World Equalized Factor Price and Integrated World Trade Space

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Baoping Guo*

Abstract – This paper studies the approach to attain a general equilibrium for the Heckscher-Ohlin model in the context of higher dimensions. The equalized factor prices at the higher dimensions are the prices that Dixit and Norman illustrated, i.e., that the world price will remain the same when allocations of factor endowments change within a higher-dimension equalized factor price set[1]. The study derives a way to access the share of GNP, the trade flows, and factor contents of trade of each country analytically.

Keywords: Factor Price Equalization, Integrated World Equilibrium, Heckscher-Ohlin Model, General Equilibrium, Cone of Commodity Price.

JEL Classification: F11

1. Introduction

The Heckscher-Ohlin model has a unique structure to show general trade equilibriums of multiple commodities, made by several factors, and trading by many countries. Paul Samuelson and Lionel McKenzie are pioneers both in general equilibrium theory and in international trade theory. The factor-price equalization is a milestone for the studies of general trade equilibriums. Dixit and Norman's (1980) integrated world equilibrium (IWE) is remarkable to present the FPE by trade equilibrium both from the supply side and from the demand side. Deardorff (1994) mentioned that the IWE is "Perhaps the most useful and enlightening approach to FPE." Helpman and Krugman (1985) popularized the IWE approach for the equilibrium analyses. Deardorff (1994) studied the FPE by applying the IWE to higher dimensions. Deardorff (1994) presented the lenses within the IWE for the FPE under multiple commodities.

The general equilibrium should reflect market processes. A country taking part in trade tends to maximize its welfare by minimizing its trade-off. A proper utility function reflecting trade competition properties is helpful to attain a general equilibrium of multi-commodity, multi-factor, and multi-country economy.

Guo (2018) provided a general equilibrium of trade for the 2 x 2 x 2 model, which shows the structure of equalized factor prices. He derived and explained his result by Helpman and Krugman's insight

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[1] It is only for the squared technology matrix (even number N=M) and it required that a technology matrix is non-singular.

idea that the difference in the factor composition is the sole basis for trade (see Helpman and Krugman, 1985, p.24)

This paper generalizes the general trade equilibrium of $2 \times 2 \times 2$ model to the analysis of higher-dimension models. It shows that the equalized factor price is also within a high-dimension FPE set. It finds a way to attain the share of GNP and the trade flows as a trade consequence. It shows the possible approaches to identify the structure of equalized factor prices.

This paper is divided into four sections. Section 2 reviews the trade equilibrium for the $2 \times 2 \times 2$ model. Section 3 investigates the general equilibrium of trade of the $3 \times 3 \times 2$ model. It provides a way to attain the share of GNP and the trade flow as trade consequences. Section 4 investigates the $N \times M \times Q$ model.

2. Review of the general equilibrium of the 2 x 2 x 2 model

We denote the Heckscher-Ohlin $2 \times 2 \times 2$ model as

$$A^h X^h = V^h \qquad (h = H, F) \tag{2-1}$$

$$(A^h)' W^h = P^h \qquad (h = H, F) \tag{2-2}$$

where A^h is the 2 x 2 matrix of factor input requirements with elements a_{ih} (r,w), i 1,2, $and\ h = H, F$; V^h is the 2 x 1 vector of factor endowments with elements K as capital and L as labor; X^h is the 2 x 1 vector of output; W^h is the 2 x 1 vector of factor prices with elements r as rental and w as wage; P^h is a 2 x 1 vector of commodity prices with elements p_1^h and p_2^h; h=H,F.

Guo (2015) introduce the trade box to the IWE to integrate the goods price diversification cone[2] with trade flows. He provided two approaches to reach the general trade equilibrium. One is by trade volume analyses proposed by Helpman and Grugman (1985, chapter 1). Another is by using a competitive GNP share of country H as[3]

$$s^h = \frac{1}{2}\frac{K^H L^w + K^w L^H}{K^w L^w} \tag{2-3}$$

Under this value, trade volume gets its maximum value, and each country's benefit gets its maximum value. Guo (2018) presented the general equilibrium of trade of the Heckscher-Ohlin model as

$$r^* = \frac{L^w}{K^w} \tag{2-4}$$

$$w^* = 1 \tag{2-5}$$

$$p_1^* = a_{k1}\frac{L^w}{K^w} + a_{L1} \tag{2-6}$$

$$p_2^* = a_{k2}\frac{L^w}{K^w} + a_{L2} \tag{2-7}$$

$$s^H = \frac{1}{2}\frac{K^H L^w + K^w L^H}{K^w L^w} \tag{2-8}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{L^w}, \qquad F_L^h = -\frac{1}{2}\frac{K^h L^w - K^w L^h}{K^w} \qquad (h = H, F) \tag{2-9}$$

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[2] See Fisher (2011) for the goods price diversification cone .

[3] Guo used a utility function to maximize each country benefits inside of trade box, to achieve the competitive share of GNP.

$$T_1^h = x_1^h - s^h x_1^w\,, \qquad T_2^h = x_2^h - s^h x_2^w \qquad (h = H, F) \tag{2-10}$$

where L^W is the world labor endowment; K^W is the world capital endowment, F_i^h is factor content of trade of country h, i=K, L. T_n^h is commodity export flow of country h, j is commodity number. r^* and w^* are equalized factor price; p_1^* and p_1^* are world commodity prices. Appendix A is details of the derivation.

