Downstream Subsidization and Upstream Privatization with a Vertically Integrated Foreign Firm

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Downstream Subsidization and Upstream Privatization

with a Vertically Integrated Foreign Firm

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Abstract
This study constructs a model with a vertical structure in which a state-owned enterprise (SOE) in an upstream market and a private firm in a downstream market compete with a vertically integrated foreign firm (VIFF). Given a cost-inefficient SOE, we examine the strategic entry decision of a VIFF that can enter either the upstream, or the downstream market, or both, under downstream subsidization and upstream privatization policies. We find that when the government implements a subsidy policy, the VIFF enters only the downstream market if the cost inefficiency is low and enters both markets otherwise; however, the social welfare of the later is always higher than that of the former. We also find that reducing the cost inefficiency might cause welfare loss when ex-ante inefficiency is intermediate, below which the VIFF might change its entry decision. Finally, we show that an upstream privatization policy reduces welfare either when the cost inefficiency ex-post privatization decreases to a lesser degree or when the ex-ante inefficiency is relatively low.

Keywords Downstream Subsidization · Upstream Privatization · Vertically Integrated Foreign Firm · Cost Inefficiency · Entry Decision · Mixed Market

JEL Classification L13 · D45 · H23

1 Introduction

Vertically integrated foreign firms (VIFFs) currently dominate global markets. For example, one of the most representative VIFFs in both the semi-conductor and smart phone markets is Samsung, a Korean company. Due to the trade war between China and the United States in recent years, Samsung stopped

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selling smart phone components to Huawei on Sept 15, 2020. That is, Samsung changed its strategy in China from supplying both markets to operating in the downstream market only. Another example in China is Baden Aniline and Soda Factory (BASF), a German chemical company. In November 2019, it began construction on a new factory in Zhanjiang, China, which includes a steam cracker with a capacity of 1 million metric tons of ethylene annually and downstream plants for consumer-oriented products. BASF will not only enter the intermediate goods market and produce engineering plastics, an intermediate inputs for cars, but also enter the final goods market to produce and sell basic chemicals and electronics.³

The entry strategies and operations of VIFFs in domestic markets will not only contribute to the development of the local economy, but also influence the production activities of domestic upstream and downstream firms. In this study, we analyze how a VIFF’s entrance strategy will affect the welfare consequences of domestic markets, especially when the VIFF can enter the upstream market, the downstream market, or both under the government’s policies.

On the contrary, state-owned enterprises (SOEs) existed for a long time as an instrument to fulfill government objectives in both developing and developed countries. They are often the key suppliers of intermediate inputs to domestic downstream firms in many industries such as the electricity, power generation, water, transportation, mining, oil, telecommunication, post, financing and banking services, and energy-related industries.⁴ Governments can control SOEs in vertical relations with downstream firms, which rely completely on buying inputs from upstream SOEs to produce (or sell directly) final goods. Simultaneously, governments usually provide various incentives to encourage their business activities. Wade (1990) also reports that foreign competition is still allowed with the existence of SOEs.

However, many developed and developing countries have been privatizing their SOEs since the 1980s for the purposes of improving cost efficiencies and/or reducing government budgets, due,  

⁴ According to an OECD report by Kowalski et al. (2013), among the 2,000 largest public companies in the world, more than 10% are either SOEs or have significant government ownership; these government-associated companies’ sales are equivalent to approximately 6% of global GDP. Further, Kurlantzick (2012) reports that SOEs control about 90% of the world’s oil and Christiansen (2011) reports that around half of all SOEs in OECD countries are in industries that can be classified as upstream industries. For evidence, see Chang and Ryu (2015) and Xu et al. (2021).
respectively, to inefficient operations and government deficits. The privatization policies in these industries attracted the attention of policy makers and economics researchers in developed, developing, and transitional economies such as Eastern Europe, Latin America, and Asia, including China. Furthermore, numerous governments in both developed and developing countries offer substantial subsidies or tax credits to domestic firms to remedy firm-level distortions and to ensure access to financial resources and reallocate resources across firms.

In the economics literature, several studies examined the vertical structure of production and privatization policy. Several works provide theoretical analyses of mixed oligopolies in which an SOE competes with private firms in vertically related markets. A few early works by Lee (2006) and Willner (2008) consider a vertical structure model in which an SOE provides both inputs and downstream services, and competes with downstream private firms. These studies show that vertical separation or the cost advantage of the independent rivals improves welfare post privatization. Further, Matsumura and Matsushima (2012) and Yang et al. (2014) consider a vertical market consisting of an upstream mixed oligopoly and a downstream private oligopoly with several private firms and examine the optimal degree of partial privatization. Liu et al. (2019) and Liu and Wang (2021) extend these works to a model with a successive mixed oligopoly market and examine the welfare effect of a retailer’s effort on the optimal privatization policy. However, these studies neither consider foreign competition nor include downstream subsidization policy.

The recent decade witnessed increasing application of international mixed oligopoly frameworks in which domestic public or/and private firms compete with foreign private firms, which is a useful instrument for analyzing policy interactions between governments and foreign firms. Chang and Ryu (2015) study the optimal privatization policy in a vertical market in which an upstream public firm competes with a foreign private rival in supplying an input to the downstream market in which domestic and foreign downstream firms compete in the domestic market. Wu et al. (2016) also consider a vertical framework.

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5 Lee and Hwang (2003) and Lee (2006) provide further discussion on the policy concerns around privatization.
6 Most governments prefer to subsidize output as output enhancement is politically more popular among consumers and less complicated than other subsidies. Studwell (2013) and Lee et al. (2017) show that subsidies, along with other policies, played an important role in the economic development of Asian countries such as Japan, South Korea, and China.
market structure involving an upstream foreign monopolist selling an essential input to an SOE and a private firm located downstream in the domestic economy, where the downstream private firm may be owned by domestic and/or foreign investors. However, the foreign private firm is not integrated and an upstream public firm has the same cost efficiency as the foreign firm.

