# Altruistic Overlapping Generations of Households and the Contribution of Human Capital to Economic Growth

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**Abstract**

We developed a dynamic deterministic general equilibrium model that accounts for human capital accumulation through both home education and schooling. The model is characterized by an altruistic link between households of succeeding generations in the sense parents, caring about their children's welfare, impart them freely some knowledge at home in addition to helping them financially when they are schooling. The education regime is private and features distinguishing our model from related works are: (1) young households are economically active and work part-time while schooling, (2) allocating time to schooling or labor entails disutility, and (3) tuition fee is proportional to the time allocated to schooling. We calibrated the model to some balanced growth facts observed between 1981 and 2019 in the Province of Quebec.

We have used our model to investigate the contribution of human capital to economic growth. To do that, we have simulated the model assuming in turn a permanent rise in the tuition and in the household's ability to learn. Each of these two shocks reveals a positive correlation between output, education, and human capital, provided the latter variable is stationary. We have also used the predictions of the model to shed a light on the student crisis Quebec witnessed in 2012 following the decision of the government to increase tuition. We have concluded that raising tuition will neither harm education nor negatively impact on students' ability to pay.

**Keywords:** education, economic growth, human capital, overlapping generations.

**JEL classification:** I25, O31, O41

**Résumé**

Nous avons développé un modèle d'équilibre général dynamique et déterministe qui explique l'accumulation du capital humain par l'enseignement à domicile et la scolarisation. Le modèle est caractérisé par un lien altruiste entre des ménages de générations successives dans le mesure où, soucieux du bien-être de leurs enfants, les parents leur transmettent sans contrainte des connaissances à domicile en plus de les aider financièrement pendant qu'ils étudient. Le système d'éducation est privé et les caractéristiques qui distinguent notre modèle des autres sont : (1) les jeunes sont économiquement actifs et travaillent pendant

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qu'ils étudient, (2) le fait de consacrer du temps aux études ou au travail occasionne des désutilités et (3) les droits de scolarité sont proportionnels au temps alloué aux études.

Nous avons utilisé notre modèle pour étudier la contribution du capital humain à la croissance économique. Pour cela, nous avons simulé le modèle en supposant à tour de rôle une hausse permanente des droits de scolarité par crédit et de la capacité d'apprentissage des ménages. Chacun de ces deux chocs révèle une corrélation positive entre la production, l'éducation et le capital humain, à condition que celui-ci soit une variable stationnaire. Par la suite, nous avons utilisé les prédictions de notre modèle pour faire la lumière sur la crise étudiante que le Québec a connue en 2012, à la suite de la décision du gouvernement d'augmenter les frais de scolarité. Nous concluons que le fait d'augmenter les frais de scolarité ne nuit ni à l'éducation ni à la capacité des étudiants de payer leurs droits de scolarité.

**Mots clés :** éducation, croissance économique, capital humain, générations imbriquées..

**Classification JEL :** I25, O31, O41

# Contents

# Non-Technical Summary

**Motivation** The empirical evidence on the role played by human capital accumulation in economic growth is contrasting. Lucas (1988), Barro (1991, 2001) and Mankiw, Romer, and Weil (1992) found supporting evidence, which Bils and Klenow (2000) questioned.

The empirical evidence on the inverse relationship (*i.e.* the impact of economic growth on human capital accumulation) is also contrasting. While DeJong and Ingram (2001) found these two variables to be correlated negatively, Fowler and Young (2004) sustained that human capital accumulation by the young is procyclical.

This divergence stems from the assumptions underlying these studies. In the model of Lucas (1988) and that of DeJong and Ingram (2001), households are infinitely-lived, unlike the models of Bils and Klenow (2000) and Fowler and Young (2004). Besides, the inputs of the human capital production function vary from one study to the other: the time allocated to education (Mankiw, Romer, and Weil, 1992; DeJong and Ingram, 2001; Fowler and Young, 2004), the time allocated to education or the work experience (Lucas, 1988), the time allocated to education, the work experience, and the quality of teachers (Bils and Klenow, 2000).

**Objectives** We seek to explain the decision of households to invest in education and, by so doing, to investigate the contribution of human capital to economic growth. We intend to use our findings to shed a light on a topical issue in the Province of Quebec: the student protests named *Maple Spring* that followed in 2012 the decision of the government to increase the tuition.

**Methodology** We have built a dynamic general equilibrium model that includes some realistic features. Households are finitely-lived and different generations of households coexist. Students work part-time to finance their education and working or studying entails disutility. We have solved numerically this model assuming a permanent rise in the tuition and the ability to learn of households.

**Findings** When human capital is assumed to be a stationary variable, raising the tuition does not affect in the long-run the time allocated to education, the hours worked, the real wage rate, and the real interest rate. But, that is not the case when human capital is assumed to be a non-stationary variable. The effects of a permanent rise in the ability to learn of households are the opposite of those of a rise in the tuition rate. When human capital is assumed to be stationary, the correlation between this latter and output turns out to be positive and high.

# Sommaire Non-Technique

**Motivation** Les preuves empiriques du rôle joué par l'accumulation du capital humain dans la croissance économique divergent. Lucas (1988), Barro (1991, 2001) et Mankiw, Romer et Weil (1992) ont trouvé des preuves justificatives, que Bils et Klenow (2000) ont remis en question.

Les preuves empiriques sur la relation inverse (c.-à-d. l'impact de la croissance économique sur l'accumulation du capital) divergent aussi. Alors que DeJong et Ingram (2001) ont trouvé que ces deux variables sont corrélées négativement, Fowler et Young (2004) soutiennent que l'accumulation du capital humain par les jeunes est procyclique.

Cette divergence provient des hypothèses sous-jacentes de ces études. Dans le modèle de Lucas (1988) et dans celui de DeJongs et Ingram (2001), les agents vivent indéfiniment, contrairement aux modèles de Bils et Klenow (2000) et de Fowler et Young (2004). Par ailleurs, les intrants de la fonction de production du capital humain varient d'une étude à l'autre : le temps alloué à l'éducation (Mankiw, Romer et Weil, 1992 ; DeJong et Ingram, 2001 ; Fowler et Young, 2004) le temps alloué à l'éducation ou l'expérience professionnelle (Lucas, 1988), le temps alloué à l'éducation, l'expérience professionnelle et la qualité des enseignants ()Bils et Klenow, 2000).

**Objectifs** Nous cherchons à expliquer la décision des ménages d'investir dans l'éducation et à étudier, ce faisant, la contribution du capital humain à la croissance économique. Nous voulons utiliser nos résultats pour éclairer un sujet d'actualité au Québec : la crise étudiante connue sous le nom de *Printemps érable* qui, en 2012, a suivi la décision du gouvernement d'augmenter les droits de scolarité.

**Methodologie** Nous avons construit un modèle dynamique d'équilibre général qui inclut des caractéristiques réalistes. Les ménages ont une durée de vie limitée et différentes générations de ménages coexistent. Les étudiants travaillent à temps partiel et le fait de travailler ou d'étudier cause des désutilités. Nous avons résolu le modèle de façon numérique en supposant une hausse permanente des frais de scolarité et de la capacité d'apprentissage des ménages.

**Résultats** Quand on suppose que le capital humain est une variable stationnaire, le fait d'augmenter les droits de scolarité n'affecte pas dans le long-terme le temps alloué à l'éducation, les heures travaillées, le taux de salaire réel et le taux d'intérêt réel. Mais, ceci n'est pas le cas quand l'on suppose que le capital humain est une variable non-stationnaire. Les effets d'une augmentation permanente de la capacité d'apprentissage des ménages sont le contraire de ceux d'une hausse des frais de scolarité. Quand on suppose que le capital humain est stationnaire, la corrélation entre ce dernier et la production s'avère être positive et élevée.

# 1 Introduction

Becker's 1964 seminal work and subsequent studies including Lucas (1988), Barro (1991), and Mankiw, Romer, and Weil (1992) shed a light on the key role played by *human capital* in economic growth. Human capital, defined as the ability to perform labor, can be acquired through education and work experience. Accumulating human capital enhances households' productivity, induces additional investment in physical capital, and favors economic growth. [1] In the same time, some authors such as Bils and Klenow (2000) questioned, on the basis of some empirical evidence, the importance given to the contribution to economic growth of human capital accumulation through formal education. By formal education, we mean schooling. Conversely, several authors investigated the impact of economic growth on human capital accumulation. DeJong and Ingram (2001) showed that college enrollments in the US used as a proxy for human capital accumulation through schooling was negatively correlated with output growth rate, over the period 1970-1996. They also found that an increase in wage induced by a positive technology shock negatively impacted on human capital accumulation. As for Fowler and Young (2004), human capital accumulation by young households is rather procyclical.

This essay specifically deals with the contribution to economic growth of human capital accumulation through both home and formal education. Home education, also known as intergenerational knowledge spill-over, is about young households inheriting without any effort some of their parents' knowledge whereas formal education involves some resource, precisely time and income, allocations. [2] Lucas (1988) advocated the modeling of home education arguing that "human capital accumulation is a *social* activity, involving *groups* of people". In modeling households' decision to invest in education, one of the following two assumptions are often made about their life span: (1) they are infinitely-lived (Razin, 1972; Lucas, 1988; DeJong and Ingram, 2001), or (2) they are finitely-lived (Tran-Nam, Truong, and Van Tu, 1995; Shimomura and Tran-Nam, 1997; Heckman, Lochner, and Taber, 1998; Sadahiro and Shimasawa, 2003). We have followed the latter class of models, *viz*, households in our model are finitely-lived and heterogeneous in their age. This framework called overlapping generations (OLG) originated from Samuelson's 1958 and Diamond's 1965 contributions. The use of the OLG framework is motivated by the fact that: (1) education is an investment that largely takes place in the earlier stage of a household's life-cycle, (2) the financing of education could involve the contribution of older generations of households. The OLG framework helps easily represent these realities.

