General Trade Equilibrium of Integrated World Economy

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ABSTRACT

The general trade equilibrium is one of the most critical topics in international economics. Even for the simplest 2×2×2 Heckscher-Ohlin model, it is not easy to reach its equilibrium. This paper studies the approaches to get the general trade equilibrium and the structure of equalized factor prices. The most straightforward and convincing process is to use the trade volume defined by domestic factor endowments that Helpman and Krugman (1985) proposed. This paper uses their idea to attain the general trade equilibrium of factor price equalization simply. The study shows that the equalized factor prices ensure gains from trade for both countries. The optimality property of the equilibrium is that the trade volume achieves its maximum value.

Keywords:
Factor content of trade, factor price equalization, General equilibrium of trade; Integrated World Equilibrium; IWE

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1. INTRODUCTION

Samuelson said, "Historically the development of economic theory owes much to the theory of international trade." (1938). International trade is a subject that mentions general equilibrium more than any other economic subject.

The Heckscher-Ohlin model is ideal for exploring the general price-trade relationship among factor prices, commodity prices, production outputs, and trade volumes. Samuelson (1948) presented the famous theorem of factor price equalization. Immediately, he made a verbal argument that the equalized factor prices will not change when factors are mobilized across countries (see Samuelson 1949). Thirty years later, Dixit and Norman (1980) provided the Integrated World Equilibrium (IWE) to illustrate the factor price equalization (the FPE), which fulfilled the factor mobility analysis perfectly. They demonstrated that the world prices remain the same when the allocation of factor endowments changes within the FPE set in the IWE. Helpman and Krugman (1985) normalized the assumption of the integrated equilibrium. Deardorff

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(1994) illustrated the conditions of the FPE for many goods, many factors, and many countries by the IWE approach. He discussed the FPE for all possible allocations of factor endowments within lenses identified.

McKenzie (1955) proposed the cone of diversification of factor endowments, which is vital to understand FPE and trade from production supply constraints. He provided a mathematical demonstration of the existence of the FPE for many factors and many goods. Fisher (2011) proposed the concept of goods price diversification cone, which is the counterpart of the diversification cone of factor endowments. He also offered another brilliant idea of the intersection of goods price cones to illustrate the price-trade relationship when countries have different technologies.

Vanek (1968) variegated the preference taste on the Heckscher-Ohlin model by the share of GNP, which engaged prices with trade and consumption. It resulted in the HOV studies to convert the assumption of homothetic taste into consumption balance.

Woodland (2013, pp39) described the importance of the general equilibrium, "General equilibrium has not only been important for a whole range of economics analyses but especially so for the study of international trade." Deardorff (1982, pp685) said, "A trade equilibrium is somewhat more complicated."

The one focus of studies on the general equilibrium for constant returns and perfect competition is the social utility function and direct and indirect trade utility function (offer curve). It is difficult either for those approaches to get a desired price-trade equilibrium. It provided a framework for solutions of equilibriums from consumption.

International economists had paid much attention to price-trade equilibriums and achieved many milestone results. Helpman and Krugman (1985, pp23) proposed a unique idea of trade volume by domestic factor endowments. They derived an insight economic logic as "the differences in factor composition are the sole basis of trade." (See Helpman and Krugman 1985, pp24). That moves an enormous step toward general trade equilibrium after Dixit and Norman's integrated world equilibrium. In this study, I extend their idea to build an approach to attain the general trade equilibrium within IWE.

The paper shows the optimality property of the equilibrium solution that the trade volume reached its maximum value when factor prices equalized. It illustrates that the world prices at equilibrium are the functions of the world factor endowments. The result is consistent with the factor mobility property of the FPE that Dixit and Norman demonstrated. It also illustrates that the equalized factor prices ensure gains from trade for countries participating in the trade.

The study provides the analytical expression of autarky prices, exactly the price that Samuelson mentioned in 1949. Samuelson's idea is excellent but straightforward that the autarky price of a continent will be the world prices if the continent is divided into two countries artificially, supposing that every other thing remains no changes (See Samuelson 1949). Methodologically, the logic of calculating the world price in the equilibrium solution of this paper can be used to calculate the autarky prices: autarky factor endowments determine the autarkey prices.

We divide this paper into six sections. Section 2 firstly identifies the Dixit-Norman constant, which shows why the world prices remain the same when the allocation of factor endowments changes within the FPE set. Then, it derives the general trade equilibrium by using the trade volume defined with domestic factor endowments by Helpman and Krugman (1985). Section 3 provides another independent approach to confirm the equilibrium solution. It illustrates that the trade volume gets its maximum value at the price-trade equilibrium. Section 4 provides a way to measure autarky prices. The idea is that the autarky factor endowments determine autarky prices. Section 5 is the equilibrium solution for the multiple-country economy. Section 6 is a brief discussion.

2. THE GENERAL TRADE EQUILIBRIUM WITH THE IWE

2.1 The Notation of the Heckscher-Ohlin Model

We take the following typical assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are immobile across countries, but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals, and (7) full employment of factor resources. We denote the Heckscher-Ohlin model as follows. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \qquad (2\text{-}1)$$

where A is the 2×2 matrix of direct factor inputs, X^h is the 2×1 vector of commodities of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular. The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \qquad (2\text{-}2)$$

where $W^h$is the 2×1 vector of factor prices, its elements are r rental for capital and w wage for labor, P^h is the 2×1 vector of commodity prices.

