Cournot-Bertrand comparison under RD competition: Output versus RD subsidies

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This study compares Cournot and Bertrand firms with research and development (R&D) competition under government policies between output and R&D subsidies. We demonstrate that firms invest more (less) in R&D and the government grants more (less) subsidies under Cournot than Bertrand competition with output (R&D) subsidy policies. We also reveal that both competition modes yield the same welfare with output subsidy while Bertrand yields higher welfare than Cournot with R&D subsidy irrespective of product substitutability. Finally, we show that firms' profits and social welfare are always higher under output subsidies in Cournot competition, while they can be higher under R&D subsidies in Bertrand competition if the product substitutability is high and the firm's R&D investment is efficient.

JEL Classifications: L13, H20

Keywords: Cournot; Bertrand; R&D investment; Output Subsidy; R&D subsidy

1. Introduction

Comparisons between Cournot and Bertrand competition in a differentiated product duopoly market have been popular in the literature on oligopoly theory since Singh and Vives (1984) and it is well-known that a one-tier market is more competitive and efficient when it is characterized by Bertrand competition wherein firms set lower prices and produce higher outputs rather than those under Cournot competition.[1]

Recent studies have also examined the relationship between different market structures and research and development (R&D) activities. Qiu (1997) considered a process R&D with cost-reducing activities and demonstrated that Cournot induces more R&D effort than Bertrand, but the price is lower and output is larger in Bertrand. Kabiraj and Roy (2002) considered different marginal costs and found that Cournot firms invest a larger amount in R&D than Bertrand firms, but Cournot price can be less than Bertrand price when the R&D technology is relatively inefficient. Hinloopen and Vandekerckhove

[1] A substantial research in the literature has been developed thereafter extending the Sigh and Vives results. For recent works, see Heckner (2000), Symeonidis (2003), Arya et al. (2008), Gosh and Mitra (2010), Matsumura and Ogawa (2012), Mukherjee et al. (2012), Chirco and Scrimitore (2013), Alipranti et al. (2014), Haraguchi and Matsumura (2016), and Basak (2017) among others.

(2009) considered the efficiency of R&D that generates input spillovers and showed that Cournot firms invest more in R&D than Bertrand but can yield lower prices than Bertrand when the R&D process is efficient, spillovers are substantial, and products are less differentiated. Basak and Wang (2019) also examined R&D competition in a mixed duopoly where the public firm competes with a private firm. They indicated that the public firm invests more R&D than the private firm and Bertrand is the equilibrium of endogenous choice between Bertrand and Cournot.

However, all these previous works did not consider the effect of R&D policy. Due to the world-wide trends of globalization and innovation, oligopolistic firms have intensified market competition and policy makers have thus enacted various policies to encourage R&D activities.[2] Recently, a number of studies have also assessed the welfare consequences of R&D activities in light of governmental intervention.[3] The extensive studies on R&D incentives and policy implications for innovation under imperfect competition are contemporary and practical.

This study examines and compares output and R&D subsidy policies between Cournot and Bertrand competitions with R&D activities. We show that (i) firms invest more (less) in R&D and the government grants more (less) subsidies under Cournot than Bertrand competition with output (R&D) subsidy policies. (ii) Cournot and Bertrand competitions yield the same social welfare with output subsidy policies while Bertrand competition yields higher social welfare than Cournot competition with R&D subsidy policies. (iii) Firms' profits and social welfare are always higher under output subsidies in Cournot competition, while they can be higher under R&D subsidies in Bertrand competition if the product substitutability is high and the R&D investment of firms is efficient.

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[2] For example, EU institutions have reaffirmed their commitment to R&D policies and consequently, the budgets of the research Framework Programs (FPs) have grown exponentially from EUR 3.3 billion in the first FP, launched in 1984, to EUR 80 billion of Horizon 2020. Further, the Research, Innovation, and Science Policy Experts (RISE) high-level group, created in 2014, has proposed doubling this budget or, at least, the maintenance of this growth rate, which would lead to a 7-year budget of more than EUR 120 billion in current prices for the next period. See Miyagiwa and Ohno (2002), Marinucci (2012), and Chen et al. (2021) for more details.

[3] For early discussions, see d'Aspremont and Jacquemin (1988), Kamien et al. (1992), Poyago-Theotoky (1995, 1998, 1999), Lee (1998), and Beath et al. (1998), among others. Recent works extend the analysis into different directions. For example, in Cournot competition, Yang and Nie (2015) and Lee and Muminov (2021) investigated R&D subsidies with asymmetric information. Kesavayuth and Zikos (2013) and Lee et al. (2017) compared output and R&D subsidy policies in a mixed market.

The remainder of this paper is organized as follows. In section 2, we formulate a differentiated duopoly model with R&D where the government can grant an output or an R&D subsidy policy. We compare the equilibrium results with output or R&D subsidy under Cournot and Bertrand competitions in sections 3 and 4, respectively. In section 5, we compare the level of social welfare under the two subsidy policies and discuss policy implications. Finally, concluding remarks are provided in section 6.

2. Basic model

Consider a duopoly market where two firms produce differentiated commodities where a quasi-linear utility function of the representative consumer is denoted by $U = a(q_1 + q_2) - \frac{(q_1{}^2 + 2bq_1q_2 + q_2{}^2)}{2} + m$, where a is a positive constant, m is the consumption of the outside goods, q_i denotes the quantity of the good i, which is produced by the firms, and $b \in (0,1)$ represents the degree of product substitutability. The inverse demand function is $p_i = a - q_i - bq_j, (i, j = 1,2; i \neq j)$, where p_i is the price of good i. Then, consumer surplus is $CS = U - p_1q_1 - p_2q_2$.

