Heckscher-Ohlin Theories from Factor Price Equalization to Factor Price Localization vs Empirical Observations from the Leontief Paradox to the Leontief Trade

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Empirical Observations from the Leontief Paradox to the Leontief Trade

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ABSTRACT - This paper attains the general trade equilibrium of factor price localizations of the Trefler Hicks-Neutral HOV model. The breakthrough is to use Helpman and Krugman's equilibrium analyses of trade volume (see Helpman and Krugman, 1985, Chapter 1). The trade consequences with factor price localization show some new results. The factor price localizations are associated with the three trade patterns. The Leontief paradox phenomenon is a regular trade pattern conceptually (this study calls it the Leontief trade). The Leontief trade not only occurs by factor intensity reversals (FIR) but also occurs without factor intensity reversals. The sign predictions based on the effective endowments (see Trefler 1995) and the virtual endowments (see Fisher and Marshall, 2008) favor both the Heckscher-Ohlin trade and the Leontief trade. The empirical studies based on them have included the Leontief trade. The study explains well the skill intensity reversals (assignment intensity reversal (see Kurokawa, 2011 and Sampson, 2016) by localized factor prices. Like the Leontief trade, the Heckscher-Ohlin trade may cause factor reward intensity reversal when countries have different productivities.

Keywords:
Localized factor prices, factor price equalization, factor price non-equalization, General equilibrium of trade, Leontief Paradox, Leontief trade

JEL Classification Code: F10, F15

1. Introduction

International trades integrate the world economy. Regardless of whether or not world commodity price is formed, free trade tends to reward different countries' factors by their relative productivity. The simple motivation of this article is to study how the price-trade equilibrium is established with factor price localization and to answer whether the trade pattern is diversified and whether the Leontief paradox can occur as trade consequences when countries have different productivities.

The general trade equilibrium is the essence of international trade theory. The Heckscher-Ohlin model is ideal for presenting the relationship among factor endowments, factor prices, commodity prices, production outputs, and trade flows. Samuelson said, "Historically, the development of economic theory owes much to the theory of international trade." (Samuelson, 1938, [1966, p.775]). International trade is a subject that studies general equilibrium more than any other economic subject. Woodland (2013, p.39) addressed that "General equilibrium has not only been important for a whole range of economics analyses but especially so for the study of international trade." Deardorff (1982, p.685) said, "A trade equilibrium is somewhat more complicated."

Samuelson (1948) presented the famous theorem of factor price equalization (FPE). Immediately, he made an argument about general trade equilibrium that the world prices do not change, supposing that a virtual world is divided into two continents artificially (see Samuelson 1949, [1996], p.882-883). Dixit and Norman (1980, chapter 4) provided the Integrated World Equilibrium (IWE) to illustrate the factor price equalization, which perfectly fulfilled the factor mobility analysis. They demonstrated that the world prices remain the same when the allocation of factor endowments

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changes within the factor price equalization (FPE) set in the IWE chart. Helpman and Krugman (1985, chapter 1) normalized the assumption of the integrated world equilibrium. Deardorff (1994) illustrated the conditions of the FPE for many goods, many factors, and many countries by the IWE approach. He demonstrated factor price equalizations for all allocations of factor endowments within the lenses he identified.

McKenzie (1955) proposed the diversification cone of factor endowments, which is essential to understand factor price equalization (FPE) and trade balance from production constraints. Fisher (2011) proposed the concept of "goods price diversification cone," which is the counterpart of factor diversification cone. He also offered another insight into the intersection of goods price cones to illustrate the price-trade relationship when countries have different technologies.

Vanek (1968) proposed the HOV model that presented factor contents of trade. The share of GNP in the HOV model engaged prices with trade and consumption. It resulted in the application issue on how to convert the assumption of homothetic taste into consumption balance.

The Leontief test (Leontief, 1953) showed that the US, as a capital-abundant country, exported its labor-intensive commodities. It counters the common sense of international economics then. The Leontief paradox impelled the HOV studies aimed to provide alternative approaches to explain it. Leontief (1953) proposed the productivity-equivalent factor (workers) to explain his test results. Trefler (1993) extended Leontief's idea and implemented it with factor-argument parameters in his model by effective (equivalent) endowments. The model is also instrumental for theoretical analyses to attain factor price non-equalizations. Fisher and Marshall (2008) provided another excellent approach to involve different technologies by the virtual endowments and the conversion matrix.

Deardorff (1986) presented the diversification cones of the FIRs. He showed the double factor intensity reversals. He suggested a way to turn any model with the FIRs into one without it, and vice versa, by simply redefining goods.

Chipman (1969), Trefler (1993), Krugman (2000), Fisher (2011), Leamer (2000), and Rassekh and Thompson (1993), and many other studies had argued the need for factor price non-equalization[1] when considering different technologies across countries.

Bertrand (1972) predicted trade equilibrium by introducing two variables to analyze the factor content of consumptions. He proposed the earliest theorem to illustrate trade patterns for factor price non-equalization.

Helpman and Krugman (1985, p.23-24) predicted trade equilibrium along with domestic factor endowments in the IWE diagram. They abstracted a unique principle as "the differences in factor composition are the sole basis of trade." This study extends their idea and method to format an approach to attain the general trade equilibriums of factor price localizations.

With the equilibrium of factor price localizations, this study obtains three trade patterns under the law of comparative advantage: the Heckscher-Ohlin trade, the FIR Leontief trades, and the mutual Leontief trade. All of them are trade consequences by different structures of production technologies. They are associated with localized factor prices. The FIR Leontief trade is caused by the factor intensity reversal, in which one country does Leontief trade, another does the Heckscher-Ohlin trade. It is a hybrid trade. The mutual Leontief trade occurs when the country's actual factor abundance conflicts with its effective factor abundance. It happens without the presence of FIR. The study presents the exact conditions when the mutual Leontief trade occurs.

Conceptually, the Leontief trades are regular trade patterns. The wondering question is that the sign predictions, based on the effective endowments and the virtual endowments, have included both the Heckscher-Ohlin trade and the Leontief trades. Both the trade patterns satisfy the logic of sign predictions that a country exports the services of its effective (or virtual) abundant factor.

Kurokawa (2011) and Sampson (2016) and a couple of other literature pieces found the phenomena of skill intensity reversal in this century. This study refers to phenomena as factor reward intensity reversal. The localized factor prices and trade patterns in this study can explain those phenomena well. This paper shows that both the Heckscher-Ohlin trade and the FIR Leontief trade can lead to factor reward intensity. When the national productivity is different, the Heckscher-Ohlin trade will also bring some trade consequences that we did not expect before.

[1] This paper uses factor price localizations and factor price non-equalizations alternatively for the phenomena that local factors are rewarded differently under the common world commodity prices when countries have different productivities.

The paper studies and reviews some of the exprimal literature on trade patterns and divides them into three categories: 1) the studies before 1990, 2) the studies by effective endowments and virtual endowments, 3) studies on skill intensity reversals. The three trade patterns proposed in this paper stand with all of them.

The author organizes this paper into six sections. Section 2 reviews the general trade equilibrium of factor price equalization by Guo (2019) and confirms it by Helpman and Krugman's (1985, chapter 1) equilibrium analysis. It also derives autarky prices, which are helpful to show trade patterns when countries are with different productivities. Section 3 derives the general trade equilibrium of factor price localizations using Helpman and Krugman's equilibrium analysis. It shows that the world effective endowments determine localized factor prices. It confirms the comparative advantage theory because localized factor prices ensure the gains from trade for both countries. Section 4 illustrates the diversification of trade patterns when countries have different productivities. It shows that conceptually there are three trade patterns: the Heckscher-Ohlin trade, the FIR Leontief trade, and the mutual Leontief trade. The localized factor prices show that both the Heckscher-Ohlin trade and the FIR Leontief trade can cause the factor reward intensity reversal. It is the first time to show the "unexpected" property of the Heckscher-Ohline trade when factors are rewarded differently across countries. Section 5 demonstrates the Leontief trades by virtual endowments (see Fisher 2011). It also presents the geometric expression of the Leontief trades. Section 6 reviews HOV empirical studies. It finds that the prediction signs, commonly used, favor all the trade patterns. It also provides a new explanation of the Leontief paradox by the FIR Leontief trade. The last section is the concluding remarks.

2. Preliminary - The General Trade Equilibrium Under the Same Technologies Confirmed by the Helpman and Krugman Analysis Result

We start by reviewing the price-trade equilibrium of factor price equalization. It is a theoretical basis of factor price localizations and trade patterns when countries' productivities are different.

2.1 The review of The General Trade Equilibrium of Factor Price Equalization

We denote the Heckscher-Ohlin model first by the typical assumptions of Heckscher-Ohlin theory. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \qquad \text{(2-1)}$$

where A is the 2×2 matrix of direct factor inputs, X^h is the 2×1 vector of commodities of country h, its elements are x_1, x_2 . V^h is the 2×1 vector of factor endowments of country h, its elements are *K, L*. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular. The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \qquad \text{(2-2)}$$

where W^h is the 2×1 vector of factor prices of country h, its elements are r rental for capital and w wage for labor, P^h is the 2×1 vector of commodity prices of country h.

Factor prices will be equalized when prices and trade reach their equilibrium. We denote the world price equations as

$$A'W^* = P^* \qquad \text{(2-3)}$$

where we use the superscript * to represent world prices.

The trade balance condition for the factor contents is

$$\frac{w^*}{r^*} = -\frac{F_K^H}{F_L^H} = -\frac{K^H - s^H K^W}{L^H - s^H L^W} \qquad \text{(2-4)}$$

where F_L^H and F_K^H are the export of factor content of trade in country H, K^W and L^W are world factor endowments, s^H is the share of the GNP of country H to the world GNP.

Denote two parameters, which are the shares of country H's factor endowments to their world factor endowments respectively,

$$\lambda_L = \frac{L^H}{L^W} \quad , \qquad \lambda_K = \frac{K^H}{K^W} \qquad \text{(2-5)}$$

(2-4) can be rewritten as

$$\frac{w^*}{r^*} = -\frac{(\lambda_K - s^H)K^W}{(\lambda_L - s^H)L^W} \qquad (2\text{-}6)$$

Dixit and Norman illustrated the whole FPE set in the IWE diagram shares the same world prices. The FPE set is concrete. The world prices should be concrete. What are the world prices? Dixit and Norman's conclusion of the FPE set implies that the wage-rental ratio $\frac{r^*}{w^*}$ is constant. Therefore, the right side of (2-6) should be a constant also. More specifying, this paper introduces

$$\varphi = \frac{(s^H - \lambda_L)}{(\lambda_K - s^H)} \qquad (2\text{-}7)$$

Substituting it into (2-6) yields

$$\frac{w^*}{r^*} = \varphi \frac{K^W}{L^W} \qquad (2\text{-}8)$$

φ is the FPE constant in IWE. We also call it the Dixit-Norman constant to honor their finding of the FPE set. This constant indicates that the world price will remain unchanged no matter how the world factor endowment is distributed within the FPE set in the IWE.

We present the solution of price-trade equilibrium by Dixit and Norman as

$$w^* = 1 \qquad (2\text{-}9)$$

$$\frac{w^*}{r^*} = \varphi \frac{K^W}{L^W} \qquad (2\text{-}10)$$

$$p_1^* = a_{k1}\varphi \frac{L^W}{K^W} + a_{L1} \qquad (2\text{-}11)$$

$$p_2^* = a_{k2}\varphi \frac{L^W}{K^W} + a_{L2} \qquad (2\text{-}12)$$

It drops one market-clearing condition by assuming $w^* = 1$.
Substituting (2-9) and (2-10) into the share of GNP in country H yields

$$s^h = \frac{w^* L^H + r^* K^H}{w^* L^W + r^* K^W} = \frac{\varphi K^h L^W + L^h K^W}{(1+\varphi)K^W L^W} \qquad (h = H, F) \qquad (2\text{-}13)$$

The equations (2-9) through (2-13) reduces the mystery of the structures of equalized factor prices, although φ is an unknown constant. We still need to know what the constant is.