Factor prices will be equalized when prices and trade reach their equilibrium. We denote the world price equations as

$$A'W^* = P^* \qquad (2\text{-}3)$$

The trade balance condition for the factor contents is

$$\frac{r^*}{w^*} = -\frac{F_L^H}{F_K^H} = -\frac{L^H - s^H L^W}{K^H - s^H K^W} \qquad (2\text{-}4)$$

where F_L^H and F_K^H are factor content of trade of country H, $K^W$and L^W are world factor endowments, s^H the share of the GNP of country H to the world GNP.

Embedded in the Heckscher-Ohlin system represented by (2-1), (2-3), and (2-4), there are seven equations with nine endogenous variables in the model which are p_1^* , p_2^* , w^* , r^* , x_1^H , x_2^H , x_1^F , x_2^F, and s^H . There are four exogenous variables K^H , L^H , K^F , and L^F . The system is not determined. By Walras' equilibrium, we can drop one of these market-clearing conditions, such as we can take one price as the numeraire to set its value to 1. That will leave only one uncertain condition for the equilibrium. If we result in that one, we will solve the equilibrium. Some optimal analyses can help with this; some economics principles or logic can help if the approaches are proper.

2.2 Trade Box on IWE Diagram and The Dixit-Norman Constant

The relative world commodity prices $\frac{p_1^*}{p_2^*}$ should lie between the rays of goods price diversification cone (see Fisher, 2011) in algebra as,

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{2-5}$$

This condition can ensure that the factor prices by unit cost equation (2-2) are positive. The range of the shares of GNP s^H, corresponding to the rays of the goods price diversification cone above, can be calculated as

$$s_b^H(p) = s\left(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^W} \tag{2-6}$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^W} \tag{2-7}$$

These are just the range of s^H Leamer (1984, pp9) first proposed as

$$\frac{K^H}{K^W} > s^H > \frac{L^H}{L^W} \tag{2-8}$$

For convenience, we denote two parameters, which are the shares of the factor endowments in the home country to their world factor endowments respectively,

$$\lambda_L = \frac{L^H}{L^W} \tag{2-9}$$

$$\lambda_K = \frac{K^H}{K^W} \tag{2-10}$$

Figure 1 is an IWE diagram added with a trade box and the equal trade volume line (see Helpman and Krugman, 1985, pp24). The dimensions of the diagram represent world factor endowments. The origin of the home country is the lower-left corner. It is the right-upper corner for the foreign country. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Helpman and Krugman (1985, pp15) call the parallelogram the FPE (Factor Price Equalization) set. Suppose that an allocation of the factor endowments is at point *E*, where the home country is capital abundant (we will use this assumption for all analyses of this study). Point *C* represents the trade equilibrium point. It shows the sizes of the consumption of the two countries. Line $\overline{HK}$ is an equal trade volume line, which is parallel to the diagonal line $\overline{OO^*}$ (we will discuss it in the following sub-section) .

We identify the trade box by the range of shares of GNP in (2-8). If a relative commodity price lies in the goods price diversification cone (2-5), the share of GNP by that price lies in the trade box.

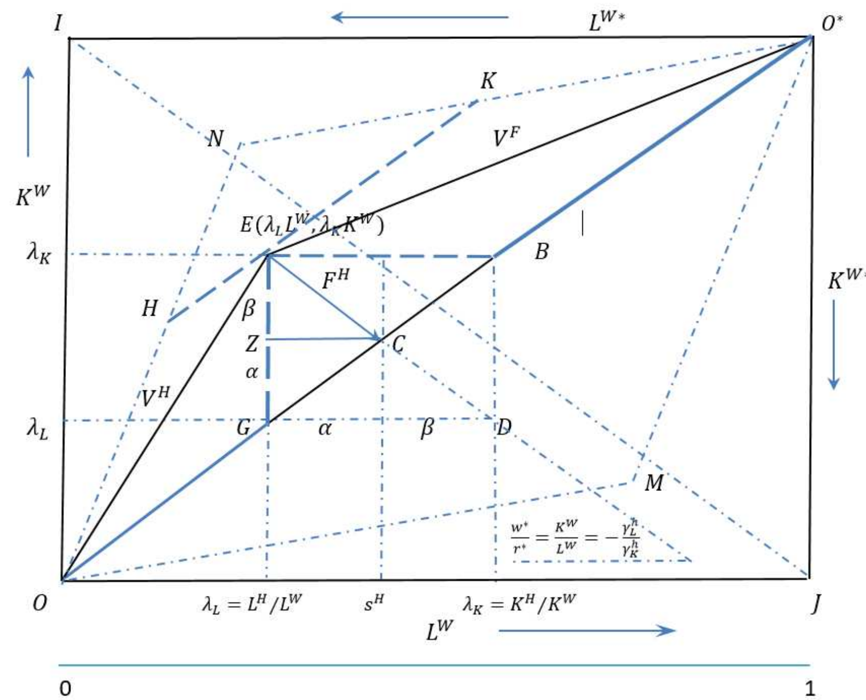

FIGURE 1
IWE Diagram with Trade Box and Equal Trade Volume Line

For a given allocation (or distribution) of factor endowments E, its equilibrium point or the consumption point C needs to fall within the diagonal line $\overline{GB}$ of the trade box.

