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Growth and Welfare Effects of Interventions in Patent Licensing Negotiations*

Shin Kishimoto,[†] Keishun Suzuki[‡]

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Abstract

Policy makers sometimes intervene in patent licensing negotiations to guide licensing fees, but the impacts of such interventions on economic growth and welfare are relatively unknown. This paper develops a novel Schumpeterian growth model featuring a cooperative game-theoretic framework that describes negotiations about licensing fees. We find that the growth effect of intervention is negative if firms can raise unlimited external funds for their R&D investment. However, when the amount of external funds available is limited, both the growth and the welfare effects of intervention can be positive. This result means that interventions are desirable when the internal funds of firms are the main source of their R&D investment.

Keywords: Patent licensing negotiations, Schumpeterian growth, Cooperative game, Patent protection, Financial constraints.

JEL Classification: C71, D45, O30.

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1 Introduction

Patent licensing fees are often determined through private negotiations between the relevant firms, but policy makers and some organizations can indirectly influence such fees through interventions. For example, competition authorities sometimes sue firms that charge unfair and anti-competitive licensing fees.¹ Similarly, standard-setting organizations request that firms that hold standard-essential patents in an industry license them under fair, reasonable, and nondiscriminatory (FRAND) conditions. We examine the impacts of such an (indirect) intervention on economic growth and welfare.

Patent licensing is an important way for patent holders to benefit from their inventions; in other words, licensing provides an incentive for potential innovators to innovate further. From this perspective, the literature on Schumpeterian growth theory has recently emphasized the role of patent licensing in economic growth (e.g., [O'donoghue and Zweimüller, 2004](#); [Chu, 2009](#); [Chu et al., 2012](#); [Chu and Pan, 2013](#); [Niwa, 2016](#); [Niwa, 2018](#); [Yang, 2018](#); [Suzuki, 2020](#)). However, most of these previous studies give little attention to the process by which the licensing fee is determined.² They simply assume that the licensing fee is an exogenous fraction of the licensee's profit. Thus, in the above literature, licensing fees are often regarded as a policy variable with the assumption that policy makers can directly control licensing fees.

We develop a novel Schumpeterian growth model in which licensing fees are endogenously determined as a result of a cooperative game representing the licensing negotiation. To the best of our knowledge, this is the first study to consider a synthesis of cooperative game theory and Schumpeterian growth theory. There are two advantages to formulating the patent licensing negotiation as a cooperative game. First, we can describe a situation in which the relative bargaining power between firms is determined by the market structure and affects the licensing fee, as shown in some empirical studies (e.g., [Sakakibara, 2010](#)). As mentioned above, most of the literature on endogenous growth theory has not considered the interactions between the relevant firms in the negotiations on the licensing fee. In this sense, our framework provides a microfoundation for licensing fees. Second, we can investigate licensing fees determined through patent licensing negotiations in which policy makers moderately intervene. Most of the literature analyzes the direct control of the licensing fee (i.e., compulsory interventions), but we consider a milder intervention often implemented in reality. Therefore, our framework complements the existing studies.

This paper compares two solution concepts from cooperative game theory to investigate the effects of interventions in patent licensing negotiations on economic growth and welfare.

¹As a recent example, the Federal Trade Commission in the United States has insisted that Qualcomm has illegally charged an excessive licensing fee in the market for smartphone chips.

²An exception is [Suzuki \(2020\)](#), which incorporates an innovator's voluntary patent licensing into a Schumpeterian growth model. In this model, unlike in previous studies, the licensing fee is endogenously determined by the innovator's maximization problem. He assumes a take-it-or-leave-it licensing offer; i.e., the patent holder first announces a licensing fee, and then other firms decide whether to become licensees. However, in actual patent licensing contracts, the innovator would not be able to unilaterally set the licensing fee because firms often negotiate to license the patent. Our paper complements this study by describing such a negotiation process.

The first solution is the bargaining set. This solution is defined based on the incentives for negotiating firms to gain more profits. Thus, the bargaining set provides a licensing fee that is determined through negotiation in the case of no intervention. The second is the Aumann-Drèze value, which provides the licensing fee in the case of intervention. The Aumann-Drèze value is an extension of the Shapley value and generates a fair profit division based on the contribution of the negotiating firms to their total profit. As noted above, policy makers are concerned about whether licensing fees are fair. Therefore, we analyze a licensing fee in the case of intervention by adopting the Aumann-Drèze value, which embeds a concept of fairness.³

Our main findings are summarized as follows. The negotiation without intervention yields an exploitative licensing fee, but the intervention decreases the licensing fee. The growth effect of the intervention is negative when firms can raise unlimited external funds for their R&D investment. However, the growth effect becomes positive when the amount of external funds available is limited. Furthermore, our quantitative analysis shows that the intervention can increase welfare when external funds are less available and patent protection is sufficiently strong. These results mean that the intervention can increase both growth and welfare when the internal funds of firms are the primary source of their R&D investment in an economy with strong patent protection.

Related literature

There are three strands in the theoretical literature on patent licensing: (i) endogenous growth theory, (ii) North-South growth theory, and (iii) game theory.

First, in endogenous growth theory, most of the literature considers a situation in which the new innovator becomes a licensee of past innovators to avoid infringement on their patents and is the exclusive user of the new patented technology (e.g., [O'donoghue and Zweimüller, 2004](#); [Chu, 2009](#); [Chu et al., 2012](#); [Chu and Pan, 2013](#); [Niwa, 2016](#); [Niwa, 2018](#); [Yang, 2018](#)). In contrast, our model considers a different situation in which not only the new innovator but also other firms utilize the new technology through patent licensing. Therefore, we complement the above literature.

Second, some studies in North-South growth theory models international patent licensing as a random process (e.g., [Grossman and Helpman \(1991, Ch.7\)](#); [Yang and Maskus \(2001, 2009\)](#); [Tanaka et al. \(2007\)](#)). For example, in [Yang and Maskus \(2001\)](#), a Northern innovator decides how much effort to devote to licensing his/her patent to Southern firms. The probability of successful licensing is endogenously determined in the steady-state equilibrium. Similar to the literature on endogenous growth theory, these studies assume the exogenous profit division rule between the Northern innovator and the Southern licensee. As an exception, [Yang and Maskus \(2009\)](#) consider Nash bargaining, which determines the shares of joint surplus from sales under licensing. However, in their study, because exogenous bargaining power directly determines the shares in Nash bargaining, in practice they also consider the profit division to

³[Layne-Farrar et al. \(2007\)](#) also use the Shapley value as a benchmark for FRAND conditions in standard-setting organizations.

be exogenous.

Third, as an application of game theory, patent licensing has been extensively studied since the seminal paper of [Kamien and Tauman \(1986\)](#). To investigate the innovator's optimal licensing contracts, which are endogenously determined, subsequent studies often formulate take-it-or-leave-it offers for licensing contracts as noncooperative games (e.g., [Muto, 1993](#); [Sen and Tauman, 2007](#); [Chen, 2017](#)). As noted above, on the other hand, licensing contracts can be determined through negotiations. From this practical viewpoint, several studies have formulated negotiations about licensing fees as cooperative games (e.g., [Tauman and Watanabe, 2007](#); [Watanabe and Muto, 2008](#)). In particular, [Kishimoto et al. \(2011\)](#) apply the model of [Watanabe and Muto \(2008\)](#) to licensing a cost-reducing innovation in a large Cournot market and show that the fair profit division represented by the Aumann-Drèze value is not necessarily one of the bargaining outcomes realized in the bargaining set. In this paper, by regarding the Aumann-Drèze value as a policy maker's intervention, we shed light on the relationship between the two solution concepts from a macroeconomic viewpoint.

Roadmap

This paper is structured as follows. Section 2 introduces our model. Section 3 considers a licensing negotiation without intervention, derives the licensing fee in the bargaining set, and solves for the steady state. Section 4 derives a fair licensing fee by applying the Aumann-Drèze value and solves for the steady state. Section 5 investigates the growth and welfare effects of the intervention in the licensing negotiation. Sections 3 to 5 assume that the R&D investment is not financially constrained. Section 6 introduces some financial constraints and reinvestigates the growth and welfare effects of the intervention. Section 7 concludes the paper.

2 A model with licensing negotiation

In this section, we build a baseline model that considers patent licensing through negotiations. Our model is based on [Grossman and Helpman \(1991, Ch.4\)](#). We extend their model by (a) introducing negotiations on licensing fees, (b) considering the R&D activities of the licensees instead of potential firms, and (c) assuming Cournot competition instead of Bertrand competition.⁴

2.1 Households

The household setup is the same as in [Grossman and Helpman \(1991, Ch.4\)](#). We consider an economy consisting of L identical and infinitely lived households. Time is continuous. There is no population growth. Each household inelastically supplies one unit of labor and earns a wage in every period. The representative household has the following intertemporal utility

⁴Under Bertrand competition, the profit of the licensees becomes zero because their goods are perfect substitutes, so they cannot pay the licensing fee to the innovator. Therefore, we consider Cournot competition so that the licensees can earn a positive profit.

function:

$$U_t = \int_0^\infty \exp(-\rho t) \ln c_t dt, \quad (1)$$

where ρ is the subjective discount rate and c_t is an index for consumption at time t . The economy has a continuum of industries indexed by $i \in [0, 1]$, and households consume final goods from across all industries. Period utility is defined as

$$\ln c_t = \int_0^1 \ln \left(\sum_{k=0}^{\tilde{k}(i)} \lambda^k x_{kt}(i) \right) di, \quad (2)$$

where $x_{kt}(i)$ is the consumption of a good, the quality of which is λ^k in industry i at time t , and $\tilde{k}(i)$ means that there are $\tilde{k}(i) + 1$ generations of the good in industry i . According to the additive specification in the abovementioned period utility function, $\tilde{k}(i) + 1$ generations of the good are perfect substitutes for households. The quality of each good is represented by the k -th power of $\lambda > 1$ ($k = 0, 1, \dots, \tilde{k}(i)$), which means that the quality of the new good is λ times higher than that of the previous good.

The budget constraint for each household is

$$\dot{a}_t = r_t a_t + w_t - e_t.$$

a_t is the real value of assets (equities), and r_t is the real interest rate. w_t is the wage rate. e_t is per capita expenditure, which is given by

$$e_t = \int_0^1 \left(\sum_{k=0}^{\tilde{k}(i)} p_{kt}(i) x_{kt}(i) \right) di,$$

where $p_{kt}(i)$ is the price of the good with quality λ^k .

We solve the utility maximization problem in two parts: the static problem and the dynamic problem. First, given instantaneous expenditure level e_t , we maximize the period utility function $\ln c_t$. Given the logarithmic utility function, households spend their budget equally across product lines $i \in [0, 1]$. Moreover, for each product line, they choose the latest good that has the lowest quality-adjusted price. Therefore, individual demand in industry i is $x_{\tilde{k}(i)t}(i) = e_t / p_{\tilde{k}(i)t}(i)$ and $x_{kt}(i) = 0$ for $k = 0, 1, \dots, \tilde{k}(i) - 1$.

