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Zhang, Xiaomeng and Palivos, Theodore and Liu, Xiangbo

Capital University of Economics and Business, Athens University of
Economics and Business, Renmin University of China

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Aging and Automation in Economies with Search Frictions

Xiaomeng Zhang

Capital University of Economics and Business

Theodore Palivos

Athens University of Economics and Business

Xiangbo Liu

Renmin University of China

Abstract

This paper investigates the impact of an increase in life expectancy on the level and the distribution of income in the presence of skill heterogeneity and automation capital. It shows analytically that an increase in life expectancy induces the replacement of low-skilled workers by automation capital and high-skilled workers. It also raises the skill premium, but has an ambiguous effect on total income. When we perform a simulation exercise, based on US data, we find that an increase in life expectancy raises the level of income but exacerbates its distribution. For this reason, we also examine redistributive policies that can mitigate some of the negative effects that follow an increase in life expectancy.

JEL Classification: J11, J24, J64, O33

Keywords: Life Expectancy, Automation, Search and Matching, Skill Heterogeneity

1 Introduction

A specter is haunting the developed world - the specter of automation. The steady growth of automation capital, such as robots, control systems and 3D-printers is ubiquitous in every developed country. It is estimated that the number of *industrial* robots in operation globally was around 1.8 million at the end of 2016, rose to over 2.4 million at the end of 2018 and is expected to approach 4.5 million by the end of 2022 (our calculations based on data in IFR 2019a). Moreover, roughly 168,000 professional *service* robots were sold globally in 2017 and this figure is expected to reach over 2.8 million in the period between 2018 and 2022 (IFR 2019b). The fear is then that automation capital will replace labor, especially the low-skilled one, thereby leading to “high rates of technological unemployment” and a worsening of wage inequality, a situation often referred to as “automation anxiety” (see Prettnner and Bloom 2020, pp. xi and xii).

A related phenomenon, as we argue below, is population aging, which has become a global phenomenon. Over the past several decades, most economies have experienced a substantial increase in life expectancy. According to the United Nations, in 2019 there were 703 million older persons aged 65 years or over, who comprised 9 percent of the world population. In particular, Europe and Northern America have the most aged population, with 18 percent being over 65 years.¹ Population aging results in an increase in the dependency ratio - the ratio of those not of working age to those of working age - lowers per capita income and puts pressure on productive population. To overcome these adverse effects and increase their productivity, it is often argued that economies must rely not only on traditional capital deepening but also on robots and automation, which, as we mention above, may affect negatively the distribution of income. Still, we claim that the connection between longevity and automation is even deeper.

Automation and robotics are fields of artificial intelligence (AI).² While AI and robots, in particular, emerged in large-scale mass manufacturing, they are now spreading to more and more application areas.³ One of them is Healthcare Informatics, which seems to be advancing in great leaps and bounds. For example, new technology, such as AI-powered Digital Workers, reduces significantly the cost of collecting, sorting and analyzing data that are important for the development and approval of new medicines. Thus, the pharmaceutical companies are in a position to bring new and better drugs to the market

¹The advanced economies are not the only ones facing rapid population aging. Some emerging economies follow closely a similar transformation. For example, in China, the share of the population aged 65 years or over has continuously increased in recent years and reached 11.9 percent in 2019.

²The economic use of AI can be divided into five categories: Deep Learning, Robotization, Dematerialization, Gig economy, and Autonomous Driving (see Wisskirchen et al. 2017)

³The first industrial robot was installed in a General Motors automobile factory in New Jersey (Brynjolfsson and McAfee 2016).

faster. Moreover, using AI-powered tools such as DAC [Deep Aging Clocks], doctors should be able to track the changes that occur every second in patients' bodies over their lifetime; hence, they should be able to assess more precisely individual health risks and design appropriate interventions and changes in lifestyle for each specific patient. Longevity medicine, a new branch of medicine, "is specifically focused on promoting healthspan and lifespan, and is powered by AI technology" (Zhavoronkov et al. 2021, p. 6).⁴

At the same time, an increase in longevity has a positive effect on saving (see, for example, Bloom et al. 2003, Zhang and Zhang 2005, Kinugasa and Mason 2007 and Li et al. 2007). It is then natural to expect that part of this saving is invested in new technologies.⁵ Hence, automation and population aging are not only synchronous phenomena, but they also reinforce each other. Research in AI, which includes automation, finds applications in medicine and raises human longevity, while longevity increases saving and investment in AI. In fact, whereas, in the past, part of the increase in longevity was driven by improvements in sanitation, housing and education, in addition to vaccines and antibiotics, a further increase would have to rely almost entirely on technological advancements. Hence, the link between longevity and AI will become even stronger.⁶

This paper analyzes the distributional effects of a change in life expectancy. Specifically, it examines the interplay between longevity and automation capital within an otherwise standard overlapping generations model that allows for labor market frictions and skill heterogeneity. We capture the increase in longevity parametrically and show how it leads to higher saving and investment in automation. Within such a framework, then, we show analytically that an increase in life expectancy induces the replacement of low-skilled workers by automation capital and high-skilled workers. It also raises the skill premium, but has an ambiguous effect on total income. When we calibrate the model to the US data, we find that an increase in life expectancy raises the level of income but exacerbates its distribution. We also extend the baseline model and allow for differential longevity between low- and high-skilled workers as well as for endogenous education and thus occupational attainment. Finally, we propose redistributive policies that can mitigate and in some cases even reverse the aforementioned adverse effects.

⁴Other important applications of AI in health include deep learning to diagnose diseases, medical robots, and AI-powered radiology assistant, to name but a few.

⁵Indeed, Gehringer and Prettnner (2019) using data from the OECD find a remarkably robust positive relation between decreasing mortality and technological progress.

⁶Recent empirical studies that examine the relation between demographics and automation include Abeliansky and Prettnner (2017) and Acemoglu and Restrepo (2018). The first study finds that an increase in population growth is associated with a reduction in the growth rate of automation density. The second study documents that countries that undergo faster aging - measured by an increase in the ratio of older to middle-aged workers - invest more in robots.

The remainder of the paper is organized as follows. The next subsection reviews some of the related literature. Section 2 presents the basic model and Section 3 analyzes its steady-state equilibrium. Section 4 calibrates the model to the US data and assesses quantitatively the effects of an increase in agents' life expectancy/longevity. Moreover, using the calibrated model, the same section assesses the implications of a redistributive policy that provides a subsidy to firms for maintaining low-skilled vacancies and finances it with a "robot tax." Section 5 extends first the basic model to allow for a) different mortality rates between low- and high-skilled workers and b) endogenous education decision. Then, it evaluates a second redistributive policy, namely, one that levies again a robot tax and uses the proceeds to subsidize the acquisition of human capital. Finally, Section 6 summarizes the main results and suggests some avenues for future research.

1.1 Related Literature

This section outlines the contribution of the paper with respect to the relevant literature. Several important papers have examined the effects of an increase in longevity on saving and economic growth; e.g., Zhang et al. (2001), Zhang and Zhang (2005), Cipriani (2014), Baldanzi et al. (2019). However, much of the existing research pays little attention to the impact of longevity on labor market outcomes. Two notable exceptions include de la Croix et al. (2013) and Friese (2016). Nevertheless, since these studies do not allow for skill heterogeneity, they are not designed to analyze any distributional effects. Moreover, most of the previous studies, when analyzing the effects of longevity, do not take into account the importance of automation capital, a factor that is highly substitutable for unskilled labor.

Our paper is closely related to the recent literature that studies the effects of automation. In fact, our analysis of life expectancy within an automation-augmented search and matching framework brings together two previously disconnected lines of research, consisting of Abeliansky and Prettnner (2017), Acemoglu and Restrepo (2018) and Stähler (2021), on the one hand, and Cords and Prettnner (2019), Guimarães and Gil (2019), Lankisch et al. (2019) and Gasteiger and Prettnner (2020), on the other. As mentioned above (see footnote 6), the papers by Abeliansky and Prettnner (2017) and Acemoglu and Restrepo (2018) show theoretically and document empirically that aging leads to greater automation. The first paper captures aging by a decrease in the population growth rate, whereas the second paper measures it as an increase in the ratio of older to middle-aged workers. The paper by Stähler (2021) analyzes a life-cycle model in which the representative firm produces a final good using four factors: routine and non-routine labor, traditional and automation capital. It shows that the positive effect of technological progress on

per capita output outweighs the negative effect of population aging; this, however, comes at the cost of increased inequality. We do not consider technological progress and focus instead on changes in the skill premium, the (un)employment rates and the distribution of income. We also allow for different survival probabilities across skill groups as well as for endogenous education decision. Finally, we analyze the role of policy in mitigating the adverse distributional effects.

Cords and Prettner (2019) develop a model with automation and a rich search and matching environment. The production side of their model is almost identical to the one that we use in this paper, namely, the aggregate production technology combines skilled and unskilled labor with physical and automation capital. Automation capital, in particular, is a perfect substitute for unskilled labor and an imperfect substitute for skilled labor. Within such a framework, they study how changes in automation capital affect the unemployment and wages of the two types of labor. When they calibrate their model to German data they find that the job creation of automation outweighs the job destruction and thus overall employment increases. Although our paper shares several common characteristics with the one by Cords and Prettner (2019), there are also several important differences between the two papers. For example, our research question is different in that we ask how a change in longevity affects labor market outcomes and especially income inequality. Moreover, the answer to this question depends crucially on our use of an overlapping generations model where agents save for the old days, whereas Cords and Prettner use an infinite horizon Mortensen-Pissarides type model where agents are risk neutral.

Guimarães and Gil (2019) develop a search and matching model in which firms choose between two different technologies, an automated and a manual-labor. They show that an automation-augmenting shock, i.e., an increase in the productivity of automation capital, increases the average wage and employment, but reduces the labor share in total income. Their model also suggests that the observed decline in the US labor share is mainly attributed to technological shocks, with institutional shocks playing an almost insignificant role. Thus, their focus is on the effect of automation capital on the labor share and not on changes in longevity or income inequality.

Lankisch et al. (2019) is another important paper in this literature. They simplify the household side of the economy by assuming a constant saving rate à la Solow and emphasize the production side by using a technology with four factors as in Cords and Prettner (2019), described above. They show the possibility of a balanced growth path with a positive growth rate of per capita output despite the absence of technical progress. Furthermore, they find that automation decreases the real wages of low-skilled workers

and increases the skill premium. Our paper differs from theirs since i) it uses an overlapping generations model with endogenous saving behavior and varying lifetime and ii) it includes, as part of the economic environment, search frictions that create involuntary unemployment. Moreover, Lankisch et al. (2019) conclude their paper with a central policy implication of their model, namely, to invest in higher education. They write that “such an education policy could dampen the effect of automation on wage inequality.” We explore in detail this policy recommendation.

Methodologically, Gasteiger and Prettnner (2020) is also closely related to our work even though it has a different focus. They show that the implications of automation for long-run growth in an overlapping generations model are different from those in a Cass-Koopmans or in a Solow model. In an overlapping generations model with automation, the economy converges to a steady-state with zero growth in per capita output, even when labor is fully replaced by automation capital and thus the production function becomes of the AK form. The reason is that, in the overlapping generations framework, households save and invest exclusively out of their first-period labor income, which diminishes with automation and eventually becomes a negligible fraction of the stock of automated capital; hence, accumulation ceases to take place.

Finally, an important recent contribution regarding the taxation of robots is Guerreiro et al. (2021). They develop an overlapping generations framework with endogenous skill acquisition and labor supply. Robots are better substitutes for routine than for non-routine labor. Moreover, the cost of producing robots falls over time as a result of technical progress. Within this framework, they solve for the optimal Mirrleesian tax structure under perfect commitment. We are not concerned with optimal taxation issues. Instead, we focus on how a change in longevity affects labor market outcomes. Also, in our investigation of the role of policy in mitigating the adverse distributional effects of an increase in longevity, we take the tax on automation capital as given.

In sum, our paper shares common ingredients with several of the above-mentioned papers. We cast our model within an overlapping generations framework, as in Gasteiger and Prettnner (2020), because of the convenience that it provides in modelling changes in life expectancy/longevity and the motive to save for the old days. Also, we introduce automation capital as a highly substitutable factor for unskilled labor, which is a common characteristic of most papers in the literature. Hence, changes in the quantity of automation capital lead to changes in the demand for unskilled labor (availability of unskilled jobs). This ingredient coupled with search and matching frictions, also found in Cords and Prettnner (2019) and Guimarães and Gil (2019), affects the rate of unemployment among unskilled workers, as a result of the way that firms react to market conditions (changes

in market tightness). The position then of unskilled labor is affected not only by changes in its price but also by changes in its quantity employed. Finally, we distinguish between two groups, skilled and unskilled workers, as in Cords and Prettner (2019) and Lankisch et al. (2019), so that we can analyze changes in the skill premium and the distribution of income.