The share of GNP s^H divides the trade box into two parts: α and β,

$$\alpha = s^H - \lambda_L \tag{2-11}$$

$$\beta = \lambda_K - s^H \tag{2-12}$$

When α increases, the home country's share of GNP increases, and the foreign country's share of GNP decreases, and vice versa. The trade competition between countries is that each country tends to maximize the factor price of its abundant factor to achieve its maximum share of GNP.

We rewrite the trade balance of factor contents (2-4) as

$$\frac{r^*}{w^*} = \frac{(s^H - \lambda_L)}{(\lambda_K - s^H)} \frac{L^W}{K^W} = \frac{\alpha}{\beta} \frac{L^W}{K^W} \tag{2-13}$$

Dixit and Norman found the factor price equalization (FPE) set in the IWE diagram, which implies that $\frac{r^*}{w^*}$ is a constant. Introduce

$$\varphi = \frac{(s^H - \lambda_L)}{(\lambda_K - s^H)} \tag{2-14}$$

Substituting it into (2-13) yields

$$\frac{r^*}{w^*} = \varphi \frac{L^W}{K^W} \tag{2-15}$$

We call φ the Dixit-Norman constant to honor their contribution on the IWE and FPE set. It interprets the factor price equalization in the IWE diagram analytically. If φ remains the same, the world prices will stay the same when the allocation of factor endowments changes within the FPE set in the IWE. The equations (2-15) reduces the mystery of the structures of world commodity prices and equalized factor prices.

2.3 General Trade Equilibrium

The range of φ corresponding (2-8) is

$$\infty > \varphi > 0$$

We need to find what is the value of this constant at equilibrium.

Helpman and Krugman (1985, pp23) defined the trade volume by commodity trades as

$$VT = 2p_1^*\left(x_1^H - s^H x_1^W\right) = -2p_2^*\left(x_2^H - s^H x_2^W\right) \tag{2-16}$$

where x_i^W is world commodity output, i=1, 2.

They also provided another expression of trade volume (see Helpman and Krugman, 1985, pp23). They illustrated that there are some variables (γ_L , γ_K) for all equal trade volumes lines, which satisfy the following relationships:

$$VT = \gamma_L L^H + \gamma_K K^H \tag{2-17}$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \tag{2-18}$$

They illustrated that the equal trade volume curves in the FPE set are straight lines, which are parallel to the diagonal line OO^* in the IWE diagram. The primary argument for the relationships above is that the trade volume is a linear function of K^H and L^Heventually (see Helpman and Krugman 1985, pp23, pp175). The two equations ensure that a higher difference in factor composition leads to a higher trade volume and that trade volume is zero if factor endowments allocate at the diagonal line OO^*. It is the first time to show that world factor endowments somehow relate to the equilibrium relationship. They identified that one of γ_L, γ_K is negative. If country H is relatively capital abundant, its two variables are $\gamma_K > 0$ and $\gamma_L < 0$.

Line $\overline{ED}$ in Figure 1 is the constraint line of the two variables by (2-18), which is parallel to the anti-diagonal line $\overline{IJ}$. Equation (2-17) is an abstract expression of trade volume. It can either serve as a reference to the price-trade equilibrium or as an independent way to solve the equilibrium.

Similarly to the idea of (2-16), the trade volume of net factor contents by factor prices can be expressed[1]

$$VT = 2\left(K^H - s^H K^W\right)r^* = 2F_K^H r^* \tag{2-19}$$

Equation (2-19) is a concrete expression of trade volume.

Two variables $\left(\gamma_L , \gamma_K\right)$ in (2-17) and (2-18) are different across countries. We denote them with a country mark as γ_L^h and γ_K^h, $h = H, F$. This study interprets them as

$$\gamma_L^H = -w^* \tag{2-20}$$

$$\gamma_K^H = r^* \tag{2-21}$$

$$\gamma_L^F = w^* \tag{2-22}$$

$$\gamma_K^F = -r^* \tag{2-23}$$

The variable, corresponding to an abundant factor of its country, takes a positive sign; otherwise, it takes a negative sign.

Substituting (2-20) and (2-21) into (2-17), for the trade volume of country H, yields

$$VT = -w^* L^H + r^* K^H \tag{2-24}$$

[1] Be aware that trade volume of commodity trade (2-16) is different from th trade volume of net factor contents of trade (2-19) quantitively.

Similarly, substituting (2-22) and (2-23) into (2-17), for the trade volume of country F, yields

$$VT = w^* L^F - r^* K^F \tag{2-25}$$

The two countries' trade volumes should be the same. Substituting (2-24) into (2-25) yields

$$-w^* L^H + r^* K^H = w^* L^F - r^* K^F \tag{2-26}$$

It yields,

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \tag{2-27}$$

It is consistent with (2-18) as

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} = -\frac{\gamma_L^H}{\gamma_K^H} = -\frac{\gamma_L^F}{\gamma_K^F} \tag{2-28}$$

It shows that the Dixit-Norman constant is 1. Substituting (2-27) into (2-4) yields

$$s^H = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) = \frac{1}{2}(\lambda_K + \lambda_L) \tag{2-29}$$

Equation (2-24) and (2-25) shows that the difference between the total cost of the abundant factor and the total cost of the scarce factor of a country equals to its trade volume of factor contents. In other words, It shows that the monetary value of the difference in factor composition of a country is its trade volume.