Guo (2005) initialed his study on this question. Guo (2019) reported his result that $\varphi = 1$. Guo added the trade box, by the goods price diversification cone, on the IWE diagram. It allows the IWE analyses to link trade to prices directly. Appendix A is the brief derivation for Guo's result. By the constant as 1, the solution of equilibrium is

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \quad , \qquad w^* = 1 \qquad (2\text{-}14)$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \, , \qquad p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \qquad (2\text{-}15)$$

$$F_K^h = K^h - s^h K^W = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = H, F) \qquad (2\text{-}16)$$

$$F_L^h = L^h - s^h L^W = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = H, F) \qquad (2\text{-}17)$$

We assume here that country H is capital abundant. The numerator of (2-16) is $K^h L^W - K^W L^h$. It shows that $F_K^H > 0$ and $F_L^H < 0$ when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$. It just states the Heckscher-Ohlin theorem. The equilibrium firmly shows the factor price equalization theorem and the Heckscher-Ohlin theorem are the same things from different views of the trade consequences.

2.2 The Confirmation of the Equilibrium of Factor Price Equalization by Helpman and Krugman Equilibrium Analyses

The general trade equilibrium and factor price equalization is a severe and classical topic. Can any other literature confirm the result φ=1? Helpman and Krugman's (1985, chapter 1) equilibrium analysis does.

Helpman and Krugman (1985, p.23) defined the trade volume for commodity trades as

$$VT = 2p_1^*\left(x_1^H - s^H x_1^W\right) = -2p_2^*\left(x_2^H - s^H x_2^W\right) \qquad (2\text{-}18)$$

They illustrated that the equal trade volume curves in the FPE set are straight lines, which are parallel to the diagonal line OO^* in the IWE diagram. They provided another unique expression of the trade volume by domestic factor endowments (see Helpman and Krugman, 1985, p.23). They clarified and proved that there are two variables γ_L and γ_K associated with all equal trade volumes lines, which satisfy the following relationships:

$$VT = v(L^h, K^h) = \gamma_L L^h + \gamma_K K^h \quad (2\text{-}19)$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \quad (2\text{-}20)$$

The primary argument for the relationships above is that the trade volume is a linear function of K^H and L^H eventually (see Helpman and Krugman 1985, p.23, p.175). The two equations make sure that a higher difference in factor composition leads to a higher trade volume. The trade volume is zero if the factor endowments allocate at the diagonal line OO^* in the IWE diagram. They also identified that one of γ_L, γ_K is negative. If country H is capital abundant, the two variables satisfy: $\gamma_K > 0$ and $\gamma_L < 0$.

Equations (2-19) is an "abstract" expression of trade volume of factor content. Like the idea of (2-18), the trade volume of net factor contents by factor prices can be expressed[2]

$$VT = 2\left(K^H - s^H K^W\right)r^* = 2F_K^H r^* \quad (2\text{-}21)$$

It is the "concrete" expression of trade volume of factor content.

Variables γ_L and γ_K in (2-19) and (2-20) are different across countries since the two countries' abundant factors are different. We denote them with a country mark as γ_L^h and γ_K^h, $h = H, F$.

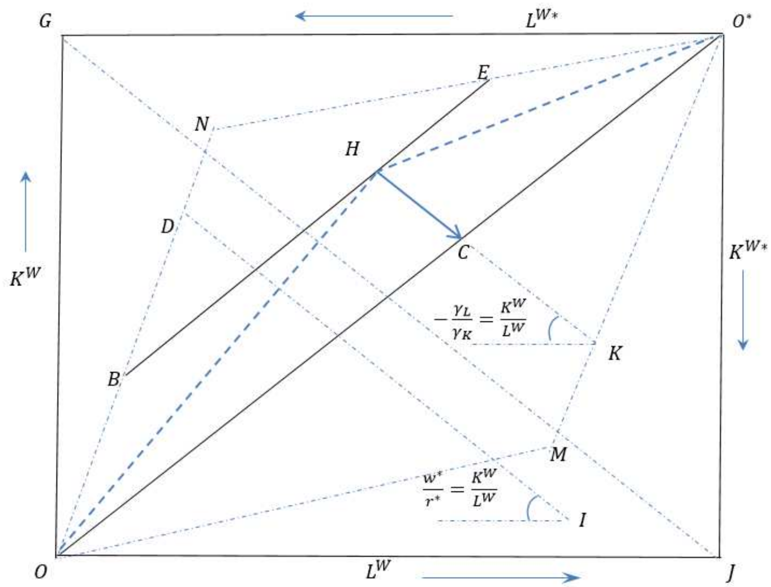

Figure 1 IWE Diagram with The FPE Set and Equal Trade Volume Line

The abstract definition of the trade volume (2-19) looks dazzling but rich in economic logic. Figure 1 is a typical IWE diagram presenting the Helpman and Krugman's equal trade volume lines. The dimensions of the diagram represent the world factor endowments. The home country's origin is the lower-left corner, and the foreign country is from the right-upper corner. $\overline{ON}$ and $\overline{OM}$ are the rays of the cone of factor diversifications. Parallelogram NO^*MO is the FPE set. Line $\overline{DI}$ is the line with an equal share of the GNP, which indicates trade direction. It is parallel to the anti-diagonal line $\overline{GJ}$ under the equilibrium solution by $\varphi = 1$. Line $\overline{BE}$ is an equal trade volume line, which is parallel to the diagonal line $\overline{OO^*}$. Line $\overline{HK}$ is constriction line of the two variables by (2-20), which is parallel to the anti-diagonal line $\overline{GJ}$.

Substituting (2-14) into (2-20), we obtain immediately

$$\frac{w^*}{r^*} = -\frac{\gamma_L^H}{\gamma_K^H} = -\frac{\gamma_L^F}{\gamma_K^F} = \frac{K^W}{L^W} \quad (2\text{-}22)$$

It leads this paper to interpret those two variables by equalized factor prices as

[2] Be aware that trade volume of commodity trade (2-18) is different from the trade volume of net factor contents of trade (2-21) quantitatively.

$$\gamma_L^H = -w^* \tag{2-23}$$
$$\gamma_K^H = r^* \tag{2-24}$$
$$\gamma_L^F = w^* \tag{2-25}$$
$$\gamma_K^F = -r^* \tag{2-26}$$

We assume that country H is capital abundant. The assumption above fit (2-19) well also. Substituting (2-23) and (2-24) into (2-19) for the trade volume of country H yields

$$VT = -w^* L^H + r^* K^H \tag{2-27}$$

Similarly, substituting (2-25) and (2-26) into (2-19) for the trade volume of country F yields

$$VT = w^* L^F - r^* K^F \tag{2-28}$$

The two countries' trade volumes should be the same. Substituting (2-27) into (2-28) yields

$$-w^* L^H + r^* K^H = w^* L^F - r^* K^F \tag{2-29}$$

Simplifying it yields

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \tag{2-30}$$

It confirms $\varphi = 1$ in (2-8). The Helpman and Krugman's equilibrium analysis by trade volume is also an independent approach to derive the equilibrium. Appendix B shows it.

Equation (2-27) and (2-28) show that the difference in local factors evaluated by equalized factor prices equals its trade volume of factor contents (or the difference between the total cost of the abundant factor and the total cost of the scarce factor of a country is its trade volume). It not only holds on all equal trade volume lines but also on the whole FPE set.

2.3 Autarky Price and Comparative Advantage

Leamer and Levinsohn (1995, p.1342) mentioned the importance of gains from trade as "Proofs of the static gains from trade fall into the unrefutable category yet these are some of the most important results in all of economics." Autarky prices are helpful not only for the original Heckscher-Ohlin theories but also for the studies of trade patterns when countries have different productivities.

The general trade equilibrium above shows that world factor endowments determine world prices. We now use this logic to evaluate the autarky prices of a country with an isolated market. The idea is that the autarky factor endowments determine its autarky prices. Samuelson (1949, [1966], p.882-883) first proposed this idea. He argued verbally that the world's autarky price would be the world prices if the world is divided into two continents artificially, supposing that every other thing remains no changes. Dixit and Norman (1980, chapter 4) implemented it as the IWE diagram perfectly. Both Samuelson's original idea and the IWE implementation imply that the way to calculate the world price is the same as calculating the autarky price of an isolated country.

Helpman and Krugman (1985, p.16) proposed a clear-sighted conclusion about the factor price equalization (FPE) set in the IWE. They addressed "This FPE set is not empty because it always contains the diagonal OO^*. Since it is a convex symmetrical set around the diagonal, its boundaries defined the limits of dissimilarity in factor composition which is consistent with factor price equalization. Hence for sufficiently similar composition, there is a factor price equalization in the trading equilibrium". It normalized the FPE set. Without it, the nearby area to the diagonal line will not be valid for the FPE[3]. It can be used to derive analytical autarky prices directly.

Let us imagine an allocation of factor endowments, C, on the diagonal line OO^* in Figure 1. At this point, both countries' factor compositions are the same, and they equal to world factor composition as

$$\frac{L^H}{K^H} = \frac{L^F}{K^F} = \frac{L^W}{K^W} \tag{2-31}$$

At that moment, we know both countries' rental/wage ratios are the same. Otherwise, it will cause trade. It implies that the world rental/wage ratio equals the autarky rental/wage ratios of the two countries as

$$\frac{r^{aH}}{w^{aH}} = \frac{r^{aF}}{w^{aF}} = \frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{2-32}$$

[3] Mathematically, it makes sure that whole FPE set is on a plane. Otherwise the FPE will be with a hole or a ditch along the diagonal line.

where superscript ah indicates the autarky price of country h, $h = H, F$. We see that the logic of autarky prices formation is the same as world prices formation.

The IWE diagram itself supports this idea analytically. Consider the allocation of factor endowments, like point H, in Figure 1. Assume that it moves close to the origin O. The factor endowments of country H will shrink to very small, the factor endowments of country F will close to be world factor endowments. The world prices then will be Country F's autarky prices. Mathematically, when the allocation $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the world relative factor price $\frac{w^*}{r^*}$ will close to the relative autarky factor price of country H. Let present the world wage/rental as

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} = \frac{K^H + K^F}{L^H + L^F} \tag{2-33}$$

Seeking the limit above yields

$$\lim_{\substack{L^H \to 0 \\ K^H \to 0}} \frac{K^H + K^F}{L^H + L^F} = \frac{K^F}{L^F} = \frac{w^{aF}}{r^{aF}} \tag{2-34}$$

Meanwell, the world commodity prices will close to the autarky output prices of country F. Equation (2-34) proved the autarky price measurement mathematically also. The limit of a two-variable function generally depends on the path to its limit. However, for (2-34), any path or direction is good to reach the same result.

With the above discussion, we present the autarky prices of two countries as

$$r^{ah} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{2-35}$$

$$w^{ah} = 1 \qquad (h = H, F) \tag{2-36}$$

$$p_1^{ah} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{2-37}$$

$$p_2^{ah} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{2-38}$$

The gains from trade are measured by

$$-W^{ah\prime}F^h > 0 \qquad (h = H, F) \tag{2-39}$$

$$-P^{ah\prime}T^h > 0 \qquad (h = H, F) \tag{2-40}$$

We add a negative sign in inequalities above since we expressed the net factor content of trade by export. In most other works of literature, they denoted the net factor content of trade by import.

Applying factor content of trade (2-16), (2-17), (2-35), and (2-36) in (2-39), we get gains from trade of both countries. Appendix C is the detail of the derivations. The quantitative or computable gains from trade are essential for international trade analyses. We can use this approach to estimate any country's gain from trade if we know its trade flows.

3. **The General Trade Equilibrium of Factor Price Non-Equalization**

The Trefler (1993) model is the first HOV model to incorporate the Heckscher-Ohlin model with different productivities across countries magnificently. We use it to illustrate the factor price localizations.

3.1 Review of Trefler Model

The central assumption in the Trefler model is to express technology differences by factor input requirements as

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F \tag{3-1}$$

where Π is a 2×2 diagonal matrix, its element π_k is the factor productivity-argument parameter, $k = K$ (capital), L(Labor). A^h is the 2×2 technology matrix of country h, its element $a_{ik}^h(w/r)$ is the input requirement of factor k needed to produce one unit of output i, i=*1,2*, k=*L*, *K*.

We adopt all Heckscher-Ohlin model assumptions, except that productivities are different across countries in the Trefler model.