3. Integrated World Equilibrium for the $3 \times 3 \times 2$ Model

3.1 The cone of commodity price

The goods price diversification cone is the counterpart of the cone of diversification of factor endowments. Fisher (2011) provided this vital concept. The cone is something about angles. When models go to higher dimensions, it can present the relationship between commodity prices and factor prices clearly in space. We first illustrate it in 2 dimensions.

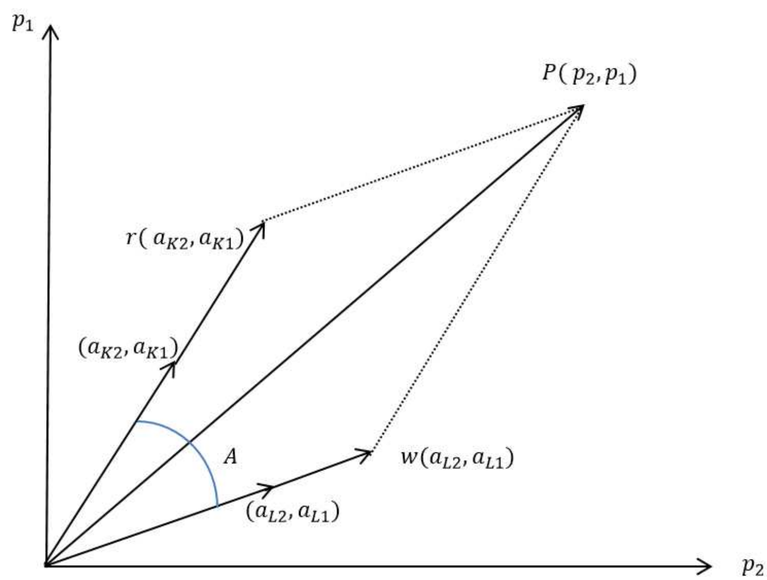

Figure 1 The Cone of Commodity Price

To illustrate the idea of the goods price diversification cone, let rewrite the non-profit cost condition (2-2) for the $2 \times 2 \times 2$ model in vectors as

$$\begin{bmatrix} a_{K1} \\ a_{K2} \end{bmatrix} r + \begin{bmatrix} a_{L1} \\ a_{L2} \end{bmatrix} w = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{3-1}$$

We place them in Figure 1. Multiplying each of these corresponding vectors by factor rewards, we obtain the unit capital costs $r(a_{K2}, a_{K1})$ and labor costs $w(a_{L2}, a_{L1})$. Summing these as in equation (3-1), we obtain the commodity price(p_2 , p_1). Space spanned by these two vectors is the goods price diversification cone, labeled by cone A in Figure 1.

3.2 The $3 \times 3 \times 2$ Model

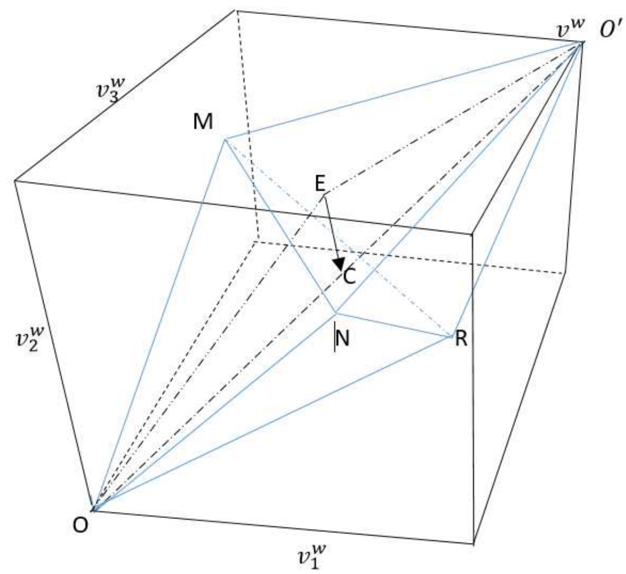

Figure 2. Integrated World Equilibrium by the 3 x 3 x2 Model

Figure 2 draws an Integrated World Equilibrium (IWE) for the 3 x 3 x 2 model. The origin for country H is the left-lower corner; country F is the right-upper corner. Tetrahedra $OMNR$ is the 3-dimension cone of factor diversification of country H, and tetrahedra $MNRO'$ is for country foreign. Point E is an allocation of factor endowments of the two countries. Point C is the equilibrium point of trade, which shows the size of the two countries.

The equal trade volume line in the $3 \times 3 \times 2$ model is a cylinder with its axis pass diagonal line OO^*. It is obvious that the diamond shape $OMNRO'$ is an equalized factor price set. When allocation E changes within the diamond shape $OMNRO'$, the world factor endowments will not change, world demand, supply, and income will not change. Therefore, world prices will remain the same[4].

We denote the technology matrix now as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{3-2}$$

where $a_{ij}\,(W)$is the technology input coefficient of sector i by factor endowment j.