Second, we solve the dynamic maximization problem. Each household decides its expenditure E_t in each period to maximize its intertemporal utility function, U_t , subject to the intertemporal budget constraint. The household's indirect period utility function is given by $\ln c_t = \ln e_t - \ln P_t$, where P_t is the ideal price index associated with the consumption index c_t , which is defined as

$$\ln P_t = \int_0^1 \ln \left(\frac{p_{\tilde{k}(i)t}(i)}{\lambda^{\tilde{k}(i)}} \right) di.$$

Given the aggregate price index, households spend to maximize their intertemporal utility.

From the maximization result, the household's optimal time path for spending is represented by $\dot{e}_t/e_t = r_t - \rho$. Following [Grossman and Helpman \(1991, Ch.4\)](#) and many subsequent studies, we treat aggregate expenditure $E_t \equiv e_t L$ as the *numéraire* by normalizing the price index in each period so that $E_t = P_t \cdot c_t L = 1$. This means that the price index falls over time at a rate equal to the growth rate of the consumption index. From this, we obtain $e_t = 1/L$ and $r_t = \rho$. Hereafter, the notation omits t and i in cases in which there is no risk of misunderstanding.

2.2 Industries

Consider an industry in which there is an innovator who holds a patent for a state-of-the-art good and potential imitators who can copy the innovator's good. Following [Goh and Olivier \(2002\)](#), we assume that the unit cost of producing an imitative good is increasing in patent breadth. Specifically, while the innovator can produce one state-of-the-art good by devoting one unit of labor to its production, the potential imitators must employ $\chi > 1$ units of labor to produce one unit of the same quality good. The cost disadvantage $\chi \in (1, \lambda)$ can be interpreted as the degree of patent breadth, which is the extent to which patent holders can legally prevent imitators from copying their patented technologies. For example, if the patent breadth is as narrow as possible ($\chi = 1$), potential imitators can perfectly imitate the production technology of the innovator.

Patent licensing through bargaining

The innovator can license his/her own patent to potential imitators. Firms that are licensed the innovator's patent (licensees) can use the innovator's production technology. Hereafter, the innovator and licensees are often referred to as technology holders.

Let the innovator be denoted by I , and let S be the set of ℓ licensees (i.e., $|S| = \ell$).⁵ The innovator and all firms in S negotiate how much the licensees will pay to the innovator as licensing fees. In the industry, infinite potential imitators exist during licensing negotiations. We formulate the negotiation on the assumption that m potential imitators exist, where $0 < m < \infty$, and by taking the limit $m \rightarrow \infty$, we specify the negotiation outcomes for the case in which infinite imitators exist.

Let $F \geq 0$ be the licensing fee paid by each licensee to the innovator in each period. Let $\bar{\pi}_{Th}$ be the gross profit that a technology holder gains from production. Then, the net profit of the innovator is $\pi_I \equiv \bar{\pi}_{Th} + \ell F$, while the net profit of each licensee is $\pi_\ell \equiv \bar{\pi}_{Th} - F$. We also use the ratio of the licensing fee to the licensee's (gross) profit, which is defined as $f \equiv F / \bar{\pi}_{Th}$.

After the licensing negotiation, the innovator, ℓ licensees, and infinite imitators engage in Cournot competition, with each knowing which firms are licensed or not. Following the literature on patent licensing through bargaining, we assume that cartels are not allowed by the competition authority in the product market.

⁵For any finite set N , $|N|$ means the cardinality of N .

R&D

The licensees can perform R&D to create a good with quality λ times higher than that of the previous good. Following [Grossman and Helpman \(1991, Ch.4\)](#), we assume that the success of the R&D investment follows a Poisson process and that its technology is linear. Specifically, if a licensee hires z researchers, he/she performs successful R&D with a small probability θz , where $\theta > 0$ is the parameter of R&D efficiency.

Note that the innovator does not perform R&D because of the Arrow replacement effect, as in [Grossman and Helpman \(1991, Ch.4\)](#) and subsequent studies. Even if the current innovator succeeds in performing R&D, his/her firm's value does not increase because the latest good is instantaneously imitated by the other firms. Empirically, the incentive to innovate tends to be weak for the innovator, even though he/she has the ability to innovate. For example, [Igami \(2017\)](#) shows that successful incumbents in the hard disk drive industry are reluctant to innovate even though they have a substantial cost advantage.

Furthermore, we assume that nonlicensees do not perform R&D activities. This assumption can be justified by assuming that the research productivity of licensees is sufficiently higher than that of nonlicensees. Empirically, [Wang et al. \(2013\)](#) find a "learning-by-licensing" process among licensees by showing that licensees perform better in subsequent innovation than nonlicensees do.

The schedule

Suppose that one of the licensees in an industry conducts successful R&D and becomes a new innovator. After the new innovator obtains a patent on the latest good, each firm in the industry behaves in accordance with the following procedure.

1. The innovator chooses the number of firms that join the licensing negotiation.⁶
2. The licensing fee F is endogenously determined through the negotiation.
3. The innovator, licensees, and imitators engage in Cournot competition.
4. Each licensee pays the licensing fee F to the innovator.
5. The licensees conduct their own R&D.
6. If a licensee conducts successful R&D, he/she becomes the new innovator (and the process begins again with step 1). Otherwise, replacement does not occur in this period (and the process begins again with step 3).

We assume that the number of licensees chosen by the new innovator is the same as that chosen by the previous innovator. Further, to avoid complexity, we assume that the new

⁶The patent holders have the statutory right to exclude others from making or selling their invention. Therefore, they can determine whether they license their patents to certain firms.

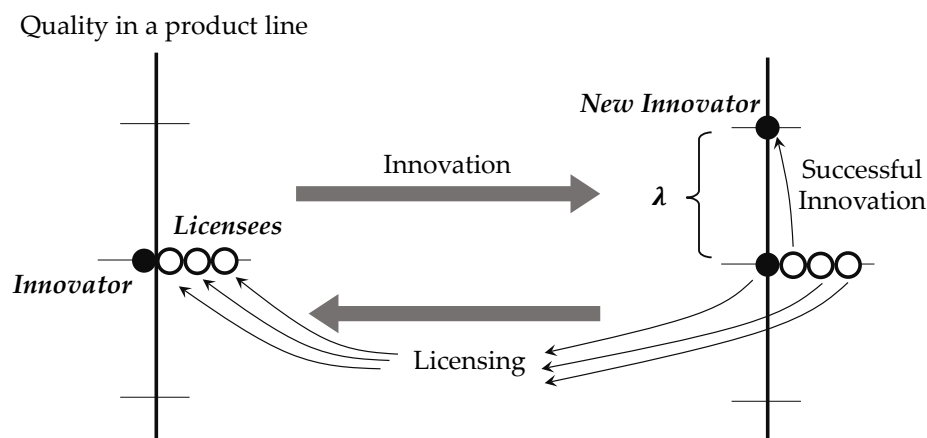


Figure 1: The model structure. The quality ladder on the left-hand side illustrates a situation in which the old technology holders are licensed through negotiation. As illustrated on the right-hand side, after one of the licensees conducts successful R&D, that licensee becomes the new innovator, who has the patent for the latest good, the quality of which is higher than that of the previous good. Then, through a new negotiation, the old innovator and old licensees are licensed. Note that this figure does not mean that the model has two different states because negotiation starts and ends instantaneously. In this model, there is no operating imitator in equilibrium, as we will see later.

innovator licenses his/her patent to the other old technology holders. In other words, we consider a stationary situation in which the group of firms holding the latest technology is the same over time.⁷ Figure 1 briefly illustrates this industrial structure.

3 The outcomes under no intervention in the licensing negotiation

This section considers a licensing negotiation in which policy makers do not intervene and in which R&D investment is assumed to not be financially constrained. The model with this assumption is referred to as the unconstrained economy.

3.1 Static outcomes in an industry

3.1.1 Cournot competition

Consider an industry in which an innovator, ℓ licensees, and m imitators produce the same good and engage in Cournot competition. Let M be a set of m imitators. (Note that the set of all firms in this industry is $\{I\} \cup S \cup M$.) Let y_j be firm j 's production level, and let $y \equiv (y_j)_{j \in \{I\} \cup S \cup M}$ be the production vector. As derived in Subsection 2.1, the inverse demand function for the latest

⁷This assumption seems to be realistic when the innovation is not very drastic. If the innovation is very drastic (e.g., the birth of general purpose technologies), the old technologies are rapidly made obsolete, and the old technologies holders are quickly excluded from the market. However, if innovation is incremental, firm exit and entry are relatively mild. Thus, our quality-ladder setup models the latter case.

good in an industry is $p(X) = 1/X$, where $X \equiv xL$ is the aggregate demand in the industry. Given the inverse demand function and the wage rate w of one unit of labor, firm j maximizes its gross profit given by

$$\bar{\pi}_j(y) \equiv \frac{1}{X}y_j - \gamma_j w y_j,$$

where γ_j is the unit cost of firm production. Note that $\gamma_j = 1$ when firm j is a technology holder (the innovator or a licensee) and $\gamma_j = \chi (> 1)$ when it is an imitator. The market clearing condition is

$$X = \sum_{j \in \{I\} \cup \text{SUM}} y_j \equiv Y,$$

where Y is the aggregate output of the industry. By the first-order condition for firm j 's profit maximization, we obtain the output of firm j as follows:

$$\begin{aligned} \frac{\partial \bar{\pi}_j}{\partial y_j}(y) = 0 &\Leftrightarrow \frac{1}{X} - \frac{1}{X^2}y_j - \gamma_j w = 0 \\ &\Leftrightarrow y_j = X - X^2 \gamma_j w. \end{aligned} \quad (3)$$

By this equation and the market clearing condition, we can derive the industry's aggregate output in Cournot equilibrium as follows:

$$Y(\ell, m) = \frac{\ell + m}{1 + \ell + m\chi} \frac{1}{w}. \quad (4)$$

Then, by $p = 1/Y$, the Cournot equilibrium price $p(\ell, m)$ is

$$p(\ell, m) = \frac{1 + \ell + m\chi}{\ell + m} w.$$

Let $\ell^* \equiv 1/(\chi - 1)$. If $\ell > \ell^*$, then $p(\ell, m) < \chi w$; that is, the Cournot equilibrium price is less than the unit cost of the imitators. Thus, all imitators are excluded from production (i.e., the Cournot equilibrium output of each imitator is zero).

Let $y_{Th}(\ell, m)$ and $y_{Im}(\ell, m)$ denote the Cournot equilibrium outputs of a technology holder (the innovator or a licensee) and an imitator, respectively. Recall that $\gamma_j = 1$ if firm j is a technology holder and $\gamma_j = \chi$ if it is an imitator. Then, by (3) and (4), we have

$$y_{Th}(\ell, m) = \begin{cases} \frac{(\ell+m)[1+m(\chi-1)]}{(1+\ell+m\chi)^2} \left(\frac{1}{w}\right) & \text{if } \ell < \ell^* \\ \frac{\ell}{(1+\ell)^2} \left(\frac{1}{w}\right) & \text{if } \ell \geq \ell^*, \end{cases}$$

and

$$y_{Im}(\ell, m) = \begin{cases} \frac{(\ell+m)[1-\ell(\chi-1)]}{(1+\ell+m\chi)^2} \left(\frac{1}{w}\right) & \text{if } \ell < \ell^* \\ 0 & \text{if } \ell \geq \ell^*. \end{cases}$$

Thus, the (gross) Cournot equilibrium profits of each technology holder and each imitator are

$$\bar{\pi}_{Th}(\ell, m) = \begin{cases} \left(1 - \frac{\ell+m}{1+\ell+m\chi}\right)^2 & \text{if } \ell < \ell^* \\ \left(\frac{1}{1+\ell}\right)^2 & \text{if } \ell \geq \ell^*, \end{cases} \quad (5)$$

and

$$\bar{\pi}_{Im}(\ell, m) = \begin{cases} \left(\frac{1-\ell(\chi-1)}{1+\ell+m\chi}\right)^2 & \text{if } \ell < \ell^* \\ 0 & \text{if } \ell \geq \ell^*, \end{cases} \quad (6)$$

respectively.