We assume that firms invest in R&D to reduce the cost of production. Specifically, the cost function for firm i is denoted as $C_i = (c - x_i)q_i$, where x_i is the R&D investment of firm i and $a > c > x_i > 0$. Each firm has to spend $\Gamma(x_i) = \frac{r}{2}x_i^2$ to implement cost-reducing R&D where the R&D investment causes decreasing returns to scale, and r represents the efficiency of R&D investment.

We assume that government grants output or R&D subsidies, s^P or s^R, respectively, where the superscripts "*P*" and "*R*" denote production output subsidy and R&D subsidy, respectively. The profit function for firm i is denoted by $\pi_i = p_iq_i - (c - x_i)q_i - \frac{r}{2}x_i^2 + s^Pq_i + s^Rx_i$. The social welfare is defined as the sum of consumer surplus and firms' profits minus total output subsidy, which is given as: $W = CS + \sum_{i=1}^{2}\pi_i - \sum_{i=1}^{2}s^Pq_i - \sum_{i=1}^{2}s^Rx_i$. Both firms are assumed to maximize their own profits while the government maximizes welfare.[4]

As a benchmark, we can obtain the first-best outcome, which yields the highest welfare from the

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[4] We focus on the comparison between an output subsidy of $\{s^P > 0$ and $s^R = 0\}$ and an R&D subsidy of $\{s^P = 0$ and or $s^R > 0\}$ under Cournot or Bertrand competition, respectively. Note that the first-best outcome can be obtained by policy mix of $\{s^P \neq 0$ and $s^R \neq 0\}$. Regularity conditions and detailed analysis for comparisons are provided in Appendix B.

direct allocation of the output productions and R&D investments.

$$x_i^F = \frac{a-c}{r+br-1},\ q_i^F = \frac{(a-c)r}{r+br-1},\ p_i^F = \frac{-a+(1+b)cr}{r+br-1},\ W^F = \frac{(a-c)^2 r}{r+br-1} \quad (1)$$

Since there is no strategic interaction, that is, $\frac{\partial q_j}{\partial x_i} = 0\ (i,j = 1,2; i \neq j)$, the first-best R&D allocation is determined at the marginal where $\frac{\partial W}{\partial x_i} = q_i - rx_i = 0$. Notice that $x_i q_i - \frac{r}{2}x_i^2 = \frac{r}{2}x_i^2$, in which net benefit of R&D on the left-hand side (total R&D outputs for reducing unit cost minus total R&D expenditures) is positive and equals the total R&D expenditures on the right-hand side.

We compare the equilibrium outcomes between Cournot or Bertrand competition under output and R&D subsidies, respectively, when both firms invest in R&D. The game structure runs as follows. In the first stage, the government grants output or R&D subsidies to maximize the social welfare. In the second stage, both firms decide R&D investment independently and simultaneously to maximize their own profits. In the third stage, both firms compete in Cournot or Bertrand competitions. We solve the subgame perfect Nash equilibrium by backward induction.

3. Analysis with output subsidy policies

We first analyze and compare the equilibrium outcomes under output subsidies between Cournot or Bertrand competition when both firms invest in R&D.

3.1 Cournot competition

In the third stage, both firms choose quantities to maximize their profits. The first order conditions provide the equilibrium quantities are as follows:

$$q_i = \frac{2(a-c+s^P+x_i) - b(a-c+s^P+x_j)}{4-b^2} \quad (2)$$

In the second stage, both firms choose R&D investments. The equilibrium results are as follows:[5]

$$x_i = \frac{4(a-c+s^P)}{(2-b)(2+b)^2 r - 4} \quad (3)$$

In the first stage, the maximization of social welfare with respect to s^P yields the following

[5] From the reaction function of each firm under Cournot competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Cournot firms.

optimal output subsidy:

$$s^{CP} = \frac{(a-c)(16r + b^4 r - 4b^2(1+2r))}{E} \quad (4)$$

where $E = 8b^2(1-r) - 16(1-r) + 16br - 8b^3 r + b^4 r + b^5 r > 0$ and the superscript "*CP*" denotes the equilibrium outcomes under production output subsidy policies in Cournot competition.

Under the regularity conditions, we have the following equilibrium outcomes:[6]

$$x_i^{CP} = \frac{4(4-b^2)(a-c)}{E} \quad (5)$$

$$q_i^{CP} = \frac{(4-b^2)^2(a-c)r}{E} \quad (6)$$

$$p_i^{CP} = \frac{(1+b)(4-b^2)^2 cr - 8a(2-b^2)}{E} \quad (7)$$

$$\pi_i^{CP} = \frac{(4-b^2)^2(a-c)^2 r((4-b^2)^2 r - 8)}{E^2} \quad (8)$$

$$W^{CP} = \frac{(4-b^2)^2(a-c)^2 r}{E} \quad (9)$$

We compare the results with no subsidy where the superscript "*C*" denotes the equilibrium outcomes under no subsidy in Cournot competition, and the first-best outcome, where the superscript "*F*" denotes the optimal levels that the government directly determines to maximize welfare.[7] The graphical relations between equilibria under Cournot competition are illustrated in Figure 1.[8]

Lemma 1. $x_i^{CP} > x_i^F > x_i^C$, $q_i^F > q_i^{CP} > q_i^C$, and $W^F > W^{CP} > W^C$ for $b \in (0,1)$.