Our main contribution to the literature is that we study the effects of longevity, which has been consistently increasing for most societal groups and in most developed countries. We argue that this increase in longevity is consistent and in fact contributes to some of the changes that we observe in the labor markets, namely, an increasing skill premium, declined real wages and replacement of the low-skilled workers by automation capital and a rising income inequality.⁷ Finally, we propose policies that can mitigate and in some cases even reverse the adverse distributional effects of an increase in longevity.

2 The Model

Consider an overlapping-generations economy inhabited by an infinite sequence of large households whose members have the potential lifetime of two periods. More specifically, the individual members of each household live with certainty during the period following their birth, but they may or may not survive to their second and last period of life. We assume that, before their survival prospect is realized, each agent gives birth to one offspring.

All agents belong to a household and all members of a household are *a priori* identical. There are two types of household: one whose members are all high-skilled (h) and another whose members are all low-skilled (l). We use the index $i \in \{h, l\}$, either as a subscript or as a superscript, to denote the skill level. We normalize the numbers of representative high- and low-skilled households to n_h and n_l , so that $n_l = 1 - n_h$. Moreover, we also normalize the size of each type of household to 1. We let $\rho \in [0, 1]$ denote the probability that a young agent survives to maturity; consequently, $1 - \rho$ is the probability that the agent dies prematurely.

During youth, agents search for employment. If they are successful in finding a match with a firm, they work (the time endowment is 1) and receive labor income. If, on the other hand, individuals cannot find a job, they remain unemployed. In the second period of their life, even if agents survive, nature does not bestow on them the ability to work. Thus, as a result of frictions in the labor market, individuals face uncertainty in income.

⁷For a summary of the stylized facts regarding the skill premium, wages, employment rates and inequality see Lankisch et al. (2019) and Prettner and Bloom (2020).

We follow the “large household” assumption (see, for example, Lucas 1990), and assume that all members in the same household pool their income together in both periods of life.

2.1 Households

Each household i seeks to maximize the average utility of its members:

$$U_t^i = \log c_{y,t}^i + \beta\rho \log c_{o,t+1}^i,$$

where $c_{y,t}^i$ and $c_{o,t+1}^i$ denote consumption in the young and in the old age, respectively, $\beta > 0$ is the discount factor and, as mentioned before, ρ is the probability that a young agent survives to maturity.⁸ The household’s problem is to choose $\{c_{y,t}^i, c_{o,t+1}^i\}$ and saving s_t^i subject to the budget constraints (one for each period):

$$c_{y,t}^i + s_t^i = w_t^i e_t^i, \tag{1}$$

$$c_{o,t+1}^i = \frac{1 + r_{t+1}}{\rho} s_t^i, \tag{2}$$

where e_t^i is the proportion of household members that are employed, w_t^i is the wage for workers with skill i , and r_{t+1} is the (common) interest rate.⁹ Solving the maximization problem outlined above yields the expressions:

$$\frac{c_{o,t+1}^i}{c_{y,t}^i} = \beta(1 + r_{t+1}), \tag{3}$$

$$s_t^i = \frac{\beta\rho w_t^i e_t^i}{1 + \beta\rho}. \tag{4}$$

It follows from equation (4) that an increase in the probability of survival to the second period of life, ρ , and hence an increase in longevity, has a positive effect on household’s saving. As mentioned in the Introduction, this finding receives strong empirical support.¹⁰

⁸This is a common way of introducing the survival probability in overlapping generations models, see, for example, Blackburn and Cipriani (2002), Chakraborty (2004), Cipriani (2014), Palivos and Varvarigos (2017) and Baltanzi et al. (2019).

⁹To simplify our analysis, we do not consider taxation and unemployment benefits.

¹⁰The logarithmic utility function that we use results in great analytical tractability; however, it yields a constant saving rate, $\beta\rho/(1 + \beta\rho)$ (the income and substitution effect of an increase in the interest rate offset each other). Thus, one cannot distinguish the effect of an increase in patience, as captured by an increase in β , from the effect of an increase in longevity, as captured by an increase in ρ , on saving. On the contrary, the two parameters have an opposite effect on future consumption $c_{o,t+1}^i$. Whereas an increase in β raises $c_{o,t+1}^i$, because the future matters more, an increase in ρ lowers it, because the opportunity cost of future consumption ($= \rho/(1 + r_{t+1})$) increases.

2.2 Firms

There is a continuum of identical firms. Each period t , the representative firm employs low-skilled labor, l_t , high-skilled labor, h_t , traditional physical capital, k_t , and automation capital, p_t , e.g., robots, control systems and other appliances with a minimal direct human operation, to produce output, y_t . More specifically, we postulate that the production technology takes the same functional form as in Cords and Prettnner (2019) and Lankisch et al. (2019):

$$y_t = Ak_t^\psi [\lambda(l_t + \phi p_t)^\sigma + (1 - \lambda)h_t^\sigma]^{\frac{1-\psi}{\sigma}}, \quad (5)$$

where $A > 0$ is a productivity parameter, $\psi \in (0, 1)$ governs capital income share, $\lambda \in (0, 1)$ governs labor income shares, $\sigma < 1$ determines the elasticity of substitution between low- and high-skilled labor and $\phi > 0$ measures the productivity of automation capital relative to unskilled labor. Accordingly, as it is common in this literature, e.g., Prettnner (2019) and Gasteiger and Prettnner (2020), automation capital is assumed to be a perfect substitute for unskilled labor (at the rate ϕ) and an imperfect substitute for skilled labor. Henceforth, we restrict our attention to the empirically relevant case where low- and high-skilled labor are gross substitutes for each other, i.e., $\sigma > 0$ and the elasticity of substitution $1/(1 - \sigma) > 1$ (see, for example, Ottaviano and Peri 2012). This is also the case analyzed in the recent literature (see Cords and Prettnner 2019 and Lankisch et al. 2019).

The marginal products of low- and high-skilled labor are given by

$$y_{l,t} = Ak_t^\psi [\lambda(l_t + \phi p_t)^\sigma + (1 - \lambda)h_t^\sigma]^{\frac{1-\psi}{\sigma}-1} (1 - \psi) \lambda (l_t + \phi p_t)^{\sigma-1}, \quad (6)$$

$$y_{h,t} = Ak_t^\psi [\lambda(l_t + \phi p_t)^\sigma + (1 - \lambda)h_t^\sigma]^{\frac{1-\psi}{\sigma}-1} (1 - \psi) (1 - \lambda) h_t^{\sigma-1}, \quad (7)$$

while the marginal product of traditional physical capital is

$$y_{k,t} = \psi Ak_t^{\psi-1} [\lambda(l_t + \phi p_t)^\sigma + (1 - \lambda)h_t^\sigma]^{\frac{1-\psi}{\sigma}} \quad (8)$$

and that of automation capital $y_{p,t} = \phi y_{l,t}$.

A firm opens a job vacancy of type i and searches for a suitable worker in the labor market. There is a cost $d_i > 0$ for maintaining a vacancy (a recruitment cost). Hence, the representative firm's profit flow, π_t , is equal to the output produced net of the cost of employing low- and high-skilled labor, the cost of renting traditional physical and automation capital, and the cost of maintaining vacancies:

$$\pi_t = y_t - w_t^l l_t - w_t^h h_t - R_{k,t} k_t - R_{p,t} p_t - d_l v_t^l - d_h v_t^h,$$

where v_t^i denotes vacancies of type i , and $R_{k,t}$ and $R_{p,t}$ denote, respectively, the gross rate paid to traditional physical capital and to automation capital. The demands for low- and high-skilled labor are given by $l_t = q_t^l v_t^l$ and $h_t = q_t^h v_t^h$, where q_t^i is the vacancy matching rate in labor market i , that is, the probability that a vacancy of type i will be filled.

The firm maximizes profits with respect to p_t , k_t , v_t^l and v_t^h . The first-order conditions for profit maximization are

$$y_{k,t} = R_{k,t}, \tag{9}$$

$$\phi y_{l,t} = R_{p,t}, \tag{10}$$

$$y_{l,t} q_t^l - q_t^l w_t^l = d_l, \tag{11}$$

$$y_{h,t} q_t^h - q_t^h w_t^h = d_h. \tag{12}$$

Equations (9) and (10) state that, at the optimum, the marginal products of traditional and automation capital equal their respective marginal cost. Equations (11) and (12), on the other hand, equate the expected marginal benefit from filling a vacancy of type i to its marginal cost d_i .

In addition, there is a no-arbitrage condition, which states that investing in traditional physical capital or in automation capital yields the same rate of return, i.e., $R_{k,t} = R_{p,t}$, $\forall t$. Setting equations (9) and (10) equal to each other, we solve for k_t

$$k_t = \frac{\psi}{1 - \psi} \frac{\lambda (l_t + \phi p_t)^\sigma + (1 - \lambda) h_t^\sigma}{\lambda (l_t + \phi p_t)^{\sigma-1} \phi}. \tag{13}$$

For simplicity, we assume that traditional and automation capital are fully depreciated within a period. As a result, $R_{j,t} = 1 + r_t$, where $j \in \{k, p\}$.

2.3 Job Matching

Each labor market exhibits search and matching frictions. We assume pair-wise random matching. Moreover, vacancies match with workers of the same type, i.e., there is no cross-skill matching. All newly born individuals are initially unemployed and, thus, the total measure of job-seekers of type i at the beginning of every period is n_i . The measure

of successful job matches in each labor market $i = l, h$ is determined by the matching functions:

$$l_t = M(v_t^l, n_l) = \mu_l (v_t^l)^\alpha (n_l)^{1-\alpha}, \quad (14)$$

$$h_t = M(v_t^h, n_h) = \mu_h (v_t^h)^\alpha (n_h)^{1-\alpha}, \quad (15)$$

where $\mu_i > 0$ measures the degree of matching efficiency and $\alpha \in (0, 1)$ denotes the elasticity of the matching function with respect to vacancies.

Define the tightness in labor market i as $\theta_t^i = v_t^i/n_i$. The job finding rate, i.e., the probability that a worker of type i finds a job, is

$$m(\theta_t^i) = \frac{M(v_t^i, n_i)}{n_i} = \mu_i \left(\frac{v_t^i}{n_i} \right)^\alpha = \mu_i (\theta_t^i)^\alpha, \quad (16)$$

whereas the vacancy matching rate is given by

$$q_t^i = \frac{M(v_t^i, n_i)}{v_t^i} = \mu_i \left(\frac{v_t^i}{n_i} \right)^{\alpha-1} = \mu_i (\theta_t^i)^{\alpha-1}. \quad (17)$$

2.4 Wage Determination

The wage rate w_t^i is determined through cooperative Nash bargaining. Workers and firms have relative bargaining power γ and $1 - \gamma$, respectively, where $\gamma \in (0, 1)$. For a firm, hiring an additional worker will create a surplus of $y_{i,t} - w_t^i$. On the other hand, for a household, accepting a job offer will raise its objective function by $\partial U_t^i / \partial e_t^i$. The outcome of the bargaining game then is the wage rate w_t^i that solves the maximization problem:

$$\max_{w_t^i} \left\{ (1 - \gamma) \log(y_{i,t} - w_t^i) + \gamma \log \left(\frac{\partial U_t^i}{\partial e_t^i} \right) \right\}.$$

Simple differentiation yields:

$$w_t^i = \gamma y_{i,t}, \quad (18)$$

i.e., workers receive a fraction of their marginal product, which is equal to their bargaining power γ .

3 Equilibrium Analysis: The Effects of an Increase in Life Expectancy

Definition 1 *The equilibrium is a sequence $\{c_{y,t}^i, c_{o,t+1}^i, s_t^i, v_t^i, k_t, p_t, l_t, h_t, w_t^i, r_t, R_{j,t}\}$, $i \in \{h, l\}$ and $j \in \{k, p\}$, such that in every period t : (a) given the factor price sequence*

$\{w_t^i, r_t\}$, the consumption and saving decisions $\{c_{y,t}^i, c_{o,t+1}^i, s_t^i\}$ maximize household's i discounted lifetime utility; (b) given $\{w_t^i, R_{j,t}\}$, the vacancy and investment decisions $\{v_t^i, k_t, p_t\}$ maximize firm's profits; (c) the measures of low- and high-skilled employed workers are given by equations (14) and (15); (d) the wage in each market is given by equation (18); (e) the no-arbitrage condition $R_{k,t} = R_{p,t}$ holds; (f) the interest rate $r_t = R_{j,t} - 1$; and (g) the market-clearing condition for loanable funds, $k_{t+1} + p_{t+1} = s_t^l n_l + s_t^h n_h$, holds.

