

## Understanding a New Keynesian Model with Liquidity

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## Understanding a New Keynesian Model with Liquidity

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#### Abstract

The Global Financial Crisis of 2007–2009 and its aftermath have called for a rethink of the role of money in shaping business cycle fluctuations. To this end, this paper studies a New Keynesian model with money (liquidity). In the model, agents hold government money and other financial assets. However, there is a "short rate disconnect" (i.e., an interest rate spread) between the policy rate on money and the interest rate on household's savings. The paper shows that there exists a meaningful "liquidity effect" that is quantitatively significant for the macroeconomy. As the spread increases, so does the price of liquidity. In a model where consumption and money are complements, such an increase in the opportunity cost of money induces agents to consume less and work less. Both the effects imply that the real wage can fall, which in turn puts downward pressures on inflation via the New Keynesian Phillips curve. The fall in inflation makes the monetary authority cut the nominal interest rates by more, but at the cost of increasing the spread even further. In addition, the paper compares the dynamic responses to technology shocks and monetary policy shocks for the model with liquidity and the standard New Keynesian model. The results show that the responses can be quantitatively different for the two models. Finally, this paper studies the interaction between the liquidity effect and monetary policy, highlighting the liquidity effect that can play in business cycles.

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**Keywords**: Liquidity, Money, New Keynesian model, Business cycle fluctuations.

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"There is hardly any issue of a more fundamental nature, with regard to monetary policy analysis, than whether such analysis can coherently be conducted in models that make no explicit reference whatsoever to any monetary aggregate."

Bennett T. McCallum (2008)

## 1 Introduction

Standard monetary models typically assume that the central bank directly influences the stochastic discount factor, and through which has great power to affect valuation of all financial assets – hence agent's intertemporal decisions. In practice, however, central banks often target the nominal interest rate on liquid assets, such as shortterm government bonds and bank deposits. Empirical studies have long argued that there exists a "short rate disconnect" between the rate of return on savings and the interest rate on liquid assets – the disconnect has been attributed to a convenience yield (liquidity yield) on short safe bonds.<sup>1</sup>

Would the short rate disconnect be quantitatively important for us to understand business cycle fluctuations? What are the novel features if one instead accounts for the short rate disconnect in a standard monetary model? To answer these questions, I study a New Keynesian DSGE model with liquidity. In the model, agents hold two types of assets: liquidity (money) and other financial assets.<sup>2</sup> The interest rate on liquidity is set by the government whereas the interest rate on other assets is endogenously determined in the economy. As a result, there exists an interest spread between the two rates – accounting for the interest rate disconnect.

In this paper, I refer money as liquidity for at least two reasons. First, I adopt a money-in-the-utility function (MIU) to capture the fact that money provides liquidity services.<sup>3</sup> In the model, money earns a liquidity yield that is above and beyond the

<sup>&</sup>lt;sup>1</sup>The short rate disconnect has been a stylized fact since Duffee (1996). See also Lenel et al. (2019) and Piazzessi et al. (2019) for recent contributions.

<sup>&</sup>lt;sup>2</sup>Money is liquidity. In this paper, I use money and liquidity interchangeably.

 $<sup>^{3}</sup>$ Note that money *per se* does not yield utility. MIU is often rationalized on three theoretical

pecuniary rate of return. Second, money in this model can be interpreted more broadly to include bank deposits that are essentially as liquid as money.<sup>4</sup> Note that money in this paper is also distinct from cash in that physical cash has a zero return, whereas the rate of return on government money in this model is set by the government, following a standard Taylor rule.

In the model, money earns a convenience yield because it enters the utility function. In particular, I adopt the model for general nonseparable utility and assume that consumption and real money balances are *complements* (see Carlstrom and Fuerst, 2003; Piazzesi et al., 2019). I show that complementarity between consumption and money has interesting implications for model dynamics through a "liquidity effect".<sup>5</sup> In this model, the marginal utility of consumption also depends on the level of real money balances. As the interest spread increases, money becomes more expensive, which leads to a reduction in the marginal utility of consumption. The increase in the price of liquidity also discourages work since consumption becomes less attractive. A fall in consumption and labor also implies that the real wage falls. This in turn leads to a fall in inflation through the New Keynesian Phillips curve. In response, the monetary authority has to cut the nominal interest rate on liquid assets, however, this leads to a further increase in the spread. Such an effect is absent from the standard New Keynesian monetary model, but can be quantitatively important for the macroeconomy. This paper aims to provide a quantitative evaluation of this mechanism through the lens of a New Keynesian DSGE model with liquidity.

grounds: (i) cash-in-advance (CIA); (ii) transaction costs (TC); (iii) shopping time technology (ST). A MIU model can be viewed as a generic specification that encompasses all the aforementioned models (i.e., CIA, TC, ST), see Carlstrom and Fuerst (2001, 2003). In essence, a MIU economy is a shortcut to model liquidity services.

 $<sup>^{4}</sup>$ The essential points of this paper would remain in a model with an explicit banking sector, see Piazzesi et al. (2019) for a discussion.

<sup>&</sup>lt;sup>5</sup>Note that the term "liquidity effect" has different meanings in the literature. For example, liquidity effect could refer to the negative correlation between money supply and nominal interest rate. In this paper, a liquidity effect refers to the impact of liquidity (also the price of liquidity) on macroeconomic dynamics.

This paper has several novel findings. First, to have a quantitative evaluation of the liquidity effect, I consider the case where the substitutability between consumption and liquidity  $\eta$  is relatively low. This means the spread is more responsive to exogenous shocks. In response to a positive technology shock, the spread (also the price of liquidity) increases by more in a low  $\eta$  environment. This makes agents to consume less and work less, leading to a smaller increase in the real wage and a bigger fall in inflation.<sup>6</sup> Such dynamic responses are shown to be quantitatively significant. The same story also holds true for a contractionary monetary policy shock, albeit in this case the spread falls by more as  $\eta$  becomes smaller.

Second, this paper compares the dynamic responses to technology shocks for the model with liquidity and the standard New Keynesian model. I find that the responses of macroeconomic variables are very different for the two models. For example, compared to the standard model, the model with liquidity displays a bigger fall in inflation (by about 30% more). This is driven by a smaller real marginal cost that results from the increase in the price of liquidity and thus a subsequent fall in employment and consumption. This suggests the liquidity effect highlighted in the current paper cannot be safely ignored in studying business cycle fluctuations.<sup>7</sup>

Third, for monetary policy shocks, however, the responses of macroeconomic variables for the two models are not quantitatively different for real variables such as output, employment, and the real wage. This is because under our baseline configuration of parameter values, the liquidity effect is relatively small. For financial variables, the differences between the two models are quantitatively non-negligible. For example, inflation increases by around 8% more in the model with liquidity, compared to the

<sup>&</sup>lt;sup>6</sup>It is interesting to note that the liquidity effect is amplified in response to technology shocks. This is because the bigger fall in inflation induces the monetary authority to cut the nominal interest rate further, which in turn builds into an even larger interest spread.

<sup>&</sup>lt;sup>7</sup>Note that the similar monetary effect has been analyzed in early flexible price macroeconomic models, see, for example, Carlstrom and Fuerst (2001). However, it has received little attention in the standard New Keynesian sticky price models.

standard model.

Forth, to engineer a larger liquidity effect, I compare the responses to technology shocks and monetary policy shocks for the two models, in a low  $\eta$  environment. The results show that as liquidity channel is strengthened, the different responses of macroeconomic variables are quantitatively significant for both of the shocks. In this experiment, even the dynamic responses of real variables to a monetary policy shock can be very different.

