Is There Really Hysteresis in OECD Countries' Unemployment Rates? New Evidence Using a Fourier Panel Unit Root Test

Omay, Tolga and Shahbaz, Muhammad and Stewart, Chris

Atilim University, Turkey, Beijing Institute of Technology, China, Kingston University, UK

1 May 2021

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Tolga Omay
Atilim University, Department of Economics, Ankara, Turkey
Montpellier Business School, Montpellier, France
Email: omay.tolga@gmail.com

Muhammad Shahbaz
Department of International Trade and Finance School of Management and Economics
Beijing Institute of Technology, Beijing, China
Email: muhdshahbaz77@gmail.com

Chris Stewart,
School of Law, Social and Behavioural Sciences, Kingston University, UK
Email: c.stewart@kingston.ac.uk

Abstract: We investigate the hysteresis hypothesis by proposing a heterogeneous panel unit root test that allows for gradually changing trends and cross-sectional dependence (CSD) among panel members using a flexible Fourier form. Inconclusive results from previous studies are potentially due to using very restrictive specifications with homogenous break structures and/or exogenously determined abrupt breaks. We seek to address these limitations by employing general specifications that are more capable of characterising the true data generation process of unemployment and by allowing for spill-over effects using a bootstrapping procedure to accommodate CSD that must be considered in a globalized world. Extensive simulations suggest that the failure to take structural breaks and CSD into account can lead to misleading conclusions about whether the unemployment rate is stationarity. We apply our test procedure to unemployment data for 23 OECD countries and find conclusive evidence against the hysteresis hypothesis for all these countries.

JEL Classification: C12; C22; C23; E24

Keywords: Smooth Break, Panel Unit Root, Cross Section Dependency, CCE, Hysteresis

I. Introduction

The labour market performance of many developed countries has remained unsatisfactory since the first oil price rise in 1973-74. In the wake of the first oil price shock, the unemployment rate in the OECD area increased from 3.3 percent in 1973 to almost 5 percent in 1978[1]. The second oil price increase in 1979-80 caused the total OECD unemployment rate to rise to 7.8 percent in 1983. After 1983 the unemployment rate fluctuated around approximately 6.7 percent during the next 23 years. Weak employment growth and persistently high levels of unemployment went hand in hand with the large numbers of long-term unemployed. This gloomy picture worsened after the financial crisis of 2008-09 with the OECD unemployment rate rising from 5.6 percent in 2007 to a post-war high of 8.4 percent in quarter 1 of 2010.

Unemployment remains persistently above its 2008-09 level in many developed countries, with the prospects especially discouraging for European countries. According to the OECD employment outlook (2015Q1), OECD unemployment remained higher than eight years earlier in 27 out of 34 countries. In this period, the highest unemployment rates were recorded in Greece (24.2%), Spain (20.9%), Portugal (12.1%), Italy (11.5%), and the Slovak Republic (10.9%). OECD projections were not promising, suggesting that in quarter 4 of 2017 only 8 OECD countries' unemployment rates would be below pre-crisis levels. The lack of a stronger labour market recovery is also reflected in a substantial increase in long-term unemployment for these countries. In the last quarter of 2015, 33% of unemployed persons had been out of work for 12 months or more across the OECD.

High and persistent unemployment can have detrimental effects on a country's welfare beyond reducing output and income. Job losses may cause a deterioration in physical and mental health of affected workers and increased crime rates. Joblessness that turns into long-term unemployment can have enduring adverse effects on human capital, permanently reducing the earnings potential of the unemployed, especially for the young (Ellwood 1982, Layard 1986, Machin and Manning 1999). Furthermore, part of the cyclical increase in labour market slack may become structural unemployment that is not absorbed during the ensuing recovery, the so-called "hysteresis" effect. Hysteresis refers to the effect of unemployment history on the natural rate of unemployment (Phelps, 1972)[2]. The hysteresis hypothesis was first econometrically

[1] See *OECD Economic Outlook: Statistics and Projections* database (http://dx.doi.org/10.1787/9572784d-en).

[2] According to Phelps, a macroeconomic disequilibrium may reduce the equilibrium unemployment rate over time due to the method of operations of labour markets and the permanent effects of the disequilibrium on the skills and

investigated by Blanchard and Summers (1986)[3, 4]. Persistently high unemployment rates in many developed countries, notably in Europe, have led economists to argue that unemployment exhibits hysteresis. Analyses using conventional unit root tests commonly verified the existence of hysteresis, especially in European countries[5]. According to the hysteresis hypothesis, all economic shocks will have permanent effects, with no tendency for unemployment rate to return to its equilibrium level, even in the long-run. Thus, high unemployment rates will persist in the long-run. If the hysteresis hypothesis is true, monetary or fiscal policies can be used to permanently reduce unemployment rates.

A key question for countries where unemployment rates have not returned to their pre-crisis levels is whether these high rates are due to cyclical factors that are expected to dissipate as the economic recovery strengthens or they represent hysteresis and will remain high even after the economy has fully recovered. The appropriate policies for reducing unemployment rates depend on finding the right answer to this question. This has been one of the most fiercely debated questions in the macroeconomics literature. A perennial question is whether changes in nominal variables such as money can affect real activity, and if they can, are these effects temporary or permanent. The first answer to this question was provided by Phelps (1967) and Friedman (1968) with the natural rate hypothesis. According to this hypothesis, there exists a natural (or equilibrium) level of unemployment to which the economy will eventually revert in the long-run. Although neither Phelps (1967) nor Friedman (1968) argued that the natural rate is a constant, assuming it to be so became a rule rather than an exception and investigating the real determinants of the natural rate was largely overlooked until the 1980s. The natural rate hypothesis applies the monetary neutrality proposition to unemployment and concludes that monetary policy can only cause temporary deviations of the unemployment rate from the natural

habits of people. These effects on the natural rate may last for a long time, and the value of the natural rate at any future date will depend on the course of history for that equilibrium.

[3] Blanchard and Summers (1986) highlighted three theories that can explain hysteresis in unemployment (physical capital, human capital and insider-outsider models) and considered the last one as the most likely explanation for hysteresis.

[4] As argued in Blanchard and Summers (1986), for unemployment to formally exhibit hysteresis, current unemployment should be a function of its past values, and the coefficients of these past values should sum to 1. However, the hysteresis that the authors refer to is where shocks have persistent, though not permanent, effects with the coefficients on the past unemployment rates summing to a number close to, if below, 1.

[5] Many studies in the literature use the terms hysteresis and persistence interchangeably. While hysteresis represents an extreme situation of a unit root, persistence occurs when unemployment rates are stationary with a near unit root. In the latter case, the shocks affecting unemployment, while protracted, will eventually die out and the unemployment rate will converge to its unique equilibrium in the long run. In this sense, studies either treat persistence as a special case of the natural rate hypothesis or term it as partial hysteresis with the case of a unit root corresponding to full, or pure, hysteresis (Lèon-Ledesma and McAdam, 2004).

rate because expectations will adjust to return it to its equilibrium. If the natural rate hypothesis is true an activist policy will not be able to achieve full employment and may be destabilizing.

Since unemployment rates in the developed world fluctuated around an average rate of 3.3 percent before oil price shocks, the natural rate hypothesis seemed consistent with the data. However, after 1980, the observed persistently higher unemployment rates suggested a reconsideration of the natural rate theory. In response, the "structuralist" school (Phelps, 1990) emerged. According to Phelps (1995, p. 24), the natural rate of unemployment is viewed as an endogenous variable determined by the economy's current structure (that is, non-monetary forces) while retaining the notion that the actual unemployment rate returns to its natural rate:

> "If we view the natural rate path as endogenous, pushed like other economic variables by non-monetary forces, and take on board the rest of the natural rate doctrine – actual unemployment tending soon to equilibrium, and all equilibrium paths approaching the natural path – we arrive at a new paradigm: a non-monetary equilibrium theory of unemployment movements – an endogenously moving-natural rate theory of movements in the actual rate of unemployment."

According to the structuralist hypothesis, both the equilibrium and actual unemployment rate will fluctuate due to shifts in the structural parameters of the model[6]. The structuralist hypothesis does not suggest passive policy and determining the appropriate policy instruments to employ during a recession is very important (Phelps, 1994, p. 361).

All three aforementioned hypotheses can be assessed by applying unit root tests to the unemployment series. The natural rate hypothesis holds if unemployment rate is mean reverting (stationary) to a unique equilibrium unemployment rate. Whereas the hysteresis hypothesis requires unemployment series to contain a unit root and have no tendency to return to an equilibrium after a shock.[7] Under the structuralist hypothesis, the unemployment rate should be mean reverting towards an occasionally changing equilibrium.

[6] For a detailed discussion on the structuralist theory of key determinants of unemployment fluctuation, see Phelps (1994, pp. 60-66).

[7] Hysteresis is commonly tested using conventional linear unit root tests; however, hysteresis is a property of nonlinear systems with heterogeneous elements (Amable et al. 1995). Further, the unemployment rate is a bounded variable and so is arguably stationary over long time periods (Cross, 1995).

