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June 2021

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June 15, 2021

Abstract

This paper deals with two intrinsically linked issues: the endogeneity of the demand regime and the personal distribution impact on aggregate demand. By microfounding the savings function, the aggregate savings rate is an increasing function of the Gini index, which in turn is decomposed as a function of the functional income distribution and the Gini indices for wages and profits. By assuming that saving is a function of personal rather than functional income distribution, an increase of the labour share is effective in boosting consumption and aggregate demand, not per se, but only as long as it reduces personal inequality. As the labour share increases, depending on the distribution of wages and profits, both the demand regime type – the *sign* of the slope of the demand schedule - and its strength- the *size* of the slope of the demand schedule - can endogenously change. Concerning the former, there can be a threshold value for the wage share beyond which there is a shift from wage-led to profit-led demand. The analysis shows that, unlike most Kaleckian models, profit inequality is just as important as wage inequality in determining the demand regime type and its strength.

Keywords: Personal distribution, functional distribution, wage-led, profit-led, non-linear demand, endogenous demand regime.

JEL Codes: B50, E11, E12, D31, D33.

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1 Introduction

One of the key features of the Kaleckian model of growth, both in its original version (Rowtorn, 1981 [20] and Dutt, 1984 [5]) and in its subsequent evolutions (Amadeo, 1987 [1] and Taylor, 1990 [23]), is a saving function based on the functional distribution of income. Both in the former simplified versions where workers do not save at all and in the latter more advanced versions where a propensity to save out of wages is added, class-based distribution is fundamental in generating a wage-led growth regime. Other variants of the model, starting from the contribution of Bhaduri and Marglin (1990) [2], modify the other model's equation, the investment function, including the profit share to take into account the profitability impact on investment decisions. With this evolution, the positive effect on consumption of a lower profit share could be offset by the negative effect on investment. Depending on which of the two effects prevails the economy will be wage-led or profit-led. However in all these models, the consequence of adopting a saving function based on the functional distribution of income is that the economy will be either universally wage-led or profit-led. As already pointed out by Nikiforos (2016) [14], this conclusion is problematic, as an economy would reach its maximum growth rate either with a labour share tending towards one or zero, depending on whether it is wage-led or profit-led.

The introduction of an endogenous demand regime that bounds the "distribution-ledness" of an economy has been the object of different contributions. In all of them the non-linearity in the demand schedule comes, explicitly or not, from a changing propensity to invest and/or to save in the labour share level. In Palley (2013) [19] both mechanisms are at work, and a non-linear demand schedule is justified, on one hand, with the presence of neoclassical capital stock adjustment costs and, on the other, with a redistribution towards top tier income households produced by a profit share increase. Only the saving channel is rather present in Palley (2015) [16], who introduces an endogenous wage bill split between workers and managers that in turn makes the demand regime endogenous: the increased economic activity following a pro-capital redistribution in a profit-led regime can trigger a redistribution of the wage bill towards workers if managers are a fixed normal cost. The redistribution generates a fall in the aggregate saving rate, due to workers propensity to save is lower than that of managers. Both channels are present in Nikiforos (2016) [14], who explicitly assumes that the propensity to invest and the propensity to save change in response to the changes in the labour share. These two mechanisms, together with an unstable distribution of income influenced by class power, "distribution-ledness" of the economy and lagged effects, generate endogenous changes between wage-led and profit-led periods.