In this study, we construct a vertical structure model where an SOE in an upstream market and a domestic private firm in a downstream market compete with a VIFF, which can enter the upstream market, downstream market, or both. We then examine the strategic entry decision of a VIFF under the downstream subsidization and upstream privatization policies, and compare the welfare effects of the entry decisions when the SOE has a cost inefficiency.

We show that when the government implements a subsidy policy, the VIFF’s strategic decision is to enter only the downstream market if the cost inefficiency is low and to enter both markets otherwise. However, the social welfare of the later is always higher than the former; thus, it is socially desirable to induce the VIFF to enter both markets, irrespective of cost inefficiency. We also show that a reduction in the SOE’s cost inefficiency might cause welfare loss when the ex-ante cost inefficiency is intermediate. Below this level, the VIFF might change its entry decision. This result implies that a certain level of cost inefficiency of the SOE is necessary for the VIFF’s strategic decision. Finally, we examine an upstream privatization policy and show that it reduces welfare unless it improves cost efficiency post privatization. Further, we find that though cost gap between the SOE and the VIFF can be reduced post privatization, it might not increase welfare if the SOE’s ex-ante cost inefficiency is relatively low.

The remainder of this paper is organized as follows. Section II provides the basic model. In Sections III and IV, we analyze the equilibrium outcomes and compare the results, respectively. In Sections V and VI, we examine the equilibrium outcome under a privatization policy and discuss the welfare effect post privatization. Section VII concludes the paper.

2 The Model

We consider a vertical structure model in which an SOE (firm U) in an upstream market provides an intermediate good used by a domestic private firm (firm D) in a downstream market. Both firms
compete with a VIFF (firm F), which produces both intermediate and final goods, and thus competes with firm U in the intermediate goods market and firm D in the final goods market. For simplicity, we assume that both firms in each market produce homogeneous products while producing one unit of the final good requires one unit of the intermediate input.

Let $q$ and $q_F$ be the output of firm D and firm F in the final goods market, respectively. In the final goods market, the inverse demand function is

$$p(Q) = a - Q,$$

where $a > 1$, $p$ is the market price, and $Q(= q + q_F)$ is the total output of final products.

Let $x$ and $x_F$ be the output of firms U and F in the intermediate goods market, respectively. The industry intermediate output is

$$X = x + x_F,$$

(2)

Since F can produce intermediate goods and transform them into final goods through internal transactions, only firm D purchases the industry output of intermediate goods. Thus, X is the total supply of intermediate goods while $q$ is the total demand for intermediate goods. Therefore, we can express $q$ as

$$q = X = x + x_F.$$

(3)

We assume that firm D has constant marginal cost with zero, for simplicity. Firm F has also constant marginal cost. We denote its cost function by

$$C_F(q_F + x_F) = q_F + x_F.$$

(4)

This setting implies that the production cost of final goods is zero, which is the same as firm D, while that of intermediate goods is constant and proportional to the total amount of firm F’s production of intermediate goods.

In contrast, we assume that firm U’s cost function is

$$C(x) = x + \frac{1}{2}cx^2,$$

(5)

where $c(\geq 0)$ represents the SOE’s cost inefficiency. Note that if $c = 0$, then the marginal cost of firm U is also constant and there is no cost gap between firms U and F. However, if $c > 0$, then the
marginal cost of firm U is increasing with its production of intermediate goods.\(^7\)

In this case, we can denote the profit function of firm U as

$$
\pi_U = vx - C(x),
$$

(6)

where \( v \) is the market-clearing price of the intermediate goods, which equates the total supply with the total demand of intermediate goods.

We also assume that the government can grant an output subsidy to domestic firm D. Therefore, the profit function of D is

$$
\pi_D = p(Q)q - vq + sq,
$$

(7)

where \( s \) is the subsidy rate, which applies to the amount of its production of final goods.

The profit function of firm F is

$$
\pi_F = p(Q)q_F + vx_F - C_F(q_F + x_F).
$$

(8)

Finally, we assume that firms D and F maximize their own profits, while firm U maximizes social welfare, which is the sum of the consumer surplus \( CS = Q^2 / 2 \) and the profits of firms U and D, minus the amount of government subsidy:

$$
W = \frac{1}{2} Q^2 + \pi_D + \pi_U - sq,
$$

(9)

We consider the entry strategies of firm F under a domestic subsidization policy and then compare the equilibrium outcomes.

3 Analysis

In this section, we examine the four entry decision cases of the VIFF: (1) no entry, (2) entry to the intermediate goods market only, (3) entry to the final goods market only, and (4) entry to both markets.

The game with three stages in each case runs as follows. In the first stage, the government chooses the optimal output subsidy rate. In the second stage, if firm F enters the upstream market, then the upstream duopoly firms U and F compete in the intermediate goods market. In the last stage, if firm F enters the downstream market, then the downstream duopoly firms D and F compete in the final goods market. We can obtain the subgame perfect Nash equilibrium outcomes by backward induction.

