); in this line of research the preference is not necessarily fluctuating but the cognitive processes can. Bounded rationality is sufficient to generate the stochastic choice in a deterministic setting, even if the indifference curves in the Marschak-Machina triangle are linear (Stoye 2015). Another approach is based on the bounded rationality of decision-maker who optimally acquires costly information (among others, see Sims 2003; Caplin, Dean, and Leahy 2019). The rational inattention models maximize a generalized expectation minus attention cost. It explains better than random utility models in decision with time variation (Webb 2019) and decisions with information acquisition. The earliest dynamic stochastic choice model generated by optimizing long-term rewards might be credited to Thompson (1933); it is plausible that stochastic choice is a result of sampling and learning (Natenzon 2017).

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