Lemma 1 states that output subsidies increase both R&D investments and outputs of the firms, which increase social welfare, compared to no subsidy. However, this induces under-production and over-investment to Cournot firms, compared to the first-best, which results in welfare loss. This is because the strategic effect of R&D in relation to output is positive under Cournot competition, that is, $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} >$

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6 Following Hinloopen and Vandekerckhove (2009), we provide regularity conditions under Cournot or Bertrand competition where the second-order conditions, positive post-innovation costs, and the stable equilibrium are examined in Appendix A.

7 Some necessary proofs of propositions and lemmas are provided in Appendix C while others are straight-forward and thus omitted.

8 We set $r = 1$ and $a - c = 1$ in all the figures.

0 $(i,j = 1,2; i \neq j)$, while the outputs are strategic substitutes.[9]

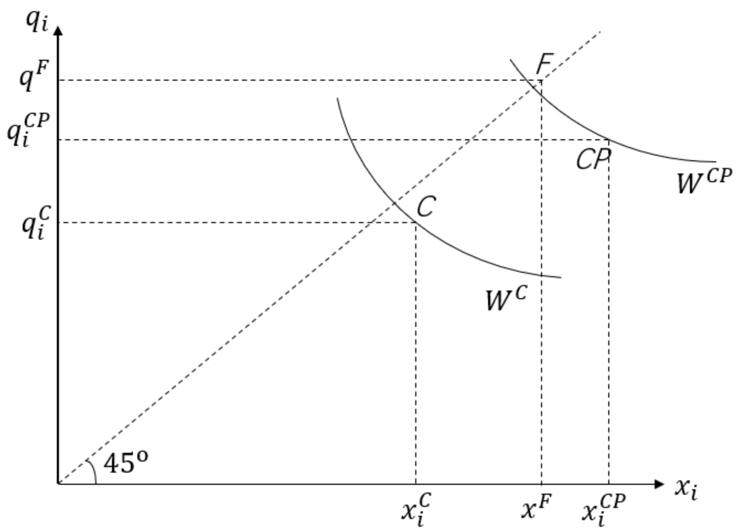

Figure 1: output subsidy vs. no subsidy under Cournot competition

3.2 Bertrand competition

In the third stage, both firms choose price to maximize their profits, where the demand function for firm i is given as $q_i = \frac{a-ab-p_i+bp_j}{1-b^2}$. The resulting equilibrium price is as follows:

$$p_i = \frac{a(2-b-b^2)+(2+b)c-2s^P-bs^P-2x_i-bx_j}{4-b^2} \quad (10)$$

In the second stage, both firms choose R&D investments. The first order conditions provide the equilibrium R&D investment as follows:[10]

$$x_i = \frac{2(2-b^2)(a-c+s^P)}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \quad (11)$$

In the first stage, the maximization of social welfare with respect to s^P yields the following optimal output subsidy:

$$s^{BP} = \frac{(a-c)(16r-b^4(2-9r)-b^6r+4b^2(1-6r))}{E} \quad (12)$$

where the superscript "*BP*" denotes the equilibrium outcomes under output subsidy policies in Bertrand

[9] See Qiu (1997) and Hinloopen and Vandekerckhove (2009) for discussions on the strategic effects of R&D between Cournot and Bertrand competitions. The positive strategic effect leads to over-investment in the absence of spillovers, see Brander and Spencer (1983).

[10] From the reaction function of each firm under Bertrand competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Bertrand firms.

competition.

Under the regularity conditions, we have the following equilibrium outcomes:

$$x_i^{BP} = \frac{2(4-b^2)(2-b^2)(a-c)}{E} \tag{13}$$

$$q_i^{BP} = \frac{(4-b^2)^2(a-c)r}{E} \tag{14}$$

$$p_i^{BP} = \frac{(1+b)(4-b^2)^2cr - 8a(2-b^2)}{E} \tag{15}$$

$$\pi_i^{BP} = \frac{(4-b^2)^2(a-c)^2r(b^4(2-9r) - 8 + 16r - b^6r + 8b^2(1-3r))}{E^2} \tag{16}$$

$$W^{BP} = \frac{(4-b^2)^2(a-c)^2r}{E} \tag{17}$$

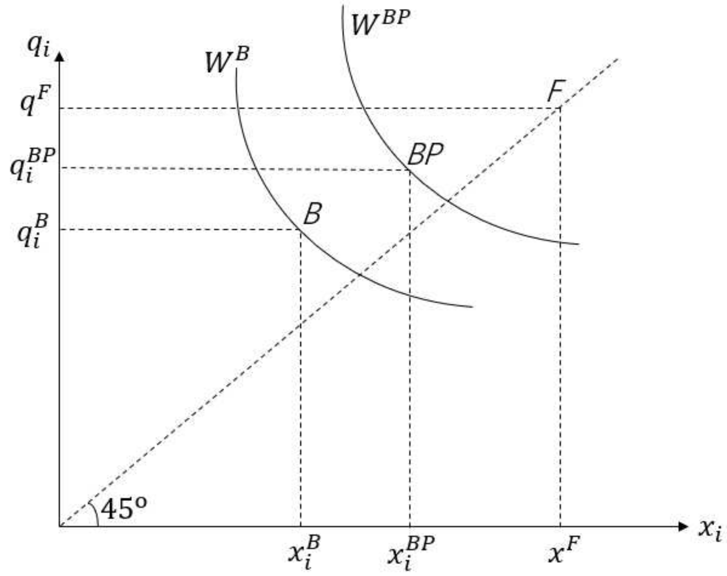

Figure 2: output subsidy vs. no subsidy under Bertrand competition

We compare the results with no subsidy where the superscript "*B*" denotes the equilibrium outcomes under no subsidy in Cournot competition, and the first-best outcome. The graphical relations between equilibria under Bertrand competition are presented in Figure 2.