\(^7\) Note that quadratic cost of the SOE ensures interior solutions in mixed markets. For more discussion, see Lee and Hwang (2003) and Kim et al. (2019).
3.1 No entry

If firm F does not enter either the intermediate goods market or final goods market, then there are two domestic firms, firms U and D, in each market. In the final goods market of the last stage, the first-order condition of firm D to maximize its profits yield the equilibrium output

\[ q^* = \frac{a+s-v}{2}, \]

(10)

where the superscript * denotes the equilibrium of the final goods market. Because \( q^* = Q = X = x \), we can rewrite (10) as follows, where we can express \( v, Q, \) and \( p \) as

\[ v = a - 2q + s = a - 2x + s, Q = q = x, p = a - Q = a - x. \]

(11)

The profits of firms D and U, and social welfare are, respectively.

\[ \pi_D = p(Q)q - vq + sq = x^2, \]

(12)

\[ \pi_U = vx - C(x) = -x + (a + s - 2x)x - \frac{cx^2}{2}, \]

(13)

\[ W = \frac{1}{2}Q^2 + \pi_D + \pi_U - sq = \frac{1}{2}(-2 + 2ax - x^2 - cx^2). \]

(14)

In the second stage, firm U decides the intermediate output to maximize social welfare. The first-order condition yields the following equilibrium outputs:

\[ x_1^* = X_1^* = q_1^* = Q_1^* = \frac{a-1}{1+c}, \]

(15)

where \( x_1^*, X_1^*, q_1^*, \) and \( Q_1^* \) denote the equilibrium outputs in case (1) of no entry.

Substituting (15) into (14) provides social welfare:

\[ W_1^* = \frac{(a-1)^2}{2(1+c)}. \]

(16)

In the first stage, the government determines the subsidy rate to maximize social welfare. Note that social welfare is independent of W and thus the optimal rate should satisfy the interior solutions of the equilibrium in (11), which is \( s_1^* > \frac{a(1-c)-2}{1+c} \). Note that the optimal output subsidy can be negative if \( c > \frac{a-2}{a} \), that is, the cost inefficiency of firm U is high. Otherwise, if there is no entry, the government always provides an output subsidy to encourage the production of the final goods.

In this case, we have the profits of firm D under no entry:

\[ \pi_{D1}^* = \frac{(a-1)^2}{(1+c)^2}. \]

(17)
3.2 Entry to the intermediate goods market only

If firm F enters the intermediate goods market only, then there is competition between firms U and F, and domestic firm D in the final goods market. In the last stage, from (10), where \( q^* = Q = X = x + x_F \), we have the following:

\[
v = a - 2q + s = a - 2x - 2x_F + s, Q = q = x + x_F, p = a - Q = a - x - x_F.
\]  
(18)

The profits of firms D, U, and F, and social welfare are, respectively:

\[
\pi_D = p(Q)q - vq + sq = x^2 + 2xx_F + x_F^2,
\]
(19)

\[
\pi_U = vx - C(x) = -x - \frac{cx^2}{2} + x(a + s - 2x - 2x_F),
\]
(20)

\[
\pi_F = vx_F - C_F(q_F + x_F) = (-1 + a + s - 2x - 2x_F)x_F,
\]
(21)

\[
W = \frac{1}{2}Q^2 + \pi_D + \pi_U - sq = \frac{1}{2}(-2x + 2ax - x^2 - cx^2 - 2sx_F + 2xx_F + 3x_F^2).
\]
(22)

In the second stage, firms F and U simultaneously choose the intermediate outputs. The first-order conditions yield the following equilibrium outputs:

\[
x^* = \frac{5(a-1)+s}{2(3+2c)}, \quad x_F^* = \frac{1-a-c+ac+s+cs}{2(3+2c)}, \quad X^* = q^* = \frac{(a-1)(4+c)(2+c)s}{6+4c}.
\]
(23)

Note that \( x^* > x_F^* \) if \( c < \frac{6(a-1)}{s+a-1} \). That is, under the output subsidy policy when firm F enters only the intermediate goods market, firm U produces more (less) intermediate goods if the SOE’s cost inefficiency is low (high).

Substituting (23) into (22), we obtain social welfare:

\[
W^* = \frac{(a-1)^2(4+c)(7+3c)-2(a-1)(-8+(-3+c)c)s-(1+c)(8+5c)s^2}{8(3+2c)^2}.
\]
(24)

In the first stage, the government determines the subsidy rate to maximize social welfare. From the first-order condition, we have

\[
s^*_2 = \frac{(a-1)(8+3c-c^2)}{(1+c)(8+5c)},
\]
(25)

where \( s^*_2 \) denotes the optimal output subsidy in case (2) of entry to the intermediate goods market only.

Note that \( s^*_2 \geq 0 \) if \( 0 \leq c \leq 4.7016 \); the subsidy becomes a tax otherwise. Thus, only when the cost inefficiency is low, if F enters the intermediate goods market only, then the government grants an output subsidy to encourage higher final goods output.

Therefore, we have the profits of firms D and F, and social welfare, respectively:
\[
\pi^*_D = \frac{(a-1)^2(8+c(7+c))}{(1+c)^2(8+5c)^2}, \quad \pi^*_F = \frac{2(a-1)^2c^2}{(8+5c)^2}, \quad \text{and} \quad W^*_2 = \frac{(a-1)^2(8+c(5+c))}{2(1+c)(8+5c)}.
\]

(26)

### 3.3 Entry to the final goods market only

If \( F \) enters the final goods market only, then there is competition between firms \( D \) and \( F \) and firm \( U \) operates in the intermediate goods market. In the last stage, each firm simultaneously chooses its output to maximize its profits. Subsequently, from the first-order condition, we have the equilibrium output:

\[
q^* = \frac{1}{3}(1 + a + 2s - 2v), \quad q^*_F = \frac{1}{3}(-2 + a - s + v), \quad Q^* = \frac{1}{3}(-1 + 2a + s - v).
\]

