Government Intervention through Informed Trading in Financial Markets

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Abstract

We develop a theoretical model of government intervention in which a government with private information trades strategically with other market participants to achieve its policy goal of stabilizing asset prices. When the government has precise information and cares much about its policy goal, both the government and the informed insider engage in reversed trading strategies, but they trade against each other. Government intervention can improve both market liquidity and price efficiency, and the effectiveness of government intervention depends crucially on the information quality of the government.

Keywords: government intervention; trading; price stability; price efficiency
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1 Introduction

Government intervention is a common way to stabilize financial markets, especially during a financial crisis or a stock market meltdown. For example, during the COVID-19 in 2020, Federal Reserve of America, Bank of Japan and other central banks massively purchase government bonds, ABS, ETF and other financial assets.[1] While the government's goal is to ensure financial stability, whether government intervention has some externalities when leaning against market fluctuations is an open question. For example, Brunnermeier, Sockin and Xiong (2020) show that government intervention reduces the informational efficiency of asset prices.

From 2015 to 2016, the China's stock market experienced three major market crashes, and the market index decreased about 50% in 6 months. The intervention of Chinese government was very aggressive during the period, by organizing a "national team" who directly purchased stocks of more than 1000 firms (Huang, Miao and Wang, 2019). It is well known that the majority of investors in China' stock market are inexperienced retail investors, and some believe those investors contribute significantly to the market crash. For this reason, Brunnermeier, Sockin and Xiong (2020) analyze the implications of government intervention to reduce price volatility induced by noise traders. However, some insiders who have superior information about the firms also trade strategically during the period of government intervention. For example, the managers of the listed firm, Mei Yan Ji Xiang, bought their own firm stocks in July, 2015 and cleared the positions after 6 months.[2] Given various investor structures, how does government intervention affect the strategic trading of informed traders? What are the corresponding market-quality implications? In this paper, we study those questions by develop a multi-period model with price impact and informed trading.

We develop a two-period Kyle (1985) model to analyze the impact of government in-

[1] Government intervention does not necessarily happen in a financial crisis. For instance, Japanese government expands its stock purchase program gradually to control deflation (Shirai, 2018).

[2] On August 04, 2015, the firm of "Mei Yan Ji Xiang" made an announcement that the China Central Huijin Investment Limited (CCH), one of the "national team," became the largest shareholder. In the next 10 trading days, the stock price increased over 250%.

tervention through direct trading in the stock market. We consider an economy with two assets, a risky and a risk-free asset, respectively. There are four types of traders: a risk-neutral insider with perfect information, a representative risk-neutral competitive market maker, noise traders and a government with imperfect information.[3] The objective function of the government has two parts. The first part is to minimize the price volatility, which is policy related. The second part is profit maximization, which is the same as that of the insider. We consider a linear equilibrium in which the trading strategies and the pricing functions are all linear. We solve the linear perfect Bayesian equilibrium and explore the trading behavior of the government and the insider as well as the effectiveness of government intervention through trading in the financial market.

Our analysis delivers two important messages. First, we find that both the government and the insider can engage in reversed trading strategies but in opposite directions, which implies that they effectively trade against each other in both periods. This situation arises when the government has very precise information and cares much about its policy goal of price stability. Specifically, in this situation, seeing strong fundamental information, the insider sells (as opposed to buys) in the first period and then buys in the second period. Meanwhile, the government buys in the first period and then sells in the second period. The intuition is primarily driven by the fact that the insider wants to conceal her information in period 1 and exploits more information advantage in period 2. If the government has very precise information and weighs heavily on its policy goal, the insider trades against the government to conceal his information in period 1, and at the same time, the government will trade against the insider to stabilize prices.

On the other hand, when the government's information quality is low, the insider is not heavily influenced by the presence of the government and so it will trade in a way similar to that in the standard Kyle model with one insider, without reversed trading strategies. Similarly, when the government does not care too much its policy goal, the model is similar

[3] We use "he/him" to refer to the insider, "she/her" to refer to the market maker, and "it/its" to refer to the government.

to a standard Kyle setting with two insiders, and again, no reversed trading strategies arise.

The second important message delivered by our analysis is that government intervention can not only stabilize the financial market, but also improve market liquidity and price efficiency simultaneously, and that the effectiveness of government intervention is positively related to the government's information quality. This result suggests that it is most effective for the government to intervene via direct trading only when it has private information with great quality. Otherwise, the effect of government trading is limited.

Specifically, in terms of market-liquidity implications, we find that relative to the standard Kyle setting, government intervention only slightly affects the period-1 market liquidity but improves the period-2 market liquidity. When the government has no policy concerns and very precise information, market liquidity is slightly smaller than that of the Kyle model in period 1, which shows that private information has a mild negative effect on market liquidity. When the government has imprecise information and cares more about price stability, the market liquidity is larger than that of the Kyle model in period 1. In period 2, the market liquidity is always larger than that of the Kyle model and does not hinge on the policy weight of the government. When the government's information quality is very low, the market liquidity measures in two periods converge to that of the Kyle model. The negative effect on market liquidity of information and the positive effect of policy concerns cancel out.

When it comes to the implications for price efficiency, government intervention effectively raises price discovery/efficiency in two periods. Because the government has information about fundamentals, its informative trading improves price discovery of the financial market. More interestingly, price discovery increases in the policy weight of the government in period 1 while decreases in the policy weight in period 2. Intuitively, in period 1, the insider trades less by hedging on the larger policy weight of the government. In order to hedge on the insider's reserved trading, the government trades more, which increases the total amount of the informational trading and hence improves price discovery. In period 2, the insider exploits the remaining information advantage and trades more aggressively to hedge on the larger policy weight. Since the government cares more about price stability, it has to trade less aggressively, so price discovery decreases in period 2. Moreover, if the government's information quality is very low, the price discovery measures in two periods are very close to and sightly less than those of the standard Kyle model.

Related Literature. Our paper contributes to the literature studying the implications of government intervention for asset markets, with a focus on China's stock market. Government intervention happens in many regions and countries, which is extensively analyzed in the literature. For example, Veronesi and Zingales (2010) analyze the costs and benefits of Paulson's plan in the United States, and Cheng, Fung and Chan (2000) and Su, Yip and Wong (2002) study the implications of the intervention of Hong Kong government during the financial crisis in 1998.

Moreover, the analysis of government intervention needs to model a stylized government with explicit policy goals. Bhattacharya and Weller (1997), Pasquariello (2017), and Pasquariello, Roush and Vega (2020) study a central bank with a policy goal to minimize the expected squared distance between the traded asset's equilibrium price and the target. In our model, the government is represented by the "national team" which directly trades in the China's stock market, and its policy goal is to minimize the expected squared distance between two equilibrium prices in different periods.

For government intervention in China's stock market in 2015, various policy tools are used to stabilize the market.[4] Chen et al. (2019) study destructive market behaviors induced the daily price limits; Liu, Xu and Zhong (2017) show that price limits and trading suspension can induce contagion; and Chen, Petukhov and Wang (2019) analyze the dark side of circuit breakers. Moreover, Bian et al. (2021) find that marginal investors are forced to resell during the market crash, and Huang, Miao and Wang (2019) show that government intervention in 2015 both creates value and improves liquidity. Our paper, complementary to the literature, analyzes how government intervention affects the informed and strategic trading behaviors

[4] More details are summarized by Song and Xiong (2018) and Brunnermeier, Sockin and Xiong (2020).

of market participants. Moreover, our theory prediction about liquidity is consistent with Huang, Miao and Wang (2019).

Our paper is closely related to Brunnermeier, Sockin and Xiong (2020) who analyze the implications of government intervention to reduce price volatility induced by noise traders (e.g., De Long, Shleifer, Summers and Waldmann,1990). In particular, Brunnermeier, Sockin and Xiong (2020) find that information efficiency of asset prices is reduced. In Brunnermeier, Sockin and Xiong (2020), the market volatility comes from noisy trading, and the government has no private information. For this reason, government intervention to reduce price volatility decreases information efficiency. By contrast, in our model, the market volatility stems from speculative insider trading and the government has information about the fundamentals, which implies that government intervention effectively stabilizes the asset prices and improves the price efficiency of the financial markets.

Our model considers price impact and informed trading, which originates from Kyle (1985). Huddart, Hughes and Levine (2001) solve a two period Kyle model that is treated as a benchmark in our paper. We solve the model by conjecturing linear trading strategies and linear pricing, which are developed by Bernhardt and Miao (2004) and Yang and Zhu (2020). Finally, for asset pricing implications, we consider market liquidity and price discovery measures that are emphasized by O'Hara (2003) and Bond, Edmans, and Goldstein (2012).

The rest of the paper is organized as follows. We first present a model of government intervention in Section 2 and solve the model in Section 3. We then present the equilibrium results in Section 4 and conduct numerical analysis in Section 5. Finally, we conclude in Section 6. All proofs and figures are provided in the Appendix.

2 A Model of Government Intervention

In this section, we develop a two-period Kyle (1985) model to analyze the impact of government intervention on the stock market. In particular, we model government trading in the financial market to capture government intervention.

2.1 The Financial Market with Government Intervention

We consider an economy with two trading periods ($t = 1, 2$). There are two assets, a risky asset and a risk-free asset, are traded in the financial market. The risky asset pays a liquidation value v at the end of period 2, and v is a normally distributed random variable with mean p_0 and variance Σ_0. The risk-free asset has an infinitely elastic supply with a constant return r (normalized to be zero) for each period.

The economy is populated by four types of traders: a risk-neutral insider (i.e., informed trader), a representative risk-neutral competitive market maker, a large government player ("national team") and noise traders. As usual, the insider submits market orders to maximize profits, noise traders provide randomness to hide the insider's private information, and the market maker sets the price. The new player is the government and its behavior serves regulation purposes.

Specifically, in each period, the government submits a market order g_t to minimize the expected value of the following loss function:

$$\phi_p (\Delta p)^2 + \phi_c c, \tag{1}$$

where ϕ_p and ϕ_c are two exogenous positive constants. The first term $(\Delta p)^2$ captures the government's policy motive, "price stability." Formally, $(\Delta p)^2 \equiv (p_2 - p_1)^2$, where p_2 and p_1 are the equilibrium prices in the two periods. This measure of price stability is a widely used objective function of government intervention (e.g., Brunnermeier, Sockin and Xiong, 2020).[5] The second component in (1), c, is the cost of intervention, which comes from the trading loss (negative of trading revenue). Specifically, we have

$$c = c_1 + c_2 \text{ with } c_t = (p_t - v)\, g_t \text{ for } t = 1, 2, \tag{2}$$

where g_t is the government's order flow submitted at date t, and $(p_t - v)\, g_t$ is its trading loss at date t. We can show that the government makes profits in equilibrium and so $c < 0$. The specification of loss function (1) is similar in spirit to Stein (1989), Bhattacharya and Weller (1997), Vitale (1999), Pasquariello (2017), and Pasquariello, Roush and Vega (2020).[6]

If $\phi_p = 0$, the government trades just as another insider who maximizes the expected profit from trading. When $\phi_p > 0$, the government cares about its policy goal. The greater ϕ_p is, the more important is the government's policy goal (financial stability). To economize notations, let us define $\phi \equiv \phi_p/\phi_c \in [0, \infty)$ and so the loss function of the government, (1) is equivalent to

$$\phi\, (\Delta p)^2 + c, \tag{3}$$

where ϕ is the relative weight placed by the government on its policy motives.