Lemma 2. $x_i^F > x_i^{BP} > x_i^B$, $q_i^F > q_i^{BP} > q_i^B$, and $W^F > W^{BP} > W^B$ for $b \in (0,1)$.

Lemma 2 states that, compared to no subsidy, output subsidies increase both R&D investments and outputs of the firms, which increase social welfare. However, this induces both under-production and under-investment to Bertrand firms, compared to the first-best, which results in welfare loss. This is because the strategic effect of R&D on the output is negative under Bertrand competition, that is,

$\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$ $(i, j = 1,2; i \neq j)$, while the prices are strategic complements.

3.3 Comparisons

We compare the Cournot and Bertrand competitions under output subsidies. Figure 3 combines Figures 1 and 2, and illustrates the graphical relations between Cournot and Bertrand competitions. It indicates that, compared to no subsidies, output subsidies increase R&D investments and outputs in both Cournot and Bertrand firms, but the R&D investments exceeds the first-best in Cournot but is less than that in Bertrand competition.

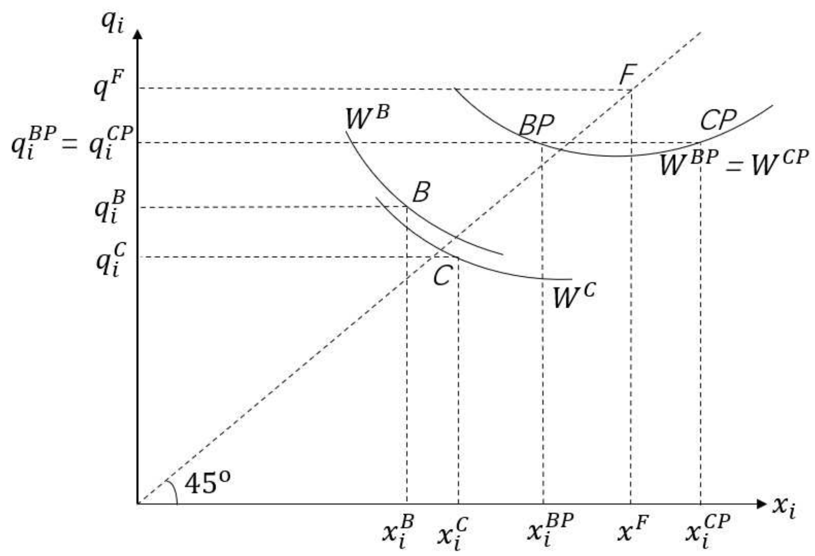

Figure 3: Welfare comparisons under output subsidies

Proposition 1. $s^{CP} > s^{BP} > 0$ for $b \in (0,1)$.

Proposition 1 states that the government grants more output subsidies to Cournot firms than Bertrand firms. In the absence of output subsidies, it is well-known that Cournot firms produce less outputs than Bertrand firms due to the strategic effects between quantities and prices, but undertake more R&D.[11] Under quantity competition with strategic substitutes, output subsidies can induce Cournot firms to undertake R&D more aggressively, which reduces its marginal cost and thus, a firm can increase output

[11] For more explanations on the strategic effects, see Singh and Vives (1984) and Qiu (1997) for the case without and with R&D activities, respectively.

which in turn decreases the output of rival firm and increase its own profit. Under price competition with strategic complements, if a Bertrand firm with output subsidies aggressively undertakes R&D, which reduces its marginal cost, then the firm can increase its outputs and reduces its price, which in turn reduces the price of rival firm. More R&D can reduce the firm's profit by undercutting the prices. The government, therefore, has an incentive to provide more output subsidy to Cournot firms, which can affect allocation efficiency by encouraging output production decisions.

Proposition 2. $x_i^{CP} > x_i^{BP}$ for $b \in (0,1)$.

Proposition 2 states that Cournot firms undertake more R&D and thus lower production cost under output subsidies. This finding confirms the previous result in the absence of output subsidies (Qiu, 1997; Hinloopen and Vandekerckhove, 2009). In the presence of output subsidy, it is interesting to observe that, to reduce costs and increase outputs to earn more profits, both Cournot and Bertrand firms' have incentives to undertake more R&D investments, which is higher for Cournot firms and can induce over-investment, compared to the first-best.

Proposition 3. $q_i^{CP} = q_i^{BP}$ and $p_i^{CP} = p_i^{BP}$ for $b \in (0,1)$.