Finally, I examine the interaction between the liquidity effect and monetary policy by altering the degree of aggressiveness of monetary policy  $\phi_{\pi}$ . The result shows that as monetary policy becomes less aggressive, inflation drops by more in response to a positive technology shock, and the real interest rate also increases, dampening the increase in output. The fall in inflation in turn makes the government supply more real money balances, which can only be met by the decrease in the price of liquidity. Cheap liquidity encourages consumption and labor work, but this is offset by the increase in the real rate. Such a result also holds for a contractionary monetary policy shock. Both the results suggest that monetary policy still has a great lever in shaping business cycles in a modelling environment with liquidity.

I also compare the responses to exogenous shocks for the model with liquidity and the standard DSGE model, in an environment with a low  $\phi_{\pi}$ . It is interesting to find that, compared with our previous analysis, the results are now different. For technology shocks, as analyzed before, a lower degree of aggressiveness of monetary policy leads to a lower interest spread and hence cheap liquidity. As a result, the responses of the spread are dampened, so is the effect of the liquidity channel. And the difference between the two models gets smaller. However, this is not the case for a contractionary monetary policy shock. Cheap liquidity in this case means the spread falls by more, which amplifies the liquidity effect. A stronger liquidity channel now generates strong aggregate demand and an increase in employment. This implies a higher real wage, which pushes up inflation. Unlike from the previous comparison between the liquidity model and the standard model for monetary policy shocks, the differences in real variables are now quantitative significant, owing to a strong liquidity effect.

**Related Literature**. This paper relates closely to a growing literature on explicitly modelling monetary aggregates in a quantitative general equilibrium macroeconomic framework (see, for example, Goodfriend and McCallum, 2007; Rognlie, 2016; Eggertsson et al., 2017; Piazzesi and Schneider, 2018; Bigio and Sannikov, 2019; Kiyotaki and Moore, 2019; Piazzesi et al., 2019; Balloch and Koby, 2020; Bianchi and Bigio, 2020; Cui and Radde, 2020). This strand of literature has evolved very fast since the Global Financial Crisis of 2007–2009, partly due to the urgent need to rethink the interaction between money and macroeconomy and capture more deeply the monetary nature of our economies (see Borio, 2014).

Goodfriend and McCallum (2007) reconsider the role of money and banking in monetary policy analysis by including money and a banking sector in an optimizing model otherwise of a standard type. They argue that the omission of a demand for money that serves to facilitate transactions can be of first-order importance for the "financial accelerator" mechanism that works via increase in the supply of collateral induced by asset price increases. In models with money, however, such increases also increase the demand for collateral as agents need additional money to facilitate the additional spending induced by the initiating shock, pointing to a "banking attenuator" effect that works in the opposite direction from the financial accelerator effect emphasized by Bernanke et al. (1999).

Rognie (2016) studies money demand and optimal monetary policy in a negative nominal interest rate environment through the lens of a continuous-time general equilibrium model with cash. He finds that negative rates help stabilize aggregate demand, but at the cost of an inefficient subsidy to cash. Near 0%, the first effect dominates, and negative rates are generically optimal whenever output averages below its efficient level. Breaking the zero lower bound with negative rates is sufficient to undo most welfare losses relative to the first best allocation. In addition, the gains from negative rates depend inversely on the level and elasticity of money demand. His results show that negative interest rate policies lower the optimal long-run inflation target, and that abolishing paper currency is only optimal when currency demand is highly elastic.

Piazzesi and Schneider (2018) focus on asset pricing and develop a monetary model that features a two layered payment system: in the end user layer, nonbanks pay for goods and securities with inside money, supplied by banks; in the bank layer, interbank payments are made with reserves – outside money, controlled by the central bank. In both layers, money is valued for its liquidity services, but its creation requires costly leverage. In their model, what happens in securities markets then has an influence on both the supply and the demand of inside money. As a result, asset prices, inflation, and policy transmission depend on the institutional details of the payment system.

Kiyotaki and Moore (2019) construct a model of monetary economy where there are differences in liquidity across assets. In their model, money circulates because it is more liquid than other assets, not because it has any special function. The model is used to study how aggregate activity and asset prices fluctuate with shocks to productivity and liquidity. In particular, their model features a standard borrowing constraint and a resalability constraint of assets. They show that the presence of these two constraints opens up the possibility for money to circulate, to lubricate the transfer of goods from savers to investors. There is then a wedge, a liquidity premium, between money and other assets that arises out of the assumed difference in their resalability, similar to the spread analyzed in this paper.

This paper is closely related to Piazzesi et al. (2019) who also study the role of

money in a similar New Keynesian monetary model. The differences are fourfold. First, they compare the dynamic responses to a contractionary monetary policy shock for the model with money (i.e., CBDC in their model) and the standard New Keynesian model. Their results show that output and inflation responses in the CBDC model are about half the size of those in the standard model. However, this paper finds that, quantitatively, output responses remain largely unchanged for the two models, whereas the difference in inflation responses is, although non-negligible, quantitatively small. Our results show that, in the benchmark comparison, the responses of real variables to a monetary policy shock are not quantitatively different for the two models.<sup>8</sup>

Second, this paper also studies the responses to technology shocks for the two models and finds that the dynamics are very different, owing to a strong liquidity effect. Third, this paper provide a quantitative evaluation of the liquidity effect. The result shows that the liquidity effect can play a significant role in shaping business cycle fluctuations. Fourth, this paper studies the interaction between the liquidity effect and monetary policy. The results show that as monetary policy becomes less aggressive, the liquidity effect is dampened in response to technology shocks, however, it is amplified in response to monetary policy shocks.

The rest of the paper is organized as follows. In Section 2, I lay out a New Keynesian model with liquidity. I also derive the equilibrium of the model. In Section 3, I calibrate the model using conventional values in the literature. Section 4 evaluates the dynamic responses of macroeconomic variables to a technology shock and a monetary policy shock. In this section, I also examine the liquidity effect in shaping business cycles, compare the model dynamics with a standard New Keynesian model, and explore the interaction between the liquidity effect and monetary policy. Section 5 offers concluding

<sup>&</sup>lt;sup>8</sup>As shown in this paper, this result depends crucially on the strength of the liquidity effect, which is sensitive to the complementarity between consumption and real money balances, as well as the operation of monetary policy.

remarks.

## 2 The model

Our basic framework is based on a canonical textbook version of a New Keynesian DSGE model with sticky prices as in Woodford (2003) and Galí (2015), augmented with the introduction of liquidity à *la* Piazzesi et al. (2019). In the model, money (liquidity) earns a convenience yield (liquidity yield) because it enters the utility function. In particular, I adopt the model for general nonseparable utility and assume that consumption and real money balances are *complements* (see Carlstrom and Fuerst, 2003; Piazzesi et al., 2019). That is, the cross partial derivative of the utility function is strictly positive.<sup>9</sup>

This also introduces a "liquidity effect" that works through the level and price of liquidity. This has interesting implications for business cycle fluctuations, as analyzed in this paper.<sup>10</sup> In a model where consumption and money are complements, an increase in the price of liquidity makes agents consume less, owing to a decrease in the marginal utility of consumption. It also induces agents to work less as consumption becomes less attractive. Both the effects imply that the real wage can fall, which drives down the real marginal costs and thus puts downward pressures on inflation via the New Keynesian Phillips curve. The fall in inflation in turn makes the monetary authority cut the nominal interest rates by more.

