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Effects of Exhaustible Resources and Declining Population on Economic Growth with Hotelling's Rule*

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Abstract

This study introduces declining population and exhaustible resources into a semi-endogenous growth model that explicitly incorporates firms' optimization behavior and investigates the relationship between the population growth rate and the growth rate of the per capita output. The main results are as follows. First, irrespective of whether the population growth rate is positive or negative, the long-run growth rate of per capita output can be positive, depending on the conditions. Second, when the population growth rate is positive, the long-run growth rate of per capita output depends positively on the saving rate, although the model belongs to the class of semi-endogenous growth without scale effects.

Keywords: exhaustible resources; declining population; endogenous growth: Hotelling's rule

JEL Classification: O13; O44; Q32; Q43

1 Introduction

This study investigates how exhaustible resources and declining populations affect economic growth by building a semi-endogenous growth model. We integrate these two fields of economic research. One is related to the relationship between exhaustible

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resources and economic growth, while the other is related to the relationship between population decline and economic growth.

First, since the Club of Rome's The Limits to Growth, it has been pointed out that exhaustible resources such as petroleum and natural gas constrain economic growth (Meadows et al., 1972). The global oil crisis of the mid-1970s raised interest in this issue. Many economists have examined the effects of exhaustible resources on sustainable economic development (Stiglitz, 1974a, 1974b; Solow, 1974; Dasgupta and Heal, 1974). They concluded that if technological progress can overcome the scarcity of resources, sustainable economic development is possible. Although technology has advanced since the limits to growth, resources will run out if human beings continue to extract them. Therefore, the depletion of resources is an important issue.

Second, Japan's first postwar experience of a fall in population occurred in 2005, with negative population growth rates following 2009 and 2011. Similarly, concerns about population decline have been increasing in Italy and Germany (World Bank, 2013). Therefore, population decline is an urgent problem in developed economies. At first sight, few countries seem to have experienced negative population growth. However, we should consider the effects of the immigrants. If we consider the rate of natural increase (i.e., the crude birth rate minus the crude death rate), several countries have experienced negative population growth. Indeed, according to the United Nations (2013), the rates of natural population increase in 17 OECD countries were negative between 2005 and 2010. The global population indeed continues to increase. However, the population of developed countries does not grow as much. Contrarily, some countries, such as Japan, experienced population decline. Table 1, taken from the United Nations, World Population Prospects 2019, shows that population growth will decelerate over time. Further, in developed countries, the population growth rate will be harmful in the future, as shown by the blue shaded columns in Table 1.

[Table 1: Forecasts of population growth (Source: United Nations, World Population Prospects 2019)]

Based on these observations, it is vital to investigate how exhaustible resources and declining populations affect economic growth.

Our study incorporates exhaustible resources into an economic growth model and investigates the relationship between population growth and per capita output growth. Therefore, we can take Stiglitz (1974b).[1] The final goods are produced by exhaustible

1) For initial contributions that consider the relationship between non-renewable resources and economic development, see also Stiglitz (1974a), Solow (1974) and Dasgupta and Heal (1974).

resources, in addition to labor and capital. Under the assumption that the production function has constant returns to scale, the saving rate is exogenously given. Further, the population growth rate is exogenously given. He investigates the stability of the steady-state and the per capita output growth along the balanced growth path. The dynamical system comprises differential equations concerning the output–capital ratio, the ratio of exhaustible resource inputs, and the stock of exhaustible resources. The perfect foresight competitive economy is a saddle point; hence, the dynamics are unstable unless the economy starts on the saddle path. However, this is a study before the rational expectation hypothesis becomes mainstream. Accordingly, the dynamics are varying because no mechanism locates the initial value on the saddle path. Contrastingly, if the rational expectation is assumed, the initial value can be located on the saddle path, and the dynamics become stable. Suppose that the economy is steady, the per capita output growth rate is positive if the population growth rate is less than a threshold value and negative if the population growth rate is more than the threshold value. However, Stiglitz (1974b) only considers positive population growth and not negative population growth.

Cigno (1981) incorporates an endogenous population growth rate into Stiglitz's (1974b) model and examines how the dynamics of the model change by this endogenization of the population growth rate. He assumes that the population growth rate increases per capita consumption and decreases in industrialization, as measured by the capital-labor ratio. The results show that the endogenization of the population growth rate can stabilize the steady-state. Like Stiglitz (1974b), Cigno (1981) assumes positive population growth and does not consider negative population growth.

The studies mentioned above investigate neoclassical growth models with exogenous technological progress and constant returns to scale production functions. In contrast to these studies, some studies investigate endogenous growth models with exhaustible resources.[2)]

Groth and Schou (2002) present a semi-endogenous growth model with increasing returns to scale production function and show that even if non-renewable resources constrain economic growth, per capita output can grow sustainably, provided that the population growth rate is positive. The production function exhibits increasing

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Malaczewski (2018) points out that capital stock and non-renewable resources are complementary and not substitutes assumed in many former studies and presents a growth model in which capital stock and non-renewable resources are complements.