2.2 Information Structure and Pricing

Similar to Kyle (1985), the insider learns v at the beginning of the first period and places market orders x_1 at $t = 1$ and x_2 at $t = 2$, respectively. Noise traders do not receive any

[5] Note that in our model, $(\Delta p)^2$ refers the squared distance between the traded asset's equilibrium prices p_2 and p_1. That is, the government only considers the price stability for one period. In fact, the government is not always participating the market directly. Government intervention only happens in a turbulent market. For this reason, we only consider the case in which the government only cares the price stability for one period. Of course, we can easily extend our model to allow the government to care about price stability for two periods. The results are not qualitatively different.

[6] In Pasquariello (2017) and Pasquariello, Roush and Vega (2020), there is only one trading period, and meanwhile, the government (central bank) has a nonpublic price target p_T as its private information and it wants to minimize the squared distance between the traded asset's equilibrium price and the target p_T. In our model, there are two trading periods, and the government minimizes the expected squared distance between two equilibrium prices as its policy goals endowed with the noisy signal about the liquidation value of the risky asset.

information, and their net demands in the two periods, u_1 and u_2, are normally distributed with mean zero and variance σ_u^2. The government is likely to have first-hand knowledge of macroeconomic fundamentals.[7] Thus, we assume that the government is endowed with a private and noisy signal about the liquidation value of the financial asset, namely,

$$s = v + \varepsilon, \tag{4}$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Random variables v, ε, u_1 and u_2 are mutually independent.

In (4), s is normally distributed with mean p_0 and variance $\Sigma_0 + \sigma_\varepsilon^2$, and hence the parameter σ_ε^2 controls the information quality of the signal. A large σ_ε^2 means less accurate information about v. In particular, we can allow σ_ε^2 to take values of 0, which corresponds to the case in which s perfectly reveals v. Moreover, when σ_ε^2 goes to ∞, s reveals nothing about v. The government places market orders g_1 with information $\{s\}$ at the beginning of period 1 and g_2 with information $\{s, p_1\}$ at the beginning of period 2.

The market maker determines the prices p_1 and p_2 at which she trades the quantity necessary to clear the market. The market maker observes the aggregated order flows $y_t = x_t + u_t + g_t$ for $t \in \{1, 2\}$. The weak-form-efficiency pricing rule of the market maker implies that the market maker sets the price equal to the posterior expectation of v given public information as follows:

$$p_1 = E(v|y_1) \text{ and } p_2 = E(v|y_1, y_2). \tag{5}$$

3 Solving the Model

Given the model described in the previous section, we look for a perfect Bayesian equilibrium, in which the insider and the government choose their trading strategies to optimize their objectives. The market maker's strategy is pinned down by (5). An equilibrium is formally

[7] In fact, many investors in the China's stock market rely on macroeconomic information, which is normally a sector for investment banks. Thus, when government trades directly, its trading may bring some macroeconomic information.

defined as follows:

Definition 1. *A perfect Bayesian equilibrium of the two-period trading game is a collection of functions*

$$\{x_1(v), x_2(v, p_1), g_1(s), g_2(s, p_1), p_1(y_1), p_2(y_1, y_2)\},$$

that satisfies:

1. *Optimization:*

$$x_2^* \in \arg\max_{\{x_2\}} E\left[(v - p_2) x_2 | v, p_1\right],$$

$$x_1^* \in \arg\max_{\{x_1\}} E\left[(v - p_1) x_1 + (v - p_2) x_2^* | v\right],$$

$$g_2^* \in \arg\min_{\{g_2^*\}} E\left[\phi (p_2 - p_1)^2 + (p_2 - v) g_2 | s, p_1\right],$$

$$g_1^* \in \arg\min_{\{g_1^*\}} E\left[\phi (p_2 - p_1)^2 + (p_1 - v) g_1 + (p_2 - v) g_2^* | s\right]$$

2. *Market efficiency: p_1 and p_2 are determined according to equation (5).*

Given the model structure, we are interested in a linear equilibrium in which the trading strategies and the pricing functions are all linear. Formally, a linear equilibrium is defined as a perfect Bayesian equilibrium in which there exist six constants

$$(\beta_1, \beta_2, \gamma_1, \gamma_2, \lambda_1, \lambda_2) \in \mathbb{R}^6,$$

such that

$$\begin{aligned}
x_1 &= \beta_1 (v - p_0), && (6)\\
x_2 &= \beta_2 [v - E(v|y_1)], && (7)\\
g_1 &= \gamma_1 (s - p_0), && (8)\\
g_2 &= \gamma_2 [s - E(s|y_1)], && (9)\\
p_1 &= p_0 + \lambda_1 y_1, \text{ with } y_1 = x_1 + g_1 + u_1, && (10)\\
p_2 &= p_1 + \lambda_2 y_2, \text{ with } y_2 = x_2 + g_2 + u_2. && (11)
\end{aligned}$$

Equations (6), (7), (8) and (9) indicate that the insider and the government trade on their information, respectively. The linear forms are motivated by Bernhardt and Miao (2004) and Yang and Zhu (2020), who specify that the trading strategy of an informed agent is a linear function of each piece of private information. The pricing equations (10) and (11) state that the price in each period is equal to the expected value of v before trading, adjusted by the information carried by the arriving aggregated order flows. Since our model has two periods, we derive the linear equilibrium of the model backwards.

3.1 The Insider's Problems

The insider trades in both periods, and so we solve his problems by backward induction. Let $\pi_t = (v - p_t) x_t$ denote the insider's profit that is directly attributable to his period-t trade, $t \in \{1, 2\}$. In period 2, the insider has information $\{v, p_1\}$ and chooses x_2 to maximize $E(\pi_2 | v, p_1)$. Using equations (9) and (11), we can compute

$$E[(v - p_2) x_2 | v, p_1] = \{v - p_1 - \lambda_2 x_2 - \lambda_2 \gamma_2 E[s - E(s|y_1) | v, y_1]\} x_2.$$

Taking the first-order-condition (FOC) results in the solution as follows:

$$x_2 = \frac{v - p_1}{2\lambda_2} - \frac{\gamma_2}{2} E\left[s - E\left(s|y_1\right)|v, y_1\right] = \frac{1}{2\lambda_2}\left(1 - \lambda_2\gamma_2\delta_1\right)\left(v - p_1\right), \tag{12}$$

where

$$\delta_1 \equiv \frac{cov\left(s, v|y_1\right)}{var\left(v|y_1\right)} = \frac{\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2}{\sigma_u^2 + \gamma_1^2\sigma_\varepsilon^2}. \tag{13}$$

The expression for the conditional expectation in equation (12), $E\left[s - E\left(s|y_1\right)|v, y_1\right]$, shows that the insider learns the government's noisy signal s by using his information set. The second-order-condition (SOC) is

$$\lambda_2 > 0. \tag{14}$$

Comparing equation (12) with the conjectured strategy (7), we have

$$\beta_2 = \frac{1}{2\lambda_2}\left(1 - \lambda_2\gamma_2\delta_1\right). \tag{15}$$

In period 1, the insider has information $\{v\}$ and chooses x_1 to maximize

$$E\left(\pi|v\right) = E\left(\pi_1 + \pi_2|v\right) = E\left[\left(v - p_1\right)x_1 + \frac{\left(1 - \lambda_2\gamma_2\delta_1\right)^2}{4\lambda_2}\left(v - p_1\right)^2|v\right]. \tag{16}$$

The last term in the bracket is by inserting (12) into $\pi_2 = \left(v - p_2\right)x_2$, which yields

$$E\left(\pi_2|v, p_1\right) = \frac{\left(1 - \lambda_2\gamma_2\delta_1\right)^2}{4\lambda_2}\left(v - p_1\right)^2. \tag{17}$$

Using (8) and (10), we can further express $E\left(\pi|v\right)$ as follows:

$$E\left(\pi|v\right) = \left(\begin{array}{c} \left[v - p_0 - \lambda_1 x_1 - \lambda_1\gamma_1 E\left(s - p_0|v\right)\right]x_1 + \\ \frac{\left(1-\lambda_2\gamma_2\delta_1\right)^2}{4\lambda_2}\left\{\begin{array}{c} \left(v - p_0\right)^2 + \lambda_1^2 x_1^2 + \lambda_1^2\gamma_1^2 E\left[\left(s - p_0\right)^2|v\right] \\ + \lambda_1^2\sigma_u^2 - 2\lambda_1 x_1\left(v - p_0\right) - \\ 2\lambda_1\gamma_1\left(v - p_0\right)E\left(s - p_0|v\right) + 2\lambda_1^2 x_1\gamma_1 E\left(s - p_0|v\right) \end{array}\right\} \end{array}\right). \tag{18}$$

Then the FOC of x_1 yields

$$x_1 = \frac{1-\lambda_1\gamma_1}{2\lambda_1}\frac{1-\frac{\lambda_1}{2\lambda_2}\left(1-\lambda_2\gamma_2\delta_1\right)^2}{1-\frac{\lambda_1}{4\lambda_2}\left(1-\lambda_2\gamma_2\delta_1\right)^2}\left(v-p_0\right).$$

Compared with the conjectured pure strategy (6), we have

$$\beta_1 = \frac{1-\lambda_1\gamma_1}{2\lambda_1}\frac{1-\frac{\lambda_1}{2\lambda_2}\left(1-\lambda_2\gamma_2\delta_1\right)^2}{1-\frac{\lambda_1}{4\lambda_2}\left(1-\lambda_2\gamma_2\delta_1\right)^2}. \tag{19}$$

The SOC is

$$\lambda_1\left[1-\frac{\lambda_1}{4\lambda_2}\left(1-\lambda_2\gamma_2\delta_1\right)^2\right] > 0. \tag{20}$$

3.2 The Government's Decisions

The government's optimization problem is also solved by backwards induction. In period 2, the government has the information $\{s, p_1\}$. Using equations (7) and (11), we can compute

$$E\left[\phi\left(p_2-p_1\right)^2+\left(p_2-v\right)g_2|s,p_1\right] = \left\{\begin{array}{c}\phi\lambda_2^2\left[\begin{array}{c}\beta_2^2E\left(\left(v-p_1\right)^2|s,y_1\right)+g_2^2+\\ \sigma_u^2+2\beta_2g_2E\left(v-p_1|s,y_1\right)\end{array}\right]+\\ \left[-\left(1-\lambda_2\beta_2\right)E\left(v-p_1|s,y_1\right)+\lambda_2g_2\right]g_2\end{array}\right\}, \tag{21}$$

where

$$E\left(v-p_1|s,y_1\right) = \delta_2\left[s-E\left(s|y_1\right)\right],$$

$$\begin{aligned}E\left(\left(v-p_1\right)^2|s,y_1\right) &= E^2\left(v-E\left(v|y_1\right)|s,y_1\right)+var\left(v-E\left(v|y_1\right)|s,y_1\right)\\ &= \delta_2^2\left[s-E\left(s|y_1\right)\right]^2+var\left(v-E\left(v|y_1\right)|s,y_1\right),\end{aligned}$$

$$\delta_2 = \frac{cov\left(v,s|y_1\right)}{var\left(s|y_1\right)} = \frac{\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0}{\left(\beta_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+\sigma_u^2\sigma_\varepsilon^2}. \tag{22}$$