In the model, central bank controls the quantity of money and is the sole supplier of liquidity.<sup>11</sup> In addition, the central bank sets the interest rate on money, as opposed

<sup>&</sup>lt;sup>9</sup>Empirical studies have also suggested that an increase in real money balances raises the marginal utility of consumption, see, for example, Koenig (1990). See also Calvo (1979) and Woodford (1994) for early theoretical contributions.

<sup>&</sup>lt;sup>10</sup>Note that standard monetary New Keynesian models adopt either a cashless approach or a separable utility in consumption and real balances, and hence the role of money is trivial.

<sup>&</sup>lt;sup>11</sup>In reality, of course, bank deposits (bank debt) are another form of money through which private

to the short interest rate of agents' stochastic discount factor, which will de adjusted endogenously to clear markets.<sup>12</sup> Note that one can think of liquidity in this model as government bonds and private agents use bonds as a vehicle to settle transactions. That is, liquidity serves as the medium of exchange in the economy, apart from being used as the unit of account and a store of value. Alternatively, one can think of liquidity as a central bank digital currency (CBDC): agents have CBDC accounts at the central bank, which controls both the nominal quantity and the interest rate (see Piazzesi et al., 2019).

## 2.1 Households

The economy is assumed to be populated by a continuum of infinitely-lived households of size one. Households appreciate consumption, real money balances (liquidity), and dislike labor. The representative household seeks to maximize a discounted sum of utilities of the form:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \frac{1}{\sigma}} (C_t^{1 - \frac{1}{\eta}} + \omega (B_t / P_t)^{1 - \frac{1}{\eta}})^{\frac{1 - \frac{1}{\sigma}}{1 - \frac{1}{\eta}}} - \frac{N_t^{1 + \varphi}}{1 + \varphi} \right], \tag{1}$$

where  $\sigma$  is the intertemporal elasticity of substitution between consumption bundles at different dates,  $\eta$  is the intratemporal elasticity of substitution between consumption and real balances,  $\varphi$  is the inverse elasticity of intertemporal substitution of labor disutility,  $\omega$  is the weight parameter for liquidity,  $C_t$  is a consumption index defined across all differentiated goods,  $b_t = B_t/P_t$  is real money balances and  $B_t$  denotes *interest-bearing* liquidity supplied by the government (central bank),  $N_t$  is labor supply, E is the expectations operator and  $0 < \beta < 1$ . Note that if  $\sigma = \eta$ , utility is separable

agents settle transactions. However, the main results can remain largely unchanged if one introduces a formal banking sector. see Piazzesi et al. (2019) for a discussion.

<sup>&</sup>lt;sup>12</sup>This is motivated by the fact that there exists a "short rate disconnect" between the two types of interest rates (see Lenel et al., 2019), as discussed in the introduction.

in consumption and real balances, as commonly assumed in New Keynesian monetary models.<sup>13</sup>

In this paper, I follow Carlstrom and Fuerst (2003) and Piazzesi et al. (2019) and adopt a general nonseparable utility function. In particular, the analysis follows the assumption that  $\sigma > \eta$ , where consumption and liquidity are complements (i.e.,  $U_{cb} > 0$ ).<sup>14</sup> Note that complementarity is not only empirically fit but also theoretically appealing. For example, a reduction in real balances, induced by, say a higher price of liquidity (or a higher opportunity cost of money), would bring down the marginal utility of consumption. It would also lower the price of leisure (relative to consumption), and induce agents to work less. This mechanism is analyzed formally below.

Liquidity is provided by the government (think of government bonds) and earn the nominal interest rate  $R_t$ . The household can also invest in other short safe assets that pay the nominal interest rate  $R_t^S$ . The cost of liquidity  $R_t^S - R_t$  is the convenience yield (liquidity yield) on money. Following Lenel et al. (2019) and Piazzesi et al. (2019), I refer to  $R_t^S$  as the shadow rate. It represents the nominal short rate in the household's stochastic discount factor and hence the first-order term in the nominal rate of return on any asset held directly by households. Note that standard monetary models typically assume that the central bank's policy rate directly affects the nominal pricing kernel and hence agent's intertemporal decisions such as savings and investment. However, in practice, central banks target interest rates on short safe government bonds. Indeed, there is a "short rate disconnect" between the interest rates on savings and safe bonds, which is attributed to a convenience yield on government bonds.

The representative household is subject each period to a flow budget constraint of

<sup>&</sup>lt;sup>13</sup>However, it is well-known that in such models, money does not affect the dynamics of macroeconomic variables as one can solve the model without money. And hence, the role of money is trivial.

<sup>&</sup>lt;sup>14</sup>Such a feature is well in line with the results of empirical studies (see Koenig, 1990; Piazzesi et al., 2019). For example, Koenig (1990) finds that an increase in real money balances raises the marginal utility of consumption.

the form:

$$P_t C_t + B_t + S_t \le W_t N_t + B_{t-1} R_{t-1} + S_{t-1} R_{t-1}^S + T_t + \Phi_t,$$
(2)

where  $B_t$  and  $S_t$  denote the holdings of liquidity and shadow assets,  $P_t$  is the aggregate price level,  $W_t N_t$  is nominal wage income,  $T_t$  denotes government transfers, and  $\Phi_t$ denotes the nominal profits received by the household from the ownership of firms.

Each household chooses money, assets, consumption, and labor supply that maximize the life-time utility (1) subject to the budget constraint (2) for  $t \ge 0$ . First, the marginal rate of substitution of consumption for liquidity has to be equal to the relative price of liquidity services provided by money, or the opportunity cost of holding money. This gives us a "money demand" function:<sup>15</sup>

$$B_t = P_t C_t \omega^{\eta} \left(\frac{R_t^S - R_t}{R_t^S}\right)^{-\eta}.$$
(3)

Since utility is homogenous of degree one in consumption and money, agents hold money in proportion to their nominal spending. The demand for money is also affected by the price of liquidity, or the opportunity cost of money, i.e., the spread  $R_t^S - R_t$ . As the spread increases, holding money becomes relatively expensive, and agents choose to hold less money. The elasticity of substitution  $\eta$  can be seen as an interest elasticity of money demand.

For illustrative purposes, it would be useful to introduce the price index for a bundle of consumption and liquidity services from money. Bundles are given by:

$$X_t \equiv (C_t^{1-\frac{1}{\eta}} + \omega (B_t/P_t)^{1-\frac{1}{\eta}})^{\frac{1}{1-\frac{1}{\eta}}}.$$

Accordingly, the price index is:

<sup>&</sup>lt;sup>15</sup>The full derivations of household first-order conditions can be found in Appendix A.

$$Q_t \equiv (1 + \omega^{\eta} (\frac{R_t^S - R_t}{R_t^S})^{1-\eta})^{\frac{1}{1-\eta}}.$$
(4)

Note that this price index is measured in units of real consumption. In addition, the price index is increasing with the spread or the price of liquidity  $Z_t \equiv \frac{R_t^S - R_t}{R_t^S}$ , if the interest elasticity of money demand is smaller than unity.<sup>16</sup>

Next, the first-order conditions imply a second intratemporal Euler equation that the real wage rate equals the marginal rate of substitution between consumption and leisure:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} Q_t^{1-\frac{\eta}{\sigma}}.$$
(5)

As consumption and money are complements, an increase in the opportunity cost of money would lower labor supply relative to consumption. This is because that a higher price of liquidity makes bundles less attractive and leads households to take more leisure goods and thus reduce their labor supply through intratemporal decisions.