2) Suzuki (1976) presents a growth model in which investment in research and development activity by firms accumulates knowledge stock, which leads to technological progress. For endogenous growth models that incorporate non-renewable resources into final goods production, see Barbier (1999) and Cabo *et al.* . (2016).

returns concerning capital stock and labor. In their model, increasing returns are merely assumed and do not occur by some mechanism.[3)]

Bretschger (2013) presents an endogenous growth model in which both final goods production and research and development activities require exhaustible resources. In this model, population growth is endogenized and takes a positive value. The purpose of this study is to show that the economy can grow even though it faces exhaustible resources and population growth that seem unfavorable for per capita output growth. The results show that if the scarcity of exhaustible resources is fully reflected in the price of exhaustible resources, as resources are exhausted by their use, population growth is compatible with using exhaustible resources. Population growth promotes research and development activities and can offset the low increase caused by depleting resources.

However, Groth and Schou (2002) and Bretschger (2013) do not consider negative population growth.[4)]

First, few economic growth models consider population decline.[5)] Christiaans (2011) is one of the few exceptions and presents a semi-endogenous growth model. Wherein production exhibits increasing returns to scale due to a positive externality effect of capital accumulation. It investigates whether positive per capita output growth is possible when the population growth rate is negative.[6)] He reveals that the long-run per capita output growth rate can be positive if the absolute value of the negative population growth rate is considerable.

Sasaki (2015) builds a small open economy growth model with negative population growth and investigates the relationship between trade patterns and economic development. Sasaki and Hoshida (2017) introduce negative population growth into Jones'

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3) Groth (2007) presents a growth model in which production exhibits increasing returns regarding capital stock, labor, and non-renewable resources because of a positive externality effect arising from capital accumulation. The results obtained are similar to those of Groth and Schou (2002).

4) Naso *et al.*. (2020) consider resource constraints and a decline in the population growth rate in a two-sector growth model. In the manufacturing sector, labor and capital are factors of production, while in the agricultural sector, labor, capital, and land are factors of production. Both the population growth rate and sectoral TFP growth rates were endogenized. The resource constraint in their study is the land, not exhaustible resources, and not a decline in population, but a decline in the growth rate of the population is considered.

5) Ritschl (1985) shows that if negative population growth is considered in Solow's (1956) model, a negative saving rate is necessary for the steady-state per capita capital stock to be favorable.

6) Sasaki (2019) presents a Solow growth model with the CES production function that considers the negative population growth rate. It further shows that the long-run growth rate of per capita output is equal to the exogenously given technological progress rate. This is if the elasticity of substitution is less than unity, which is reasonable in light of empirical studies. Christiaans (2017) built a two-sector growth model with negative population growth in which labor moves from a rural sector to an urban sector.

(1995) semi-endogenous growth model. These studies find that per capita output can grow sustainably even if population growth is negative.

Jones (2020) also investigated negative population growth in an endogenous growth model. He builds a model whose growth engine is knowledge production by R&D and examines the long-run consequences of population decline. When the fertility rate is negative, two steady states exist. One is a steady-state where the population, knowledge, and standard of living continue to increase exponentially. The other is a steady state in which the population continues to decrease, and knowledge production and living standards are stagnant. The crucial difference between his study and our study lies in the treatment of capital accumulation. He emphasizes technological progress driven by knowledge production and abstracts capital accumulation. In contrast, we emphasize capital accumulation and abstract technological progress. One of our purposes is to compare our results with the results of existing studies that assume a constant saving rate, and hence, we focus on capital accumulation.

However, the studies mentioned above of negative population growth do not consider exhaustible resources in production.

Considering these issues, Sasaki (2021) presents a growth model that considers exhaustible resources and a declining population and investigates whether the long-run growth rate of per capita output can be positive. This shows that even if the population growth rate is negative, the long-run growth rate of per capita output is positive as long as the absolute value of the population decline rate is relatively large. His model is based on the semi-endogenous growth model of Groth and Schou (2002), which considers exhaustible resources and positive population growth.

In Sasaki (2021), to make the input ratio of exhaustible resources a policy variable, the input ratio of exhaustible resources is given exogenously. However, theoretical models in this field usually use Hotelling's rule obtained from firms' dynamic profit maximization to obtain the input ratio of exhaustible resources (Stiglitz, 1974b).