The contribution of older generations of households to education financing could be modeled in several ways depending on whether the education regime entertained is public or private. Under a public regime, education is free and financed out of income tax revenue (Glomm and Ravikumar, 1992; Tran-Nam, Truong, and Van Tu, 1995). Under a private regime, it costs to get educated and altruistic parents directly pay for their children's education (Glomm and Ravikumar, 1992) whereas selfish parents just

[1] Aghion and Howitt (1998, 2009) reviewed some popular theories and empirics on this subject.

[2] Henceforth, whenever we mention education without any further precision, we are referring to formal education. The expressions education and schooling will therefore be used interchangeably.

grant them a loan they reimburse when they finish schooling. (Tran-Nam, Truong, and Van Tu, 1995; Shimomura and Tran-Nam, 1997). In the model we have developed herein, the education regime is private and young households receive generously some financial transfers from their parents. We allow the transfer a young household receives to differ from the tuition fee he pays. Some features distinguishing our model from that of Glomm and Ravikumar are: (1) we have allowed young households to work part-time while schooling, (2) the way we have modeled *altruism*, and (3) the tuition fee is proportional to the time allocated to schooling. Students working part-time on or off campus to finance their needs is nowadays an overwhelming reality that we want our model to take into account. Besides, students, especially those pursuing a university degree, are economically active agents and do not necessarily live under the same roof as their parents. To take this into account, we have modeled the consumption decision of the young separately from that of their parents.

In the literature, altruism means caring about one's offspring welfare. There are two possible ways of modeling it. The first way, known as pure altruism, is to posit that parents value their children's welfare, as Barro (1974) did. The second way, known as warm-glow giving, is to say parents value the quality of education passed on to their children, as Glomm and Ravikumar (1992) did. With Barro's recursive modeling of altruism, one ends up expressing a household's welfare as a weighted sum of his life-cycle utility and that of each of his descendants. In our OLG framework, altruism is in the sense of Barro. All households therefore have their preferences defined over their own consumption and leisure and their children's welfare. As a consequence, allocating time to schooling or labor entails disutility. Most growth models with endogenous accumulation of human capital that we surveyed abstracted from these disutilities for convenience reasons. We have represented the preferences over consumption and leisure by a logarithmic utility function.

There are interesting features in other models that we have not taken into account. For instance, Tran-Nam, Truong, and Van Tu (1995) Shimomura and Tran-Nam (1997) introduced uncertainty in the outcome of education, *i.e.*, a student may or may not succeed in education. For convenience reasons, we have modeled a deterministic way the human capital production sector. In our model,, the human capital accumulated by a household depends positively and with certainty on his ability to learn, the time he has allocated to education, and the level of his parents' human capital. Similar approaches include, on the one hand, Lucas (1988) who modeled human capital as a cumulative outcome of the time allocated each period to education and, on the other hand, Glomm and Ravikumar (1992) who used a Cobb-Douglas technology to model the human capital accumulation process with as inputs the time allocated to education, the educational expenses, and the human capital inherited from parents. Whereas the quality of education is held constant in the former model, in the latter one, it depends on the household's educational expenses. In our model, the household's ability to learn is a time-dependent parameter whose motion may depend on several factors including the household's personal aptitude, the total factor productivity as Fowler and Young (2004) did, the quality of the available didactic resources as well as that of teachers.

The other production sector in our model is the one manufacturing the final output.

We have modeled this sector using a Cobb-Douglas technology with effective labor and physical capital, as inputs. The effective labor is the raw labor multiplied by the human capital stock. While such authors as Heckman, Lochner, and Taber (1998), Sadahiro and Shimasawa (2003), and Fowler and Young (2004) included physical capital as input other authors such as Glomm and Ravikumar (1992), Tran-Nam, Truong, and Van Tu (1995), and Shimomura and Tran-Nam (1997), seeking a tractable analytical solution, abstracted from this input. Our choice to include physical capital is motivated by the fact that households' decision to accumulate human capital through schooling impacts on their savings and consequently on physical capital accumulation because schooling entails allocating less time to labor in addition to paying tuition fees.

The rest of this paper consists of four sections. In the next section, which is Section 2, we have sketched our model. It is made up of two (production) sectors: the human capital sector operated by some overlapping generations of households and the final output sector operated by business firms. The model is characterized by an altruistic link between members of succeeding generations. In addition to educating their children at home, parents could financially help them while they are schooling.
Some theoretical results emerging from the optimizing behavior of households and firms are presented in Section 3. It appears that a young household substitutes education for labor at a rate greater than unity. Furthermore, within an altruistic economy, a young household always allocates more time to leisure than his parents do.
In Section 4, we have calibrated the dynamic deterministic general equilibrium (DDGE) model that we have built to some balanced growth facts observed in the Province of Quebec between 1981 and 2019. Then, we have solved numerically this model and simulated its transition paths assuming in turn a permanent rise in the tuition rate and in the household' ability to learn. The purpose of these simulations is to find out the long-run relationship between education, human capital, output, and some other key variables. These two shocks reveal a positive correlation between output and education and human capital, when the latter variable is assumed to be stationary.
Finally, in Section 5, we have used the predictions from the model to investigate the student crisis referred to as Maple Spring that Quebec witnessed in 2012. [3] As a matter of fact, on March 17, 2011, Quebec's former Liberal Finance Minister, Mr Raymond Bachand, announced in his 2011-2012 budget speech an increase in university tuition. From Fall 2012 till 2017, the full-time tuition would increase each year by \$ 325 to reach \$ 3,793 in 2017. To protest against this decision, students started on February 13, 2012 what became the longest student strike in Quebec's history. Our model predicts that students should not worry too much about the rise in tuition, since it will induce a rise in the educational transfer they receive along with an increase in their human capital stock. We also made some policy recommendations in that final section.

[3]The reference Maple Spring was made in relation to the popular uprisings named Arab spring that were going on in the Arab world at the same time. Maple, which is called *Érable* and is a homophone of *Arabe* in French, is a common tree in Quebec that produces syrup in the beginning of Spring

# 2 The Model

The economy consists of two agents: (1) some overlapping generations of households and (2) firms.

## 2.1 The households

Each generation of households lives for three periods of time. During the first period where they are young, households invest some time in formal education to accumulate human capital and work part-time, at the same time. Then, they become mature, are full-time employed, and procreate. During the third period, they are pensioned off and pass away later on. Population grows exponentially at the exogenous rate $0 < n < 1$. Households are altruistic and have preferences defined over consumption, leisure, and their children's welfare. Therefore, following Barro (1974), the welfare of a household born at time $t = 0, 1, 2, \ldots$ can be defined as

$$u_t = \sum_{g=0}^{2} \frac{1}{(1+\rho)^g} \left[\ln c_{g,t+g} + \sigma \ln \lambda_{g,t+g}\right] + \frac{1+n}{(1+\phi)(1+\rho)} u_{t+1},$$

hence the following (linear-in-life-cycle-utility) social welfare

$$u_s = \sum_{t=0}^{\infty} \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \sum_{g=0}^{2} \frac{1}{(1+\rho)^g} \left[\ln c_{g,t+g} + \sigma \ln \lambda_{g,t+g}\right], \quad \text{with } 0 < (1+n)/(1+\phi)(1+\rho) < 1. \tag{2.1}$$

The parameters $-1 < \phi < 1$, $0 < \rho < 1$, and $\sigma > 0$ are respectively the degree of parental altruism, the time preference rate, and the leisure weight. When $\phi < 0$, one has $\rho > \rho + \phi(1+\rho)$, *viz* the time preference rate $\rho$ used by the household to discount his own life-cycle utility is greater than $\rho + \phi(1+\rho)$, the intergenerational time preference rate. He is then said to be altruistic. On the other hand, a positive $\phi$ is a sign of selfishness. The variables $c_{g,t+g}$ and $0 < \lambda_{g,t+g} < 1$ are respectively the consumption and the leisure at time $t+g$ of a household aged $g$. A household faces the following constraints

$$\lambda_{gt} = 1 - e_{gt} - l_{gt}, \quad 0 \le e_{gt},\ l_{gt},\ \lambda_{gt} \le 1 \tag{2.2a}$$

$$h_{1t+1} = (1 + \psi_t e_{0t})\, h_{0t}, \qquad \psi_t > 0 \tag{2.2b}$$

$$h_{0t} = \frac{\gamma}{1+n} h_{1t}, \quad 0 < \gamma < 1 \tag{2.2c}$$

$$a_{1t+1} = w_t l_{0t} h_{0t} + \epsilon_{1t} - c_{0t} - f_t e_{0t} \tag{2.2d}$$

$$a_{2t+2} = w_{t+1} l_{1t+1} h_{1t+1} + (1 + r_{t+1}) a_{1t+1} - c_{1t+1} - (1+n)\epsilon_{1t+1} \tag{2.2e}$$

$$c_{2t+2} = (1 + r_{t+2}) a_{2t+2}. \tag{2.2f}$$

Constraint (2.2a) states a household shares his time endowment normalized to unity between leisure, education $e_{0t}$, and labor $l_{g,t+g}$. Constraint (2.2b) relates $h_{1t+1}$, the human capital stock after graduation, to the amount of time he allocated to education. The parameter $\psi_t$ in that constraint is the household's ability to learn. Relation (2.2c) is about home education. It says, parents by raising their children hand down to them some of their knowledge. [4] The share $\gamma$ is referred to as the intergenerational knowledge spill-over coefficient.

Relations (2.2d) through (2.2f) are the representative household's life cycle budget constraints. During the first period of his life, he receives both labor income and an educational transfer $\epsilon_{1t}$ from his parents. The variable $w_t$ denotes the real wage at time $t$. The labor income depends on both his hours worked and human capital stock. Out of his income, he finances his consumption and tuition; what is left is invested in financial assets. The tuition paid, $f_t e_{0t}$, is proportional to the time allocated to education. During the second period, the household receives both labor and financial assets income. The variable $r_{t+1}$ denotes the real interest rate at time $t+1$. From the budget constraint (2.2f), one could see that, during retirement, the household lives only on his financial assets income and leaves no bequest at the end of his life.