A higher price of liquidity also makes consumption less desirable as marginal utility of consumption now depends on the bundle of consumption and money and is decreasing with the opportunity cost of money:

$$U_{c,t} = X_t^{(\frac{1}{\eta} - \frac{1}{\sigma})} C_t^{-\frac{1}{\eta}} = Q_t^{\frac{\eta}{\sigma} - 1} C_t^{-\frac{1}{\sigma}}.$$
 (6)

Note that I have used the fact that  $X_t \equiv (C_t^{1-\frac{1}{\eta}} + \omega(B_t/P_t)^{1-\frac{1}{\eta}})^{\frac{1}{1-\frac{1}{\eta}}} = [1 + \omega^{\eta}(\frac{R_t^S - R_t}{R_t^S})^{1-\eta}]^{\frac{1}{1-\frac{1}{\eta}}}C_t = Q_t^{-\eta}C_t$  in the above equation.

The intertemporal Euler equation for the shadow rate relates the marginal utilities of consumption at different dates to interest rates:

<sup>&</sup>lt;sup>16</sup>Existing literature typically uses a small interest elasticity  $\eta < 1$ , see Mankiw and Summers (1986), Koenig (1990), Carlstrom and Fuerst (2003), and Piazzesi et al. (2019).

$$\beta R_t^S E_t [(\frac{Q_t}{Q_{t+1}})^{1-\frac{\eta}{\sigma}} (\frac{C_t}{C_{t+1}})^{\frac{1}{\sigma}} (\frac{1}{\Pi_{t+1}})] = 1,$$
(7)

where  $\Pi_{t+1} \equiv \frac{P_{t+1}}{p_t}$  denotes the gross inflation rate from period t to t + 1. It is clear from the above equation that the real stochastic discount factor is now affected by the price of liquidity (or the price of bundles). As the price of liquidity decreases, agents see a drop in the stochastic discount factor and discount the future by more. This in turn facilitates current consumption.

Finally, combining Eq (3) and Eq (7), one could write an analogous Euler equation for money:

$$\beta R_t E_t [(\frac{Q_t}{Q_{t+1}})^{1-\frac{\eta}{\sigma}} (\frac{C_t}{C_{t+1}})^{\frac{1}{\sigma}} (\frac{1}{\Pi_{t+1}})] + \omega (\frac{P_t C_t}{B_t})^{\frac{1}{\eta}} = 1.$$
(8)

The total return on money now consists of two terms (a pecuniary term and a nonpecuniary one). First, as a store of value, money is valued for its pecuniary rate of return. Second, as a medium of exchange, money is valued for its liquidity property. In other words, money earns a convenience yield, above and beyond its payoff. This convenience yield depends on total spending relative to money, as well as the interest rate elasticity of money demand. For example, if nominal spending is high relative to money, money is relatively scarce and its marginal benefit is therefore higher. The convenience yield is also higher if the interest elasticity is lower.

## 2.2 Firms

The supply side of the economy is standard. Assume a continuum of firms indexed by  $i \in [0, 1]$ . Each firm produces a differentiated good, but they all use an identical technology, represented by the production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha},\tag{9}$$

where  $A_t$  is the level of technology, assumed to be common to all firms and to evolve exogenously over time according to:

$$A_t = \bar{A}^{1-\rho_a} A_{t-1}^{\rho_a} e^{\epsilon_t^a},\tag{10}$$

where  $\rho_a$  is the first-order autocorrelation,  $\bar{A} = 1$  is the steady state value of technology, and the standard deviation of  $\epsilon_t^a$  is  $\sigma_{\epsilon}^a$ .

Firms set their prices subject to a Calvo (1983) price rigidity. Each firm may reset its price only with probability  $1 - \theta$  in any given period, independent of the time elapsed since it last adjusted its price. Since the problem is symmetric, every firm faces the same decision problem and will choose the same optimal price  $P_t^*$ . This pricing behavior implies the law of motion for the aggregate price index:

$$P_t = [(1-\theta)(P_t^*)^{1-\epsilon} + \theta(P_{t-1})^{1-\epsilon}]^{\frac{1}{\epsilon-1}}.$$
(11)

A firm reoptimizing in period t will choose the price  $P_t^*$  that maximizes the current market value of the profits generated while that price remains effective. This corresponds to solving the problem:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} [P_t^* Y_{t+k|t} - (1/\mu^s) \Psi_{t+k}(Y_{t+k|t})],$$
(12)

subject to the sequence of demand constraints:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k},$$
(13)

where  $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$  is the nominal stochastic discount factor. Note that

since we have assumed that firms and households use the same discount factor, the stochastic discount factor now is affected by the price of liquidity, as suggested by (6).  $Y_{t+k|t}$  denotes output in period t + k for a firm that last reset its price in period t,  $\Psi_t$  is the nominal cost function, and  $\mu^s = \frac{\epsilon}{\epsilon - 1}$  is time-invariant employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition. In addition, minimizing labor costs yields the expression for the real marginal cost:  $mc_t = \frac{W_t}{A_t P_t}$ .

The optimality condition associated with the problem above satisfies:

$$\left(\frac{P_t^*}{P_t}\right) = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j (Q_{t+j}^{\frac{\eta}{\sigma}-1} C_{t+j}^{-\frac{1}{\sigma}}) (\frac{P_{t+j}}{P_t})^{\epsilon} m c_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j (Q_{t+j}^{\frac{\eta}{\sigma}-1} C_{t+j}^{-\frac{1}{\sigma}}) (\frac{P_{t+j}}{P_t})^{\epsilon-1} Y_{t+j}} = \frac{K_t}{F_t},$$
(14)

where  $K_t$  and  $F_t$  are aggregate variables that satisfy the recursive relations:

$$K_t = Q_t^{\frac{\eta}{\sigma} - 1} C_t^{-\frac{1}{\sigma}} m c_t Y_t + \beta \theta E_t K_{t+1} \Pi_{t+1}^{\epsilon}$$
(15)

$$F_t = Q_t^{\frac{\eta}{\sigma} - 1} C_t^{-\frac{1}{\sigma}} Y_t + \beta \theta E_t F_{t+1} \Pi_{t+1}^{\epsilon - 1}.$$
 (16)

Also, it follows from (11) and (14):

$$\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{\epsilon-1}} = \frac{K_t}{F_t}.$$
(17)

### 2.3 The government

In this paper, I consider a consolidated monetary/fiscal authority, which I call "the government". The government implements two policy instruments: the interest rate on money  $R_t$  and the supply of government bonds  $B_t$ . In addition, assume that the government levies lump sum taxes  $T_t$  to satisfy its budget constraint:

$$B_t + P_t T_t = B_{t-1} R_{t-1}.$$
 (18)

For the interest rate on liquidity, I consider a standard Taylor rule, and assume:

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} V_t,\tag{19}$$

where  $\bar{R}$  and  $\bar{Y}$  are steady-state values of nominal interest rates and output, and  $\Pi$ is the central bank's headline inflation target, which is assumed to be one.  $\phi_{\pi}, \phi_{y}$ are the relative weights measuring the response of interest rate to inflation deviations and output gap, respectively.  $V_t$  is an exogenous monetary policy shock that evolves according to:

$$V_t = \bar{V}^{1-\rho_v} V_{t-1}^{\rho_v} e^{\epsilon_t^v},\tag{20}$$

where  $\rho_v$  is the first-order autocorrelation,  $\bar{V} = 1$  is the steady state value of the shock, and the standard deviation of  $\epsilon_t^v$  is  $\sigma_{\epsilon}^v$ .