This paper aims to the test the hysteresis hypothesis for a panel of 23 OECD countries, including European countries with a more pronounced risk of hysteresis. This paper contributes to the literature by developing and applying a novel panel unit root testing method that is specifically designed to capture the structural features of the unemployment rate and remove the limitations of the testing methods typically used in the literature. The three notable generalisations that can help avoid misleading conclusions from more usually applied testing procedures are the following. First, the test is implemented using Chang's (2004) sieve bootstrap algorithm so that the unrealistic assumption of cross-sectional independence typically made in panel unit root tests can be avoided. Second, it allows the equilibrium unemployment rate to slowly change (the structuralist hypothesis) by allowing for structural breaks using a flexible Fourier form for each country rather than specifying restrictive (exogenous) abrupt and/or homogeneous shifts across countries, as is typical in the literature. Third, the sequential panel selection methodology (SPSM) of Chortareas and Kapetanios (2009) is used to identify the countries where the unemployment rate is stationarity (the natural rate hypothesis or structuralist hypothesis) and those where it is nonstationary (hysteresis hypothesis). Such an exact identification of countries into each category is rare in the literature. We are not aware of any previous application of such a generalised panel unit root testing method to OECD unemployment rates to test the hysteresis hypothesis and therefore fill this gap in the literature. Using this unique technique that addresses the limitations of the methods employed in the previous literature we are the first to reject the hysteresis hypothesis for all 23 OECD countries in our panel.

The remainder of the paper is organized as follows. Section II surveys the empirical literature on the stationarity properties of the unemployment rate. Section III describes the theoretical background of the model and testing procedure. Section IV details data collection and empirical results. Section V draws conclusions.

II. Empirical Literature

The empirical literature on unemployment hysteresis typically falls into three main categories according to the econometric methodology applied. The first strand used standard univariate unit root tests, including the augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Kwiatkowski et al. (1992), and/or the Elliot et al. (1996) tests. Using such tests Nelson and Plosser (1982), Perron (1988) and Evans (1989) found the U.S. unemployment rate to be stationary while Perron (1989) did not test this variable for a unit root due to the belief that unemployment rates are intrinsically stationary. In contrast, Mitchell (1993), Breitung (1994), Hatanaka (1996), Song and Wu (1997, 1998) and Leon-Ledesma (2002) failed to reject the hysteresis hypothesis in the U.S. and its states using ADF and PP tests. Most European countries' unemployment rates were found to be nonstationary using conventional univariate unit root tests (Blanchard and Summers 1986, Groenewold and Taylor 1992, Mitchell 1993, Roed 1996, Song and Wu 1998, Leon-Ledesma 2000, Camarero and Tamarit, 2004).

However, standard unit root tests are biased toward non-rejection of the unit root null hypothesis when there are structural breaks that OECD unemployment rates are typically subject to (including the 1970s oil price shocks and the general rise in real interest rates and fall in capital accumulation of the early 1980s, see Arestis and Mariscal, 1999). Studies within the first group of univariate unit root tests include those that model possible structural breaks in unemployment in the deterministic components. Some with parameters changing gradually over time using smooth transition models (Becker et al., 2004, 2006; Enders and Lee, 2012a, 2012b; Bierens, 1997, and Leybourne et al., 1998). Others where the change in parameters occurs instantaneously employing discrete instantaneous change models (Perron, 1989, 1990; Rappoport and Rechlin, 1989; Zivot and Andrews, 1992; Lumsdaine and Papell, 1997, Bai and Perron, 1998, and Lee and Strazicich, 2003, 2004). These studies that accommodate structural breaks using univariate tests yield ambiguous conclusions on the hysteresis hypothesis. For example, the following reject the unit root null: Baddeley et al. (1998), Arestis and Mariscal (1999), Bjornland (1999), Papell et al. (2000), Ewing and Wunnava (2001), Leon-Ledesma and McAdam (2004), Camarero and Ordonez (2006), Lee and Chang (2008), Ayala et al. (2012), Jo and Kim (2014), Furuoka (2014a). In contrast, many studies cannot reject hysteresis, even when accounting for structural breaks: Mitchell (1993), Song and Wu (1997), Arestis and Mariscal (1999), Gomes and da Silva (2007), Chang et al. (2007), Chang (2011), Pérez-Alonso, Pérez-Alonso and Di Sanzo (2011), Cuestas et al. (2011), Cheng et al. (2014), Furuoka (2014b).

Conventional univariate unit root tests also have low power in small samples, particularly for near-unit root processes. A second strand of studies has, therefore, employed panel data techniques to increase the power of unit root tests. For example, Song and Wu (1997, 1998) apply the Levin and Lin (1992) panel unit root test to analyze hysteresis in U.S. states and OECD countries, respectively. The unit root null was rejected for the entire panel of 48 U.S. states, 15 OECD countries and 11 European countries. However, these panel tests did not account for cross-sectional dependence (CSD), that can lead to biased estimates and misleading inference when neglected (Barbieri, 2009), therefore their evidence against the hysteresis hypothesis must be treated with caution.

Consequently, many authors have applied panel unit root tests that take CSD into account. For instance, Leon-Ledesma (2002) employed the Im et al. (2003) test (hereafter, IPS) with cross-sectional demeaning to adjust for CSD and rejected the unit root null in unemployment rates for the panel of 51 U.S. states. However, because the alternative hypothesis for the Im et al. (2003) test is that that at least one state's unemployment rate in the panel is stationary the rejection of the unit root null does not extend to all states (this criticism applies to many of the papers using panel tests discussed below). Further, Leon-Ledesma (2002) could not reject the unit root null for a panel of 12 European Union (EU) countries. Using a similar (slightly expanded) data set Romero-Avila and Usabiaga (2009) could not reject the hysteresis hypothesis for either U.S. states or EU country panels using the Hadri (2000) test and controlling for CSD. Similarly, Dregers and Reimers (2009), also utilizing a similar data set and using panel unit root tests that control for CSD provided mixed evidence on whether EU and U.S. unemployment rates were stationary. While Camarero and Tamarit (2004) rejected the hysteresis hypothesis for an entire panel of 19 OECD countries using the Taylor and Sarno (1998) test, which accounts for CSD.[8]

Applying the SURADF test Chang et al. (2005) could not reject the hysteresis hypothesis for 8 out of 10 European countries while Furuoka, (2014a) rejected hysteresis in only 2 out of 12 East Asian Pacific countries. Camarero et al. (2006) could not reject the unit root null for the entire panel of 19 OECD countries using the IPS, Maddala and Wu (1999) and Hadri (2000) panel unit root tests (both with and without adjustment for CSD). Camarero et al. (2008) corroborated Camarero et al.'s (2006) findings for 8 Central and Eastern European countries (CEECs). Conversely, Leon-Ledesma and McAdam (2004) strongly rejected the hysteresis hypothesis for a panel of 12 CEECs by implementing IPS, Chang (2004) and Taylor and Sarno (1998) tests while accounting for CSD. Other studies that reject the hysteresis hypothesis using panel unit root tests that control for CSD include Chang et al. (2007) for Taiwan and Lanzafame (2012) for Italy.

[8] The Taylor and Sarno test can become unreliable for large dimension VARs (certainly above 10 countries) especially with a small time-series dimension and evidence from it using panels with many cross-sectional units should be treated with caution.

Studies that have rejected the hysteresis hypothesis whilst accounting for instantaneous structural breaks for different panels of OECD, EU and CEEC countries include, Murray and Papell (2000), Leon-Ledesma and Mc Adam (2004), Camarero et al. (2006), Camarero et al. (2008), Lee et al. (2009), Fallahi and Rodriguez (2011), Lanzafame (2012). However, these papers typically employ tests that do not account for CSD and/or reject hysteresis only for at least one country in the panel (the number and identity of countries where the hypothesis is rejected is unclear). In contrast, Romero-Avila and Usabiaga (2007, 2008), Garcia-del-Barrio and Gil-Alana (2009) cannot reject the unit root null in Spanish unemployment using panel unit root tests with and without structural breaks. Romero-Avila and Usabiaga (2009) could not reject the unit root null in unemployment rates for an EU panel whilst controlling for endogenous structural breaks and CSD, if U.S. states' unemployment was found to exhibit regime-wise stationarity. Lee et al. (2010), Lanzafame (2010) and Liu et al. (2012) corroborated the hysteresis hypothesis for East Asian countries, Italy and Australia using panel unit root tests that account for both structural breaks and CSD.[9]

The tests discussed above assume linear and symmetric adjustment in unemployment rates, however; many macroeconomic variables may exhibit nonlinear behaviour. For example, downward wage rigidity and sharper downturns than recoveries in the business cycle for output and employment. Thus, a third strand of the literature considers unit root tests that allow for nonlinear adjustment (such as, threshold autoregressive, TAR, smooth transition autoregressive, STAR, exponential smooth transition autoregressive, ESTAR and Markow switching processes) in unemployment. Studies employing such univariate nonlinear tests find unemployment rates are nonstationary for the majority of European countries, if for the U.S. the evidence typically favoured the natural rate hypothesis (Gustavsson and Österholm 2006, Ghosh and Dutt 2008, Lin et al. 2008, Chang and Lee 2011, Tiwari 2014)[10].