It is worth noting that households have perfect foresight and both formal education (*viz* attending a university) and home education are the only ways of accumulating human capital. The tuition rate (*i.e.* the tuition fee per credit hour) $f_t$ is an exogenous variable. The representative household maximizes (2.1), the social welfare, subject to constraints (2.2a) through (2.2f).

## 2.2 The Firms

The aggregate production technology operated by firms is of Cobb-Douglas type and is defined by

$$Y_t = K_t^{\alpha} \left[\exp(xt) H_t\right]^{1-\alpha}, \quad 0 < \alpha < 1. \tag{2.3}$$

The variables $Y_t$, $K_t$, and $H_t$ are respectively the aggregate output, physical capital, and effective labor. The parameters $x$ and $\alpha$ are respectively the exogenous rate of the labor-augmenting technological progress and the share of physical capital income in the aggregate output.

The firms maximize their profit defined as

$$\max_{K_t,\ H_t} K_t^{\alpha} \left[\exp(xt) H_t\right]^{1-\alpha} - (1 + r_t) K_t - w_t H_t.$$

In the above expression, $1+r_t$, the rental price of the aggregate physical capital, indicates that this input completely depreciates or becomes out-of-date at the end of each time period. Recall that a time period corresponds to a generation span.

[4]The expressions children, students, and young households are interchangeably used herein. *Idem* for the expressions mature households and parents. Given, on the one hand, the relationship between the households and, on the hand, their occupations, it has been difficult to stick to one expression.

# 3 The General Equilibrium and the Balanced Growth

Before defining the general equilibrium (*i.e.* the simultaneous equilibrium on all the markets), we present the equations that we have derived from the optimizing behavior of households and firms. Then, we will describe the behavior of the model along the balanced growth path (*i.e.* its long-run equilibrium behavior).

## 3.1 The General Equilibrium

The optimality conditions from households' utility maximization problem are (these results are detailed in Appendix A):

$$\sigma c_{0t} = w_t h_{0t}(1 - e_{0t} - l_{0t}) \tag{3.1a}$$

$$\sigma c_{1t} = w_t h_{1t}(1 - l_{1t}) \tag{3.1b}$$

$$(1 + r_{t+1})c_{0t} = (1 + \rho)c_{1t+1} \tag{3.1c}$$

$$(1 + r_{t+1})c_{1t} = (1 + \rho)c_{2t+1} \tag{3.1d}$$

$$c_{1t} = (1 + \phi)c_{0t} \tag{3.1e}$$

$$\begin{aligned}\gamma \left[ l_{0t} + \left(1 + \frac{f_t}{w_t h_{0t}}\right) e_{0t} \right] + l_{1t} = {} & \frac{1 + r_t}{\psi_{t-1}} \frac{w_{t-1}}{w_t} \left(1 + \frac{f_{t-1}}{w_{t-1} h_{0,t-1}}\right) \\ & - \frac{\gamma}{\psi_t} \left(1 + \frac{f_t}{w_t h_{0t}}\right).\end{aligned} \tag{3.1f}$$

The optimalaity conditions (3.1a) and (3.1b) show respectively the trade–off between consumption and leisure during the first and second stages of the life-cycle. The Euler equations (3.1c) and (3.1d) govern the inter-temporal choices of consumption. Relation (3.1e) explains the difference between a mature household and his children's contemporaneous consumption in terms of altruism ($\phi < 0$) or selfishness ($\phi > 0$). Finally, (3.1f) guides the intra-temporal trade-off between education and labor. Some theoretical evidence that emerges from the model are now highlighted.

**Proposition 3.1** (Education-Labor Trade-off)**.** *When the tuition is proportional to the time allocated to schooling, a unit increase (decrease) in the latter activity, ceteris paribus, results in a greater decrease (increase) in the time allocated to labor.*

*Proof.* It transpires from (3.1f) that the marginal rate of substitution of education for labor is greater than one in absolute value

$$\frac{\partial l_{0t+1}}{\partial e_{0t+1}} = -\left(1 + \frac{f_{t+1}}{w_{t+1} h_{0t+1}}\right).$$

□

Proposition 3.1 comes from the fact that increasing the time allocated to schooling occasions two costs: the forgone wage and the additional tuition fee. If tuition were lump-sum or formal education were free, the household would instead decrease the time allocated to labor on a one-one basis, when the time allocated to schooling increased.

**Proposition 3.2** (Altruistic Parents and their Children's Leisure)**.** *Within an altruistic economy, children allocate more time to leisure than their parents do.*

*Proof.* Consider (3.1a) describing the intra-temporal trade-off between consumption and leisure that a young household faces. Divide this by (3.1b) to get

$$\frac{c_{0t}}{c_{1t}} = \frac{h_{0t}}{h_{1t}}\frac{\lambda_{0t}}{\lambda_{1t}}.$$

Using (3.1e) to replace $c_{1t}$ in the above relation by $(1+\phi)c_{0t}$ and then calling on (2.2c), one gets

$$\frac{\lambda_{0t}}{\lambda_{1t}} = \frac{1+n}{\gamma(1+\phi)} > 1, \quad \text{for} -1 < \phi < 0, \tag{3.2}$$

which establishes the claim made in Proposition 3.2. □

Note that even if parents were selfish, *i.e.*, $\phi > 0$, (3.2) would still hold so long as $\phi < (1+n)/\gamma - 1$.

At equilibrium, both physical and human capital are remunerated at their marginal productivity.

$$\alpha\frac{Y_t}{K_t} = 1 + r_t \tag{3.3a}$$

$$(1-\alpha)\frac{Y_t}{H_t} = w_t \tag{3.3b}$$

**Definition 3.3** (General Equilibrium)**.** *It consists of prices* $\{(f_t, r_t, w_t)\}_{t=0}^{\infty}$*, an aggregate state of the world* $\{(\psi_t, \exp(xt))\}_{t=0}^{\infty}$*, and allocations:*

- $\{(a_{1t+1},\ c_{0t},\ e_{0t},\ l_{0t})\}_{t=0}^{\infty}$ *for young households,*
- $\{(a_{2t+1},\ c_{1t},\ \epsilon_{1t},\ h_{1t},\ l_{1t})\}_{t=0}^{\infty}$ *for mature households,*
- $\{(c_{2t})\}_{t=0}^{\infty}$ *for elderly households, and*
- $\{(H_t,\ K_t, Y_t)\}_{v=0}^{\infty}$ *for firms,*

*solving simultaneously:*

1. *the households' optimization problem, i.e., relations* (3.1a) *through* (3.1f) *along with the constraints* (2.2a) *through* (2.2f)*,*
2. *the firms' optimization problem, i.e., relations* (2.3)*,* (3.3b) *and* (3.3a)*,*

*and clearing*

3. *the financial and labor markets, i.e.,*

$$K_t = \left(a_{1t} + \frac{a_{2t}}{1+n}\right)\frac{N_{0t}}{1+n}$$
$$H_t = (\gamma l_{0t} + l_{1t})\,\frac{N_{0t}}{1+n}h_{1t},$$

*where* $N_{0t}$ *is the number of households born at time* $t$*,* $(t = 1, 2, \dots)$*.*

## 3.2 The Balanced Growth

The time households allocate to education and labor, respectively $e_{0t}$, $l_{0t}$, and $l_{1t}$, are stationary variables, *i.e.*, they fluctuate around their constant means $e_0$, $l_0$, and $l_1$. Following the literature (DeJong and Ingram 2001 and Fowler and Young 2004, among others), we have considered the household's ability to learn $\psi_t$ as a stationary parameter.

While human capital is a non-stationary variable in the model of Lucas (1988), Romer (1990) sustained that "a college-educated engineer working today and one working 100 years ago have the same human capital, which is measured in terms of years of forgone participation in the labor market". The stationarity of human capital is the most common assumption (see Glomm and Ravikumar, 1992; Heckman, Lochner, and Taber, 1998; DeJong and Ingram, 2001; Fowler and Young, 2004, among others).

One can express recursively the human capital stock after graduation, using (2.2b) and (2.2c).

$$h_{1t+1} = \frac{(1+\psi_t e_{0t})\gamma}{1+n} h_{1t}$$

It transpires that the long-run gross growth rate of $h_{1t}$ is equal to $(1+\psi e_0)\gamma/(1+n)$, which we denote $\nu$. If the human capital stock after graduation is a stationary variable, one will have $h_{1t+1} = h_{1t}$ along the balanced growth path (BGP), which implies the autoregressive parameter $\nu$ will equal one. This autoregressive parameter will be greater than one, if human capital is a non-stationary variable. We have considered these two possibilities (*i.e.* the stationary and the non-stationary of $h_{1t}$) in our investigations.

In Definition 3.3, the equilibrium condition in the labor market suggests the gross growth rate of $H_t$ along the BGP is $(1+n)\nu$. Given the gross growth rate of the aggregate effective labor and the labor-augmenting technological progress, the aggregate output and physical capital are constrained to grow along the BGP at the gross rate $(1+n)\nu\exp(x)$. Then, it turns out that the real wage $w_t$ grows at the same rate as the technology factor, whereas the real interest rate is stationary. The tuition rate $f_t$ and all the other per capita variables grow by the factor $\nu\exp(x)$.