For the supply of government bonds, I follow Piazzesi et al. (2019) and assume a rule such that:

$$B_t = \mu B_{t-1} + P_t G_t. (21)$$

That is, I assume the government increases or shrinks the nominal money supply by a factor  $1 - \mu$ , and issues new money *worth*  $G_t$  consumption goods. Note that one special case is that the government commits to a path for the nominal money supply (i.e.,  $G_t = 0$ ). The government then provides a "nominal anchor" for the economy. Another special case is to assume that  $\mu = 0$  and  $G_t > 0$ . The government then commits to a path for real balances. In this paper, I consider an intermediate case where  $\mu \in (0, 1)$ : the government is assumed to gradually move real balances by retiring a share  $1 - \mu$  of the nominal money supply and adding money worth  $G_t$  goods. The path of transition is rewritten in real terms:

$$\frac{B_t}{P_t} = \mu \frac{B_{t-1}}{P_{t-1}} \frac{1}{\Pi_t} + \bar{G},$$
(22)

where I have also assumed that  $G_t = \overline{G}$ , for simplicity. Note that  $\mu$  can be naturally viewed as a measure of nominal rigidity in the money supply.

### 2.4 Equilibrium

The market clearing condition of goods market is given by:

$$Y_t = C_t, \tag{23}$$

and market clearing in labor market requires:

$$Y_t = \frac{A_t N_t}{\Delta_t},\tag{24}$$

where price dispersion  $\Delta_t \equiv \int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di$  evolves according to:

$$\Delta_t = (1-\theta) \left(\frac{1-\theta \Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_t^{\epsilon} \Delta_{t-1}.$$
(25)

We define a rational expectations equilibrium is a collection of stochastic processes  $\{w_t, C_t, N_t, Q_t, R_t, R_t^s, B_t/P_t, \Pi_t, K_t, F_t, Y_t, \Delta_t\}_{t=0}^{\infty}$ , satisfying each of the equilibrium conditions in equations (3), (4), (5), (7), (15), (16), (17), (19), (22), (23), (24), (25), consistently with the stochastic processes for the exogenous disturbances  $\{\epsilon_t^a\}, \{\epsilon_t^v\}$ , and initial conditions. Equilibrium equations and the derivations of steady state are given

in Appendix B.

## **3** Parameterization

The model is based on a stylized New Keynesian DSGE model. To study the dynamic properties of the model I parameterize it using standard values in the literature. For the parameters that are related to liquidity, I set the values following Piazzesi et al. (2019). The model is parameterized at a quarterly frequency. The discount factor  $\beta$  is set at 0.99, which gives a steady state annualized interest rate of 4%. Note that  $1/\beta$  is the steady state value of  $R^S$ . I follow Piazzesi et al. (2019) and choose the deposit spread (i.e.,  $R^S - R$ ) to be 2.4% per year. This implies the annualized return on government money is 1.6%. The intertemporal elasticity of substitution between bundles is set to one ( $\sigma = 1$ ). I choose  $\varphi = 5$ , which implies a Frisch elasticity of labour supply of 0.2 (see Galí, 2015). The elasticity of substitution between intermediate goods among themselves,  $\epsilon$ , is set equal to 9, which implies a value for the steady state mark-up rate,  $\epsilon/(\epsilon - 1)$ , of approximately 12.5%. The price stickiness parameter,  $\theta$ , is set at 0.75, which corresponds to the average duration of price contracts of about four quarters.

Regarding the parameters characterizing government policies, I set  $\phi_{\pi} = 1.5$ ,  $\phi_y = 0$ , and  $\mu = 0.8$  (Piazzesi et al., 2019). I choose the autoregressive coefficients of technology shock  $\rho_a$  and monetary policy shock  $\rho_v$ , to 0.9 and 0.8, respectively. The weight parameter on liquidity  $\omega$  is set to 0.14, which implies an average velocity of 0.5. Finally, I set the interest elasticity of money demand  $\eta = 0.22$ , following Piazzesi et al. (2019) who estimate this parameter using U.S. data. Note that empirical studies have found different values for the parameter, ranging from 0.05 to 0.35 (see, for example, Mankiw and Summers, 1986; Koenig, 1990). In the model experiments below, I will also perform sensitivity analysis regarding to this parameter.

## 4 Model analysis

#### 4.1 Responses to exogenous shocks

In this paper, I study the macroeconomic impact of technology shocks and monetary policy shocks.<sup>17</sup> I start by describing the dynamic effects of an expansionary technology shock on a number of macroeconomic variables, as shown in Figure 1. The level of technology is assumed to increase by one percent. The increase in technology leads to an immediate increase in output and a fall in inflation. Following the Taylor rule, the government cuts the nominal interest rate, i.e., the return on liquidity  $R_t$ . This causes the spread  $(R_t^S - R_t)$  to increase  $(R_t^S$  also increases, not shown), at around 8 basis points.<sup>18</sup> As liquidity becomes more expensive, households tend to reduce their demand for money. However, such an effect is dominated by the increase in money demand that results from a higher aggregate demand. As a result, real money balances increase. Note that the increase in real balances is also matched by the increase in real money supply, due to the fall in inflation. Following Piazzesi et al. (2019), suppose that the nominal money supply is constant, such an increase in real balances would imply that the price level has to decline. In addition, as both the price of liquidity and aggregate demand increase, the marginal utility of consumption falls unambiguously, which implies a higher real wage. At the same time, agents consume more leisure and work less, albeit the fall in employment is relatively small.

Figure 2 depicts the dynamic responses of monetary policy shocks. The shock takes the form of an increase of 25 basis points in  $\epsilon_t^v$ . Tightening monetary policy generates a decrease in inflation, output, and thus employment. Note that under our configuration of parameter values, the nominal interest rate falls, due to the dominant influence of

<sup>&</sup>lt;sup>17</sup>Since the effects of preference shocks are similar to those of monetary policy shocks, I relegate the analysis for preference shocks to Appendix C for interested readers.

<sup>&</sup>lt;sup>18</sup>Note that in this model,  $R_t^S$  is solved by using the money demand function.

lower inflation in shaping the government's response.<sup>19</sup> However, the real interest rate would unambiguously increase. A lower inflation also means that the government has to increase real money balances, engineered by a fall in the price level. Such an increase in real balances can only be matched by a lower interest spread and cheap liquidity. The decline in the opportunity cost of holding money, together with a drop in aggregate demand, also increases the marginal utility of consumption. Combined with the fall in disutility of labor, the real wage falls unambiguously.

## 4.2 The case of a low $\eta$

One crucial feature of the model is the complementarity between consumption and liquidity services by assuming  $\eta < \sigma$  in the utility function. As discussed earlier, this also introduces a meaningful "liquidity effect" that shapes business cycles. This section aims to quantitatively evaluate the strength of the liquidity effect by considering a lower interest elasticity  $\eta$ . In the low  $\eta$  environment, I set  $\eta = 0.05$ . Such a value is estimated by Mankiw and Summers (1986).<sup>20</sup> Figure 3 compares the dynamic responses to a positive technology shock for the benchmark model ( $\eta = 0.22$ , blue-solid lines) and the model with a low  $\eta$  ( $\eta = 0.05$ , red-dashed lines). A lower  $\eta$  increases the convenience yield and pushes up the price of liquidity. One can see that the spread increases from less than 10 basis points to more than 30 basis points. This implies a strong liquidity effect, which is shown to have important implications for the macroeconomy.

Such a change has immediate implications for the dynamics of consumption (not shown) and employment. Since the price of liquidity increases, agents would consume less and also work less, due to a lower marginal utility of consumption. As households

<sup>&</sup>lt;sup>19</sup>This is a well-known result even for the standard three-equation DSGE model with linear production function. If one adopts decreasing return to scale in the production function, one can have the nominal interest rate to increase in response to a contractionary monetary policy shock.