Using equations (4), (11), (12) and (18) and imposing steady-state conditions, we can reduce the equilibrium system to:

$$p + k = \frac{\beta\rho}{1 + \beta\rho} \gamma \left[y_l \mu_l (\theta^l)^\alpha n_l + y_h \mu_h (\theta^h)^\alpha n_h \right], \quad (19)$$

$$(1 - \gamma)y_l = \frac{d_l}{\mu_l} (\theta^l)^{1-\alpha}, \quad (20)$$

$$(1 - \gamma)y_h = \frac{d_h}{\mu_h} (\theta^h)^{1-\alpha}, \quad (21)$$

where y_l , y_h , k , l , and h are given by equations (6), (7), (13), (14) and (15).

Proposition 1 *An increase in life expectancy, i.e., an increase in ρ , results in: (a) a positive effect on the steady-state level of traditional and automation capital, the employment and wage of high-skilled workers and the skill premium; (b) a negative effect on the employment and wage of low-skilled workers; and (c) an ambiguous effect on output.*

Proof: See the Appendix.

Consider first the effect of an increase in ρ , which corresponds to an increase in life expectancy or equivalently in longevity, on traditional and automation capital. The right-hand side of equation (19), which captures households' saving, goes up. Intuitively, an increase in the probability of surviving to retirement motivates individuals to save more for old-age consumption. More saving then translates into higher levels of traditional and automation capital, since besides factors of production, they are also assets (stores of value). Second, it follows from equation (7) that the increase in traditional or automation capital raises the marginal product of high-skilled labor, which, in turn, raises firms' demand for high-skilled labor and increases the wage of high-skilled workers (equation 18). On the contrary, from equation (6), the effects of an increase in traditional and automation capital on variables that are related to low-skilled labor work in opposite directions. In particular, on the one hand, an increase in automation capital reduces the marginal product of low-skilled workers, which decreases firm's demand for low-skilled labor and

lowers the wage of low-skilled workers (see equation 18). On the other hand, an increase in traditional physical capital raises the marginal product of low-skilled workers. As a result, the employment and the wage of low-skilled workers tend to increase. Nevertheless, the former effect, which operates through automation capital, dominates the latter one, which operates via traditional capital. Therefore, an increase in life expectancy has a negative effect on the employment level and the wage of low-skilled workers. Next, the increase in w^h and the drop in w^l result in an increase in the skill premium, defined as w^h/w^l . Finally, the effect of an increase in life expectancy on output is ambiguous. This is so because, on the one hand, the employment of low-skilled workers goes down, but, on the other hand, the level of traditional physical capital, automation capital and the employment of high-skilled workers go up.

4 Quantitative Analysis

In this section, we calibrate the model to the US data and obtain quantitative results regarding the effects of an increase in life expectancy/longevity. We are primarily interested in the effects of an increase in life expectancy on output, automation capital, wages, employment of skilled and unskilled labor, skill premium and income distribution.

4.1 Calibration

There are 14 parameters in the model: the discount factor β , the elasticity of the matching function with respect to the measure of vacancies α , the matching efficiency parameters μ_h and μ_l , the workers' bargaining power γ , the capital income share ψ , the share of high-skilled labor force n_h , the vacancy costs d_h and d_l , the production parameters A , σ and λ , the probability of survival to the old age ρ , and the relative productivity of automation capital ϕ . One period in our model lasts for 30 years.

First, we use the annual discount factor of 0.98, which implies $\beta = 0.545$. Second, following common practice, we set $\alpha = \gamma = 0.5$, and $\psi = 0.3$. Third, following, among others, Chassamboulli and Palivos (2014) and Prettnner and Strulik (2020), we define as skilled a worker with at least a Bachelor's degree and set the percentage of skilled workers n_h equal to 0.323. Fourth, based on Acemoglu and Restrepo (2020), we use the value of $\phi = 3$.¹¹ Fifth, ρ is set equal to 0.6, so that the life expectancy obtained is consistent with that in the data (78 years). Finally, based on the estimates of Ottaviano and Peri (2012), we set the production parameter σ equal to 0.5, implying an elasticity of substitution

¹¹Nevertheless, our results are robust to lower values of ϕ .

between high- and low-skilled labor equal to 2.0.¹²

The remaining parameters are jointly calibrated to match the following 6 targets obtained from the US data: *a*) the average employment rates of workers with at least a Bachelor’s degree (skilled labor) and of workers with less than a Bachelor’s degree (unskilled labor) equal 0.976 and 0.939, respectively; *b*) the skill premium is 1.97; *c*) the vacancy to unemployment ratios equal 0.620; *d*) the robots to labor ratio is 2%. The values of the calibrated parameters are presented in Table 1.

Table 1: Values of the Calibrated Parameters

<i>Value</i>	<i>Interpretation</i>
$\lambda = 0.427$	Labor income share parameter
$A = 6.830$	Production efficiency parameter
$\mu_l = 1.203, \mu_h = 1.240$	Matching efficiency parameters
$d_l = 0.975, d_h = 1.979$	Vacancy costs

4.2 Results

To assess the effects of longevity we perform a simulation exercise. More specifically, we let ρ increase gradually from its baseline value 0.6 (life expectancy = 78 years) to 0.9 (life expectancy = 87 years). The results regarding the level of automation capital (p), the employment levels (l and h) and wages (w_l and w_h) for low- and high-skilled workers, and output (y) are presented in Figure 1 (dashed line).¹³ As can be seen, consistent with our theoretical results, an increase in life expectancy has a positive effect on automation capital, the wage and the employment level of high-skilled workers and the skill premium. On the contrary, it has a negative effect on the wage and the employment level of low-skilled workers.

¹²Our results are also qualitatively robust to changes in σ (see Subsection A.3 in the Appendix). For high values of σ the adjustment in the quantities and prices of labor are relatively small. In fact, when $\sigma = 1$, the two types of labor become perfect substitutes. In this case, the ratios of their marginal products (wages) remain constant ($= \lambda/1 - \lambda$) and their levels of employment cease to respond to changes in longevity.

¹³In Subsection A.4 we also present all the results in a tabular form both in levels and in percentage changes.

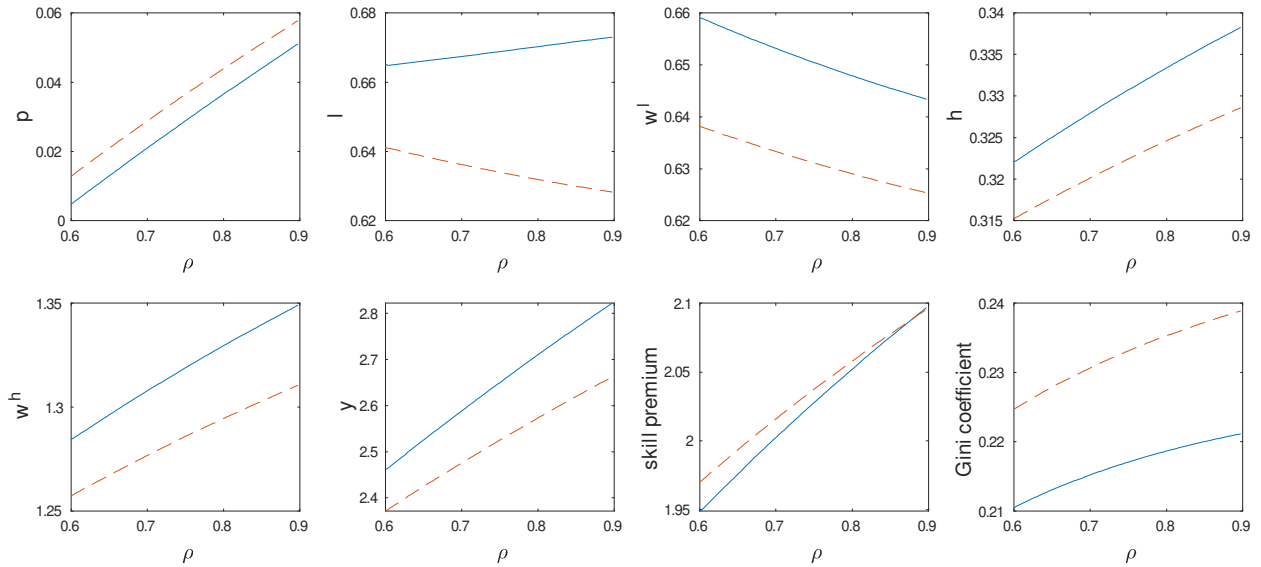


Figure 1: The effects of longevity

dashed line: baseline model, no subsidy, solid line: a vacancy-maintenance subsidy financed by a robot tax $\tau = 0.1$.

As a consequence of the aforementioned effects, there is an adverse change in the distribution of income. More specifically, based on the criteria of age and skill, there are four income groups in this economy: young low-skilled, young high-skilled, old low-skilled and old high-skilled. The income level of each group is affected by the changes in wages and the levels of employment. The change in the Gini coefficient, presented in Figure 1, indicates that, as life expectancy increases, income inequality goes up.

Finally, the effect of an increase in life expectancy on output is positive, meaning that the effects via traditional physical capital, automation capital and the employment of high-skilled workers dominate on the effect through the employment of low-skilled workers. In Subsection A.2 of the Appendix, we also present the effects on traditional capital (k) and the tightness in each labor market (θ^i). Quantitatively, the effects range from small (on wages and employment levels) to modest (on the skill premium and the Gini coefficient) to substantial (on output and automation capital).

4.3 A Vacancy-maintenance Subsidy

In this subsection, we consider a redistributive policy towards low-skilled workers. More specifically, the government provides to firms a subsidy for maintaining low-skilled vacancies and finances this subsidy with a “robot tax,” i.e., a tax on the use of automation

capital (see Gasteiger and Prettner 2020).¹⁴ As a result, the profit of the representative firm becomes:

$$\pi_t = y_t - w_t^l l_t - w_t^h h_t - R_{k,t} k_t - (1 + \tau) R_{p,t} p_t - (d_l - \chi_t) v_t^l - d_h v_t^h,$$

where τ is the rate of the robot tax and χ_t is the subsidy for maintaining a vacancy of type l . The first-order conditions with respect to p_t and v_t^l are

$$\phi y_{l,t} = (1 + \tau) R_{p,t}, \quad (22)$$

$$y_{l,t} q_t^l - q_t^l w_t^l = d_l - \chi_t. \quad (23)$$

Setting equations (9) and (22) equal to each other, we solve for k_t

$$k_t = (1 + \tau) \frac{\psi}{1 - \psi} \frac{\lambda (l_t + \phi p_t)^\sigma + (1 - \lambda) h_t^\sigma}{\lambda (l_t + \phi p_t)^{\sigma-1} \phi}.$$

Equation (20) changes now to

$$(1 - \gamma) y_l = \frac{d_l - \chi_t}{\mu_l} (\theta^l)^{1-\alpha}.$$

Finally, assuming that the government budget constraint is balanced in every period, we have

$$\tau R_{p,t} p_t = \chi_t v_t^l. \quad (24)$$

Acemoglu et al. (2020) find that the tax rate on equipment and software capital is around 10%. The same number is used by Guerreiro et al. (2021). Thus, we set $\tau = 10\%$ and we let the government budget constraint (24) determine the subsidy χ_t .¹⁵ The resulting subsidy is presented in Subsection A.2 and in Table A.5 (Subsection A.4). As a percentage of the cost of maintaining a low-skilled vacancy, the subsidy ranges from 0.4% (when $\rho = 0.6$) to 4.0% (when $\rho = 0.9$).¹⁶

The results appear in Figure 1 (solid line). The subsidy lowers the cost of maintaining a low-skilled vacancy, spurs firms to open more low-skilled vacancies and increases the market tightness for low-skilled workers. At the same time, the robot tax discourages the

¹⁴A robot tax is often suggested as a way to mitigate the negative effects of automation (see Gasteiger and Prettner 2020 and Prettner and Bloom 2020 for details).

¹⁵As shown in Subsection A.5.1, our results are robust with respect to changes in τ .

¹⁶In Subsection A.6, we consider the case where a vacancy-maintenance subsidy at a constant rate is financed by a constant robot tax and additional lump-sum taxation. The results are qualitatively the same.

accumulation of automation capital. If the subsidy is combined with the robot tax then, *for any given ρ* , automation capital, the skill premium and the Gini coefficient decrease. On the contrary, traditional capital, the wages and the quantities employed of both types of labor as well as total output increase.