3.1.2 Bargaining outcomes for patent licensing under the bargaining set

We analyze the patent licensing negotiation by the following procedure. First, according to [Watanabe and Muto \(2008\)](#), on the assumption that the number of potential imitators m is finite, we formulate the negotiation as a cooperative game with a coalition structure. Second, we introduce the bargaining set for a coalition structure, which represents the bargaining outcomes under no intervention. Finally, we derive the bargaining outcomes in patent licensing for the case in which the number of potential imitators increases to infinity ($m \rightarrow \infty$) because infinitely many imitators exist within the industry.

Formulation as a cooperative game

We formalize patent licensing through bargaining as a cooperative game with a coalition structure. Recall that the innovator is denoted by I , and let S and M be the sets of ℓ licensees and m potential imitators, respectively (i.e., $|S| = \ell$ and $|M| = m$). The set of all players in this cooperative game is $\{I\} \cup S \cup M$. In the following argument, let $n \equiv \ell + m$ for notational ease.

A nonempty subset of $\{I\} \cup S \cup M$ is called a coalition. The imitators, who do not belong to $\{I\} \cup S$, cannot participate in the licensing negotiation but play a relevant role in determining the outside options of the firms in $\{I\} \cup S$. Furthermore, each firm in $\{I\} \cup S$ should claim credible outside options in the negotiation process. Therefore, for each coalition, we need to provide the worth of the coalition, which is the total profit that the firms belonging to the coalition can guarantee for themselves in the worst case.⁸ We define the worth of each coalition, which is generally called the characteristic function, as the sum of the (gross) Cournot equilibrium profits of the firms in the coalition. Then, the characteristic function $v : 2^{\{I\} \cup S \cup M} \rightarrow \mathbb{R}$ is given by

$$\begin{aligned} v(\emptyset) &= 0, \\ v(\{I\} \cup T) &= (1+t)\bar{\pi}_{Th}(t, n-t), \\ v(T) &= t\bar{\pi}_{Im}(n-t, t) \end{aligned}$$

⁸According to [von Neumann and Morgenstern \(1944\)](#), the worth of each coalition is defined from a pessimistic viewpoint. This definition does not play an important role in deriving our propositions.

for each $T \subseteq S \cup M$ with $t = |T|$ ($t = 0$ if $T = \emptyset$). When the innovator stops licensing to the firms in S and the patented technology is transferred to the firms in T ($\subseteq S \cup M$), the other $n - t$ firms are not licensed, and the total (gross) Cournot equilibrium profit of the innovator and the new licensees in T is $(1 + t)\bar{\pi}_{Th}(t, n - t)$; thus, $v(\{I\} \cup T) = (1 + t)\bar{\pi}_{Th}(t, n - t)$. $v(T)$ is the total (gross) Cournot equilibrium profit that the firms in T can guarantee for themselves in the worst case. When some (or all) firms in S jointly break off the negotiation and form a coalition T that does not include the innovator, the worst case in our model is that the firms in T become imitators and all the other $n - t$ firms are licensed. Thus, $v(T) = t\bar{\pi}_{Im}(n - t, t)$.⁹

For a set of licensees S , the permissible coalition structure is denoted by $P(S) \equiv \{\{I\} \cup S\} \cup \{\{j\} | j \in M\}$. The coalition structure $P(S)$ means that firms in $\{I\} \cup S$ can communicate with one another, but imitators in M are not allowed to communicate with any firms. $(\{I\} \cup S \cup M, v, P(S))$ denotes a cooperative game with coalition structure $P(S)$, which we sometimes call a cooperative game if no confusion could arise.

Bargaining outcomes for patent licensing under the bargaining set

We consider the bargaining set of the cooperative game $(\{I\} \cup S \cup M, v, P(S))$.¹⁰ In what follows, we consider only a nonempty set of licensees S (i.e., $S \neq \emptyset$) and fix it because the outcome is uniquely determined if $S = \emptyset$ (i.e., no licensing occurs).

Let $\pi \equiv (\pi_I, (\pi_i)_{i \in S}, (\pi_j)_{j \in M})$, which means a (net) profit vector. Then, the set of imputations under the coalition structure $P(S)$ is defined as

$$\Pi(S) \equiv \left\{ \pi \in \mathbb{R}^{n+1} \left| \begin{array}{l} \pi_I + \sum_{i \in S} \pi_i = (1 + \ell)\bar{\pi}_{Th}(\ell, m), \pi_I \geq \bar{\pi}_{Th}(0, n), \\ \pi_i \geq \bar{\pi}_{Im}(n - 1, 1) \forall i \in S, \text{ and } \pi_j = \bar{\pi}_{Im}(\ell, m) \forall j \in M \end{array} \right. \right\}$$

$\Pi(S)$ implies the set of outcomes for patent licensing through negotiation among the innovator and ℓ licensees. Firms in $\{I\} \cup S$ divide the total (gross) Cournot equilibrium profit of the innovator and ℓ licensees with each $i \in \{I\} \cup S$ being guaranteed the worst payoff $v(\{i\})$. Each imitator in M obtains the (gross) equilibrium profit $\bar{\pi}_{Im}(\ell, m)$ because ℓ firms are licensed.

Let $i, j \in \{I\} \cup S$ and $\pi \in \Pi(S)$. We say that i has an objection (π', T) against j at π if $i \in T$, $j \notin T$, $T \subseteq \{I\} \cup S \cup M$, $\pi'_k > \pi_k$ for any $k \in T$ and $\sum_{k \in T} \pi'_k \leq v(T)$ and that j has a counter-objection (π'', R) to i 's objection (π', T) if $j \in R$, $i \notin R$, $R \subseteq \{I\} \cup S \cup M$, $\pi''_k \geq \pi_k$ for any $k \in R$, $\pi''_k \geq \pi'_k$ for any $k \in R \cap T$ and $\sum_{k \in R} \pi''_k \leq v(R)$. Furthermore, we say that i has a justified objection (π', T) at π if there exists no counter-objection to i 's objection (π', T) . Then, the bargaining set for the coalition structure $P(S)$ is defined as

$$\mathcal{B}^S \equiv \{\pi \in \Pi(S) \mid \text{no firm in } \{I\} \cup S \text{ has a justified objection at } \pi\}.$$

⁹Actually, $v(T)$ is redundant for each $T \subseteq M$ because no firm in such a coalition participates in the negotiation. By convention, however, we define the worth of all coalitions.

¹⁰In cooperative game theory, one of the standard solution concepts is the core. In our model, however, the core for the coalition structure $P(S)$ is empty if $m \geq 1$. [Watanabe and Muto \(2008\)](#) also show the emptiness of the core in their model, similar to ours. In general, for each coalition structure, the core is included in the bargaining set; thus, we use the bargaining set.

The imputations that belong to \mathcal{B}^S are regarded as stable bargaining outcomes because there exists no firm in $\{I\} \cup S$ that has a justified objection to them.¹¹ Note that the coalitions proposed in objections (or counter-objections) do not actually form because coalition $\{I\} \cup S$ eventually forms; that is, in the negotiation, each firm in $\{I\} \cup S$ makes objections (or counter-objections) against another firm via a coalition T (or R) that is feasible but does not actually form.

We analyze the bargaining set for the coalition structure $P(S)$ under the condition that the number of potential imitators is infinitely large ($m \rightarrow \infty$). For notational ease, let $\bar{\pi}_{Th}(\ell)$ denote the gross Cournot equilibrium profit of each technology holder for the case in which ℓ firms are licensed and infinitely many potential imitators exist; that is, $\bar{\pi}_{Th}(\ell) \equiv \lim_{m \rightarrow \infty} \bar{\pi}_{Th}(\ell, m)$. Then, the next proposition holds.

Proposition 1. *Consider the case in which an infinite number of potential imitators exist, and let $F^B(\ell)$ be the licensing fee that corresponds to the imputations in the bargaining set for coalition structure $P(S)$, where $|S| = \ell$. Then, for each number of licensees ℓ , $F^B(\ell) = \bar{\pi}_{Th}(\ell)$, and the optimal number of licensees for the innovator is ℓ^* ; that is, the innovator licenses to ℓ^* firms, and each licensee pays its entire profit as the fee.*

Proof. See Appendix. □

In the patent licensing negotiation based on the bargaining set, the ratio of the licensing fee to (gross) profit is $f^B \equiv F^B(\ell) / \bar{\pi}_{Th}(\ell) = 1$ for each number of licensees ℓ . This means that the innovator can extract the entire profit of all licensees in the bargaining outcomes. Intuitively, when the number of imitators becomes infinitely large, it is harder for each licensee in S to make objections and counter-objections against the innovator because the nonlicensees' (gross) profits tend to be zero. In other words, the bargaining power of each licensee in S decreases as the number of potential imitators increases. As a result, the net profits of the innovator and each licensee are $\pi_I^B(\ell) = (1 + \ell)\bar{\pi}_{Th}(\ell)$ and $\pi_\ell^B(\ell) = 0$, respectively, and the innovator's net profit is maximized when $\ell = \ell^*$.

3.2 Dynamic outcomes in an industry and general equilibrium

Bellman equations

Let V_I and V_ℓ be the innovator's firm value and the licensee's firm value, respectively. We assume that there is a perfectly risk-free asset market and that the interest rate on the safe asset at period t is r_t . For notational convenience, we define the aggregate R&D intensity as $Z \equiv \ell z$.

First, we consider a licensee's maximization problem for z . As a result of the licensing negotiation, a licensee's net profit is zero ($\pi_\ell^B(\ell) = 0$). However, the licensee can raise its firm value to V_I from V_ℓ with a probability of θz by paying zw as its R&D cost. Then, standard

¹¹The bargaining set for a coalition structure is always nonempty, which is shown by [Davis and Maschler \(1967\)](#) and [Peleg \(1967\)](#). Furthermore, for each coalition structure, the bargaining set includes the kernel and the nucleolus, which are regarded as normative solutions based on the complaints of the coalitions about each imputation.

arguments imply that the licensee's firm value satisfies the following Bellman equation:¹²

$$rV_\ell - \dot{V}_\ell = \max_z \{ \theta z(V_I - V_\ell) - zw \}. \quad (7)$$

In the R&D investment decision, the licensee chooses z to maximize the right-hand side (RHS) of (7), taking V_I and w as given.