For this reason, the present study extends Sasaki (2021) to endogenize the input ratio of exhaustible resources by deriving Hotelling's rule and compares the results obtained with those of Sasaki (2021). This study attempts to conduct a more detailed analysis to consider firms' dynamic optimization in a decentralized economy. Our study proves that the ratio of inputs of exhaustible resources to the stock of exhaustible resources converges to a constant value. We also know that the input ratio of exhaustible resources depends on the parameters of the model. In addition, we find that the importance of the optimal input ratio of exhaustible resources differs for a positive population growth case and a negative population growth case. Sasaki

(2021) uses a fixed input ratio of exhaustible resources and assumes that it can take the same value irrespective of the population growth rate. However, from our analysis, the optimal input ratio of exhaustible resources is uniquely determined by the model parameters.

Our study abstracts exogenous technological progress but instead considers increasing returns to scale due to the externality of capital accumulation. Stiglitz (1974b) considers positive population growth, but we consider negative population growth and positive population growth. In Stiglitz (1974b), the smaller the population growth rate, the more favorable is the per capita output growth. In our model, the smaller the population growth rate, the better the per capita output growth if the population growth rate is lower than a threshold value depending on the conditions.

The main results are as follows. When the population growth rate is more significant than a threshold value that takes a negative value, the steady-state value of the output–capital ratio is positive, and the long-run growth rate of per capita output is positive, depending on the conditions. In addition, this growth rate is increasing in the savings rate of households. In contrast, when the population growth rate is lower than the threshold value, the steady-state value of the output–capital ratio is zero. The long-run growth rate of per capita output is favorable depending on conditions, even if the population growth rate is negative.

The remainder of this paper is organized as follows. Section 2 presents our model and derives the optimal conditions to obtain the dynamical equations. Section 3 examines the model dynamics in detail, whereas Section 4 investigates the growth rate of per capita output. Finally, Section 5 concludes the paper.

2 Firms' profit maximization

Suppose an economy in which final goods are produced with capital, labor, and exhaustible resources. All the production factors were fully employed. There are three economic agents in this economy: final goods-producing firms, resource extracting firms, and households. Households own both absolute goods-producing firms and resource-extracting firms. All the goods markets and factor markets are competitive. Both final goods-producing firms and resource extracting firms maximize their profits with the price of final goods, resources, and factor prices given. In what follows, we specify the behavior of final goods-producing firms, resource extracting firms, and households in order.

2.1 Final goods-producing firms

The production function of a final representative goods-producing firm is given by

$$Y_t = A\bar{K}_t^{\gamma} K_t^{\alpha} L_t^{\beta} R_t^{1-\alpha-\beta}, \quad A > 0,\ 0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < \gamma < 1,\ \alpha + \beta < 1, \tag{1}$$

where Y_t denotes output, A is the level of total factor productivity, $\bar{K}_t$, aggregate capital stock, K_t, capital stock of the representative firm, L_t, inputs of labor, and R_t are inputs of exhaustible resources. The variable $\bar{K}_t$ captures the positive externality due to capital accumulation. The parametric restriction $0 < \gamma < 1$ shows that the externality of capital is not very large. This production function has constant returns to scale with respect to K_t, L_t, and R_t with $\bar{K}_t$ given while increasing returns to scale with respect to K_t, L_t, and R_t. At equilibrium, the relation $\bar{K}_t = K_t$ holds, and then, we obtain

$$Y_t = AK_t^{\alpha+\gamma} L_t^{\beta} R_t^{1-\alpha-\beta}. \tag{2}$$

We assume that α and γ satisfy the following restriction.

$$\alpha + \gamma < 1. \tag{3}$$

This implies that the production function decreases returns to scale concerning capital stock, including capital externality, which is necessary for obtaining semi-endogenous growth.

In addition, we assume that γ and β satisfy the following restriction:

$$\gamma < \beta. \tag{4}$$

This restriction states that the labor share of income β is larger than the capital externality γ, which is reasonable.[7] As shown below, this assumption ensures the existence of a steady state.

The representative final goods producing firm maximizes the total sum of the discounted present value of net cash flow, which is given by

$$\max \int_0^{\infty} \exp\left(-\int_0^t r_s\, ds\right) (AK_t^{\alpha}\bar{K}_t^{\gamma} L_t^{\beta} R_t^{1-\alpha-\beta} - w_t L_t - I_t)\, dt, \tag{5}$$

$$\text{s.t. } \dot{K}_t = I_t - \delta K_t, \quad K_0 = \text{given} > 0, \tag{6}$$

7) Graham and Temple (2006) use $\gamma = 0.3$ in their numerical simulations. The labor share of income is usually assumed to be $2/3$. Therefore, $\gamma < \beta$ is realistic.

where r_t denotes the real rental price of capital, w_t is the real wage rate, and p_t is the price of exhaustible resources in terms of final goods. Given that capital externality is not internalized (the Marshallian externality), the representative final goods producing firm maximizes the profit given by $\bar{K}_t$.