Since \( q^* = X = x \), we have the following:

\[
v = \frac{1}{2}(1 + a + 2s - 3x), \quad q^*_F = \frac{1}{2}(-1 + a - x)
\]

\[
Q = \frac{1}{2}(-1 + a + x), \quad p = a - Q = \frac{1}{2}(1 + a - x).
\]

The profits of firms \( D \) and \( U \), and social welfare are, respectively:

\[
\pi_D = p(Q)q - vq + sq = x^2,
\]

(29)

\[
\pi_U = vx - C(x) = \frac{1}{2}(-x + ax + 2sx - 3x^2 - cx^2),
\]

(30)

\[
W = \frac{1}{2}Q^2 + \pi_D + \pi_U - sq = \frac{1}{8}(1 - 2a + a^2 - 6x + 6ax - 3x^2 - 4cx^2).
\]

(31)

In the second stage, firm \( U \) decides the intermediate output to maximize social welfare. The first-order condition yields the following equilibrium outputs:

\[
x^*_3 = X^*_3 = q^*_3 = \frac{3(a-1)}{3+4c}, \quad q^*_F3 = \frac{2(a-1)c}{3+4c}, \quad \text{and} \quad Q^*_3 = \frac{(a-1)(3+2c)}{3+4c}.
\]

(32)

where \( x^*_3, X^*_3, q^*_3, q^*_F3, \) and \( Q^*_3 \) denote the equilibrium outputs in case (3) of entry to the final goods market only.

Substituting (32) into (31), we obtain social welfare:

\[
W^*_3 = \frac{(a-1)^2(3+c)}{6+8c}.
\]

(33)

In the first stage, the government determines the subsidy rate, but social welfare is independent of \( W \); thus, the optimal rate should satisfy the interior solutions of the equilibrium in (28), which are

\[
s^*_3 > \frac{3(a-2)-2c(1+a)}{3+4c}.
\]

Note that the optimal output subsidy can be negative if \( c > \frac{3(a-2)}{2(1+a)} \); that is, the cost inefficiency of firm \( U \) is high. Otherwise, if firm \( F \) enters the final goods market only, then the government always provides an output subsidy to encourage the production of final goods.
Thus, we have the profits of firms D and F:
\[
\pi^*_D = \frac{9(a-1)^2}{(3+4c)^2}, \quad \pi^*_F = \frac{4(a-1)^2c^2}{(3+4c)^2}.
\tag{34}
\]

### 3.4 Entry to both markets

If F enters both markets, then there is duopolistic competition in each market. In the last stage, from (27), where \( q^* = X = x + x_F \), we have the following:

\[
v = \frac{1}{2}(1 + a + 2s - 3x - 3x_F), \quad q_F = \frac{1}{2}(-1 + a - x - x_F) \tag{35}
\]

\[
Q = \frac{1}{2}(-1 + a + x + x_F), \quad p = a - Q = \frac{1}{2}(1 + a - x - x_F).
\]

The profits of firms D, U, and F, and social welfare are, respectively:

\[
\pi_D = p(Q)q - vq + sq = x^2 + 2xx_F + x_F^2, \tag{36}
\]

\[
\pi_U = vx - C(x) = \frac{1}{2}(-x + ax + 2sx - 3x^2 - cx^2 - 3xx_F), \tag{37}
\]

\[
\pi_F = p(Q)q_F + vq_F - C_F(q_F + x_F) = \frac{1}{4}((1 - a + x)^2 + 4(s - x)x_F - 5x_F^2), \tag{38}
\]

\[
W = \frac{1}{8}(1 - 2a + a^2 - 6x + 6ax - 3x^2 - 4cx^2 - 2x_F + 2ax_F - 8sx_F + 6xx_F + 9x_F^2). \tag{39}
\]

In the second stage, firms F and U simultaneously choose the intermediate outputs. The first-order conditions yield the following equilibrium outputs:

\[
x^* = \frac{3(-5+5a+2s)}{21+20c}, \quad x_F^* = \frac{2(3-3a+3s+4cs)}{21+20c}, \quad X^* = q^* = \frac{9(a-1)+4(3+2c)s}{21+20c}, \tag{40}
\]

\[
q_F^* = \frac{2(a-1)(3+5c)-2(3+2c)s}{21+20c}, \quad \text{and } Q^* = \frac{(3+2c)(5a-5+2s)}{21+20c}.
\]

Note that \( x^* > x_F^* \) if \( c > \frac{21(a-1)}{8s} \). That is, when firm F enters only the intermediate goods market under the output subsidy, firm U produces more (less) intermediate goods if the cost inefficiency is low (high).

Substituting (40) into (39), we obtain social welfare:

\[
W^* = \frac{((a-1)^2(297+25c(15+4c))+4(a-1)(72+c(87+20c))s-4(3+4c)(12+11c)s^2)}{2(21+20c)^2}. \tag{41}
\]

In the first stage, the government determines the following subsidy:

\[
s^*_4 = \frac{(a-1)(72+c(87+20c)^2)}{2(3+4c)(12+11c)} \geq 0. \tag{42}
\]

where \( s^*_4 \) denotes the optimal output subsidy in case (4) of entry to both markets.
Therefore, we have the profits of firms D and F, and social welfare:

\[ \pi^*_D = \frac{(a-1)^2(36c+39+4c)^2}{(36+81c+44c^2)^2}, \quad \pi^*_F = \frac{3(a-1)^2c^2(345+8c(85+42c))}{2(36+81c+44c^2)^2}, \quad \text{and} \quad W^*_4 = \frac{3(a-1)^2(12+c(15+4c))}{2(3+4c)(12+11c)}. \]

(43)

4 Comparisons

We compare the equilibrium outcomes and then provide the policy implications of firm F’s entry decision.

Lemma 1: \( s^*_2 \leq s^*_4 \).