Second, we consider the innovator's maximization problem in terms of ℓ . The innovator earns a net profit $\pi_I^B(\ell) = (1 + \ell)\bar{\pi}_{Th}(\ell)$ in every period. However, the innovator's stock partially loses its value due to creative destruction (happening at the rate of θZ). The innovator's Bellman equation is

$$rV_I - \dot{V}_I = \max_\ell \left\{ \begin{array}{l} (1 - 1/\chi)^2 \text{ if } \ell = 0 \\ (1 + \ell)\bar{\pi}_{Th}(\ell) - \theta Z(V_I - V_\ell) \text{ if } \ell > 0 \end{array} \right\}. \quad (8)$$

The innovator maximizes the RHS of (8) for ℓ . In the maximization, the innovator takes Z as given when $\ell > 0$.¹³ However, we assume that the innovator knows that $Z = 0$ holds when the innovator chooses $\ell = 0$. We show a parameter condition that implies that the innovator chooses $\ell = \ell^* > 0$ later. Hereafter, we solve for the equilibrium by using this result in advance.

Labor market equilibrium

Labor is devoted to production and R&D. In the labor market equilibrium, aggregate labor demand must be equal to labor supply L . Let y_{Th}^* denote the Cournot equilibrium quantity of each technology holder for the case in which ℓ^* firms are licensed. When ℓ^* firms are licensed, the infinite imitators are driven out of the market. Then, $(1 + \ell^*)y_{Th}^* = 1/(\chi w)$ holds. Hence, the condition for labor market equilibrium is

$$\frac{1}{\chi w} + Z = L. \quad (9)$$

Steady state

In the economy, all variables are constant over time in the steady state. Using $\dot{V}_I = \dot{V}_\ell = 0$, the optimization conditions, and the equilibrium conditions, we solve for the steady state value of aggregate R&D intensity in the economy.

With $\dot{V}_I = 0$, $r = \rho$, and $\ell = \ell^*$, we obtain

$$\dot{V}_I = 0 \Leftrightarrow V_I = \frac{\pi_I^B(\ell^*) + \theta Z V_\ell}{\rho + \theta Z}, \quad (10)$$

where $\pi_I^B(\ell^*) = (1 + \ell^*)\bar{\pi}_{Th}(\ell^*)$.

¹²Recall that a licensee can be licensed again after another licensee succeeds in R&D. Therefore, the licensee's stock does not lose its value even if another licensee succeeds in the R&D.

¹³Since the R&D technology is linear, z is *indeterminate* in the model and is determined in the steady state. Therefore, $Z = \ell z$ is uncontrollable for the innovator.

Next, we consider the steady state value of V_ℓ . Let z^* be the optimal value in (7). If $\theta(V_I - V_\ell) > w$, then z^* goes to infinity. However, in the steady state, because V_I converges to V_ℓ when $z^* \rightarrow \infty$ by (10), the case of $\theta(V_I - V_\ell) > w$ never happens as long as the wage is strictly positive; thus, $\theta(V_I - V_\ell) \leq w$ holds. If $\theta(V_I - V_\ell) < w$, then $z^* = 0$, which is trivial, so we focus on the case of

$$\theta(V_I - V_\ell) = w. \quad (11)$$

Then, by $\pi_\ell^B(\ell^*) = 0$, (7), and (11), we obtain

$$\dot{V}_\ell = 0 \Leftrightarrow V_\ell = 0. \quad (12)$$

The steady state is unique, as in [Grossman and Helpman \(1991, Ch.4\)](#). Since there are only jumpable variables in the dynamics, there are no transitional dynamics in the model. Hence, we obtain the following proposition.

Proposition 2. *Suppose that policy makers do not intervene in licensing negotiations. Then, the economy immediately jumps to the unique steady state at $t = 0$ and remains there forever.*

Proof. See Appendix. □

Let us solve for aggregate R&D intensity in the steady state. Substituting (5), (12), and $\ell^* = 1/(\chi - 1)$ into (10), we obtain

$$V_I = \frac{1 - 1/\chi}{\rho + \theta Z}. \quad (13)$$

By (9), (11), and (12), we obtain

$$V_I = \frac{1}{\theta\chi(L - Z)}. \quad (14)$$

By (13) and (14), we obtain aggregate R&D intensity under the bargaining set in the unconstrained economy as follows:

$$Z^* = \frac{(\chi - 1)L - \rho/\theta}{\chi} \equiv Z_U^B. \quad (15)$$

Optimal number of licensees

We consider the optimal number of licensees from the dynamic viewpoint of the innovator. In Proposition 1, we already showed that ℓ^* is statically optimal. However, in the long run, an innovator faces a trade-off between licensing revenue and the risk of replacement caused by a licensee's further innovation. Therefore, dynamically, the innovator may have an incentive to choose a smaller ℓ . We show a parameter condition that ensures that ℓ^* is also dynamically optimal for the innovator.

Proposition 3.

$$\frac{(\lambda - 1)^2}{2\lambda - 1} < \frac{\rho}{\theta L} \quad (16)$$

is a sufficient condition for $\ell^* \equiv 1/(\chi - 1)$ to be dynamically optimal for the innovator.

Proof. See Appendix. □

Parameter condition (16) is likely to be satisfied when the risk of further innovation is small (small θ) or the innovator is sufficiently myopic (large ρ).

4 The outcomes under intervention in the licensing negotiation

In the previous section, we considered the bargaining set as the solution concept in the patent licensing negotiation. The bargaining set can be interpreted as representing the negotiation outcomes based on the firms' own incentives without any intervention from the outside. As discussed in Section 3, such laissez-faire negotiation yields an extremely unfair licensing fee ($f^B = 1$) because the innovator's bargaining power is strong. In reality, as stated in Section 1, policy makers may intervene in patent licensing negotiations to correct unfair licensing fees. Therefore, this section considers a situation in which policy makers intervene in the patent licensing negotiations between the innovator and ℓ^* licensees.

In this section, we consider profit division according to the Aumann-Drèze value, which is an extension of the Shapley value for cooperative games with coalition structures. [Aumann and Drèze \(1974\)](#) provide a set of axioms that characterizes the value, and the Aumann-Drèze value is regarded as a "fair" profit division among the firms in the patent licensing negotiation. After we obtain the Aumann-Drèze value as the static outcome in an industry, we solve for the steady state in the unconstrained economy. Then, in the next section, we compare the results from adopting the Aumann-Drèze value with those from adopting the bargaining set from the perspectives of the growth rate and of welfare.

4.1 Static outcomes in an industry

Note that the Cournot equilibrium outcomes and the formulation of the cooperative game are exactly the same as in the previous section.

Bargaining outcomes for patent licensing under the Aumann-Drèze value

Let $\varphi^S (\in \mathbb{R}^{n+1})$ denote the Aumann-Drèze value of our cooperative game with coalition structure $P(S)$. Recall that $\ell = |S|$, and for each $i \in \{I\} \cup S$, let $t' = |T'|$ for $T' \subseteq (\{I\} \cup S) \setminus \{i\}$ ($t' = 0$ if $T' = \emptyset$). The Aumann-Drèze value φ_i^S for a technology holder $i \in \{I\} \cup S$ is represented by

$$\varphi_i^S \equiv \sum_{T' \subseteq (\{I\} \cup S) \setminus \{i\}} \frac{t'!(\ell - t')!}{(1 + \ell)!} (v(\{i\} \cup T') - v(T')), \quad (17)$$

and we define $\varphi_j^S \equiv \bar{\pi}_{Im}(\ell, m)$ for all $j \in M$.¹⁴ By the above definition, $\varphi_I^S + \sum_{i \in S} \varphi_i^S = v(\{I\} \cup S) (= (1 + \ell)\bar{\pi}_{Th}(\ell, m))$, and $\varphi_i^S = \varphi_j^S$ for all $i, j \in S$; thus, we let φ_ℓ^S denote the Aumann-Drèze value for each licensee.

The interpretation of the Aumann-Drèze value is as follows. For each $i \in \{I\} \cup S$ and for each $T' \subseteq (\{I\} \cup S) \setminus \{i\}$, $v(\{i\} \cup T') - v(T')$ is called the marginal contribution of firm i to coalition T' . In the formation process for $\{I\} \cup S$, when a coalition T' already forms and a firm i joins that coalition, the firm demands and is promised to gain its marginal contribution to T' . If the order in which each firm joins is determined with equal probability, then the Aumann-Drèze value for each technology holder i in coalition $\{I\} \cup S$ is the average of i 's marginal contribution to the formation of $\{I\} \cup S$. In this sense, the Aumann-Drèze value represents a fair profit division among $\{I\} \cup S$.

When the number of licensees is ℓ^* and the number of potential imitators is infinitely large, the Aumann-Drèze value is specified as follows.

Proposition 4. *Consider the case in which an infinite number of potential imitators exist, and let $F^{AD}(\ell^*)$ be the licensing fee that corresponds to the Aumann-Drèze value under coalition structure $P(S^*)$, where $|S^*| = \ell^*$. Then, $F^{AD}(\ell^*) = \bar{\pi}_{Th}(\ell^*)/2$; that is, each licensee pays half of its entire profit as the fee paid to the innovator who licenses to ℓ^* firms.*

Proof. See Appendix. □

Applying the Aumann-Drèze value, we define the ratio of the licensing fee to (gross) profit as $f^{AD} \equiv F^{AD}(\ell^*)/\bar{\pi}_{Th}(\ell^*)$. Then, by Proposition 4, $f^{AD} = 1/2$, and the net profits of the innovator and each licensee are $\pi_I^{AD}(\ell^*) (= \lim_{m \rightarrow \infty} \varphi_I^{S^*}) = (1 + \ell^*/2)\bar{\pi}_{Th}(\ell^*)$ and $\pi_\ell^{AD}(\ell^*) (= \lim_{m \rightarrow \infty} \varphi_\ell^{S^*}) = \bar{\pi}_{Th}(\ell^*)/2$, respectively; that is, in the Aumann-Drèze value, each licensee gains positive net profit.

When infinitely many imitators exist, nonlicensees' (gross) profits are always zero. Thus, the innovator's participation in nonlicensee coalitions contributes to the large increase in total profit through the licensing of the patented technology. On the other hand, for an infinitely large number of imitators, the formation of $\{I\} \cup S^*$ maximizes the total profit of the innovator and the licensees. From this viewpoint, not only the innovator but also each licensee contributes to the realization of the maximum total profit. Therefore, each licensee obtains a positive profit in the Aumann-Drèze value.¹⁵

¹⁴When we follow the original definition of the Aumann-Drèze value, φ_j^S is given as $\varphi_j^S = v(\{j\}) = \bar{\pi}_{Im}(n-1, 1)$ for all $j \in M$. However, to suit our model, we define $\varphi_j^S \equiv \bar{\pi}_{Im}(\ell, m)$ for all $j \in M$.

¹⁵In the following argument, we assume that the number of licensees is ℓ^* in order to compare the outcome when the Aumann-Drèze value is adopted as the solution with that when the bargaining set is adopted. However, we can show that the optimal number of licensees for the innovator when the Aumann-Drèze value is adopted is at least as large as ℓ^* ; that is, the Aumann-Drèze value for the innovator is not always maximized when ℓ^* firms are licensed.