As the policy of a maintenance subsidy in combination with a robot tax continues and longevity increases, the behavior of all the variables follows the same pattern as before, starting either from higher or lower value, except for that of low-skilled labor employed. The measure of employed low-skilled workers increases with the policy and continues to do so as ρ rises. Thus, the trend of a high rate of technological unemployment for low-skilled workers is not only mitigated, but reversed. The reason behind this result is that the subsidy that follows from equation (24) is rising with ρ , as a given robot tax rate is applied on a higher stock of automation capital. Regarding the skill premium, notice in Figure 1 that when the subsidy is applied the skill premium drops instantaneously because of the increase in market tightness θ^l and the concomitant increase in the wage rate for low-skilled labor. As then ρ increases and the subsidy remains, the skill premium starts rising as before and eventually, for high values of longevity and hence high values of tightness in the market for low-skilled labor, it surpasses the one without the subsidy. Thus, there is a value of ρ below (above) which the skill premium with the subsidy is below (above) the one without the subsidy. Finally, the Gini coefficient in the presence of the subsidy remains far below the one without the subsidy.¹⁷

5 Extensions of the Baseline Model

Next, we consider two extensions of the basic model: (a) the existence of different survival probabilities between the two skill groups and (b) the presence of endogenous participation decision in tertiary education.

5.1 Differential Survival Probability

Recent empirical evidence suggests that there is a positive association between longevity and education. For instance, Sasson and Hayward (2019) find that the estimated life expectancy at age 25 years in the US between 2010 and 2017 declined among persons without a Bachelor's degree and increased among college-educated persons. They attribute these findings to the unhealthy lifestyle followed by people with lower educational

¹⁷As mentioned by Gasteiger and Prettnner (2020), in an open-economy world, the success of a robot tax requires its coordinated implementation in many countries to avoid the reallocation of capital to jurisdictions that do not impose such a tax.

background.¹⁸ We therefore extend our model to allow for different survival probabilities between the two skill groups. In particular, we let $\rho_i \in [0, 1]$ denote the probability that a young agent with skill i survives to maturity.

Each household's i behavior follows now from the maximization of

$$U_t^i = \log c_{y,t}^i + \beta \rho_i \log c_{o,t+1}^i,$$

subject to the first-period budget constraint (1) and

$$c_{o,t+1}^i = \frac{1 + r_{t+1}}{\rho_i} s_t^i.$$

The first-order conditions are equation (3) and the expression for saving:

$$s_t^i = \frac{\beta \rho_i w_t^i e_t^i}{1 + \beta \rho_i}.$$

5.1.1 Quantitative Analysis

In this subsection, we perform a simulation exercise regarding the effects of an increase in either ρ_l or ρ_h . There are now 15 parameter values; the previous 13 and the two probabilities of survival to the old age, ρ_l and ρ_h . For the parameters $\{\beta, \alpha, \gamma, \psi, n_h, \sigma, \phi\}$, we use the same values as in the baseline model. Following Sasson and Hayward (2019), we set $\rho_l = 0.57$ and $\rho_h = 0.82$, so that the life expectancies obtained are consistent with the data (77.1 years for unskilled labor and 84.6 for skilled labor). Finally, the remaining 6 parameters $\{\lambda, A, \mu_h, \mu_l, d_h, d_l\}$ are recalibrated to match the above-mentioned 6 targets obtained from the US data. The calibrated parameter values are presented in Table 2.

Table 2: Values of the Calibrated Parameters

<i>Value</i>	<i>Interpretation</i>
$\lambda = 0.427$	Labor income share parameter
$A = 6.180$	Production efficiency parameter
$\mu_l = 1.203, \mu_h = 1.240$	Matching efficiency parameters
$d_l = 0.882, d_h = 1.791$	Vacancy costs

The results, presented in Figure 2, indicate that an increase in either ρ_l or ρ_h has always a positive effect on high-skilled labor and a negative effect on low-skilled labor. The effect

¹⁸This is not the first time that we observe a decrease in the life expectancy of certain groups. For example, the average life expectancy among American women without a high school diploma declined from 78.5 years in 1990 to 73.5 years in 2008 (Olshansky et al. 2012).

on output is also positive.¹⁹ The intuition is the same as the one described before when the survival probability was the same between the two skill groups. The interesting finding is that the welfare of the low-skilled group declines even as their prospects for survival to the old age improve. Furthermore, the only difference in the effects resulting from a change in the two survival probabilities is with respect to the Gini coefficient for income distribution. As seen in Figure 2 (solid line), an increase in the survival probability of the low-skilled workers raises income inequality, since more agents are in the second lowest income group.²⁰ On the other hand, an increase in the survival probability of the high-skilled workers lowers income inequality, since more agents are in the highest income group (dashed line).

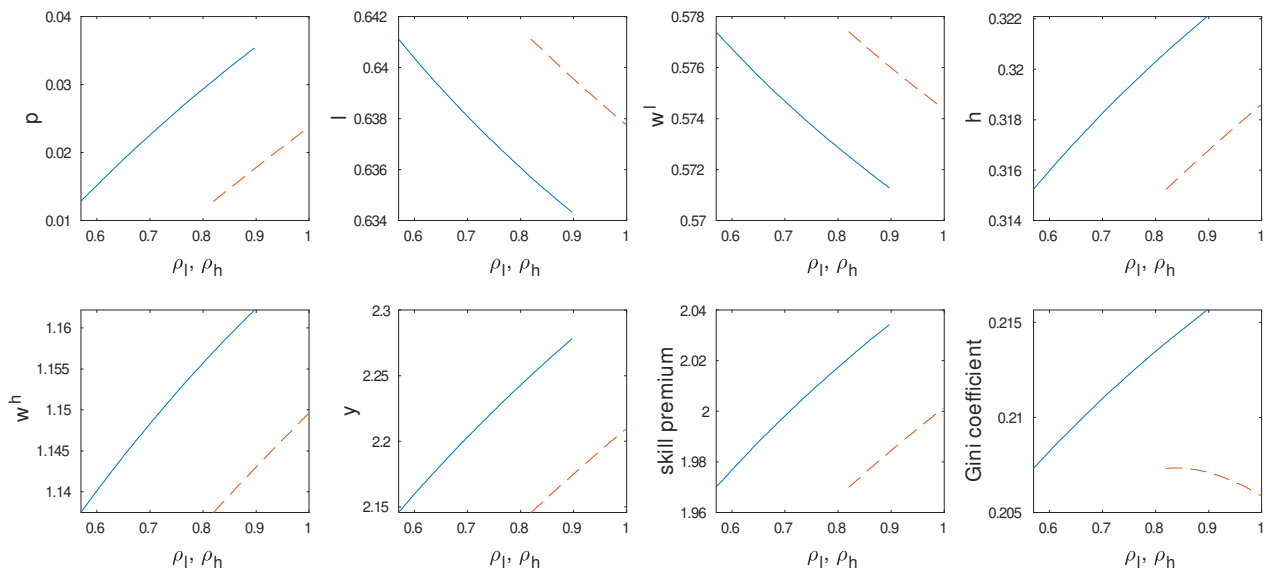


Figure 2: Differential survival probability

dashed line: the effects of an increase in ρ_h , solid line: the effects of an increase in ρ_l .

5.2 Endogenous Investment in Tertiary Education

As we have shown in the previous sections, an increase in life expectancy has positive effects on high-skilled and negative effects on low-skilled households. Therefore, one may expect that more households are willing to invest in education. In this subsection, we pursue this extension and allow for endogenous education decision.

¹⁹In addition, automation capital and traditional physical capital increase as either ρ_h or ρ_l goes up (see Subsection A.2 in the Appendix).

²⁰Note that the income of the old high-skilled is the highest among the four groups, followed by the income of the young high-skilled, then by the income of the old low-skilled and finally by the income of the young low-skilled.

Following Prettner and Strulik (2020), we assume that each household's i utility is now given by:

$$U_t^i = \log c_{y,t}^i + \beta\rho \log c_{o,t+1}^i - 1_{[i=h]}v(a), \quad (25)$$

where $1_{[i=h]}$ denotes an indicator function that takes the value 1 if $i = h$ and zero otherwise; i.e., it takes the value 1 for individuals that obtain a Bachelor's degree. Moreover, agents are assumed to be heterogeneous in ability, a , and more able individuals need to spend less effort on obtaining a university degree: $v'(a) < 0$. The last term then in (25) captures the disutility from the effort that is required to obtain a university degree. As in Prettner and Strulik (2020), we assume that the effort function takes the form $v(a) = \theta \log\left(\frac{\xi}{a - a_{\min}}\right)$ for $a \geq a_{\min}$ and $v(a) = \infty$ otherwise (individuals with an ability level below a_{\min} must spend infinite effort). The positive parameters θ and ξ are used below to calibrate the ability function. The household's budget constraint in the first-period of life changes to:

$$c_{y,t}^i + s_t^i = w_t^i e_t^i (1 - \eta_i) + T_t^i,$$

where η_i is the constant investment of time spent on education; hence, $\eta_i = 0$ for $i = l$ (no college degree) and $\eta_i = \eta$, for $i = h$ (college degree), where $0 < \eta < 1$. Also, $T_t^i > 0$, if $i = h$ and $T_t^i = 0$ if $i = l$; hence, T_t^i is a transfer/subsidy towards those that choose to invest in education. Utility maximization yields the following expressions for consumption:

$$c_{y,t}^i = \frac{w_t^i e_t^i (1 - \eta_i) + T_t^i}{1 + \beta\rho}. \quad (26)$$

$$c_{o,t+1}^i = \frac{\beta(1 + r_{t+1}) [w_t^i e_t^i (1 - \eta_i) + T_t^i]}{1 + \beta\rho}. \quad (27)$$

Substituting (26) and (27) in equation (25), we obtain the indirect utility function for any given education decision. Household members compare the utility levels with and without a college degree and choose to invest in higher education if

$$v(a) \leq (1 + \beta\rho) \log \left[\frac{w_t^h e_t^h (1 - \eta) + T_t^h}{w_t^l e_t^l} \right]. \quad (28)$$

Substituting the effort function $v(a)$, we solve (28) with equality to obtain the threshold ability level, a_t^* , above which household members obtain a college degree:

$$a_t^* = \xi \left[\frac{w_t^h e_t^h (1 - \eta) + T_t^h}{w_t^l e_t^l} \right]^{-\frac{1+\beta\rho}{\theta}} + a_{\min}.$$

If we let $F(a)$ denote the cumulative distribution function of ability, then the share of high-

skilled households $n_{h,t} = 1 - F(a_t^*)$. Note that, ceteris paribus, a higher life expectancy strengthens individuals' incentives to pursue a college degree and hence $n_{h,t}$ increases.

The government finances the education subsidy with a robot tax and is subject to the following budget constraint:

$$\tau R_{p,t} p_t = n_{h,t} T_t^h. \quad (29)$$

5.2.1 Quantitative Analysis

We first calibrate the model assuming away any tax or subsidy. We then introduce an education subsidy that is financed by a robot tax and study the effects on output, employment and distribution. In this version of the model, there are 17 parameter values to be determined; the previous 13, that is, the 14 parameters that are present in the baseline model except for n_h , which now becomes an endogenous variable; the time spent on education η ; and the parameters of the effort function θ , ξ , and a_{\min} . For the parameters $\{\beta, \alpha, \gamma, \psi, \sigma, \phi, \rho\}$, we use the same values as in the baseline model. Following Prettner and Strulik (2020), we set the time spent on education $\eta = 0.11$ and the ability level $a_{\min} = 100$; moreover, we assume that ability follows a normal distribution with a mean of 100 and a standard deviation of 15.²¹ Finally, the remaining 8 parameters $\{\lambda, A, \mu_h, \mu_l, d_h, d_l, \theta, \xi\}$ are recalibrated to match the above-mentioned 6 targets as well as two additional targets obtained from the US data: the percentage of individuals with at least a Bachelor's degree, which is 32.3%, and the elasticity of college attendance with respect to its price, which is 1.5 (Dynarski, 2003). The calibrated parameter values are presented in Table 3.

Table 3: Values of the Calibrated Parameters

<i>Value</i>	<i>Interpretation</i>
$\lambda = 0.441$	Labor income share parameter
$A = 6.809$	Production efficiency parameter
$\mu_l = 1.203, \mu_h = 1.240$	Matching efficiency parameters
$d_l = 0.980, d_h = 1.990$	Vacancy costs
$\theta = 0.452, \xi = 39$	Ability function parameters

²¹As explained in detail in Prettner and Strulik (2020), these numbers are based on the empirical approximation of the ability distribution with the IQ distribution.