The expressions for conditional moments in (21), $E\left(\left(v-p_1\right)^2|s,y_1\right)$, $E\left(v-p_1|s,y_1\right)$, show that the government learns the private information of the insider, v, by using its information set $\{s, y_1\}$.[8] The FOC of g_2 gives

$$g_2 = \frac{1 - \lambda_2 \beta_2 - 2\phi \lambda_2^2 \beta_2}{2\lambda_2 + 2\phi \lambda_2^2} \delta_2 \left[s - E\left(s | y_1 \right) \right]. \tag{23}$$

Combining (23) with the conjectured trading strategy (9) leads to

$$\gamma_2 = \frac{1 - \lambda_2 \beta_2 - 2\phi \lambda_2^2 \beta_2}{2\lambda_2 + 2\phi \lambda_2^2} \delta_2. \tag{24}$$

The SOC is $2\phi\lambda_2^2 + 2\lambda_2 > 0$, which holds accordingly if (14) holds.

In period 1, the government chooses g_1 to minimize

$$E\left[\phi \left(p_2 - p_1 \right)^2 + \left(p_1 - v \right) g_1 + \left(p_2 - v \right) g_2 | s \right]. \tag{25}$$

Inserting (9) into $E\left[\left(p_2 - v \right) g_2 | v, p_1 \right]$, the objective function becomes

$$E \left\{ \left[\phi \left(p_2 - p_1 \right)^2 + \left(p_1 - v \right) g_1 + \left[- \left(1 - \lambda_2 \beta_2 \right) \gamma_2 \delta_2 + \lambda_2 \gamma_2^2 \right] \left[s - E\left(s | y_1 \right) \right]^2 \right] | s \right\}. \tag{26}$$

Using (7), (9), and (11), and applying the projection theorem repeatedly, we can compute

[8] Equation (10) shows that the information sets $\{p_1\}$ and $\{y_1\}$ are informationally equivalent.

(26) as a polynomial of g_1 as follows:

$$\left(\begin{array}{c}\phi\lambda_2^2\left\{\begin{array}{c}\beta_2^2\left[\left((1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1 g_1\right)^2+var(v-p_1|s)\right]\\ \gamma_2^2\left[\begin{array}{c}(s-p_0)^2+\beta_1^2\delta_3^2E\left((v-p_0)^2|s\right)+\delta_3^2g_1^2+\sigma_u^2\delta_3^2-2\delta_3g_1(s-p_0)\\ -2\beta_1\delta_3(s-p_0)E(v-p_0|s)+2\delta_3^2g_1\beta_1E(v-p_0|s)\end{array}\right]+\sigma_u^2+\\ 2\beta_2\gamma_2\left[\begin{array}{c}(1-\delta_4\beta_1)(s-p_0)E(v-p_0|s)-\delta_3\beta_1(1-\delta_4\beta_1)E\left((v-p_0)^2|s\right)\\ -\delta_4g_1(s-p_0)-\delta_3g_1(1-\delta_4\beta_1)E(v-p_0|s)\\ +\delta_3\delta_4g_1\beta_1E(v-p_0|s)+\delta_3\delta_4g_1^2+\delta_3\delta_4\sigma_u^2\end{array}\right]\end{array}\right\}\\ -g_1\left[(1-\lambda_1\beta_1)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}(s-p_0)-\lambda_1g_1\right]+\\ \left[\lambda_2\gamma_2^2-(1-\lambda_2\beta_2)\gamma_2\delta_2\right]\left\{\begin{array}{c}(s-p_0)^2+\delta_3^2\beta_1^2E\left((v-p_0)^2|s\right)+\\ \delta_3^2g_1^2+\delta_3^2\sigma_u^2-2\delta_3\beta_1(s-p_0)E(v-p_0|s)\\ -2\delta_3g_1(s-p_0)+2\delta_3^2g_1\beta_1E(v-p_0|s)\end{array}\right\}\end{array}\right). \tag{27}$$

Then we conduct FOC with respect to g_1 and derive

$$g_1=\frac{\left\{\begin{array}{c}\left[\begin{array}{c}(1-\lambda_1\beta_1)\left(1+2\phi\lambda_1\lambda_2^2\beta_2^2\right)+2\phi\lambda_2^2\gamma_2\theta_3(\beta_2-\beta_1\gamma_2\theta_3-2\beta_1\beta_2\theta_4)\\ +2\beta_1\theta_3^2(\gamma_2\theta_2-\lambda_2\gamma_2^2-\lambda_2\gamma_2\beta_2\theta_2)\end{array}\right]\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\\ +2\phi\lambda_2^2\gamma_2(\gamma_2\theta_3+\beta_2\theta_4)+2\theta_3(\lambda_2\gamma_2^2-\gamma_2\theta_2+\lambda_2\beta_2\gamma_2\theta_2)\end{array}\right\}}{2\phi\lambda_2^2\left(\lambda_1^2\beta_2^2+\gamma_2^2\theta_3^2+2\beta_2\gamma_2\theta_3\theta_4\right)+2\lambda_1+2\theta_3^2(\lambda_2\gamma_2^2-\gamma_2\theta_2+\lambda_2\beta_2\gamma_2\theta_2)}(s-p_0).$$

Combined with the conjectured pure strategy (8), we have

$$\gamma_1=\frac{\left\{\begin{array}{c}\left[\begin{array}{c}(1-\lambda_1\beta_1)\left(1+2\phi\lambda_1\lambda_2^2\beta_2^2\right)+2\phi\lambda_2^2\gamma_2\delta_3(\beta_2-\beta_1\gamma_2\delta_3-2\beta_1\beta_2\delta_4)\\ +2\beta_1\delta_3^2(\gamma_2\delta_2-\lambda_2\gamma_2^2-\lambda_2\gamma_2\beta_2\delta_2)\end{array}\right]\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\\ +2\phi\lambda_2^2\gamma_2(\gamma_2\delta_3+\beta_2\delta_4)+2\delta_3(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2)\end{array}\right\}}{2\phi\lambda_2^2\left(\lambda_1^2\beta_2^2+\gamma_2^2\delta_3^2+2\beta_2\gamma_2\delta_3\delta_4\right)+2\lambda_1+2\delta_3^2(\lambda_2\gamma_2^2-\gamma_2\delta_2+\lambda_2\beta_2\gamma_2\delta_2)}, \tag{28}$$

where

$$\delta_3\equiv\frac{cov(s,y_1)}{var(y_1)}=\frac{(\beta_1+\gamma_1)\Sigma_0+\gamma_1\sigma_\varepsilon^2}{(\beta_1+\gamma_1)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \tag{29}$$

$$\delta_4 \equiv \frac{cov\left(v, y_1\right)}{var\left(y_1\right)} = \frac{\left(\beta_1 + \gamma_1\right)\Sigma_0}{\left(\beta_1 + \gamma_1\right)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2}. \tag{30}$$

The SOC is

$$\phi\lambda_2^2 \left(2\lambda_1^2\beta_2^2 + 2\gamma_2^2\delta_3^2 + 4\beta_2\gamma_2\delta_3\delta_4\right) + 2\lambda_1 + 2\delta_3^2 \left(\lambda_2\gamma_2^2 - \gamma_2\delta_2 + \lambda_2\beta_2\gamma_2\delta_2\right) > 0. \tag{31}$$

3.3 The Market Maker's Decisions

In period 1, the market maker observes the aggregate order flow y_1 and sets $p_1 = E\left(v|y_1\right)$. By equation (5) and the projection theorem, we can compute

$$\lambda_1 = \frac{\left(\beta_1 + \gamma_1\right)\Sigma_0}{\left(\beta_1 + \gamma_1\right)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2}. \tag{32}$$

Similarly, in period 2, the market maker observes $\{y_1, y_2\}$ and sets $p_2 = E\left(v|y_1, y_2\right)$. By equations (5), (6), (6), (7), (8), (9) and (11), and applying the projection theorem, we have

$$\lambda_2 = \frac{cov\left(v, y_2|y_1\right)}{var\left(y_2|y_1\right)} = \frac{\left(\beta_2 + \gamma_2\right)\left(\gamma_1^2\sigma_\varepsilon^2 + \sigma_u^2\right)\Sigma_0 - \left(\beta_1 + \gamma_1\right)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{\left(\begin{array}{c} \beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2 + \sigma_u^2\right)\Sigma_0 + 2\beta_2\gamma_2\left(\sigma_u^2 - \beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0 + \\ \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0 + \sigma_u^2\sigma_\varepsilon^2 + \sigma_u^2\Sigma_0\right) + \sigma_u^2\left[\left(\beta_1 + \gamma_1\right)^2\Sigma_0 + \gamma_1^2\sigma_\varepsilon^2 + \sigma_u^2\right] \end{array}\right)}. \tag{33}$$

4 Equilibrium Characterization

Following the procedure in the previous section, we characterize the perfect Bayesian equilibrium in this section. The linear equilibrium is defined by six unknowns which are solutions of six equations. In general, the model cannot be solved in closed form and so we have to rely on numerical analysis. To examine the asset pricing implications numerically, we focus on several variables, including expected price volatility, price discovery/efficiency, the expected lifetime profits of the insider and expected lifetime costs of the government, the correlation coefficients between the trading positions of the insider, the government and the market maker, respectively. The equilibrium variables are formally characterized by the following proposition.