4.2 Dynamic outcomes in an industry and general equilibrium

Bellman equations

The licensee's firm value and the innovator's firm value satisfy the following Bellman equations:¹⁶

$$rV_\ell - \dot{V}_\ell = \max_z \left\{ \frac{\bar{\pi}_{Th}(\ell^*)}{2} + \theta z(V_I - V_\ell) - zw \right\}. \quad (18)$$

$$rV_I - \dot{V}_I = \left(1 + \frac{\ell^*}{2}\right) \bar{\pi}_{Th}(\ell^*) - \theta Z(V_I - V_\ell). \quad (19)$$

Steady state

We solve for the steady state of this economy by using $\dot{V}_I = \dot{V}_\ell = 0$ again. The labor market equilibrium condition is the same as (9). We derive another important equation that characterizes the steady state. In the steady state, (19) implies that

$$\dot{V}_I = 0 \Leftrightarrow V_I = \frac{\bar{\pi}_{Th}(\ell^*) + \ell^* \bar{\pi}_{Th}(\ell^*)/2 + \theta Z V_\ell}{\rho + \theta Z}. \quad (20)$$

Next, we solve for the steady state value of V_ℓ . For the same reason as in the previous section, $\theta(V_I - V_\ell) = w$ holds. Then, by $\pi_\ell^{AD}(\ell^*) = \bar{\pi}_{Th}(\ell^*)/2$ and (18), the following equation holds in the steady state:

$$\dot{V}_\ell = 0 \Leftrightarrow V_\ell = \frac{\bar{\pi}_{Th}(\ell^*)}{2\rho}. \quad (21)$$

The appendix shows that there are no transitional dynamics and that the economy immediately jumps to the steady state at $t = 0$, as shown in Proposition 2.

Let us solve for aggregate R&D intensity in the steady state. Substituting (21) for (20), we obtain

$$V_I = \frac{\bar{\pi}_{Th}(\ell^*) + \ell^* \bar{\pi}_{Th}(\ell^*)/2 + \theta Z \bar{\pi}_{Th}(\ell^*)/(2\rho)}{\rho + \theta Z}. \quad (22)$$

Furthermore, from (9), (11), and (21), we obtain

$$V_I = \frac{1}{\theta\chi(L - Z)} + \frac{\bar{\pi}_{Th}(\ell^*)}{2\rho}. \quad (23)$$

By solving (22) and (23), we obtain the aggregate R&D intensity for the case in which the Aumann-Drèze value is the solution in the unconstrained economy as follows:

$$Z^* = \frac{(\chi - 1)L - 2\rho/\theta}{\chi + 1} \equiv Z_U^{AD}. \quad (24)$$

¹⁶Note that ℓ is fixed at ℓ^* because we consider a situation in which policy makers intervene in the patent licensing negotiations between the innovator and the ℓ^* licensees.

5 Growth and welfare effects of intervention in the unconstrained economy

In this section, we investigate the effects of an intervention in the licensing negotiation on growth and welfare by using Z_U^B and Z_U^{AD} , derived in the previous section.

5.1 Growth effect

Suppose that $Z_U^{AD} > 0$ holds. Then, from (15) and (24) $Z_U^B > Z_U^{AD}$ holds. Since the economic growth rate $g = \dot{c}/c$ is calculated as $g = \theta Z \ln \lambda$, we obtain the following proposition.

Proposition 5. *Consider the unconstrained economy. Then, an intervention in licensing negotiations decreases the growth rate.*

Proposition 5 suggests that policy makers should not intervene in licensing negotiations from the perspective of economic growth. The intuition is simple. First, the exploitative licensing fee ($f^B = 1$) maximizes the innovator's licensing revenue. This naturally stimulates the licensee's incentive to innovate (*the Schumpeterian effect*). Second, because a higher licensing fee decreases the net profits of licensees, they attempt to escape from the current severe situation by succeeding in R&D.

5.2 Welfare effect

To compare $f^B = 1$ with $f^{AD} = 1/2$ from the perspective of welfare, we first derive the first-best allocation in the unconstrained economy. We evaluate welfare in the case in which the economy starts in the steady state at $t = 0$. By the utility function (2) and the labor market equilibrium condition (9), we have

$$\ln c_t = g \cdot t + \ln(L - Z) - \ln L.$$

By integrating the lifetime utility function (1) with respect to time, we obtain the representative household's welfare as follows:

$$W = \frac{1}{\rho} \left[\frac{\theta Z \ln \lambda}{\rho} + \ln(L - Z) - \ln L \right]. \quad (25)$$

By differentiating (25) with respect to Z , we obtain the welfare-maximizing aggregate R&D intensity as follows:

$$Z^S = \max \left\{ L - \frac{\rho}{\theta \ln \lambda}, 0 \right\}.$$

We assume that $Z^S > 0$ because $Z^S = 0$ is a trivial case in an innovation-driven growth economy. Then, by (25), the relationship between W and Z is an inverted U-shape.

There is no guarantee that aggregate R&D intensity in the unconstrained economy (Z_U^B or Z_U^{AD}) coincides with the socially optimal level Z^S . Depending on the parameters, Z can be either socially excessive or insufficient. This comes from several market failures in our

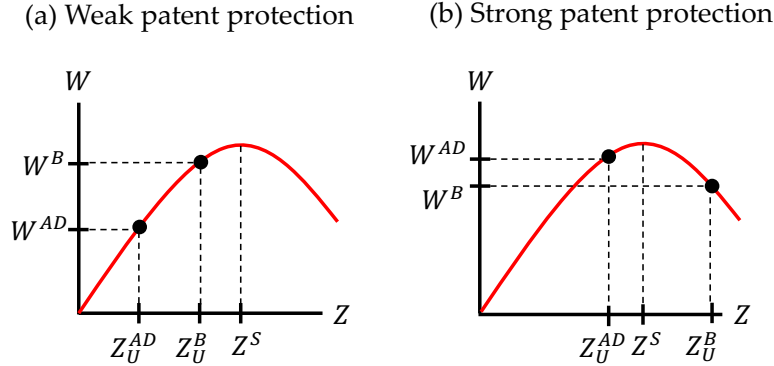


Figure 2: Welfare and patent protection.

model. First, there is a business stealing effect that leads firms to engage in more R&D than the socially optimal level.¹⁷ Second, a positive externality exists because firms choose to invest in R&D without taking into account the welfare of the household. Third, the number of producers (firms) is also determined regardless of welfare. Therefore, labor can be allocated to the production sector either excessively or insufficiently. Finally, the license fee distorts the allocation of labor by influencing the incentive to innovate.

Let W^B and W^{AD} be welfare under no intervention (the bargaining set) and welfare under intervention (the Aumann-Drèze value), respectively. Recall that Z_U^B and Z_U^{AD} are increasing in χ and $Z_U^{AD} < Z_U^B$ always holds. Then, $Z_U^{AD} < Z_U^B < Z^S$ holds when χ is sufficiently small. In this case, as shown in Panel (a) of Figure 2, $W^B > W^{AD}$ holds. However, when χ is sufficiently large, Z_U^B may exceed Z^S . In this case, since the relationship between welfare and Z is an inverted U-shape, $W^{AD} > W^B$ may hold, as in Panel (b) of Figure 2. Figure 3 is a numerical example that shows that such a parameter range exists.

The above discussion can be summarized as follows:

Proposition 6. *Consider the unconstrained economy. When patent protection is sufficiently weak, intervention in licensing negotiations is harmful to welfare. However, when patent protection is sufficiently strong, intervention may improve welfare.*

The economic intuition behind Proposition 6 is simple. Under weak patent protection, since the reward from innovation is small, the licensee's incentive to innovate is also small. Then, the degree of innovation is likely to be lower than the socially optimal level. In this case, the exploitative licensing fee is socially desirable because it encourages licensees to engage in R&D by rising the reward for innovation. In contrast, under strong patent protection, the reward for

¹⁷The business stealing effect originally meant that a new entrant decreases the output of incumbent firms in Mankiw and Whinston (1986). In our model, a licensee's successful R&D also decreases the innovator's output. In this sense, we call this the business stealing effect.

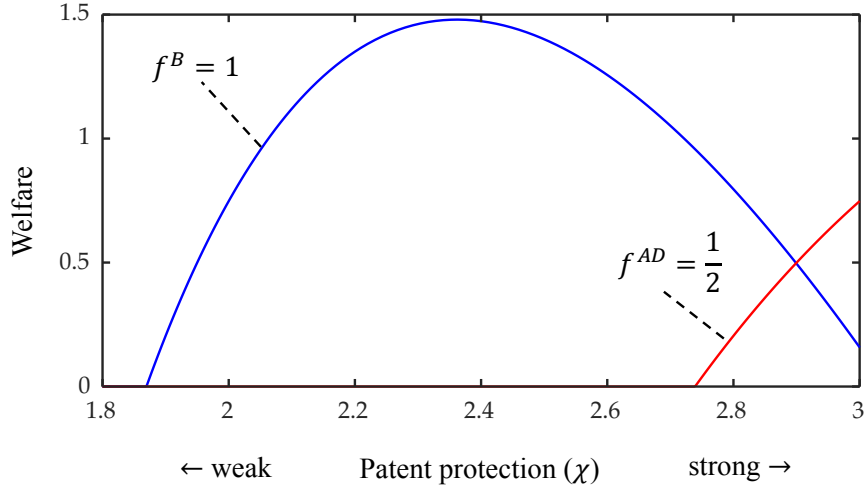


Figure 3: A numerical example of Proposition 6. The parameters are $L = 1$, $\lambda = 3$, $\rho = 0.02$, and $\theta = 0.023$. The condition in (16) is satisfied.

innovation is already large. Then, under the exploitative licensing fee, innovation may exceed the socially optimal level. In this case, a fair licensing fee is socially better.

6 Growth and welfare effects of intervention in a constrained economy

For simplicity, we considered two unrealistic assumptions in the unconstrained economy. First, we assumed that licensees can raise unlimited external funds for R&D investment by issuing stocks. Second, we assumed that licensees distribute all of their profit to their shareholders. However, in reality, the amount of external funds available is usually limited due to financial market frictions created by information asymmetries, uncertainty in R&D investment, and transaction costs.¹⁸ Furthermore, in the literature on corporate finance, it is well known that firms prefer to rely on their internal funds rather than on external funds for R&D investment.¹⁹ This firm behavior is also consistent with the *pecking order theory* of Myers and Majluf (1984). Namely, firms prefer to use internal financing first, while issuing new equity is a last resort. These facts require some modifications to the previous model. The model that imposes financial constraints is referred to as the constrained economy.

¹⁸Additionally, R&D-intensive firms may face financial frictions because they tend to have few collateralizable assets. A large part of R&D expenditures is wages for researchers, and obviously, their human capital cannot be collateralized. Furthermore, equity financing entails costs in the sense that the stock price drops at the announcement of a new equity issue (see Eckbo et al. (2007)).

¹⁹See Hall and Lerner (2010) for this point.

6.1 The growth effect

In this section, we consider a constrained economy with two financial constraints. First, we assume that the external funds available to each licensee are limited to a finite value $\kappa \geq 0$. We can interpret the unconstrained economy as the case in which $\kappa \rightarrow \infty$. Let z_E be the number of researchers whose costs are financed through external funds. Then, each licensee faces the following *external* financial constraint:

$$z_E w \leq \kappa. \quad (26)$$

Second, we assume that the licensees can use their internal funds (i.e., their profit flow) for R&D investment. Let z_I be the number of researchers whose costs are financed through internal funds. Then, each licensee faces the following *internal* financial constraint:

$$z_I w \leq \pi_\ell, \quad (27)$$

where π_ℓ is the licensee's net profit. In this case, each licensee's Bellman equation becomes

$$rV_\ell - \dot{V}_\ell = \max_{z_I, z_E} [\pi_\ell - z_I w + \theta(z_I + z_E)(V_I - V_\ell) - z_E w] \quad (28)$$

subject to (26) and (27).