We have removed the trend from the non-stationary variables by dividing them by their growth components, which gives rise to the variables $\hat{y}_t = Y_t/[N_{0t}\nu^t\exp(xt)]$, $\hat{k}_t = K_t/[N_{0t}\nu^t\exp(xt)]$, $\bar{h}_t = H_t/[N_{0t}\nu^t]$, $\tilde{w}_t = w_t/\exp(xt)$, $\bar{h}_{1t} = h_{1t}/\nu^t$, and $\hat{\mathbf{z}}_t = \mathbf{z}_t/[\nu^t\exp(xt)]$ with $\mathbf{z}_t = \{a_{1t}, a_{2t}, c_{0t}, c_{1t}, c_{2t}, f_t, \epsilon_{1t}\}$. The expressions that follow define some variables and ratios along the BGP.

$$r = (1+\phi)(1+\rho)\nu\exp(x) - 1 \tag{3.4a}$$

$$\tilde{w} = (1-\alpha)\left[\frac{\alpha}{(1+\phi)(1+\rho)\nu\exp(x)}\right]^{\frac{\alpha}{1-\alpha}} \tag{3.4b}$$

$$\frac{\hat{k}}{\hat{y}} = \frac{\alpha}{(1+\phi)(1+\rho)\nu\exp(x)} \tag{3.4c}$$

$$\frac{\hat{k}}{\bar{h}} = \left[\frac{\alpha}{(1+\phi)(1+\rho)\nu\exp(x)}\right]^{\frac{1}{1-\alpha}} \tag{3.4d}$$

$$e_0 = \frac{1}{\psi}\left(\frac{1+n}{\gamma}\nu - 1\right) \tag{3.4e}$$

$$\bar{h} = \frac{\nu}{\psi}\left[\frac{(1+\phi)(1+\rho)}{1+n} - 1\right]\left(\bar{h}_1 + \frac{1+n}{\gamma}\frac{\hat{f}}{\tilde{w}}\right) \tag{3.4f}$$

According to (3.4a) and (3.4b), an increase in the rate of the labor-augmenting technological progress $x$, the degree of parental altruism $\phi$, the time preference rate $\rho$, or the long-run growth rate of human capital $\nu$, *ceteris paribus*, raises the BGP interest rate and has the opposite effect on the real wage. This is due to the induced decrease in physical capital as (3.4c) or (3.4d) shows and to the induced increase in labor supply as (3.4f) suggests.

An increase in the population growth rate $n$, *ceteris paribus*, lessens the hours worked by households, as (3.4f) suggests. As for the time a household allocates to schooling, it increases as $n$ increases, according to (3.4e). A rise in the intergenerational knowledge spill-over coefficient $\gamma$ means young households are inheriting more human capital from their parents, which causes them to reduce the time they allocate to schooling. An improvement in $\psi$, the household's ability to learn, will also result in a decrease in the time allocated to education. This latter impact as well as that of an increase in the tuition rate are furthered in Subsection 4.2.

# 4 The Numerical Solution

We have normalized the life span of a household to nine years and each of the three stages of the life-cycle lasts three years. Fowler and Young (2004) took a similar approach in solving numerically their model. The rest of the model is calibrated to match some balanced growth facts observed in Quebec over the sample period 1981-2019.

## 4.1 The Calibration

**The Population Growth Rate** The population $N_t$ is made up of young, mature, and retired households. It grows exponentially at the rate $n$, *i.e.*, $N_t = N_0(1+n)^t$. To have an estimate of $n$, we have regressed the natural logarithm of the population aged 15 and over on an intercept and a linear time trend. The data used are from Statistics Canada and the ordinary least squares (OLS) estimate of the annual population growth rate is .009. The equivalent compound triennial rate is 2.7%.

$$\begin{array}{rcccc} \widehat{\ln N_t} & = & 15.445 & + & .009t \\ & & (7987) & & (104) \\ \multicolumn{5}{c}{R^2 = .997, t_{5\%}(37) = 1.69} \end{array} \tag{4.1}$$

According to (4.1), the log-linear model $\ln N_t = \ln N_0 + t\ln(1+n)$ explains 99.7% of the observed variability in the population aged 15 and over. All the coefficients of the regression are statistically significant, as the $t$-ratios in parentheses are greater than their 5% critical value.

**The Physical Capital's Share** We have used Statistics Canada's income based gross domestic product (GDP) to compute $\alpha$, the share of physical capital in the aggregate output, following Cooley and Prescott (1995) and Gomme and Rupert (2007). This share is defined as the ratio of the *unambiguous physical capital income* to the total unambiguous incomes

$$\alpha = \frac{\text{unambiguous physical capital income}}{\text{GDP - ambiguous income}}.$$

The unambiguous capital income is made up of the income-based GDP estimates that are considered as remunerating specifically the physical capital input. It consists of corporate profits before tax, interest and miscellaneous investment income, and the capital depreciation allowance. As for the ambiguous income, it remunerates indistinctly both labor and capital. It comprises the net income of farm operators and unincorporated business, the taxes on factors of production and products, and the statistical discrepancies. The average of this share is .309.

**The Rate of the Labor-Augmenting Technological Progress** We have computed this parameter doing some growth accounting. Log-differentiating (2.3) gives

$$x = \frac{\Delta \ln Y_t}{1-\alpha} - \frac{\alpha}{1-\alpha}\Delta \ln K_t - \Delta \ln H_t.$$

We have used the real GDP, the real gross capital stock, and the total hours worked as the respective measures of $Y_t$, $K_t$, and $H_t$. The average annual rate of technological progress turns out to be .01, which is equivalent to .03 over three years.

**The Long-Run Gross Growth Rate of Human Capital** The long-run growth rate of the aggregate output is $(1+n)\nu\exp(x)-1$. We have run a log-linear regression of the real output on an intercept and a time trend using annual data from Statistics Canada. From now on, we define output as the sum of household final consumption expenditure and business gross fixed capital formation.

$$\begin{array}{rcccc} \widehat{\ln Y_t} & = & 11.648 & + & .026t \\ & & (651) & & (20) \\ \multicolumn{5}{c}{R^2 = .951, t_{5\%}(37) = 1.69} \end{array} \tag{4.2}$$

This econometric model (4.2) fits 95.1% of the observed data. The slope parameter, which equals .026, is the OLS estimate of the annual long-run output growth rate. This growth rate is equivalent to 8% over three years. One solves for $\nu$, the long-run human capital growth rate, by equating this latter estimate to the expression $(1+n)\nu\exp(x)-1$, which yields $\nu = 1.022$. When we will be assuming that a household's human capital is stationary, $\nu$ will be set at one, as explained in Subsection 3.2.

**The Other Parameters** According to the Labor Force Survey of Statistics Canada, between 1981 and 2019, households working part-time in Quebec and those working full-time allocated respectively, on average, 16.9 and 36.5 hours a week to labor. It emerges from the six Censuses of the population conducted between 1981 and 2007 that the average weeks worked mostly part-time and full-time in Quebec are respectively 32.9 and 44.1. Besides, according to the General Social Survey (GSS) of Statistics Canada, between 1986 and 2015, students and workers in Quebec allocated respectively, on average, 10.9 and 10.5 hours a day to personal care, which includes night sleep and meals at home. Therefore, we have computed $l_0$ and $l_1$ as the ratios of the total hours worked by the households in part-time and full-time employment to their respective total *discretionary time*, which gives .117 and .327 respectively. The discretionary time is the number of hours that are not allocated to personal care. The GSS data reveal also that students allocated, on average, 6.1 hours a day to education and related activities. Knowing that an academic year represents about the two-thirds of a calendar year, the share of time allocated to education, $e_0$, equals .249.

It turns out that a mature household allocates a greater share of his time to leisure than a young (67.3% of the discretionary time for a mature household versus 63.4% for a young). According to Proposition 3.2, this announces that the degree of parental altruism parameter is positive (*viz* parents care more about their own welfare).

The average investment-output ratio is .208 (recall that output is defined as the sum of consumption and private investment). Since physical capital completely depreciates from one period to the other, one has $K_{t+1} = I_t$, where $I_t$ denotes the aggregate investment. [5] As the model constrains these two variables to grow at the same rate, one has along the BGP $(1+n)\nu \exp(x)\hat{k}/\hat{y} = \hat{i}/\hat{y}$. This means the implied long-run capital-output ratio, $\hat{k}/\hat{y}$, is .192.

The value of a household's human capital stock only affects the level of the variables and has no impact on the parameters and the ratios. Therefore, we have normalized $\bar{h}_1$ to unity. As for the intergenerational knowledge spill-over coefficient $\gamma$, the highest value it can assume is .79. Otherwise the tuition rate would be negative. This means one has to pay a household before he forgoes labor or leisure to allocate time to education, because he is already well instructed. For $\gamma$ lower than .79, the share of tuition fees in the aggregate output implied by the model will be much higher than the actual one, which is about .001. The value assigned to $\gamma$ impacts on such parameters as the time preference rate $\rho$, the leisure weight $\sigma$, and the degree of parental altruism $\phi$. Whereas $\rho$ increases along with $\gamma$, both $\sigma$ and $\phi$ decrease. Along the initial BGP, schooling accounts for 29.9%, *i.e.* $(1+n)/\gamma - 1$, of a household's human capital.

Table 4.1 displays the values assigned to all the parameters when human capital is assumed to be stationary (the case where $\nu = 1$) and when it is assumed to be non-stationary (the case where $\nu = 1022$). The calibration exercise points out a high selfishness. Given the high values of $\phi$ and $\rho$ displayed in Table 4.1, .379 and .107 respec-

[5] The annual depreciation rate of physical capital averages 5.2%, between 1981 and 2019. This means physical capital lasts about 19.2 years. Thus, assuming that physical capital depreciates completely within one time interval primarily means a complete depreciation over a generation span.

Table 4.1: The Parameters of the Model

| | | | $\boldsymbol{\nu} = 1$ | $\boldsymbol{\nu} = 1.022$ |
|---|---|---|---|---|
| **Households** | $n$ | Population growth rate | .027 | 027 |
| | $\gamma$ | Knowledge spill-over coefficient | .790 | .790 |
| | $\rho$ | Time preference rate | .107 | .107 |
| | $\sigma$ | Leisure weight | 4.464 | 4.333 |
| | $\phi$ | Degree of parental altruism | .379 | .379 |
| | $\psi$ | Household ability to learn | 1.313 | 1.201 |
| **Firms** | $x$ | Rate of technological progress | .030 | .030 |
| | $\alpha$ | Physical capital share | .309 | .309 |

tively, it appears that households put more weight on their own current consumption than on their children's. Along the initial BGP, even though the educational transfer exceed the tuition fees, students still have to work and borrow money to finance their consumption. One can interpret the educational transfer as a lump-sum tax raised on mature households' income and entirely transferred to the younger generation as de la Croix and Michel (2002, pp 129-30) did. In this particular case, the analysis of the initial BGP indicates that students cannot pay their tuition and live on the grants or loans they receive from the government. They end up with debts after their education.