It is interesting that output subsidies make outputs and prices be the same under Cournot and Bertrand firms even though the government induces Cournot firms to undertake more R&D. We can explain these results as follows. First, it is noteworthy that when firms do not invest in R&D, the quantities and prices of firms are the same under both Cournot and Bertrand competitions if there are optimal output subsidy policies.[12] Second, we can show that the net benefits from R&D between both types of firms are the same and positive under the output subsidies, that is, $x_i^{CP} q_i^{CP} - \frac{r}{2}\left(x_i^{CP}\right)^2 = x_i^{BP} q_i^{BP} - \frac{r}{2}\left(x_i^{BP}\right)^2$. This is because output subsidies have opposite effects on the R&D decisions of the firms. In other words, the strategic effect of R&D on Cournot firms is positive, that is, $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} > 0$, while the effect of R&D on Bertrand firms is negative, that is, $\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$. Both effects are off-set so that the net benefits from

[12] Regarding the efficiency properties of output subsidies, Kim and Lee (1995) and Lee (1997) analyzed the different oligopolistic incentives under asymmetric information and demonstrated that output subsidies can still obtain the first-best allocation.

R&D are the same under the output subsidies. As a result, the quantities and prices can also be the same in both types of competition.[13] Therefore, output subsidies can rearrange firms' production efficiency to redistribute the allocations of firms' outputs, given the different levels of R&D investments.[14] This result also implies that consumer surplus under Cournot firms is same as that under Bertrand firms.

Proposition 4. $\pi_i^{CP} > \pi_i^{BP}$ for $b \in (0,1)$.

Proposition 4 states that Cournot firms gain more profit since they undertake more R&D (to reduce the unit cost) and earn more output subsidy, which induces the lower unit cost of output production under the same prices and quantities as Bertrand firms. This finding implies that irrespective of product substitutability, Cournot competition can be an equilibrium if firms can choose the market mode between quantity and price competitions.[15]

Proposition 5. $W^{CP} = W^{BP}$ for $b \in (0,1)$.

It is also interesting to find that both Cournot and Bertrand firms yield the same welfare under output subsidies. From Proposition 3, the same consumer surplus between both Cournot and Bertrand competitions and the effect of output subsidies on the profits of both firms is exactly off-set in the social welfare even though Cournot firm gains more profit.

4. Analysis with R&D subsidy policies

We also analyze and compare the equilibrium outcomes under R&D subsidies between Cournot and Bertrand competition when both firms invest in R&D.

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[13] Note that this result holds under the linear marginal cost between the firms. However, if we consider a quadratic cost between the firms, it does not hold even under output subsidies.

[14] Note that under the same R&D activities, for example, if $x_i = x_i^F$, output subsidies can attain the first-best outputs irrespective of Cournot or Bertrand competition. However, under different R&D activities, output subsidies can yield the same output that is lower than the first-best outputs irrespective of Cournot or Bertrand competition.

[15] For some related discussions on the endogenous choice of market structure, see Häckner (2000), Symeonidis (2003), Matsumura and Ogawa (2012), and Basak (2017).

4.1 Cournot competition

In the third stage, the first order conditions provide the equilibrium quantities as follows:

$$q_i = \frac{(a-c)(2-b) + 2x_i - bx_j}{4-b^2} \tag{18}$$

In the second stage, the resulting equilibrium R&D investments are as follows:[16]

$$x_i = \frac{4(a-c) + (2-b)(2+b)^2 s^R}{(2-b)(2+b)^2 r - 4} \tag{19}$$

In the first stage, the maximization of social welfare with respect to s^R yields the output subsidy as follows:

$$s^{CR} = \frac{(1-b)(2+b)(a-c)r}{(2-b)M} \tag{20}$$

where the superscript "*CR*" denotes the equilibrium outcomes under R&D subsidy policies in Cournot competition and $M = 4r + b^2 r - b(1-4r) - 3 > 0$.

Then the resulting equilibrium outcomes are given as follows:

$$x_i^{CR} = \frac{(3+b)(a-c)}{M} \tag{21}$$

$$q_i^{CR} = \frac{(2+b)(a-c)r}{M} \tag{22}$$

$$p_i^{CR} = \frac{(2+3b+b^2)cr + a(-3+b(-1+r)+2r)}{M} \tag{23}$$

$$\pi_i^{CR} = \frac{(a-c)^2 r(6+b(5-8r) - 16r + 4b^2(1+r) + b^3(1+2r))}{2(-2+b)M^2} \tag{24}$$

$$W^{CR} = \frac{(3+b)(a-c)^2 r}{M} \tag{25}$$

The graphical relations between equilibria under Cournot competition are presented in Figure 4.

Lemma 3. $x_i^F > x_i^{CR} > x_i^C$, $q_i^F > q_i^{CR} > q_i^C$, and $W^F > W^{CR} > W^C$ for $b \in (0,1)$.

Lemma 3 states that R&D subsidies increase both R&D investments and outputs of the firms, which increase social welfare, compared to no subsidy. However, this induces both under-production and under-investment in Cournot firms, compared to the first-best, which results in welfare loss. This is also

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[16] From the reaction function of each firm under Cournot competition, we can see R&D investments are strategic substitutes for both firms. However, R&D subsidies monotonically increase both quantities and R&D investments of Cournot firms.

because the strategic effect of R&D on the output is positive under Cournot competition, that is, $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} > 0$ $(i,j = 1,2; i \neq j)$, while the outputs are strategic substitutes.