However, since a universally wage/profit-led demand is a direct consequence of a saving function based on functional income distribution, this paper proceeds by simply shifting from functional to personal distribution as the key distributional variable determining the saving rate. This shift of perspective is not merely instrumental to generate a non-monotonic demand schedule. It seems more plausible that the individual propensity to save depends on the individual position in total income ranking, i.e. on personal income distribution[1], rather than on the type of income earned - wage or profit. The saving function is microfounded following Carvalho and Rezai (2016) [3]: aggregating individual saving decisions, where individual consumption depends on the income deviation from the median individual, yields a positive relation between the aggregate saving rate and the Gini index. The Gini index is then decomposed, following Lerman and Yitzhaki (1985) [12], as a function of the Gini indices for wage and profit and of the functional distribution. From this decomposition it derives that wage inequality and profit inequality play the same, but symmetric, role in determining the demand regime type - the *sign* of the slope of the demand schedule - and its strength - the *size* of the slope of the demand schedule. Moreover, since saving is a function of the personal income distribution rather than the functional one, raising the wage share is effective in reducing the aggregate saving rate, not per se, but only as long as it reduces personal inequality. As the labour share increases, depending on how it affects the personal distribution, both the demand regime type and its strength can endogenously change. In particular, there can be a threshold value of the wage share beyond which a further increase raises inequality rather than reducing it. This generates a shift from wage-led to profit-led demand. For this reason, when the saving rate is determined by personal distribution rather than functional distribution, a declining propensity to save in the labour share level can arise naturally, generating a non-monotonic demand schedule, without the need to make additional assumptions[2].

If the economy is populated by a majority of low-income workers and a small number of high-income capitalists with very low within-class inequality, the two types of distributions - functional and personal - may be similar[3] and the functional distribution could act as a good proxy for personal distribution. However, these restrictions - that could be reasonable for the nineteenth-century early capitalism - are far from being convincing in an advanced economy.

[1]Dynan et al. (2004) [6]) provide empirical evidence of the positive relationship between saving rates and lifetime income - based on data from the Panel Study of Income Dynamics (PSID), the Survey of Consumer Finances (SCF) and the Consumer Expenditure Survey (CEX) - along with an extensive examination of empirical and theoretical debate on the issue. Evidence based on recent data from Consumer Expenditure Survey can be also found in Carvalho and Rezai (2016) [3].

[2]Obviously, this does not mean that there are not other mechanisms that contribute to the endogeneity of the demand regime.

[3]At least for not too high labour share levels.

Indeed, while measures of personal inequality have steadily increased since the 70s, only a part of this can be explained by an increase in the profit share, which - despite a significant increase - has remained much more stable than the personal income inequality (Figure 1[4]).