## 4.2 The Findings

We are investigating the responses of our model economy to a rise in the tuition rate and in the household's ability to learn. These two exogenous variables influence households' decision to invest in education. We are also interested in knowing if these responses depend on the assumption made about the time series characteristics of human capital: stationarity versus non-stationarity.

For this purpose, we have solved numerically and simulated the dynamic paths of our model using the software platform Dynare (`https://www.dynare.org/`). To simulate the model, we have generated in turn an exogenous rise of one percent in the tuition rate and in the household's ability to learn. This rise is permanent and expected in the first period. In each case, we have simulated the model over ten periods of time and we have sketched the transitional dynamics of such key variables as the time allocated to education and labor, the human and the physical capital stocks, the educational transfer, the wage, and the output.

### The Model with a Stationary Human Capital

In this case, the long-run gross growth rate of human capital, $\nu$, is set at one. How a new BGP is reached after a one percent permanent increase in the tuition rate is plotted in blue line in Figure 4.1. When education becomes less affordable, students initially decrease the time they allocate to education (panel 1) to increase the time they allocate

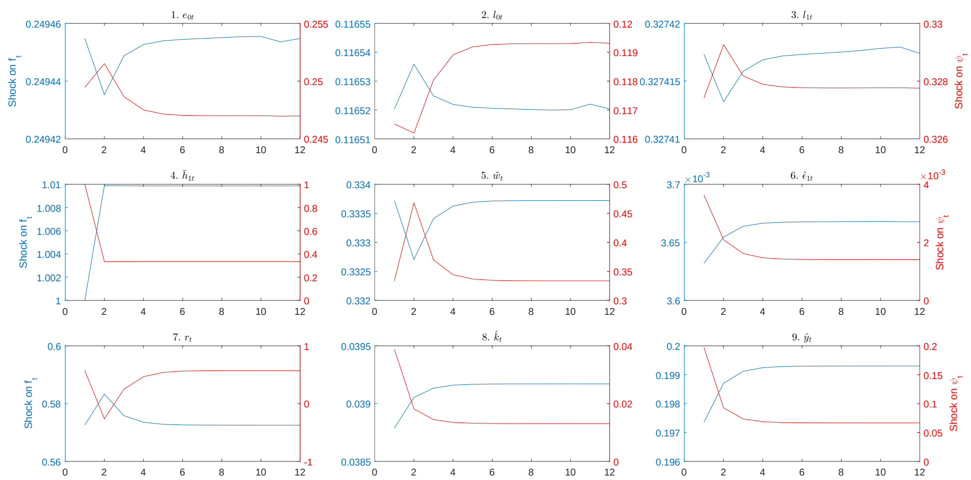


Figure 4.1: The Model with a Stationary Human Capital: Dynamic Paths of some Key Variables after a Permanent 1% Rise in the Tuition Rate (left-hand side axes of the panels) and a .1% Rise in the Household's Ability to Learn (right-hand side axes)

Table 4.2: The Model with a Stationary Human Capital: Simulated Correlation with Output

| | $e_{0t}$ | $h_{1t}$ | $\hat{\epsilon}_{1t}$ |
|---|---|---|---|
| Shock on $\hat{f}_t$ | .177 | .952 | .998 |
| Shock on $\psi_t$ | .548 | .979 | .994 |

to labor (panel 2). At the same time, mature households decrease the time they allocate to labor (panel 3) because wage has decreased (panel 5) and interest rate has increased (panel 7). The rise in interest rate raises mature households' wealth (panel 8), which induces a rise in the educational transfer they grant their children (panel 6). Then, this grant encourages students to start working less and to allocate more time to education. When the new BGP is reached, the time allocated to education returns to its initial level, because in relation (3.4e) $\nu$ is fixed at one. So do the hours worked, the wage rate, and the interest rate. But, the stocks of human capital, the aggregate output, and the educational transfer remain higher. This enables us to conclude that, when human capital is stationary, the increase in the tuition rate favors education and growth.

The red lines in Figure 4.1 show the transitional dynamics after a .one percent rise in the household's ability to learn. In most cases, the shapes of these transitional dynamics are the opposite of those generated by the rise in the tuition rate. The increase in the household's ability to learn occasions an economic downturn. As one could predict from relation (3.4e), households end up allocating less time to education. The level of human capital stock, output, and educational transfer also drop, along the final BGP.

For each of the two shocks (*i.e.* the rise in $f_t$ and the one in $\psi_t$), Table 4.2 displays

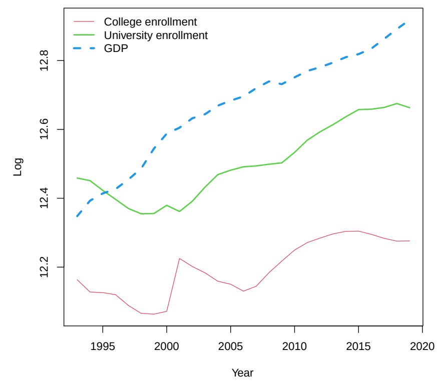


Figure 4.2: Post-secondary Enrollments and GDP, Quebec, 1992-2019

the correlation coefficients of the simulated output series with the time allocated to education, the stock of human capital, and the educational transfer. All the two shocks reveal a positive and high correlation between the aggregate output and the variables of interest. Figure 4.2 plots the natural logarithm of the college and university enrollments, and the real GDP in Quebec. In the literature, college enrollment is used as a proxy for human capital accumulation (DeJong and Ingram, 2001; Fowler and Young, 2004). It shows a positive relation between the latter variable and the real GDP. Their correlation coefficient is .789.

### The Model with a Non-Stationary Human Capital

The long-run gross growth rate of human capital is now a stationary endogenous variable defined as

$$\nu_t = \gamma \frac{1 + \psi_t e_{0t}^*}{1 + n},$$

where $e_{0t}^*$ denotes the value of $e_{0t}$ along the current BGP.

The blue lines in Figure 4.3 represent the response of the variables to a one-percent permanent rise in the tuition rate, when human capital is non-stationary. It appears in the first panel that the time allocated to education falls and does not return to its initial level as in Figure 4.1. Thus, students substitute labor for education definitely (panel 2). The mature households also decrease their labor supply for good (panel 3), due to the permanent rise in the real wage rate (panel 5). The educational transfer parents grant their children has fallen (panel 6), due to the negative income effect generated by the decrease in the real interest rate (panel 7). It follows that the stock of human capital returns to its initial level after rising (panel 4), but its long-run gross growth rate falls. However, the economy ends ups with a higher level of physical capital stock (panel 8) and output (panel 9), as it also appears in Figure 4.1.

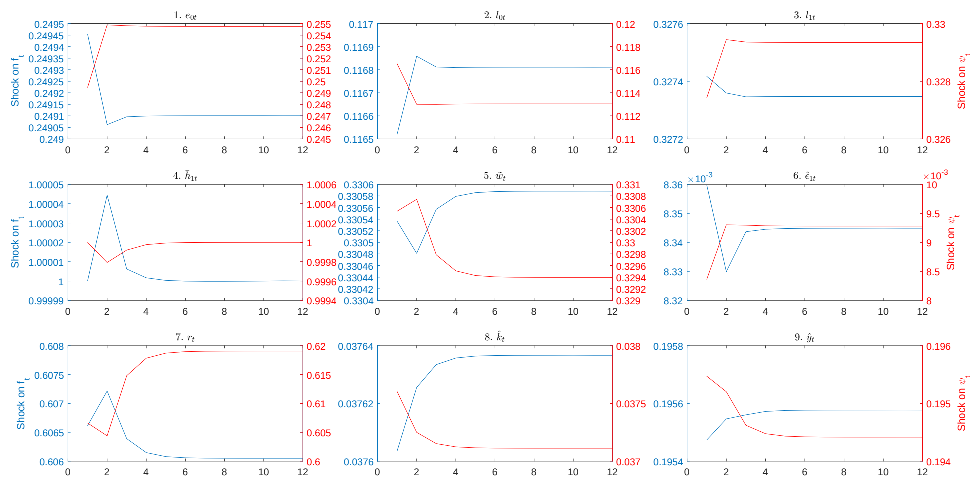


Figure 4.3: The Model with a Non-Stationary Human Capital: Dynamic Paths of some Key Variables after a Permanent 1% Rise in the Tuition Rate (left-hand side axes of the panels) and a .1% Rise in the Household's Ability to Learn (right-hand side axes)

Table 4.3 shows that the percentage changes in output, consumption, and leisure increase along with the magnitude of the exogenous rise in the tuition rate. Whether human capital is assumed to be stationary or not, our model predicts that a rise in the tuition rate promotes economic growth and raises the social welfare (the utility of households is a function of consumption and leisure).

After the one-percent rise in the household's ability to learn, the value of the long-run gross growth rate of human capital, $\nu_t$, increases from 1.022 to 1.03 and the time allocated to education rises for good (panel 1). As a result, the real interest rate also rises definitely, consistently with relation (3.4a). The rise in the time allocated to education by students and the subsequent drop in their labor market activity that appears in

Table 4.3: The Model with a Non-Stationary Human Capital: Percentage Change in some Key Variables a Rise in Tuition

| % Change in Tuition | Consumption | Leisure | Human Capital Growth $\nu_t$ | Output |
|---|---|---|---|---|
| 1 | .026 | .010 | -.035 | .053 |
| 5 | .131 | .052 | -.175 | .265 |
| 10 | .263 | .105 | -.350 | .529 |
| 15 | .396 | .159 | -.525 | .792 |
| 20 | .529 | .214 | -.701 | 1.053 |
| 25 | .664 | .269 | -.876 | 1.313 |
| 50 | 1.353 | .554 | -1.752 | 2.592 |
| 100 | 2.804 | 1.176 | -3.505 | 5.048 |

Figure 4.3 is opposite to what appears in Figure 4.1. The response of the wage rate, the real interest rate, the physical capital, and the output to a shock on $\psi_t$ are stronger, when human capital is assumed to be stationary.