Proposition 1 *A linear pure strategy equilibrium is defined by six unknowns* $\beta_1, \beta_2, \gamma_1, \gamma_2, \lambda_1$, and λ_2, *which are characterized by six equations (15), (19), (24), (28), (32), and (33), together with three SOCs ((14), (20), (31)). In equilibrium, the expected price volatility is*

$$E(p_2 - p_1)^2 = \frac{\lambda_2^2 \left\{ \begin{array}{c} \beta_2^2 \left(\gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2\right) \Sigma_0 + \gamma_2^2 \left(\beta_1^2 \sigma_\varepsilon^2 \Sigma_0 + \sigma_u^2 \Sigma_0 + \sigma_\varepsilon^2 \sigma_u^2\right) + \\ 2\beta_2 \gamma_2 \left(\sigma_u^2 - \beta_1 \gamma_1 \sigma_\varepsilon^2\right) \Sigma_0 + \sigma_u^2 \left[(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2\right] \end{array} \right\}}{(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2}.$$

The price discovery/efficiency variables are

$$\Sigma_1 = var(v|y_1) = E(v - y_1)^2 = \frac{\left(\gamma_1 \sigma_\varepsilon^2 + \sigma_u^2\right) \Sigma_0}{(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2},$$

$$\Sigma_2 = var(v|y_1, y_2) = E(v - y_2)^2 = \frac{(1 - \lambda_2 \beta_2 - \lambda_2 \gamma_2)\left(\gamma_1 \sigma_\varepsilon^2 + \sigma_u^2\right) \Sigma_0 + \lambda_2 (\beta_1 + \gamma_1) \gamma_1 \gamma_2 \sigma_\varepsilon^2 \Sigma_0}{(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2}.$$

The expected lifetime profits of the insider and expected lifetime costs of the government are, respectively,

$$E(\pi) = (1 - \lambda_1 \beta_1 - \lambda_1 \gamma_1) \beta_1 \Sigma_0 + \frac{\left[(1 - \lambda_2 \beta_2)\left(\gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2\right) - \lambda_2 \gamma_2 \left(\sigma_u^2 - \beta_1 \gamma_1 \sigma_\varepsilon^2\right)\right] \beta_2 \Sigma_0}{(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2},$$

$$\begin{aligned} E(c) &= \gamma_1 \left[\lambda_1 \gamma_1 \sigma_\varepsilon^2 - (\lambda_1 \beta_1 + \lambda_1 \gamma_1 - 1) \Sigma_0\right] - \\ & \frac{\gamma_2 \left[(1 - \lambda_2 \beta_2)\left(\sigma_u^2 - \beta_1 \gamma_1 \sigma_\varepsilon^2\right) \Sigma_0 - \lambda_2 \gamma_2 \left(\beta_1^2 \sigma_\varepsilon^2 \Sigma_0 + \sigma_u^2 \Sigma_0 + \sigma_\varepsilon^2 \sigma_u^2\right)\right]}{(\beta_1 + \gamma_1)^2 \Sigma_0 + \gamma_1^2 \sigma_\varepsilon^2 + \sigma_u^2}. \end{aligned}$$

The correlation coefficients between the trading positions of the insider and the govern-

ment are

$$corr\left(x_1, g_1\right) = \frac{\beta_1\gamma_1\Sigma_0}{\sqrt{\beta_1^2\gamma_1^2\Sigma_0\left(\Sigma_0+\sigma_\varepsilon^2\right)}},$$

$$corr\left(x_2, g_2\right) = \frac{\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0}{\sqrt{\beta_2^2\gamma_2^2\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)}}.$$

The correlation coefficients between the trading positions of the government and the ones of the market maker are

$$corr\left(g_1, y_1\right) = \frac{\beta_1\gamma_1\Sigma_0+\gamma_1^2\left(\Sigma_0+\sigma_\varepsilon^2\right)}{\sqrt{\gamma_1^2\left(\Sigma_0+\sigma_\varepsilon^2\right)\left[\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]}},$$

$$corr\left(g_2, y_2\right) = \frac{\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0+\gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)}{\sqrt{\gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)\left[\begin{array}{c}\beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0\\ +\gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)+\\ 2\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0+\\ \sigma_u^2\left[\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]\end{array}\right]}}.$$

Proof *The proof is in Appendix A.* □

For the purpose of comparison, we consider two degenerate economies: the economy with $\sigma_\varepsilon^2 = 0$ and the economy with $\sigma_\varepsilon^2 = +\infty$ (i.e., the standard Kyle setting). The first economy corresponds to the case in which the government has perfect information about the future liquidation value of the risky asset (i.e., $s = v$). In this case, the government and the insider have the same information and the equation system (composed of (15), (19), (24), (28), (32), and (33)) can be further simplified as a polynomial of a single variable λ_2. In the second economy, the government has no information and does not participate in the market. Thus, the model is essentially the standard two-period Kyle model. We summarize the results of the two special cases in Corollary 1 and Corollary 2, respectively.

Corollary 1 *If $\sigma_\varepsilon^2 = 0$, the government has perfect information about the liquidation value* *of the risky asset, and the equation system describing the linear pure strategy equilibrium degenerates to a polynomial of* λ_2*. To be specific,* λ_2 *solves the following polynomials:*

$$a_{10}\lambda_2^{10} + a_9\lambda_2^9 + a_8\lambda_2^8 + a_7\lambda_2^7 + a_6\lambda_2^6 + a_5\lambda_2^5 + a_4\lambda_2^4 + a_3\lambda_2^3 + a_2\lambda_2^2 + a_1\lambda_2 + a_0 = 0, \quad (34)$$

where

$$\begin{aligned}
a_{10} &= 23040\theta^2\phi^6 + 2560\theta^3\phi^4, a_9 = 161280\theta^2\phi^5 + 15360\theta^3\phi^3, \\
a_8 &= 455040\theta^2\phi^4 + 34560\theta^3\phi^2, a_7 = 654080\theta^2\phi^3 - 15360\theta\phi^5 + 34560\theta^3\phi, \\
a_6 &= 494680\theta^2\phi^2 - 69120\theta\phi^4 + 12960\theta^3, a_5 = 184800\theta^2\phi - 115200\theta\phi^3, \\
a_4 &= 26280\theta^2 - 88320\theta\phi^2 + 256\phi^4, a_3 = -3168\theta\phi + 512\phi^3, \\
a_2 &= -432\theta + 384\phi^2, a_1 = 128\phi, a_0 = 16.
\end{aligned}$$

All the other variables can be given as expressions of λ_2 as follows:

$$\beta_2 = \frac{1 + 2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2}, \gamma_2 = \frac{1 - 2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2}, \lambda_1 = \frac{3\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2 - (2 + 4\phi\lambda_2)/\theta}{4\lambda_2},$$

$$\beta_1 = \frac{1}{\lambda_1}\left[1 - \lambda_1\left(3 - \frac{(2 + 4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\left[1 - \frac{\lambda_1(2 + 4\phi\lambda_2)^2}{2\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right],$$

$$\gamma_1 = \frac{1}{\lambda_1}\left[1 - \lambda_1\left(3 - \frac{(2 + 4\phi\lambda_2)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\left[1 + \frac{2\lambda_1\lambda_2\left(4\phi^2\lambda_2^2 + 4\phi\lambda_2 - 1\right)^2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right],$$

where $\theta \equiv \sigma_u^2/\Sigma_0$*. Then the expected price volatility is*

$$E(p_2 - p_1)^2 = \frac{(3 + 2\phi\lambda_2)}{1 + 2\phi\lambda_2}\lambda_2^2\sigma_u^2.$$

The measures for price discovery/efficiency are

$$\Sigma_1 \equiv var\left(v|y_1\right) = E\left(v - p_1\right)^2 = \frac{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}{2 + 4\phi\lambda_2}\sigma_u^2,$$

$$\Sigma_2 \equiv var\left(v|y_1, y_2\right) = E\left(v - p_2\right)^2 = \frac{\left(3 + 2\phi\lambda_2\right)}{2}\lambda_2^2\sigma_u^2.$$

The expected lifetime profits of the insider and expected lifetime costs of the government are, respectively,

$$E\left(\pi\right) = \beta_1\left[1 - \lambda_1\left(3 - \frac{\left(2 + 4\phi\lambda_2\right)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\Sigma_0 + \beta_2\left[1 - \lambda_2\left(\beta_2 + \gamma_2\right)\right]\frac{\sigma_u^2\Sigma_0}{\left(\beta_1 + \gamma_1\right)^2\Sigma_0 + \sigma_u^2},$$

$$E\left(c\right) = -\gamma_1\left[1 - \lambda_1\left(3 - \frac{\left(2 + 4\phi\lambda_2\right)^2}{4\theta\lambda_2\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right)\right]\Sigma_0 - \gamma_2\left[1 - \lambda_2\left(\beta_2 + \gamma_2\right)\right]\frac{\sigma_u^2\Sigma_0}{\left(\beta_1 + \gamma_1\right)^2\Sigma_0 + \sigma_u^2}.$$

The correlation coefficients between the trading positions of the insider and the government are

$$corr\left(x_1, g_1\right) = \frac{\beta_1\gamma_1}{\sqrt{\beta_1^2\gamma_1^2}} \text{ and } corr\left(x_2, g_2\right) = \frac{\beta_2\gamma_2}{\sqrt{\beta_2^2\gamma_2^2}}.$$

The correlation coefficients between the trading positions of the government and the market maker are

$$corr\left(g_1, y_1\right) = \frac{\gamma_1\left(\beta_1 + \gamma_1\right)}{\sqrt{\gamma_1^2}}\sqrt{\frac{\Sigma_0}{\left[\left(\beta_1 + \gamma_1\right)^2\Sigma_0 + \sigma_u^2\right]}},$$

$$corr\left(g_2, y_2\right) = \frac{\gamma_2\left(\beta_2 + \gamma_2\right)}{\sqrt{\gamma_2^2\left[\left(\beta_2 + \gamma_2\right)^2 + \left(\beta_1 + \gamma_1\right)^2 + \theta\right]}}.$$

Proof *The proof is in Appendix B.* □

As is shown in Corollary 1, when the government has perfect information about the future liquidation value of the risky asset as the insider, the learning processes between the insider and the government degenerate. In particular, four learning variables defined in (13), (22), (29), and (30) are degenerated as $\delta_1 = \delta_2 = 1$ and $\delta_3 = \delta_4 = \lambda_1$. The equation system describing the equilibrium is greatly simplified and can be solved as a 10-th order polynomial about λ_2.

Corollary 2 (Two-Period Kyle Model) *If $\sigma_\varepsilon^2 = +\infty$, the government has no information about the fundamentals and does not trade in the financial market. The general model degenerates to the standard two-period Kyle model. In this case, a subgame perfect linear equilibrium exists in which*

$$x_t = \beta_t \left(v - p_{t-1}\right), t \in \{1, 2\}, \tag{35}$$

$$p_t = p_{t-1} + \lambda_t y_t, t \in \{1, 2\}, \tag{36}$$

$$\beta_1 = \sqrt{\frac{2k-1}{2k}} \frac{\sigma_u}{\sqrt{\Sigma_0}}, \beta_2 = \sqrt{\frac{4k-1}{2k}} \frac{\sigma_u}{\sqrt{\Sigma_0}}, \tag{37}$$

$$\lambda_1 = \frac{\sqrt{2k\left(2k-1\right)}}{4k-1} \frac{\sqrt{\Sigma_0}}{\sigma_u}, \lambda_2 = \sqrt{\frac{k}{2\left(4k-1\right)}} \frac{\sqrt{\Sigma_0}}{\sigma_u}, \tag{38}$$

$$E\left(\pi\right) = \left[\frac{\sqrt{2k\left(2k-1\right)}}{4k-1} + \frac{1}{2}\sqrt{\frac{2k}{4k-1}}\right] \sigma_u \sqrt{\Sigma_0}, \tag{39}$$

$$E\left(p_2 - p_1\right)^2 = \frac{k}{4k-1} \Sigma_0, \tag{40}$$

$$\Sigma_1 = E\left(v - p_1\right)^2 = \frac{2k}{4k-1} \Sigma_0, \Sigma_2 = E\left(v - p_2\right)^2 = \frac{k}{4k-1} \Sigma_0, \tag{41}$$

where

$$k \equiv \frac{\lambda_2}{\lambda_1} = \frac{1}{6}\left[1 + 2\sqrt{7}\cos\left(\frac{1}{3}\left(\pi - \arctan 3\sqrt{3}\right)\right)\right] \approx 0.901,$$

and two associated SOCs are $\lambda_1 > 0$, $\lambda_2 > 0$.[9]

Corollary 2 shows that when $\sigma_\varepsilon^2 = +\infty$, the general model becomes a two-period Kyle (1985) benchmark that can be solved explicitly (see Huddart, Hughes and Levine, 2001).