3.3 Trade Box Specified by the Cone of Goods Price Diversification through Shares of GNP

For the $3 \times 3 \times 2$ model, the goods price diversification cone (briefly, cone of commodity prices) is also a tetrahedron shape. Figure 2 shows the tetrahedron for the cone of commodity prices. Commodity price vectors lie within the tetrahedron will ensure positive factor prices. To derive the

[4] See Woodland (2013)

general trade equilibrium solution, we firstly identify the cone of commodity prices. We rewrite the unit cost function as

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} w_1 + \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} w_2 + \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} w_3 = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad (3\text{-}3)$$

Each column of $A'(W)$ represents the optimal unit coefficients from a single factor. Denote

$$\theta^1 = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}, \theta^2 = \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}, \theta^3 = \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} \quad (3\text{-}4)$$

Those three vectors are the three rays or ridges that compose the price tetrahedron in Figure 3.

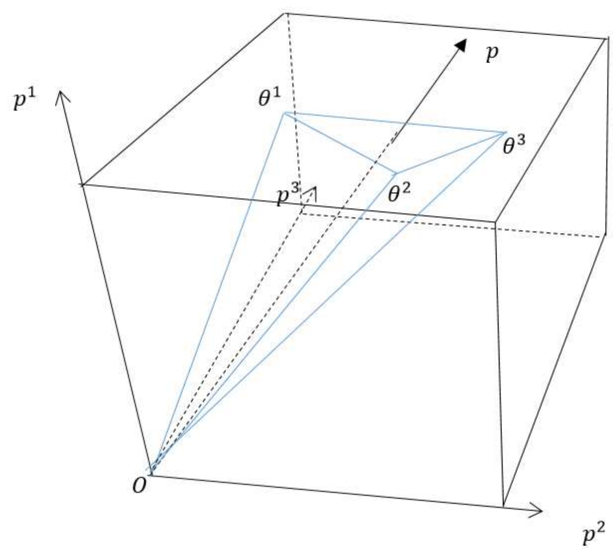

Figure 3 The Cone of Commodity Price by 3 Commodities and 3 Factors

When a price lies on any ridge of the tetrahedron, such as

$$p = \theta^1 \quad (3\text{-}5)$$

There are no rewards for factor 2 and factor 3 as

$$w_2 = 0, \qquad w_3 = 0 \quad (3\text{-}6)$$

When a price lies on any face (or surface) of the tetrahedron, such as

$$P = \theta^1 + \theta^2 \quad (3\text{-}7)$$

There is no reward for factor 3. We see now that commodity price cannot lie in any face of the price tetrahedron. It must lie within the tetrahedron.

The definition of the share of GNP of a country is

$$s^h = \frac{w^* \cdot V^h}{w^* \cdot V^W} \qquad (h = H, F) \quad (3\text{-}8)$$

Or

$$s^h = \frac{P^* \cdot X^h}{P^* \cdot X^W} \qquad (h = H, F) \quad (3\text{-}9)$$

Associated a commodity price alone a ridge in the cone of commodity prices, We can present the boundary of the share of GNP of a country. let $P = \theta^{i}$, and substituting it into (3-9), we obtain the first boundary of the share of GNP as

$$s_1^H(\theta^1 \ ) = \frac{\theta^1 \cdot X^h}{\theta^1 \cdot X^W} = \frac{v_1^H}{v_1^W} \tag{3-10}$$

Similarly, for $P = \theta^2$ and $P = \theta^3$, we have

$$s_2^H(\theta^2) = \frac{v_2^H}{v_2^W} \tag{3-11}$$

$$s_3^H(\theta^3) = \frac{v_3^H}{v_3^W} \tag{3-12}$$

We present the boundaries of the shares of GNP in Figure 4. Using the three shares of GNP above, we draw a trade box indicated by NEMJRQ. We call $\angle QEM$ the factor trade diversification cone. Like EC, all possible trade vectors should end in the diagonal line QM. And all possible trade vectors should be on the surface QEM.

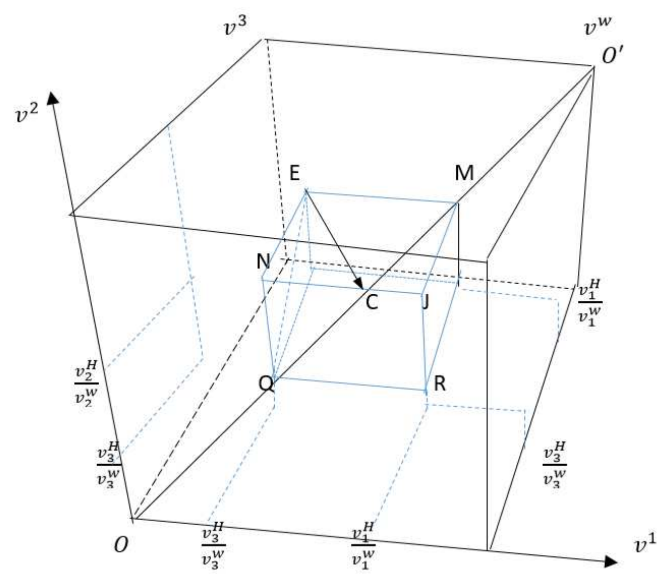

Figure 4 Tarde Box Identified By the Cone of Commodity Price

3.4 GNP redistribution by trade

As the analyses in 2-dimension IWE, the diagonal line in the 3-dimension IWE can indicate the share of GNP. The size of $\overline{OQ}$ is the share of GNP of country H, which matched world consumption composition. We call it the no-redistributed share of GNP. The size of $\overline{QC}$ is the share of GNP in country H built by trade. We call it the redistributed share of GNP, which also indicates the trade volume in country H.