# 5 Discussion

We have used herein an altruistic OLG model in which young households receive from their parents some home education and financial support. Then, they add to their human capital stock by completing some formal education while working part-time. Some theoretical evidence emerge from the model: (1) when the tuition is proportional to the time allocated to education, the marginal rate of substitution of education for labor is greater than unity in absolute value and (2) within an altruistic economy, young households allocate more time to leisure than their parents do.

We have calibrated the DDGE model that we have built to match some balanced growth facts observed in the Province of Quebec over the period 1981-2019. The calibration results indicate *inter alia* that parents do not care enough about the welfare of their children and that education accounts for thirty percents of a household's human capital. Thereafter, we have used the calibrated model to investigate the contribution to economic growth of human capital accumulation through education. To do that, first, we have identified two exogenous variables that could affect a household's decision to invest in education. These variables are the the tuition rate and the household's ability to learn. Then, we have solved the model numerically and simulated its transition paths assuming in turn a permanent increase in the tuition rate and the household's ability to learn.

In our investigations, we have not lost the sight of fact that, in the literature, while most authors assume implicitly that human capital is stationary, others model it is a non-stationary variable. Without regard to this assumption, our model predicts that a rise in the tuition favors economic growth and improves the social welfare. As for the rise in the household's ability to learn, it discourages economic growth. However, the correlation between output and human capital accumulation that transpire from the simulations are contingent on the assumption made about the stationarity of the latter variable. When human capital is stationary, this correlation is positive, which is consistent with observations. Therefore, the findings from our model with a stationary human capital can be used to address some topical issues in Quebec.

Should the government increase the tuition fees as the former Liberal Finance Minister Mr Raymond Bachang announced in his 2011-2012 budget speech? Were students right to protest against this decision, were their worries justified? Our model predicts that, when human capital is stationary, an increase in the tuition rate will not at all affect, in the *long-run*, the time allocated to education. The time allocated to education and to labor as well as the real wage rate will move back to their initial levels after falling. Human capital stocks and output will increase. Would education become less affordable after the increase in tuition, as some of the protesters sustained? Our model predicts that the rise in the tuition rate will not harm students' ability to pay for their education since during the transition to the new BGP, parents (or the society) will adjust accordingly the financial support they grant them.

We will use now our model to assess three education policy measures: (1) free university education in Quebec as proposed by some students union leaders, (2) indexing tuition to the rate of growth of households' disposable income, as the Parti Québécois led by Mrs Pauline Marois recommended in February 2013 during the submit on higher education, and (3) investing in cultural and sportive activities on campuses and acquiring up-to-date didactic resources.
Free education means a negative shock to the tuition. The effects of this measure will be the opposite of those illustrated in the previous section. At first, education will become popular insofar as the share of time allocated to this activity will rise. After some periods, this share will move down to its initial level. Output and human capital will fall. On the other hand, indexing the tuition rate is equivalent to maintaining the *status quo* in the *normalized* model. The economy will keep moving along its initial BGP.
As for investing in extra-curricular activities on campuses or in new didactic resources, this will improve students' ability to learn. With less effort, they could accumulate the same amount of knowledge as the previous cohorts. But, a substantial reduction in the time allocated to education will end up reducing the level of the human capital stock. When this policy is implemented, a way of avoiding this fall in the human capital stock would be not to reduce the financial help granted to students.

To finish with, we address a modeling issue. A household's human capital depends on both the time he has allocated to education and his ability to learn. We have not included in this production function the tuition or the educational expenses, as some authors did. A reason why we have chosen not to include these inputs is the following. Since the time the Parti Québécois abolished the decision to increase the tuition the way the Liberal Party proposed, universities have been able to cut their budget and to deliver the same quality of education. Therefore, we have concluded that it would not be efficient to include at this stage the tuition in the human capital production function. It will be quite straightforward to include the tuition in the human capital production function of our model. One has just to redefine the current household ability to learn as the product of the tuition rate and a human capital productivity parameter, which are all exogenous variables.

## References

Aghion, P., and P. Howitt (1998): *Endogenous Growth Theory*. The MIT Press, Cambridge, MA, US.

——— (2009): *The Economics of Growth*. The MIT Press, Cambridge, MA, US.

Barro, R. J. (1974): "Are government bonds net wealth?," *Journal of political economy*, 82(6), 1095–1117.

——— (1991): "Economic growth in a cross section of countries," *The quarterly journal of economics*, 106(2), 407–443.

——— (2001): "Human Capital: Growth, History, and Policy - A Session to Honor Stanley Engerman," *The American Economic Review*, 91(2), 12–7.

Becker, G. (1964): "Human capital," *National Bureau for Economic Research, New York*.

Bils, M., and P. J. Klenow (2000): "Does schooling cause growth?," *American economic review*, 90(5), 1160–1183.

Cooley, T. F., and E. C. Prescott (1995): "Economic growth and business cycles," *Frontiers of business cycle research*, 1.

de la Croix, D., and P. Michel (2002): *A theory of economic growth: dynamics and policy in overlapping generations*. Cambridge University Press.

DeJong, D. N., and B. F. Ingram (2001): "The cyclical behavior of skill acquisition," *Review of Economic Dynamics*, 4(3), 536–561.

Diamond, P. A. (1965): "National debt in a neoclassical growth model," *The American Economic Review*, 55(5), 1126–1150.

Fowler, S. J., and E. R. Young (2004): "The Acquisition of Skills Over the Life-Cycle," Discussion paper.

Glomm, G., and B. Ravikumar (1992): "Public versus private investment in human capital: endogenous growth and income inequality," *Journal of political economy*, 100(4), 818–834.

Gomme, P., and P. Rupert (2007): "Theory, measurement and calibration of macroeconomic models," *Journal of Monetary Economics*, 54(2), 460–497.

Heckman, J. J., L. Lochner, and C. Taber (1998): "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," *Review of Economic Dynamics*, 1, 1–58.

Lucas, R. E. (1988): "On the mechanics of economic development," *Journal of Monetary Economics*, (22), 3–42.

Mankiw, N. G., D. Romer, and D. N. Weil (1992): "A Contribution to the Empirics of Economic Growth," *The Quarterly Journal of Economics*, 107(2), 407–43.

Razin, A. (1972): "Optimum investment in human capital," *The Review of Economic Studies*, 39(4), 455–460.

Romer, P. M. (1990): "Endogenous technological change," *Journal of political Economy*, 98(5, Part 2), S71–S102.

Sadahiro, A., and M. Shimasawa (2003): "The computable overlapping generations model with an endogenous growth mechanism," *Economic Modelling*, 20(1), 1–24.

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with and without the Social Contrivance of Money," *The Journal of Political Economy*, 66(6), 467–82.

Shimomura, K., and B. Tran-Nam (1997): "Education, Human Capital, and Economic Growth in an Overlapping Generations Model," *Journal of Economics and Business Administration (Kokumin-Keizai Zasshi)*, 175, 63–79.

Tran-Nam, B., C. N. Truong, and P. N. Van Tu (1995): "Human capital and economic growth in an overlapping generations model," *Journal of Economics*, 61(2), 147–173.

# Appendices

## A The Households' Optimization Problem

As mentioned in Subsection 2.1, the household born at time $t$ seeks to maximize his life-cycle utility

$$u_s = \sum_{t=0}^{\infty} \left[ \frac{1+n}{(1+\phi)(1+\rho)} \right]^{t-s} \sum_{g=0}^{2} \frac{1}{(1+\rho)^g} \left[ \ln c_{gt+g} + \sigma \ln \lambda_{gt+g} \right].$$

subject to:

$$\begin{aligned}
\lambda_{gt} &= 1 - e_{gt} - l_{gt}, \quad 0 \leq e_{gt},\ l_{gt},\ \lambda_{gt} \leq 1 \\
h_{1t+1} &= (1 + \psi_t e_{0t})\, h_{0t}, \qquad \psi > 0 \\
h_{0t} &= \frac{\gamma}{1+n} h_{1t}, \quad 0 < \gamma < 1 \\
a_{1t+1} &= w_t h_{0t} l_{0t} + \epsilon_{1t} - c_{0t} - f_t e_{0t} \\
a_{2t+2} &= w_{t+1} h_{1t+1} l_{1t+1} + (1 + r_{t+1}) a_{1t+1} - c_{1t+1} - (1+n)\epsilon_{1t+1} \\
c_{2t+2} &= (1 + r_{t+2}) a_{2t+2},
\end{aligned}$$

given $f_t, r_t$, $w_t$, and $\psi_t$.

This optimization problem can be solved using either the method of Lagrange (*i.e.* for a single generation of households over the three periods of the life-cycle) or the equation of Bellman (*i.e.* for a cross-section of three contemporaneous generations of households). We have derived the first order conditions (FOCs) and the Euler equations using each of these two methods, in turn. Beforehand, we have reduced the number of constraints from eight to four by substituting the three time constraints in the objective function and replacing $h_{0t}$, the human capital stock inherited at birth, by its expression in the rest of the constraints.