Nonlinear panel unit root tests which account for CSD have been developed to improve the power of the tests. Lanzafame (2010) rejected the hysteresis hypothesis for at least one region in a panel of Italian regional unemployment rates using a nonlinear panel unit root test. Using Ucar and Omay's (2009), UO, nonlinear panel unit root test with the Chortareas and Kapetanios (2009) method, Lee (2010) provided evidence in favour of the natural rate hypothesis in 23 out

[9] Most of the studies we discuss consider (panel) unit root tests that allow for instantaneous structural breaks. Hence, we focus on methods that allow for the more plausible breaks where adjustment occurs over several periods.
[10] Exceptionally Cevik and Dibooglu (2013) find that the U.S. unemployment rate is nonstationary in the recession regime using the Markov Switching ADF test of Hall et al. (1999).

of 29 OECD countries. Bolat et al. (2014) applied the UO test with a Fourier function to model possible structural breaks and nonlinearity simultaneously and could not reject the natural rate hypothesis in 11 of the 17 Euro area countries considered. Enders and Lee (2012a) suggest that state dependent nonlinearity (ESTAR) is also captured by a Fourier function. Hence, our testing strategy that uses a Fourier function is general because it captures both time varying and state dependent nonlinearity.

III. Theoretical Background of Model and Testing Procedure

III.I Theoretical Background

The three hypotheses discussed above can be tested using (1):

$$u_t = \alpha + \beta u_{t-1} + \varepsilon_t \tag{1}$$

where the unemployment series u_t is assumed to follow a first-order autoregressive process, AR(1), and ε_t is a white noise error term[11]. If the null hypothesis $H_0 : \beta = 1$ in (1) cannot be rejected there is hysteresis whereas non-rejection of the alternative $H_0 : \beta < 1$ indicates that the natural rate hypothesis holds in its *weak* form and the unemployment rate reverts to its long-run equilibrium value, $\alpha/1-\beta$ (Song and Wu, 1997). For the natural rate hypothesis to hold in its strong form, the unemployment rate should not be a function of its past values, requiring the null $H_0 : \beta = 0$ to hold. If this null cannot be rejected the unemployment rate will remain at the constant level, α. Although much of the literature use the terms hysteresis and persistence interchangeably, strictly hysteresis means a unit root ($\beta = 1$) while persistence occurs when unemployment rates are stationary with β close to one (a near unit root). In the latter case, shocks to unemployment will eventually dissipate and the unemployment rate will converge to its unique equilibrium value in the long-run. Hence, studies either treat persistence as a special case of the natural rate hypothesis or call it partial hysteresis with $\beta = 1$ being full or pure hysteresis (Lèon-Ledesma and McAdam, 2004).

The natural rate and hysteresis hypotheses can be tested using the conventional unit root testing equation obtained by rewriting (1) as:

[11] This equation can be obtained as a reduced form from models of structural unemployment (Srinivasan and Mitra, 2014) or macroeconomic models of the Phillips curve (Song and Wu, 1997, 1998). Higher order AR processes can also be used without changing the main conclusions.

$$\Delta u_t = \alpha + \gamma u_{t-1} + \varepsilon_t \quad (2)$$

where $\gamma = \beta - 1$. Hence, $H_0 : \beta = 1$ in (1) is equivalent to $H_0 : \gamma = 0$ in (2), where (2) is the basis of the Dickey-Fuller test which can be modified to include deterministic regressors, lagged changes in Δu_t or allow different error structures in ε_t.

Testing the structuralist hypothesis necessitates the use of unit root tests that allow for structural breaks or regime-wise (or state dependent) nonlinearity through the appropriate modification of (1) and (2). The structuralist hypothesis holds if the unit root null can only be rejected after allowing for breaks or nonlinearity. Regime wise nonlinearity and multiple equilibria could be introduced using Markov-switching models (Lèon-Ledesma and McAdam 2004, Camarero et al. 2006). However, these models assume abrupt or instantaneous transition between different equilibria, which is only possible if agents reacted at the same time and in the same direction to economic shocks affecting the natural rate. With an economy consisting of many heterogeneous agents acting at slightly different times, a gradual or smooth path is more plausible. Therefore, it is more reasonable to test for the structuralist hypothesis using unit root tests that allow for a smooth transition around an occasionally changing natural rate. Thus, STAR models, either with an ESTAR or LSTAR transition function could be used. Chappell and Peel (1998) suggest an ESTAR function, which can exhibit multiple equilibria and complex dynamics, should be favoured. Nonetheless, to consider smooth breaks in the deterministic components of a series, unit root tests have alternatively been developed based on Gallant's (1981) flexible Fourier form which has advantages that include the following. First, being able to capture the behaviour of a deterministic function of unknown form even if the function itself is not periodic (Gallant 1981, Davies 1987, Bierens 1997, Becker et al. 2006, Enders and Lee 2012a, b). Second, that it works better than dummy variable methods irrespective of whether the breaks are instantaneous or smooth (Enders and Lee, 2012a, b). Third, it avoids the problems of selecting the dates, number and form of breaks (Enders and Lee 2012a, 2012b, Rodrigues and Taylor 2009)[12]. For these reasons the test we develop employs a flexible Fourier form and, as mentioned above, Enders and Lee (2012a) showed that state dependent nonlinearity (ESTAR) can also be captured by a

[12] For examples of panel unit root testing using smooth transition models with one structural break see Omay et al. (2017), Corakçı et al. (2017) and Omay et al. (2018). We do not compare these testing procedures further as they assume one structural break and we are interested in multiple structural breaks.

Fourier function. Thus, the test used in this study also controls for state dependent nonlinearity using a Fourier function.

III.II The Model and Testing Procedure

Let the unemployment rate, y_{it}, be a panel autoregressive process on the time domain t = 1,2,…,T for the cross-section units $i = 1,\ldots,N$. We assume that y_{it} are generated by the following process with the fixed effect parameter α_i:

$$y_{it} = \alpha_i + d_i(t) + \phi_{i,1} y_{i,t-1} + \lambda_i t + \varepsilon_{it} \qquad (3)$$

where $\varepsilon_{i,t}$ is a stationary disturbance with variance σ_i^2, and $d_i(t)$ is a deterministic function of t. The initial value is assumed to be fixed, and $\varepsilon_{i,t}$ is weakly dependent as in Enders and Lee (2012a,b). Enders and Lee (2012a,b) note that if the functional form of $d_i(t)$ is known (3) can be estimated straightforwardly and the unit root null tested. However, when the form of $d_i(t)$ is unknown, any test for $\phi_{i,1} = 1$ is problematic if $d_i(t)$ is misidentified. Following Enders and Lee (2012a,b) our test uses the Fourier expansion to approximate $d_i(t)$:

$$d_i(t) = \alpha_{i,0} + \alpha_{i,k} \sin\left(\frac{2\pi k_i t}{T}\right) + \beta_{i,k} \cos\left(\frac{2\pi k_i t}{T}\right) \qquad (4)$$

where k_i indicates a particular frequency. When $\alpha_{i,k} = \beta_{i,k} = 0$ there is no nonlinear trend, and the test becomes the IPS test. For selecting the best fitting single frequency, we follow the data driven method used by Davies (1987). The grid search method is as follows. Estimate (4) for each of the 6 integer frequencies of (k_i) 0, 1, 2, 3, 4 and 5. Select the single $k_i = \hat{k}_i$ that minimizes the sum of squared residuals (SSR). Calculate the $F\left(\hat{k}_i\right) = MaxF\left(k_i\right)$ test statistics following Enders and Lee (2012a,b) and Im et al. (2003). The testing regression is:

$$\Delta y_{i,t} = \rho_i y_{i,t-1} + c_{i,1} + c_{i,2} t + c_{i,3} \sin\left(\frac{2\pi k_i t}{T}\right) + c_4 \cos\left(\frac{2\pi k_i t}{T}\right) + e_{i,t} \qquad (5)$$

The test is constructed by standardizing the average of the individual Enders and Lee (2012b), EL, statistics across the whole panel. To this end, construct the panel EL test for the i^{th} individual as a *t*-statistic for testing $\rho_i = 0$ in (5), thus:

$$t_{i,\mathrm{F}}(N,T) = \frac{\Delta y_i^{'}\mathrm{M}_z y_{i,-1}}{\hat{\sigma}_{i,fr}\left(y_{i,-1}^{'}\mathrm{M}_z y_{i,-1}\right)^{1/2}} \tag{6}$$

where $\Delta y_i = \left(\Delta y_{i,1}, \Delta y_{i,2}, ..., \Delta y_{i,\mathrm{T}}\right)^{'}$, $y_{i,-1} = \left(y_{i,0}, y_{i,1}, ..., y_{i,\mathrm{T}-1}\right)^{'}$, $\mathrm{M}_z = I_T - Z\left(Z^{'}Z\right)^{-1}Z^{'}$, $Z = \left(\tau, \Upsilon_1, \Upsilon_2\right)$, $\tau = \left(1,1,...,1\right)^{'}$, $\Upsilon_1 = \left(\sin\left(2\pi k/\mathrm{T}\right), \sin\left(2\pi k 2/\mathrm{T}\right), ..., \sin\left(2\pi k \mathrm{T}/\mathrm{T}\right)\right)^{'}$ $\Upsilon_2 = \left(\cos\left(2\pi k/\mathrm{T}\right), \cos\left(2\pi k 2/\mathrm{T}\right), ..., \cos\left(2\pi k \mathrm{T}/\mathrm{T}\right)\right)^{'}$, and $\hat{\sigma}_i$ is the consistent estimator such that $\hat{\sigma}_i^2 = \Delta y_i^{'}\mathrm{M}_{i,z}\Delta y_i / \left(T-4\right)$ in which $\mathrm{M}_{i,z} = I_T - G_i\left(G_i^{'}G_i\right)^{-1}G_i^{'}$ and $G_i = \left(Z, y_{i,-1}\right)$.