With (2-28), we get the complete equilibrium solution of the Heckscher-Ohlin model as

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^W} + \frac{L^h}{L^W}\right) \qquad (h = H, F) \tag{2-30}$$

$$r^* = \frac{L^W}{K^W} \tag{2-31}$$

$$w^* = 1 \tag{2-32}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{2-33}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{2-34}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = H, F) \tag{2-35}$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = H, F) \tag{2-36}$$

In equation (2-32), we assume $w^* = 1$ to drop one market condition. The factor content of trade (2-35) shows that when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$, then $F_K^H > 0$ and $F_L^H < 0$. It just states the Heckscher-Ohlin theorem.

We now derive the equilibrium by equation (2-17) geometrically to show its economic meaning from a different angle. It will avoid the assumptions (2-20) through (2-23), which is a kind of jump.

The factor endowment vector V^H in country H can be written by Figure 1, as,

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \overrightarrow{OG} + \overrightarrow{EG} \tag{2-37}$$

$\overline{OG}$ represents the part of the factor endowments of country H, which is with the proportion of world factor consumptions as

$$\overrightarrow{OG} = \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} \tag{2-38}$$

$\overline{EG}$ is the excessive capital services, which is out of the proportion of world factor consumptions. It is the difference of factor composition described by Helpman and Krugman. Its value is

$$\overrightarrow{EG} = \begin{pmatrix} (\alpha + \beta)K^W \\ 0 \end{pmatrix} \tag{2-39}$$

The trade volume (2-17) can be rewritten as a dot product of V^H and the pair of the variables $(\gamma_K^H \quad \gamma_L^H)$

$$VT^H = (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} K^H \\ L^H \end{pmatrix} \tag{2-40}$$

Substituting (2-37) into the above yields

$$VT^H = (\gamma_K^H \quad \gamma_L^H) \cdot (\overrightarrow{OG} + \overrightarrow{EG}) = (\gamma_K^H \quad \gamma_L^H) \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} + (\gamma_K^H \quad \gamma_L^H) \begin{pmatrix} (\alpha+\beta)K^W \\ 0 \end{pmatrix} \quad (2\text{-}41)$$

The first term on the right side above is zero by (2-18),

$$(\gamma_K^H \quad \gamma_L^H) \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} = 0 \quad (2\text{-}42)$$

Simplify (2-41) as

$$VT^H = (\alpha+\beta) K^W \gamma_K^H \quad (2\text{-}43)$$

Similarly, the trade volume for country F is

$$VT^F = (\alpha+\beta) L^W \gamma_L^F \quad (2\text{-}44)$$

Substituting (2-43) into (2-44) yields

$$\frac{K^W}{L^W} = \frac{\gamma_L^F}{\gamma_K^H} \quad (2\text{-}45)$$

Rewrite trade balance (2-13) as

$$\frac{\beta w^*}{\alpha r^*} = \frac{K^W}{L^W} \quad (2\text{-}46)$$

Substituting it into (2-45) yields

$$\frac{\gamma_L^F}{\gamma_K^H} = \frac{\beta w^*}{\alpha r^*} \quad (2\text{-}47)$$

Assume

$$\gamma_K^H = r^* \quad (2\text{-}48)$$

Substituting it into (2-47) yields

$$\gamma_L^F = \frac{\beta}{\alpha} w^* \quad (2\text{-}49)$$

Substituting (2-48) into (2-43) yields

$$VT^H = (\alpha+\beta) K^W r^* \quad (2\text{-}50)$$

The trade volume in country H can be expressed as

$$VT^H = 2\beta K^W r^* \quad (2\text{-}51)$$

Substituting it into (2-50) yields

$$(\alpha+\beta) = 2\beta \quad (2\text{-}52)$$

It implies

$$\alpha = \beta \quad (2\text{-}53)$$

Substituting it into (2-46) yields

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \quad (2\text{-}54)$$

2.4 The difference in the composition of factor endowment

The difference in the composition of factor endowment

In the trade box, $\overline{EG}$ is the part of the capital services of country H, which cannot be self-matched for the preference taste,

$$\overline{EG} = \left(\frac{K^H}{K^W} - \frac{L^H}{L^W}\right) K^W \quad (2\text{-}55)$$

The size (or percentage) of $\overline{EG}$ to K^W is

$$\lambda_K - \lambda_L = \alpha + \beta \quad (2\text{-}56)$$

Similarly, $\overline{EB}$ is the labor services of the country F, which cannot be self-matched for the preference taste,

$$\overline{EB} = \left(\frac{K^H}{K^W} - \frac{L^H}{L^W}\right) L^W = (\lambda_K - \lambda_L) L^W \quad (2\text{-}57)$$

ΔEBG is similar to ΔOIO^* , we have the relationship,

$$\frac{\overline{EG}}{\overline{EB}} = \frac{K^W}{L^W} \tag{2-58}$$

The $\overline{EG}$ and $\overline{EB}$ represent the differences in the composition of factor endowments in each country. Helpman and Krugman illustrated that they are the sole source for trade. It is an extreme reason why the Dixit-Norman constant is 1.