### A.1 Solving the Problem Using the Method of Lagrange

$$\begin{aligned}
\mathcal{L}_t = \max & \sum_{t=0}^{\infty} \left[ \frac{1+n}{(1+\phi)(1+\rho)} \right]^{t-s} \left[ \ln c_{0t} + \sigma \ln(1 - e_{0t} - l_{0t}) \right] + \\
& \sum_{t=0}^{\infty} \left[ \frac{1+n}{(1+\phi)(1+\rho)} \right]^{t-s} \left\{ \frac{1}{(1+\rho)} \left[ \ln c_{1t+1} + \sigma \ln(1 - l_{1t+1}) \right] + \frac{1}{(1+\rho)^2} \ln c_{2t+2} \right\} \\
& + \mu_{1t} \left[ \frac{\gamma}{1+n} w_t h_{1t} l_{0t} + \epsilon_{1t} - c_{0t} - f_t e_{0t} - a_{1t+1} \right] \\
& + \mu_{2t} \left[ w_{t+1} h_{1t+1} l_{1t+1} + (1 + r_{t+1}) a_{1t+1} - c_{1t+1} - (1+n)\epsilon_{1t+1} - a_{2t+2} \right] \\
& + \mu_{3t} \left[ (1 + r_{t+2} a_{2t+2} - c_{2t+2} \right] + \mu_{4t} \left[ (1 + \psi_t e_{0t}) \frac{\gamma}{1+n} h_{1t} - h_{1t+1} \right],
\end{aligned}$$

where the variables $\mu_{1t}, \ldots, \mu_{4t}$ are the Lagrange multipliers or shadow prices and the variables $f_t$, $r_t$, $w_t$, and $\psi_t$ are given.

### The FOCs

$$c_{0t}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{1}{c_{0t}} = \mu_{1t} \tag{A.1a}$$

$$c_{1t+1}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{1}{1+\rho}\frac{1}{c_{1t+1}} = \mu_{2t} \tag{A.1b}$$

$$c_{2t+2}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{1}{(1+\rho)^2}\frac{1}{c_{2t+2}} = \mu_{3t} \tag{A.1c}$$

$$e_{0t}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{\sigma}{1-e_{0t}-l_{0t}} + f_t\mu_{1t} = \psi_t\frac{\gamma}{1+n}h_{1t}\mu_{4t} \tag{A.1d}$$

$$l_{0t}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{\sigma}{1-e_{0t}-l_{0t}} = \frac{\gamma}{1+n}w_t h_{1t}\mu_{1t} \tag{A.1e}$$

$$l_{1t+1}: \quad \left[\frac{1+n}{(1+\phi)(1+\rho)}\right]^{t-s} \frac{1}{1+\rho}\frac{\sigma}{1-l_{1t+1}} = w_{t+1}h_{1t+1}\mu_{2t} \tag{A.1f}$$

$$\begin{aligned} h_{1t+1}: w_{t+1}l_{1t+1}\mu_{2t} &+ \frac{\gamma}{1+n}w_{t+1}l_{0t+1}\mu_{1t+1} \\ &+(1+\psi_{t+1}e_{0t+1})\frac{\gamma}{1+n}\mu_{4t+1} = \mu_{4t} \end{aligned} \tag{A.1g}$$

$$a_{1t+1}: \quad (1+r_{t+1})\mu_{2t} = \mu_{1t} \tag{A.1h}$$

$$a_{2t+2}: \quad (1+r_{t+2})\mu_{3t} = \mu_{2t} \tag{A.1i}$$

$$\epsilon_{1t+1}: \quad (1+n)\mu_{2t} = \mu_{1t+1} \tag{A.1j}$$

$$\mu_{1t}: \quad \frac{\gamma}{1+n}w_t h_{1t} l_{ot} + \epsilon_{1t} - c_{0t} - f_t e_{0t} = a_{1t+1} \tag{A.1k}$$

$$\begin{aligned} \mu_{2t}: \quad & w_{t+1}h_{1t+1}l_{1t+1} + (1+r_{t+1})a_{1t+1} \\ & -c_{1t+1} - (1+n)\epsilon_{1t+1} = a_{2t+2} \end{aligned} \tag{A.1l}$$

$$\mu_{3t}: \quad (1+r_{t+2}a_{2t+2} = c_{2t+2} \tag{A.1m}$$

$$\mu_{4t}: \quad (1+\psi_t e_{0t})\frac{\gamma}{1+n}h_{1t} = h_{1t+1} \tag{A.1n}$$

The leads in the second and third elements on the left-hand side of (A.1g) come from the fact that a fraction of the level of human capital stock $h_{1t+1}$ chosen by a young household at time $t$ is inherited by his children during the next time period. The lead of the Lagrange multiplier on the right-hand side of (A.1j) points to the fact that the educational transfer is granted by a household born at time $t$ and cashed by households born at $t+1$.

**The Optimality Conditions**

We can get rid of the Lagrange multipliers in the FOCs (A.1a) through (A.1j), by substituting some equations or their first lead into others.

$$\sigma c_{0t} = \frac{\gamma}{1+n} w_t h_{1t}(1 - e_{0t} - l_{0t}) \tag{A.2a}$$

$$\sigma c_{1t+1} = w_{t+1} h_{1t+1}(1 - l_{1t+1}) \tag{A.2b}$$

$$(1 + r_{t+1}) c_{0t} = (1 + \rho) c_{1t+1} \tag{A.2c}$$

$$(1 + r_{t+2}) c_{1t+1} = (1 + \rho) c_{2t+2} \tag{A.2d}$$

$$c_{1t+1} = (1 + \phi) c_{0t+1} \tag{A.2e}$$

$$\begin{aligned}\gamma \left[ l_{0t+1} + \left(1 + \frac{1+n}{\gamma} \frac{f_{t+1}}{w_{t+1} h_{1t+1}}\right) e_{0t+1} \right] + l_{1t+1} &= \frac{1 + r_{t+1}}{\psi_t} \frac{w_t}{w_{t+1}} \left(1 + \frac{1+n}{\gamma} \frac{f_t}{w_t h_{1t}}\right) \\ &\quad - \frac{\gamma}{\psi_{t+1}} \left(1 + \frac{1+n}{\gamma} \frac{f_{t+1}}{w_{t+1} h_{1t+1}}\right)\end{aligned} \tag{A.2f}$$

Relation (A.2a), for instance, is obtained after plugging (A.1a) into (A.1e). Now, we focus on showing how one gets (A.2f). Combine (A.1d) and (A.1e), to express $\mu_{4t}$ as a function of $\mu_{1t}$.

$$\mu_{4t} = \left(w_t + \frac{1+n}{\gamma} \frac{f_t}{h_{1t}}\right) \frac{\mu_{1t}}{\psi_t}$$

Plugging this latter expression and its first lead into (A.1g) yields

$$\begin{aligned} &w_{t+1} l_{1t+1} \mu_{2t} + \frac{\gamma}{1+n} w_{t+1} l_{0t+1} \mu_{1t+1} \\ &+ \frac{\gamma}{1+n}(1 + \psi_{t+1} e_{0t+1}) \left(w_{t+1} + \frac{1+n}{\gamma} \frac{f_{t+1}}{h_{1t+1}}\right) \frac{\mu_{1t+1}}{\psi_{t+1}} = \left(w_t + \frac{1+n}{\gamma} \frac{f_t}{h_{1t}}\right) \frac{\mu_{1t}}{\psi_t}. \end{aligned}$$

Replacing in the above equation $\mu_{1t}$ and $\mu_{1t+1}$ by their expressions as they appear respectively in (A.1h) and (A.1j) helps get rid of all the Lagrange multipliers.

$$\begin{aligned} &\left[ w_{t+1} l_{1t+1} + \gamma w_{t+1} l_{0t+1} + \frac{\gamma}{\psi_{t+1}} (1 + \psi_{t+1} e_{0t+1}) \left(w_{t+1} + \frac{1+n}{\gamma} \frac{f_{t+1}}{h_{1t+1}}\right) \right] \mu_{2t} \\ &\qquad = \frac{1 + r_{t+1}}{\psi_t} \left(w_t + \frac{1+n}{\gamma} \frac{f_t}{h_{1t}}\right) \mu_{2t} \end{aligned}$$

One gets (A.2f), after rearranging.

## A.2 Solving the Problem Using the Equation of Bellman

The *state* variables in our dynamic model are $a_{1t}$, $a_{2t}$, and $h_{1t}$. These three variables are enough to determine the level of the social welfare at time $t$. Likewise, their future values determine the future level of the social welfare. The Bellman equation enables to link recursively the social welfare of the three generations of households coexisting at time $t$ to that of the next coexisting generations of households.

$$\begin{aligned}
\mathcal{V}(a_{1t}, a_{2t}, h_{1t}) = \max\ & \ln c_{0t} + \sigma \ln(1 - e_{0t} - l_{0t}) + \frac{1+\phi}{1+n}\left[\ln c_{1t} + \sigma \ln(1 - l_{1t})\right] \\
& + \left(\frac{1+\phi}{1+n}\right)^2 \ln c_{2t} + \frac{1+n}{(1+\phi)(1+\rho)} \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1}) \\
& + \tilde{\mu}_{1t}\left[\frac{\gamma}{1+n} w_t h_{1t} l_{0t} + \epsilon_{1t} - c_{0t} - f_t e_{0t} - a_{1t+1}\right] \\
& + \tilde{\mu}_{2t}\left[w_t h_{1t} l_{1t} + (1+r_t) a_{1t} - c_{1t} - (1+n)\epsilon_{1t} - a_{2t+1}\right] \\
& + \tilde{\mu}_{3t}\left[(1+r_t) a_{2t} - c_{2t}\right] + \tilde{\mu}_{4t}\left[(1 + \psi_t e_{0t}) \frac{\gamma}{1+n} h_{1t} - h_{1t+1}\right]
\end{aligned}$$