For a fixed T the panel test statistic is:

$$\overline{t}_{FIPS} = N^{-1}\sum_{i=1}^{N} t_{i,F} \tag{7}$$

In the Appendix we demonstrate that this test has good size and power.

III.IIII Sieve Bootstrap Algorithm under Cross-Sectional Dependence

One frequently encountered problem in panel regression models is the presence of cross-sectional dependence (CSD). The limiting distribution of panel unit root test statistics is not valid in the presence of correlated errors. Banerjee et al. (2004, 2005) assess the finite sample performance of available panel unit root tests and find that they all experience severe size distortions when the errors are cross-sectionally correlated. To overcome this issue, tests based on including unobserved and/or observed factors as additional regressors in the test equations have been suggested (Moon and Perron 2004, Bai and Ng 2004, Pesaran 2007, Omay and Kan 2010, Kapetanios et al. 2011, Pesaran et al. 2013). Alternatively, Maddala and Wu (1999), Chang (2004), Ucar and Omay (2009), Palm et al. (2011) and Emirmahmutoglu and Omay (2014) use bootstrap methods to obtain good size properties of their tests. The main aim is to approximate a test statistic's distribution by sieve bootstrap resampling to preserve the pattern of CSD in the panel (Pesaran et al. 2013).

We follow Chang's (2004) sieve bootstrap methodology to address the CSD problem.[13] New critical values are generated when using the sieve bootstrap methodology, since those tabulated in Tables A.1 and A.2 in the Appendix are invalid under CSD.[14] The sieve bootstrap scheme we use follows a five-step algorithm, that is:

(i) For each country estimate (10) by OLS allowing a different lag p_i and frequency order k_i for each i:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + d_i(t) + \sum_{j=1}^{p_i} \delta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{10}$$

After first determining the frequency $\hat{k}_i$ (see Section III.II), we select the lag orders using the Schwartz criterion with the maximum lag set at $p_i = 6$.

(ii) Following Basawa et al. (1991), the unit root null hypothesis is imposed to generate bootstrap sample residuals using:

$$\hat{\varepsilon}_{i,t} = \Delta y_{i,t} - \sum_{j=1}^{p_i} \hat{\delta}_{i,j} \Delta y_{i,t-j} \tag{11}$$

(iii) We follow Stine (1987) and centre the residuals as follows:

$$\tilde{\varepsilon}_t = \hat{\varepsilon}_t - (T-p-1)^{-1} \sum_{t=p+1}^{T} \hat{\varepsilon}_t \tag{12}$$

[13] The CSD problem arises in Chang's (2004) investigation of real interest rates because the OECD countries considered are highly integrated.

[14] In the Appendix we employ Monte Carlo simulations to illustrate the finite sample properties of the proposed test using the homogenous frequency version. By eliminating all other effects, such as employing a different estimator, we show the detrimental effect of taking the frequency term to be homogenous in heterogeneous panel unit root testing. This power analysis can be compared to the test of Lee et al. (2016), as discussed in the Appendix, as we are taking a homogenous frequency for comparison. Further, Omay et al. (2018) demonstrate the superior power performance of a bootstrap with respect to a CCE estimator in a structural break panel unit root test setting.

where $p = \max_i(p_i)$, $\hat{\varepsilon}_t = \left(\hat{\varepsilon}_{1,t}, \hat{\varepsilon}_{2,t}, ..., \hat{\varepsilon}_{N,t}\right)'$ and we develop the $NxT\left[\tilde{\varepsilon}_{i,t}\right]$ matrix from these residuals. We randomly select a full column (with replacement) from this matrix each time, preserving the cross-covariance structure of the errors. We denote the bootstrap residual as $\tilde{\varepsilon}^*_{i,t}$ where $t = 1, 2, ..., T^*$ and $T^* = T + m$ in order to ensure the stationarity of $\Delta y^*_{i,t}$. *m* denotes the number of bootstrap residuals at the start of the sample that we discard, where we set *m*=*T*.

(iv) We produce the bootstrap $\Delta y^*_{i,t}$ data recursively from

$$\Delta y^*_{i,t} = \sum_{j=1}^{p_i} \hat{\delta}_{i,j} \Delta y^*_{i,t-j} + \tilde{\varepsilon}^*_{i,t} \tag{13}$$

where $\hat{\delta}_{i,j}$ are the estimates used in step (ii) and $\Delta y^*_{i,t-p_i} = 0$ for $p_i = 1, 2, ..., 6$.

(v) We generate nonstationary samples from the partial sums:

$$y^*_{i,t} = \sum_{j=1}^{t} \Delta y^*_{i,j} \tag{14}$$

The bootstrap statistics $\overline{t}^*_{fr}$ are computed for each replication by running the regression:

$$\Delta y^*_{i,t} = \theta_i y^*_{i,t-1} + d_i(t) + \sum_{j=1}^{p_i} \mu_{i,j} \Delta y^*_{i,t-j} + \upsilon_{i,t} \tag{15}$$

while noting that the last *T* observations of $y^*_{i,t}$ and $\Delta y^*_{i,t}$ are used in this regression. The bootstrap empirical distribution of the $\overline{t}^*_{FIPS}$ statistic is generated by employing 2000 replications.

To investigate whether there is CSD in the unemployment panel data, we implement three alternative tests. The first is the following CD test proposed by Pesaran (2004), which is based on the average of pairwise correlation coefficients of the OLS residuals from the individual regressions in the panel:

$$CD_{LM1} = \sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right) \tag{16}$$

where $\hat{\rho}_{ij}$ are the sample estimates of the pair-wise correlation of the OLS residuals e_{it} from (5), thus:

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it}e_{jt}}{\left(\sum_{t=1}^{T} e_{it}^2\right)^{1/2}\left(\sum_{t=1}^{T} e_{jt}^2\right)^{1/2}} \tag{17}$$

The second test is the Breusch and Pagan LM test, below:

$$CD_{LM2} = T.\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}^2\right) \tag{18}$$

The third test is the scaled version of the Breusch and Pagan LM test, that is:

$$CD_{LM3} = \sqrt{\frac{1}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(T.\hat{\rho}_{ij}^2 - 1)\right) \tag{19}$$

IV. The Data and Empirical Results

Since all our hypotheses refer to the long-run, we use a relatively long sample of annual unemployment data for the period 1960-2016 for 23 OECD countries. These include 14 European Union (EU) countries (Austria, Belgium, Denmark, Finland, France, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and the United Kingdom (UK)). The other 9 OECD countries are Australia, Canada, Japan, Iceland, New Zealand, Norway, Switzerland, Turkey and the United States (U.S.). All data are obtained from the AMECO online database.

We only use panel unit root tests because they are more powerful than their time-series counterparts. However, we first report the CSD test results outlined above in Table 1.

Table 1. Cross Sectional Dependence Tests

CD_{LM1}	CD_{LM2}	CD_{LM3}
65.196 (0.000)	229.758 (0.000)	5421.285 (0.000)
Note: The p-values of the tests are given in parentheses.		

Table 1 indicates that severe CSD is exhibited by the data. This is expected in our panel data that includes highly integrated EU countries that coordinate their national economic policies and Euro area countries whose monetary policies are managed by a single monetary authority. We use Chang's (2004) sieve bootstrap to account for this evident CSD and therefore improve the power of our panel tests.

In typical applications of panel unit root tests, the unit root null can be rejected if only one country's series in the panel is stationary. Hence, we apply our panel unit root tests using the sequential panel selection method (SPSM) proposed by Chortareas and Kapetanios (2009) to identify the stationary and nonstationary components of the panel. Further, from (1) and (2) the rejection of the hysteresis hypothesis involves unemployment rates that are stationary around the natural rate. Therefore, when testing the null hypothesis of a unit root, the deterministic specification should contain only a constant term. As argued in Camarero et al. (2006), if a time trend is also included it would hide the existence of structural breaks that cause shifts in the natural rate. Hence, we only include a constant term in our test regressions.