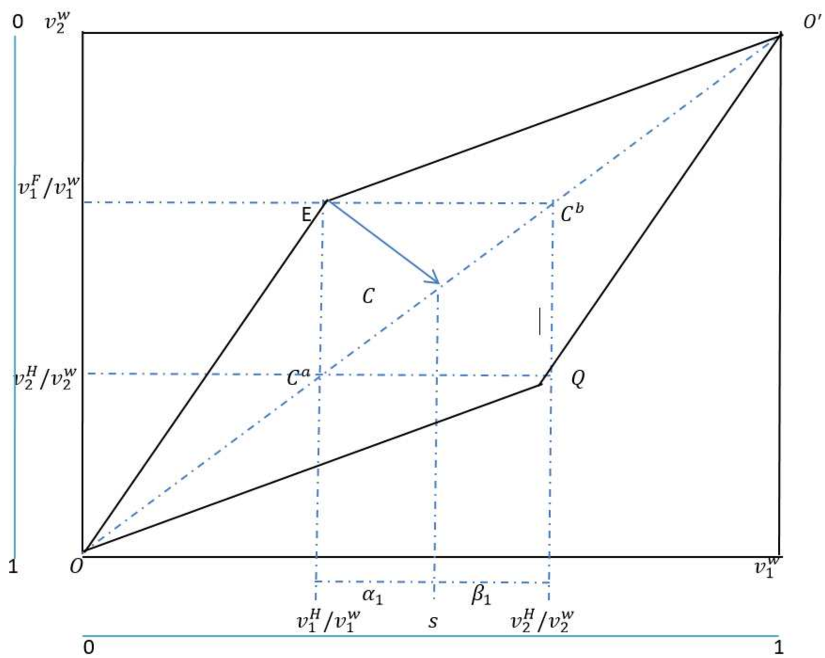

Figure 5 the projection of 3-dimension IWE on the plane by v_1^w and v_2^w

Figure 5 is the projection of the 3-dimension IWE in Figure 2, on the plane by v_1^W and v_2^W ($v_1^W v_2^W$-plane). It shows the allocation of factor endowment 1 and factor endowment 2. The trade box EC^bQC^a is the projection of the trade box in Figure 4. α_1 is redistributable the share of GNP for country H, and β_1 is for country F. α_1 represents the size of consumption built by trade in country H. β_1 is the size of consumption built by trade in country F. For the $2 \times 2 \times 2$ model, the redistributed share of GNP is easier to understand. See it in appendix A.

When α_1 increases, the distributable share of GNP of country H will increase. Also, when β_1 increases, the distributable share of GNP of country F will increase. The analyses will be similar when we project the 3-dimension IWE on $v_2^W v_3^W$-plane and on $v_1^W v_3^W$-plane. Figure 5 only presents the competitive relationship in factor 1 and factor 2. Trade competition is essential to settle the share of GNP under the equilibrium of prices and trade flows.

The lengths of redistributable GNP for country H in three planes ($v_1^W v_2^W$-plane, $v_2^W v_3^W$-plane, and $v_1^W v_3^W$-plane) are

$$\alpha_1 = \left(s^h - \frac{v_1^H}{v_1^W}\right), \quad \alpha_2 = \left(v - \frac{v_2^H}{v_2^W}\right), \quad \alpha_3 = \left(s^h - \frac{v_3^H}{v_3^W}\right) \tag{3-13}$$

The lengths of redistributable GNP for the home countries are

$$\beta_1 = \left(\frac{v_1^H}{v_1^W} - s^h\right), \quad \beta_2 = \left(\frac{v_2^H}{v_2^W} - s^h\right), \quad \beta_3 = \left(\frac{v_3^H}{v_3^W} - s^h\right) \tag{3-14}$$

We propose a utility function for two factors in figure 5 as

$$\mu_1 = \alpha_1 \beta_1 \tag{3-15}$$

It reflects the interests or benefits of both countries by trade. A similar relationship occurs on each surface or plane. There are three surfaces for three factors. We propose a utility function for $3 \times 3 \times 2$ model as

$$\mu = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 \tag{3-16}$$

It reflects the competitive relations of three pairs of factor content of trade together. A country participating in trade tends to obtain better welfare.