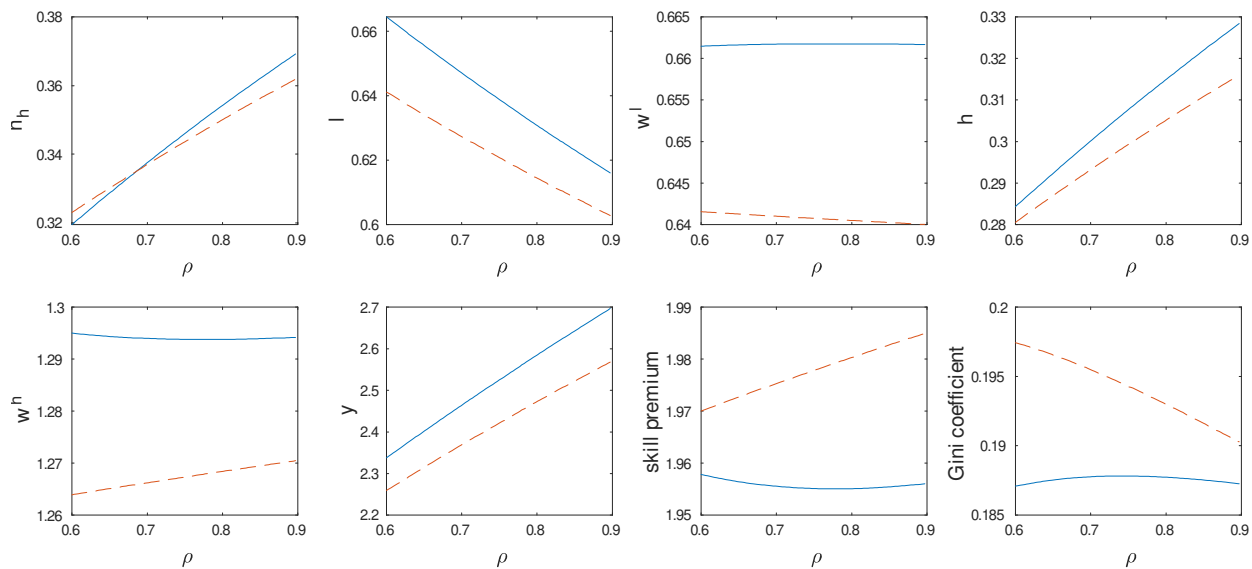


Figure 3: The effects of an education subsidy

dashed line: baseline model, no subsidy, solid line: an education subsidy financed by a robot tax $\tau = 0.1$.

The results, shown in Figure 3 (dashed line), indicate that when education investment is endogenized, an increase in life expectancy raises the number of high-skilled households. Due to this change, an increase in life expectancy still has positive effects on automation capital, the employment and the wage of high-skilled workers and output (the results on automation capital, traditional physical capital and market tightness in the two labor markets are presented in Appendix A.2). As for the low-skilled workers, their employment and their wage decrease. Consequently, the skill premium increases. Finally, given the above changes in the wages and the employment levels of the two skill groups as well as the changes in the relative frequencies, the distribution of income gets better, while at the same time the position of those who remain low-skilled worsens in both absolute and in relative terms.

Next, based on the work of Acemoglu et al. (2020) and Guerreiro et al. (2021) mentioned above in the case of the maintenance subsidy, we introduce a tax on robots at the rate $\tau = 0.1$ and let the government budget constraint (29) determine the education subsidy T_t^h .²² The resulting subsidy is presented in Subsection A.2 and in Table A.9 (Subsection A.4). As a percentage of the labor income of high-skilled household, the subsidy ranges from 0.5% (when $\rho = 0.6$) to 4.8% (when $\rho = 0.9$).²³

²²As shown in Subsection A.5.2, our results are robust with respect to changes in τ .

²³In Subsection A.6, we consider the case where an education subsidy at a constant rate is financed by a constant robot tax and additional lump-sum taxation. The results are qualitatively the same.

The effects of longevity in the presence of the redistributive policy are again shown in Figure 3 (solid line). *For any given level of longevity*, the robot tax lowers the level of automated capital and raises the level of traditional capital. On the other hand, the education subsidy induces, *ceteris paribus*, individuals to invest in education. These adjustments have countervailing effects on the wages and employment levels. The immediate (i.e., for a given ρ) decrease in automation capital and the increase in traditional capital drive the initial increase in the wage of the low-skilled workers and the decrease in the number of educated high-skilled workers in comparison with the situation before the policy is instigated. Nevertheless, as the education subsidy and the robot tax continue to be applied and longevity increases, the number of high-skilled workers increases and the level of automation capital starts rising again (see Appendix A.2). Among the most notable changes then in the effects of longevity, in the presence of the redistributive policy, are that the wages of low- and high-skilled workers are stabilized at higher levels. At the same time, the skill premium and the Gini coefficient jump to a lower level and remain essentially insensitive to changes in longevity. Thus, as before, the redistributive policy that we analyzed in this subsection mitigates some of the negative effects that follow from an increase in life expectancy.

6 Conclusion

We have analyzed the effects of an increase in life expectancy or, equivalently, longevity, on output, employment and income distribution in the presence of automation as well as traditional capital. We have shown that an increase in life expectancy raises the level of automation capital, the employment and the wage of high-skilled labor, as well as the skill premium. On the other hand, it lowers the employment and the wage of low-skilled labor. Finally, it has an ambiguous effect on output. When calibrating the model to the US data, our simulation analysis shows that output goes up, but the distribution of income deteriorates. Thus, changes in life expectancy have significant distributional effects. Most of these results remain qualitatively the same when we extend the model to allow for different mortality rates between low- and high-skilled workers or for an endogenous education decision. We have also examined the effects of redistributive policies, such as a subsidy towards the maintenance of low-skilled vacancies or education that is financed by a robot tax, and have shown that such policies can alleviate some of the negative effects of increased life expectancy and automation capital.

Our analysis is subject to several qualifications that call for further research. For the sake of brevity we outline just two of them. First, in our analysis, the increase in

life expectancy occurs as a parametric shift and is not related to any medical R&D that occurs in the economy. It is likely, however, that advances in both artificial intelligence and in longevity medicine are the results of systematic research efforts. To study then more deeply the interplay between digital health technologies and automation, one would have to introduce an R&D sector, as in Prettnner and Strulik (2020), where new discoveries affect both the production and the healthcare sectors.

Another caveat of our model is that changes in longevity as well as in automation do not influence the length of agents' working life. It will be an interesting extension to endogenize the retirement age in the presence of automation. In fact, in the current framework, changes in longevity and in automation will result in different retirement age for each skill group. This is an additional dimension that will influence not only the distribution of income but also the sustainability of the public pension system. We leave these, and other extensions, for future work.

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A Appendix

A.1 Proof of Proposition 1

Taking logarithm of the system defined by equations (19)-(21) yields

$$\ln\left(\frac{\beta\rho}{1+\beta\rho}\right) + \ln\gamma + \ln(y_l l + y_h h) - \ln(p+k) = 0, \quad (\text{A.1})$$

$$\ln(1-\gamma) + \ln y_l - \ln d_l + \ln \mu_l - (1-\alpha)\ln\theta^l = 0, \quad (\text{A.2})$$

$$\ln(1-\gamma) + \ln y_h - \ln d_h + \ln \mu_h - (1-\alpha)\ln\theta^h = 0. \quad (\text{A.3})$$

The Jacobian of the system is

$$\Delta = \begin{pmatrix} \frac{\partial \ln J_1}{\partial \ln p} & \frac{\partial \ln J_1}{\partial \ln \theta^l} & \frac{\partial \ln J_1}{\partial \ln \theta^h} \\ \frac{\partial \ln y_l}{\partial \ln p} & \frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} - (1-\alpha) & \frac{\partial \ln y_l}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} \\ \frac{\partial \ln y_h}{\partial \ln p} & \frac{\partial \ln y_h}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} & \frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} - (1-\alpha) \end{pmatrix}, \quad (\text{A.4})$$

where

$$\frac{\partial \ln J_1}{\partial \ln p} = \Omega \frac{\partial \ln y_l}{\partial \ln p} + (1-\Omega) \frac{\partial \ln y_h}{\partial \ln p} - \frac{p}{p+k} - \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln p},$$

$$\frac{\partial \ln J_1}{\partial \ln \theta^l} = \left[\Omega \left(\frac{\partial \ln y_l}{\partial \ln l} + 1 \right) + (1-\Omega) \frac{\partial \ln y_h}{\partial \ln l} - \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln l} \right] \frac{\partial \ln l}{\partial \ln \theta^l},$$

$$\frac{\partial \ln J_1}{\partial \ln \theta^h} = \left[\Omega \frac{\partial \ln y_l}{\partial \ln h} + (1-\Omega) \left(\frac{\partial \ln y_h}{\partial \ln h} + 1 \right) - \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln h} \right] \frac{\partial \ln h}{\partial \ln \theta^h},$$

and

$$\Omega = \frac{y_l l}{y_l l + y_h h},$$

$$\Omega_l = \frac{\lambda(l+\phi p)^\sigma}{\lambda(l+\phi p)^\sigma + (1-\lambda)h^\sigma},$$

$$\frac{\partial \ln y_l}{\partial \ln p} = -(1-\psi)(1-\sigma)(1-\Omega_l) \frac{\phi p}{l+\phi p} < 0,$$

$$\frac{\partial \ln y_l}{\partial \ln l} = -(1-\psi)(1-\sigma)(1-\Omega_l) \frac{l}{l+\phi p} < 0,$$

$$\frac{\partial \ln y_l}{\partial \ln h} = (1 - \psi)(1 - \sigma)(1 - \Omega_l) > 0,$$

$$\frac{\partial \ln y_h}{\partial \ln p} = (1 - \sigma)[1 - (1 - \psi)(1 - \Omega_l)] \frac{\phi p}{l + \phi p} > 0,$$

$$\frac{\partial \ln y_h}{\partial \ln l} = (1 - \sigma)[1 - (1 - \psi)(1 - \Omega_l)] \frac{l}{l + \phi p} > 0,$$

$$\frac{\partial \ln y_h}{\partial \ln h} = -(1 - \sigma)[1 - (1 - \psi)(1 - \Omega_l)] < 0.$$

From equation (13), we have

$$\ln k = \ln \psi - \ln(1 - \psi) + \ln[\lambda(l + \phi p)^\sigma + (1 - \lambda)h^\sigma] - \ln \lambda - (\sigma - 1) \ln(l + \phi p) - \ln \phi.$$

Thus,

$$\frac{\partial \ln k}{\partial \ln p} = [1 - \sigma(1 - \Omega_l)] \frac{\phi p}{l + \phi p} > 0,$$

$$\frac{\partial \ln k}{\partial \ln l} = [1 - \sigma(1 - \Omega_l)] \frac{l}{l + \phi p} > 0,$$

$$\frac{\partial \ln k}{\partial \ln h} = \sigma(1 - \Omega_l) > 0, \text{ if } \sigma > 0.$$

Given by equations (14) and (15), we have

$$\ln l = \ln \mu_l + \alpha \ln \theta^l + \ln n_l,$$

$$\ln h = \ln \mu_h + \alpha \ln \theta^h + \ln n_h.$$

Thus,

$$\frac{d \ln l}{d \ln \theta^l} = \alpha,$$

$$\frac{d \ln h}{d \ln \theta^h} = \alpha.$$

The determinant of the Jacobian is

$$|\Delta| = (1 - \alpha) \left[\begin{array}{c} \Omega \frac{\partial \ln y_l}{\partial \ln p} + (1 - \Omega) \frac{\partial \ln y_h}{\partial \ln p} + \frac{p}{p+k} \left(\alpha \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \right) \\ + \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln p} \left(\alpha \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \right) - \alpha \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln l} \frac{\partial \ln y_l}{\partial \ln p} - \alpha \frac{k}{p+k} \frac{\partial \ln k}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} \end{array} \right], \quad (\text{A.5})$$

where

$$\frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln k}{\partial \ln p} = - (1 - \psi) (1 - \sigma) (1 - \Omega_l) \frac{l}{l + \phi p} [1 - \sigma (1 - \Omega_l)] \frac{\phi p}{l + \phi p},$$

$$\frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln k}{\partial \ln p} = - (1 - \sigma) [1 - (1 - \psi) (1 - \Omega_l)] [1 - \sigma (1 - \Omega_l)] \frac{\phi p}{l + \phi p},$$

$$\frac{\partial \ln y_l}{\partial \ln p} \frac{\partial \ln k}{\partial \ln l} = - (1 - \psi) (1 - \sigma) (1 - \Omega_l) \frac{\phi p}{l + \phi p} [1 - \sigma (1 - \Omega_l)] \frac{l p}{l + \phi p},$$

$$\frac{\partial \ln y_h}{\partial \ln p} \frac{\partial \ln k}{\partial \ln h} = (1 - \sigma) [1 - (1 - \psi) (1 - \Omega_l)] \frac{\phi p}{l + \phi p} \sigma (1 - \Omega_l).$$