[9] The proof of Corollary 2 can be found in Huddart, Hughes and Levine (2001). In addition, since there is no government in the standard Kyle model, the correlation coefficients ($corr\left(x_i, g_i\right)$, $corr\left(y_i, g_i\right)$) are all zero.

All results are intuitive: The trading intensities (β_1, β_2) increase in the amount of noisy trading per unit of private information (defining as $\theta \equiv \sigma_u^2/\Sigma_0$); the market liquidity ($1/\lambda_1$, $1/\lambda_2$) increases in the amount of noisy trading per unit of private information;, the expected lifetime profit of the insider, $E(\pi)$, increases both in the amount of noisy trading (σ_u^2) and in the amount of private information (Σ_0); and as equation (41) shows, the equilibrium prices reveal information gradually.

Note that, as shown in equation (40), the expected squared price change, $E(p_2 - p_1)^2$, increases in the amount of private information, Σ_0, and does not depend on noisy trading, σ_u^2. Thus, in the Kyle-type models, price stability is driven by the speculative trading of the insider with private information and does not relate to noisy trading. De Long, Shleifer, Summers and Waldmann (1990) and Brunnermeier, Sockin and Xiong (2020) both show that stock market turbulence originates from the noisy trading, and Brunnermeier, Sockin and Xiong (2020) also consider government intervention to reduce price volatility. Our paper complements theirs by providing an alternative origin of stock market turbulence.

5 Numerical Results

There are four exogenous variables in the model: the variance of the liquidation value of the risky asset, Σ_0, the variance of the noise trading in each period, σ_u^2, the variance of the information noise of the government, σ_ε^2, and the policy weight of the government, ϕ. For analytical convenience, we make several specifications about parameters. First, we define $\theta \equiv \sigma_u^2/\Sigma_0$ as the amount of noisy trading per unit of private information and change its values in $[0, 1]$ continuously. Second, we choose three possible values for $\sigma_\varepsilon^2 : \{0, 2, 10\}$. When $\sigma_\varepsilon^2 = 0$, the government has perfect information about the liquidation value of the risky asset. When $\sigma_\varepsilon^2 = 2$, the government's information quality is relatively high, and when $\sigma_\varepsilon^2 = 10$, the government's information quality is low. Third, we choose three possible values for ϕ : $\{0, 1, 3\}$. When $\phi = 0$, the government is another insider. When $\phi = 1$, the government puts an equal weight on its policy goal and profit maximization. When $\phi = 3$, the government cares more about the policy goals than about profit maximization.

5.1 The Insider's Behavior

Figure 1 describes the insider's trading intensities in two periods and his expected lifetime profits. For any given values of σ_ε^2 and ϕ, the trading intensities of the insider in two periods, (β_1, β_2), increase in the amount of noisy trading per unit of private information. Since the insider is maximizing his profits, the larger trading intensities are associated with more expected lifetime profits. Hence, the expected lifetime profits also increase in the amount of noisy trading per unit of private information, θ.

[**Insert Figure 1 about here**]

We want to highlight two messages. First, as a very striking result, the insider may trade against his signal in period 1 (i.e., $\beta_1 < 0$). This will happen when the government has perfect information and cares a lot about its policy goal (i.e., $\sigma_\varepsilon^2 = 0$ and $\phi = 3$). In this case, seeing strong information, the insider will sell (as opposed to buy) in period 1 and buy a lot in period 2, i.e., β_1 is negative and β_2 is positive and large. This is because in the presence of a very informed government player who cares about price stability, the insider wants to hide his information in period 1 and then trade aggressively in period 2 to exploit his uncovered information and maximize profits.

Second, we can compare our results to the standard Kyle model to highlight the implications of government intervention. When the government's information is imperfect but its quality is relatively high (i.e., $\sigma_\varepsilon^2 = 2$), compared to the standard Kyle model, the insider trades less aggressively (lower β_1) in period 1 but more aggressively (higher β_2) in period 2 for any given values of σ_ε^2 and θ.[10] Intuitively, when the government's information quality is

[10] Note that if the government has perfect information ($\sigma_\varepsilon^2 = 0$) and cares only about profits ($\phi = 0$), the insider's trading intensities in two periods are less than that in the standard Kyle model.

relatively high, the insider tries to conceal his information by trading less aggressively in period 1. In period 2, however, the insider exploits all of his information advantage and trades more aggressively than he would do in the standard Kyle model. Moreover, the trading intensity of the insider in period 1 decreases in the policy weight of the government, ϕ, and the trading intensity in period 2 increases in ϕ for any given values of σ_ε^2 and θ. As shown by the third column of Figure 1, when the government's information quality increases, it is harder for the insider to earn profits.

If the government's information quality is very low (i.e., $\sigma_\varepsilon^2 = 10$), the willingness of the insider to conceal his information is very weak, and in both periods, he trades just like a standard Kyle insider. Due to the low information quality, the government trades like a noise trader and provides more liquidity for the insider.[11] Thus, in this case, the insider is likely to earn more profits than he does in the standard Kyle model.

5.2 The Government's Behavior

Figure 2 displays the government's trading intensities in two periods (γ_1, γ_2), as well as the two elements in its objective function, the government's expected lifetime costs $E(c)$ and expected squared price change $E(p_2 - p_1)^2$. The first two columns show that for any given values of σ_ε^2 and ϕ, the government's trading intensities in two periods (γ_1, γ_2) increase in the amount of noisy trading per unit of private information (θ). Echoing the insider's trading behavior, a striking result here is that the government's trading patterns depend crucially on the weight of the policy goal in its objective function. In particular, when the government cares a lot its policy goal (i.e., $\phi = 3$), it will engage in reverse trading: seeing strong information, the government buys in period 1 but sells in period 2 (i.e., $\gamma_1 > 0$ and $\gamma_2 < 0$). Combining with the result on the insider's trading, this implies that when the government has very precise information and cares a lot its policy goal (i.e., $\sigma_\varepsilon^2 = 0$ and $\phi = 3$), the government and the insider are trading against each other in both periods.

[11]If the government makes money in this situation, the noise traders will lose more money. In this case, it is optimal for the government to quit the financial market.

[Insert Figure 2 about here]

As shown in the third column of Figure 2, the government always makes money when it trades in the financial market. On one hand, it is intuitive to see that the government's expected lifetime profits is lower when it puts more weight on policy goals relative to profit concerns. On the other hand, the expected lifetime profits of the government increase in its information quality. Empirical evidence of the model prediction is shown by Huang, Miao and Wang (2019). They estimate the value creation of the government intervention that increases the value of the rescued non-financial firms by RMB 206 billion after netting out the average purchase cost, which is about one percent of the Chinese GDP in 2014.[12]

The fourth column in Figure 2 demonstrates the resulting price stability due to government intervention. We observe that relative to the standard Kyle model, government intervention effectively lowers price volatility for all parameter values, which implies that government intervention is effective in enhancing price stability. Moreover, the price volatility $E(p_2 - p_1)^2$ increases in σ_ε^2 and decreases in ϕ for good information quality. When information quality is low ($\sigma_\varepsilon^2 = 10$), the price volatility is insensitive to ϕ.[13] Thus, the price-stability effect on financial market of government intervention hinges crucially on information quality. If the government's information quality is high, the government stabilizes the financial market effectively. If the government's information quality is low, government intervention is not effective no matter how the government cares about financial stability. Finally, the intervention effect is less effective when noisy trading is high, since price volatility increases with noise trading. This result is consistent with that derived by Brunnermeier, Sockin and Xiong (2020) but through a different mechanism.

[12] The value estimated is for the stocks purchased by the Chinese government between the period starting with the market crash in mid-June of 2015 and the market recovery in September.

[13] When σ_ε^2 approaches infinite, the equilibrium $E(p_2 - p_1)^2$ will converge to its value of the standard Kyle model, 0.346, as shown in Corollary 2.

5.3 Position Correlations

As the analysis in the previous two subsections shows, the insider and the government can trade against each other, which is true when the government has precise information and cares a lot about its policy goal. In this subsection, we further sharpen this result by examining the correlations among the positions of the government, the insider, and the market maker (or equivalently, the total order flows).

The first two columns in Figure 3 show the correlation coefficients between the government's and the insider's trading positions in the two periods. In period 1, if the government has perfect information ($\sigma_{\varepsilon}^2 = 0$) and cares more about policy goals ($\phi = 3$), the insider and the government trade exactly against each other with opposite directions ($corr\,(x_1, g_1) = -1$). If the government is less concerned about policy goals or has imperfect information, it trades in the same direction as the insider ($corr\,(x_1, g_1) > 0$). In period 2, if the government cares more about policy goals ($\phi = 3$), it trades in the opposite directions of the insider. If the government cares more about profits ($\phi = 0$), it trades in the same direction as the insider. If the government puts on an equal footing on these two goals ($\phi = 1$), the trading correlation depends on the amount of noisy trading per unit of private information (θ). When θ is below a certain threshold, the government and the insider trade in the opposite directions. When θ is above the threshold, the government and the insider trade in the same direction. Moreover, the value of the threshold decreases in the information quality of the government.

[Insert Figure 3 about here]

The last two columns in Figure 3 show the correlation coefficients between the government's trading positions and the total order flows. In period 1, the correlation coefficient between the government's trading positions and the total order flow is positive and decreases in the information quality of the government. In period 2, similarly, if the government cares more about policy goals, the correlation is negative. If the government cares more about profits, the correlation is positive. If the government puts on an equal footings on these two goals, there is a threshold in which the sign of the correlation can switch. Moreover, given σ_ε^2, the switching points for $corr(x_2, g_2)$ and $corr(g_2, y_2)$ are the same, the government, as a large player in the financial market, dominates the market maker (with trading volumes $-y_i, i = 1, 2$) to trade against the insider.