2.2 Resource extracting firms

Suppose that resource-extracting firms can extract exhaustible resources without costs. Then, the representative resource-extracting firm maximizes the total sum of the discounted present value of its revenue, which is given by:

$$\max_{R_t} \int_0^\infty D_t p_t R_t \, dt, \tag{7}$$

$$\text{s.t. } \dot{S}_t = -R_t, \quad S_0 = \text{given} > 0, \tag{8}$$

where p_t denotes the price of the extracted resources and S_t is the stock of exhaustible resources.

2.3 Households

Households own both final goods-producing firms and resource-extracting firms. Suppose that households lend and borrow funds to each other by trading bonds whose return is r_t. Suppose that the ownership of capital stock supports bonds. Let B_t be the stock of bonds, and C_t be the consumption of households. Then, the budget constraint of households is given by

$$\dot{B}_t = r_t B_t + w_t L_t + p_t R_t - C_t. \tag{9}$$

From the assumption of the representative household, the bond market clearing condition leads to

$$B_t = K_t. \tag{10}$$

Let $s \in (0, 1)$ denote the savings rate of the households. Then, we obtain household consumption.

$$C_t = (1 - s)(r_t B_t + w_t L_t + p_t R_t). \tag{11}$$

We assume that s is constant.

2.4 Optimization conditions

For final goods producing firms, we set the Hamiltonian function as follows:

$$\mathcal{H}_t^f = D_t(A\bar{K}_t^{\gamma} K_t^{\alpha} L_t^{\beta} R_t^{1-\alpha-\beta} - w_t L_t - I_t - p_t R_t) + q_t^f (I_t - \delta K_t), \tag{12}$$

where $D_t = \exp\left(-\int_0^t r_s \, ds\right)$, and q_t^f is the co-state variable.

The first-order conditions for optimization are as follows:

$$\frac{\partial \mathcal{H}_t^f}{\partial L_t} = 0 \Longrightarrow \beta \frac{Y_t}{L_t} = w_t, \tag{13}$$

$$\frac{\partial \mathcal{H}_t^f}{\partial R_t} = 0 \Longrightarrow (1-\alpha-\beta) \frac{Y_t}{R_t} = p_t, \tag{14}$$

$$\frac{\partial \mathcal{H}_t^f}{\partial I_t} = 0 \Longrightarrow D_t = q_t^f, \tag{15}$$

$$-\frac{\partial \mathcal{H}_t^f}{\partial K_t} = \dot{q}_t^f \Longrightarrow \dot{q}_t^f = -D_t \alpha \frac{Y_t}{K_t} + \delta q_t^f.. \tag{16}$$

From equation (15), we obtain $\dot{q}_t^f / q_t^f = \dot{D}_t / D_t = -r_t$, and from equations (15) and (16), we obtain

$$r_t = \alpha \frac{Y_t}{K_t} - \delta. \tag{17}$$

The real rental price of capital is equal to the marginal product of capital minus the capital depreciation rate.

For resource extracting firms, we set the Hamiltonian function as follows:

$$\mathcal{H}_t^e = D_t p_t R_t - q_t^e R_t, \tag{18}$$

where q_t^e denotes the co-state variable.

The first-order conditions for optimization are as follows:

$$\frac{\partial \mathcal{H}_t^e}{\partial R_t} = 0 \Longrightarrow D_t p_t = q_t^e, \tag{19}$$

$$-\frac{\partial \mathcal{H}_t^e}{\partial S_t} = \dot{q}_t^e \Longrightarrow \dot{q}_t^e = 0. \tag{20}$$

From equations (19) and (20), we obtain $\frac{\dot{D}_t}{D_t} + \frac{\dot{p}_t}{p_t} = 0$, from which $\frac{\dot{D}_t}{D_t} = -r_t$, we obtain

$$\frac{\dot{p}_t}{p_t} = r_t. \tag{21}$$

The rate of change in the price of extracted resources is equal to the real rental price of capital, known as Hotelling's rule (Hotelling, 1931).

From equation (14), we obtain $\frac{\dot{p}_t}{p_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{R}_t}{R_t}$, and from this equation and equation (21), we obtain the rate of change in exhaustible inputs as follows:

$$\frac{\dot{R}_t}{R_t} = -r_t + \frac{\dot{Y}_t}{Y_t}. \tag{22}$$

If we obtain $\frac{\dot{Y}_t}{Y_t}$ along the balanced growth path (BGP), we can find $\frac{\dot{R}_t}{R_t}$ along the BGP. Sasaki (2021) assumes that $R_t = s_R S_t$, where s_R denotes a constant input ratio of exhaustible resources. In this case, $\frac{\dot{R}_t}{R_t} = -s_R < 0$ holds.