Thus,

$$\begin{aligned} |\Delta| &< \frac{p}{p+k} \left(\frac{\partial \ln y_h}{\partial \ln p} + \alpha \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \right) \\ &+ \frac{k}{p+k} \left(\frac{\partial \ln y_h}{\partial \ln p} + \alpha \frac{\partial \ln k}{\partial \ln p} \frac{\partial \ln y_h}{\partial \ln h} - \alpha \frac{\partial \ln k}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} - (1 - \alpha) \frac{\partial \ln k}{\partial \ln p} \right) \\ &< \frac{p}{p+k} ((1 - \alpha) (1 - \sigma) [1 - (1 - \psi) (1 - \Omega_l)] - (1 - \alpha)) \\ &+ \frac{k}{p+k} \frac{\phi p}{l + \phi p} ((1 - \alpha) (1 - \sigma) [1 - (1 - \psi) (1 - \Omega_l)] - (1 - \alpha) [1 - \sigma (1 - \Omega_l)]) \\ &= - \frac{p}{p+k} (1 - \alpha) [\sigma + (1 - \sigma) (1 - \psi) (1 - \Omega_l)] \\ &\quad - \frac{k}{p+k} \frac{\phi p}{l + \phi p} (1 - \alpha) [(1 - \sigma) (1 - \psi) (1 - \Omega_l) + \sigma \Omega_l] \\ &< 0, \text{ if } \sigma > 0. \end{aligned}$$

To analyze the effects of increased longevity on automation capital, we replace the first

column of the Jacobian by the partial derivatives of equations (A.1)-(A.3) with respect to $\ln \rho$:

$$\Delta_{p,\rho} = \begin{pmatrix} \frac{1}{1+\beta\rho} & \frac{\partial \ln J_1}{\partial \ln \theta^l} & \frac{\partial \ln J_1}{\partial \ln \theta^h} \\ 0 & \frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} - (1-\alpha) & \frac{\partial \ln y_l}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} \\ 0 & \frac{\partial \ln y_h}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} & \frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} - (1-\alpha) \end{pmatrix}. \quad (\text{A.6})$$

The determinant of the matrix in equation (A.6) is positive

$$|\Delta_{p,\rho}| = -\frac{1}{1+\beta\rho} (1-\alpha) \left[\alpha \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln y_h}{\partial \ln h} - (1-\alpha) \right] > 0. \quad (\text{A.7})$$

Applying Cramer's rule yields

$$\frac{d \ln p}{d \ln \rho} = -\frac{|\Delta_{p,\rho}|}{|\Delta|} > 0.$$

To analyze the effects of longevity on low-skilled labor market tightness, we substitute the second column of the Jacobian by the partial derivatives of equations (A.1)-(A.3) with respect to $\ln \rho$:

$$\Delta_{\theta^l,\rho} = \begin{pmatrix} \frac{\partial \ln J_1}{\partial \ln p} & \frac{1}{1+\beta\rho} & \frac{\partial \ln J_1}{\partial \ln \theta^h} \\ \frac{\partial \ln y_l}{\partial \ln p} & 0 & \frac{\partial \ln y_l}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} \\ \frac{\partial \ln y_h}{\partial \ln p} & 0 & \frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} - (1-\alpha) \end{pmatrix}. \quad (\text{A.8})$$

The determinant of the matrix in equation (A.8) is positive

$$|\Delta_{\theta^l,\rho}| = \frac{1}{1+\beta\rho} (1-\alpha) \frac{\partial \ln y_l}{\partial \ln p} < 0.$$

Therefore

$$\frac{d \ln \theta^l}{d \ln \rho} = -\frac{|\Delta_{\theta^l,\rho}|}{|\Delta|} < 0.$$

To analyze the effects of longevity on the tightness of high-skilled labor market, we replace the third column of the Jacobian by the partial derivatives of equations (A.1)-(A.3) with respect to $\ln \rho$, such that

$$\Delta_{\theta^h,\rho} = \begin{pmatrix} \frac{\partial \ln J_1}{\partial \ln p} & \frac{\partial \ln J_1}{\partial \ln \theta^l} & \frac{1}{1+\beta\rho} \\ \frac{\partial \ln y_l}{\partial \ln p} & \frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} - (1-\alpha) & 0 \\ \frac{\partial \ln y_h}{\partial \ln p} & \frac{\partial \ln y_h}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} & 0 \end{pmatrix}. \quad (\text{A.9})$$

The determinant of the matrix in equation (A.9) is positive

$$|\Delta_{\theta^h, \rho}| = \frac{1}{1 + \beta\rho} (1 - \alpha) \frac{\partial \ln y_h}{\partial \ln p} > 0.$$

Thus,

$$\frac{d \ln \theta^h}{d \ln \rho} = -\frac{|\Delta_{\theta^h, \rho}|}{|\Delta|} > 0.$$

$$\begin{aligned} \frac{d \ln k}{d \ln \rho} &= \frac{\partial \ln k}{\partial \ln p} \frac{d \ln p}{d \ln \rho} + \frac{\partial \ln k}{\partial \ln l} \frac{d \ln l}{d \ln \theta^l} \frac{d \ln \theta^l}{d \ln \rho} + \frac{\partial \ln k}{\partial \ln h} \frac{d \ln h}{d \ln \theta^h} \frac{d \ln \theta^h}{d \ln \rho} \\ &= \frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha) \left[\begin{array}{c} \alpha \frac{\partial \ln k}{\partial \ln p} \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln k}{\partial \ln p} \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \frac{\partial \ln k}{\partial \ln p} \\ -\alpha \frac{\partial \ln k}{\partial \ln l} \frac{\partial \ln y_l}{\partial \ln p} - \alpha \frac{\partial \ln k}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} \end{array} \right] \\ &= \frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha) \left[\alpha \frac{\partial \ln k}{\partial \ln p} \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \frac{\partial \ln k}{\partial \ln p} - \alpha \frac{\partial \ln k}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} \right] > 0, \end{aligned}$$

$$\begin{aligned} \frac{d \ln y_l}{d \ln \rho} &= \frac{\partial \ln y_l}{\partial \ln p} \frac{d \ln p}{d \ln \rho} + \frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} \frac{d \ln \theta^l}{d \ln \rho} + \frac{\partial \ln y_l}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} \frac{d \ln \theta^h}{d \ln \rho} \\ &= \frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha) \left[\begin{array}{c} \alpha \frac{\partial \ln y_l}{\partial \ln p} \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln y_l}{\partial \ln p} \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \frac{\partial \ln y_l}{\partial \ln p} \\ -\alpha \frac{\partial \ln y_l}{\partial \ln l} \frac{\partial \ln y_l}{\partial \ln p} - \alpha \frac{\partial \ln y_l}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} \end{array} \right] \\ &= -\frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha)^2 \frac{\partial \ln y_l}{\partial \ln p} < 0, \end{aligned}$$

$$\begin{aligned} \frac{d \ln y_h}{d \ln \rho} &= \frac{\partial \ln y_h}{\partial \ln p} \frac{d \ln p}{d \ln \rho} + \frac{\partial \ln y_h}{\partial \ln l} \frac{\partial \ln l}{\partial \ln \theta^l} \frac{d \ln \theta^l}{d \ln \rho} + \frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln h}{\partial \ln \theta^h} \frac{d \ln \theta^h}{d \ln \rho} \\ &= \frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha) \left[\begin{array}{c} \alpha \frac{\partial \ln y_h}{\partial \ln p} \frac{\partial \ln y_l}{\partial \ln l} + \alpha \frac{\partial \ln y_h}{\partial \ln p} \frac{\partial \ln y_h}{\partial \ln h} - (1 - \alpha) \frac{\partial \ln y_h}{\partial \ln p} \\ -\alpha \frac{\partial \ln y_h}{\partial \ln l} \frac{\partial \ln y_l}{\partial \ln p} - \alpha \frac{\partial \ln y_h}{\partial \ln h} \frac{\partial \ln y_h}{\partial \ln p} \end{array} \right] \\ &= -\frac{1}{|\Delta|} \frac{1}{1 + \beta\rho} (1 - \alpha)^2 \frac{\partial \ln y_h}{\partial \ln p} > 0. \end{aligned}$$

A.2 Additional Results

In this subsection, we present the effects of longevity on traditional and automation capital, the market tightness for low- and high- skilled workers and the resulting subsidy.

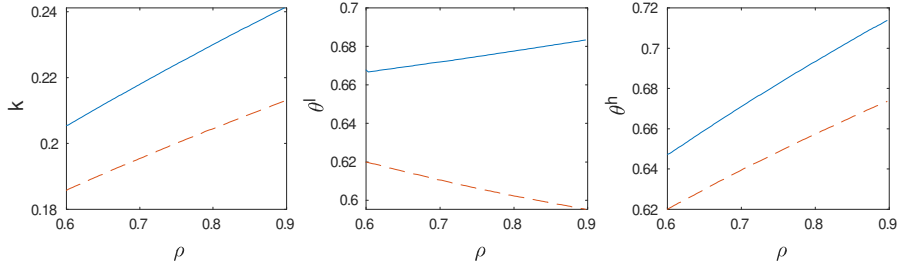


Figure A1: The effects of longevity

dashed line: baseline model, no subsidy, solid line: a vacancy-maintenance subsidy financed by a robot tax $\tau = 0.1$.

Vacancy-maintenance subsidy as a proportion of the cost of maintaining a low-skilled vacancy:

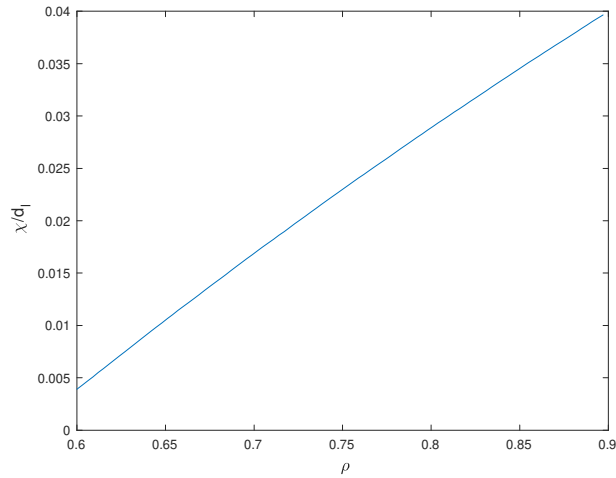


Figure A2: Size of vacancy-maintenance subsidy
a vacancy-maintenance subsidy financed by a robot tax $\tau = 0.1$.

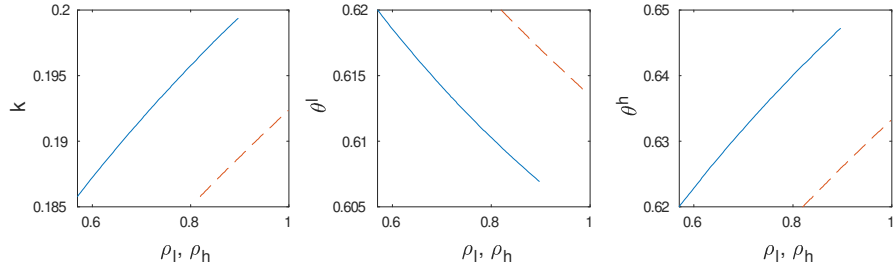


Figure A3: Differential survival probability

dashed line: the effects of an increase in ρ_h , solid line: the effects of an increase in ρ_l .

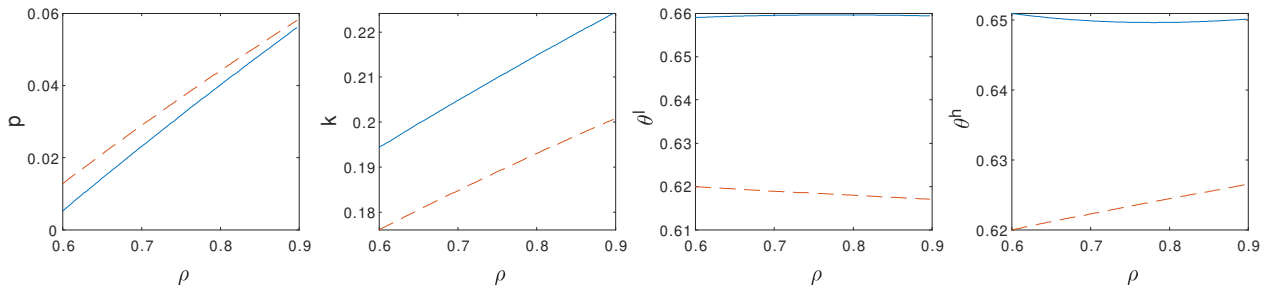


Figure A4: The effects of an education subsidy

dashed line: baseline model, no subsidy, solid line: an education subsidy financed by a robot tax $\tau = 0.1$.

Education subsidy as a proportion of the labor income of a high-skilled household.