Production constraint function and the unit cost function for country H are

$$A^H X^H = V^H \tag{3-2}$$

$$(A^H)'W^H = P^H \tag{3-3}$$

For country F, they are

$$\Pi^{-1}A^H X^F = V^F \tag{3-4}$$

$$(\Pi^{-1}A^H)'W^F = P^F \tag{3-5}$$

where V^h is the 2×1 vector of factor endowments with elements K as capital and L as labor; X^h is the 2×1 vector of commodity output; W^h is the 2×1 vector of factor prices with elements r as rental and w as wage; P^h is the 2×1 vector of commodity prices with elements p_1^h and p_2^h; $h = H, F$ for countries.

The Trefler model is with a single cone of goods price diversifications[4]. Its factor cost ratio ranks, which show the rays of the cone of goods prices in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = (\frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{K1}^H/\pi_K}{a_{K2}^H/\pi_K}) > \frac{P_1^*}{P_2^*} > \frac{a_{L1}^H}{a_{L2}^H} = (\frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{L1}^H/\pi_L}{a_{L2}^H/\pi_L}) \tag{3-6}$$

where we assume both countries are capital intensive on sector 1. The single cone of goods price diversifications reduces the difficulties of analyses of the price-trade equilibrium. The Trefler model does have two cones of the factor endowment diversifications, which show different productivities across countries.

Bernhofen (2011, p104) mentioned, "A country's factor content is defined using the country's domestic technology matrix." His idea is a critical point in analyses of trade equilibrium when countries with different productivities. It is a start point to understand why some conclusions in the factor price localizations are different from factor price equalization. To express a country's factor contents, we need to use the respective world effective endowments. We denote them as

$$K^{hW} = K^H + \pi_K K^F , \qquad L^{hW} = L^H + \pi_L L^F \tag{3-7}$$

$$K^{fW} = K^F + K^H/\pi_K , \qquad L^{fW} = L^F + L^H/\pi_L \tag{3-8}$$

where K^{hW} and L^{hW} are the effective endowments of the world by referring to the productivity of country *h* to produce world commodities, *h, h=h, f.* We use the lowercase character *h* to indict the country referred to its productivities.

3.2 Factor Price Localizations by Helpman and Krugman's Equilibrium Analyses

The Helpman and Krugman's idea that the differences in factor composition are the sole source of trade is accurate even for factor price non-equalization. When countries have different productivities, the differences in factor composition evaluated by effective endowments are the sole basis of trade.

[4] See Fisher (2011) for the the cone of goods price diversification.

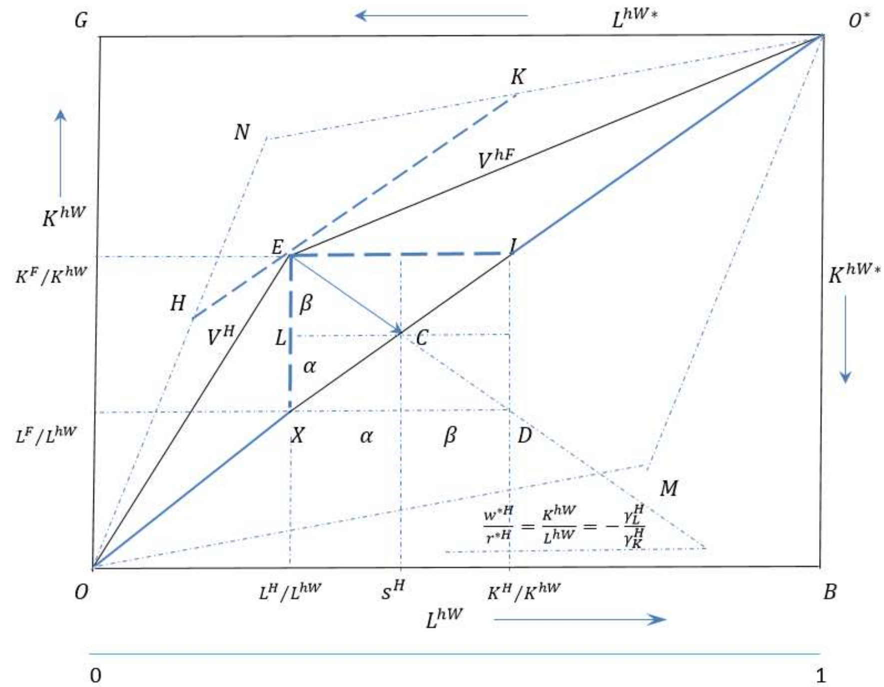

Figure 2 Trade Box and Equal Trade Volume Line by Effective Factor Endowments

Figure 2 is the IWE diagram extended to present effective endowments. The dimensions of the diagram represent the effective endowments of the IWE world measured by referring to the productivities of the home country to produce world commodities. The home country's origin is the lower-left corner, and the foreign country is from the right-upper corner. ON and OM are the rays of the cone of factor diversifications in the home country. Any point within the parallelogram formed by ONO^*M is an available allocation of effective endowments of two countries. V^{hF} is the vector of effective endowments of country F measured by the productivities of country H.

Suppose that E is the allocation describing the distribution of the world effective endowments. Country H is effective capital abundant at this allocation (we will use this assumption for all analyses of this study). Point C represents the trade equilibrium point. It shows the sizes of the consumption of the two countries.

We propose that each country's trade volume is the function of the local (or domestic) factor endowments and localized factor prices by using Helpman and Krugman's idea. We slightly change (2-20) by using the world effective endowments in the following,

$$VT^h = \gamma_L^h L^h + \gamma_L^h K^h \qquad (h = H, F) \qquad (3\text{-}9)$$

$$-\frac{\gamma_L^h}{\gamma_K^h} = \frac{K^{hW}}{L^{hW}} \qquad (h = H, F) \qquad (3\text{-}10)$$

In Figure2, $\overline{HK}$ is an equal trade volume line by effective endowments. We interpret these two variables γ_L^h, γ_K^h as localized factor prices,

$$\gamma_L^H = -w^{*H} \qquad (3\text{-}11)$$

$$\gamma_K^H = r^{*H} \qquad (3\text{-}12)$$

$$\gamma_L^F = w^{*F} \qquad (3\text{-}13)$$

$$\gamma_K^F = -r^{*F} \qquad (3\text{-}14)$$

If a variable corresponds to an effective-abundant factor in its country, it takes a positive sign. Otherwise, it takes a negative sign.

Substituting equations (3-11) and (3-12) into (3-9) yields

$$VT^H = -w^{*H} L^H + r^{*H} K^H \qquad (3\text{-}15)$$

Similarly, substituting equations (3-13) and (3-14) into (3-9) yields

$$VT^F = w^{*F} L^F - r^{*F} K^F \qquad (3\text{-}16)$$

The Trefler model supposed that the factor price of country F can be expressed as

$$w^{*F} = \pi_L w^{*H} \tag{3-17}$$
$$r^{*F} = \pi_K r^{*H} \tag{3-18}$$

Substituting them into (3-16) yields

$$VT^F = \pi_L w^{*H} L^F - \pi_K r^{*H} K^F \tag{3-19}$$

One condition we need to use is

$$VT^F = VT^H \tag{3-20}$$

It is true also when countries are with different productivities. Appendix D provides proof of it. Substituting (3-15) and (3-19) into (3-20) yields

$$-w^{*H} L^H + r^{*H} K^H = \pi_L w^{*H} L^F - \pi_K r^{*H} K^F \tag{3-21}$$

Simplify it as

$$\frac{w^{*H}}{r^{*H}} = \frac{K^H + \pi_K K^F}{L^H + \pi_L L^F} = \frac{K^{hW}}{L^{hW}} \tag{3-22}$$

Similarly, we can obtain

$$\frac{w^{*F}}{r^{*F}} = \frac{K^H/\pi_K + K^F}{L^H/\pi_L + L^F} = \frac{K^{fW}}{L^{fW}} \tag{3-23}$$

At the equilibrium, $\Delta ELC \sim \Delta OO^*B \sim \Delta EIX$. The trade volume in country H is the value of the factor represented by $\overline{EX}$. For country F, it is $\overline{EI}$.

Using (3-22), assume $w^{*H} = 1$ to drop one market clear condition, we attain the general trade equilibrium of the Trefler model as

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^{hW}} + \frac{L^h}{L^{hW}}\right) \qquad h = (H, F) \tag{3-24}$$

$$W^{*F} = \begin{bmatrix} \frac{L^{hW}}{K^{hW}} \\ 1 \end{bmatrix} \tag{3-25}$$

$$P^* = (A^H)' \, W^{*H} \tag{3-26}$$

$$W^{*F} = \Pi W^{*H} \tag{3-27}$$

The wage in country H, $w^{*H} = 1$, serves as "benchmark" price to be referred to by the other three factors' prices and two world commodity prices. The factor contents of trade and trade flows are

$$F_K^h = \frac{1}{2}\frac{K^h L^{hW} - K^{hW} L^h}{L^{hW}} \qquad h = (H, F) \tag{3-28}$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^{hW} - K^{hW} L^h}{K^{hW}} \qquad h = (H, F) \tag{3-29}$$

$$T_1^h = x_1^h - s^h \, x_1^W \qquad (h = H, F) \tag{3-30}$$

$$T_2^h = x_2^h - \, s^h x_2^W \qquad (h = H, F) \tag{3-31}$$

The numerators of (3-28) and (3-29) show that when

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}} \tag{3-32}$$

then $F_K^H > 0$ and $F_L^H < 0$. It just states that a country exports the services of its effective abundant factor.

(3-30) through (3-31) imply that a country being effective-capital abundance will export its capital-intensive goods and import its labor-intensive goods. For caution, Appendix E proves it in detail. We will use it late in this paper.

Equations (3-25) through (3-27) show that the world effective endowments determine world prices. The world effective factor composition equals world consumption composition. It implies eventually that the effective consumption composition of the world determines world prices.

The approach by Helpman and Grugmen's trade volume analyses is a way of derivations by economic concepts. There is another simple way to derive it. Trefler mentioned that the factor price equalization hypothesis and HOV theorem hold in his equivalent-productivities system. We can extend the equilibrium of factor price equalization mathematically to the equilibrium of factor price localization. Measuring factor endowments and factor prices in country F by referencing the productivities of country H, we obtain a Heckscher-Ohlin system mapped. The factor prices by the equilibrium solution of equalized factor prices are the local prices in country H. Doing the same by measuring factor endowments and factor prices in country H by referencing the productivities of country F, we can obtain the localized factor prices in country H.

4. Trade Patterns Associated with Localized Factor Prices When Countries Have Different Productivities

Like factor price localizations, trade patterns are trade consequences also. The general trade equilibrium can show them clearly. Something interesting is that factor price localizations are associated with the diversification of trade patterns. We offer that there exist three trade patterns when countries have different productivities conceptually in this section.

The first trade pattern is the well-known Heckscher-Ohlin trade, in which each country exports the commodity that is produced by using its actual-abundant factor intensively. In this trade pattern, the actual-abundant factor is just the effective-abundant factor (for the term of the effective factor abundance, see equation (4-2) in this section).

The second pattern is the mutual Leontief trade, in which both countries make the Leontief trade. Each country exports the commodity that is produced by using its effective abundant factor intensively. It occurs when effective factor abundance reverses the actual factor abundance. It is not caused by factor intensity reversal. It is just the phenomena of the Leontief paradox arising in both countries.

The last trade pattern is the FIR Leontief trade, in which one country does Leontief trade; another does the Heckscher-Ohlin trade. It is caused by factor intensity reversal.

We present the three trade patterns both by effective factor abundance and by localized factor prices. Each trade pattern corresponds to a price pattern.

4.1 Mutual Leontief Tarde

Besides the Heckscher-Ohlin theorem, the Leamer theorem is an alternative to judge if a trade pattern is the Heckscher-Ohlin trade or not. The Leamer theorem says that if capital is abundant relative to labor in a country, then the HOV theorem implies that the capital/labor ratio embodied in production for the country exceeds the capital/labor ratio embodied in consumption.

Feenstra and Taylor (2012, p102-103) defined effective endowments with localized factor prices. They proposed measuring the factor abundance by comparing a country's share of its effective factor with its share of world GDP and specified that effective endowments determine trade direction. It is an insight opinion about trade pattern. Naturally, if the trade direction by actual factor abundance is different from the trade direction of effective factor abundance in a country, the Leontief trade occurs. We illustrate how it appears.