<sup>&</sup>lt;sup>20</sup>However, different values of  $\eta$  have been used in the past studies. For example, Carlstrom and Fuerst (2003) use  $\eta = 0.1$ ; Koenig (1990) estimates the interest elasticity to be around 0.35.

consume less bundles (consumption and real balances) and work less, the real wage also falls by about 40%. This reduces the real marginal cost substantially, which in turn puts downward pressures on inflation via the traditional New Keynesian Phillips curve. As inflation falls by more, the government has to lower the nominal interest rate by more, following the Taylor rule. However, such a drop in the nominal interest rate would feed into a further increase in the spread, and hence the price of liquidity.

In addition, the fall in inflation implies that the government needs to increase real money balances, through a decline in the price level. Note that the increase in the real money supply is matched by an increase in money demand. Although both the decrease in consumption and the increase in the price of liquidity tend to reduce money demand, a lower  $\eta$  makes money demand less responsive to the spread. The aggregate effect thus leads to an increase in the real demand for liquidity.

Figure 4 reports the dynamic responses to a contractionary monetary policy shock for the benchmark model and the model with a low  $\eta$ . A lower interest elasticity of money demand pushes down the convenience yield even further. Cheap liquidity makes agents to consume more and work more. It also pushes up the real wage, but the magnitude is very small, due to the decline in the price of liquidity. As shown in Figure 4, inflation increases under a lower  $\eta$  because of the increase in the real wage as well as the increase in expected inflation. Since the real interest rate does not increase much (not shown), expected inflation has to increase to offset the big increase in the nominal interest rate. Finally, a higher inflation implies that the government supplies less real money balances, through a higher price level. In sum, the above two experiments highlight the liquidity effect that plays in shaping the dynamics for both real and financial variables.

## 4.3 Comparison with the standard model

Next, it would be interesting to compare the business cycle dynamics of this model with the standard New Keynesian DSGE model (for example, Galí, 2015, Chapter 3).<sup>21</sup> This is also meant to capture the role of liquidity that can play in business cycle fluctuations, both qualitatively and quantitatively. Figure 5 compares the dynamic responses to a technology shock for the benchmark model (blue-solid lines) and the standard New Keynesian model (red-dashed lines). Note that the spread in the standard model would just be the nominal interest rate set by the government, as in such a model the government is assumed to directly influence the stochastic discount factor and the price of all the financial assets held by households. Thus, in the standard model, the spread drops due to the fall in the policy rate, whereas the spread increases as liquidity becomes more expensive in the benchmark model.

In the liquidity model, as the opportunity cost of money increases, agents would consume less and work less. The first effect is quantitatively negligible. For employment, however, the liquidity effect discourages work by about 50% more. This leads to a bigger drop in the real wage in the liquidity model, at around 10%. Driven by the fall in real wages, inflation decreases by more in the liquidity model, more than 20%, compared to the standard model. Following the Taylor rule, the government also cuts the policy rate by a lot more in the liquidity model. It is clear from the above analysis that the dynamic responses of macroeconomic variables are quantitatively very different for the two models. These results also suggest that the liquidity effect cannot be safely ignored in studying business cycles.<sup>22</sup>

<sup>&</sup>lt;sup>21</sup>Note that for simplicity, I assume a linear production function for the standard New Keynesian model. In addition, the standard model also includes money (cash) in the utility function (in a separable fashion), see Galí (2015, Chapter 3). It is well recognized that the role of money is trivial in such a model, and money does not affect model dynamics as the model can be solved without money.

 $<sup>^{22}</sup>$ It would also be interesting to quantitatively study the liquidity effect in a more complex DSGE model (e.g., with capital formation or financial frictions). This is however beyond the scope of this paper.

Figure 6 reports the dynamic responses to a monetary policy shock for the two models. As can be seen, responses are quantitatively unchanged for output, employment, and the real wage, especially for the first a few quarters. This is due to the fact that, under our current configuration of parameter values, the liquidity effect is estimated to be quantitatively small. One can see from our previous Figure 2 that the spread does fall in the benchmark model. But the magnitude is small, at around 4 basis points initially. Such a drop in the price of liquidity, although does encourage consumption and work to some extent, turns out to be quantitatively negligible. However, it is interesting to note that the responses for inflation and the nominal interest rate are quantitatively different across the two models. In particular, inflation increases in the benchmark model, compared to the standard model. Such an increase in inflation is mainly driven by the increase in expected future inflation, given that the real marginal cost hardly moves.

In addition, note that our results for monetary policy shocks are in contrast with the ones obtained by Piazzesi et al. (2019). Using the same model, Piazzesi et al. (2019) also study the dynamic responses to a contractionary monetary policy shock for the New Keynesian model with CBDC (i.e., liquidity in this model) and the standard DSGE model. They find that output and inflation responses in the CBDC model are only about half the size of those in the standard model. However, this paper finds that output responses are quantitatively unchanged for the two models. Inflation does drop by less in the model with liquidity, but the magnitude is less than 10%. The small differences are largely due to the small liquidity effect.

What if we make the liquidity effect stronger by having the spread more responsive to exogenous shocks? Next, I consider the dynamic responses of macroeconomic variables to both technology shocks and monetary policy shocks for the two models, in a low  $\eta$  environment (i.e.,  $\eta = 0.05$ ). Results are reported in Figure 7 and Figure 8. As expected, the spread increases by more for technology shocks (from 10 basis points to 30 basis points) and decreases by more for monetary policy shocks (from 4 basis points to 15 basis points). For the responses to technology shocks, note that the differences across the two models were already quantitatively significant, they become even larger in a low  $\eta$  environment, due to a large liquidity effect. For example, one can see that output falls by around 10% more in the benchmark model. Inflation falls by more than 60%, compared to the standard model. For the responses to monetary policy shocks, the differences between the two models are now readily seen. With a strong liquidity effect, real variables (e.g., output, employment, real wage) respond to the shock differently, although it is not true for the initial periods. Inflation now increases (compared to the standard model) by about one third in the model with liquidity.

In sum, there are several points that are worth noting. First, the dynamic responses of macroeconomic variables to technology shocks are quantitatively different for the model with liquidity and the standard New Keynesian model. This, again, calls for a reconsideration of liquidity and the role it can play in shaping business cycles. Second, for monetary policy shocks, the responses across the two models are not so different for many variables, such as output, employment, the real wage. The responses for inflation are, however, different. Third, the different dynamic responses to shocks are influenced largely by the liquidity effect. As the spread becomes more responsive to shocks, macroeconomic dynamics across the two models can be significantly different, even for monetary policy shocks.

## 4.4 The aggressiveness of monetary policy

Finally, to quantitatively examine the interaction between the liquidity effect and monetary policy, I explore how alternative monetary rules with different degrees of aggressiveness alter the model dynamics. Figure 9 compares the impulse responses to a technology shock, as the benchmark case being  $\phi_{\pi} = 1.5$  (blue-solid lines) and the low aggressiveness case being  $\phi_{\pi} = 1.1$  (red-dashed lines). By reacting less aggressively to inflation, monetary policy ensures that inflation falls by a lot more in the low  $\phi_{\pi}$  environment. This in turn causes a big decline in the nominal interest rate. It also reduces the fall in the real interest rate, tempering output increases. In addition, a larger fall in inflation makes the government supply more real balances. Such an increase in real money supply can only be met by cheap liquidity, thus the interest spread falls. Note that a lower opportunity cost of money should in principle make agents consume more and work more through the liquidity effect, but this is offset by the effects of a less aggressive monetary policy. The same story holds for the monetary policy shock, if one compares the dynamic responses for the benchmark case with the low  $\phi_{\pi}$  case, see Figure 10.