Lemma 1 states that the government grants a higher subsidy to firm D when firm F enters both markets compared to the case when firm F enters the intermediate goods market only. This is because the government intends to support firm D in the final goods market under competition with firm F, which substitutes the production of firm D, and thus competition under higher subsidization is beneficial to the domestic firm and consumers.

Lemma 2 (i) \( x^*_3 \leq x^*_4 \leq x^*_1 \leq x^*_2 \); (ii) \( x^*_F4 \leq x^*_F2 \).

Lemma 2 (i) implies that the entry of firm F to the intermediate goods market will stimulate firm U to produce more outputs (\( x^*_3 \leq x^*_4 \) and \( x^*_1 \leq x^*_2 \)). Additionally, when firm F does not enter the domestic market, more outputs will be produced than when firm F enters both markets (\( x^*_4 \leq x^*_1 \)). This is because competition with firm F induces a welfare-maximizing firm U to be aggressive in production and protect the final goods market. Lemma 2 (ii) implies that the intermediate output of firm F if it enters the intermediate goods market only is always higher than that when it enters both markets. This is because firm D can be a competitor in the final goods market when firm F enters the final goods market. Thus, if firm F provides the intermediate goods to firm D when it enters the final goods market, it will reduce its production of intermediate goods to support the price in the final goods market.

Lemma 3 (i) \( q^*_3 \leq q^*_4 \leq q^*_1 \leq q^*_2 \) if \( 0 \leq c \leq 1.3561 \); \( q^*_3 \leq q^*_1 \leq q^*_4 \leq q^*_2 \) otherwise. (ii) \( q^*_F4 \leq q^*_F3 \).

\(^8\) Note that \( s^*_1 \) and \( s^*_3 \) are indeterminate and thus the government can set the same rate with \( s^*_2 \) or \( s^*_4 \), which results in the same welfare level in each case.
(iii) \[ Q'_1 \leq Q'_2 \leq Q'_3 \leq Q'_4. \]

Lemma 3 (i) implies that the entry of firm F to the intermediate goods market will stimulate firm D to produce more outputs \((q'_4 \leq q'_4 \text{ and } q'_1 \leq q'_2)\). That is, there is a competition effect. However, when firm F enter both markets, firm D produces less outputs compared to the case when F enters the intermediate goods market only \((q'_4 \leq q'_2)\). This is because firm F will reduce its production of intermediate goods when it provides not only the intermediate goods to firm D but also final goods. (Lemma 2(ii).) Nevertheless, firm D produces higher (lower) output under firm F’s entry to both markets than under no entry when the SOE is inefficient (efficient). This is because higher production can reduce the SOE’s cost because the intermediate goods are substitutes. Lemma 3 (ii) also implies that the output of firm F if it enters the final goods market only is always higher than if it enters both markets. This is because firm D always produces more outputs under competition when the final outputs are strategic substitutes. Finally, Lemma 3 (iii) implies that the entry of firm F in the final goods market results in a market-competition effect, which increases the total industry output.

**Lemma 4**

(i) \[ v'_2 \leq v'_4; \]  
(ii) \[ p'_4 \leq p'_3 \leq p'_2 \leq p'_1. \]

Lemma 4 (i)\(^9\) comes directly from Lemma 3, where \[ q'_4 = X'_4 = x'_4 + x'_{F4} \leq q'_2 = X'_2 = x'_2 + x'_{F2}, \]
while Lemma 4 (ii) comes directly from Lemma 3(iii), where \[ Q'_1 \leq Q'_2 \leq Q'_3 \leq Q'_4. \] This setting implies that firm F’s entry increases output competition, which decreases the final goods market price, while the entry effect on the final goods market is stronger than in the intermediate goods market. Thus, firm F’s entry to both markets yields the lowest final goods price. This result also implies that consumers are better off when firm F enters both markets.

**Lemma 5**

(i) \[ \pi'_{D3} \leq \pi'_{D4} \leq \pi'_{D1} \leq \pi'_{D2} \text{ if } 0 \leq c \leq 1.3561; \]  
\[ \pi'_{D3} \leq \pi'_{D4} \leq \pi'_{D1} \leq \pi'_{D2} \text{ otherwise.} \]

(ii) \[ \pi'_{F2} \leq \pi'_{F4} \leq \pi'_{F3} \text{ if } 0 \leq c \leq 2.8326; \]  
\[ \pi'_{F2} \leq \pi'_{F3} \leq \pi'_{F4} \text{ otherwise.} \]

Lemma 5 (i) implies that the profit of firm D depends on whether firm F enters only one market or both

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\(^9\) Note that \[ v'_1 = \frac{2c}{1+c} + s'_1 \] and \[ v'_3 = \frac{6-3a+2c+azc}{3+4c} + s'_3, \] where \( s'_1 \) and \( s'_3 \) are indeterminate and thus the government can set the same rate with \( s'_2 \) or \( s'_4 \), which results in the same market price of intermediate goods in each case.
markets. In particular, firm D profits the most when firm F enters the intermediate goods market only and profits the least when firm F enters the final goods market only. Thus, firm D’s profit under firm F’s entry to both markets is always between that when firm F enters a single market. However, from Lemma 3, the profit comparison of firm D in the no entry and entry to both markets cases depends on the cost inefficiency of firm U. If the cost inefficiency is low, firm D can reduce its production and increase its profit but the result is reversed otherwise. Lemma 5 (ii) implies that firm F’s profit under entry to the intermediate goods market only is always the lowest, and thus the profitability of the final goods market is detrimental for the entry decision. In particular, firm F enters both markets when the SOE is inefficient and enters the final goods market only otherwise. This result implies that the competition in the final goods market is also significant to firm F’s profit comparison. Note that as the SOE’s cost inefficiency increases, the profits of firm F increase; that is, \( \frac{\partial \pi_k^*}{\partial c} > 0, k = 2,3,4 \).