At the equlibrium, the value of $\overline{EG}$ is

$$(\lambda_K - \lambda_L)K^W r^* = (\alpha + \beta)K^W r^* \tag{2-59}$$

It equals trade volume.

The consumption built by trade

Triangle ΔZGC in Figure 1 represents the consumption of country H, which is established by free trade. Country H exports the excessive capital services, βK^W and imports labor services, αL^W. Its consumption volume within the trade box is

$$CV = \alpha K^W r^* + \alpha L^W w^* \tag{2-60}$$

The consumption size, built by trade, respective to the world GNP, is α. We see it in the following way clearly,

$$\alpha = \frac{VC}{K^W r^* + L^W w^*} = \frac{\alpha K^W r^* + \alpha L^W w^*}{K^W r^* + L^W w^*} \tag{2-61}$$

Similarly, the size of the consumption volume of country F in the trade box is β. And $\alpha = \beta$.

We see that the consumption volume built by trade equals the trade volume as

$$CV = VT \tag{2-62}$$

3. Optimality Property of the General Trade Equilibrium

Lionel McKenzie (1987, pp29) described the task of general equilibrium as

> "Walras set of major objectives of general equilibrium theory as they have remained ever since. First, it was necessary to prove in any model of general equilibrium that the equilibrium exists. Then its optimality properties should be demonstrated. Next, it should be shown how the equilibrium would be attained; that is, the stability of the equilibrium and its uniqueness should be studied. Finally, it should be shown how the equilibrium will change when conditions of demand, technology, or resources are varied."

We presented the equilibrium solution above. What is the optimal property of the equilibrium? We illustrate that the trade volume reaches its maximum value at the equilibrium. It implies that both countries get their full benefits through free trade.

Triangle ΔEZC in figure 1 displays the trade flows of factor contents. The trade volume in country H is

$$VT = (\lambda_K - s^H)K^W r^* + (s^H - \lambda_L)L^W w^* \tag{3-1}$$

We assume, by (2-13),

$$r^* = (s^H - \lambda_L)L^W \tag{3-2}$$

It implies

$$w^* = (\lambda_K - s^H)K^W \tag{3-3}$$

Substituting them to (3-1) yields

$$VT = 2\left(\lambda_K - s^H\right)\left(s^H - \lambda_L\right)L^W K^W \tag{3-4}$$

It shows that VT is a quadratic function of s^H.

We introduce a utility function μ just as the trade volume,

$$\mu = 2(\lambda_K - s^H)(s^H - \lambda_L)L^W K^W \tag{3-5}$$

It reaches its maximum value as $\frac{1}{2}(\lambda_K - \lambda_L)$ when $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$. See Appendix A for its optimal solution.

It confirms the equilibrium solution in the last subsection. It is also an independent approach to reach equilibrium.

The utility function μ is complete with market logic. It is defined as

$$\mu = 2F_K^H r^* = 2 \cdot (\lambda_K - s^H)K^W \cdot (s^H - \lambda_L)L^W \tag{3-6}$$

It shows that the import of labor serves as the price of the export of capital. If the export of capital services increases as s^H increases, its price decreases, vice versa.

In Figure 1, only the share of GNP inside the trade box is redistributable by trade,

$$\lambda_K > s^H > \lambda_L \tag{3-7}$$

If α increases, the share of GNP of country H will increase; and the share of GNP of country F will decrease, vice versa. We call α as a redistributable share of GNP for country H, and β is one for country F[2]. The utility function (3-5) can also be explained as the product of two countries' redistributable shares of GNP. In the two-country integration and competition, the ultimate evaluation of trade benefit and welfare of a country is its share of GNP to the world GNP. At the equilibrium, both countries reach their maximum redistributed share of GNP. It is equivalent to maximize the consumption of both countries. α is the size of consumption formed by trade for country H. β is the size for country F.

4. Autarky Price and Comparative Advantage

Leamer and Levinsohn (1995, p.1342) mentioned the importance of gains from trade as "Proofs of the static gains from trade fall into the unrefutable category yet these are some of the most important results in all of economics."

The general trade equilibrium above shows that world factor endowments determine world prices. We now apply it to evaluate the autarky prices of a country under an isolated market. The idea is that the autarky factor endowments determine its autarky prices. The IWE diagram itself supports this extension analytically. Consider the allocation of factor endowments, point *E,* in Figure 1. Assume that it moves closer to the origin O. The factor endowments of country H will shrink to very small, the factor endowments of country F will close to be world factor endowments. The autarky prices in country F are then world prices. Mathematically, when the allocation $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the world relative factor price r^* will close to the relative autarky factor price of country H. We present the relative rental price as

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \tag{4-1}$$

Seeking the limit above yields

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[2] Originally, I uses the this idea to illustrates the general trade equilbrium.

$$\lim_{\substack{L^H \to 0 \\ K^H \to 0}} \frac{L^H + L^F}{K^H + K^F} = \frac{L^F}{K^F} = r^{Fa} \tag{4-2}$$

At the same time, the world commodity prices will close to the autarky output prices of country F. We proved the autarky price measurement mathematically. Samuelson (1949) argued this idea. He mentioned that the autarky prices are the world prices if the country (or continent) is divided into two countries geographically (or artificially), supposing that all other things are unchanged. Now we know world prices; the calculation or the measurement of world prices can calculate autarky prices.