5.4 Market Liquidity and Price Efficiency

Figure 4 examines the market-quality implications of government intervention, and for market-quality measures, we mainly focus on market liquidity and price discovery (e.g., O'Hara, 2003; Bond, Edmans, and Goldstein, 2012; Goldstein and Yang, 2017). Market liquidity is measured by the inverse of Kyle's lambda $(1/\lambda_1, 1/\lambda_2)$, and a lower λ_t means that the period-t market is deeper and more liquid.[14] Price discovery measures how much information about the asset value v is revealed in prices. Given that price functions (10) and (11) are linear functions of aggregate order flows (y_1 and y_2), price discovery is measured by the market maker's posterior variances of v in periods 1 and 2: $\Sigma_1 = var(v|y_1)$, $\Sigma_2 = var(v|y_1, y_2)$. A lower Σ_t implies a more informative period-t price about v, for $t \in \{1, 2\}$.

[Insert Figure 4 about here]

The first two columns of Figure 4 present the equilibrium market liquidities in two periods. First, as in the standard Kyle models, for any given σ_ε^2 and ϕ, the market liquidity measures in two periods $(1/\lambda_1, 1/\lambda_2)$ increase in θ, the amount of noisy trading per unit of private information. Second, relative to the standard Kyle model, government intervention has mild effects on the market liquidity in period 1, but raises the market liquidity in period 2. If the government has no policy concerns ($\phi = 0$) and perfect information ($\sigma_\varepsilon^2 = 0$), the market liquidity is slightly smaller than that of the Kyle model in period 1, which shows that private

[14] One important reason to care about market liquidity is that it is related to the welfare of noise traders, who can be interpreted as investors trading for non-informational, liquidity or hedging reasons that are decided outside the financial markets. In general, noise traders are better off in a more liquid market.

information has mild negative effect on market liquidity. If the government has imperfect information ($\sigma_\varepsilon^2 \neq 0$) and cares about price stability ($\phi > 0$), the market liquidity is slightly larger than that of the Kyle model in period 1. In period 2, the market liquidity is larger than that of the Kyle model and does not hinge on the policy weight of the government. Third, if the government's information quality is very low ($\sigma_\varepsilon^2 = 10$), the market liquidity measures in two periods converge to that of the Kyle model. The negative effect on market liquidity of information and the positive effect of policy concerns cancel out. This, again, suggests that the effectiveness of government intervention crucially hinges on the information quality of the government.

The last two columns of Figure 4 show that government intervention effectively raises price discovery in two periods. Because the government has information about fundamentals, its informative trading improves price discovery/efficiency of the financial market. Thus, in contrast to the results in Brunnermeier, Sockin and Xiong (2020), Figure 4 shows that government intervention improves price stability and price efficiency simultaneously. In Brunnermeier, Sockin and Xiong (2020), the market volatility comes from noisy trading and the government has no private information so government intervention reducing price volatility decreases information efficiency. However, in our model, the market volatility stems from speculative insider trading and the government has information about the fundamentals. For this reason, government intervention effectively stabilizes the asset prices and improves the price efficiency of the financial markets.

More interestingly, price discovery increases in the policy weight of the government in period 1 while decreases in the policy weight in period 2. Intuitively, in period 1, the insider trades less by hedging on the larger policy weight of the government. In order to hedge on the insider's reserved trading, the government trades more, which increases the total amount of the informational trading and hence improves price discovery. In period 2, the insider exploits the remaining information advantage and trades more aggressively to hedge on the larger policy weight. Since the government cares more about price stability, it has to trade less aggressively, so price discovery decreases in period 2. Moreover, if the government's information quality is very low ($\sigma_\varepsilon^2 = 10$), the price discovery measures in two periods are very close to and sightly less than those of the standard Kyle model.

6 Conclusions

In this paper, we explore the implications of government intervention in a two period Kyle (1985) model in which a government with private information directly trades in financial markets to achieve its policy goal of stabilizing the financial market. We find that when the government has very precise information and cares much about price stability, it effectively trades against the informed insider in the financial markets, and both the government and the insider engage in reversed trading strategies, although in different directions. In terms of market quality implications, we find that in general, government intervention can effectively stabilize the financial markets and improve the price efficiency, but the effectiveness crucially depends on the information quality of the government. The higher the information quality, the more effective the government intervention is. If the government's information quality is very low, government intervention becomes ineffective. Our analysis also makes other predictions that are consistent with the empirical findings. For instance, the government makes trading profits in equilibrium; price volatility increases with the noise trading in the financial markets.

Appendix

Proof of Proposition 1

Proof of Proposition 1. The insider's problem in period 2 is solved in the text. The objective function of the insider in period 1, (18), is derived by substituting (8) and (10) into (16),

$$
\begin{aligned}
& E(\pi|v) \\
= \; & E\left[(v-p_1)x_1 + \frac{(1-\lambda_2\gamma_2\theta_1)^2}{4\lambda_2}(v-p_1)^2|v\right] \\
= \; & E\left\{\begin{array}{c} [v-p_0-\lambda_1(x_1+\gamma_1(s-p_0)+u_1)]x_1+ \\ \frac{(1-\lambda_2\gamma_2\theta_1)^2}{4\lambda_2}\left[(v-p_0-\lambda_1(x_1+\gamma_1(s-p_0)+u_1))^2|v\right] \end{array}\right\} \\
= \; & [v-p_0-\lambda_1x_1-\lambda_1\gamma_1E(s-p_0|v)]x_1 + \\
& \frac{(1-\lambda_2\gamma_2\theta_1)^2}{4\lambda_2}E\left\{[v-p_0-\lambda_1x_1-\lambda_1\gamma_1(s-p_0)-\lambda_1u_1]^2|v\right\} \\
= \; & [v-p_0-\lambda_1x_1-\lambda_1\gamma_1E(s-p_0|v)]x_1 + \\
& \frac{(1-\lambda_2\gamma_2\theta_1)^2}{4\lambda_2}\left\{\begin{array}{c} (v-p_0)^2+\lambda_1^2x_1^2+\lambda_1^2\gamma_1^2E\left[(s-p_0)^2|v\right]-2(v-p_0)\lambda_1x_1 \\ +\lambda_1^2\sigma_u^2-2\lambda_1\gamma_1(v-p_0)E(s-p_0|v)+2\lambda_1^2\gamma_1x_1E(s-p_0|v) \end{array}\right\}.
\end{aligned}
$$

Then we derive the FOC and the SOC in the main text.

The government's problem in period 2 is derived in the main text. It is hard to derive the objective function in period 1. For this purpose, using equations (7), (9) and (11), we have

$$
\begin{aligned}
& E\left\{\left[\phi(p_2-p_1)^2+(p_1-v)g_1+\left(-(1-\lambda_2\beta_2)\gamma_2\theta_2+\lambda_2\gamma_2^2\right)(s-E(s|y_1))^2\right]|s\right\} \qquad (42) \\
= \; & E\left\{\left[\begin{array}{c} \phi\lambda_2^2(\beta_2(v-p_1)+\gamma_2(s-E(s|y_1))+u_2)^2- \\ (v-p_1)g_1+(\lambda_2\gamma_2^2-(1-\lambda_2\beta_2)\gamma_2\theta_2)(s-E(s|y_1))^2 \end{array}\right]|s\right\} \\
= \; & \phi\lambda_2^2\left\{\beta_2^2E\left[(v-p_1)^2|s\right]+\gamma_2^2E\left[s-E(s|y_1)^2|s\right]+\sigma_u^2+2\beta_2\gamma_2E\left[(v-E(v|y_1))(s-E(s|y_1))|s\right]\right\} \\
& -g_1E(v-p_1|s)+\left[\lambda_2\gamma_2^2-(1-\lambda_2\beta_2)\gamma_2\theta_2\right]E\left[(s-E(s|y_1))^2|s\right],
\end{aligned}
$$

where, using the projection theorem repeatedly,

$$E\left(v-p_1|s\right)=\left(1-\lambda_1\beta_1\right)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\left(s-p_0\right)-\lambda_1 g_1,$$

$$\begin{aligned}
var\left(v-p_1|s\right) &= var\left(v-p_1\right)-\frac{cov\left(v-p_1,s\right)^2}{var\left(s\right)} \\
&= var\left(v|y_1\right)-\frac{cov\left(v-p_1,s\right)^2}{var\left(s\right)} \\
&= \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}-\frac{\left[\left(1-\lambda_1\beta_1-\lambda_1\gamma_1\right)\Sigma_0-\lambda_1\gamma_1\sigma_\varepsilon^2\right]^2}{\Sigma_0+\sigma_\varepsilon^2},
\end{aligned}$$

$$\begin{aligned}
& E\left[\left(v-p_1\right)^2|s\right] \\
= \; & E^2\left(v-p_1|s\right)+var\left(v-p_1|s\right) \\
= \; & \left[\left(1-\lambda_1\beta_1\right)\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\left(s-p_0\right)-\lambda_1 g_1\right]^2+ \\
& \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}-\frac{\left[\left(1-\lambda_1\beta_1-\lambda_1\gamma_1\right)\Sigma_0-\lambda_1\gamma_1\sigma_\varepsilon^2\right]^2}{\Sigma_0+\sigma_\varepsilon^2},
\end{aligned}$$

$$\begin{aligned}
& E\left[\left(v-E\left(v|y_1\right)\right)\left(s-E\left(s|y_1\right)\right)|s\right] \\
= \; & E\left[\left(v-Ev-\theta_4\left(y_1-Ey_1\right)\right)\left(s-Es-\theta_3\left(y_1-Ey_1\right)\right)|s\right] \\
= \; & \left\{\begin{array}{c}
\left(1-\theta_4\beta_1\right)\left(s-p_0\right)E\left(v-p_0|s\right)-\theta_3\beta_1\left(1-\theta_4\beta_1\right)E\left[\left(v-p_0\right)^2|s\right]- \\
\theta_3 g_1\left(1-\theta_4\beta_1\right)E\left(v-p_0|s\right)-\theta_4 g_1\left(s-p_0\right)+\theta_4\theta_3 g_1\beta_1 E\left(v-p_0|s\right)+\theta_4\theta_3 g_1^2+\theta_4\theta_3\sigma_u^2
\end{array}\right\},
\end{aligned}$$

$$E\left(v-p_0|s\right)=\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\left(s-p_0\right),$$

$$var\left(v-p_0|s\right)=\frac{\Sigma_0\sigma_\varepsilon^2}{\Sigma_0+\sigma_\varepsilon^2},$$

$$E\left[(v-p_0)^2|s\right] = E^2(v-p_0|s)+var(v-p_0|s)$$
$$= \left(\frac{\Sigma_0}{\Sigma_0+\sigma_\varepsilon^2}\right)^2(s-p_0)^2+\frac{\Sigma_0\sigma_\varepsilon^2}{\Sigma_0+\sigma_\varepsilon^2},$$

$$\begin{aligned}
& E\left[(s-E(s|y_1))^2|s\right] \\
= \; & E\left\{\left[s-E(s)-\frac{cov(s,y_1)}{var(y_1)}(y_1-E(y_1))\right]^2|s\right\} \\
= \; & E\left\{[s-p_0-\theta_3(\beta_1(v-p_0)+g_1+u_1)]^2|s\right\} \\
= \; & E\left\{[s-p_0-\theta_3\beta_1(v-p_0)-\theta_3 g_1-\theta_3 u_1]^2|s\right\} \\
= \; & \left[\begin{array}{c} (s-p_0)^2+\theta_3^2\beta_1^2E\left[(v-p_0)^2|s\right]+\theta_3^2g_1^2+\theta_3^2\sigma_u^2- \\ 2\theta_3\beta_1(s-p_0)E(v-p_0|s)-2\theta_3g_1(s-p_0)+2\theta_3^2g_1\beta_1E(v-p_0|s)\end{array}\right].
\end{aligned}$$

Substituting the above expressions into (42) leads to the government's period-1 objective function (27). Then we can derive the FOC and SOC in the main text.