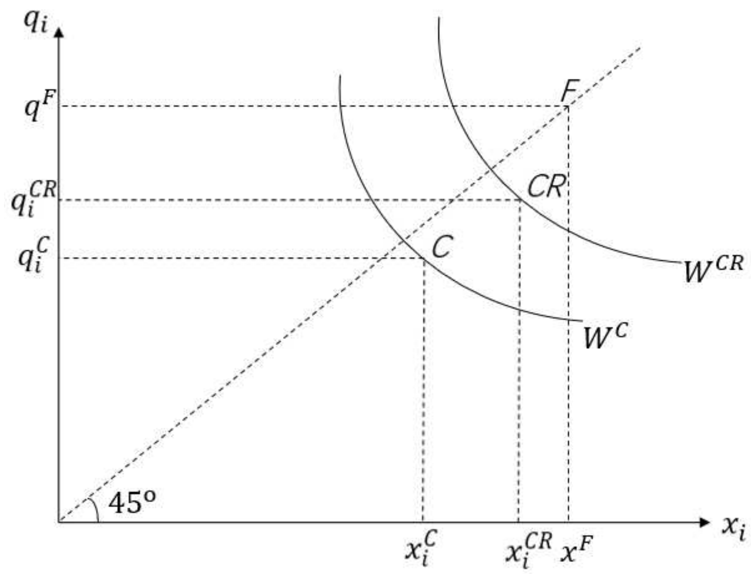

Figure 4: R&D subsidy vs. no subsidy under Cournot competition

4.2 Bertrand competition

In this case, the equilibrium prices are as follows:

$$p_i = \frac{a(2-b-b^2)+(2+b)c-2x_1-bx_2}{4-b^2} \tag{26}$$

In the second stage, the equilibrium R&D investments are as follows:[17]

$$x_i = \frac{2(2-b^2)(a-c)+(2-b)^2(2+3b+b^2)s^R}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \tag{27}$$

In the first stage, the maximization of social welfare with respect to s^R yields the output subsidy:

$$s^{BR} = \frac{(2-b)(a-c)r}{(2+b)H} \tag{28}$$

where the superscript "*BR*" denotes the equilibrium outcomes under R&D subsidy policies in Bertrand competition and $H = 2b+4r-3b^2r+b^3r-3 > 0$.

The resulting equilibrium outcomes are given as follows:

$$x_i^{BR} = \frac{(3-2b)(a-c)}{H} \tag{29}$$

[17] From the reaction function of each firm under Bertrand competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Bertrand firms.

$$q_i^{BR} = \frac{(2-b)(a-c)r}{H} \tag{30}$$

$$p_i^{BR} = \frac{(2+b-b^2)cr + a(-3+b(2-r)+2r-2b^2r+b^3r)}{H} \tag{31}$$

$$\pi_i^{BR} = \frac{(a-c)^2 r(16r+4b^4r-2b^5r+4b^2(2-5r)+b(1-8r)-6-b^3(4-10r))}{2(2+b)H^2} \tag{32}$$

$$W^{BR} = \frac{(3-2b)(a-c)^2 r}{H} \tag{33}$$

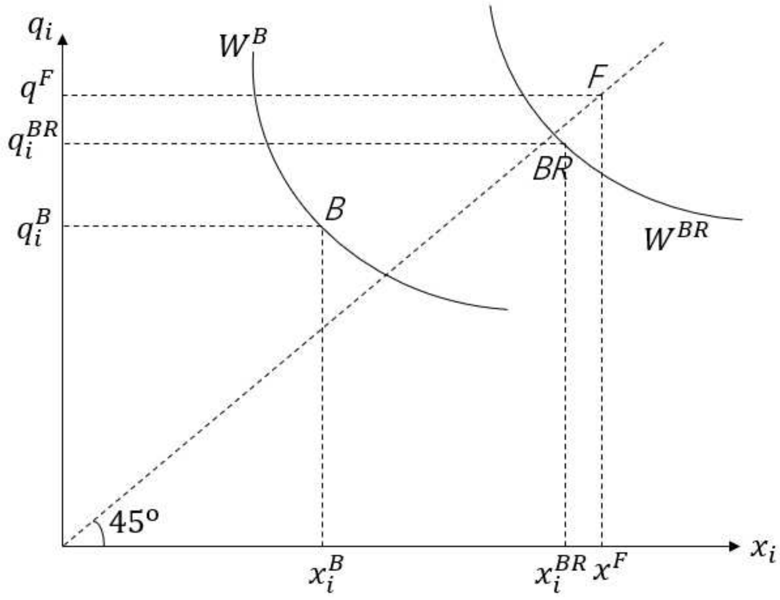

Figure 5: R&D subsidy vs. no subsidy under Bertrand competition

The graphical relations between equilibria under Bertrand competition are illustrated in Figure 5.

Lemma 4. $x_i^F > x_i^{BR} > x_i^B$, $q_i^F > q_i^{BR} > q_i^B$ and $W^F > W^{BR} > W^B$ for $b \in (0,1)$.

Lemma 4 states that R&D subsidies increase both R&D investments and outputs of the firms, which increase social welfare, compared to no subsidy. However, this induces both under-production and under-investment in Bertrand firms, compared to the first-best, which results in welfare loss. This is also because the strategic effect of R&D on the output is negative under Bertrand competition, that is, $\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$ $(i, j = 1,2; i \neq j)$, while the prices are strategic complements.

4.3 Comparisons

We compare the Cournot and Bertrand competitions under R&D subsidies. Figure 6 combines

Figures 4 and 5, and shows the graphical relations between Cournot and Bertrand competitions. It indicates that R&D subsidies increase R&D investments and outputs in both Cournot and Bertrand firms, compared to no subsidies. However, with R&D subsidy policies, the R&D investments are less than the first-best in both Cournot and Bertrand competitions.