We assume that country H be actual-capital abundant as[5]

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{4-1}$$

We also assume that country H is effective-capital abundant as[6]

$$\frac{K^H}{L^H} > \frac{K^{hF}}{L^{hF}} = \frac{\pi_K K^F}{\pi_L L^F} \tag{4-2}$$

---

[5] Actual capital abundance of home country can be expressed by $\frac{K^H}{L^H} > \frac{K^F}{L^F}$ or $\frac{K^H}{L^H} > \frac{K^W}{L^W}$. Actually, there are relationships

$$\frac{K^H}{L^H} > \frac{K^W}{L^W} > \frac{K^F}{L^F}.$$

[6] The equivalent or effective capital abundance of home country is defined by $\frac{K^H}{L^H} > \frac{K^{hF}}{L^{hF}}$ or $\frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}$. Actually, there are relationships

$$\frac{K^H}{L^H} > \frac{K^{hW}}{L^{fW}} > \frac{K^{hF}}{L^{hF}}.$$

where K^{hF} and L^{hF} are the effective endowments in country F, measured by referring to the productivities of country H. We use the lowercase character in the superscript to indicate the country referred for productivities. Condition (4-1) and (4-2) show that country H will make the Heckscher-Ohlin trade. The actual factor abundance (4-1) and the effect factor abundance (4-2) are in the same direction. Rewrite (4-2) as

$$\frac{K^H L^F}{L^H K^F} > \frac{\pi_K}{\pi_L} \tag{4-3}$$

If inequality (4-2) holds, inequality (4-3) holds.

However, if inequality (4-3) is under the opposite direction, merely by the changes of π_K and π_L ,

$$\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L} \tag{4-4}$$

The mutual Leontief trade will occur while (4-1) remains the same. (4-4) can be rewritten as the following two expressions as

$$\frac{K^H}{L^H} < \frac{\pi_K K^F}{\pi_L L^F} = \frac{K^{fF}}{L^{fF}} \tag{4-5}$$

$$\frac{K^F}{L^F} > \frac{K^H/\pi_K}{L^H/\pi_L} = \frac{K^{fH}}{L^{fH}} \tag{4-6}$$

Under (4-1), (4-5), and (4-6), both countries' actual factor abundances are conflict with their effective factor abundances. It implies that both countries make the Leontief trade.

Merely the productivity argument parameters change, the Leontief trade may occur within the Trefler model. It happens without the existence of the FIR. The existing studies only described that the FIRs are a cause for the Leontief trade. The scope of the presence of the Leontief trade is much larger than what we expected before.

4.2 The FIR Leontief Trade - Factor Conversion Trade

The Trefler model is also helpful to present FIRs. We now specify the Trefler model a little bit differently by assuming that technological matrices of the two countries be

$$A^H = \psi A^F = \begin{bmatrix} 0 & \theta_K \\ \theta_L & 0 \end{bmatrix} A^F \tag{4-7}$$

where ψ is a 2×2 anti-diagonal matrix, its element θ_k is the productivity-across-factor-argument parameter, $k = K, L$. Denote

$$A^H = \begin{bmatrix} a_{K1}^H & a_{K2}^H \\ a_{L1}^H & a_{L2}^H \end{bmatrix} \tag{4-8}$$

The technology matrix for country F is

$$A^F = \psi^{-1} A^H = \begin{bmatrix} \frac{1}{\theta_L} a_{L1}^H & \frac{1}{\theta_L} a_{L2}^H \\ \frac{1}{\theta_K} a_{K1}^H & \frac{1}{\theta_K} a_{K2}^H \end{bmatrix} \tag{4-9}$$

Those compose a model with the FIRs as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{4-10}$$

$$\psi^{-1} A^H X^F = V^F, \qquad (\psi^{-1} A^H)' W^F = P^F \tag{4-11}$$

The world effective endowments by referring to home productivity are

$$K^{hW} = K^H + \theta_L L^F \ , \qquad L^{hW} = L^H + \theta_K K^F \tag{4-12}$$

The world effective endowments by referring to foreign productivity are

$$K^{fW} = K^F + L^F / \theta_L \ , \qquad L^{fW} = L^F + K^F / \theta_K \tag{4-13}$$

The cost requirement ratios, which indicate the rays of goods price diversification cone in algebra, are

$$\frac{a_{K1}^H}{a_{K2}^H} = \left(\frac{a_{L1}^F}{a_{L2}^F} = \frac{\frac{a_{K1}^H}{\theta_L}}{\frac{a_{K2}^H}{\theta_L}}\right) \ , \quad \frac{a_{L1}^H}{a_{L2}^H} = \left(\frac{a_{K1}^F}{a_{K2}^F} = \frac{\frac{a_{L1}^H}{\theta_K}}{\frac{a_{L2}^H}{\theta_K}}\right) \tag{4-14}$$

It is also the case of the single cone of goods prices. If country H is capital-intensive to produce commodity 1, country F will be capital-intensive to produce commodity 2, vice versa. The model is with the existence of the FIR. When $\theta_L = 1$ and $\theta_k = 1$, it will be the classical model of the FIR in the Heckscher-Ohlin theories. We derive the FIR model here just to explore the typical characteristics of the FIR trade.

One feature of the model with the FIR is that both countries are effective abundance in the same factor. Both countries export commodities that are produced by using the same factor intensively.

Supposing that country H be both actual capital abundant and effective capital abundant as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{4-15}$$

$$\frac{K^H}{L^H} > \frac{K^{hF}}{L^{hF}} = \frac{\theta_L L^F}{\theta_K K^F} \tag{4-16}$$

(4-16) can be rewritten as

$$\frac{K^F}{L^F} > \frac{L^H/\theta_K}{K^H/\theta_L} = \frac{K^{fH}}{L^{fH}} \tag{4-17}$$

It implies that country F also is effective capital abundant. It presents a supersized phenomenon that both countries export the services of the same factor and import the services of the same factor. We call it also factor conversion trade. By (4-16), country H will export commodity 1 since country H is capital intensive in producing product 1. By (4-17), the foreign country will export commodity 2 since country F is capital intensive at product 2. In this case, both countries export their capital-intensive commodities and import their labor-intensive commodities. The commodity trades equilibrate in the normal way as $T^H = -T^F$.

The Trefler FIR model is essentially a Trefler (1993) model mathematically. The result of general trade equilibrium (3-24) through (3-31) can be applied directly to the Trefler FIRs model.

With factor content reversals, both countries will consume more on their effective scarce factor, embodied in the trade flows.

Appendix F shows how the FIR Leontief trade occurs under higher dimension cases.

4.3 ***Factor Price Definitions of the trade patterns***

Heckscher and Ohlin initially used prices to define factor abundance. It is not as popular as the physical definition of factor abundance since both autarky prices and world prices are not available at their time. We use localized factor prices and autarky prices to define the three trade patterns since they are available now.

The localized wage-rental ratio at factor price localization for a country is

$$\frac{w^{*h}}{r^{*h}} = \frac{K^{hW}}{L^{hW}} \qquad (h = H, F) \tag{4-18}$$

Autarky prices of countries can be written as

$$\frac{w^{ah}}{r^{ah}} = \frac{K^h}{L^h} \qquad (h = H, F) \tag{4-19}$$

The physical factor abundance defines the Heckscher-Ohlin trade as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}\ , \qquad \frac{K^H}{L^H} > \frac{K^{hW}}{L^{hW}}\ , \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-20}$$

Substituting (4-18) and (4-19) into the above yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}\ , \qquad \frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}}\ , \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-21}$$

It illustrates that the trade will benefit capital services in country H and labor in country F. Trades benefit the effective-abundant factors, which are actual abundant factors also. (From autarky wage-rental ratios of two countries, exporting the capital services and import labor services by country H is directly consistent with the price definition of the Heckscher-Ohlin trade)

From (4-21), two possible (and typical) wage-rental ratio ranks are

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-22}$$

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aF}}{r^{aF}} \tag{4-23}$$

(4-23) shows

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} \tag{4-24}$$

If autarky wage-rental ratio ranks are $\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}$ and world wage-rental ratio rank after trade is $\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}}$, we say this is the reversal of factor reward intensity. that it may occur with the Heckscher-Ohlin trade if $\frac{K^{fW}}{L^{fW}} > \frac{K^{hW}}{L^{hW}}$.

The mutual Leontief trade by physical factor abundance is

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{4-25}$$

Substituting (4-18) and (4-19) into them yields,

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-26}$$

It shows that the trade will benefit labor in country H and capital in country F. They are actual scarce factors of each country. However, they are effective abundant factors of each country. Rewrite (4-26) as

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} > \frac{w^{*F}}{r^{*F}} \tag{4-27}$$

It shows that there is no chance for the factor reward reversal for the mutual Leontief trade.

The FIR Leontief trade, by definition, is

$$\frac{K^H}{L^H} > \frac{K^F}{L^F}, \qquad \frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}}, \qquad \frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{4-28}$$

Substituting (4-18) and (4-19) into them yields

$$\frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}}, \qquad \frac{w^{aH}}{r^{aH}} < \frac{w^{*H}}{r^{*H}}, \qquad \frac{w^{aF}}{r^{aF}} < \frac{w^{*F}}{r^{*F}} \tag{4-29}$$

It depicts that the trade will benefit labor in both countries, which are effective-abundant factor worldwide. And capital is the effective-scarce factor worldwide.

From (4-29), two possible wage-rent ratio ranks in many possible combinations are

$$\frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{*F}}{r^{*F}} > \frac{w^{aF}}{r^{aF}} \tag{4-30}$$

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} > \frac{w^{aH}}{r^{aH}} > \frac{w^{aF}}{r^{aF}} \tag{4-31}$$

(4-31) also shows

$$\frac{w^{*F}}{r^{*F}} > \frac{w^{*H}}{r^{*H}} \tag{4-32}$$

The reversal of factor reward intensity may occur in the FIR Leontief trade.

No matter what kind of trade patterns above, the relative commodity prices $\frac{p_1^*}{p_2^*}$ in the equilibrium will fall inside the cone by the rays of autarky relative commodity prices $\frac{p_1^{aH}}{p_2^{aH}}$ and $\frac{p_1^{aF}}{p_2^{aF}}$, which make sure of gains from trade for both countries.

The price definitions of trade patterns present the views of trade consequences. They provide a new way to observe practices of international trade. The trade patterns are beneficial for studying international trade policies.

Autarky factor prices and autarky commodity prices themself are not sufficient to derive the trade direction due to the complexity of comparative advantages when countries have different productivities. We can evaluate the gains from trade to see how they benefit each country.

4.4 The Dual between Trade Pattern and Localized Factor Prices.

We now summarize the discussions above to a theorem.

The theorem –World Prices and Trade Patterns By the Law of Comparative Advantage

Suppose that two countries are engaged in free trade, having an identical homothetic taste but different productivities and different (or same) factor endowments by the model with different productivities.

When the world commodity prices are formulated, the two countries' factor prices will be localized with their trade pattern, respectively. A country will export a commodity that they have a comparative advantage to produce.

This commodity is produced by using their effective abundant factor intensively. That leads to three trade patterns, the Heckscher-Ohlin trade, the FIR Leontief trade, and the mutual Leontief trade under different production productivity structures with actual factor endowments.

The world consumption composition determines world prices (commodity prices and localized factor prices) and trade patterns. The world consumption composition equals the composition of the world effective endowments. The world prices always make sure that both countries gain from trade.

Proof

Equations (3-25) through (3-27) show the equilibrium prices. Equations (3-28) and (3-29) show trade directions of factor services. Equations (4-20) through (4-32) establish the dual relationship between trade patterns and price patterns. Each trade pattern is associated with its pattern of localized factor prices associated with autarky prices. Appendix E shows the derivation that a country exports its commodity produced by using their effective abundant factor intensively.

The world prices are determined by the composition of world consumption of factor services. They are unique for effective world endowments given. Repeat the derivation in Appendix C with effective endowments, we can demonstrate the gains from trade by the localized factor prices.