We show another way to illustrate autarky prices.

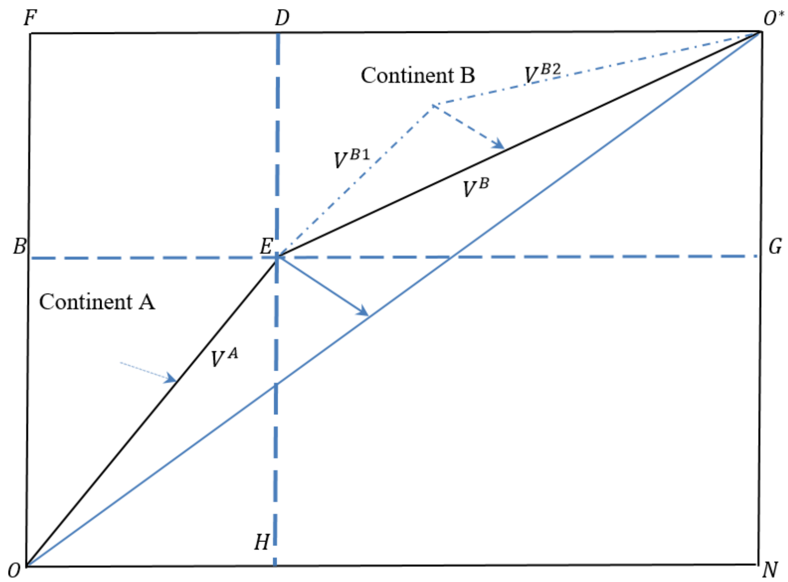

FIGURE 2
IWE Diagram of Two Continents Showing Autarky Prices

Suppose that there are two geographic continents: continent A and continent B, separated by an ocean. Continent A is a single country. Continent B is with two free-trade countries: B1 and B2. When transportation conditions are more available, two continents make free trade by no-cost shipping. We draw the scenario in figure 2. The rectangle $BEHO$ is the IWE diagram for continent A. The rectangle DO^*GE is the IWE diagram for continent B. The rectangle FO^*NO is the IWE diagram for the two-continent world. The continent prices for continent B can be decided with V^B by world prices (2-31) through (2-34), which can serve as the autarky price for continent B. The autarky prices of continent A can be decided by V^A too, even that it is a single country. We can determine a continent or a country's autarky prices by its factor endowments.

Helpman and Krugman (1985, p.16) proposed a clear-sighted conclusion about the factor price equalization (FPE) set in the IWE. They addressed "This FPE set is not empty because it always contains the diagonal OO^*. Since it is a convex symmetrical set around the diagonal, its boundaries defined the limits of dissimilarity in factor composition which is consistent with factor price equalization. Hence for sufficiently similar composition, there is a factor price equalization in the trading equilibrium". It normalized the FPE set. Without it, the nearby area to the diagonal line will not be valid for the FPE[3]. It can be used to derive autarky prices directly also.

Let us imagine an allocation of factor endowments, C, on the diagonal line OO^* in Figure 1. At this point, The factor compositions of the two countries are the same, and they equal to world factor composition as

$$\frac{L^H}{K^H} = \frac{L^F}{K^F} = \frac{L^W}{K^W} \tag{4-3}$$

At that moment, we know both countries' rental/wage ratios are the same. Otherwise, it will cause trade. It implies that the world rental/wage ratio equals the autarky rental/wage ratios of the two countries as

$$\frac{r^{aH}}{w^{aH}} = \frac{r^{aF}}{w^{aF}} = \frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{4-4}$$

where superscript ah indicates the autarky price of country h. At point C, the two countries' autarky prices are the same, and the autarky prices are world prices. We see that the logic of autarky prices formation is the same as world prices formation.

Based on the above discussion, we present the autarky prices of two countries as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{4-5}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{4-6}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{4-7}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{4-8}$$

The gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{4-9}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{4-10}$$

We add a negative sign in inequalities above since we expressed the factor content of trade by net export. In most other works of literature, they denoted factor trade by net import.

We express the gains from trade for the home country as

$$-(W^{Ha})'F^H > 0 \tag{4-11}$$

Adding trade balance condition $W^{*\prime}F^H = 0$ on (4-11) yields

$$-((W^{Ha})' - W^{*\prime})F^H > 0 \tag{4-12}$$

W^{Ha} and W^* are

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix} \quad , \quad W^* = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix} \tag{4-13}$$

Substituting them into (4-12) yields,

$$-\begin{bmatrix} \frac{L^H}{K^H} - \frac{L^W}{K^W} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \\ -\frac{1}{2}\frac{K^H L^W - K^W L^H}{K^W} \end{bmatrix} > 0 \tag{4-14}$$

It can be rewritten to

[3] Mathematically, it makes sure that whole FPE set is on a plane. Otherwise the FPE will be with a hole even a ditch along the diagonal line.

$$-\left(\frac{L^H}{K^H}-\frac{L^W}{K^W}\right)\times\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}>0 \qquad (4\text{-}15)$$

Simplify the above to

$$\frac{(K^HL^W-K^WL^H)^2}{2L^WK^WK^H}>0 \qquad (4\text{-}16)$$

It is true. So that (4-11) holds. Similarly, we can obtain

$$-W^{Fa'}F^F=\frac{(K^H L^W-K^WL^H)^2}{2L^WK^WK^F}>0 \qquad (4\text{-}17)$$

It implies that the world prices at the equilibrium ensure the gains from trade for both countries. The quantitative or computable gains from trade are essential for international trade analyses.