**Lemma 6** \( W_1^* \leq W_2^* \leq W_3^* \leq W_4^* \).

Lemma 6 compares the welfare rankings, which are in the same order as the final outputs rankings in Lemma 3 (iii). This result implies that output competition in the final goods market is also significant for the welfare comparison. Note that as the SOE’s cost inefficiency increases, welfare decreases; that is, \( \frac{\partial W_m^*}{\partial c} < 0, m = 1,2,3,4 \).

![Figure 1]: Entry choice of the VIFF and welfare
From Lemmas 5 and 6, we can derive the following findings. When the government provides an output subsidy to firm D, Lemma 5 (ii) shows that the decision of firm F is to enter only the downstream market if the cost inefficiency is low, while it enters both markets otherwise. However, Lemma 6 shows that the social welfare of the latter is always higher than the former. We depict this relation in [Figure 1].

**Proposition 1** Firm F enters only the final goods market if the SOE’s cost inefficiency is low; that is, $0 < c < 2.8326$, but it decreases domestic welfare.

Proposition 1 implies that if the SOE’s cost inefficiency is low; that is, $0 < c < 2.8326$, the government should take some measures to encourage firm F to enter both markets. Even when the government cannot change firm F’s entry choice, it would be better to reduce the SOE’s inefficiency to improve welfare. However, if the inefficiency is intermediate; that is, $2.5592 < c < 2.8326$ as in Figure 1, which is below the threshold of firm F’s entry decision, reducing the SOE’s cost inefficiency will reduce welfare.

**Proposition 2** If the government reduces the SOE’s cost inefficiency when the ex-ante inefficiency is above the threshold, then $c = 2.8326$, below which the VIFF might change its entry decision and could cause welfare loss for some ranges of cost inefficiency; that is, $2.5592 < c < 2.8326$.

Proposition 2 implies that it is not always socially desirable to implement some measures to reduce the SOE’s cost inefficiency, especially when this implementation can induce firm F to withdraw its decision to enter the intermediate goods market. Therefore, there is an appropriate threshold for the level of cost inefficiency if the government cannot fully eliminate the cost inefficiency.

### 5 Privatization policy

#### 5.1 Analysis with privatization

In this section, we consider a privatization policy in which the SOE becomes a domestic private firm that can maximize its profit, and then examine the post-privatization welfare effect. We also compare the outcomes if the government can implement the privatization policy before firm F’s entry decision,
and thus compare the equilibrium outcomes of four cases. For the further analysis of privatization, we assume that the SOE’s cost inefficiency can be improved by $d$ post privatization, where $0 \leq d \leq c$.

First, in case (i) of no entry, the analysis of the last stage is the same as before, while in the second stage, firm $U$ maximizes the following profits:

$$\pi_U = vx - C(x) = (a + s - 2x)x - x - \frac{dx^2}{2}.$$  

The first-order condition yields the equilibrium output:

$$x^{**} = X^{**} = q^{**} = \frac{a-1+s}{4+d},$$  

where the superscript ** denotes the equilibrium of the intermediate goods market under privatization. Substituting (45) into (14) provides social welfare:

$$W^{**} = \frac{(a-1+s)((a-1)(7+d)-(1+d)s)}{2(4+d)^2}.$$  

In the first stage, the equilibrium subsidy rate is

$$s^{**}_1 = \frac{3(a-1)}{1+d} > 0,$$

where $s^{**}_1$ denotes the optimal output subsidy in case (1) of no entry under the privatization policy.

Therefore, the profits of firms $D$ and $U$, and social welfare are

$$\pi_{U1}^{**} = \frac{(a-1)^2(4+d)}{2(1+d)^2}, \pi_{D1}^{**} = \frac{(a-1)^2}{(1+d)^2}, W_{1}^{**} = \frac{(a-1)^2}{2(1+d)}.$$  

Second, in case (ii) of firm $F$’s entry to only the intermediate goods market, the profit of firm $U$ in the second stage becomes

$$\pi_U = vx - C(x) = x(a + s - 2x - 2x_F) - x - \frac{dx^2}{2}.$$  

Subsequently, firms $U$ and $F$ simultaneously and independently choose their outputs to maximize profit. The first-order conditions yield the equilibrium intermediate output.

$$x^{**} = \frac{a-1+s}{2(3+d)}, x_F^{**} = \frac{(2+d)(a-1+s)}{4(3+d)}, X^{**} = q^{**} = \frac{a-1+s}{4(3+d)}.$$  

Note that $x^{**} < x_F^{**}$ if $a > 1$, where the government grants a positive subsidy. That is, when firm $F$ enters only the intermediate goods market, then the privatized firm $U$ produces fewer intermediate goods, irrespective of the cost inefficiency even under the subsidy policy. Substituting (50) into (22), we obtain social welfare:

$$W^{**} = \frac{(4+d)(a-1+s)((a-1)(16+3d)-(8+5d)s)}{32(3+d)^2}.$$
In the first stage, the optimal subsidy rate becomes

\[ s^*_2 = \frac{(a-1)(4-d)}{8+5d}, \]  

where \( s^*_2 \) denotes the optimal output subsidy in case (2) of entry to only the intermediate goods market under the privatization policy. Therefore, we have the profits of firms U, D, and F, and social welfare:

\[
\pi^*_U = \frac{2(a-1)^2(4+d)}{(8+5d)^2}, \pi^*_D = \frac{(a-1)^2(4+d)^2}{(8+5d)^2}, \\
\pi^*_F = \frac{2(a-1)^2(2+d)^2}{(8+5d)^2}, W^*_U = \frac{(a-1)^2(4+d)}{2(8+5d)}.
\]