End Proof

The world prices make sure gains from trade for both countries. It is a basic requirement for an equilibrium solution in the Heckscher-Ohlin theories, along with comparative advantage. The localized factor prices also satisfied with Helpman (1984) restrictions between factor price differences and factor content of trade

$$(w^j - w^i)'F^{ij} > 0 \tag{4-23}$$

$$(w^j - w^i)'(F^{ij} - F^{ji}) > 0 \tag{4-24}$$

where w^j is the vector of payment in country j and $F^{ij}$is the vector of factor content of trade exported from country j to country i, $i=1,2$, and $j=1,2$. This can be displayed numerically for the three trade patterns.

The parallelogram formed by ONO^*M is an equal localized factor price set. It is with similar property to Dixit and Norman's FPE set. It shows that world commodity prices and the localized factor prices remain the same when the allocation of effective endowments changes within the factor price localization set.

Guo(2015) shows the general trade equilibrium for two factors, two commodities, and multiple countries under the same technologies between countries. It can be extended to the equilibrium with different productivities.

The FIR Leontief trade can occur for higher dimension cases. It can be demonstrated numerically. Appendix F shows how to simulate an FIR that occurs in the higher dimension.

5. Analyses of Trade Patterns by the Virtual Endowments

The idea of the virtual endowments presented the full technologies difference across countries in the Heckscher-Ohlin framework (see Fisher and Marshall, 2008). It is more complex in model structure and equilibrium analyses, although its mathematical expression is still concise. Fisher (2011) proposed the interception of goods diversification cones, which explored the most challenging part of the model's general trade equilibrium with virtual endowments. Fisher (2011) also mentioned that under the virtual endowments assumptions, the classical Heckscher-Ohlin theory holds when technologies and factor prices are identical to those of the reference country. The virtual endowment model's behaviors are similar to or same to the Trefler model's behaviors on the trade patterns.

The $2 \times 2 \times 2$ model with virtual endowments can be expressed as

$$A^h X^h = V^h \qquad (h = H, F) \tag{5-1}$$

$$(A^h)'W^h = P^h \qquad (h = H, F) \tag{5-2}$$

where $A^H \neq A^F$ in general. We refer the model to the Fisher-Marshall model[7] or the Heckscher-Ohlin-Ricordo model[8]. The world virtual endowments referring to the home country's technology can be expressed by the conversion matrix as

[7] In their original notaion, they consider the indirect primary factors by intermediate input in their empirical analysis, such as $A = B(I - \tilde{A})$, where B is input-output matrix, $\tilde{A}$ is directe factor requirement matrix.

[8] Davis (1995) studied this model, referred it as Heckscher-Ohlin-Ricardo model.

$$V^{hW} = V^H + (A^H)^{-1} A^F V^F \quad (5\text{-}3)$$

The world virtual endowments referring to the foreign country's technology can be expressed by the conversion matrix as

$$V^{fW} = V^F + (A^F)^{-1} A^H V^H \quad (5\text{-}4)$$

where $V^{hW}$is the vector of is the factor services needed to produce world commodity x^wusing a reference to the technology matrix of country h as A^h , $h = h, f$. We use the same notation as one used in the effective endowments.

5.1 The intersection of goods price diversification cone and the Intersection of the trade boxes

Technology differences across countries lead to the diversification of trade patterns naturally, which is related to different factor intensities across countries, the actual factor abundance, and the country's virtual factor abundance.

Unlike the Trefler model, the model of virtual endowments is with two price diversification cones. The two rays of the price diversification cone of a country can be expressed as

$$p_K^h = \begin{bmatrix} a_{K1}^h \\ a_{K2}^h \end{bmatrix}, \quad p_L^h = \begin{bmatrix} a_{L1}^h \\ a_{L2}^h \end{bmatrix} \qquad (h = H, F) \quad (5\text{-}5)$$

If both countries are capital intensity in producing commodity 1, the cones of goods price diversification can be expressed in algebra as

$$\frac{a_{K1}^h}{a_{K2}^h} > \frac{p_1^h}{p_2^h} > \frac{a_{L1}^h}{a_{L2}^h} \qquad (h = H, F) \quad (5\text{-}6)$$

This kind of cone structure does not present FIR. However, it can still deliver a mutual Leontief trade, if virtual factor abundance is different from actual factor abundance.

If country H is capital intensive in commodity 1 and country F is labor-intensive in commodity 1, the ranks of rays of two goods price diversification cones are

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^H}{a_{L2}^H} \quad , \quad \frac{a_{K1}^F}{a_{K2}^F} < \frac{a_{L1}^F}{a_{L2}^F} \quad (5\text{-}7)$$

It is the case of factor intensity reversal. The FIR Leontief trade will (always) occurs.

Assume that country H is virtual-capital abundance and country F is virtual-labor abundance. Besides, Assume no FIR presents. There is a range of the share of GNP of country H by the goods price diversification cones (5-4) as

$$s_K^H\left(\begin{bmatrix} a_{K1}^H \\ a_{K2}^H \end{bmatrix}\right) > \quad s^H \quad > \quad s_L^H\left(\begin{bmatrix} a_{L1}^H \\ a_{L2}^H \end{bmatrix}\right) \quad (5\text{-}7)$$

where

$$s_K^H = \frac{a_{K1}^H x_1^H + a_{K2}^H x_2^H}{a_{K1}^H x_1^W + a_{K2}^H x_2^W} = \frac{K^H}{K^{hW}} \quad (5\text{-}8)$$

$$s_L^H = \frac{a_{L1}^H x_1^H + a_{L2}^H x_2^H}{a_{L1}^H x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^{hW}} \quad (5\text{-}9)$$

Substituting (5-8) and (5-9) into (5-7) yields[9]

$$\frac{K^H}{K^{hW}} > \quad s^H \quad > \frac{L^H}{L^{hW}} \quad (5\text{-}10)$$

Similarly, for country F, the range of the share of GNP is

$$\frac{L^F}{L^{fW}} > \quad s^F \quad > \frac{K^F}{K^{fW}} \quad (5\text{-}11)$$

(5-10) and (5-11) implies that country H is virtual-capital abundance and country F is virtual-labor abundant.

Figure 3 is a generalized IWE diagram with virtual endowments. It draws a two-scale diagram that merges the two diagrams. The densities of each diagram's scales are different. The lower-left corner is the two origins for the home country. The upper-right corner is the two origins for the foreign country. Dimension O^1O^{1*} is the world virtual factor endowments measured by technologies of referencing country H to produce the world commodities. Dimension O^2O^{2*} is the world virtual factor endowments measured by technologies of country F. The diagram dimensions just fit V^{hW} and V^{fW}, although $V^{hW} \neq V^{fW}$. The goal is to make subtle changes to each scale's feature density to avoid distortion of the factor content of trade and overall message.

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[9] Leamer (1980, p.502) first mentioned that $\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{L1}^H}{a_{L2}^H}$ is equivalent to $\frac{K^H}{K^W} > \frac{L^H}{L^W}$.

For the simple, we do not analyze actual factor abundances of the two countries in Figure 3.

The figure draws two trade boxes by (5-10) and (5-11). The solid-line box is for the home country; the dash-line box is for the foreign country. The share of GNP is a convex function of commodity prices and virtual factor endowments. The intersection of the two trade boxes, indicated by the diagonal line C^2C^3, reflects the intersection cone of two commodity price cones in the figure.

Point C will change within C^2C^3 when given different commodity prices. However, the signs of F_L^H, F_K^H, F_L^F and F_K^F will not change. Such as F_L^H is always negative, which means the import of labor services. F^H can end at any point within C^2C^3, the trade direction F_L^H and F_K^H remain the same. Therefore, any share of GNP within the diagonal line C^2C^3 can present the right trade direction of factor contents.

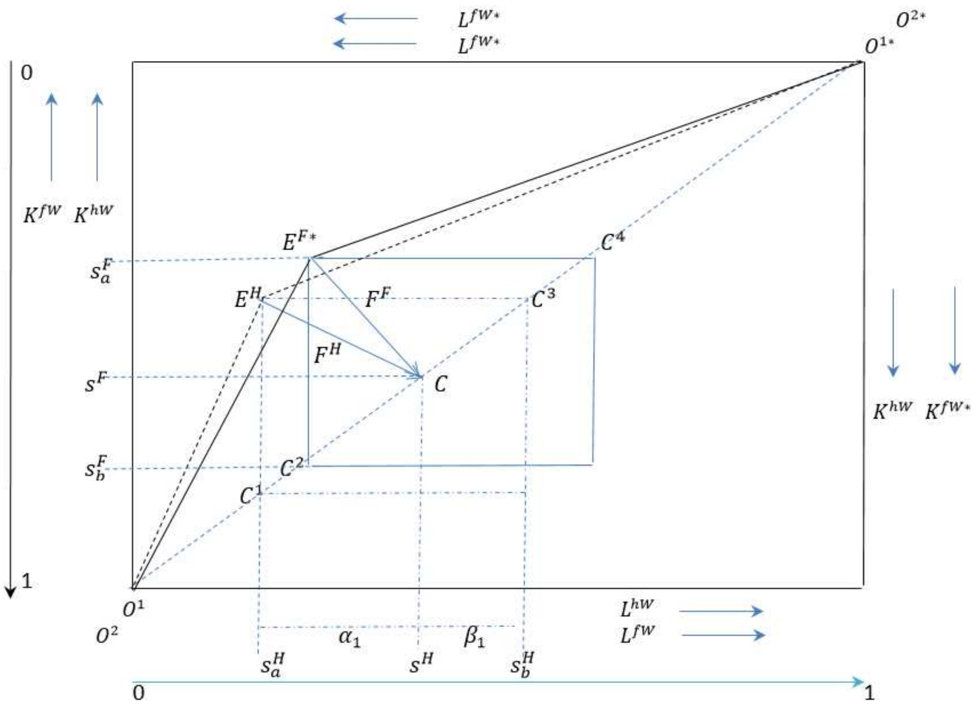

Figure 3 Twoscale IWE Diagram with the Two Trade Boxes

Giving a share of GNP in the range of the two trade boxes' intersection can be used to predict trade direction. It provides a way to demonstrate the three trade patterns numerically. The median of the ranges of the share of GNP (5-10) and (5-11) is a suitable candidate to display the trade direction numerically. Appendix H is a numerical example of the FIR Leontief trade by virtual endowments.

An issue to attain the solution of the general trade equilibrium of the model is that we get two sets of solutions by using the two sets of virtual endowments separately, such as V^{hW} and V^{fW} in (5-10) and (5-11). It leads to two sets of different solutions. They are slightly different. For empirical studies, it is Okay. For the theoretical analysis, this paper leaves it open.

5.2 *Geometric Presentations of Trade Patterns*

Davis and Weinstein (2000) talked about the new perspective of Integrated World Equilibrium (IWE). They mentioned, "A breakdown of FPE and a multiple-cone view of the world will importantly inform additional work on the Heckscher-Ohlin-Vanek model." We move a step in this direction. We use the IWE with the virtual endowments to illustrate both the Leontief trades' geometric presentation and the Heckscher-Ohlin trade's presentation.