Because we have $r_t = \alpha \frac{Y_t}{K_t} - \delta$, $w_t = \beta \frac{Y_t}{L_t}$, $p_t = (1 - \alpha - \beta) \frac{Y_t}{R_t}$, and $B_t = K_t$ from equations (17), (13), (14), and (10), respectively. By substituting these values into the budget constraint of households given by equation (9), we obtain

$$\dot{K}_t = sY_t - \delta K_t. \tag{23}$$

This is an equation of motion of capital stock.

3 Analysis of dynamics

For analysis of dynamics, we introduce the following new variables.

$$z_t = \frac{Y_t}{K_t}, \quad x_t = \frac{R_t}{S_t}, \tag{24}$$

where z_t denotes the output–capital ratio, and x_t is the ratio of exhaustible resource inputs to the stock of exhaustible resources. The time derivatives of z_t and x_t yield

$$\dot{z}_t = z_t \left(\frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \right), \tag{25}$$

$$\dot{x}_t = x_t \left(\frac{\dot{R}_t}{R_t} - \frac{\dot{S}_t}{S_t} \right). \tag{26}$$

We obtain $\frac{\dot{Y}_t}{Y_t}$, $\frac{\dot{K}_t}{K_t}$, $\frac{\dot{R}_t}{R_t}$, $\frac{\dot{S}_t}{S_t}$ from equations (2), (23), (22), and (8), respectively. Then, substituting these values into equations (25) and (26), we obtain the differential equations of z_t and x_t as follows:

$$\dot{z}_t = -az_t^2 + bz_t, \quad a \equiv \frac{s(\beta-\gamma)+\alpha(1-\alpha-\beta)}{\alpha+\beta} > 0,\ b \equiv \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta} \gtreqless 0, \tag{27}$$

$$\dot{x}_t = (cz_t + b)x_t + x_t^2, \quad c \equiv \frac{(\alpha+\gamma)s-\alpha}{\alpha+\beta} \gtreqless 0. \tag{28}$$

If the population growth rate is positive, we obtain $b > 0$. In contrast, if the population growth rate is negative and its absolute value is large, we obtain $b < 0$. In the following analysis, we classify the cases according to the sign of b:

3.1 Case of $b > 0$

We have $b > 0$ if the following condition holds.

$$\beta n + (1-\alpha-\gamma)\delta > 0. \tag{29}$$

Note that this condition allows $n < 0$, even if $n < 0$, we obtain $b > 0$ as long as its absolute value is small.

From $\dot{z}_t = 0$, we obtain

$$z_t = \frac{b}{a} > 0. \tag{30}$$

From $\dot{x}_t = 0$, we obtain

$$x_t = -cz_t - b. \tag{31}$$

The intersection of $\dot{z}_t = 0$ and $\dot{x}_t = 0$ exists on the (z_t, x_t)-plane, provided that the following two conditions are simultaneously satisfied:

$$c < 0 \Longrightarrow (\alpha+\gamma)s < \alpha, \tag{32}$$

$$s < \alpha. \tag{33}$$

If $s < \alpha$ holds, then $(\alpha+\gamma)s < \alpha$ also holds. Therefore, we obtain

$$s < \alpha. \tag{34}$$

This restriction was also considered by Stiglitz (1974b) and Cigno (1981). If α corresponds to the capital share of income, the value $\alpha = 0.3$ is reasonable. Since the saving rate of households in developed countries is less than 0.3, this assumption is realistic.

Under the above assumption, the steady state values are as follows:

$$z^* = \frac{\beta n + (1-\alpha-\gamma)\delta}{s(\beta-\gamma) + \alpha(1-\alpha-\beta)} > 0, \tag{35}$$

$$x^* = \frac{(\alpha - s)[\beta n + (1-\alpha-\gamma)\delta]}{s(\beta-\gamma) + \alpha(1-\alpha-\beta)} > 0. \tag{36}$$

The steady-state input ratio of exhaustible resources x^* is endogenously determined by the savings rate, capital depreciation rate, population growth rate, and production function parameters.

Proposition 1. *Suppose that the parameters satisfy $\beta n + (1-\alpha-\gamma)\delta > 0$. Then, there exists a steady state $z^* > 0$ and $x^* > 0$ if $s < \alpha$.*

The initial values were considered. From $z_t = \frac{AK_t^{\alpha+\gamma}L_t^{\beta}R_t^{1-\alpha-\beta}}{K_t}$ and $x_t = \frac{R_t}{S_t}$, we obtain

$$x_t = \phi z_t^{\frac{1}{1-\alpha-\beta}}, \quad \phi \equiv S_t^{-1}(A^{-1}K_t^{1-\alpha-\gamma}L_t^{-\beta})^{\frac{1}{1-\alpha-\beta}}. \tag{37}$$

We assume perfect foresight. Because only R_t is a jump variable, the initial values of z_t and x_t must satisfy the following relation:

$$x_0 = \phi z_0^{\frac{1}{1-\alpha-\beta}}, \quad \phi \equiv S_0^{-1}(A^{-1}K_0^{1-\alpha-\gamma}L_0^{-\beta})^{\frac{1}{1-\alpha-\beta}}. \tag{38}$$

Figure 1 shows the phase diagram, from which we find that there exists the unique saddle path along which an economy converges to the steady-state: the economy starts at the intersection of equation (38) and the saddle path, and then, converges to the steady-state.