Combining (5) and (10) and applying the projection theorem, we have (32). Since $E(y_2|y_1)=0$, by (5) and (11), using the projection theorem, we know that

$$\lambda_2=\frac{cov(v,y_2|y_1)}{var(y_2|y_1)}. \tag{43}$$

Using the projection theorem, we have that

$$\begin{aligned}
var(y_2|y_1) &= var(y_2)-\frac{cov(y_2,y_1)^2}{var(y_1)} \\
&= var(y_2) \\
&= var(\beta_2(v-E(v|y_1))+\gamma_2(s-E(s|y_1))+u_2) \\
&= \left[\begin{array}{c}\beta_2^2var(v-p_1)+2\beta_2\gamma_2cov(v-E(v|y_1),s-E(s|y_1)) \\ +\gamma_2^2var(s-E(s|y_1))+\sigma_u^2\end{array}\right],
\end{aligned} \tag{44}$$

where

$$var\left(v-p_1\right)=\frac{\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \tag{45}$$

$$\begin{aligned} & cov\left(v-E\left(v|y_1\right),s-E\left(s|y_1\right)\right) \\ = \; & E\left(v-E\left(v|y_1\right),s-E\left(s|y_1\right)\right) \\ = \; & \left(1-\beta_1\theta_4-\gamma_1\theta_4\right)\left(1-\beta_1\theta_3-\gamma_1\theta_3\right)\Sigma_0-\gamma_1\theta_4\left(1-\gamma_1\theta_3\right)\sigma_\varepsilon^2+\theta_3\theta_4\sigma_u^2, \end{aligned} \tag{46}$$

$$var\left(s-E\left(s|y_1\right)\right)=var\left(s|y_1\right)=\frac{\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_u^2\sigma_\varepsilon^2}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{47}$$

Substituting (45), (46) and (47) into (44) gives rise to

$$var\left(y_2|y_1\right)=\frac{\left(\begin{array}{c}\beta_2^2\Sigma_0\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)+2\beta_2\gamma_2\left(\sigma_u^2-\beta_1\gamma_1\sigma_\varepsilon^2\right)\Sigma_0+ \\ \gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_u^2\sigma_\varepsilon^2\right)+\sigma_u^2\left[\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right]\end{array}\right)}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{48}$$

Using (5), (11), (7) and (9), we derive

$$\begin{aligned} cov\left(v,y_2|y_1\right) &= E\left(v-E\left(v|y_1\right)\right)\left(y_2-E\left(y_2|y_1\right)\right) \\ &= E\left(v-E\left(v|y_1\right)\right)\left(\beta_2\left(v-E\left(v|y_1\right)\right)+\gamma_2\left(s-E\left(s|y_1\right)\right)+u_2\right) \\ &= \left(\beta_2+\gamma_2\right)var\left(v|y_1\right)+\gamma_2E\left(v-E\left(v|y_1\right)\right)\left(s-E\left(s|y_1\right)\right), \end{aligned} \tag{49}$$

where

$$\begin{aligned} var\left(v|y_1\right) &= var\left(v\right)-\frac{cov\left(v,y_1\right)^2}{var\left(y_1\right)} \\ &= \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}, \end{aligned} \tag{50}$$

$$\begin{aligned}
& E\left(v-E\left(v|y_1\right)\right)\left(s-E\left(s|y_1\right)\right) \\
= & E\left[v-Ev-\theta_4\left(y_1-Ey_1\right)\right]\left[\varepsilon-E\varepsilon-\frac{cov\left(\varepsilon,y_1\right)}{var\left(y_1\right)}\left(y_1-Ey_1\right)\right] \\
= & -\frac{\left(\beta_1+\gamma_1\right)\gamma_1\sigma_\varepsilon^2\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.
\end{aligned} \tag{51}$$

Plugging (50) and (51) into (49) leads to

$$cov\left(v,y_2|y_1\right)=\frac{\left(\beta_2+\gamma_2\right)\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0-\left(\beta_1+\gamma_1\right)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}. \tag{52}$$

Putting (48) and (52) in (43) leads to (33).

By substitution and the projection theorem, we can derive the moments listed in Proposition 1, namely, $E\left(p_2-p_1\right)^2$, Σ_1, Σ_2, $E\left(\pi\right)$ and $E\left(c\right)$. In particular,

$$\begin{aligned}
& E\left(p_2-p_1\right)^2 \\
= & \lambda_2^2 E y_2^2 \\
= & \lambda_2^2 E\left[\beta_2\left(v-p_1\right)+\gamma_2\left(s-E\left(s|y_1\right)\right)+u_2\right]^2 \\
= & \lambda_2^2\left\{\beta_2^2 var\left(v|y_1\right)+\gamma_2^2 var\left(s|y_1\right)+\sigma_u^2+2\beta_2\gamma_2 E\left(v-p_1\right)\left(s-E\left(s|y_1\right)\right)\right\} \\
= & \left\{\begin{array}{c}
\frac{\lambda_2^2\beta_2^2\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+\lambda_2^2\gamma_2^2\left(\beta_1^2\sigma_\varepsilon^2\Sigma_0+\sigma_u^2\Sigma_0+\sigma_\varepsilon^2\sigma_u^2\right)}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}+\lambda_2^2\sigma_u^2+ \\
2\beta_2\gamma_2\lambda_2^2\left[\begin{array}{c}
\left(1-\theta_4\beta_1-\theta_4\gamma_1\right)\left(1-\theta_3\beta_1-\theta_3\gamma_1\right)\Sigma_0 \\
-\gamma_1\theta_4\left(1-\theta_3\gamma_1\right)\sigma_\varepsilon^2+\theta_4\theta_3\sigma_u^2
\end{array}\right]
\end{array}\right\},
\end{aligned}$$

where the last equality is obtained by substitution of Equations (47), (50), and (51).

By definition and (50), we have that

$$\begin{aligned}
\Sigma_1 & \equiv var\left(v|y_1\right) \\
& = E\left(v-p_1\right)^2 \\
& = \frac{\left(\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.
\end{aligned}$$

By definition and the projection theorem, we obtain

$$\begin{aligned}
\Sigma_2 &\equiv var\left(v|y_1, y_2\right) \\
&= var\left(v|y_1\right) - \frac{cov\left(v, y_2|y_1\right)^2}{var\left(y_2|y_1\right)} \\
&= \frac{\left(1-\lambda_2\beta_2-\lambda_2\gamma_2\right)\left(\gamma_1\sigma_\varepsilon^2+\sigma_u^2\right)\Sigma_0+\lambda_2\left(\beta_1+\gamma_1\right)\gamma_1\gamma_2\sigma_\varepsilon^2\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\gamma_1^2\sigma_\varepsilon^2+\sigma_u^2}.
\end{aligned}$$

where the last equality comes from plugging Equations (48), (49), (50), and (51).□

Proof of Corollary 1

Proof of Corollary 1. If $\sigma_\varepsilon^2 = 0$, then the government has the same perfect information about the liquidation value of the risky asset as the insider. The four θ's describing the learning processes between the insider and the government are degenerated as: $\theta_1 = \theta_2 = 1$, $\theta_3 = \theta_4 = \lambda_1$. Setting $\sigma_\varepsilon^2 = 0$ in (15), (19), (24), (28), (32), and (33), we obtain the degenerated equation system

$$\beta_2 = \frac{1}{2\lambda_2}\left(1-\lambda_2\gamma_2\right), \tag{53}$$

$$\beta_1 = \frac{1-\lambda_1\gamma_1}{2\lambda_1}\frac{1-\frac{\lambda_1}{2\lambda_2}\left(1-\lambda_2\gamma_2\right)^2}{1-\frac{\lambda_1}{4\lambda_2}\left(1-\lambda_2\gamma_2\right)^2}, \tag{54}$$

$$\gamma_2 = \frac{1-\lambda_2\beta_2-2\phi\lambda_2^2\beta_2}{2\lambda_2+2\phi\lambda_2^2}, \tag{55}$$

$$\gamma_1 = \frac{1+2\lambda_1\left[\phi\lambda_2^2\left(\beta_2+\gamma_2\right)^2+\lambda_2\gamma_2\left(\beta_2+\gamma_2\right)-\gamma_2\right]}{1+\lambda_1\left[\phi\lambda_2^2\left(\beta_2+\gamma_2\right)^2+\lambda_2\gamma_2\left(\beta_2+\gamma_2\right)-\gamma_2\right]}\frac{1-\lambda_1\beta_1}{2\lambda_1}, \tag{56}$$

$$\lambda_1 = \frac{\left(\beta_1+\gamma_1\right)\Sigma_0}{\left(\beta_1+\gamma_1\right)^2\Sigma_0+\sigma_u^2}, \tag{57}$$

$$\lambda_2 = \frac{\left(\beta_2+\gamma_2\right)\Sigma_0}{\left(\beta_2+\gamma_2\right)^2\Sigma_0+\left(\beta_1+\gamma_1\right)^2\Sigma_0+\sigma_u^2}, \tag{58}$$

with three SOCs:

$$\lambda_2 > 0,$$

$$\lambda_1 \left[1 - \frac{\lambda_1}{4\lambda_2} (1 - \lambda_2\gamma_2)^2\right] > 0,$$

$$2\lambda_1^2 \left[\phi\lambda_2^2 (\beta_2 + \gamma_2)^2 + \lambda_2\gamma_2 (\beta_2 + \gamma_2) - \gamma_2\right] + 2\lambda_1 > 0.$$

Solving the linear equation system composed of (15) and (24) gives rise to

$$\beta_2 = \frac{1 + 2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2}, \gamma_2 = \frac{1 - 2\phi\lambda_2}{3\lambda_2 + 2\phi\lambda_2^2}. \tag{59}$$

Substituting (59) into (54), (56), and (57), respectively, we obtain

$$\frac{\lambda_1\beta_1}{1 - \lambda_1 (\beta_1 + \gamma_1)} = 1 - \frac{\lambda_1}{2\lambda_2} \left(\frac{2 + 4\phi\lambda_2}{3 + 2\phi\lambda_2}\right)^2, \tag{60}$$

$$\frac{\lambda_1\gamma_1}{1 - \lambda_1 (\beta_1 + \gamma_1)} = 1 + \frac{2\lambda_1\lambda_2 \left(4\phi^2\lambda_2^2 + 4\phi\lambda_2 - 1\right)}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}, \tag{61}$$

$$\frac{\lambda_1 (\beta_1 + \gamma_1)}{1 - \lambda_1 (\beta_1 + \gamma_1)} = \frac{(\beta_1 + \gamma_1)^2 \Sigma_0}{\sigma_u^2}. \tag{62}$$