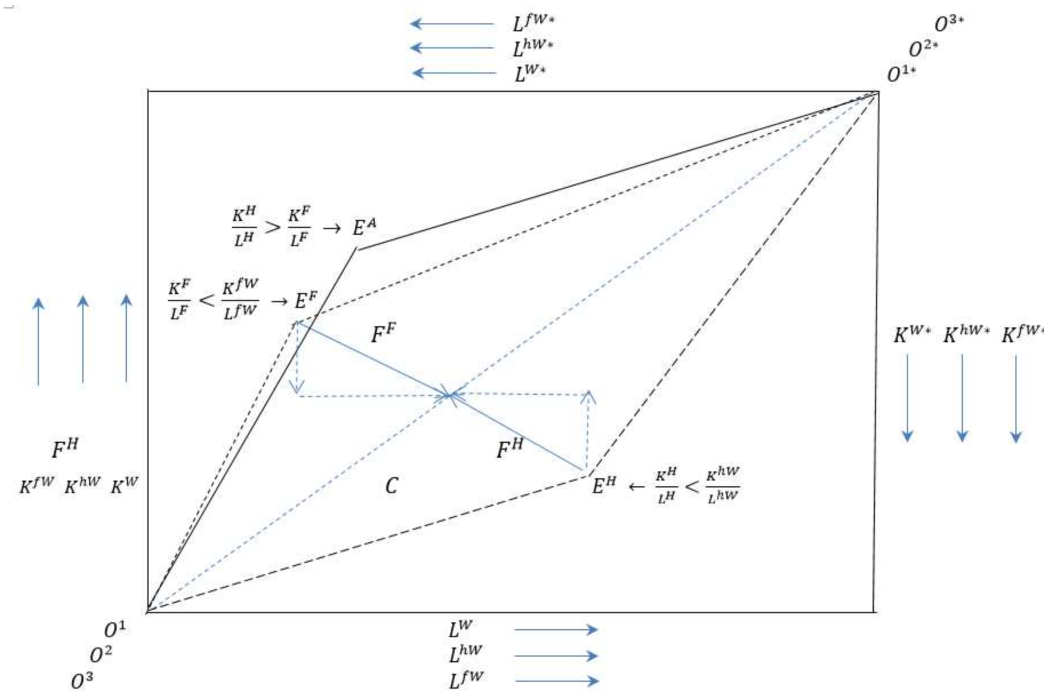

Figure 4 Multiscale IWE Diagram for the FIR Leontief Trade

Figure 4 draws an IWE diagram for the FIR Leontief trade. It is a multiscale diagram that merges the three charts. The densities of each diagram's scales are different. The right-upper corner is three origins for the foreign country. The lower-left corner is three origins for the home country. Dimension O^1O^{1*} is for world actual factor endowments. Dimension O^2O^{2*} is for virtual factor endowments measured by referring to the home country's technology. Dimension O^3O^{3*} is for virtual factor endowments measured by referring to the foreign country's technology.

For a given allocation of actual factor endowments of two countries at E^A, there are two respective allocations of virtual factor endowments E^H and E^{F*}. Allocation E^A is the vector from the home origin O^1. It is above the diagonal line. It indicates that country H is actual capital abundance as

$$\frac{K^H}{L^H} > \frac{K^W}{L^W} \tag{5-13}$$

E^H is the vector from home origin. It indicates the allocation of virtual factor endowments of two countries, measured by country H's technology. It is below the diagonal line. It signifies that country H is virtual labor abundance as

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{5-14}$$

E^F is from foreign origin, it indicates the allocation of the virtual factor endowments of two countries, measured by referring to the foreign country's technology. It is below the diagonal line from the view of foreign origin. It implies that the foreign country is virtual labor abundance as

$$\frac{K^F}{L^F} < \frac{K^{fW}}{L^{fW}} \tag{5-15}$$

Vectors F^F and F^H indicates that both countries export labor services and import capital services.

Figure 5 draws an IWE diagram for the mutual Leontief trade.

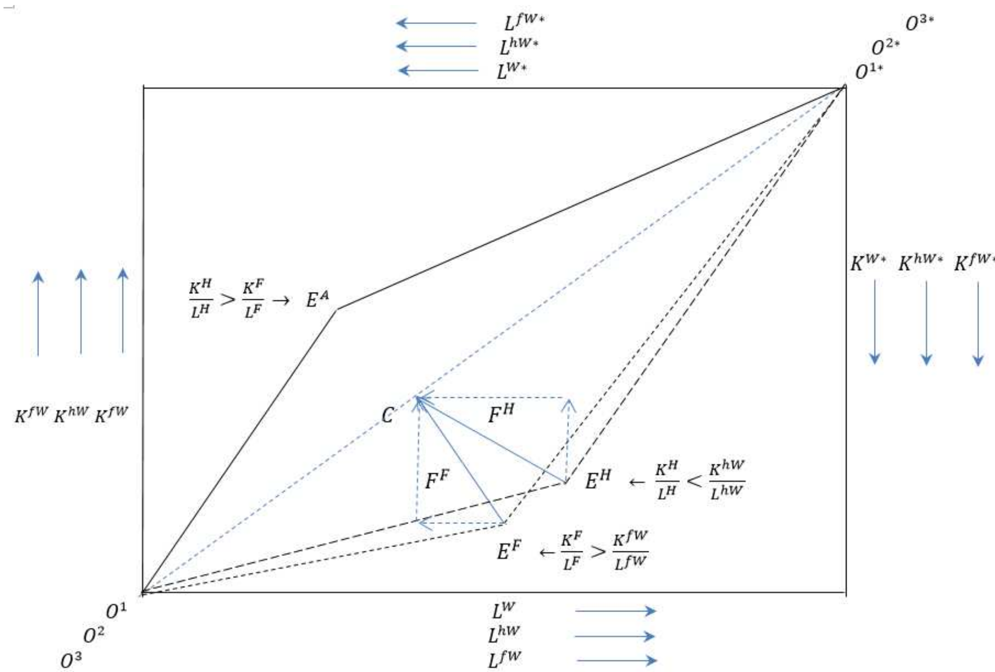

Figure 5. Multiscale IWE Diagram for the Mutual Leontief Trade

Allocation E^A is the vector from the home origin O^1. It indicates that country H is actual capital abundance as

$$\frac{K^H}{L^H} > \frac{K^W}{L^W} \tag{5-16}$$

Point E^H is below the diagonal line. It signifies that country H is virtual labor abundance as

$$\frac{K^H}{L^H} < \frac{K^{hW}}{L^{hW}} \tag{5-17}$$

Point E^F is below the diagonal line from the view of foreign origin. It signifies that the foreign country is virtual capital abundance as

$$\frac{K^F}{L^F} > \frac{K^{fW}}{L^{fW}} \tag{5-18}$$

There are two vectors of factor content of trade, F^H and F^F, in Figure 5. Vector F^H indicates that country H, as an actual capital abundant country, exports labor services; and imports capital services. Similarly, vector F^F indicates that the foreign country, as an actual labor-abundant country, exports capital services; and imports labor services.

When the factor endowments of E^A allocated below the diagonal line and E^H and E^F remain at their allocations, it will be the Heckscher-Ohlin trade.

6. Discussions of Leontief Trade and Empirical Studies of Leontief Paradox

Long-time and extensive discussions of the Leontief paradox and empirical studies have not altogether concluded, but much progress was made in the field. Leontief's input-output studies inspired the international economist to observe and express the factor content of trade. The paradox urges the studies of factor price non-equalizations. Leontief (1953) first proposed the equivalent-productivity unit to describe different productivities across countries. The effective endowments and the virtual endowments are excellent that enrich the Heckscher-Ohlin theories. This paper carries on those schemes to general trade equilibrium analyses and trade pattern analyses.

The factor intensity reversal is the basics of FIR Leontief trade, which displays a comparative advantage on global consumption. Both countries export more effective abundant factors and consume more effective scarce factors. Free trade converts the global effective abundant factor into a global effective scarce factor embodied in the commodity trade flows. The FIR Leontief trade phenomenon is a little bit like the "black hole" [10] in astronomy. The free trade trap or absorber the global effective abundant factor, which cannot "escape" from the market. Meanwhile, free trade is also like the "white hole"[11] that releases or delivers the global effective scarce factor to both countries. It displays a different kind of gains from trade, the gains from the more consumption on global scarce factor.

Some scholars thought that the Leontief paradox is over and that the paradox is explained well. Most scholars believed that the Leontief test pattern's conclusion is not proper even considering technology differences across countries. One reason is that the Leontief trade conceptually is not correct.

With the Leontief trades being a regular trade pattern, we review all empirical results and demonstrate the three trade patterns are consistent with all of them.

6.1 **The sign predictions by the effective endowments and virtual endowments favor both Leontief trade and Heckscher-Ohlin Trade**

In the HOV empirical studies, the sign prediction for the Trefler model can be written as

$$(V_k^i - s^i \sum_j \pi_k^i V_k^j) F_k^i > 0 \tag{6-1}$$

where V_k^j is the element of vector V^j which is defined as $V^j = \Pi_j^{-1} A^0 y^j$. And V_k^j is the factor service needed to product country j's commodity y^j using a reference to productivity in country i as A^0. F_k^i is the factor services exported by country i.

The sign prediction for virtual endowments is

$$(V_k^{vi} - s^i \sum_j V_k^{vj}) F_k^{vi} > 0 \tag{6-2}$$

where V_k^{vj} is the element of vector V^{vj} which is defined as $V^{vj} = A^0 y^j$. V_k^{vj} is the factor service needed to product country j's commodity y^j using a reference to technology matrix in country i as A^0. F_k^{vi} is the factor service exported by country i.

Both the Heckscher-Ohlin trade and the Leontief trades under the logic (6-1) and (6-2). The Leontief trades are derived from the concepts of effective endowments and virtual endowments. Therefore, the two signs above favor both the Heckscher-Ohlin trade and the Leontief trades.

Many empirical HOV studies predicted the trade direction successfully in the last three decades. They illustrate that the concepts of effective endowments and virtual endowments are correct and efficient to present different technologies across countries. The accuracy improvements of those predictions are sourced by including the Leontief trades, which are rejected initially by the Heckscher-Ohlin sign prediction. However, both the prediction signs above are right from the views of this paper. Something mentioned here is that it is not sufficient to use those test results to clear the Leontief paradox issue. The empirical studies by those two signs actually proved the existence of the Leontief trade. The improved accuracy indicates the Leontief trades.

The models with effective endowments and the virtual endowments show the factor price equalization in their systems. They are some mathematical mapping from the real-world of localized factor prices into the effective or virtual equalized factor prices. In the real world of each country, economists and policymakers pay attention to local factor prices and local factor content of trade. It is still the view that Leontief observed the trade contents.

6.2 Reviewing the Early Empirical Studies

Kwok and Yu (2005) investigated the 52 countries' data by using differentiated factor intensity techniques and concluded that the Leontief paradox "is found to be either disappeared or eased."

More than a hundred of the econometric literature about the Leontief paradox had been published between the 1960s and the 1990s. Half of them concluded that the paradox persists, but half of them were instead consistent with the Heckscher-Ohlin theory. The half to half results confused economists, and they cannot be judged whether the

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[10] Black hole in astronomy is defined as that a region of space having a gravitational field so intense that no matter or radiation can escape.

[11] In general relativity, a white hole is a hypothetical region of spacetime, which cannot be entered from the outside, although matter and light can escape from it. In this sense, it is the reverse of a black hole, which can only be entered from the outside and from which matter and light cannot escape.

Leontief trade patterns are correct in the observations or the Heschescher-Ohlin trade patterns are correct. It looks that all the tests are still meaningful from the view of this paper.

The early empirical studies (before 1990) mostly used sign prediction based on the same technology assumption. The reality is that countries are with different productivities. However, we still use the sign prediction by the same technologies to judge if it is the Leontief trade (see (4-20), (4-25), and (4-28)). If some studies show the Heckscher-Ohlin trade by their sign prediction, it is correct because it is okay both for the models from different technologies and the same technologies. If some tests show that no-match with the Heckscher-Ohlin trade, it is also okay because effective (or virtual) abundance may reverse the actual factor abundance in trade direction. We can say that both test results of trade patterns are correct. We do not need to expect a (fixed) percentage of the countries tested with the Leontief trade or with the Heckscher-Ohlin trade. The half to half for the results shows that half are the Leontief trade and the other half are the Leontief trade for the countries observed.

Let cite fewer of them. Keesing(1966) inspected the factor contents of trade in some OECD countries and reported that US exports have relatively higher skill input than their imports. Heller(1976) studied the Japanese economy and documented the changes in trade factor contents. Roskamp (1963) noted that in 1954 West German experts were more labor-intensive than imports. Baldwin (1971) showed that U.S. imports were 27% more capital-intensive than U.S. exports in the 1962 trade data, using a measure similar to Leontief's. All of those studies cannot be ignored for the simple reason that they lack an adequate conceptual foundation. They observed the Leontief trade. The effective (or virtual) endowment takes effect to determine the trade direction of factor contents of trade for those studies.

6.3 Reviewing the Recent Empirical Studies on the skill intensity reversal

In this century, some studies show pieces of evidence of the Leontief trade again. Kurokawa (2011) showed "clear-cut evidence for the existence of the skill intensity reversal" in his empirical study of the USA-Mexico economy. Sampson (2016) interpreted his assignment reversals of skill workforce between North and South by factor intensity reversal. Takahashi (2004) studied the postwar Japanese economy. He interpreted Japan's economic growth as a capital-intensity reversal. Reshef (2007) looked at the model with factor intensity reversals in skill, explaining the North-South skill premia increase well. Kozo and Yoshinori (2017) found the existence of factor intensity reversals in their study as well. They wrote, "Using newly developed region-level data; however, we argue that the abandonment of factor intensity reversals in the empirical analysis has been premature. Specifically, we find that the degree of the factor intensity reversals is higher than that found in previous studies on average". This study shows that conceptually their findings are with the background of international economics.