Substituting (3-13) and (3-14) into (3-16) yields

$$\mu = \left(\frac{v_1^H}{v_1^W} - s^h\right)\left(s^h - \frac{v_2^H}{v_2^W}\right) + \left(\frac{v_3^H}{v_3^W} - s^h\right)\left(s^h - \frac{v_2^H}{v_2^W}\right) + \left(\frac{v_1^H}{v_1^W} - s^h\right)\left(s^h - \frac{v_3^H}{v_3^W}\right) \tag{3-17}$$

The optimal solution is by the first-order condition

$$\frac{d\mu}{ds} = -2s^h + \left(\frac{v_1^H}{v_1^W} + \frac{v_2^H}{v_2^W}\right) - 2s^h + \left(\frac{v_1^H}{v_1^W} + \frac{v_2^H}{v_2^W}\right) - 2s^h + \left(\frac{v_1^H}{v_1^W} + \frac{v_2^H}{v_2^W}\right) = 0 \tag{3-18}$$

It yields

$$s^h = \frac{1}{3}\left(\frac{v_1^H}{v_1^W} + \frac{v_2^H}{v_2^W} + \frac{v_3^H}{v_3^W}\right) \tag{3-19}$$

Therefore, the optimal competitive share of GNP of country H allocated at the point $s(s^H, s^F)$ in Figure 5, which is the intersection of medians or centroid of the triangle. With this simple competitive solution, both countries reach their maximum values of GNP shares.

For the $2 \times 2 \times 2$ model, the share of GNP of country H, at the equilibrium, fits in the middle of the two boundaries of shares of GNP. There are two explanations of why it is filled in the middle. One is that at this point, the redistributed shares of GNP of the two countries get their maximum value. Another explanation is that trade volume gets its maximum value. Both descriptions still fit for the 3 x 3 x 2 model.

Figure 5 displays the GNP distributions of the two countries by the trade box. The vertical axis is the share of GNP of country foreign, and the horizontal axis is the share of GNP of country H. The triangle ABC shows all possible GNPs of two countries, corresponding to all possible commodity prices within the three-dimension cone described by Tetrahedra OMNR in figure 2. . We call it the GNP redistribution triangle. At any allocation of shares of GNP in the triangle ABC, there is always a relationship,

$$s^H + s^F = 1 \tag{3-20}$$

The point s is the centroid of the triangle i.e. intersection of medians. Its allocation in the triangle is

$$s = (s^H, s^F) \tag{3-21}$$

where

$$s^H = \frac{1}{3}\left(\frac{v_1^H}{v_1^W} + \frac{v_2^H}{v_2^W} + \frac{v_3^H}{v_3^W}\right) \tag{3-22}$$

$$s^F = \frac{1}{3}\left(\frac{v_1^F}{v_1^W} + \frac{v_2^F}{v_2^W} + \frac{v_3^F}{v_3^W}\right) \tag{3-23}$$

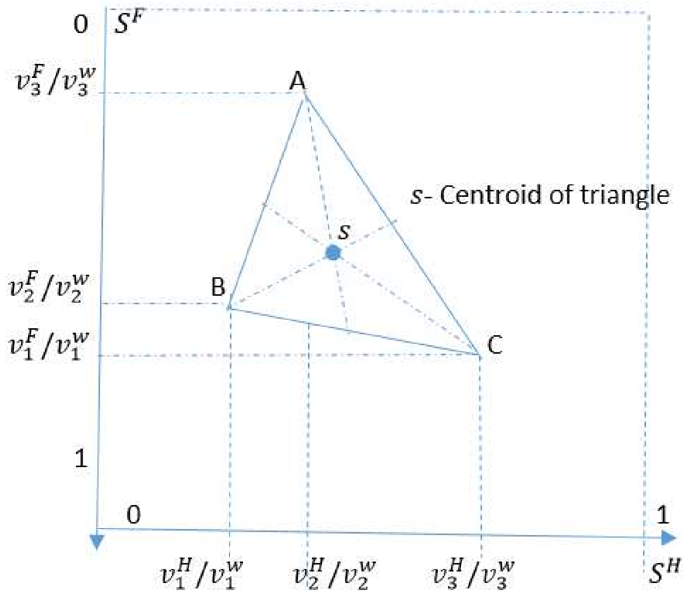

Figure 5 GNP-Share Map

Trades do redistribute welfare. The national welfare is measured by shares of GNP.

3.5 Trade flows and Equalized Factor Prices

Factor content of trade for 3 × 3 × 3 is

$$F^H = V^H - s^H V^W \tag{3-24}$$

Substituting (3-22) into the above, we obtain F^H.
Susituiting F^H into the balance condition of factor content of trade yields

$$w^* \cdot F^H = 0 \tag{3-25}$$

or

$$w^1 F_1^H + w^2 F_2^H + w^3 F_3^H = 0 \tag{3-26}$$

For the two-factor model, we can use trade balance to get a wage-capital ratio as the term of factor content of trade. By equation (3-25), we cannot attain it since there are three unknown variables, even when we know the factor content of trade by (3-24). For higher dimensions (N>=3), the trade balance is one single equation with multiple factors, like (3-25). For N=3, we miss one more condition for the solution of equilibrium. For N=4, we lack two more conditions. For N=n, we miss N-2 conditions. It means that the equilibrium solution is not determined from the mathematical view. No literature explored this issue before.

w^* and $F^H$are orthogonal. They are perpendicular to each other. There exists an infinite number of vectors in 3-dimension that are perpendicular to a fixed or given a vector. However, we can solve for the different variables. If we have one vector than the infinite amount of perpendicular vectors will form a plane that is perpendicular to F^H.