It is interesting to note that, thanks to a smaller increase in the spread, the liquidity effect is dampened for technology shocks as monetary policy becomes less aggressive. It is, however, amplified for monetary policy shocks, since the fall in the spread becomes larger in the low  $\phi_{\pi}$  environment. For example, the spread falls by around 4 basis points more when  $\phi_{\pi}$  is low. Such an asymmetric response of the liquidity effect would have important implications for model dynamics, as discussed below.

I then consider the responses to a technology shock for the benchmark model and the standard DSGE model, in a low  $\phi_{\pi}$  environment, as shown in Figure 11. If one compares the results of Figure 11 with those of Figure 5, one can see that our previous analysis largely carries over. One notable difference is that in Figure 11 inflation increases in the model with liquidity, compared to the standard model. The increase in inflation is mainly driven by the increase in expected inflation, not by the changes in the real wage. Such an increase in inflation causes the nominal interest rate to increase, not to fall as in Figure 5. Figure 12 reports the responses to a monetary policy shock for the two models, in a low  $\phi_{\pi}$  environment. Again, if one compares the results of Figure 12 with those of Figure 6, it is interesting to see the dynamic responses across the two models now become quantitatively different, even for variables like output, employment, and the real wage. This is due to the amplified liquidity effect in the low  $\phi_{\pi}$  environment. For example, one can see clearly that as the price of liquidity falls, agents tend to consume more and work more, which also leads to the increase in the real wage. Higher real marginal costs also push up inflation, feeding into the increase in the nominal interest rate. Such a mechanism is absent from the standard New Keynesian model, and it is more evident for a low  $\phi_{\pi}$  environment for monetary policy shocks.

## 5 Concluding remarks

The Global Financial Crisis of 2007–2009 and its aftermath have called for a rethink of the role of money in shaping business cycle fluctuations. To this end, this paper studies a New Keynesian model with money (liquidity). In the model, agents hold government money and other financial assets. However, there is a "short rate disconnect" (i.e., an interest rate spread) between the policy rate on money and the interest rate on household's savings, a stylized fact that has been well documented by past empirical studies. The paper shows that there exists a meaningful "liquidity effect" that is quantitatively significant. As the spread increases, so does the price of liquidity. In a model where consumption and money are complements, such an increase in the opportunity cost of money makes agents consume less, owing to a decrease in the marginal utility of consumption. It also discourages agent's work as consumption becomes less attractive. Both the effects imply that the real wage can fall, which in turn puts downward pressures on inflation via the New Keynesian Phillips curve. The fall in inflation makes the monetary authority cut the nominal interest rates by more, but at the cost of increasing the spread even further.

In addition, the paper compares the dynamic responses to technology shocks and monetary policy shocks for the model with liquidity and the standard New Keynesian DSGE model. The results show that the responses to technology shocks are quantitatively different for the two models. However, the differences in response to monetary policy shocks are negligible for real variables, due to a relatively small liquidity effect. The paper also shows that as the liquidity effect is engineered to be stronger, the dynamic responses for the two models become quantitatively different, for both technology shocks and monetary policy shocks. Finally, this paper studies the interaction between the liquidity effect and monetary policy. The results show that as monetary policy becomes less aggressive, the liquidity effect is dampened for technology shocks but it is amplified for monetary policy shocks, which leads to interesting dynamic effects. Overall, the paper highlights a liquidity effect that is shown to be quantitatively significant to understand business cycle fluctuations and contributes to the discussion on the role of money and macroeconomy.

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Figure 1 Dynamic Responses to a Technology Shock The Benchmark Model

Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.



Figure 2 Dynamic Responses to a Monetary Policy Shock The Benchmark Model

Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.



Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

# Figure 3 Dynamic Responses to a Technology Shock The Benchmark Model and the Case of a Low $\eta$



Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

# Figure 4 Dynamic Responses to a Monetary Policy Shock The Benchmark Model and the Case of a Low $\eta$



Figure 5 Dynamic Responses to a Technology Shock The Benchmark Model and the Standard Model

Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.



Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

#### Figure 6 Dynamic Responses to a Monetary Policy Shock The Benchmark Model and the Standard Model

Figure 7 Dynamic Responses to a Technology Shock The Benchmark Model (low  $\eta$ ) and the Standard Model (low  $\eta$ )



Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

#### Figure 8 Dynamic Responses to a Monetary Policy Shock The Benchmark Model (low $\eta$ ) and the Standard Model (low $\eta$ )



Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.



Figure 9 Dynamic Responses to a Technology Shock The Benchmark Model and the Case of a Low  $\phi_{\pi}$ 

Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.



Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

# Figure 10 Dynamic Responses to a Monetary Policy Shock The Benchmark Model and the Case of a Low $\phi_{\pi}$

Figure 11 Dynamic Responses to a Technology Shock The Benchmark Model (low  $\phi_{\pi}$ ) and the Standard Model (low  $\phi_{\pi}$ )



Notes: impulse responses to a one percent increase in technology. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

#### Figure 12 Dynamic Responses to a Monetary Policy Shock The Benchmark Model (low $\phi_{\pi}$ ) and the Standard Model (low $\phi_{\pi}$ )



Notes: impulse responses to a 25 basis points monetary policy shock. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.

## 1 Appendix A: Household first-order conditions

This appendix derives household first-order conditions. The maximization problem of the household is:

$$\max_{\{C_t, B_t, N_t, S_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \frac{1}{\sigma}} \left( C_t^{1 - \frac{1}{\eta}} + \omega (B_t / P_t)^{1 - \frac{1}{\eta}} \right)^{\frac{1 - \frac{1}{\sigma}}{1 - \frac{1}{\eta}}} - \frac{N_t^{1 + \varphi}}{1 + \varphi} \right],$$

subject to:

$$C_t + \frac{B_t}{P_t} + \frac{S_t}{P_t} \le w_t N_t + \frac{B_{t-1}R_{t-1}}{P_t} + \frac{S_{t-1}R_{t-1}^S}{P_t} + \frac{T_t}{P_t} + \frac{\Phi_t}{P_t}$$

In addition, let  $X_t$  denote the bundle of consumption and liquidity services:

$$X_t \equiv (C_t^{1-\frac{1}{\eta}} + \omega (B_t/P_t)^{1-\frac{1}{\eta}})^{\frac{1}{1-\frac{1}{\eta}}}$$

Solving the above maximization problem yields the first-order conditions for consumption, money (liquidity), other assets, and labor:

$$X_t^{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)} C_t^{-\frac{1}{\eta}} = \lambda_t,$$

$$X_t^{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)} \omega (B_t/P_t)^{-\frac{1}{\eta}} = \lambda_t - \beta R_t E_t [\lambda_{t+1} \frac{1}{\Pi_{t+1}}],$$

$$\lambda_t = \beta R_t^S E_t [\lambda_{t+1} \frac{1}{\Pi_{t+1}}],$$

$$N_t^{\varphi} = \lambda_t w_t.$$

Note that the marginal utility of consumption depends on the level of real money balances through variable  $X_t$ . Denote  $Q_t \equiv (1+\omega^{\eta}(B_t/P_t)^{1-\eta})^{\frac{1}{1-\eta}}$  as the price index of the bundle, measured in units of consumption. One can rewrite the marginal utility of consumption as:

$$Q_t^{\left(\frac{\eta}{\sigma}-1\right)}C_t^{-\frac{1}{\sigma}} = \lambda_t$$

This implies a higher price of liquidity (also a higher price of the bundle) leads a fall in the marginal utility of consumption. Finally, plugging the above equation into other first-order conditions yields the money demand function, the Euler equation, and the labor supply equation:

$$\begin{split} B_t/P_t &= C_t \omega^{\eta} \big( \frac{R_t^s - R_t}{R_t^s} \big)^{-\eta}, \\ \beta R_t^S E_t[(\frac{Q_t}{Q_{t+1}})^{1 - \frac{\eta}{\sigma}} (\frac{C_t}{C_{t+1}})^{\frac{1}{\sigma}} (\frac{1}{\Pi_{t+1}})] = 1, \\ w_t &= C_t^{\frac{1}{\sigma}} N_t^{\varphi} Q_t^{1 - \frac{\eta}{\sigma}}. \end{split}$$