In an unconstrained economy, intervention in patent licensing negotiations is always harmful to economic growth. However, the result changes when imposing the above two financial constraints.

First, we consider the constrained economy under no intervention ($f^B = 1$). Then, since $\pi_\ell = 0$, licensees cannot use their internal funds for their R&D investment ($z_I = 0$). They must conduct their R&D by using only external funds. Then, (28) becomes

$$rV_\ell - \dot{V}_\ell = \max_{z_E} [\theta z_E (V_I - V_\ell) - z_E w].$$

This is exactly the same as the Bellman equation (7) in the unconstrained economy if we replace z_E with z . Therefore, when the external financial constraint is not binding (i.e., $\kappa > 0$ is sufficiently large), aggregate R&D intensity becomes (15). When the external financial constraint is binding (i.e., $\kappa > 0$ is sufficiently small), $z_E = \kappa/w$ holds. From this and the labor market equilibrium (9), we obtain

$$\ell^* z_E^* = \frac{L}{(\chi - 1)/(\chi\kappa) + 1} \equiv Z_C^B. \quad (29)$$

Note that $Z_C^B = 0$ holds when $\kappa = 0$ and that Z_C^B is increasing in κ .

Aggregate R&D intensity in the constrained economy under no intervention (the bargaining set) is

$$Z^B = \min \{Z_U^B, Z_C^B\}. \quad (30)$$

Let κ^* be the critical value such that $Z_U^B = Z_C^B$ holds. Then, as shown in Figure 4, Z^B is strictly increasing in $\kappa \in [0, \kappa^*]$ and is constant in $\kappa \in (\kappa^*, \infty)$.

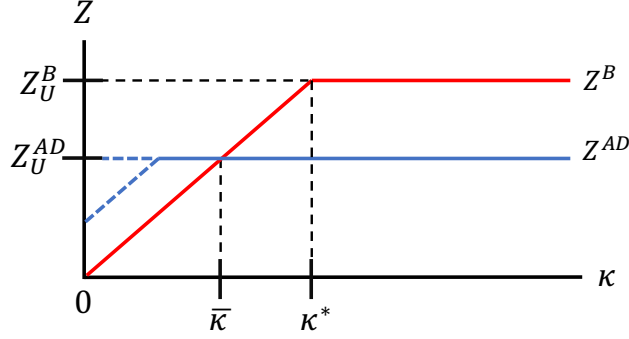


Figure 4: Z^B and Z^{AD} in the constrained economy. κ is the availability of external funds.

Second, we consider the constrained economy when interventions occur. In this economy, since $\pi_\ell = \bar{\pi}_{Th}(\ell^*)/2 > 0$, licensees can use their internal funds for their R&D investment. Note that in the licensee's Bellman equation (28), the licensee is indifferent to the choice between financing with internal funds and external funds (the sum $z_I + z_E$ is what matters). For simplicity, we assume that licensees prefer to use internal funds rather than external funds. Then, there is no case in which the external financial constraint is binding and the internal financial constraint is not binding.

If $\kappa > 0$ is sufficiently large, the external financial constraint is not binding. Then, aggregate R&D intensity $\ell^*(z_I^* + z_E^*)$ is equal to (24) in the unconstrained economy. Suppose that $\kappa > 0$ is sufficiently small and the external financial constraint is binding. Then, by the above assumption, the internal financial constraint is also binding. Therefore, we obtain

$$(z_I^* + z_E^*)w = \frac{\bar{\pi}_{Th}(\ell^*)}{2} + \kappa.$$

From this equation, $\ell^* = 1/(\chi - 1)$, (5) and (9), we obtain

$$\ell^*(z_I^* + z_E^*) = \frac{L}{2(\chi - 1)/[(\chi - 1)^2/\chi + 2\kappa\chi] + 1} \equiv Z_C^{AD}.$$

Note that $Z_C^B < Z_C^{AD}$ holds since $\chi > 1$. Aggregate R&D intensity in the constrained economy in the case of intervention (when using the Aumann-Drèze value) is

$$Z^{AD} = \min \left\{ Z_U^{AD}, Z_C^{AD} \right\}.$$

Depending on the parameters, $Z^{AD} = Z_U^{AD}$ holds for all $\kappa \geq 0$ or only for a large κ . However, regardless of the parameters, $Z^{AD} < Z^B$ holds when κ is sufficiently large, and $0 = Z^B < Z^{AD}$ holds when $\kappa = 0$, as shown in Figure 4. Thus, by the intermediate value theorem, we can find the critical value $\bar{\kappa} > 0$ such that $Z^B < Z^{AD}$ holds for $\kappa \in [0, \bar{\kappa})$. This argument can be

summarized as follows:

Proposition 7. *Consider the constrained economy. Intervention in licensing negotiations reduces growth when external funds are readily available. In contrast, intervention enhances growth when external funds are less available.*

The intuition behind Proposition 7 is the same as that behind Proposition 5 when external funds are readily available. However, when external funds are less available, licensees finance a large part of their R&D investment with internal funds. Since intervention in licensing negotiations increases the internal funds of the licensees, it can enhance their R&D investment. In contrast, since negotiation without intervention does not give internal funds to licensees, it prevents them from improving the licensed technology.

6.2 The welfare effect

Let us investigate the welfare effect of the intervention. Suppose that external funds are sufficiently available such that $Z^B = Z_U^B$ and $Z^{AD} = Z_U^{AD}$ hold. Then, $Z^{AD} < Z^B$ holds as shown in Figure 4. In this case, as in the unconstrained economy, the welfare effect of the intervention depends on the strength of patent protection; that is, the welfare effect is exactly the same as in Proposition 6.

The case in which external funds are less available is analytically complex. Therefore, we numerically evaluate the welfare effect of the intervention by calibrating our model to aggregate data on the U.S. economy. We set the discount rate ρ to the conventional value of 0.03. According to Hashmi (2013), the average of the Lerner index in the U.S. is 0.24. In our model, the Lerner index is $1/(1 + \ell^*) = 1 - 1/\chi$. Then, we obtain $\chi \simeq 1.32$. Because $\lim_{m \rightarrow \infty} p(\ell^*, m) = \chi w$, in our model, χ also means the markup of price over marginal cost. $\chi = 1.32$ is in an empirically plausible range (see Jones and Williams (2000)). We assume that patent protection in the U.S. is almost perfect ($\lambda = 1.32$). This corresponds to Park (2008), who shows that the patent rights index in the U.S. is 4.88 (the maximum is 5), the highest in the world. The Conference Board Total Economy Database reports that the average total factor productivity (TFP) growth rate in the U.S. is about 0.6%. According to the Business R&D and Innovation Survey, approximately 6% of domestic employment in the U.S. is engaged in R&D. We set L and θ by targeting these two facts. Then, $L = 1$ and $\theta = 0.35$ yield $g \simeq 0.006$ and $Z/L \simeq 0.06$.²⁰ These parameters satisfy (16).

Figure 5 numerically compares W^B with W^{AD} . The difference $W^{AD} - W^B$ shows the welfare effect of the intervention. W^{AD} is constant for all κ because $Z^{AD} = Z_U^{AD}$ always holds in the numerical analysis. Figure 5 shows that the welfare effect of the intervention is positive when external funds are less available. The intuition behind this result is the same as in Proposition 7. When external funds are less available, intervention in licensing negotiations increases the

²⁰In this calibration, we consider a scenario in which $g \simeq 0.006$ and $Z/L \simeq 0.06$ in the unconstrained economy (large κ) under the bargaining set ($f^B = 1$). Recall that Z (and also g) is increasing in κ and $Z_U^B > Z_U^{AD}$. Therefore, in other words, we calibrate L and θ so that the maximum values of g and Z/L are equal to the target values.

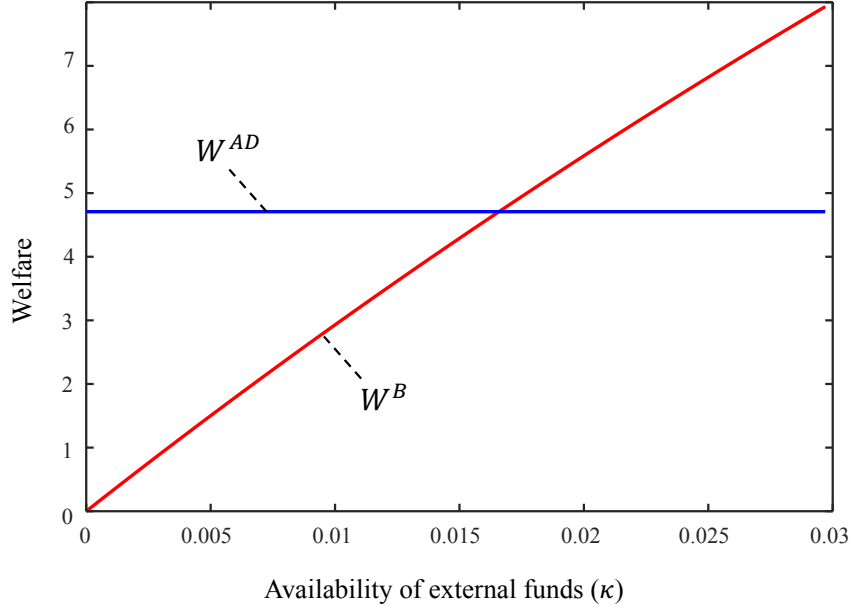


Figure 5: A quantitative analysis. The parameters are $L = 1, \lambda = \chi = 1.32, \rho = 0.03$, and $\theta = 0.35$. These parameters satisfy (16).

internal funds of the licensees. Therefore, since intervention can enhance R&D investment, welfare increases.

We also conduct another numerical analysis under imperfect patent protection ($\chi \leq \lambda$). We can interpret Figure 5 as the result of a special case of $\chi = \lambda$. Figure 6 presents the results. The value in the color bar in Panel (c) is $W^{AD} - W^B$, which is the welfare effect of the intervention. Therefore, the red region in Panel (c) is the area in which intervention increases welfare. Figure 6 shows that the welfare effect of intervention is likely to be positive as patent protection becomes stronger. This result implies that there is complementarity between a pro-patent policy and intervention in licensing negotiations.

Why does patent protection strengthen the welfare effects of the intervention? This is because stronger patent protection increases welfare under the Aumann-Drèze value (Panel (a)) but decreases welfare under the bargaining set (Panel (b)). In this numerical analysis, $Z^{AD} = Z_U^{AD}$ holds for all κ and χ . Since stronger patent protection increases Z_U^{AD} , it also increases welfare under the Aumann-Drèze value. However, in this numerical analysis, $Z^B = Z_C^B$ holds when κ is small and χ is large (i.e., the financial constraint is binding). This implies that stronger patent protection decreases ℓ^* but does not increase z_E^* much; that is, Z_C^B is decreasing in χ . Therefore, when the financial constraint is binding, stronger patent protection decreases welfare under the bargaining set because it decreases aggregate R&D intensity.