We start by testing the pure linear hysteresis hypothesis against the weak natural rate alternative hypothesis by applying the IPS conventional panel unit root test with Chang's (2004) bootstrap methodology to correct for CSD. The IPS test allows for heterogeneous panels and is based on the average of the individual ADF unit root tests computed from each time-series. The null hypothesis is that all individual time-series contain a unit root, while the alternative allows for some, though not all, individual series to have unit roots. Applying this IPS test to the full panel of 23 countries, the null hypothesis of a unit root in the unemployment rates cannot be rejected at all conventional significance levels (the test statistic is -1.867). This supports the hysteresis hypothesis and rejects the weak natural rate hypothesis. Since the null hypothesis of a unit root is not rejected in the first stage for all 23 countries, the SPSM is not applied further[15].

Next, we test the hysteresis hypothesis against the alternative structuralist hypothesis using two different ESTAR panel unit root tests. The first is the Ucar and Omay (2009), hereafter UO, test that generalizes the Kapetanios et al. (2003), KSS, test to heterogeneous panels using the panel unit root testing framework of Im et al. (2003). However, this test implicitly assumes symmetric mean reversion (Sollis, 2009) which may be overly restrictive for many important economic variables, including the unemployment rate. The unemployment rate may display asymmetric adjustment over the business cycle, rising quickly in recessions and falling slowly during expansions[16]. Sollis (2009) developed a unit root test that allows for asymmetric ESTAR nonlinearity under the alternative and Emirmahmutoglu and Omay (2014), hereafter EO, have extended this to a panel framework. Therefore, the second ESTAR panel unit root test that we utilize is the EO test.

The ESTAR panel unit root test results (both applied using Chang's (2004) Sieve bootstrap method) are reported in Table 2. Panel A presents the sequence (denoted "Seq") of results of the UO test applied using the SPSM procedure, where the individual (single country) minimum KSS statistics are used to decide which individual series are dropped from the panel in each step of the sequence. The countries where the series are stationary are thus identified in Table. Panel B reports the results of the EO test, also applied using the SPSM procedure, where the individual maximum $F_{AE,\mu}$ statistics (the univariate counterpart of the EO test) are used to determine the individual series to be removed from the panel in each iteration of the sequence. The countries where the series are stationary are identified in Table 2. In each panel, the countries given in the column headed "Countries" are sorted in order according to the univariate counterparts of each panel unit root test. Hence, for both ESTAR tests Greece is the first country

[15] When the IPS statistic is specified with a time trend included the unit root null is rejected for the full sample. By using the SPSM method, the unemployment rates of Austria, Greece, Iceland, Japan, Luxembourg, Sweden, Switzerland, Turkey and the U.S. are found to be stationary. Thus, the hysteresis hypothesis is rejected in favour of the natural rate theory in only 9 out of 23 countries (about 39% of rejections). However, as noted above, this deterministic specification can hide the presence of structural breaks, thus shifting the natural rate. The results are available upon request.
[16] For further discussion see Enders and Granger (1998) and the references therein.

excluded from the panel of 23 countries and the panel test is repeated with only the remaining 22 countries included in the next iteration of the sequence. To further divide the panel into its stationary and nonstationary components, the SPSM procedure was implemented until the respective UO and EO panel tests failed to reject the unit root null hypothesis at the 10% significance level. This occurred for both tests when the unemployment rates of Greece and Turkey were removed from the panel. Since the SPSM procedure stops the first time the unit root null cannot be rejected we only report the first three steps in the sequence because the unit root null cannot be rejected at the third step. Therefore, the hysteresis hypothesis can only be rejected in 2 countries (Greece and Turkey) out of the 23 included in the panel. For the remaining 21 (91% of) countries, the hysteresis hypothesis cannot be rejected[17]. Thus, allowing for nonlinear dynamics and multiple equilibria in unemployment by employing the ESTAR class of models the hysteresis hypothesis could not be refuted for the majority of OECD countries.

Overall, our application of linear and nonlinear panel unit root tests does not change the conclusions generally obtained in the previous literature. That is, that OECD countries typically exhibit hysteresis and need urgent reforms to restructure their labour markets if they want to cure their current joblessness situation and create new jobs for the youth. However, an important weakness of the panel unit root tests applied so far is that they ignore the possibility of structural breaks.

Table 2. Nonlinear Panel Unit Root Analysis

Panel A: Symmetric Non-Linear Unit				**Panel B: Asymmetric Non-Linear Unit**			
Seq	**Countries**	**UO**	**Min.**	**Seq**	**Series**	**EO**	**Max.**
1	**Greece**	-2.291**	-5.864	1	**Greece**	3.704**	16.935
2	**Turkey**	-2.167**	-5.146	2	**Turkey**	3.102*	13.017
3		-2.025	-3.821	3		2.630	8.827

[17] It is important to note that these results are obtained by including only a constant term in the test regressions. The number of stationary countries increases considerably if both the UO and EO test statistics are specified with a trend term. When the UO test is applied together with the SPSM, the number of unit root rejections increases to 8, with the hysteresis null now rejected additionally in the Netherlands, the U.S., Sweden, Japan, Denmark and Australia. Using the EO test the number of rejections also increases to 8 with Iceland, the Netherlands, Switzerland, Luxembourg, the U.S. and Denmark constituting the stationary countries. The different inference from these two sets of results may be thought of as an indicator of structural breaks. The results are available upon request.

Note: In all cases, *, **, and *** denote rejection of the unit root null hypothesis at the 10%, 5% and 1% significance levels, respectively. We include only a constant term in the test regressions. Test results are obtained by employing 2000 bootstrap replications. UO represents Ucar and Omay' (2009) symmetric ESTAR nonlinear panel unit root test while EO denotes Emirmahmutoglu and Omay's (2014) asymmetric ESTAR nonlinear panel unit root test.

To address the possibility of structural breaks within a panel context we apply the flexible Fourier function to the IPS unit root test (hereafter, FIPS). This is a powerful way to test for a unit root in the presence of smooth structural breaks. We show below that the FIPS test captures the stochastic properties of the unemployment rate quite well. To best of our knowledge, this is the first time that this test has been applied to investigate hysteresis in unemployment rates. The results of the FIPS unit root test applied with Chang's (2004) sieve bootstrap method to deal with CSD and utilizing the SPSM procedure are tabulated in Table 3.

The null hypothesis of a unit root in the unemployment rate was rejected for each iteration in the SPSM sequence of panel tests. Hence, the unit root null is rejected for all 23 countries in our panel. This gives conclusive evidence that the unemployment rates for all our 23 OECD countries are stationary around a mean with slowly evolving structural breaks. This result provides overwhelming evidence in favour of the structuralist hypothesis for our 14 EU countries and our 9 remaining OECD countries, including the U.S. Hence, this is the first time that the hysteresis hypothesis is decisively refuted for these countries. The result arises because we apply a new panel unit root test that accounts for multiple smooth structural breaks (that does not require prior knowledge of the structure of the break), deals with the evident CSD and can identify the countries where the series are stationary and those where they are nonstationary. Hence, our results are not subject to the biases and inability to distinguish the cross-sections that reject the null from those that do not that has affected previous work that does not account for all these factors. Previous findings that support hysteresis for these countries seem to stem from the use of low power unit root tests that fail to account for structural breaks and/or CSD in an appropriate way. Further, when more powerful panel unit root tests that account for CSD reject the unit root null in the previous literature they typically only do so for some (unidentified) countries in the panel and this cannot be taken as evidence against the hysteresis hypothesis for the whole panel that they consider. Our methodology addresses all these drawbacks.

Table 3. Fourier IPS Unit Root Analysis

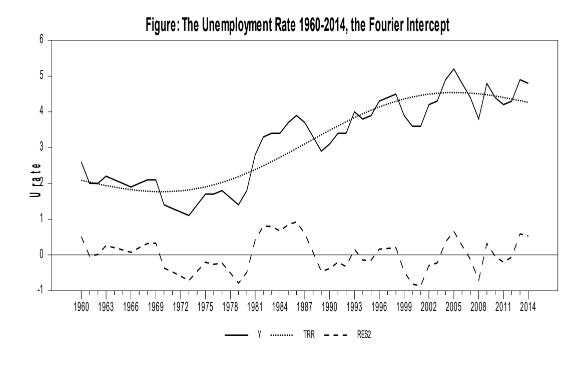
Countries	***Frequency***	***F-test***	***Individual EL***	Panel EL

Finland	1	81.342	-4.943	-3.936***
Canada	1	12.810	-4.912	-3.890***
Switzerland	1	37.413	-4.903	-3.841***
U.S.	1	31.189	-4.762	-3.788***
Sweden	1	24.026	-4.663	-3.737***
Luxembourg	1	10.198	-4.523	-3.686***
Norway	1	64.765	-4.409	-3.636***
Iceland	2	9.218	-4.363	-3.588***
Australia	3	5.582	-4.299	-3.536***
Turkey	1	15.576	-4.123	-3.482***
Denmark	1	10.542	-3.906	-3.432***
Netherlands	3	13.839	-3.868	-3.393***
Belgium	2	21.096	-3.837	-3.350***
UK	1	37.492	-3.796	-3.301***
Portugal	1	23.373	-3.736	-3.246***
Austria	1	23.410	-3.710	-3.185***
Japan	2	22.972	-3.633	-3.110***
Ireland	2	17.582	-3.270	-3.022***
France	2	8.542	-3.208	-2.973***
New Zealand	1	17.668	-3.185	-2.914***
Spain	1	8.454	-2.914	-2.823***
Italy	1	35.599	-2.855	-2.778***
Greece	2	15.886	-2.700	-2.700***