3.2 Case of $b < 0$

We obtain $b < 0$ if the population growth rate n is negative and its absolute value is large; that is,

$$\beta n + (1-\alpha-\gamma)\delta < 0. \tag{39}$$

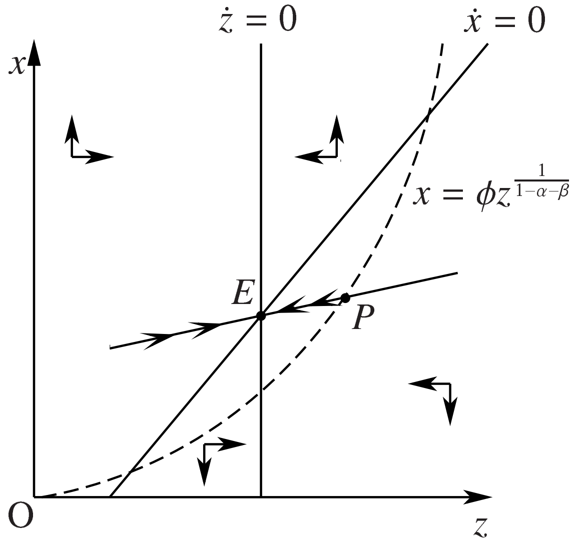

Figure 1: Phase diagram when $b > 0$ and $s < \alpha$

The locus $\dot{z}_t = 0$ corresponds to the vertical axis. The intercept of locus $\dot{x}_t = 0$ is positive. The slope of the locus $\dot{x}_t = 0$ is positive when $c < 0$ and negative when $c > 0$. Figure 2 shows the phase diagram when $c < 0$, while Figure 3 shows the phase diagram when $c > 0$. In either case, there exists a saddle path along which an economy converges to a steady state. The economy starts at the intersection of equation (38) and the saddle path, and then converges to the steady state.

$$z^{**} = 0, \tag{40}$$

$$x^{**} = -\frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta} > 0. \tag{41}$$

Proposition 2. *Suppose that the parameters satisfy $\beta n + (1-\alpha-\gamma)\delta < 0$. Then, there exists a steady state $z^{**} = 0$ and $x^{**} > 0$.*

4 Growth rate of per capita output

We investigate whether the growth rate of per capita output is positive. Let $y_t = Y_t/L_t$ denote the per capita output. Then, the growth rate of y_t is as follows:

$$g_{y,t} = \frac{s(\alpha+\gamma) - \alpha(1-\alpha-\beta)}{\alpha+\beta} z_t - \frac{\alpha}{\alpha+\beta} n + \frac{(1-\alpha-\beta)-(\alpha+\gamma)}{\alpha+\beta}\delta. \tag{42}$$

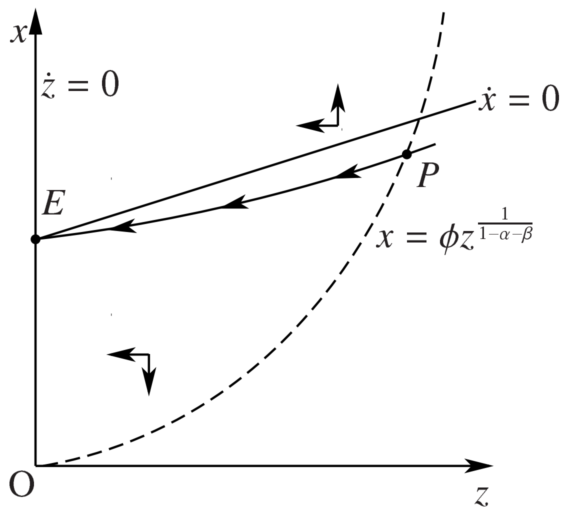

Figure 2: Phase diagram when $b < 0$ and $c < 0$

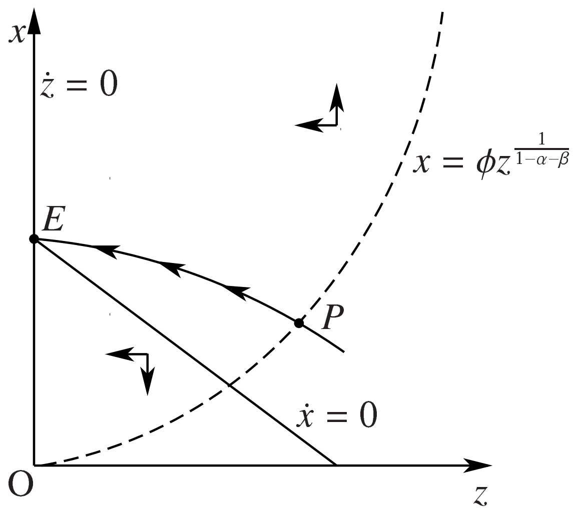

Figure 3: Phase diagram when $b < 0$ and $c > 0$

The growth rate of per capita output along the BGP is obtained by substituting the steady-state value of z, that is, equation (35) or equation (40) into equation (42).