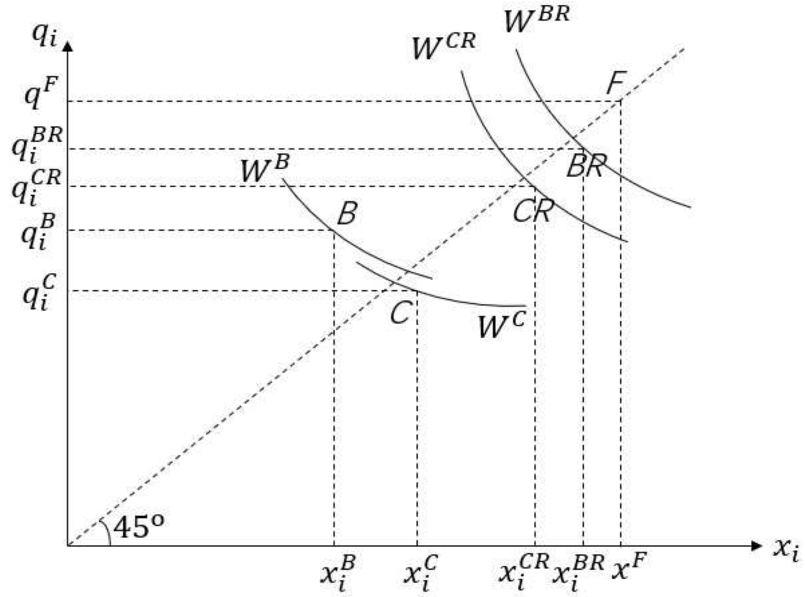

Figure 6: Welfare comparisons under R&D subsidies

Proposition 6. $s^{BR} > s^{CR} > 0$ for $b \in (0,1)$.

Proposition 6 states that the government grants less R&D subsidies to Cournot firms than Bertrand firms. This is contrary to the results with output subsidy policies (Proposition 1). In the absence of R&D subsidies, Bertrand firms invest lower R&D than Cournot firms, which are also lower than the first-best. To increase R&D directly, the government has an incentive to provide more R&D subsidies to Bertrand firms to increase social welfare.

Proposition 7. $x_i^{CR} < x_i^{BR}$ for $b \in (0,1)$.

Proposition 7 states that Bertrand firms undertake more R&D and thus lower production cost under R&D subsidies. This is also contrary to the results with output subsidy policies (Proposition 2). Even though Bertrand firms can reduce their costs and increase output to earn more profits under R&D subsidies, they choose under-investment, compared to the first-best.

Proposition 8. $q_i^{CR} < q_i^{BR}$ and $p_i^{CR} > p_i^{BR}$ for $b \in (0,1)$.

Proposition 8 states that R&D subsidies can induce Bertrand firms to increase output and lower prices more effectively compared with Cournot firms since the government induces the former to undertake more R&D. This is also contrary to the results with output subsidy policies (Proposition 3). Note that $x_i^{CR} q_i^{CR} - \frac{r}{2}\left(x_i^{CR}\right)^2 < x_i^{BR} q_i^{BR} - \frac{r}{2}\left(x_i^{BR}\right)^2$. Even though R&D output subsidies have opposite effects on the firms' R&D decisions, both effects are not off-set so that the net benefits from R&D for Bertrand firms are higher than that for Cournot firms. This results in outputs (prices) being higher (lower) for Bertrand firms compared with Cournot firms under R&D subsidies. This result also implies that consumer surplus is higher under Bertrand competition.

Proposition 9. If $b > b_2$, then $\pi_i^{CR} > \pi_i^{BR}$. However, if $b < b_2$, then $\bar{r}$ exists so that $\pi_i^{BR} \gtreqless \pi_i^{CR}$ if $r \lesseqgtr \bar{r}$, where b_2 is provided in the Appendix.

Proposition 9 states that the profits of Cournot firms are higher than Bertrand firms if the product substitutability is high, while they will be lower than Bertrand firms if the product substitutability is low and R&D investment is relatively efficient. It implies that depending on the substitutability and the efficiency of R&D investment, Cournot or Bertrand competition can be an equilibrium if both firms choose competition mode between quantity and price competitions.

We can explain these results as follows. First, note that both output and R&D subsidies increase the outputs and R&D investments of the firms under both Cournot and Bertrand competition. On the one hand, we have that $\frac{\partial s^{CR}}{\partial b} < 0$ and $\frac{\partial q_i^{CR}}{\partial b} < 0$ for any *b*. Thus, lower substitutability increases output subsidy, which increases the outputs of Cournot firms. However, Bertrand firms are more sensitive to the product substitutability, which induces such firms to set a lower price than Cournot firms. Therefore, if the product substitutability is sufficiently high, Cournot firms will earn more profit than Bertrand firms.[18] On the other hand, it can be shown that $\frac{\partial s^{BR}}{\partial b} \gtreqless 0$ and $\frac{\partial q_i^{BR}}{\partial b} \gtreqless 0$ if $b \gtreqless b_2$. Thus, if the product substitutability is high (for a larger *b*), higher substitutability increases R&D subsidy, which increases

[18] See also Tremblay and Tremblay (2011), Correa-Lopez and Naylor (2004), among others.

the output of Bertrand firms. Therefore, if the product substitutability is sufficiently high, such firms set a lower price than Cournot firms and the latter will earn more profit. However, if the product substitutability is low (for a small b), then lower substitutability decreases R&D subsidies, which also decreases the outputs of both Cournot and Bertrand firms. If firms invest R&D efficiently (lower r), Bertrand firms still have a higher subsidy (see Proposition 6) and will earn more profit than Cournot firms. In contrast, if firms invest R&D inefficiently (higher r), then Bertrand firms will face more cost loss because they invest more R&D than Cournot firms (see Proposition 7). Therefore, Cournot firms will again earn more profit than Bertrand firms if R&D investment is inefficient.