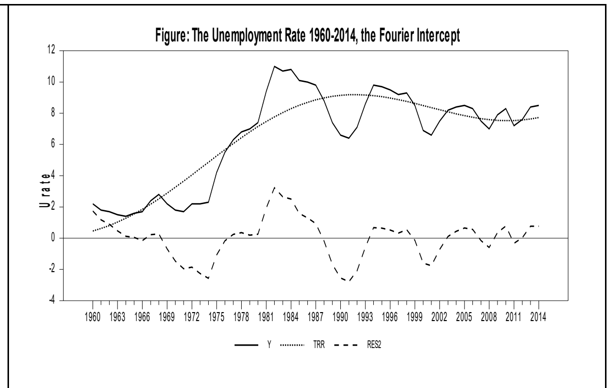
Note: In all cases, *, **, and *** denote rejection of the unit root null hypothesis at the 10%, 5% and 1% significance levels, respectively. We include only a constant term in the FIPS test regression.

As we find support for unemployment rates being stationary around a slowly evolving mean, which is consistent with the structuralist hypothesis, we plot the equilibrium unemployment rates that are obtained from the Fourier regressions in Figure 1. Visual inspection of graphs shows that all the countries included in this study are subject to smooth structural breaks that are shifting their natural rates. These graphs also suggest that the natural rate towards which unemployment is attracted has increased over time for the vast majority of countries that we consider.

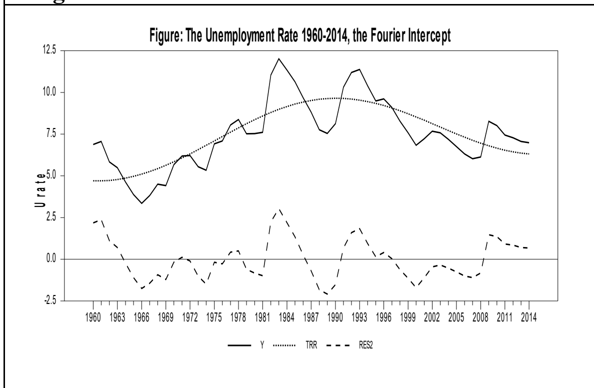
Figure 1. Nonlinear Fourier trends and the unemployment rates of countries



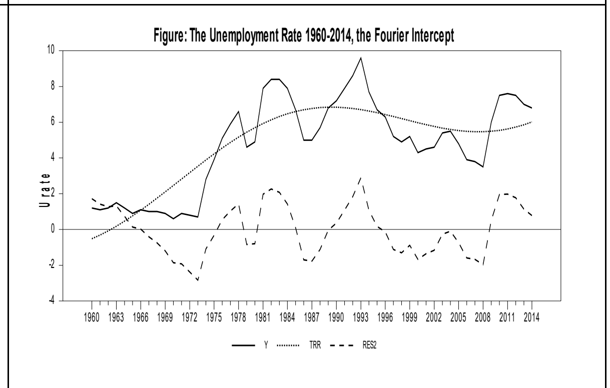
Belgium



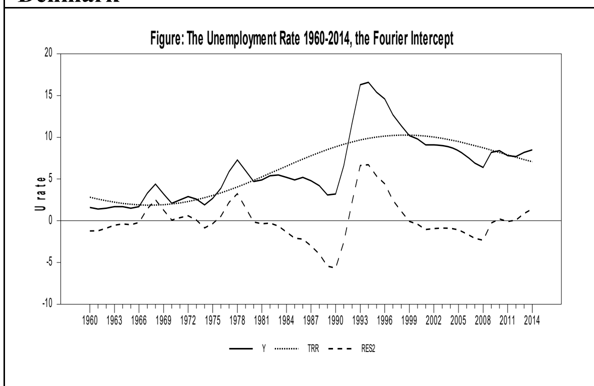
Canada



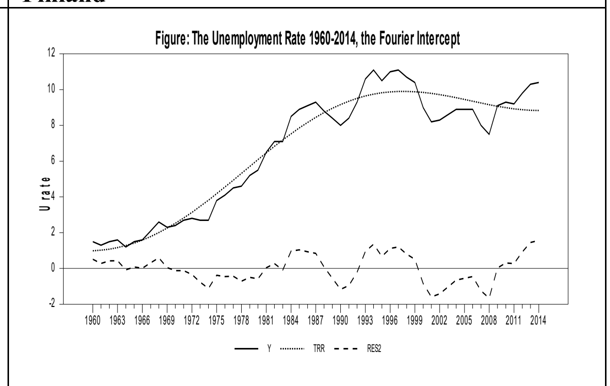
Denmark



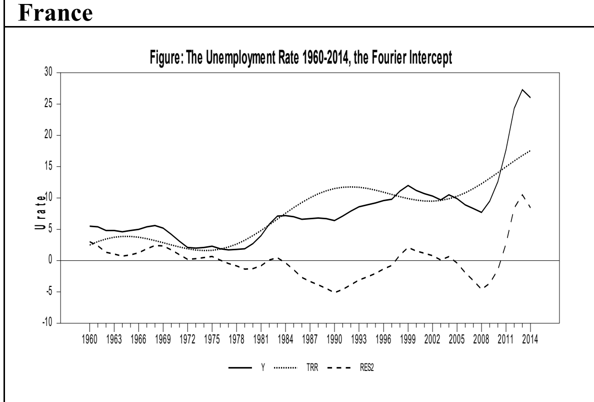
Finland



France



Greece



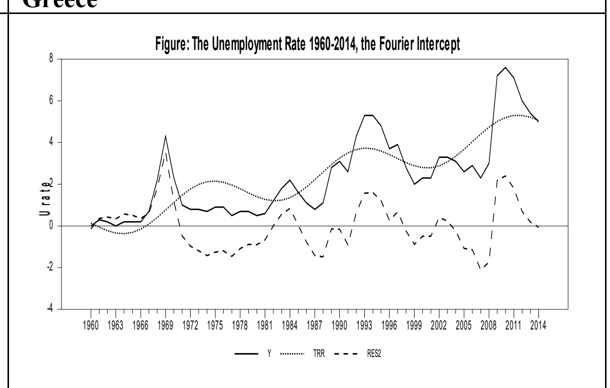
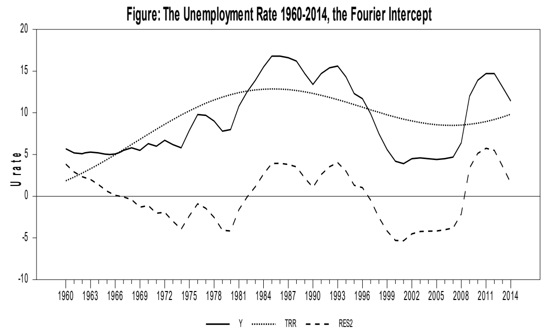
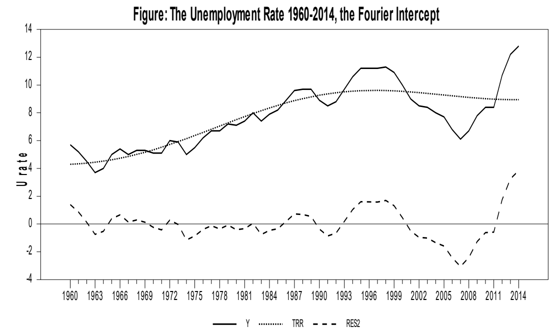


Figure 1 (continued).