We summarize the content of this section as the theorem of comparative advantage.

Theorem – The equalized factor prices make sure of gains from trade for both countries

The world factor endowments, fully employed, determine world prices, which assure the gains from trade for countries taking part in free trade.

Proof

The price solution (2-31) through (2-34) shows the structure of world prices. The Dixit-Norman constant shows why world prices remain the same with mobile factor endowments within the FPE set in the IWE. The relative factor price w/r presents an angle by K^W/L^W in Figure 1. The angle is unique for a given IWE. Therefore, the solution is unique. The FPE is true and unchanged within the FPE solution set with any given distribution of world factor endowments. Equations (4-11) through (4-17) illustrate the gains from trade by the equalized factor prices for both countries.

At the equilibrium, the composition of world factor consumptions equals the composition of world factor endowments. We can also say the world consumptions determine world prices.

End Proof

5. General equilibrium of trade of two factors, two commodities, and multiple countries

In a $2\times2\times2$ system, country H and country F are trade partners with each other. In a multi-country system, who is the trade partner with whom? Leamer (1984, preface page xiii) addressed this issue as “This theorem, in its most general form, states that a country’s trade relations with the rest of the world depend on its endowments of productive factors...”. The designated trade in this study is a transaction of goods between a country and the rest of the world. The trade relations are pretty simple by this specification. It just likes the scenario of the $2\times2\times2$ system from the view of analyses.

Figure 3 draws an IWE diagram for three countries. The dimension box represents world factor endowments. The vector $V^h(L^h, K^h)$ represents the factor endowments of country h, h=1, 2, and 3. The origin of country 1 is arranged to start at the left-bottom corner. The origin of the rest of the world is from the upper-right corner. The vector of factor endowments of country 1 is V^1; and the vector of factor endowments of the rest of the world is $V^2 + V^3$.

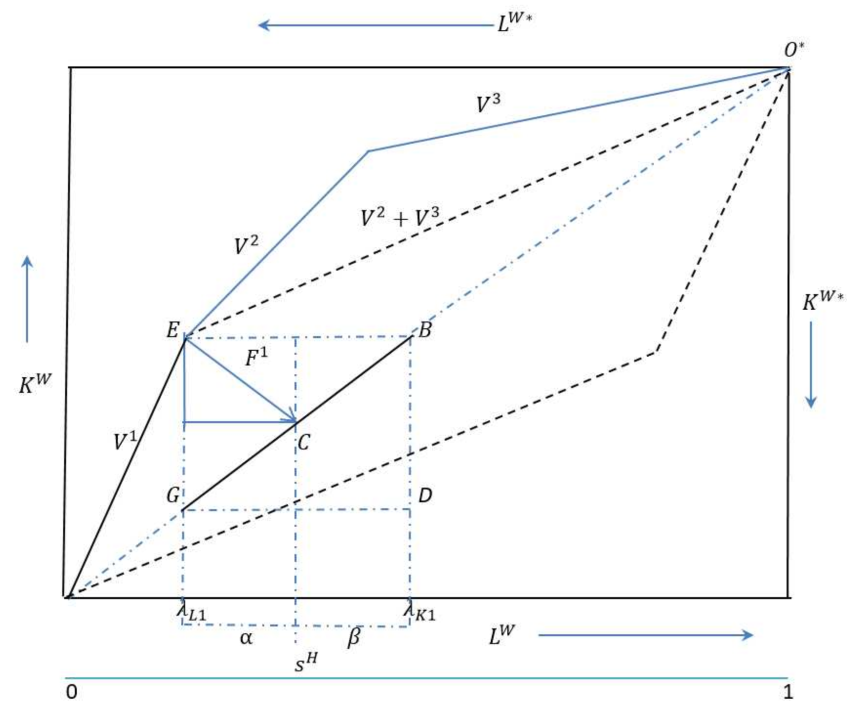

Figure 3 The Three-Country IWE Diagram

The system notation for the 2 x 2 x M model is as same as equations (2-1) and (2-2); the only difference is the country number. The country number now goes from 1 to M (In Figure 3, we present three countries for illustration).

We now introduce two lists of parameters, which are the shares of factor endowments of country h to their world factor endowments, respectively as

$$0 \le \lambda_{Lh} \le 1, \quad 0 \le \lambda_{Kh} \le 1 \qquad (h = 1,2, \dots, M) \tag{5-1}$$

$$\sum_{h=1}^{M} \lambda_{Lh} = 1 \quad , \qquad \sum_{h=1}^{M} \lambda_{Kh} = 1 \tag{5-2}$$

The factor endowments of country h can be denoted as

$$L^h = \lambda_{Lh} L^W \qquad (h = 1,2, \dots, M) \tag{5-3}$$

$$K^h = \lambda_{Kh} K^W \qquad (h = 1,2, \dots, M) \tag{5-4}$$

The allocation of factor endowments of country 1 in Figure 3 is $E(\lambda_{L1} L^W, \lambda_{K1} K^W)$. It shows how country 1 trades with the rest of the world by its factor endowments.