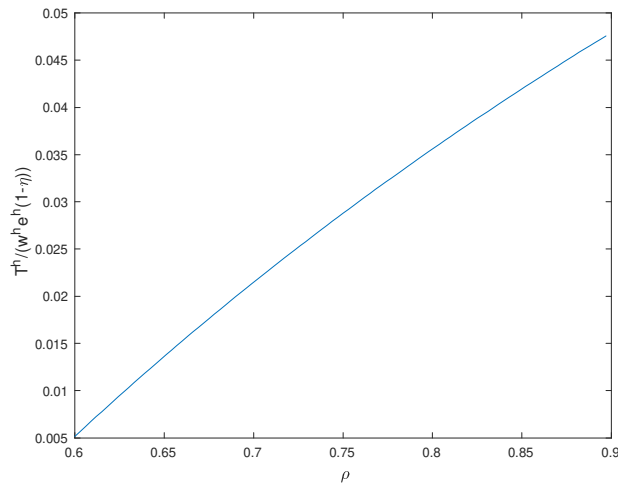


Figure A5: Size of education subsidy

an education subsidy financed by a robot tax $\tau = 0.1$.

A.3 Sensitivity Analysis with respect to σ

In this subsection, we check whether our results are robust with respect to changes in the elasticity of substitution between low- and high-skilled workers. Recall that the elasticity of substitution is equal to $1/(1 - \sigma)$. Indeed, if low- and high-skilled labor are gross substitutes for each other, i.e., $\sigma > 0$, an increase in longevity has a positive effect on the steady-state levels of traditional and automation capital, the employment and wages of high-skilled workers, output, the skill premium and the Gini coefficient. On the other hand, it has a negative effect on the employment and wages of low-skilled workers.

Table A.1: $\sigma = 0.1$

ρ	0.600	0.700	0.800	0.900
p	0.004	0.019	0.034	0.048
k	0.198	0.210	0.221	0.232
l	0.599	0.591	0.584	0.577
w_l	0.596	0.588	0.581	0.575
h	0.340	0.348	0.354	0.361
w_h	1.356	1.385	1.412	1.438
y	1.058	1.124	1.188	1.250
skill premium	2.273	2.355	2.431	2.502
Gini coefficient	0.240	0.258	0.273	0.287

Table A.2: $\sigma = 0.5$

ρ	0.600	0.700	0.800	0.900
p	0.013	0.029	0.044	0.058
k	0.186	0.195	0.205	0.213
l	0.641	0.636	0.632	0.628
w_l	0.638	0.633	0.629	0.625
h	0.315	0.320	0.325	0.329
w_h	1.257	1.277	1.294	1.311
y	2.794	2.904	3.007	3.104
skill premium	1.970	2.016	2.058	2.096
Gini coefficient	0.169	0.181	0.191	0.200

Table A.3: $\sigma = 0.9$

ρ	0.600	0.700	0.800	0.900
p	0.025	0.041	0.057	0.071
k	0.186	0.195	0.205	0.213
l	0.714	0.713	0.712	0.711
w_l	0.710	0.709	0.708	0.708
h	0.267	0.268	0.269	0.270
w_h	1.062	1.067	1.071	1.074
y	3.141	3.247	3.345	3.436
skill premium	1.496	1.504	1.511	1.518
Gini coefficient	0.035	0.038	0.040	0.042

A.4 Tabular Presentation of the Results

This Subsection reports the results presented in the main text (Figures 1, 2 and 3) in a tabular form. Related to Figure 1:

Table A.4: The effects of longevity

	Values				Percentage Changes		
ρ	0.600	0.700	0.800	0.900	0.700	0.800	0.900
p	0.013	0.029	0.044	0.058	124.9	242.5	353.3
l	0.641	0.636	0.632	0.628	-0.764	-1.433	-2.026
w^l	0.638	0.633	0.629	0.625	-0.764	-1.433	-2.026
h	0.315	0.320	0.325	0.329	1.553	2.969	4.266
w^h	1.257	1.277	1.295	1.311	1.553	2.969	4.266
y	2.372	2.476	2.573	2.665	4.388	8.500	12.36
skill premium	1.970	2.016	2.058	2.097	2.335	4.466	6.421
Gini coefficient	0.225	0.231	0.235	0.239	2.666	4.727	6.361
k	0.186	0.195	0.205	0.213	5.192	10.08	14.68
θ^l	0.620	0.611	0.602	0.595	-1.522	-2.846	-4.010
θ^h	0.620	0.639	0.657	0.674	3.130	6.025	8.713

Table A.5: The effects of longevity in the presence of a vacancy-maintenance subsidy

	Values				Percentage Changes		
ρ	0.600	0.700	0.800	0.900	0.700	0.800	0.900
p	0.005	0.021	0.037	0.051	337.0	660.2	969.2
l	0.665	0.667	0.670	0.673	0.387	0.818	1.263
w^l	0.659	0.653	0.648	0.643	-0.905	-1.703	-2.407
h	0.322	0.328	0.333	0.338	1.829	3.519	5.075
w^h	1.284	1.308	1.330	1.350	1.832	3.522	5.078
y	2.461	2.589	2.710	2.826	5.183	10.13	14.81
skill premium	1.949	2.002	2.052	2.098	2.762	5.315	7.669
Gini coefficient	0.211	0.215	0.219	0.221	2.236	3.861	5.055
k	0.205	0.218	0.230	0.242	6.144	12.03	17.64
θ^l	0.667	0.672	0.678	0.684	0.775	1.643	2.541
θ^h	0.647	0.671	0.693	0.714	3.692	7.162	10.41
$\frac{\chi}{d_i}$	0.004	0.017	0.029	0.040	329.7	635.2	917.6

Related to Figure 2:

Table A.6: The effects of ρ_l

	Values				Percentage Changes		
ρ_l	0.570	0.700	0.800	0.900	0.700	0.800	0.900
p	0.013	0.023	0.029	0.036	76.18	128.9	177.2
l	0.641	0.638	0.636	0.634	-0.473	-0.787	-1.067
w^l	0.577	0.575	0.573	0.571	-0.473	-0.787	-1.067
h	0.315	0.318	0.320	0.322	0.953	1.602	2.188
w^h	1.138	1.148	1.156	1.162	0.953	1.602	2.188
y	2.146	2.203	2.243	2.279	2.678	4.527	6.220
skill premium	1.970	1.998	2.017	2.035	1.433	2.408	3.291
Gini coefficient	0.207	0.211	0.213	0.216	1.764	2.960	4.041
k	0.186	0.192	0.196	0.199	3.166	5.357	7.366
θ^l	0.620	0.614	0.610	0.607	-0.943	-1.568	-2.123
θ^h	0.620	0.632	0.640	0.647	1.916	3.229	4.424

Table A.7: The effects of ρ_h

	Values				Percentage Changes		
ρ_h	0.820	0.880	0.940	1.000	0.880	0.940	1.000
p	0.013	0.017	0.020	0.024	29.28	57.71	85.33
l	0.641	0.640	0.639	0.638	-0.184	-0.360	-0.528
w^l	0.577	0.576	0.575	0.574	-0.184	-0.360	-0.528
h	0.315	0.316	0.318	0.319	0.369	0.724	1.066
w^h	1.138	1.142	1.146	1.150	0.369	0.724	1.066
y	2.146	2.168	2.189	2.210	1.030	2.030	3.000
skill premium	1.970	1.981	1.991	2.002	0.554	1.088	1.603
Gini coefficient	0.207	0.207	0.207	0.206	-0.045	-0.289	-0.707
k	0.186	0.188	0.190	0.192	1.217	2.398	3.546
θ^l	0.620	0.618	0.616	0.613	-0.368	-0.719	-1.053
θ^h	0.620	0.625	0.629	0.633	0.739	1.453	2.144

Related to Figure 3:

Table A.8: The effects of longevity

	Values				Percentage Changes		
ρ	0.600	0.700	0.800	0.900	0.700	0.800	0.900
n_h	0.323	0.337	0.350	0.362	4.334	8.377	12.15
l	0.641	0.627	0.614	0.603	-2.150	-4.154	-6.021
w^l	0.642	0.641	0.641	0.640	-0.084	-0.164	-0.239
h	0.281	0.293	0.305	0.316	4.527	8.766	12.74
w^h	1.264	1.266	1.268	1.271	0.185	0.359	0.526
y	2.259	2.369	2.473	2.571	4.876	9.471	13.81
skill premium	1.970	1.975	1.980	1.985	0.270	0.523	0.767
Gini coefficient	0.197	0.196	0.193	0.190	-0.979	-2.248	-3.664
p	0.013	0.029	0.044	0.058	125.8	244.3	356.0
k	0.176	0.185	0.193	0.201	4.964	9.650	14.08
θ^l	0.620	0.619	0.618	0.617	-0.169	-0.327	-0.478
θ^h	0.620	0.622	0.624	0.627	0.370	0.719	1.055

Table A.9: The effects of longevity in the presence of an education subsidy

	Values				Percentage Changes		
ρ	0.600	0.700	0.800	0.900	0.700	0.800	0.900
n_h	0.319	0.338	0.354	0.370	4.334	8.377	12.15
l	0.664	0.647	0.631	0.616	-2.618	-5.065	-7.345
w^l	0.661	0.662	0.662	0.662	0.037	0.045	0.028
h	0.284	0.300	0.315	0.329	5.568	10.77	15.63
w^h	1.295	1.294	1.294	1.294	-0.081	-0.098	-0.061
y	2.338	2.464	2.585	2.700	5.414	10.58	15.50
skill premium	1.958	1.956	1.955	1.956	-0.118	-0.143	-0.089
Gini coefficient	0.187	0.188	0.188	0.187	0.371	0.351	0.093
p	0.005	0.023	0.040	0.057	339.0	663.0	972.8
k	0.194	0.205	0.215	0.224	5.375	10.53	15.47
θ^l	0.659	0.660	0.660	0.659	0.075	0.090	0.056
θ^h	0.651	0.650	0.650	0.650	-0.162	-0.196	-0.122
$\frac{T^h}{w^h e^h (1-\eta)}$	0.005	0.021	0.036	0.048	316.3	589.8	828.6

A.5 Sensitivity Analysis with respect to τ

A.5.1 Maintenance Subsidy

In Subsection 4.3, to ensure that the amount of automation capital is positive, the rate of robot tax τ cannot exceed 0.15. Figure A6 presents the results for $\tau = 0.07, 0.1$ and 0.13 . As can be seen below, the results are robust with respect to changes in the value of τ .

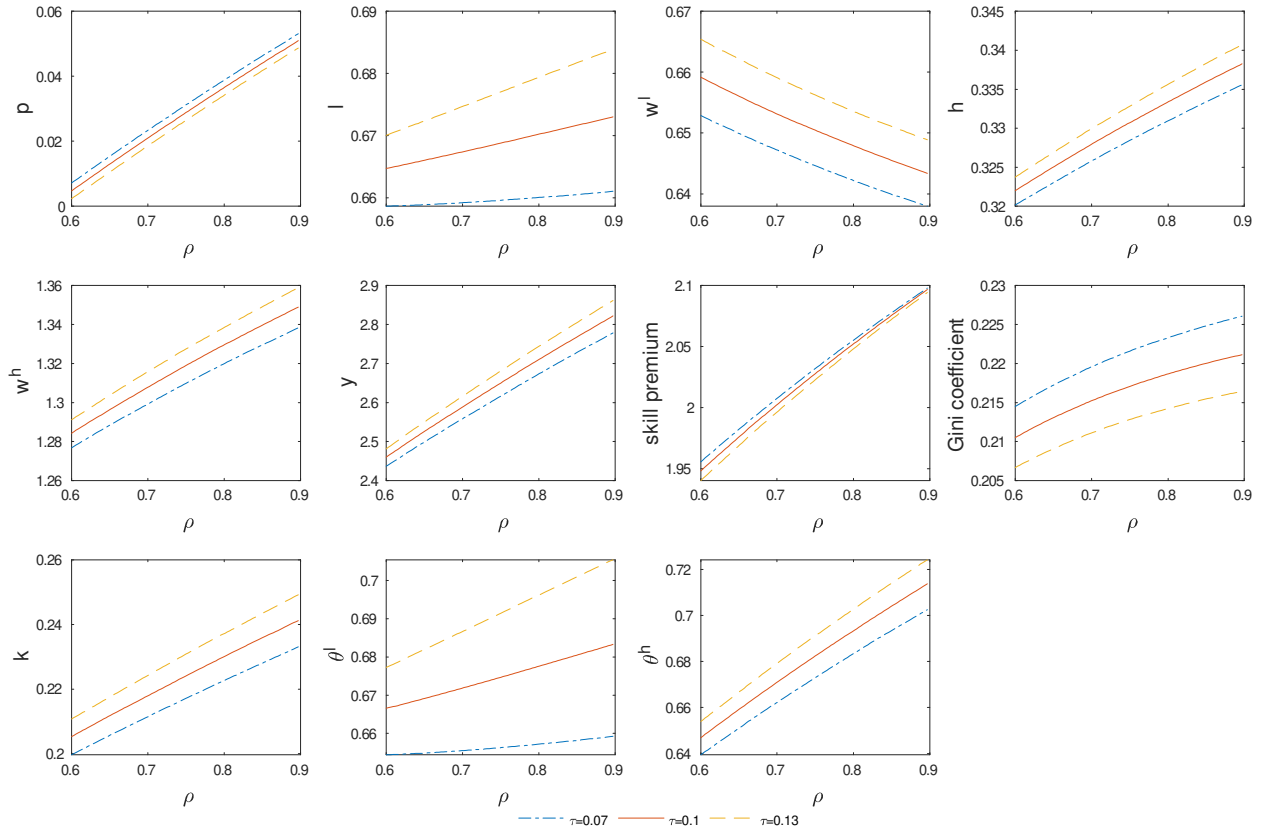


Figure A6: The effects of longevity

A.5.2 Education Subsidy

In Section 5.2, to ensure that the amount of automation capital is positive, the rate of robot tax τ cannot exceed 0.16. Figure A7 presents the results for $\tau = 0.07, 0.1$ and 0.13 . As can be seen below, the results are robust with respect to changes in the value of τ .