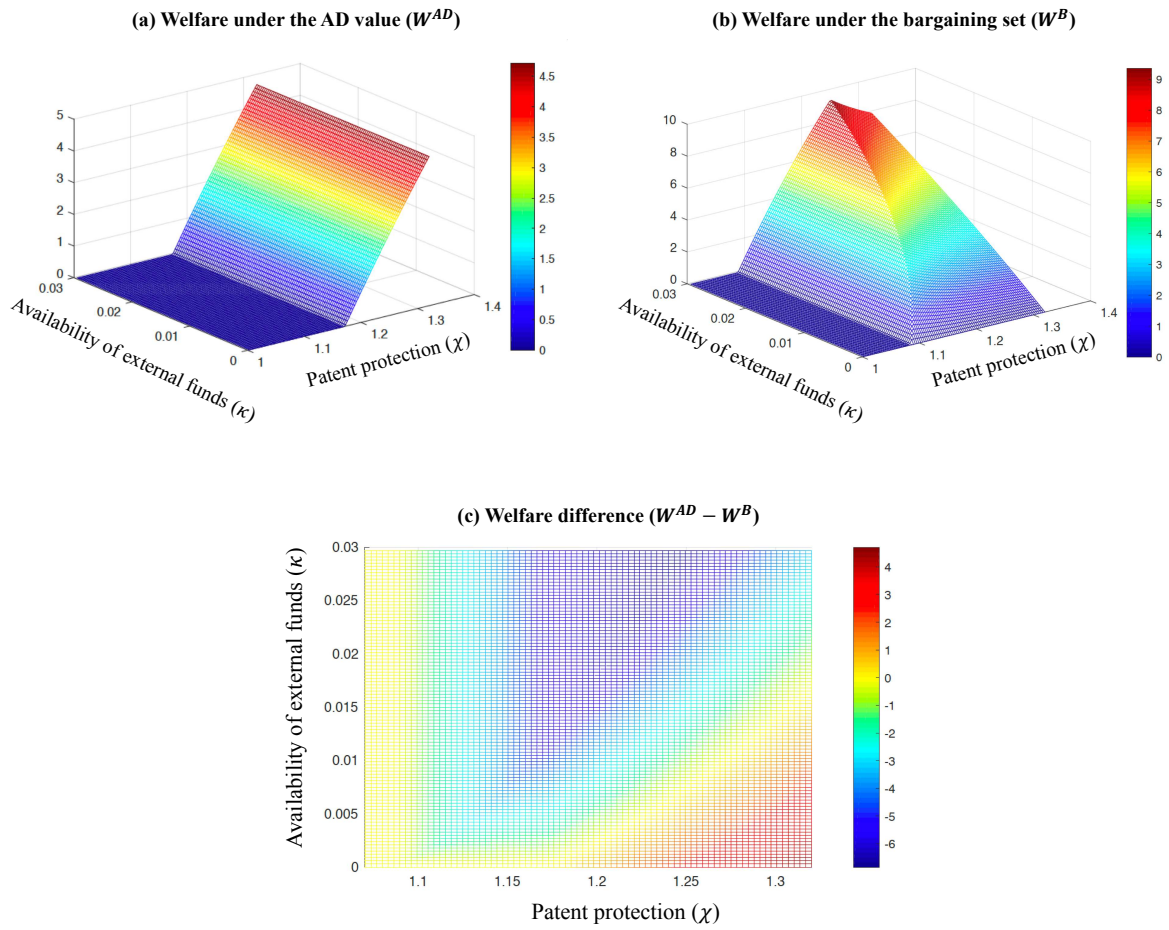


Figure 6: The welfare effect of the intervention under imperfect patent protection. In Panel (c), the red region is the area in which the intervention increases welfare. The parameters are $L = 1, \lambda = 1.32, \rho = 0.03$, and $\theta = 0.35$.

7 Conclusion

The growth and welfare effects of intervention in licensing negotiations have been less well known in the literature on endogenous growth theory. Although typical studies consider an exogenous profit division rule, in reality, licensing fees are often determined through private negotiations between the relevant firms. Generally, it is difficult for policy makers and some organizations to directly control such licensing fees, but they can indirectly influence them through certain interventions. We investigate the macroeconomic impacts of an indirect intervention in licensing negotiations by considering a synthesis of cooperative game theory and Schumpeterian growth theory. We introduce two solution concepts, the bargaining set and the Aumann-Drèze value, in a Schumpeterian growth model. The bargaining set describes free negotiation, while the Aumann-Drèze value represents an intervention from the outside.

Our main findings are summarized as follows. First, we show that negotiations without intervention yield an exploitative licensing fee, while an intervention yields a fair licensing fee. Second, we find that the growth effect of intervention is negative when firms can raise unlimited external funds for their R&D investment. However, the growth effect becomes positive when the amount of external funds available is limited. Finally, our quantitative analysis shows that intervention can increase welfare when external funds are less available and patent protection is sufficiently strong. These results imply that intervention can increase both growth and welfare when the internal funds of firms are the main source of their R&D investment in an economy with strong patent protection. In other words, there is complementarity between a pro-patent policy and interventions in licensing negotiations.

Appendix

The proof of Proposition 1

Take any $\pi \in \mathcal{B}^S$ with $|S| = \ell (< \infty)$. We first show that $\lim_{m \rightarrow \infty} \pi_i = 0$ for each $i \in S$. Let $i' \in \arg \max_{i \in S} \pi_i$, and consider the following two cases.

Case (i): $\pi_{i'} > \bar{\pi}_{Im}(\ell, m)$. Order all the $\ell + m$ firms according to their profits in nondecreasing order, and take the first ℓ firms. Let T be the set of the first ℓ firms. Note that $\pi_j = \bar{\pi}_{Im}(\ell, m)$ for $j \in M$ because $\pi \in \Pi(S)$. Then, the innovator has objection $(\pi', \{I\} \cup T)$ against i' because $\pi_I + \sum_{i \in T} \pi_i < \pi_I + \sum_{i \in S} \pi_i = (1 + \ell)\bar{\pi}_{Th}(\ell, m) = v(\{I\} \cup T)$. However, i' can have counter-objection (π'', R) to the innovator's objection $(\pi', \{I\} \cup T)$ because $\pi \in \mathcal{B}^S$. Note that $i' \in R \subseteq S \cup M$ by the definition of a counter-objection, and $0 \leq \pi_i$ for all $i \in S \cup M$ because $\pi \in \Pi(S)$. Thus, for each $i \in S$, $0 \leq \pi_i \leq \pi_{i'} \leq \sum_{i \in R} \pi_i \leq v(R) = r\bar{\pi}_{Im}(\ell + m - r, r) \leq (\ell + m)\bar{\pi}_{Im}(0, \ell + m)$, where $r = |R| \leq \ell + m$, by the definition of i' and (6). Because

$$\lim_{m \rightarrow \infty} (\ell + m)\bar{\pi}_{Im}(0, \ell + m) = \lim_{m \rightarrow \infty} (\ell + m) \left(\frac{1}{1 + (\ell + m)\chi} \right)^2 = 0,$$

we have $\lim_{m \rightarrow \infty} \pi_i = 0$ for each $i \in S$ by the squeeze theorem.

Case (ii): $\pi_{i'} \leq \bar{\pi}_{Im}(\ell, m)$. In this case, by the definition of i' , $\pi_i \leq \bar{\pi}_{Im}(\ell, m)$ for all $i \in S$. Note that $0 \leq \pi_i$ for all $i \in S$ because $\pi \in \Pi(S)$. If $\ell > \ell^*$, then $\pi_i = 0$ for each $i \in S$ by (6); thus, $\lim_{m \rightarrow \infty} \pi_i = 0$ for each $i \in S$. Suppose that $\ell \leq \ell^*$. Then, for each $i \in S$, $0 \leq \pi_i \leq \bar{\pi}_{Im}(\ell, m)$. Because $\lim_{m \rightarrow \infty} \bar{\pi}_{Im}(\ell, m) = 0$ under the supposition in (6), we also have $\lim_{m \rightarrow \infty} \pi_i = 0$ for each $i \in S$ by the squeeze theorem.

We next show that $\lim_{m \rightarrow \infty} \pi_I = \lim_{m \rightarrow \infty} (1 + \ell) \bar{\pi}_{Th}(\ell, m)$. Because S is fixed (i.e., S does not depend on m) and $\lim_{m \rightarrow \infty} \pi_i = 0$ for all $i \in S$, $\lim_{m \rightarrow \infty} \sum_{i \in S} \pi_i = 0$. By the definition of $\Pi(S)$, $\pi_I = (1 + \ell) \bar{\pi}_{Th}(\ell, m) - \sum_{i \in S} \pi_i$. Therefore,

$$\lim_{m \rightarrow \infty} \pi_I = \lim_{m \rightarrow \infty} \left[(1 + \ell) \bar{\pi}_{Th}(\ell, m) - \sum_{i \in S} \pi_i \right] = \lim_{m \rightarrow \infty} (1 + \ell) \bar{\pi}_{Th}(\ell, m).$$

Let $F^B(\ell)$ be the licensing fee that corresponds to the imputations in the bargaining set for coalition structure $P(S)$, where $|S| = \ell$, and recall that $\bar{\pi}_{Th}(\ell) \equiv \lim_{m \rightarrow \infty} \bar{\pi}_{Th}(\ell, m)$. Then, in the bargaining set for coalition structure $P(S)$, the innovator's (net) profit is $\pi_I^B(\ell) = \bar{\pi}_{Th}(\ell) + \ell F^B(\ell) = \lim_{m \rightarrow \infty} \pi_I = (1 + \ell) \bar{\pi}_{Th}(\ell)$; that is, $F^B(\ell) = \bar{\pi}_{Th}(\ell)$. Furthermore, by (5),

$$\pi_I^B(\ell) = (1 + \ell) \bar{\pi}_{Th}(\ell) = \begin{cases} (1 + \ell) (1 - 1/\chi)^2 & \text{if } \ell < \ell^* \\ 1/(1 + \ell) & \text{if } \ell \geq \ell^*, \end{cases}$$

which implies that $\pi_I^B(\ell)$ is increasing in $\ell < \ell^*$ but is decreasing in $\ell \geq \ell^*$. Thus, the innovator's (net) profit is maximized at $\ell = \ell^*$.

The proof of Proposition 2

The RHS of (13) decreases in Z and is illustrated as the downward sloping curve "NAC" in Figure 7. The RHS of (14) is increasing in Z and is illustrated as the upward sloping curve "LME" in Figure 7. The intersection of the two curves in Figure 7 represents the steady state of this economy.

Suppose firm value grows without bound along the LME curve in Figure 7 above the steady state level. In this case, from (8) and the normalization $r = \rho$, we have

$$\rho < \frac{\bar{\pi}_{Th}(\ell)}{V_I} + \frac{\dot{V}_I}{V_I} - \theta Z \left(1 - \frac{V_\ell}{V_I} \right). \quad (31)$$

Note that $\bar{\pi}_{Th}(\ell)$ is a positive constant. As V_I grows infinitely, the first term on the RHS of (31) converges to zero, and the third term on the RHS of (31) converges to $-\theta Z < 0$. Then, the growth rate of firm value, \dot{V}_I/V_I , will eventually be larger than the discount rate.