The factor contents of trade of country h are

$$F_K^h = K^h - s^h K^W = \left(\lambda_{Kh} - s^h\right) K^W \qquad (h = 1,2, \dots, M) \tag{5-5}$$

$$F_L^h = L^h - s^h L^W = \left(\lambda_{Lh} - s^h\right) L^W \qquad (h = 1,2, \dots, M) \tag{5-6}$$

The trade balance of factor contents for country h is

$$\frac{r^{*h}}{w^{*h}} = \frac{(s^h - \lambda_{Lh}) L^W}{(\lambda_{Kh} - s^h) K^W} \qquad (h = 1,2, \dots, M) \tag{5-7}$$

where r^{*h} is the equalized rental in country h, w^{*h} is the equalized wage in country h. It displays the trade balance between country h and the rest of the world. Extending the result of the Dixit-Norman constant as 1 in the last section to the equation above, we have

$$\frac{(s^h-\lambda_{Lh})}{(\lambda_{Kh}-s^h)} = 1 \qquad (h = 1,2,\dots,M) \tag{5-8}$$

$$\frac{w^{*h}}{r^{*h}} = \frac{K^W}{L^W} \qquad (h = 1,2,\dots,M) \tag{5-9}$$

This means that the relative factor price (rental-wage ratio) is the same for all countries.

$$\frac{w^{*h}}{r^{*h}} = \frac{K^W}{L^W} = \frac{w^*}{r^*} \tag{5-10}$$

By assuming $w^* = 1$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} \qquad (h = 1,2,\dots,M) \tag{5-11}$$

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{5-12}$$

$$w^* = 1 \tag{5-13}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{5-14}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{5-15}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = 1,2,\dots,M) \tag{5-16}$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = 1,2,\dots,M) \tag{5-17}$$

$$x_1^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W \qquad (h = 1,2,\dots,M) \tag{5-18}$$

$$x_2^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W \qquad (h = 1,2,\dots,M) \tag{5-19}$$

We see that

$$\sum_{h=1}^{H} s^h = \sum_{h=1}^{H}\frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} = 1 \tag{5-20}$$

Those are the equilibrium solution for the $2 \times 2 \times M$ model. We can demonstrate that all countries participating in trade gain from trade. It showed that world factor endowments determine world prices in the multi-country economy.

6. RELATED DISCUSSIONS

At the equilibrium, the ratio of factor content of trade of a country equals its factor consumption ratio. It reflects Leamer theorem (Leamer, 1980). We provide a chain of inequalities to illustrate the Heckscher-Ohlin theory,

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} = \frac{w^{Ha}}{r^{Ha}} > \frac{K^H - F_K^H}{L^H - F_L^H} = \frac{K^W}{L^W} = \frac{w^*}{r^*} = -\frac{F_K^H}{F_L^H} = \frac{K^F - F_K^F}{L^F - F_L^F} > \frac{K^F}{L^F} = \frac{w^{Fa}}{r^{Fa}} > \frac{a_{K2}}{a_{L2}} \tag{5-1}$$

It presents the Heckscher-Ohlin theorem, the Leamer theorem, the factor price equalization theorem, the Dixit and Norman IWE price, autarky prices, and comparative advantages. It is a comprehensive but straightforward expression.

The factor price equalization theorem reflects the world prices as the trade consequence. The Heckscher-Ohlin theorem reflects trade direction as the trade consequence. The general trade equilibrium shows those characters. It is a Pareto optimal solution since the trade box shows how social trade-off played. It is a balanced trade that the share of GNP of a country equals its share in world income.

The trade box illustrates how free trade redistributes benefits into each country.

Conclusion

This paper presents the general trade equilibrium and the world price structures of the Heckscher-Ohlin model. The equilibrium is consistent with Dixit and Norman's conclusion of the FPE set. The optimality of the solution is that the trade volume gets its maximum value at the equilbrium.

Dixit (2010) mentioned, "The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, 'it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.' That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor price at the equilibrium of this study presented the Heckscher-Ohlin theorem.

The study illustrates that world factor endowments determine world prices. Its first application is to identify the measurement of autarky prices: the autarky factor endowments determine autarky prices. The autarky price proves the comparative advantage finally.

The Rybczynski trade effect and the Stolper-Samuelson trade effect are partial equilibrium analyses. The equilibrium provides the way to do a complete analysis of price changes or the factor endowments changes on the world economy.

The equalized factor prices provide the theoretical basis for further analyses of factor price none-equalization when countries have different productivities.

Appendix A

For the function

$$\mu = 2(s^H - \lambda_L)(\lambda_K - s^H)K^W L^W \tag{A-1}$$

to find its maximum or minimum value, we take differential of (A-1) with respective to s^H yields

$$\frac{du}{ds^H} = 2\big(-2s^H + (\lambda_K + \lambda_L)\big)K^W L^W \tag{A-2}$$

Let it equal to 0, we get $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$.

Take the second differential of (A-2) with respective to s^H yields

$$\frac{d}{ds^H}\left(\frac{du}{ds^H}\right) = -4K^W L^W \quad \text{(A-3)}$$

It is less than 0. By the secondary condition, μ is with its maximum value at $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$.

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