We described that tetrahedra $OMNR$ and tetrahedra $MNRO'$ compose an equal factor price set. It implies that w^* is a function of world factor endowments. That also will reduce the set of the solution for facto prices w^*. It is still quite open to having a unique solution for the equalized factor prices.

4 Integrated World Equilibrium for the $3 \times 3 \times Q$ Model

4.1 Integrated World Equilibrium for the $3 \times 3 \times Q$ *Model*

In a $3 \times 3 \times 2$ system, country H and country F are trade partners with each other. In a multi-country system, who is the trade partner with whom? Leamer (1984, preface page xiii) addressed this issue as “This theorem, in its most general form, states that a country’s trade relations with the rest of the world depend on its endowments of productive factors...”. We suppose that the trade for a country is a transaction of goods between this country and the rest of the world. The trade relationships are pretty simple by this specification. It just likes the scenario of the $3 \times 3 \times 2$ system from the view of analyses.

Let study the trade relationship between country 1 and the rest world. Suppose that the nth factor endowment in the rest of the world is

$$v_n^{RW} = \sum_{h=2}^{Q} v_n^h \qquad (n = 1, 2, 3) \tag{4-1}$$

The world nth factor is

$$v_n^W = v_n^1 + v_n^{RW} \tag{4-2}$$

Substituting into (4-2) for country 1 yields

$$s^1 = \frac{1}{3}\left(\frac{v_1^1}{v_1^W} + \frac{v_2^1}{v_2^W} + \frac{v_3^1}{v_3^W}\right) \tag{4-3}$$

Similarly, we have the share of GNP for country h

$$s^h = \frac{1}{3}\left(\frac{v_1^h}{v_1^W} + \frac{v_2^h}{v_2^W} + \frac{v_3^h}{v_3^W}\right) \qquad (h = 1,2,\ldots,Q) \tag{4-4}$$

4.2 Integrated World Equilibrium for the $N \times M \times Q$ *Model*

For the general case of N factors and Q countries, we can extend the (4-4) to

$$s^h = \frac{1}{3}\left(\frac{v_1^h}{v_1^W} + \frac{v_2^h}{v_2^W} + \cdots + \frac{v_N^h}{v_N^W}\right) \qquad (h = 1,2,\ldots,Q) \tag{4-5}$$

It does not deal with commodity number. It is suitable for any number of commodities.
By (4-5), we can get trade flows for country h by

$$F^h = V^h - s^h V^W \qquad (h = 1,2,\ldots,Q) \tag{4-6}$$

$$X^h = X^h - s^h X^W \qquad (h = 1,2,\ldots,Q) \tag{4-7}$$

Appendix B is the detailed derivation for the trade flows on $\mathrm{N} \times \mathrm{M} \times \mathrm{Q}$ Model. Appendix C is a numerical example for the 4 x 5 x 3 model.

Conclusion

The Heckscher-Ohlin trade theory, in particular, has frequently been criticized for the restriction to the lower dimension presentations. This study provides a new understanding of trade-price relationships for the higher dimensions context.

It shows that the Heckscher-Ohlin model and theories can reflect international trade in the framework of higher dimensions from trade flows, world prices, and gains from trade.

It shows that world trade equilibrium at the higher dimension analyses is much complex than we expected - particularly for the solution of equalized facto prices. This paper illustrates a way to access the share of GNP, trade flows, and factor contents of trade analytically.

Appendix A – The General Trade Equilibrium of Factor Price Equalization for 2 × 2 × 2 Model

We first use the IWE diagram to present a trade box in the IWE diagram to include the price constraint on the trade balance.
The relative world commodity prices $\frac{p_1^*}{p_2^*}$ should lie between the rays of goods price diversification cone (see Fisher, 2011) in algebra as[5],

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{A-1}$$

This condition makes sure that the factor prices by the unit cost equation (2-2) are positive. The boundaries of the share of GNP, s^H, corresponding to the rays of the goods price diversification cone above, can be calculated as

$$s_{max}^H(p) = s\left(\begin{bmatrix} a_{K1}^H \\ a_{K2}^H \end{bmatrix}\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^W} \tag{A-2}$$

$$s_{min}^H(p) = s\left(\begin{bmatrix} a_{L1}^H \\ a_{L2}^H \end{bmatrix}\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^W} \tag{A-3}$$

These compose the range of s^H, which Leamer (1984, p.9) first proposed in another analytical way, as