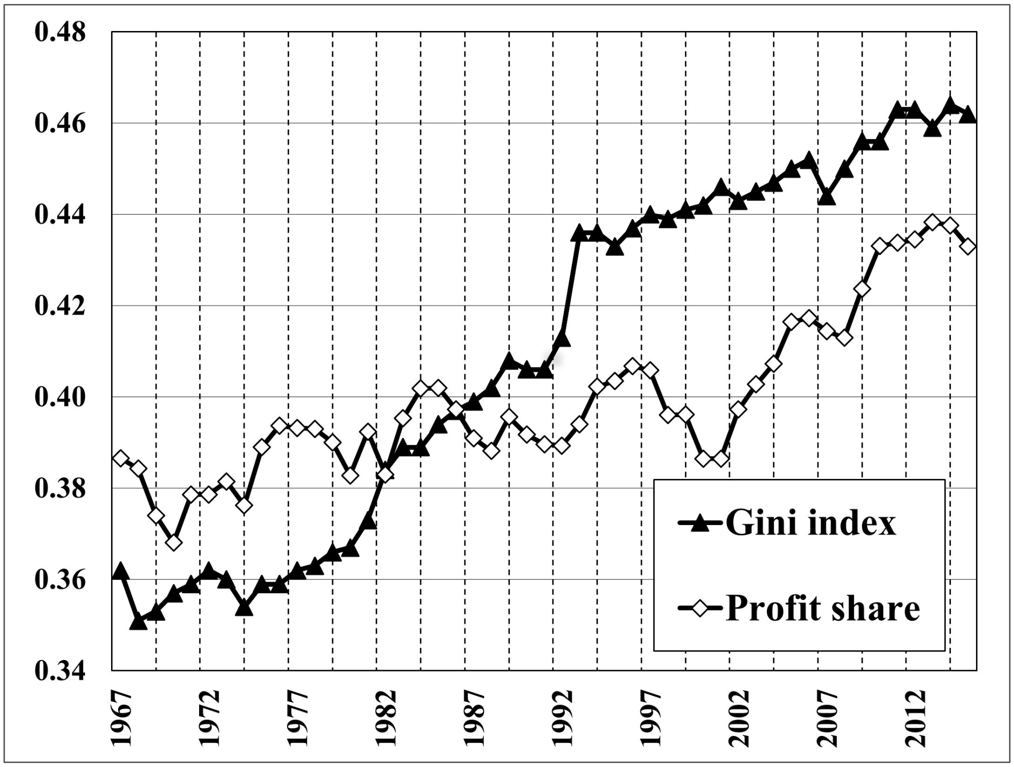

Figure 1: Gini index and Profit share in the USA

There are two main reasons behind the difficulty of functional income distribution to capture personal inequality in nowadays economies. Primarily, in modern economies there is a sizable within-class inequality, from the part-time worker's low wage to the very high top manager pay, and from the small business profit income to the major shareholders of large retailer and tech companies. Secondly, the class concept itself sometimes may be difficult to handle. Despite the majority of people earn most of their income either from labour or from capital, a certain number of people draw their income from both income sources. Although it typically represents a small part of their income, it is common for workers to draw income from interest paid on their savings. Another, more prominent, case is that of self-employed, as their income is partly imputed to labour

[4]The profit share is computed as the complement to one of the labour share series at constant prices taken from AMECO database. Gini index data are taken from *Income and Poverty in the United States* (2015), U.S. Census Bureau, Table A-3.

and partly to capital in official statistics.

Another approach to introduce personal inequality within the Kaleckian model, adopted - among others[5] - by Lavoie (1996) [11], Palley (2014 and 2015) [15] [16] and Tavani and Vasudevan (2014) [22], consists in introducing an unproductive managerial class, which shares wage income with workers. Inequality between these two classes of wage earners is used as a proxy for personal inequality. On one hand, this approach to personal distribution is only confined to deal with wage inequality, neglecting the other side of the coin: profit inequality. On the other hand, it measures only the "between" inequality, as inequality within workers, managers and capitalists is not taken into account. These downsides may be applied to a similar approach, taken by Palley (2017a) [17] and (2017b) [18], which however realistically considers that both workers and capitalists earn either capital and labour income. Carvalho and Rezai (2016) [3], show that aggregating workers' individual saving functions where saving depends on the deviation between the individual's income and the median income yields a positive relationship between the aggregate saving rate and the Gini index. Although this approach takes into account within class inequality, personal income inequality is here still limited to inequality within wage earners as the distribution of profit income is not taken into account.

The view of consumption upheld by this paper, where the main determinant of individual saving rate differences is the individual position in income ranking, is linked to the strand of literature, which draws primarily on Veblen's (1899) [25] 'conspicuous consumption' and Duesenberry's (1949)[4] 'relative income hypothesis ' [6], that sees consumption demand mainly as a social phenomenon: individuals primarily consume to keep up with a certain social standard of well-being, reflected in a given consumption level and, thus, saving is mainly determined as a residual component once those needs have been attained. Within this setup, a critical role is played by the determination of the consumption level targeted by individuals: depending on how this target is formulated, an *increasing* or *decreasing* relationship between the saving rate and personal inequality arises. The 'expenditure cascades' hypothesis proposed by Frank et al. (2014) [13] predicts that, through a sort of trickle-down consumption mechanism, an increase in inequality within a group makes its saving rate decrease. According to this idea, individuals try to emulate the consumption behaviour of those just above them in the income rank. Consequently, an increase in income and consumption of those at the top of income distribution, generates a reduction of saving rates along all the distribution in the attempt 'to keep up with the Joneses'. Along this line, the contributions of Setterfield and Kim (2016) [21], Kapeller and Schütz (2015) [10] and

[5] For an extensive review of personal inequality in Kaleckian models see Hein (2018) [8].

[6] For an extensive review of this strand of literature see Trezzini (2005) [24].