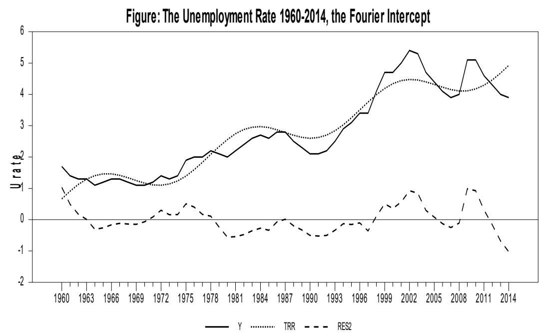
Iceland



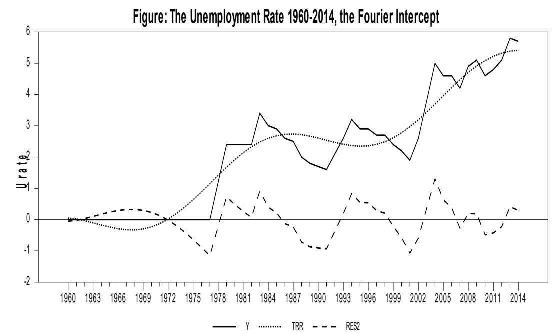
Ireland



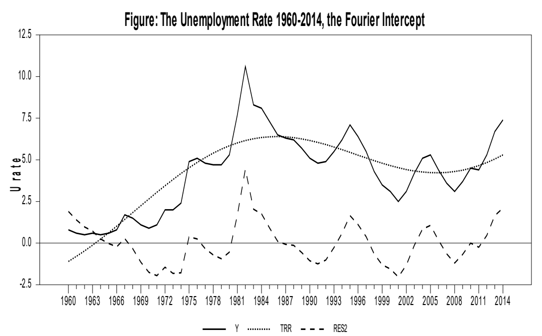
Italy



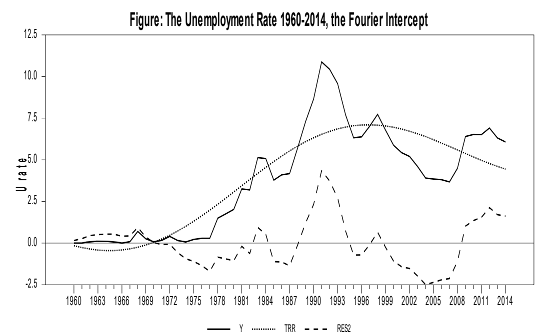
Japan



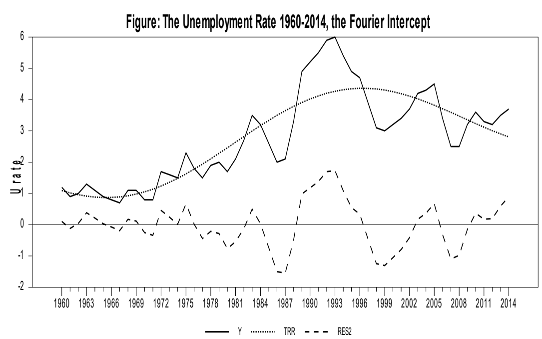
Luxembourg



Netherlands



New Zealand



Norway

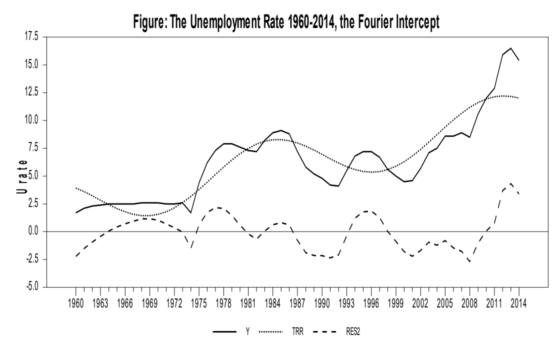
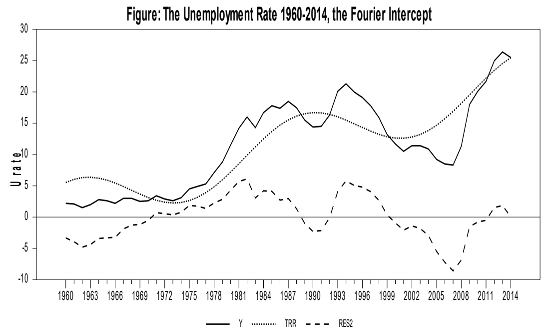
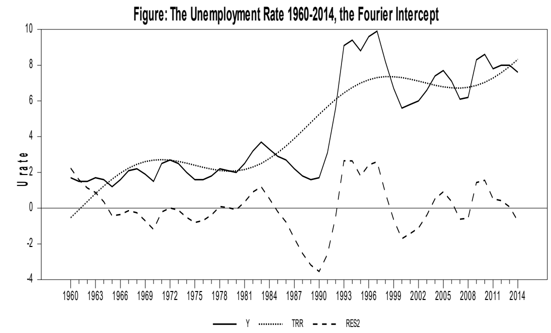


Figure 1 (continued).

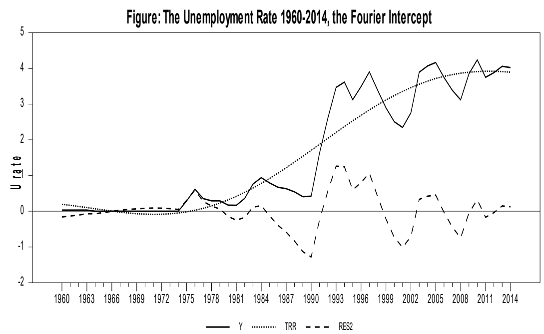
Portugal



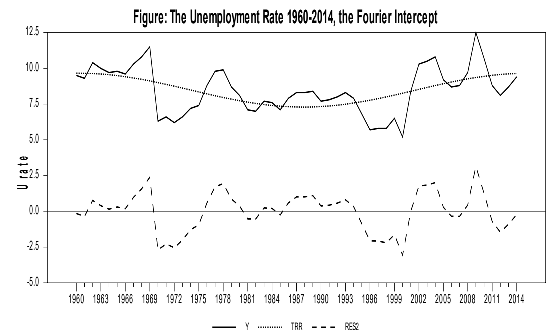
Spain



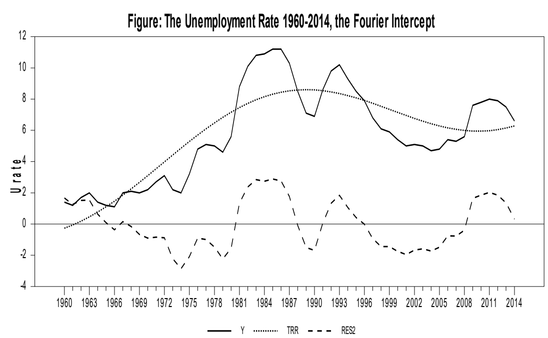
Sweden



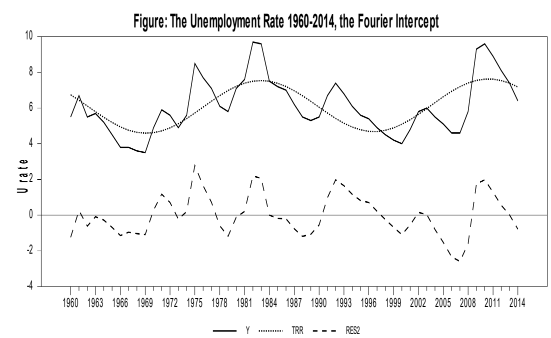
Switzerland



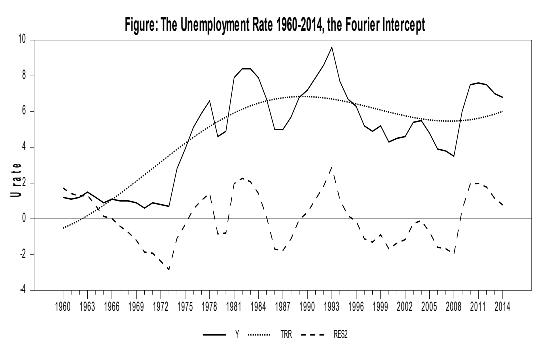
Turkey



UK



U.S.



V. Conclusion

This study analyzed the possibility 'that part of the cyclical increase in unemployment' becomes structural. The OECD estimates of the NAIRU (non-accelerating inflation rate of unemployment) suggest that it has tended to increase since the start of the financial crisis in several OECD countries, particularly Greece, Portugal and Spain. There is also evidence that a growing share of the unemployed experience increased difficulties in finding work – even after the reduced number of job vacancies is taken into account.

To this end, we developed a new panel unit root test using the Fourier function. This test rejects the hysteresis hypothesis for our entire panel of 23 OECD countries. However, hysteresis is not rejected in favour of the natural rate hypothesis (where the unemployment rate will eventually return to a constant natural rate), instead we favour the structuralist hypothesis (where the unemployment rate is continually forced towards an evolving natural rate). Our results also suggest that natural rate has increased in the vast majority of OECD countries as a result of structural factors.

In the short term, an improvement in labour market conditions is largely dependent upon a broader economic recovery and is thus shaped by factors that labour market authorities cannot directly control. For example, the recent divergence between declining unemployment in the United States and rising unemployment in many countries in the Euro area reflects the impact of the banking and sovereign debt crisis in a number of European countries. The resulting stress in European financial markets, coupled with a sharp shift toward fiscal consolidation, is depressing aggregate demand and job creation in Europe. Countries must respond to these developments with appropriate macroeconomic policy measures, including taking immediate steps to stabilize the European banking system. There is also a case for some further fiscal policy easing in those countries that still have some flexibility in this area, although this must be grounded in a credible medium-term strategy of fiscal consolidation. In addition, further monetary policy easing can play a crucial role in Europe to support growth in the short term.

Overall, promoting demand should remain a key policy objective where the recovery has been less robust, accompanied by reinforced measures to combat structural unemployment. Priority should be given to employment and training measures for the long-term unemployed who typically face significant barriers to finding work and are most likely to quit the labour force.