Along with the BGP, $z = \frac{Y}{K}$ is constant, and hence, the growth rate of Y_t is equal to that of K_t, which is given by the growth rate of y_t plus n. When the population growth rate is negative, $g_Y = g_y + n$ can be damaging, even if the growth rate of y_t is positive.

4.1 Case of $b > 0$ and $s < \alpha$

When $z = z^* > 0$, by substituting equation (35) into equation (42), we obtain the BGP growth rate of per capita output:

$$g_y^* = \frac{s\gamma - \alpha(1-\alpha-\beta)}{s(\beta-\gamma)+\alpha(1-\alpha-\beta)}\,n + \frac{(s-\alpha)(1-\alpha-\beta)}{s(\beta-\gamma)+\alpha(1-\alpha-\beta)}\,\delta. \tag{43}$$

Taking the partial derivative of g_y^* with respect to the saving rate s, we obtain

$$\operatorname{sgn}\frac{\partial g_y^*}{\partial s} = \operatorname{sgn}\,\alpha(1-\alpha-\beta)[\beta n + (1-\alpha-\gamma)\delta] > 0. \tag{44}$$

Proposition 3. *Suppose that the parameters satisfy $\beta n + (1-\alpha-\gamma)\delta > 0$. Then, an increase in households' savings rate increases the BGP growth rate of per capita output.*

The BGP input ratio of the exhaustible resources is given by

$$\begin{aligned}\frac{\dot{R}}{R} &= \frac{\dot{S}}{S} = -\frac{R}{S} = -x^* \\ &= -\frac{(\alpha-s)[\beta n + (1-\alpha-\gamma)\delta]}{s(\beta-\gamma)+\alpha(1-\alpha-\beta)} < 0.\end{aligned} \tag{45}$$

The condition under which $g_y^* > 0$ holds is as follows:

$$[s\gamma - \alpha(1-\alpha-\beta)]n > -(s-\alpha)(1-\alpha-\beta)\delta, \tag{46}$$

Here, we focus on the size of the population growth rate to investigate whether $g_y^* > 0$ is obtained.

First, when $s\gamma > \alpha(1-\alpha-\beta)$ holds, the condition under which $g_y^* > 0$ holds is given by

$$n > -\frac{(s-\alpha)(1-\alpha-\beta)\delta}{s\gamma-\alpha(1-\alpha-\beta)} \equiv \Lambda. \tag{47}$$

Because $s < \alpha$, we have $\Lambda > 0$. Accordingly, we check whether $s\gamma > \alpha(1-\alpha-\beta)$ is compatible with $s < \alpha$.

When $s < \alpha$, we have $\Lambda > 0$. Then, for $s\gamma > \alpha(1-\alpha-\beta)$ to be compatible with $s < \alpha$, it is necessary that $\alpha+\beta+\gamma > 1$ holds, which states that the production function exhibits increasing returns to scale with respect to labor and capital, including capital externality. In this case, if $n > \Lambda > 0$, we obtain $g_y^* > 0$.

Proposition 4. *Suppose that the parameters satisfy $\beta n + (1-\alpha-\gamma)\delta > 0$. Suppose also that $s\gamma > \alpha(1-\alpha-\beta)$ holds. If the production function exhibits increasing returns to scale with respect to labor and capital, including capital externality, and if the population growth rate satisfies $n > \Lambda\,(> 0)$, then the BGP growth rate of per capita output is positive.*

Second, when $s\gamma < \alpha(1-\alpha-\beta)$ holds. the condition under which $g_y^* > 0$ is given by

$$n < -\frac{(s-\alpha)(1-\alpha-\beta)\delta}{s\gamma-\alpha(1-\alpha-\beta)} \equiv \Lambda. \tag{48}$$

When $s < \alpha$, we have that $\Lambda < 0$. Then, if $\alpha+\beta+\gamma > 1$ holds, the restriction $s\gamma < \alpha(1-\alpha-\beta)$ is valid, whereas if $\alpha+\beta+\gamma < 1$ holds, the restriction $s < \alpha$ is valid. In this case, if the population growth rate lies within the range $-\frac{(1-\alpha-\gamma)\delta}{\beta} < n < \Lambda < 0$, we obtain $g_y^* > 0$.