The households' transversality condition is given by

$$\lim_{t \rightarrow \infty} \exp(-r_t t) A_t = 0,$$

where $A_t \equiv a_t \cdot L$. In this model, households accumulate their assets by purchasing equity with

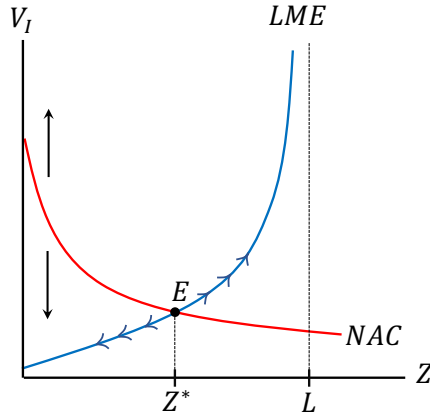


Figure 7: The steady state under no intervention in licensing negotiations.

value V_I (note that $V_\ell = 0$). From the asset market equilibrium condition ($A_t = V_I$) and $r_t = \rho$, the transversality condition can be rewritten as

$$\lim_{t \rightarrow \infty} \exp(-\rho t) V_I = 0.$$

This condition states that the growth rate of V_I cannot be larger than the discount rate. Therefore, the LME curve above the steady state is a path that eventually violates the transversality condition.

Suppose, by contrast, that firm value decreases along the LME curve below the steady-state level. Then, V_I and Z converge to zero in the phase diagram in Figure 7. This implies that all equities will have zero value in the future. However, in Cournot competition with finite firms, the innovator's profit is strictly positive. Moreover, as $Z = 0$, a household with the innovator's equity can earn a positive dividend forever. Therefore, this path entails unfulfilled expectations.

As a result, the economy must immediately jump to the steady state at $t = 0$ and remain there forever. This is the unique path that satisfies the transversality condition and rational expectations.

The proof of Proposition 3

In this proof, we demonstrate that ℓ^* is also dynamically optimal under the parameter condition given in (16). In Proposition 1, we already showed that ℓ^* is statically optimal. Note that if the innovator chooses $\ell > 0$, then ℓ^* maximizes the RHS of (8) because the innovator takes Z as given. Therefore, the innovator does not have an incentive to choose a positive $\ell \neq \ell^*$ in (8) because it decreases his/her net profit and does not decrease the risk of creative destruction (Z). However, the innovator may have an incentive to choose $\ell = 0$ because $Z = 0$ holds. In this proof, we show that the RHS of (8) when $\ell = \ell^*$ is larger than the RHS when $\ell = 0$. Note

that $f^B = 1$ is independent from ℓ by Proposition 1.

Suppose that the innovator chooses $\ell = \ell^*$. Then, the RHS of (8) in the steady state is

$$\begin{aligned} & 1 - \frac{1}{\chi} - \frac{Z^*}{\chi(L - Z^*)} \\ \Leftrightarrow & 1 - \frac{1}{\chi} - \frac{(\chi - 1)L - \rho/\theta}{\chi^2[L - ((\chi - 1)L - \rho/\theta)/\chi]} \\ \Leftrightarrow & 1 - \frac{1}{\chi} - \frac{(\chi - 1)L - \rho/\theta}{\chi(L + \rho/\theta)}. \end{aligned}$$

Therefore, by (8), the optimal number of licensees is ℓ^* if

$$\begin{aligned} & \left(1 - \frac{1}{\chi}\right)^2 < 1 - \frac{1}{\chi} - \frac{(\chi - 1)L - \rho/\theta}{\chi(L + \rho/\theta)} \\ \Leftrightarrow & \frac{(\chi - 1)^2}{2\chi - 1} < \frac{\rho}{\theta L}. \end{aligned}$$

The left-hand side of the last inequality is increasing in $\chi \in [1, \lambda)$. Therefore, (16) is a sufficient condition for the dynamic optimality of ℓ^* .

The proof of Proposition 4

Note that $\ell^* = |S^*|$, and let $t = |T|$ for $T \subseteq S^*$ ($t = 0$ if $T = \emptyset$). By (17), the Aumann-Drèze value $\varphi_I^{S^*}$ for the innovator is

$$\varphi_I^{S^*} = \sum_{T \subseteq S^*} \frac{t!(\ell^* - t)!}{(1 + \ell^*)!} (v(\{I\} \cup T) - v(T)).$$

When t is fixed, there are $\ell^*!/(t!(\ell^* - t)!)$ orderings with the same marginal contribution $v(\{I\} \cup T) - v(T) = (1 + t)\bar{\pi}_{Th}(t, \ell^* + m - t) - t\bar{\pi}_{Im}(\ell^* + m - t, t)$ by the innovator because licensees in S^* are identical. Thus, the Aumann-Drèze value $\varphi_I^{S^*}$ for the innovator is given by

$$\varphi_I^{S^*} = \frac{1}{1 + \ell^*} \sum_{t=0}^{\ell^*} [(1 + t)\bar{\pi}_{Th}(t, \ell^* + m - t) - t\bar{\pi}_{Im}(\ell^* + m - t, t)]. \quad (32)$$

For each $t = 0, 1, 2, \dots, \ell^*$, $m > t$ when m is sufficiently large. Then, $\bar{\pi}_{Im}(\ell^* + m - t, t) = 0$ by (6), so we have $\lim_{m \rightarrow \infty} t\bar{\pi}_{Im}(\ell^* + m - t, t) = 0$ for each $t = 0, 1, 2, \dots, \ell^*$.

Note that for each $t = 0, 1, 2, \dots, \ell^*$,

$$(1 + t)\bar{\pi}_{Th}(t, \ell^* + m - t) = (1 + t) \left(1 - \frac{\ell^* + m}{1 + t + (\ell^* + m - t)\chi}\right)^2,$$

and $\bar{\pi}_{Th}(\ell^*) = (1 - 1/\chi)^2$. Then, by (32) and the above facts, when infinitely many imitators

exist, the Aumann-Drèze value for the innovator under coalition structure $P(S^*)$ is

$$\begin{aligned}
\lim_{m \rightarrow \infty} \varphi_I^{S^*} &= \frac{1}{1 + \ell^*} \sum_{t=0}^{\ell^*} \lim_{m \rightarrow \infty} [(1+t)\bar{\pi}_{Th}(t, \ell^* + m - t) - t\bar{\pi}_{Im}(\ell^* + m - t, t)] \\
&= \frac{1}{1 + \ell^*} \sum_{t=0}^{\ell^*} (1+t) \left(1 - \frac{1}{\chi}\right)^2 \\
&= \frac{1}{1 + \ell^*} \frac{(2 + \ell^*)(1 + \ell^*)}{2} \left(1 - \frac{1}{\chi}\right)^2 \\
&= \bar{\pi}_{Th}(\ell^*) + \frac{\ell^*}{2} \bar{\pi}_{Th}(\ell^*).
\end{aligned}$$

Let $F^{AD}(\ell^*)$ be the licensing fee that corresponds to the Aumann-Drèze value under the coalition structure $P(S^*)$, where $|S^*| = \ell^*$. Then, by the above Aumann-Drèze value, the innovator's (net) profit is $\pi_I^{AD}(\ell^*) = \bar{\pi}_{Th}(\ell^*) + \ell^* F^{AD}(\ell^*) = \lim_{m \rightarrow \infty} \varphi_I^{S^*} = \bar{\pi}_{Th}(\ell^*) + \ell^* \bar{\pi}_{Th}(\ell^*)/2$; that is, $F^{AD}(\ell^*) = \bar{\pi}_{Th}(\ell^*)/2$.

The equilibrium path under intervention in licensing negotiations

From (9) and $w = \theta(V_I - V_\ell)$, we obtain

$$Z = L - \frac{1}{\theta\chi(V_I - V_\ell)},$$

which is increasing in V_I and decreasing in V_ℓ . The RHS of (20) is decreasing in Z .²¹ Therefore, when V_I rises, the RHS of (20) decreases while the LHS of (20) increases. Then, V_ℓ must increase to equate both sides of (20). This implies that the $\dot{V}_I = 0$ curve is increasing in V_ℓ , as shown in Figure 8. In line with (21), $\dot{V}_\ell = 0$ is represented as a vertical line in Figure 8.

Figure 8 shows that the steady state is unique. If the economy does not begin at the steady state, there are four possible paths in Figure 8. First, we consider the northeast case ($\dot{V}_I > 0, \dot{V}_\ell > 0$). In this case, V_I and V_ℓ grow infinitely. In the proof of Proposition 2, we already show that \dot{V}_I/V_I will be larger than the discount rate. We obtain the same result in this case.²² Furthermore, the transversality condition can be rewritten as $\lim_{t \rightarrow \infty} \exp(-\rho t)(V_I + \ell^* V_\ell) = 0$. This implies that the growth rate of $V_I + \ell^* V_\ell$ must be smaller than the discount rate. Therefore, the northeast case violates the transversality condition. Second, we consider the northwest case ($\dot{V}_I > 0, \dot{V}_\ell < 0$). As in the previous case, V_I grows infinitely, and the growth rate becomes higher than the discount rate. Therefore, for the same reason, the northwest case violates the transversality condition. Third, we consider the southeast case ($\dot{V}_I < 0, \dot{V}_\ell > 0$). In this case, V_I decreases, and eventually, $0 = V_I < V_\ell$ holds. This means that R&D activities are not profitable. Hence, $Z = 0$ holds. However, the innovator can earn a positive profit forever because there is no risk of creative destruction. Therefore, a household that owns the innovator's equity can earn

²¹By the zero profit condition $w = \theta(V_I - V_\ell)$, as long as the equilibrium wage rate is positive, $V_I > V_\ell$ holds. Then, the RHS of (20) is decreasing in Z .

²²Note that $V_\ell/V_I < 1$ always holds in this case.

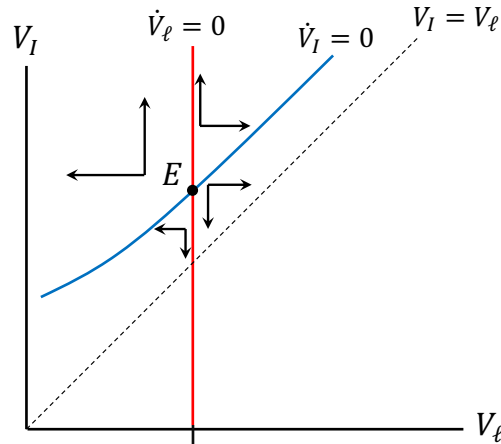


Figure 8: The steady state under interventions in licensing negotiations.

a positive dividend forever. This implies that the southeast case entails unfulfilled expectations. Finally, we consider the southwest case ($\dot{V}_I < 0, \dot{V}_\ell < 0$). In this case, the economy transitions to the origin ($V_\ell = V_I = 0$). For the same reason as in the southeast case, this entails unfulfilled expectations.

Therefore, there is a unique equilibrium path in which the economy immediately jumps to the steady state at $t = 0$ and remains there forever. Since there are only jumpable variables in the dynamics, there are no transitional dynamics in the model.

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