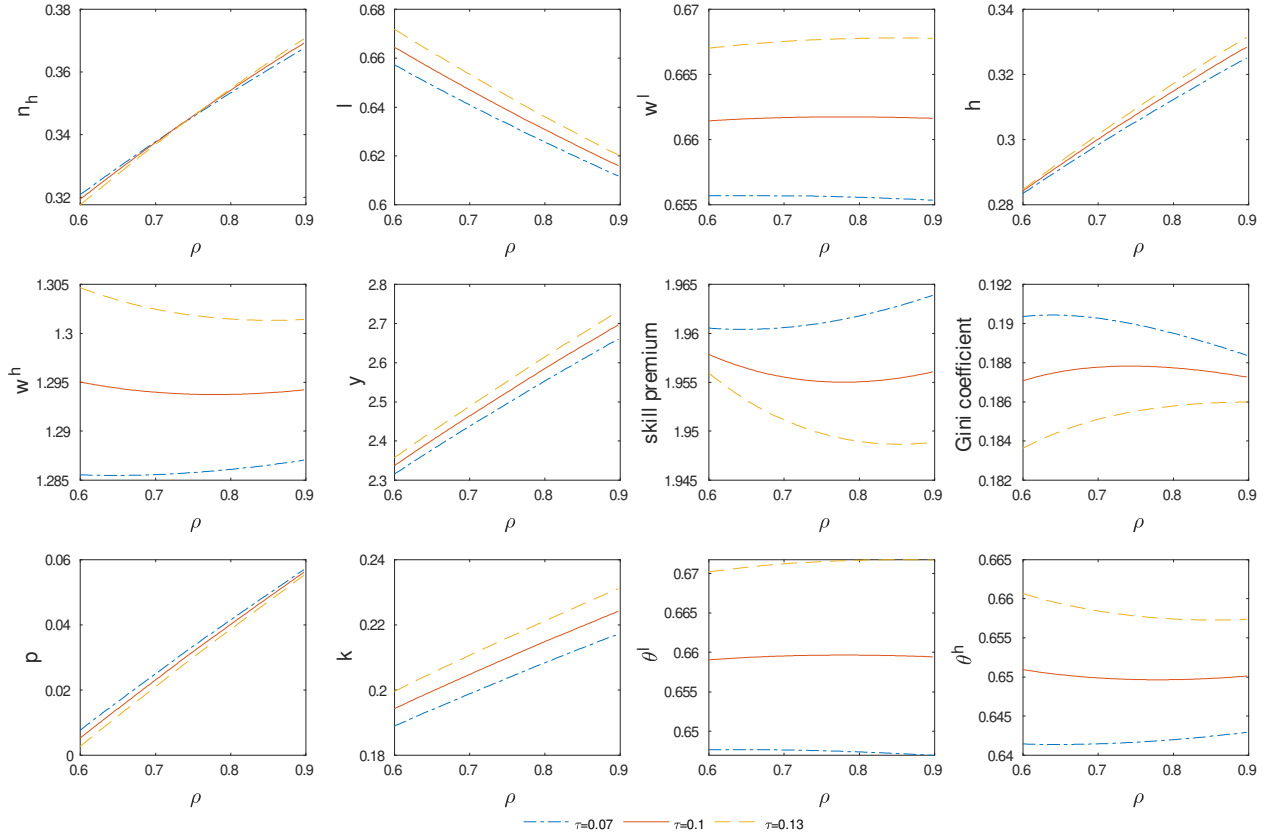


Figure A7: The effects of education subsidy

A.6 Lump-sum Taxation

In this subsection, we consider the case where the two subsidies, i.e., the maintenance subsidy and the education subsidy, are financed by a robot tax and an additional lump-sum taxation.

A.6.1 Vacancy-maintenance subsidy

We assume a constant vacancy-maintenance subsidy $\chi = 0.01d_l$ financed by a proportional robot tax $\tau = 0.1$ and an additional lump-sum taxation T_t . The profit of the representative firm becomes

$$\pi_t = y_t - w_t^l l_t - w_t^h h_t - R_{k,t} k_t - (1 + \tau) R_{p,t} p_t - (d_l - \chi) v_t^l - d_h v_t^h - T_t,$$

The government budget constraint becomes

$$\tau R_{p,t}p_t + T_t = \chi v_t^l.$$

Figure A8 presents the results.

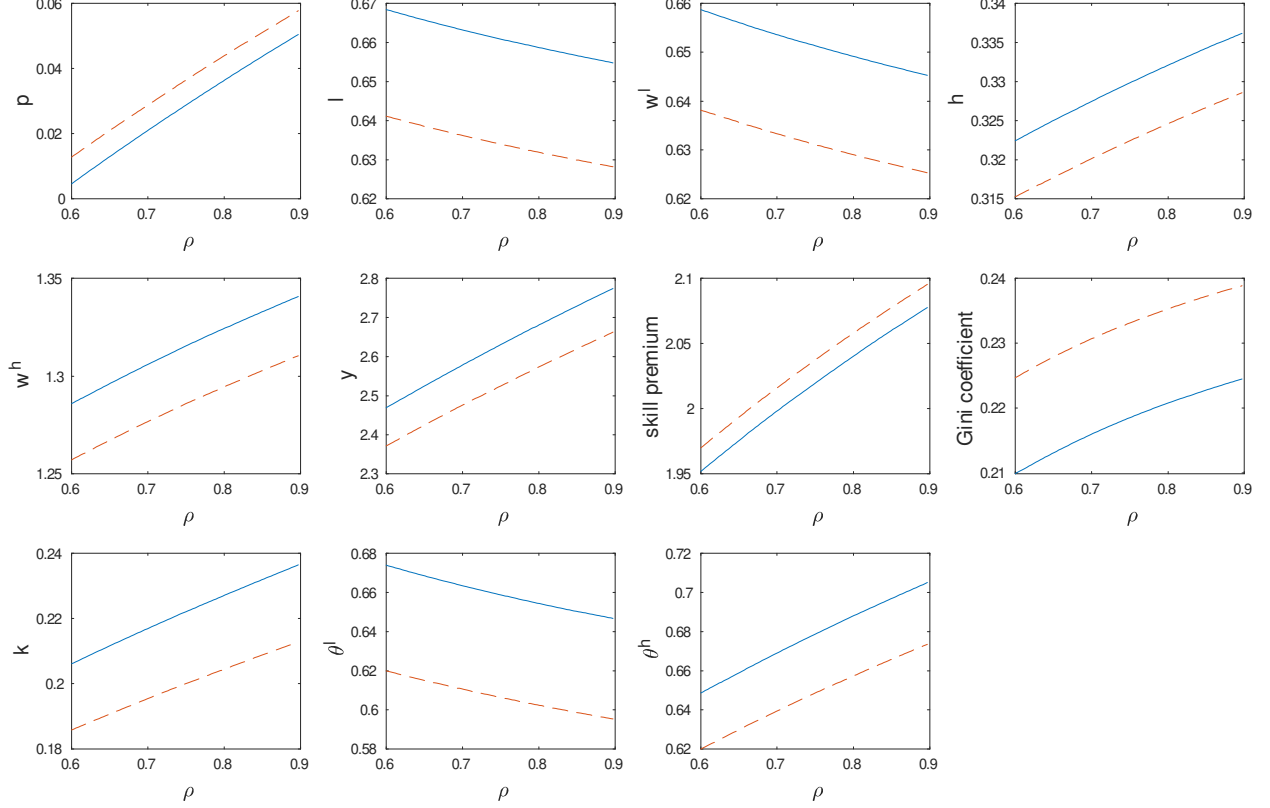


Figure A8: The effects of a constant vacancy-maintenance subsidy

dashed line: baseline model, no subsidy, solid line: a vacancy-maintenance subsidy $\chi = 0.01d_l$ financed by a robot tax $\tau = 0.1$ and a lump-sum tax T_t .

A.6.2 Education Subsidy

We assume a constant education subsidy $T^h = 0.05$ financed by a proportional robot tax $\tau = 0.1$ and an additional lump-sum tax T_t . The profit of the representative firm becomes

$$\pi_t = y_t - w_t^l l_t - w_t^h h_t - R_{k,t} k_t - (1 + \tau) R_{p,t} p_t - d_l v_t^l - d_h v_t^h - T_t,$$

The government budget constraint becomes

$$\tau R_{p,t} p_t + T_t = n_{h,t} T^h.$$

Figure A9 presents the results.

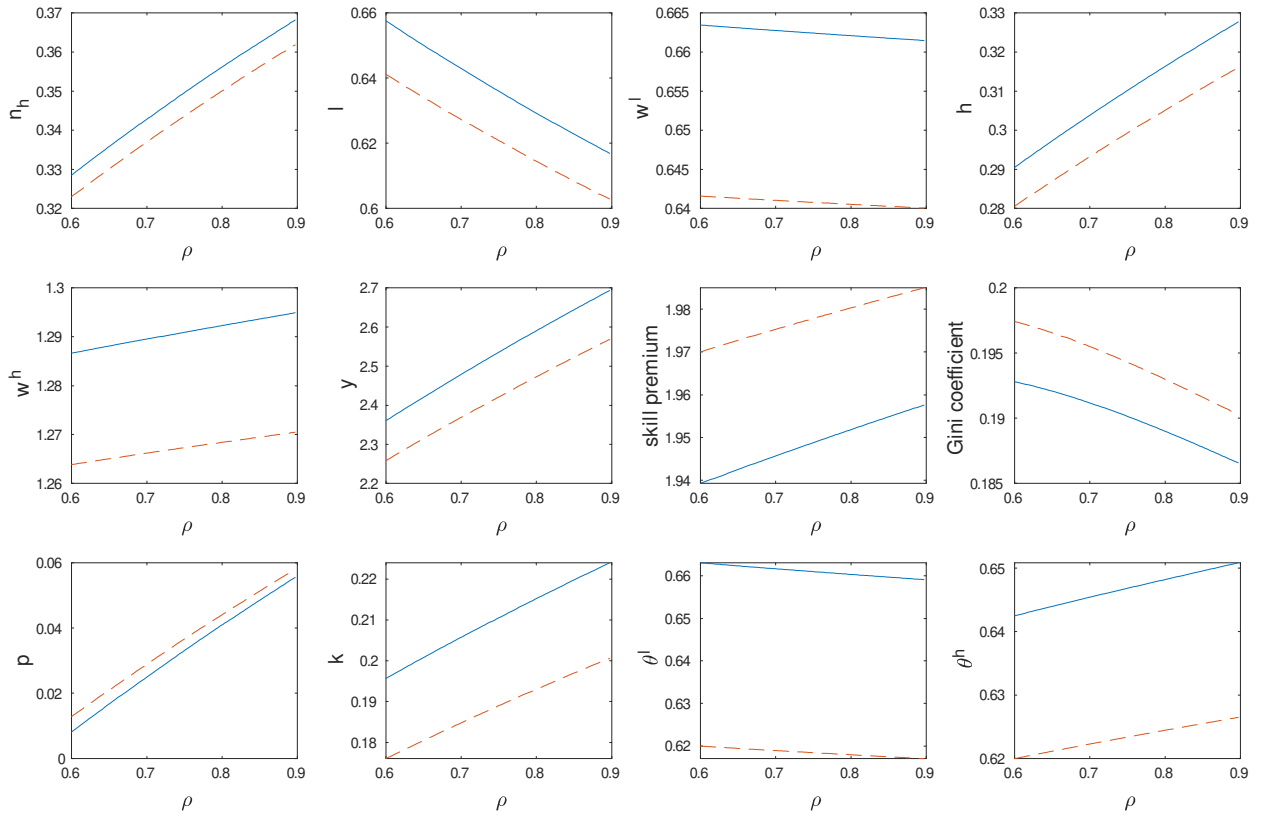


Figure A9: The effects of a constant education subsidy

dashed line: baseline model, no subsidy, solid line: a education subsidy $T^h = 0.05$ financed by a robot tax $\tau = 0.1$ and a lump-sum tax T_t .