## 1 Appendix B: Equilibrium equations

## 1.1 Equilibrium

This appendix summarizes the equilibrium conditions of the New Keynesian model with liquidity, as in the text:

$$w_t = C_t^{\frac{1}{\sigma}} N_t^{\varphi} Q_t^{1-\frac{\eta}{\sigma}} \tag{1}$$

$$Q_t = (1 + \omega^{\eta} (\frac{R_t^s - R_t}{R_t^s})^{1-\eta})^{\frac{1}{1-\eta}}$$
(2)

$$b_t = C_t \omega^{\eta} \left(\frac{R_t^s - R_t}{R_t^s}\right)^{-\eta} \tag{3}$$

$$\beta R_t^S E_t [(\frac{Q_t}{Q_{t+1}})^{1-\frac{\eta}{\sigma}} (\frac{C_t}{C_{t+1}})^{\frac{1}{\sigma}} (\frac{1}{\Pi_{t+1}})] = 1$$
(4)

$$K_t = Q_t^{\frac{n}{\sigma} - 1} C_t^{-\frac{1}{\sigma}} (\frac{w_t}{A_t}) Y_t + \beta \theta E_t K_{t+1} \Pi_{t+1}^{\epsilon}$$
(5)

$$F_t = Q_t^{\frac{\eta}{\sigma} - 1} C_t^{-\frac{1}{\sigma}} Y_t + \beta \theta E_t F_{t+1} \Pi_{t+1}^{\epsilon - 1}$$

$$\tag{6}$$

$$\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} = \frac{K_t}{F_t} \tag{7}$$

$$Y_t = C_t \tag{8}$$

$$Y_t = \frac{A_t N_t}{\Delta_t} \tag{9}$$

$$\Delta_t = (1-\theta) \left(\frac{1-\theta \Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_t^{\epsilon} \Delta_{t-1}$$
(10)

$$\left(\frac{R_t}{\bar{R}}\right) = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_{\pi}} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} V_t \tag{11}$$

$$b_t = \mu b_{t-1} \frac{1}{\Pi_t} + \bar{G}.$$
 (12)

This is a system of 12 equations in 12 unknowns  $(w_t, C_t, N_t, Q_t, R_t, R_t^s, b_t, \Pi_t, K_t, F_t, Y_t, \Delta_t)$ , which can be used to solve for the equilibrium. The processes for the shocks are given by:

$$A_{t} = \bar{A}^{1-\rho_{a}} A_{t-1}^{\rho_{a}} e^{\epsilon_{t}^{a}}$$
$$V_{t} = \bar{V}^{1-\rho_{v}} V_{t-1}^{\rho_{v}} e^{\epsilon_{v}^{v}}.$$

In addition, I choose  $\bar{A} = 1$  and  $\bar{V} = 1$  as a normalization.

#### 1.2 Steady state

In the steady state, zero inflation is assumed, i.e.,  $\bar{\Pi} = 1$ . We then have  $\bar{R}^s = 1/\beta$  and  $\bar{\Delta} = 1$ . Note that  $\bar{R} = R^D$  is the target policy rate, we then have  $\bar{Q} = (1 + \omega^\eta (\frac{1/\beta - R^D}{1/\beta})^{1-\eta})^{\frac{1}{1-\eta}}$ . The supply side of the economy implies that  $\bar{K} = \bar{F} = \frac{\bar{Y}^{1-\frac{1}{\sigma}}\bar{Q}\frac{\eta}{\sigma}^{-1}}{1-\beta\theta}$ , and  $\bar{w} = 1$ . From the market clearing conditions, we have  $\bar{C} = \bar{Y}$ , and  $\bar{N} = \bar{Y}$ . From the labor supply equation, we can solve for  $\bar{Y}$ :  $\bar{Y} = (\bar{Q}\frac{\eta}{\sigma}^{-1})^{\frac{1}{\varphi+\frac{1}{\sigma}}}$ . Finally, the money demand function indicates that  $\bar{b} = \bar{Y}\omega^{\eta}(\frac{1/\beta - R^D}{1/\beta})^{-\eta}$ . Note that, the system implies that the real level of government spending satisfies  $\bar{G} = (1 - \mu)\bar{b}$ , which is also our target.

## 1 Appendix C: Impulse responses to a preference shock

In this section, I report impulse responses to a one percent increase in preference. The results are shown in Figure A1. Preference shocks enter household's utility function:  $U(C_t, \frac{B_t}{P_t}, N_t; Z_t) = \left[\frac{1}{1-\frac{1}{\sigma}}(C_t^{1-\frac{1}{\eta}} + \omega(\frac{B_t}{P_t})^{1-\frac{1}{\eta}})^{\frac{1-\frac{1}{\sigma}}{1-\frac{\eta}{\eta}}} - \frac{N_t^{1+\varphi}}{1+\varphi}\right]Z_t$ , where  $Z_t$  denotes the preference shock. Assume that  $z_t \equiv \log Z_t$  follows an exogenous AR(1) process:  $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ , with  $\rho_z = 0.8$ . The dynamic system would remain the same, except for the Euler equation, which now needs to include the

$$\beta R_t^s E_t[(\frac{Q_t}{Q_{t+1}})^{1-\frac{\eta}{\sigma}} (\frac{C_t}{C_{t+1}})^{\frac{1}{\sigma}} (\frac{1}{\Pi_{t+1}}) (\frac{Z_{t+1}}{Z_t})] = 1.$$

preference shock:

The dynamic responses of different variables would be qualitatively very similar to an expansionary monetary policy shock. As agents give more weight to current utility, relative to future utility, the shift in preferences induces an increase in consumption and hence in aggregate demand. To match the high level of demand, the increase in  $Z_t$  leads to an increase in output, employment, and real wages. Higher output pushes up inflation and the price level, which leads to a decline in real money balances. In addition, following a Taylor rule, the central bank raises the policy rate. Finally, as nominal spending increases, the price of liquidity goes up and the spread (also the convenience yield of liquidity) increases.

Figure A1 also displays the responses of macro variables to a preference shock when  $\eta$  is low (i.e.,  $\eta = 0.05$ ). Compared to the benchmark model, a low  $\eta$  implies that the convenience yield of liquidity would increase by more. A higher spread increases the real return on assets, and through intertemporal decisions households consume less and save more. The spread also affects agents' labor supply decisions: it encourages households to demand more leisure and less consumption goods. Both effects lead to a lower increase in consumption. Such a decline in aggregate demand (compared to the benchmark case), is matched by smaller increases in output, employment, real wages. This also holds true for inflation, the price level, and real money balances.



Figure A1 Dynamic Responses to a Preference Shock The Benchmark Model and the Case of a Low  $\eta$ 

Notes: impulse responses to a one percent increase in preference. Horizontal axes indicate quarters. Variables are expressed in percentage point deviations from steady state. The responses of inflation, policy rate, and spreads are annualized percentage point changes. Spreads are differences between shadow rate and policy rate.