Proposition 5. *Suppose that the parameters satisfy $\beta n+(1-\alpha-\gamma)\delta > 0$. Suppose also that $s\gamma < \alpha(1-\alpha-\beta)$ holds. Then, if (i), the production function exhibits increasing returns to scale with respect to labor and capital, including capital externality. If $s\gamma < \alpha(1-\alpha-\beta)$ holds, or if (ii), the production function exhibits decreasing returns to scale with respect to labor and capital, including capital externality, and if $s < \alpha$ holds, the BGP growth rate of per capita output is positive when the population growth rate lies within the range $-\frac{(1-\alpha-\gamma)\delta}{\beta} < n < \Lambda < 0$.*

4.2 Case of $b < 0$

When $z^{**} = 0$, by substituting equation (40) into equation (42), we obtain the BGP growth rate of per capita output.

$$g_y^{**} = -\frac{\alpha}{\alpha+\beta}\,n + \frac{(1-\alpha-\beta)-(\alpha+\gamma)}{\alpha+\beta}\,\delta. \tag{49}$$

We see that g_y^{**} is independent of s, which contrasts with the result for g_y^{*}.

The input ratio of exhaustible resources is given by

$$\begin{aligned}\frac{\dot{R}}{R} &= -x^{**} \\ &= \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta} < 0.\end{aligned} \tag{50}$$

We investigate the condition under which $g_y^{**} > 0$ holds.

First, when $\alpha+\beta+\gamma > 1$, the growth rate of per capita output is positive if the population growth rate satisfies the following condition:

$$n < \frac{(1-\alpha-\gamma)-(\alpha+\beta)}{\alpha}\,\delta. \tag{51}$$

Second, when $\alpha+\beta+\gamma < 1$, the growth rate of per capita output is positive if the population growth rate satisfies the following condition:

$$n < -\frac{1-\alpha-\gamma}{\beta}\,\delta. \tag{52}$$

Proposition 6. *Suppose that the parameters satisfy $\beta n + (1-\alpha-\gamma)\delta < 0$. Then, ($i$) if the production function exhibits increasing returns to scale with respect to labor and capital, including capital externality, and if the population growth rate satisfies $n < \frac{(1-\alpha-\gamma)-(\alpha+\beta)}{\alpha}\,\delta$, the BGP growth rate of per capita output is positive. In addition, (ii) if the production function exhibits decreasing returns to scale with respect to labor and capital, including capital externality, and if the population growth rate satisfies $n < -\frac{1-\alpha-\gamma}{\beta}\,\delta$, the BGP growth rate of per capita output is positive.*

5 Conclusions

This study has built a semi-endogenous growth model that considers exhaustible resources and a declining population to investigate whether the long-run growth rate of per capita output can be positive. We have explicitly considered the dynamic optimization of firms and endogenized the input ratio of exhaustible resources to obtain Hotelling's rule.

Our results show that an economy converges to a steady-state irrespective of whether the population growth rate is positive or negative. The output–capital ratio and the input ratio of exhaustible resources remain constant. Even if the population growth rate is negative, the long-run growth rate of per capita output is favorable depending on the conditions. In addition, if the population growth rate is positive, the long-run growth rate of per capita output depends positively on the saving rate of households. However, our model belongs to the class of semi-endogenous growth models.

For ease of analysis, we assume that the savings rate of households is fixed. However, it is more reasonable to think that the saving rate of households depends on other economic factors, and hence, it should be endogenized. Building a growth model with households' dynamic optimization that considers exhaustible resources and a declining population will be left for future research.

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Table 1: Forecasts of population growth

Medium variant

	World Bank income groups			UN development groups	
	High-income countries	Middle-income countries	Low-income countries	More developed regions	Less developed regions
2020-2025	0.31	0.89	2.58	0.13	1.14
2025-2030	0.25	0.77	2.38	0.07	1.02
2030-2035	0.19	0.66	2.19	0.03	0.91
2035-2040	0.12	0.56	2.04	-0.01	0.81
2040-2045	0.06	0.47	1.90	-0.04	0.71
2045-2050	0.01	0.38	1.75	-0.07	0.62
2050-2055	-0.02	0.29	1.61	-0.09	0.53
2055-2060	-0.05	0.21	1.47	-0.11	0.46
2060-2065	-0.06	0.14	1.33	-0.11	0.39
2065-2070	-0.06	0.09	1.21	-0.10	0.32
2070-2075	-0.05	0.04	1.08	-0.08	0.27
2075-2080	-0.04	-0.01	0.96	-0.05	0.21
2080-2085	-0.03	-0.04	0.85	-0.03	0.17
2085-2090	-0.02	-0.07	0.74	-0.01	0.13
2090-2095	-0.00	-0.10	0.64	0.00	0.09
2095-2100	0.01	-0.13	0.55	0.01	0.05

Soruce: United Nations, World Population Prospects 2019