Combining (60), (61) and (62) leads to

$$(\beta_1 + \gamma_1)^2 = \frac{\sigma_u^2}{\Sigma_0} \left[2 - \frac{4\lambda_1\lambda_2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}\right]. \tag{63}$$

Solving (32) for $\beta_1 + \gamma_1$ and plugging (63) in it, we obtain

$$\beta_1 + \gamma_1 = \lambda_1 \frac{\sigma_u^2}{\Sigma_0} \frac{3\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2 - 4\lambda_1\lambda_2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}. \tag{64}$$

Solving (33) for λ_2 and putting (63) in it, we solve for

$$\lambda_1 = \frac{3\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2 - (2 + 4\phi\lambda_2) \frac{\Sigma_0}{\sigma_u^2}}{4\lambda_2}. \tag{65}$$

Substituting (65) into (64) leads to

$$\beta_1 + \gamma_1 = \left[3 - \frac{(2 + 4\phi\lambda_2) \frac{\Sigma_0}{\sigma_u^2}}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2} \right] \frac{2 + 4\phi\lambda_2}{4\lambda_2}. \tag{66}$$

Substituting (65) into (63) gives rise to

$$(\beta_1 + \gamma_1)^2 = -\frac{\sigma_u^2}{\Sigma_0} + \frac{2 + 4\phi\lambda_2}{\left(3\lambda_2 + 2\phi\lambda_2^2\right)^2}. \tag{67}$$

Combining (66) and (67) gives us the polynomial listed in Corollary 1, (34). The expressions for all other endogenous variables can be derived by substitution and utilizing the projection theorem. □

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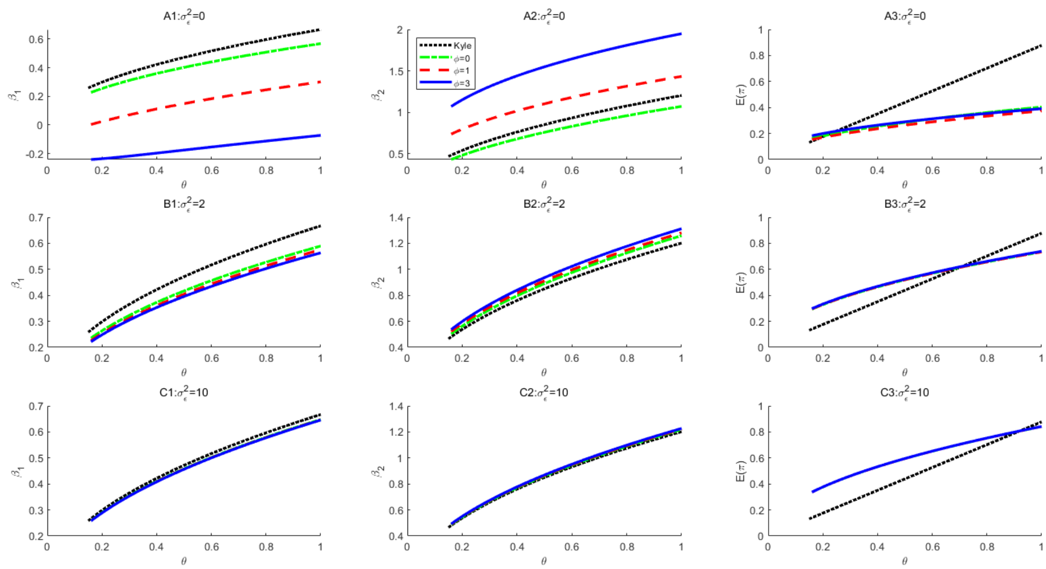

Figure 1: Insider's trading intensities, β_1, β_2, and expected lifetime profits, $E(\pi)$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dot dash green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

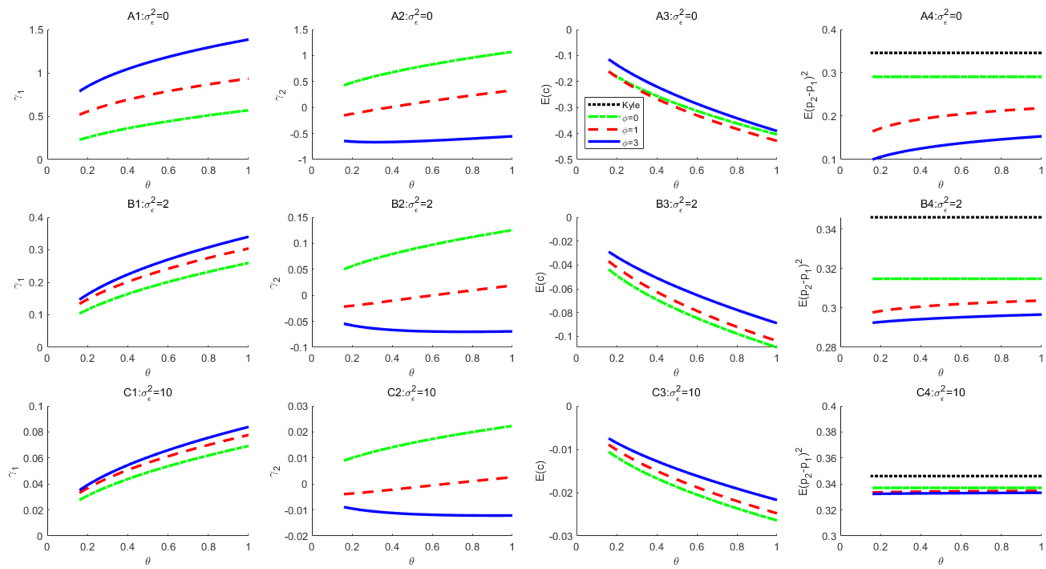

Figure 2: The government's trading intensities, γ_1, γ_2, the expected lifetime profits, $E(c)$, and the expected squared price change, $E(p_2 - p_1)^2$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dot dash green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

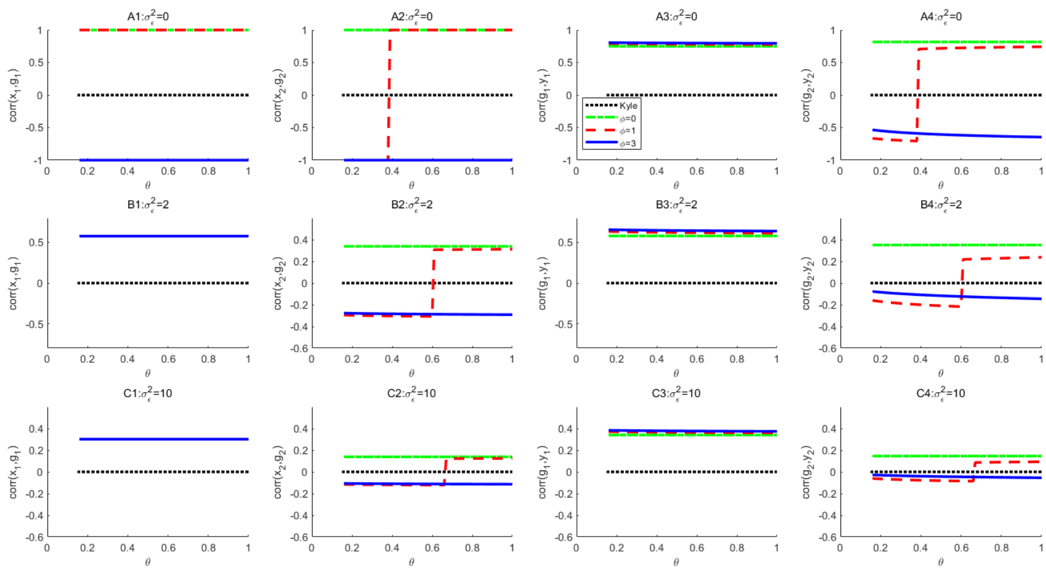

Figure 3: The correlation coefficients between the government's and the insider's trading positions in the two periods, $corr\,(x_1, g_1)$, $corr\,(x_2, g_2)$, and the correlation coefficients between the government's trading positions and the total order flows in the two periods, $corr\,(g_1, y_1)$, $corr\,(g_2, y_2)$, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dot dash green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.

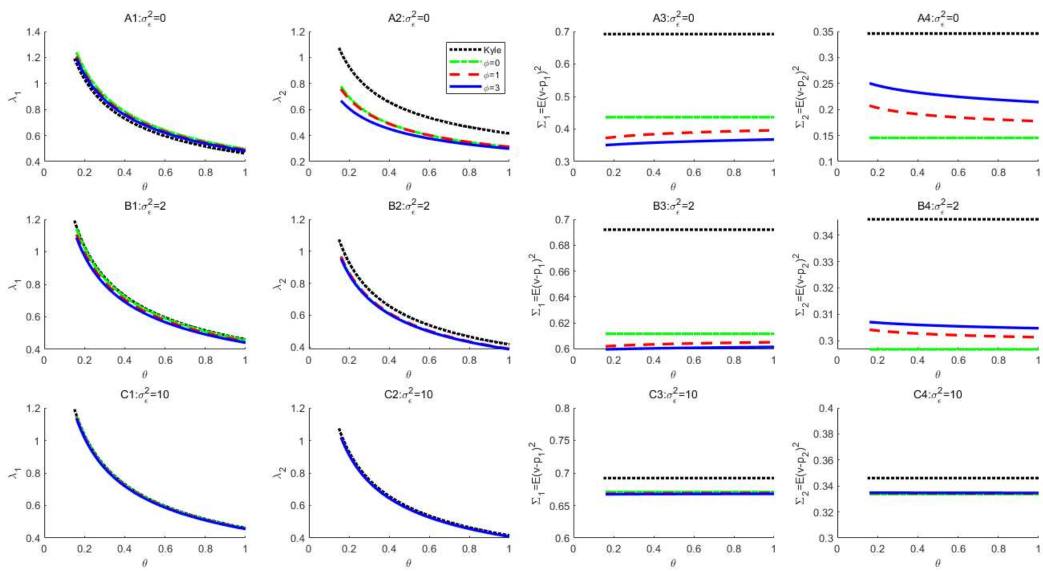

Figure 4: The market liquidities in two periods, $1/\lambda_1$, $1/\lambda_2$, and the price discoveries/efficiencies in two periods, Σ_1, Σ_2, for $\sigma_\varepsilon^2 = 0$, 2, and 10, respectively. In each panel, the dotted black line represents the standard Kyle equilibrium without the government intervention, the dot dash green line represents the equilibrium with policy weight $\phi = 0$, the dashed red line represents the equilibrium with policy weight $\phi = 1$, and the solid blue line represents the equilibrium with policy weight $\phi = 3$.