$$\frac{K^H}{K^W} > s^H > \frac{L^H}{L^W} \tag{A-4}$$

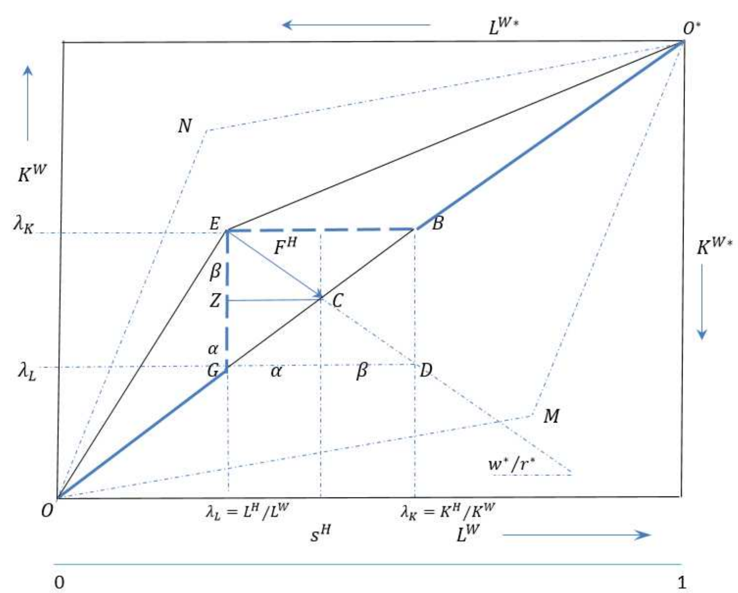

Figure A IWE Diagram with Trade Box

[5] It soppuses that country H is capital in tensive in producing commodity 1.

Figure A is an IWE diagram added with a trade box. The dimensions of the diagram represent world factor endowments. The origin of the home is the lower-left corner, and the foreign country is from the right-upper corner. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by $ONO^{*}M$ is an available allocation of factor endowments of two countries. Helpman and Krugman (1985, p.15) call the parallelogram the FPE set.

Suppose that allocation E describes the distribution of world factor endowments. Country H is capital abundant at this point (we will use this assumption for all analyses of this study). Point C represents the trade equilibrium point. It shows the sizes of the consumption of the two countries.
We identify the trade box by using GNP's range of share in (2-8). If a relative commodity price lies in the price diversification cone (2-5), the share of GNP by the price lies within the trade box $EBDG$.
For a given allocation E, its equilibrium point C needs to fall within the line $\overline{GB}$ on the diagonal line of the trade box, which implies the constraint of the goods price diversification cone.
The share of GNP, s^{H}, divides the trade box into two parts: α and β,

$$\alpha = s^{H} - \lambda_{L} \quad \text{(A-7)}$$

$$\beta = \lambda_{K} - s^{H} \quad \text{(A-8)}$$

When α increases, country H's share of GNP increases, and country F's share of GNP decreases, and vice versa. In trade competitions between countries, each of them wants to take its comparative advantage to export their commodity that used their abundant factor intensively. And each country seeks to maximize the factor price of its abundant factor to achieve its maximum share of GNP of the world. However, only the share of GNP inside the trade box is redistributable by trade. We call α as a redistributable share of GNP for country H, and β is one for country F.
We rewrite the trade balance of factor contents of trade (2-4) as

$$\frac{w^{*}}{r^{*}} = \frac{(\lambda_{K}-s^{H})}{(s^{H}-\lambda_{L})}\frac{K^{W}}{L^{W}} = \frac{\beta}{\alpha}\frac{K^{W}}{L^{W}} \quad \text{(A-9)}$$

where superscript * indicates world price.
Triangle ΔEZC in figure A represents the trade flows of factor contents. Its trade volume is

$$VT = (\lambda_{K} - s^{H})K^{W}r^{*} + (s^{H} - \lambda_{L})L^{W}w^{*} \quad \text{(A-10)}$$

Based on (A-9), suppose

$$w^{*} = (\lambda_{K} - s^{H})K^{W} \quad \text{(A-11)}$$

We then express r^{*} as[6]

$$r^{*} = (s^{H} - \lambda_{L})L^{W} \quad \text{(A-12)}$$

Substituting them to (A-10) yields

$$\mu = 2(\lambda_{K} - s^{H})(s^{H} - \lambda_{L})L^{W}K^{W} \quad \text{(A-13)}$$

It shows that the trade volume VT is a quadratic function of s^{H}. μ reaches its maximum value as $\frac{1}{2}(\lambda_{K} - \lambda_{L})$ when $s^{H} = \frac{1}{2}(\lambda_{K} + \lambda_{L})$.

Appendix B - World Trade Space and Trade Flows for Integrated World Economy

For the model of M factors, N commodities, and Q countries ($N \times M \times Q$), the technology matrix for N commodity and M factor can be expressed as

[6] It actually uses the Walras equilibrium law to drop one market clearing condition.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \tag{B-1}$$

The vector of commodity and the vector of commodity price are the $M \times 1$ vectors as

$$P^w = \begin{bmatrix} p_1^h \\ p_2^h \\ \vdots \\ p_m^h \end{bmatrix}, \quad X^h = \begin{bmatrix} x_1^h \\ x_2^h \\ \vdots \\ x_m^h \end{bmatrix} \qquad h = (1,2,\cdots,\mathrm{q}) \tag{B-2}$$

where h indicates countries.