Third, in case (iii) of firm F’s entry to only the final goods market, the profit of firm U in the second stage becomes

\[ \pi_U = vx - C(x) = \frac{1}{2}(-x + ax + 2sx - 3x^2 - dx^2). \]

The first-order condition yields the equilibrium output:

\[ x^* = X^* = q^* = \frac{a-1+2s}{2(3+d)}, q_F^* = \frac{(a-1)(5+2d)+2s}{4(3+d)}, Q^* = \frac{(a-1)(7+2d)+2s}{4(3+d)}. \]

Substituting (55) into (31), we obtain social welfare:

\[ W^*_U = \frac{(a-1)^2(69+4d(8+d))+4(a-1)(15+2d)s-4(3+4d)s^2}{32(3+d)^2}. \]

In the first stage, the optimal subsidy rate becomes

\[ s^*_3 = \frac{(a-1)(15+2d)}{2(3+4d)}, \]

where \( s^*_3 \) denotes the optimal output subsidy in case (3) of firm F’s entry to only the final goods market under the privatization policy. Therefore, we have the profits of firms U, D, and F, and social welfare:

\[
\pi^*_U = \frac{9(a-1)^2(3+d)}{2(3+4d)^2}, \pi^*_D = \frac{9(a-1)^2}{(3+4d)^2}, \pi^*_F = \frac{4(a-1)^2d^2}{(3+4d)^2}, \text{and } W^*_U = \frac{(a-1)^2(3+d)}{6+8d}.
\]

Finally, in case (iv) of firm F’s entry to both markets, the profit of firm U in the second stage becomes

\[ \pi_U = vx - C(x) = \frac{1}{2}(-x + ax + 2sx - 3x^2 - dx^2 - 3xx_F). \]

Subsequently, firms U and F simultaneously and independently choose their outputs to maximize profit.

From the first-order conditions, we have the equilibrium intermediate output of firms U and F:
\[
\begin{align*}
    x^{**} &= \frac{5(a-1)+4s}{2(12+5d)}, \quad x_F^{**} = \frac{1-a+4s+2ds}{12+5d}, \quad X^{**} = \frac{3(a-1)+4(3+d)s}{24+10d}, \\
    q_F^{**} &= \frac{(a-1)(21+10d)-4(3+d)s}{48+20d}, \quad Q^{**} = \frac{(a-1)(27+10d)+4(3+d)s}{48+20d}.
\end{align*}
\]

Note that \(x^{**} > x_F^{**}\) if \(s < \frac{7(a-1)}{4+4d}\) or \(d > \frac{7(a-1)-4s}{4s}\). That is, under the output subsidy policy when firm F enters both markets, the privatized firm U produces more (less) intermediate goods if the subsidy is low (high) or the inefficiency is low (high). Substituting (60) into (39), we obtain social welfare:

\[
W^{**} = \frac{(a-1)^2(1101+20d(32+5d))+8(a-1)(141+d(71+10d))s-16(3+d)(17+11d)s^2}{32(12+5d)^2}.
\]

(61)

In the first stage, the optimal subsidy rate becomes

\[
s_4^{**} = \frac{(a-1)(141+71d+10d^2)}{4(3+d)(17+11d)},
\]

where \(s_4^{**}\) denotes the optimal output subsidy in case (4) of firm F’s entry to both markets under the privatization policy. Therefore, we have the profits of firms U, D, and F, and social welfare:

\[
\begin{align*}
    \pi_U^{**} &= \frac{(a-1)^2(33+13d)^2}{8(3+d)(17+11d)^2}, \quad \pi_D^{**} = \frac{(a-1)^2(8+d)^2}{(17+11d)^2}, \\
    \pi_F^{**} &= \frac{3(a-1)^2(981+d(2226+d(1885+668d+84d^2))))}{8(3+d)^2(17+11d)^2}, \quad W_F^{**} = \frac{(a-1)^2(132+d(77+12d))}{8(3+d)(17+11d)}.\end{align*}
\]

(63)

5.2 Comparison with privatization

We now compare the profit of firm F and then provide the policy implications regarding the post-privatization entry decisions of F.\(^{10}\)

**Lemma 7**

(i) \(\pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) if \(0 \leq c \leq 1.3028; \quad \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) if \(1.3028 \leq c \leq 2; \quad \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) if \(2 \leq c \leq 2.36; \quad \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) if \(2.36 \leq c \leq 4.1623; \quad \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) otherwise.

(ii) \(\pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**} \leq \pi_D^{**}\) otherwise.

(iii) \(\pi_F^{**} \leq \pi_F^{**} \leq \pi_F^{**} \leq \pi_F^{**}\) if \(0 \leq d \leq 1.35; \quad \pi_F^{**} \leq \pi_F^{**} \leq \pi_F^{**} \leq \pi_F^{**}\) otherwise.

Lemma 7 (i) implies that firm D has the highest profits in the no-entry case only if the cost inefficiency of firm U post privatization is low; that is, \(d < 2\). Otherwise, firm D has the highest outputs when firm

\(^{10}\) We also provide some findings regarding the subsidy rate, equilibrium outputs, and prices of the intermediate and final goods in the Appendix.
F enters the intermediate goods market only. Thus, competition with firm F is not beneficial to firm D if firm U is an efficient monopolist. Further, the profits of firm D under firm F’s entry to both markets is lowest only if the cost inefficiency of firm U post privatization is low; that is, \( d < 2.36 \). Otherwise, the output of firm D is the lowest under firm F’s entry to the final goods market only. Again, competition with firm F is not beneficial to firm D if firm U is an efficient monopolist. These findings are in contrast to Lemma 5 (i), where firm U is an SOE. Lemma 7 (ii) also implies that the profit of firm U under no entry is always the highest while competition in the intermediate goods market is the lowest. This is because firm U can lose its profit from the direct competition. Finally, Lemma 7 (iii) shows the change in firm F’s profits under the privatization policy. Contrary to Lemma 5 (ii), Lemma 7 (iii) implies that firm F always enters both markets if the government implements the privatization policy before it determines its entry strategy, irrespective of the cost gap between firms F and U.