The localized factor prices (4-24) and (4-32) show the factor reward intensity reversal as trade consequences. We add that the phenomena of skill intensity reversal found can conceptually occur in the FIR Leontief trade (see (4-32)) and in the Heckscher-Ohlin trade (see (4-24)).

Sampson (2016) specially mentioned in his study, "Therefore, assignment reversals offer a new explanation for why trade liberalization has led to increased wage inequality not only in the relative skill abundant North but also in the relative skill scarce South." It sounds that there is the FIR Loentief trade between North and South. Formula (4-32) explains the scenario that it is not only with assignment intensive reversal but also with relative wage increasing both in North and South[12].

This paper's trade patterns stand with all three categories of the empirical studies reviewed above.

6.4 Review of the US test (1954)

The Leontief test (1954) shows that the US, as a capital-abundant country, exported labor-intensive commodities. This study provides a new explanation about the test by the property of the FIR Leontief trade. Suppose that the world economy is composed of the US and the rest of the world. The structure of the economy is with the presence of the FIR. Suppose that the US is actual capital abundant; however, labor is the effective-abundant factor, and capital is the

[12] Fewer studies explained their test results by assumeing that a country may export both factors for two factor model. This may lead another thing like another paradox. It is not consistent with the general theories of the Heckscher-Ohlin framework.

effective-scarce factor for both partners. For the US, effective labor abundance reverses actual capital abundance. Both the US and the rest of the world export labor-intensive commodities and import capital-intensive commodities. The trade tends to improve the payments of workers of the entire world. The trades allow both trade partners to enjoy more capital services embodied in the trade flows. Capital is a globally effective-scarce factor, closer to the world economy's reality in the 1950s.

Conclusion

A lot of subjects of sciences developed by the new result of observations. The Leontief test observed an unexpected trading pattern, which inspired the development of the trade theory. His equivalent productivity unit laid the foundation for factor price localization, effective endowment and virtual endowment. More than half of the empirical studies reported the discovery of the existence of the Leontief paradox. These results cannot be ignored conceptually. This article provides three different approaches to substantiate Leontief trades theoretically.

The paper first presents the general trade equilibrium with unknown constant by Dixit and Norman’s idea directly, then confirms Guo (2015) solution by Helpman and Krugman’s equilibrium approach. The equilibrium of factor price localization is extended from the solution of factor price equalization. The progress is rooted in the Heckscher-Ohlin theories step by step.

The three trade patterns associated with localized factor prices are under the law of comparative advantage. It shows the legacy of comparative advantage of the Heckscher-Ohlin model incorporating different productivities. It explores a comparative advantage both by the differences in factor compositions and productivity differences across countries.

This paper demonstrated that the Leontief trades are regular trade patterns analytically. It also provides the approach to access the autarky prices analytically. The study shows that world effective endowments determine world commodity prices, localized factor prices, and trade patterns when countries have different productivities.

Both the Heckscher-Ohlin trade and the Leontief trade are derived from the concept of effective endowments and virtual endowments. The Leontief trades satisfy the Heckscher-Ohlin framework's core idea that effective or virtual factor abundance determined trade directions. Technically, the study shows two kinds of Leontief trades: the FIR Leontief trade and the mutual Leontief trade. The research shows a surprising feature of the FIR Leontief trade as that both countries export the same factor services and consume more on other factor services. It displays that the Heckscher-Ohlin trade can present factor reward intensity reversal (skill intensity reversal) when countries have different productivities. It shows some views, which are much different from the original Heckscher-Ohlin theory, merely by introducing different productivities across countries.

The study shows that the sign predictions, commonly used in empirical studies by effective endowments and the virtual endowments, favor both the Heckscher-Ohlin trade and the Leontief trades. Therefore, accuracy improvements by effective endowments and virtual endowments do not mean there is no room for the Leontief trades. On the contrary, the Leontief trades are presented in the tests because the accuracy improvements of sign predictions are majorly by including the Leontief trades. It may be hard to be accepted by many audiences. This study roughly cites and reviews most of the empirical tests existing. This paper can explain most of them well, including the sign predictions by effective endowments and virtual endowments. The empirical studies have approved Leontief trades; they are only not realized.

Each trade pattern associates its factor price structures among countries. They illustrate which factor is benefited more by international trade. It is helpful to review international trade policies.

Demonstrating the Leontief trade conceptually by geometric or numerical methods with the intersection of goods price diversification cones is straightforward. They are two independent approaches to show the Leontief trade.

Appendix A – The General Trade Equilibrium of Factor Price Equalization

We first use the IWE diagram to present a trade box in the IWE diagram to include the price constraint on the trade balance.

The relative world commodity prices $\frac{p_1^*}{p_2^*}$ should lie between the rays of goods price diversification cone (see Fisher, 2011) in algebra as[13],

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{A-1}$$

This condition makes sure that the factor prices by the unit cost equation (2-2) are positive. The boundaries of the share of GNP, s^H, corresponding to the rays of the goods price diversification cone above, can be calculated as

$$s_{max}^H(p) = s\left(\begin{bmatrix} a_{K1}^H \\ a_{K2}^H \end{bmatrix}\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^W} \tag{A-2}$$

$$s_{min}^H(p) = s\left(\begin{bmatrix} a_{L1}^H \\ a_{L2}^H \end{bmatrix}\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^W} \tag{A-3}$$

These compose the range of s^H, which Leamer (1984, p.9) first proposed in another analytical way, as

$$\frac{K^H}{K^W} > s^H > \frac{L^H}{L^W} \tag{A-4}$$

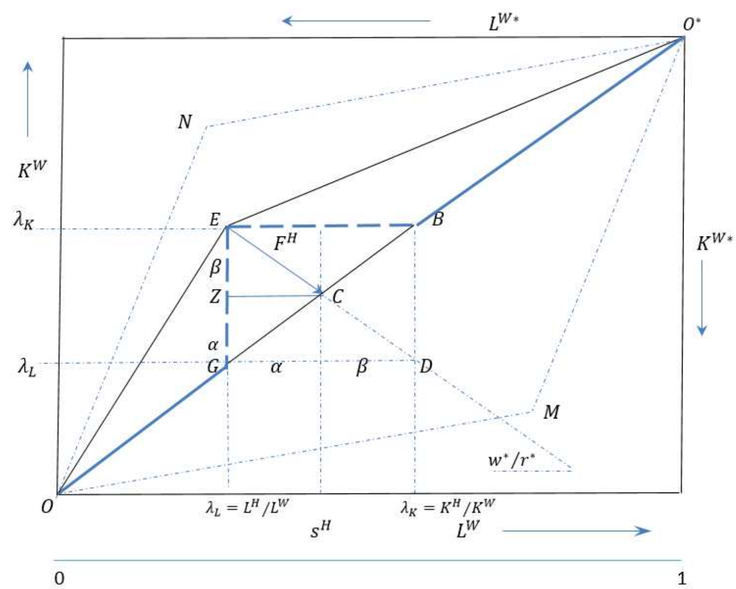

Figure A IWE Diagram with Trade Box

Figure A is an IWE diagram added with a trade box. The dimensions of the diagram represent world factor endowments. The lower-left corner is the origin fro the home country, and the foreign country is from the right-upper corner. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Helpman and Krugman (1985, p.15) call the parallelogram the FPE set.

Suppose that allocation E is a distribution of world factor endowments. Country H is capital abundant at this point Point C represents the trade equilibrium point. It shows the sizes of the consumption of the two countries.

We identify the trade box by using GNP's range of share in (2-8). If a relative commodity price lies in the price diversification cone (2-5), the share of GNP by the price lies within the trade box $EBDG$.

[13] It soppuses that country H is capital in tensive in producing commodity 1.

For a given allocation E, its equilibrium point C needs to fall within the line $\overline{GB}$ on the diagonal line of the trade box, which implies the constraint of the goods price diversification cone.

The share of GNP, s^H, divides the trade box into two parts: α and β,

$$\alpha = s^H - \lambda_L \quad \text{(A-7)}$$

$$\beta = \lambda_K - s^H \quad \text{(A-8)}$$

When α increases, country H's share of GNP increases, and country F's share of GNP decreases, and vice versa. In trade competitions between countries, each of them wants to take its comparative advantage to export their commodity that used their abundant factor intensively. And each country seeks to maximize the factor price of its abundant factor to achieve its maximum share of GNP of the world. However, only the share of GNP inside the trade box is redistributable by trade. We call α as a redistributable share of GNP for country H, and β is one for country F.

We rewrite the trade balance of factor contents of trade (2-4) as

$$\frac{w^*}{r^*} = \frac{(\lambda_K - s^H)}{(s^H - \lambda_L)}\frac{K^W}{L^W} = \frac{\beta}{\alpha}\frac{K^W}{L^W} \quad \text{(A-9)}$$

where superscript * indicates world price.

Triangle ΔEZC in figure A represents the factor contents of trade. The trade volume for country H is

$$VT = (\lambda_K - s^H)K^W r^* + (s^H - \lambda_L)L^W w^* \quad \text{(A-10)}$$

Based on (A-9), suppose

$$w^* = (\lambda_K - s^H)K^W \quad \text{(A-11)}$$

We then express r^* as[14]

$$r^* = (s^H - \lambda_L)L^W \quad \text{(A-12)}$$

Substituting them to (A-10) yields

$$\mu = 2(\lambda_K - s^H)(s^H - \lambda_L)L^W K^W \quad \text{(A-13)}$$

It shows that the trade volume VT is a quadratic function of s^H. μ reaches its maximum value when $s^H = \frac{1}{2}(\lambda_K + \lambda_L)$, which is the equilibrium share of GNP.

Appendix B – The general trade equilibrium of factor price equalization derived by Helpman and Krugman equilibrium approach

The traditional trade volume is defined with the factor content of trade as

$$VT = 2F_K^H r^* = -2F_L^H w^* \quad \text{(B-1)}$$

We suppose there that country H is capital abundant. Use notation in (A-9) and rewrite (B-1) as

$$VT = 2\beta K^W r^* \quad \text{(B-2)}$$

$$VT = 2\alpha L^W w^* \quad \text{(B-3)}$$

The "abstract" trade volume is defined with the domestic factor endowments,

$$VT = \gamma_H L^H + \gamma_K K^H \quad \text{(B-4)}$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \quad \text{(B-5)}$$

The country H's factor endowment vector V^H can be written as (seeing Figure A),

$$V^H = \begin{pmatrix} K^H \\ L^H \end{pmatrix} = \overrightarrow{OG} + \overrightarrow{EG} \quad \text{(B-6)}$$

$\overline{OG}$ represents the part of the factor endowments of country H, which is with the proportion of world factor consumptions as

$$\overrightarrow{OG} = \begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} \quad \text{(B-7)}$$

$\overline{EG}$ is the excessive capital services, which is out of the proportion of world factor consumptions

$$\overrightarrow{EG} = \begin{pmatrix} (\alpha + \beta)K^W \\ 0 \end{pmatrix} \quad \text{(B-8)}$$

The trade volume (B-2) can be rewritten as a dot product of V^H and the pair of the variables $(\gamma_K \quad \gamma_L)$

[14] It actually uses the Walras equilibrium law to drop one market clearing condition.

$$VT^H = (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} K^H \\ L^H \end{pmatrix} \tag{B-9}$$

Substituting (B-6) into it yields

$$TV^H = (\gamma_K^H \quad \gamma_L^H) \cdot (\overrightarrow{OG} + \overrightarrow{EG}) = (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} + (\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} (\alpha+\beta)K^W \\ 0 \end{pmatrix} \tag{B-10}$$

The first term on the right side above is zero by (B-5),

$$(\gamma_K^H \quad \gamma_L^H)\begin{pmatrix} \lambda_L K^W \\ \lambda_L L^W \end{pmatrix} = 0 \tag{B-11}$$

Simplify (B-10) as

$$VT^H = (\alpha+\beta)K^W \gamma_K^H \tag{B-12}$$

Similarly, the trade volume for country F is

$$VT^F = (\alpha+\beta)L^W \gamma_L^F \tag{B-13}$$

Substituting (2-12) into (2-13) yields

$$\frac{K^W}{L^W} = \frac{\gamma_L^F}{\gamma_K^H} \tag{B-14}$$

Rewrite (A-9) as

$$\frac{\beta w^*}{\alpha r^*} = \frac{K^W}{L^W} \tag{B-15}$$

Substituting it into (B-14) yields

$$\frac{\gamma_L^F}{\gamma_K^H} = \frac{\beta w^*}{\alpha r^*} \tag{B-16}$$

Assume

$$\gamma_K^H = r^* \tag{B-17}$$

Substituting it into (B-16) yields

$$\gamma_L^F = \frac{\beta}{\alpha} w^* \tag{B-18}$$

Substituting (B-17) into (B-12) yields

$$VT^H = (\alpha+\beta)K^W r^* \tag{B-19}$$

Letting (B-18) equal the trade volume (B-2) yields

$$(\alpha+\beta)K^W r^* = 2\beta K^W r^* \tag{B-20}$$

It yields

$$\alpha = \beta \tag{B-21}$$

Substituting it into (B-15) yields

$$\frac{w^*}{r^*} = \frac{K^W}{L^W} \tag{B-22}$$

Appendix C - Gains from Tarde

The gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{C-1}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{C-2}$$

We add a negative sign in inequalities above since we expressed the net factor content of trade by export. In most other works of literature, they denoted the net factor content of trade by import.