**The FOCs**

$$c_{0t}: \quad \frac{1}{c_{0t}} = \tilde{\mu}_{1t} \tag{A.3a}$$

$$c_{1t}: \quad \frac{1+\phi}{1+n}\frac{1}{c_{1t}} = \tilde{\mu}_{2t} \tag{A.3b}$$

$$c_{2t}: \quad \left(\frac{1+\phi}{1+n}\right)^2 \frac{1}{c_{2t}} = \tilde{\mu}_{3t} \tag{A.3c}$$

$$e_{0t}: \quad \frac{\sigma}{1 - e_{0t} - l_{0t}} + f_t \tilde{\mu}_{1t} = \psi_t \frac{\gamma}{1+n} h_{1t} \tilde{\mu}_{4t} \tag{A.3d}$$

$$l_{0t}: \quad \frac{\sigma}{1 - e_{0t} - l_{0t}} = \frac{\gamma}{1+n} w_t h_{1t} \tilde{\mu}_{1t} \tag{A.3e}$$

$$l_{1t}: \quad \frac{1+\phi}{1+n}\frac{\sigma}{1 - l_{1t}} = w_t h_{1t} \tilde{\mu}_{2t} \tag{A.3f}$$

$$h_{1t+1}: \quad \frac{1+n}{(1+\phi)(1+\rho)} \frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial h_{1t+1}} = \tilde{\mu}_{4t} \tag{A.3g}$$

$$a_{1t+1}: \quad \frac{1+n}{(1+\phi)(1+\rho)} \frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial a_{1t+1}} = \tilde{\mu}_{1t} \tag{A.3h}$$

$$a_{2t+2}: \quad \frac{1+n}{(1+\phi)(1+\rho)} \frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial a_{2t+1}} = \tilde{\mu}_{2t} \tag{A.3i}$$

$$\epsilon_{1t}: \quad (1+n)\tilde{\mu}_{2t} = \tilde{\mu}_{1t} \tag{A.3j}$$

We dropped the FOCs with respect to the shadow prices $\tilde{\mu}_{1t}$, …, $\tilde{\mu}_{4t}$.

**The envelope conditions**

The derivatives $\partial\mathcal{V}()/\partial h_{1t+1}$, $\partial\mathcal{V}()/\partial a_{1t+1}$, $\partial\mathcal{V}()/\partial a_{2t+1}$, which appears respectively in the FOCs (A.3g), (A.3h), and (A.3i), are unknown and will be found using the envelope conditions.

$$\begin{aligned}
\frac{\partial \mathcal{V}(a_{1t}, a_{2t}, h_{1t})}{\partial h_{1t}} &= w_t l_{1t} \tilde{\mu}_{2t} + \frac{\gamma}{1+n} w_t l_{0t} \tilde{\mu}_{1t} \quad + (1 + \psi_t e_{0t}) \frac{\gamma}{1+n} \tilde{\mu}_{4t} \Rightarrow \\
\frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial h_{1t+1}} &= w_{t+1} l_{1t+1} \tilde{\mu}_{2t+1} + \frac{\gamma}{1+n} w_{t+1} l_{0t+1} \tilde{\mu}_{1t+1} \\
&\quad + (1 + \psi_{t+1} e_{0t+1}) \frac{\gamma}{1+n} \tilde{\mu}_{4t+1} \qquad \text{(A.4a)} \\
\frac{\partial \mathcal{V}(a_{1t}, a_{2t}, h_{1t})}{\partial a_{1t}} &= (1 + r_t) \tilde{\mu}_{2t} \Rightarrow \\
\frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial a_{2t+1}} &= (1 + r_{t+1}) \tilde{\mu}_{2t+1} \qquad \text{(A.4b)} \\
\frac{\partial \mathcal{V}(a_{1t}, a_{2t}, h_{1t})}{\partial a_{2t}} &= (1 + r_t) \tilde{\mu}_{3t} \Rightarrow \\
\frac{\partial \mathcal{V}(a_{1t+1}, a_{2t+1}, h_{1t+1})}{\partial a_{2t+1}} &= (1 + r_{t+1}) \tilde{\mu}_{3t+1} \qquad \text{(A.4c)}
\end{aligned}$$

**The Optimality Conditions**

Now, the envelope conditions (A.4a) through (A.4c) can be plugged respectively into the FOCs (A.3g), (A.3h), and (A.3i). After that, one gets rid of the shadow prices in the FOCs to have

$$\begin{aligned}
\sigma c_{0t} &= \frac{\gamma}{1+n} w_t h_{1t} (1 - e_{0t} - l_{0t}) \qquad \text{(A.5a)} \\
\sigma c_{1t} &= w_t h_{1t} (1 - l_{1t}) \qquad \text{(A.5b)} \\
(1 + r_{t+1}) c_{0t} &= (1 + \rho) c_{1t+1} \qquad \text{(A.5c)} \\
(1 + r_{t+1}) c_{1t} &= (1 + \rho) c_{2t+1} \qquad \text{(A.5d)} \\
c_{1t} &= (1 + \phi) c_{0t} \qquad \text{(A.5e)}
\end{aligned}$$

$$\begin{aligned}
\gamma \left[ l_{0t+1} + \left( 1 + \frac{1+n}{\gamma} \frac{f_{t+1}}{w_{t+1} h_{1t+1}} \right) e_{0t+1} \right] + l_{1t+1} &= \frac{1 + r_{t+1}}{\psi_t} \frac{w_t}{w_{t+1}} \left( 1 + \frac{1+n}{\gamma} \frac{f_t}{w_t h_{1t}} \right) \\
&\quad - \frac{\gamma}{\psi_{t+1}} \left( 1 + \frac{1+n}{\gamma} \frac{f_{t+1}}{w_{t+1} h_{1t+1}} \right) \qquad \text{(A.5f)}
\end{aligned}$$

# B The General Equilibrium Model

Our dynamic deterministic general equilibrium model is made up of fifteen equations and has seventeen variables. Two of these seventeen variables, $f_t$ and $\psi_t$, are *exogenous*, *i.e.*, they are set outside the model. The other fifteen variables are *endogenous* in the sense they are determined by the behavior of optimizing agents. The fifteen equations are listed below. The non-stationary variables are normalized, *i.e.*, they are divided by their growth components. This gives rise to the variables $\hat{y}_t = Y_t / [N_{0t} \nu^t \exp(xt)]$,

$\hat{k}_t = K_t/[N_{0t}\nu^t \exp(xt)]$, $\bar{h}_t = H_t/[N_{0t}\nu^t]$, $\tilde{w}_t = w_t/\exp(xt)$, $\bar{h}_{1t} = h_{1t}/\nu^t$, and $\hat{\mathbf{z}}_t = \mathbf{z}_t/[\nu^t \exp(xt)]$ with $\mathbf{z}_t = \{a_{1t}, a_{2t}, c_{0t}, c_{1t}, c_{2t}, f_t, \epsilon_{1t}\}$. Note that in solving numerically the model, the variables $\hat{a}_{1t}$, $\hat{a}_{2t}$, $h_t$, and $\hat{k}_t$ are considered as *predetermined*, *i.e.*, their values at $t+1$ is chosen at time $t$ by agents.

$$\sigma\hat{c}_{0t} = \frac{\gamma}{1+n}\tilde{w}_t\bar{h}_{1t}(1-e_{0t}-l_{0t}) \tag{B.1a}$$

$$\sigma\hat{c}_{1t} = \tilde{w}_t\bar{h}_{1t}(1-l_{1t}) \tag{B.1b}$$

$$(1+r_{t+1})\hat{c}_{0t} = (1+\rho)\nu\exp(x)\hat{c}_{1t+1} \tag{B.1c}$$

$$(1+r_{t+1})\hat{c}_{1t} = (1+\rho)\nu\exp(x)\hat{c}_{2t+1} \tag{B.1d}$$

$$\hat{c}_{1t} = (1+\phi)\hat{c}_{0t} \tag{B.1e}$$

$$\begin{aligned}\gamma\left[l_{0t} + \left(1+\frac{1+n}{\gamma}\frac{\hat{f}_t}{\tilde{w}_t\bar{h}_{1t}}\right)e_{0t}\right] + l_{1t} &= -\frac{\gamma}{\psi_t}\left(1+\frac{1+n}{\gamma}\frac{\hat{f}_t}{\tilde{w}_t\bar{h}_{1t}}\right)\\ &+\frac{1+r_t}{exp(x)\psi_{t-1}}\frac{\tilde{w}_{t-1}}{\tilde{w}_t}\left(1+\frac{1+n}{\gamma}\frac{\hat{f}_{t-1}}{\tilde{w}_{t-1}\bar{h}_{1t-1}}\right)\end{aligned} \tag{B.1f}$$

$$(1+\psi_t e_{0t})\frac{\gamma}{1+n}\bar{h}_{1t} = \nu\bar{h}_{1t+1} \tag{B.1g}$$

$$\frac{\gamma}{1+n}\tilde{w}_t l_{0t}\bar{h}_{1t} + \hat{\epsilon}_{1t} - \hat{c}_{0t} - \hat{f}_t e_{0t} = \nu\exp(x)\hat{a}_{1t+1} \tag{B.1h}$$

$$\tilde{w}_t l_{1t}\bar{h}_{1t} + (1+r_t)\hat{a}_{1t} - \hat{c}_{1t} - (1+n)\hat{\epsilon}_{1t} = \nu\exp(x)\hat{a}_{2t+1} \tag{B.1i}$$

$$(1+r_t)\hat{a}_{2t} = \hat{c}_{2t} \tag{B.1j}$$

$$\hat{k}_t^{\alpha}\bar{h}_t^{1-\alpha} = \hat{y}_t \tag{B.1k}$$

$$\alpha\frac{\hat{y}_t}{\hat{k}_t} = 1+r_t \tag{B.1l}$$

$$(1-\alpha)\frac{\hat{y}_t}{\bar{h}_t} = \tilde{w}_t \tag{B.1m}$$

$$\frac{\hat{a}_{1t}}{1+n} + \frac{\hat{a}_{2t}}{(1+n)^2} = \hat{k}_t \tag{B.1n}$$

$$(\gamma l_{0t} + l_{1t})\frac{\bar{h}_{1t}}{1+n} = \bar{h}_t \tag{B.1o}$$