Proposition 10. $W^{CR} < W^{BR}$ for $b \in (0,1)$.

Proposition 10 states that Bertrand competition yields higher social welfare than Cournot with R&D subsidy policies. This is also contrary to the results with output subsidies (Proposition 5). This is because consumer surplus is always higher under Bertrand competition, which exceed the profit changes. This finding implies that irrespective of product substitutability, the society is better off under Cournot competition if the government can choose the market mode between quantity and price competitions.

5. Discussion

We now compare the equilibrium outcomes between output and R&D subsidy policies with Cournot and Bertrand firms, respectively.

Proposition 11.

(i) $s^{CP} > s^{CR}$ and $s^{BP} \gtreqless s^{BR}$ if $b \lesseqgtr b_3 \equiv \frac{4a+c-\sqrt{16a^2-8ac+9c^2}}{2c}$.

(ii) $x_i^{CP} > x_i^{CR}$ and $x_i^{BP} \gtreqless x_i^{BR}$ if $b \lesseqgtr b_3$;

(iii) $q_i^{CP} > q_i^{CR}$ and $q_i^{BP} > q_i^{BR}$ for $b \in (0,1)$.

Proposition 11 states that under Cournot competition, the government always sets higher output subsidy to Cournot firms, compared to R&D subsidies, which induce higher outputs and R&D investments to such firms. However, under Bertrand competition, the government sets higher output subsidy to Bertrand firms if the product substitutability is low. Thus, Bertrand firms always produce more output under output subsidies, but they may undertake less R&D under output subsidy if the product substitutability is high.

Finally, we compare the profits and welfares between output and R&D subsidies under Cournot and Bertrand competition, respectively. It is difficult to find the explicit ranges that support regularity conditions and thus, we provide a simulation in the Appendix and test the specific numbers as examples. From the numerical simulation, we can provide the following propositions.

Proposition 12.

(i) $\pi_i^{CP} > \pi_i^{CR}$ for $b \in (0,1)$ and $\pi_i^{BP} > \pi_i^{BR}$ if $b < b_3$. However, if $b > b_3$, then $\hat{b} > b_3$ and $\hat{r}$ exist such that $\pi_i^{BP} < \pi_i^{BR}$ if $b > \hat{b}$ and $r < \hat{r}$, while $\pi_i^{BP} > \pi_i^{BR}$ if $b < \hat{b}$ and $r > \hat{r}$.

(ii) $W^{CP} > W^{CR}$ for $b \in (0,1)$ and $W^{BP} > W^{BR}$ if $b < b_3$. However, if $b > b_3$, then $\hat{b} > b_3$ and $\hat{r}$ exist such that $W^{BP} < W^{BR}$ if $b > \hat{b}$ and $r < \hat{r}$, while $W^{BP} > W^{BR}$ if $b < \hat{b}$ and $r > \hat{r}$.

Proposition 12 states that output subsidy policies to Cournot firms always yield higher profits and social welfare than R&D subsidy policies. This result of quantity competition confirms the findings of Kesavayuth and Zikos (2013) and Lee et al. (2017), who examined Cournot duopoly with homogeneous products and with different objectives for firms. However, Bertrand firms in price competition indicate that the effect of the subsidy policies depends on the product substitutability and the efficiency of R&D investment. When the substitutability is high and the R&D investment of firms is efficient (lower r), less marginal cost is more beneficial and thus, R&D subsidy policies play a more important role in improving firms' profits and social welfare. However, when the product substitutability is low and the R&D investment of firms is inefficient (higher r), more production is more beneficial and thus, output subsidy policies can improve firms' profits and social welfare.

6. Conclusions

This study examined and compared the effects of output and R&D subsidy policies on the competition mode between Cournot and Bertrand in a differentiated product duopoly market. We demonstrated that firms invest more R&D, and the government grants more subsidies under Cournot than Bertrand with output subsidies, but the results are reversed with R&D subsidies. We also found that firms earn more profits under Cournot than Bertrand with output subsidy while the profits can be higher under Bertrand than Cournot if the product substitutability and the efficiency of firms' R&D investment is low with R&D subsidy. As a result, the level of social welfare is the same in both Cournot and Bertrand competitions with output subsidy policies, while the level of social welfare is always lower under Cournot competition. Finally, we reveal that firms' profits and social welfare are always higher under output subsidies in Cournot competition, while they can be higher under R&D subsidies in Bertrand competition if the products substitutability is high and firms' R&D investment is efficient.

We suggest some topics for future research. First, we can extend this model into an oligopolistic competition.[19] Second, we can consider the effect of R&D spillovers. Finally, we can examine a mixed market where the objectives between the firms are different.[20]

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[19] For example, see Hsu and Wang (2005), Tremblay and Tremblay (2011), Haraguchi and Matsumura (2016), among others.

[20] Recent studies have also analyzed different objectives of the firms with/without subsidy policies. For example, see Gil-Molto et al. (2011), Matsumura et al. (2013), Lee and Tomaru (2017), and Leal et al. (2021).

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