We express the gains from trade for the home country as

$$-(W^{Ha})'F^H > 0 \tag{C-3}$$

Adding trade balance condition $W^{*\prime}F^H = 0$ on the two sides of (C-3) yields

$$-((W^{Ha})' - W^{*\prime})F^H > 0 \tag{C-4}$$

where W^{Ha} and W^* are

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix}, \qquad W^* = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix} \tag{C-5}$$

Substituting them into (C-4) yields,

$$-\begin{bmatrix}\frac{L^H}{K^H}-\frac{L^W}{K^W} & 0\end{bmatrix}\begin{bmatrix}\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}\\ -\frac{1}{2}\frac{K^HL^W-K^WL^H}{K^W}\end{bmatrix}>0 \quad \text{(C-6)}$$

It can be rewritten to

$$-(\frac{L^H}{K^H}-\frac{L^W}{K^W})\times\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}>0 \quad \text{(C-7)}$$

Simplify the above to

$$\frac{(K^HL^W-K^WL^H)^2}{2L^WK^WK^H}>0 \quad \text{(C-8)}$$

So that (C-1) holds. Similarly, we can obtain

$$-W^{Fa'}F^F=\frac{(K^HL^W-K^WL^H)^2}{2L^WK^WK^F}>0 \quad \text{(C-9)}$$

It implies that the world prices at equilibrium will ensure the gains from trade for both countries. The quantitative or computable gains from trade are essential for international trade analyses.

At the equilibrium, each country exports the commodity in which it has a comparative advantage to produce. The world factor endowments, fully employed, determine world prices, ensuring trade gains for countries taking part in free trade.

Appendix D Trade Volumes of Factor Contents of Two Countries Are the Same When Productivities Are Different Across Countries

This appendix illustrates that trade volumes of the two countries' factor contents are the same for the Trefler model. The factor contents of the two countries are

$$T^H=(A^H)^{-1}F^H \quad \text{(D-1)}$$

$$T^F=(\Pi A^H)^{-1}F^H \quad \text{(D-2)}$$

By trade flows relationship $T^H=-T^F$, we get

$$\begin{bmatrix}F_1^F\\F_2^F\end{bmatrix}=-\begin{bmatrix}\frac{1}{\pi_K}F_1^H\\ \frac{1}{\pi_K}F_2^H\end{bmatrix} \quad \text{(D-3)}$$

(3-27) says

$$\begin{bmatrix}r^{*F}\\w^{*F}\end{bmatrix}=\begin{bmatrix}\pi_Kr^{*H}\\ \pi_Lw^{*H}\end{bmatrix} \quad \text{(D-4)}$$

The trade volume for country H is

$$VT^H=2|F_1^H|r^{*H} \quad \text{(D-5)}$$

The trade volume for country F is

$$VT^F=2|F_1^F|r^{*H} \quad \text{(D-6)}$$

Substituting $F_1^F=\frac{1}{\pi_K}F_1^H$ and $r^{*F}=\pi_Kr^{*H}$ into (D-6), we see $VT^H=VT^F$. The trade volumes of net factor contents of the two countries are the same.

Appendix E – A Country exports its commodity that is produced by using its effective abundant factor intensively.

Suppose country H is effective capital abundance. It will export capital services and import labor services. Therefore, the vector of factor content of trade is with signs

$$F^H=\begin{bmatrix}+\\-\end{bmatrix} \quad \text{(E-1)}$$

The signs of trade flow of country H from equation (5-13) will be[15]

[15] Leamer used the inversion matrix of technology matrix as

$$A^{-1}=\begin{bmatrix}a_{K1} & a_{K2}\\a_{L1} & a_{L2}\end{bmatrix}^{-1}=\begin{bmatrix}a_{L2} & -a_{K2}\\-a_{L1} & a_{K1}\end{bmatrix}/|A|$$

$$T^H = (A^H)^{-1}F^H = \begin{bmatrix} + & - \\ - & + \end{bmatrix}\begin{bmatrix} + \\ - \end{bmatrix}=\begin{bmatrix} + \\ - \end{bmatrix} \tag{E-2}$$

This is due to country H is capital intensive in commodity 1 by $|A^H| > 0$. If country H is labor-intensive in commodity 1, there is

$$T^H = (A^H)^{-1}F^H = \begin{bmatrix} - & + \\ + & - \end{bmatrix}\begin{bmatrix} + \\ - \end{bmatrix}=\begin{bmatrix} - \\ + \end{bmatrix} \tag{E-3}$$

Appendix F – The FIR Leontief Trade for Many Factors and Many Commodities

The FIR Leontief trade also occurs in the models with many commodities and many factors and many countries. A straightforward way to specify a FIRs model in high dimensions is by switching a pair of rows in its technology matrix. Row-switching matrix S_{ij}, like the following, switches all matrix elements on row *i* with their counterparts on row *j*.

$$S_{ij} = \begin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & 0 & 0 & 1 & & & \\ & & & 0 & \ddots & 0 & & & \\ & & & 1 & 0 & 0 & & & \\ & & & & & & 1 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 1 \end{bmatrix}$$

The corresponding elementary matrix is obtained by swapping row *i* and row *j* of the identity matrix. Since the determinant of the identity matrix is unity, $\det[S_{ij}] = -1$. It follows that for any square matrix *A* (of the correct size), we have $\det[S_{ij}A] = -\det[A]$. Using a row-switching operation, we can implement a FIRs model. It is also available for non-square (not even) technology matrix. The conversion trade not only occurs for the even model (factor number equals to output number) but also for the non-even model. To specify a non-even FIR model, just use a square Row-switching matrix S_{ij}.

We present a numerical example to display a conversion trade for $4 \times 4 \times 2$ model. The technological matrix for country H is

$$A^H = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 \\ 1.1 & 2.0 & 0.9 & 1.4 \\ 0.7 & 1.5 & 2.1 & 1.0 \\ 1.6 & 1.7 & 0.8 & 1.5 \end{bmatrix}$$

The technology matrix for country F is

$$A^F = \psi \;\; A^H$$

where

$$\psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A^F = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 \\ 1.1 & 2.0 & 0.9 & 1.4 \\ 1.6 & 1.7 & 0.8 & 1.5 \\ 0.7 & 1.5 & 2.1 & 1.0 \end{bmatrix}$$

The third row and fourth row A^F are switched from A^H.

The factor endowments of the two countries are

$$V^H = \begin{bmatrix} 4253 \\ 4189 \\ 3631 \\ 4098 \end{bmatrix}, \qquad V^F = \begin{bmatrix} 3690 \\ 4975 \\ 3865 \\ 4080 \end{bmatrix}$$

The world effective abundant by the home productivities are

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where $|A| = a_{L1}a_{L2}\left({}^{a_{K1}}/_{a_{L1}} - {}^{a_{K2}}/_{a_{L2}}\right) > 0$. the sign of A^{-1} will be $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

$$V^{hW} = \begin{bmatrix} 8333 \\ 8054 \\ 8606 \\ 7788 \end{bmatrix}$$

The world effective abundant by foreign productivities are

$$V^{fW} = \begin{bmatrix} 8333 \\ 8054 \\ 7788 \\ 8606 \end{bmatrix}$$

We see that the values of V_3^{fW} and V_4^{fW} are reversals of V_3^{hW} and V_4^{hW}. Both countries are effective abundant at factor 4 related to factor 3

$$\frac{v_3^H}{v_4^H} = \frac{3631}{4098} = 0.886 < \frac{v_3^{hW}}{v_4^{hW}} = \frac{8606}{7788} = 1.105$$

$$\frac{v_3^F}{v_4^F} = \frac{3864}{4080} = 0.947 < \frac{v_3^{fW}}{v_4^{fW}} = \frac{7788}{8606} = 0.949$$

That will cause the factor content reversals between factor 3 and factor 4.

Appendix G– Numerical Example of the FIR Leontief Trade

We suppose a numerical example with the presence of FIRS. The technological matrice for the two countries are

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix} \qquad A^F = \begin{bmatrix} 1.3125 & 2.125 \\ 3.1875 & 0.875 \end{bmatrix}$$

The factor intensities of the two countries are

$$\frac{a_{K1}^H}{a_{K2}^H} = 3 > \frac{a_{L1}^H}{a_{L2}^H} = 0.75, \qquad \frac{a_{K1}^F}{a_{K2}^F} = 0.6176 < \frac{a_{L1}^F}{a_{L2}^F} = 3.6428$$

The home country is capital intensive in sector 1, and the foreign country is capital intensive in sector 2. It is a case with the presence of FIRs.

We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 4200 \\ 3000 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 2568.75 \\ 2381.25 \end{bmatrix}$$

The home country is actual capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \quad \frac{K^F}{L^F} = \frac{2568.75}{2381.25} = 1.0787$$

The home country is effective capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \quad \frac{K^{hF}}{L^{hF}} = \frac{2400}{2550} = 0.9417$$

The foreign country is effective capital abundant also as

$$\frac{K^F}{L^F} = \frac{2568.75}{2381.25} = 1.0787 > \quad \frac{K^{fH}}{L^{fH}} = \frac{2859}{4350} = 0.6517$$

Therefore, the home country exports the net excess of capital and exports commodity1, capital-intensive product. The foreign country exports the excess of capital and exports commodity 2, the capital-intensive product. The home country will make the Leontief trade, and the foreign country will make the Heckscher-Ohlin trade. The output of the two countries are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 1200.0 \\ 600.0 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 500.0 \\ 900.0 \end{bmatrix}$$

As we discussed that any share of GNP within the intersection of trade boxes in figure 5 could illustrate trade direction, we take the median of the ranges of the shares of GNP (5-10) and (5-11). It is calculated as

$$s^H = \frac{1}{4}\left(\frac{K^H}{K^{fW}} + \frac{L^H}{L^{fW}} + \frac{K^{fH}}{K^{fW}} + \frac{L^{fH}}{L^{fW}}\right)$$

By this formula, we obtain

$$s^H = 0.5872.$$

The export volumes and the factor contents of trades by the share of GNP above are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 201.63 \\ -280.91 \end{bmatrix} = - \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix}$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} 323.29 \\ -259.37 \end{bmatrix}, \quad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} 332.29 \\ -396.91 \end{bmatrix}$$

We see that both countries export capital services and import labor services. The trade converts the globally effective abundant factor into the globally scarce factor. At the equilibrium, the world prices and the localized factor prices are

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.9016 \\ 2.8400 \end{bmatrix}, \quad \begin{bmatrix} r^{*H} \\ w^{*H} \end{bmatrix} = \begin{bmatrix} 0.8005 \\ 1.0 \end{bmatrix}, \quad \begin{bmatrix} r^{*F} \\ w^{*F} \end{bmatrix} = \begin{bmatrix} 0.9800 \\ 0.8204 \end{bmatrix}$$

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