Factor endowments and factor prices are the $N \times 1$ vectors as

$$V^h = \begin{bmatrix} v_1^h \\ v_2^h \\ \vdots \\ p_n^h \end{bmatrix}, \qquad W^h = \begin{bmatrix} w_1^h \\ w_2^h \\ \vdots \\ w_n^h \end{bmatrix} \qquad h = (1,2,\cdots,\mathrm{q}) \tag{B-3}$$

Production constraint is

$$AX^h = V^h \qquad h = (1,2,\cdots,\mathrm{q}) \tag{B-4}$$

Unit cost function at factor price equalization is

$$A' \ W^* = P^* \tag{B-5}$$

To establish the trade equilibrium, we start at identifying N boundaries of shares of GNP of country h. Denote

$$\theta_1 = \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1m} \end{bmatrix}, \qquad \theta_2 = \begin{bmatrix} a_{21} \\ a_{22} \\ \vdots \\ a_{2m} \end{bmatrix}, \qquad .... \qquad \theta_n = \begin{bmatrix} a_{n1} \\ a_{n2} \\ \vdots \\ a_{nm} \end{bmatrix} \tag{B-6}$$

Substituting them into the definition of the share of GNP like (3-10) for country h yields

$$s_1^h(\theta_1) = \frac{v_1^h}{v_1^W} \ , \quad s_2^h(\theta_2) = \frac{v_2^h}{v_2^W} \ , \ .... \ , \quad s_n^h(\theta_n) = \frac{v_n^h}{v_n^W} \ , \quad h = (1,2,\cdots,q) \tag{B-7}$$

Generalizing the utility function (4-15) on the $N \times M \times Q$ model.

$$\mu = \alpha_1\beta_1 + \alpha_2\beta_2 + \cdots + \alpha_N\beta_N \tag{B-8}$$

Using the first order condition for μ, we can obtain the share of GNP in the following,

$$s^h = \frac{1}{n}\sum_{i=1}^{n}\frac{v_i^H}{v_i^W} \tag{B-8}$$

The trade volume for commodity j for country h is

$$T_j^h = x_j^h - s^h x_j^W = x_j^h - \left(\frac{1}{n}\sum_{i=1}^{n}\frac{v_i^h}{v_i^W}\right)x_j^W \tag{B-14}$$

The factor content of trade for factor j in for country h is

$$F_j^h = v_j^h - s^h v_j^W = v_j^h - \left(\frac{1}{n}\sum_{i=1}^{n}\frac{v_i^h}{v_i^W}\right)v_j^W \tag{B-15}$$

The shares of GNP by (B-8) for all countries are harmony; summing them together equals 1 as

$$\sum_{h=1}^{q}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{v_i^h}{v_i^W}\right) = 1 \tag{B-16}$$

The commodity price (B-12) does not need the assumption that technological matrix A is squared.

Appendix C- Trade Flows for the 4 x 5 x 3 model

Let see a numerical example for the 4 x 5 x 3 model. The identical technology matrix in this example is

$$A = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 & 0.7 \\ 1.1 & 2 & 1.1 & 1.1 & 1.0 \\ 0.8 & 1.1 & 2.1 & 1.0 & 1.2 \\ 1.3 & 1.0 & 0.8 & 1.5 & 1.1 \end{bmatrix}$$

The commodity outputs of three countries by full employment of factor resources are given in advance as

$$X^1 = \begin{bmatrix} 600 \\ 1300 \\ 410 \\ 400 \\ 560 \end{bmatrix}, \qquad X^2 = \begin{bmatrix} 250 \\ 540 \\ 1490 \\ 600 \\ 800 \end{bmatrix}, X^3 = \begin{bmatrix} 900 \\ 600 \\ 500 \\ 1000 \\ 1500 \end{bmatrix}$$

The factor endowments for the three countries correspondingly are

$$\mathrm{V}^1 = \begin{bmatrix} 4655 \\ 4711 \\ 3843 \\ 3624 \end{bmatrix}, \quad \mathrm{V}^2 = \begin{bmatrix} 4435 \\ 4454 \\ 5483 \\ 3837 \end{bmatrix}, \quad \mathrm{V}^3 = \begin{bmatrix} 6020 \\ 5340 \\ 5230 \\ 5320 \end{bmatrix}$$

Calculating the factor price directly from (3-20) through (3-22) yields

$$W^* = \begin{bmatrix} 0.8464 \\ 0.8811 \\ 0.8780 \\ 1 \end{bmatrix}$$

Based on the equalized factor price above, we can obtain the world common commodity price as

$$P^* = \begin{bmatrix} 5.5109 \\ 4.7438 \\ 4.7135 \\ 4.1090 \\ 3.6273 \end{bmatrix}$$

With the prices above, we can calculate the share of GNP of each country as

$$s^1 = 0.2949$$
$$s^2 = 0.3194$$
$$s^3 = 0.3855$$

We can also use (4-6) to calculate the shares of GNP; the results are the same as above.

The exports and factor contents of exports will be

$$T^1 = \begin{bmatrix} 83.77 \\ 583.22 \\ -297.98 \\ -189.98 \\ -238.67 \end{bmatrix}, \quad T^2 = \begin{bmatrix} -308.98 \\ -239.38 \\ 723.40 \\ -38.83 \\ -113.53 \end{bmatrix}, \quad T^3 = \begin{bmatrix} 225.21 \\ -340.85 \\ -425.42 \\ 228.82 \\ 397.21 \end{bmatrix}$$

$$F^1 = \begin{bmatrix} 190.64 \\ 432.16 \\ -450.87 \\ -146.27 \end{bmatrix}, \quad F^2 = \begin{bmatrix} -388.20 \\ -179.14 \\ 833.56 \\ -245.47 \end{bmatrix}. \quad F^3 = \begin{bmatrix} 197.55 \\ -253.01 \\ -382.68 \\ 391.74 \end{bmatrix}$$

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