Appendix

Example critical values (for k=1 and that do not account for CSD) of $\overline{t}_{FIPS}$ are given in Table A.1 and Table A.2.

Table A.1 Critical Values of $\overline{t}_{\alpha}$ Statistic Fourier k = 1

T/N	5	10	15	20	25	50	100
1%							
30	-3.456	-3.147	-3.011	-2.937	-2.876	-2.743	-2.649
50	-3.365	-3.085	-2.943	-2.873	-2.828	-2.693	-2.600
70	-3.330	-3.053	-2.924	-2.840	-2.799	-2.668	-2.580
100	-3.306	-3.021	-2.905	-2.835	-2.784	-2.655	-2.563
5%							
30	-3.147	-2.935	-2.843	-2.781	-2.742	-2.648	-2.582
50	-3.076	-2.874	-2.779	-2.729	-2.695	-2.600	-2.536
70	-3.050	-2.849	-2.760	-2.703	-2.672	-2.578	-2.515
100	-3.028	-2.830	-2.744	-2.690	-2.654	-2.564	-2.500
10%							
30	-2.982	-2.819	-2.748	-2.701-	-2.671	-2.599	-2.547
50	-2.925	-2.764	-2.690	-2.653	-2.625	-2.552	-2.500
70	-2.902	-2.742	-2.674	-2.630	-2.604	-2.529	-2.481
100	-2.882	-2.724	-2.656	-2.612	-2.586	-2.517	-2.465

Notes: The $\overline{t}_{FIPS}$ test statistics (for k=1 and that do not account for CSD) are computed as simple averages of individual test statistics. Simulated 1%, 5% and 10% critical values are reported. Simulations are based on 50,000 replications.

Table A.2 Critical Values of the $\overline{t}_{\alpha}$ + Trend Statistic k = 1

T/N	5	10	15	20	25	50	100
1%							
30	-4.194	-3.905	-3.788	-3.335	-3.669	-3.553	-3.563
50	-4.030	-3.775	-3.672	-3.607	-3.560	-3.453	-3.379
70	-3.982	-3.729	-3.627	-3.566	-3.524	-3.415	-3.343
100	-3.919	-3.694	-3.586	-3.532	-3.490	-3.391	-3.320
5%							
30	-3.902	-3.711	-3.628	-3.581	-3.552	-3.471	-3.412
50	-3.773	-3.602	-3.530	-3.480	-3.450	-3.377	-3.326
70	-3.717	-3.560	-3.485	-3.444	-3.415	-3.341	-3.293
100	-3.683	-3.527	-3.459	-3.415	-3.388	-3.318	-3.268
10%							
30	-3.747	-3.608	-3.549	-3.512	-3.488	-3.427	-3.383
50	-3.636	-3.510	-3.455	-3.418	-3.394	-3.338	-3.300
70	-3.590	-3.471	-3.414	-3.383	-3.361	-3.304	-3.266
100	-3.563	-3.440	-3.389	-3.356	-3.335	-3.281	-3.241

Notes: The $\overline{t}_{FIPS}$ test statistics (for k=1 and that do not account for CSD) are computed as simple averages of individual test statistics. Simulated 1%, 5% and 10% critical values are reported. Simulations are based on 50,000 replications.

To evaluate the size of $\overline{t}_{FIPS}$, we use the following data generating process (DGP):

$$y_{i,t} = y_{i,t-1} + \varepsilon_{i,t} \quad \text{(A.1)}$$

To assess the power of $\bar{t}_{FIPS}$, we build the stationary data around a gradually changing mean using:

$$y_{i,t} = \alpha_{i,0} + \alpha_{i,k} \sin\left(\frac{2\pi k_i t}{T}\right) + \beta_{i,k} \cos\left(\frac{2\pi k_i t}{T}\right) + u_{i,t} \quad \text{(A.2)}$$

$$u_{i,t} = 0.8u_{i,t-1} + \varepsilon_{i,t} ,$$

with $u_{i,0} = 0$, $\varepsilon_{i,t} \sim NID\left(0, \sigma_i^2\right)$, and $\sigma_i^2 \sim iidU\{0.5, 1.5\}$

Table A.3 presents the empirical size for combinations of N = 5, 25, 50, and 100 and T = 30, 50, and 100. We observe that the empirical size of the proposed test is quite close to the 5% nominal size.

Table A.3 Empirical Size of the Fourier IPS Test

T/N	5	25	50	100
30	0.051	0.049	0.055	0.055
50	0.050	0.047	0.048	0.049
100	0.049	0.048	0.052	0.051

Table A.4 presents the power of the Fourier IPS test statistic (k=1) for various combinations of N and T and different values of sine and cosine parameters, $\alpha_{i,k}$ and $\beta_{i,k}$ γ_i, respectively. Table A.4 suggests that the power performance of the proposed test is very good. Notably, for our sample the maximum T is 57 (even allowing for lags and differencing T would not be below 50) which suggests power very close to 1 for N = 23 (approximately 25) and good power for small N (say N = 5), as would be used when applying the SPSM procedure.

Table A.4 Empirical Power of the Fourier IPS Test (k=1)

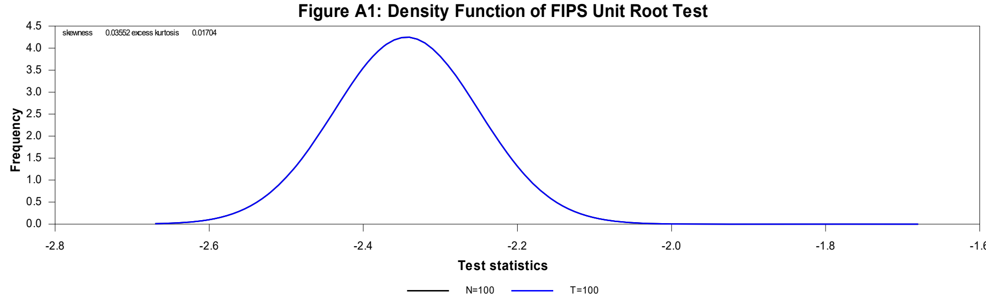
T/N	5	15	25	50
	$\alpha_{i,k}=0.0$ $\beta_{i,k}=5.0$			
30	0.315	0.759	0.944	1.000
50	0.751	0.998	1.000	1.000
100	1.000	1.000	1.000	1.000
	$\alpha_k=3.0$ $\beta_k=0.0$			
30	0.481	0.914	0.994	1.000
50	0.807	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000
	$\alpha_k=3.0$ $\beta_k=5.0$			
30	0.274	0.707	0.906	1.000
50	0.754	0.996	1.000	1.000
100	1.000	1.000	1.000	1.000

For the power and size analysis, the homogeneity assumption of the Lee, Wu, and Yang (2016) test is interesting to consider because it was the first panel unit root test based on the Fourier function in the presence of a multifactor error structure and assumes homogeneous frequency. However, Omay and İren (2020) argue that economic data will have heterogeneous structural breaks arising from different economic structures in various countries. For example, global shocks, spillover effects, and country-specific political conditions can cause the timing of structural breaks in different countries to be different. They also suggest that as the time dimension in a panel rises the heterogeneity bias afflicting panel unit root tests increases. Omay and İren's (2020) size and power analysis demonstrate the problems caused by the homogeneity assumption. Therefore, if heterogeneity is ignored, panel unit root tests involving structural breaks may yield false conclusions.

The structure discussed by Omay and İren (2020) is the fractional frequency Fourier approach. This is also used by Omay, Emirmahmutoğlu and Shahzad (2021) to demonstrate the heterogeneity bias more clearly. Omay and İren (2020) also examined this structure within the residual-based cointegration test framework. They discuss in detail heterogeneity bias and the problems arising from the homogeneity assumption. Their power and size results provide excellent explanations about the problems associated with the homogeneous integer frequency assumption. For further information see Omay and İren (2020), Cai and Omay (2021), Omay, Emirmahmutoğlu and Shahzad (2021), Omay, Emirmahmutoğlu, Shahzad and Nor (2021), and Omay and Baleanu (2021).

Figure A1 below graphs the density function of $\bar{t}_{FIPS}$ for $T=100$ $N=100$, with 50,000 replications.

Figure A1: Density Function of FIPS Unit Root Test



The derivation of the density function for the panel integer frequency Fourier form unit root test obtained above is available in Omay, Emirmahmutoğlu and Shahzad (2021) for fractional frequencies. Derivation of this asymptotic distribution for an integer frequency time series unit root test is given in Emirmahmutoğlu, Omay Shahzad, and Nor (2021). If Emirmahmutoğlu, Omay Shahzad, and Nor's (2021) Proposition 1 is followed by Omay, Emirmahmutoğlu and Shahzad's (2021) Theorem 1 and Appendix B, the asymptotic distribution specified by equation A.3 is obtained.

$$t_{i,FIPS}(N,T) \xrightarrow{d} \frac{\int_0^1 W_i(k,r)\,dW_i(r)}{\left(\int_0^1 W_i(k,r)^2\,dr\right)^{0.5}} \qquad \text{(A.3)}$$

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