**Lemma 8**

\[
W_2^{**} \leq W_4^{**} \leq W_3^{**} \leq W_1^{**} \quad \text{if} \quad d \leq 1.04; \quad W_2^{**} \leq W_4^{**} \leq W_3^{**} \leq W_1^{**} \quad \text{if} \quad 1.04 \leq d \leq 2; \quad W_1^{**} \leq W_2^{**} \leq W_4^{**} \leq W_3^{**} \quad \text{if} \quad 2 \leq d \leq 3.77; \quad W_1^{**} \leq W_2^{**} \leq W_3^{**} \leq W_4^{**} \quad \text{if} \quad d \geq 3.79.
\]

Lemma 8 implies that social welfare is the highest when firm F enters both markets only if the cost inefficiency of firm U post privatization is high; that is, \( d > 3.77 \). Otherwise, social welfare is the highest when firm F enters the final goods market only. Thus, privatization can induce different welfare effects depending on the ex-post cost inefficiency.

Therefore, Lemmas 7 (iii) and 8 imply that when the government implements a privatization policy, firm F will decide to enter both markets, while social welfare depends on the cost inefficiency of firm U post privatization.

**Proposition 3** *Firm F enters both markets irrespective of the cost inefficiency of firm U post privatization, but it decreases domestic welfare if \( d < 3.77 \).*

Proposition 3 implies that if the cost inefficiency is low \((0 < d < 3.77)\), then the government should not allow firm F to enter the intermediate goods market under the privatization policy.

6 Policy discussion
We now compare the welfare changes before and after the government chooses the privatization policy. We focus on the welfare effect of a (positive) output subsidy by assuming that \( a \geq 2 \) in the comparisons. Subsequently, from Propositions 1 and 3, we obtain the equilibrium welfare levels, which depend on firm F’s entry decisions and whether firm U is an SOE or a fully privatized firm. That is,

\[
W^* = \begin{cases} 
W_3^* = \frac{3+c}{6+8c} & \text{if } 0 < c < 2.8326 \\
W_4^* = \frac{3(12+c(15+4c))}{2(3+4c)(12+11c)} & \text{if } c > 2.8326 
\end{cases}
\]

while \( W^{**} = W_4^{**} = \frac{132+d(77+12d)}{8(3+d)(17+11d)} \).

If we define \( \Delta = W^* - W^{**} \), we have:

\[
\Delta = \begin{cases} 
-8c(81 + d(27 + d)) + 6(72 + d(123 + 32d)) & \text{if } 0 < c < 2.8326 \\
-336c(9 + 20c) + 2(2763 - 988c)cd + 288(8 + 7c)d^2 + 216(24 + 41d) & \text{otherwise}
\end{cases}
\]

Thus, the size of \( \Delta \) depends on the cost improvement between ex-ante and ex-post privatization. Figure 2 depicts the size of \( \Delta \).

![Figure 2: Privatization policy and welfare changes](image)

We offer the following remarks on the policy implications. First, if we set \( c = d \), where privatization does not change firm U’s cost inefficiency, we have \( \Delta > 0 \). In that case, the privatization
policy never improves welfare. Thus, reducing the cost inefficiency of firm U is a necessary condition of welfare-improving privatization. Second, if \( d = 0 \), where the inefficiency disappears ex-post privatization, then the privatization policy improves welfare only when \( c > \frac{2}{3} \). Thus, the ex-ante cost inefficiency of firm U should be low to result in welfare-improving privatization. Finally, if \( d \) resides in the hatched area in Figure 3, then welfare can improve post privatization because privatization can induce firm F to enter both markets. Therefore, non-negligible cost improvement is important for the privatization policy with a VIFF to improve domestic welfare.

7 Conclusion

We considered a vertically separated structure model consisting of an upstream domestic SOE, a downstream domestic private firm, and a VIFF that can produce both intermediate and final goods. In the presence of an output subsidy and the cost inefficiency of the SOE, we examined the welfare effect of downstream subsidization and upstream privatization policies and showed that the VIFF enters only the downstream market if the inefficiency is low and enters both markets otherwise. However, the latter always yields higher social welfare than the former case. We also showed that reducing the SOE’s cost inefficiency might cause welfare loss when the ex-ante inefficiency is at an intermediate level, below which the VIFF might change its entry decision. Finally, we showed that an upstream privatization policy reduces welfare when either the post-privatization cost inefficiency does not decrease or the ex-ante inefficiency of the SOE is relatively low.

Although we derived some interesting findings from the VIFF’s entry decisions in a vertical model, the analysis should be extended for further examination. For example, we can consider the optimal degree of privatization for the SOE. Second, the government can introduce other policy options, such as a subsidy for firm U post privatization or the imposition of a strategic tax or import tariff regarding the VIFF’s decisions on foreign direct investment or exports. Third, the VIFFs can differentiate its products in an intermediate or final goods market to relax price competition and change the subsidy rate. Finally, foreign ownership matters in determining any welfare consequences. It is thus worthwhile to examine the role of foreign penetration in the share of VIFFs. These remain future research areas.
References