Kapeller et al. (2018) [9] show that, if a redistribution from labour to capital is coupled with an expenditure cascade effect, it can boost demand through a *consumption-driven profit-led regime*. On the contrary, the individual consumption function of Carvalho and Rezai (2016) [3] embeds a *positive* relationship between inequality and the aggregate saving rate. Each individual compares its consumption, not with the income group immediately above in the income rank, but with the median individual of the whole distribution. Instead of a continuum of consumption levels targeted by people, there is only one target for all, which should be seen as a threshold of social satisfaction rather than a target. If we see the use of the functional income distribution in all the traditional Kaleckian and neo-Kaleckian models as a simple proxy for the personal distribution, it can be said that this literature falls under the Carvalho and Rezai category, as it predicts a drop in the aggregate saving rate following a redistribution from capital to wage. This paper can be led to this last approach, as its main goal is bounding the 'distribution-ledness' of traditional neo-kaleckian models, in which a wage-led aggregate demand is in principle possible through the positive effect on consumption of a pro-labour redistribution. Nevertheless, as we will see later, relying on a saving function where the aggregate saving rate is a decreasing function of personal inequality does not significantly alter the main findings of this paper.

The paper proceeds as follows. On one hand, the linkage between the saving rate and personal distribution is based on the work of Carvalho and Rezai (2016) [3], which embeds a positive relationship between the aggregate saving rate and the Gini index. However, differently from Carvalho and Rezai (2016) [3], the Gini index refers to total income. On the other hand, the decomposition of the Gini index developed by Lerman and Yitzhaki (1985) [12] provides the relation between the functional and personal distribution of income. Along this line, this paper tries to offer a more comprehensive framework that, linking functional to personal distribution of income, addresses the issue of the distribution impact on aggregate demand and, in particular, on consumption. As the main interest of this paper is the impact of distribution on aggregate demand, the other side of the coin, the so-called *distribution schedule*, is simply taken as flat. The paper has a fourfold contribution. First, if the aggregate saving rate is a function of personal income distribution rather than functional distribution, it shows how endogenous regime changes may arise naturally without the need of further assumptions regarding the investment function. Second, even if such regime change does not occur, it shows that the degree of wage or profit-ledness of the demand regime - its 'strength' - may not be constant. Third, it shows that in determining the regime *type* and its *strength* a symmetric role is played by wage and profit inequality, despite the role of the latter is neglected in the literature. Lastly, it offers a more general and comprehensive way of dealing with personal distribution as any kind of distribution can be virtually represented within the model.

The paper is organized as follows. Section 2 discusses the general model and its properties. As the model's features are strictly dependent on the particular distribution assumed for individual wages and profits, in Section 3 four particular distributions, among the infinite possibilities, are simulated in order to better understand the characteristics and implications of the model. Section 4 analyses what happens if the saving function embeds a decreasing relationship between personal inequality and the aggregate saving rate. Section 5 concludes.

2 The Model

Investment is described by a standard Kaleckian investment function:

$$g^i = \gamma + \gamma_u(u - u_n) \tag{1}$$

where $g^i = \frac{I}{K}$, K is the stock of capital, γ is the growth rate of autonomous investment, u is the rate of capacity utilization, u_n is the normal rate of capacity utilization and γ_u is the sensitivity of investment to deviation of capacity utilization from its normal rate. The choice not to include any distributional variables in the investment function comes from the will of focusing on consumption demand as the transmission channel from distribution to aggregate demand. The saving function is described in the following way:

$$g^s = s\frac{u}{v} \tag{2}$$

Where $g^s = \frac{S}{K}$, $v = \frac{K}{Y}$ and s, the average propensity to save, is microfounded as follows. Each individual income Y_i is equal to the sum of labour income and capital income:

$$Y_i = w_i\omega Y + p_i(1-\omega)Y \tag{3}$$

Where ω is the labour share, $1-\omega$ is the profit share and w_i and p_i are, respectively, the portion of total wage mass and total profit mass earned by individual i. These individual shares are assumed to be constant over time[7]. Individuals take their saving decisions as follows:

$$S_i = a_0 Y_i + a_1 (Y_i - Y_m) \tag{4}$$

Saving S_i depends partly on individual income Y_i, through the coefficient a_0, and partly on its deviation from the median income Y_m, through the coefficient a_1, which is indeed a measure of how much saving decisions are affected by distribution. The more affluent the individual is, the higher his saving rate. Aggregating the individual saving functions and assuming a Pareto distribution for income, Carvalho and Rezai (2016) [3] show that the aggregate average saving rate can be written as:

$$s = a_0 + a_1 \left(1 - 4^{\frac{G_y}{1+G_y}} \frac{1 - G_y}{1 + G_y}\right) \tag{5}$$

See Appendix D. Where G_y is the Gini index of total income. According to this equation, the saving rate and personal inequality are linked by a unique positive relationship: every decrease in personal inequality always reduces the saving rate, which in turn boosts aggregate demand. Notice that:

$$\lim_{G_y \to 0} s = a_0 \tag{6}$$

$$\lim_{G_y \to 1} s = a_0 + a_1 \tag{7}$$

If there is no personal income inequality at all, all individuals save the same portion of their income and everyone has a saving rate equal to a_0. There is no difference between the individual and the aggregate saving rate. At the opposite end, if personal inequality is at its maximum, then all the individual saving rates can differ from each other, according to the individual position in income ranking; and the aggregate saving rate will be equal to the sum of a_0 and a_1.

The personal and functional income distribution are linked following the Lerman and Yitzhaki

[7] This assumption implies that the flows of savings do not sum up to wealth over time; hence, the model must be intended in a short to medium-run perspective, as for a long-run analysis the assumption that p_i is constant must be released. In the long-run $\dot{p_i} = \frac{S_i}{K} - gp_i$, where K is the stock of wealth/capital. In other words, the wealth share is constant when the individual saves exactly the amount that corresponds to his share in the increase in total wealth; see Ederer and Rehm (2019) [7] and Palley (2017a)[17]. This issue is common to all contributions that, on one hand, have wage earners with a positive saving rate and, on the other hand, do not consider wealth distribution.

(1985)[12] Gini index decomposition[8]:

$$G_y = \rho_w G_w \omega + \rho_p G_p (1 - \omega) \tag{8}$$

Where G_y is Gini index for total income, G_w and G_p are the Gini indeces for wages and profits respectively, ω is the labour share and $1-\omega$ is the profit share. G_w is equal to zero when all people earn the same wage level and is equal to 1 when one individual earns all the wage mass. The same is true for G_p. Note that, if one of the two factor is equally distributed within the population, increasing (decreasing) the share of this factor in total income reduces (increases) total inequality (G_y). Finally, ρ_w and ρ_p are the Gini correlation coefficients for wages and profits, respectively. ρ_w measures the correlation between the wage level and the income ranking and is defined as follows:

$$\rho_w = \frac{cov(W, f(Y))}{cov(W, f(W))} = \frac{\sum(W_j - \overline{W})(f(Y_j) - \overline{f}(Y)}{\sum(W_i - \overline{W})(f(W_i) - \overline{f}(W)} \tag{9}$$

Where W_j and Y_j (numerator) are ordered from the lowest *income* to the highest, while W_i and Y_i (denominator) are ordered from the lowest *wage* to the highest. W_j is the wage level of individual j, $\overline{W}$ is the mean wage, $f(Y_j)$ is the cumulative distribution of income at W_j and $\overline{f}(Y)$ is its mean. Likewise $f(W_i)$ is the cumulative distribution of wages at W_i and $\overline{f}(W)$ is its mean. ρ_w will equal 1 (-1) when the wage level is an increasing (decreasing) function of income ranking, while it will be positive (negative), but smaller (greater) than 1 (-1), when it is only *on average* an increasing (decreasing) function of income ranking. It will be instead equal to zero when there is no correlation between wage income and the income rank.

Likewise, ρ_p is a measure of the correlation between the profit level and the income ranking and a positive (negative) value indicates that capital income is *on average* an increasing (decreasing) function of the income ranking.

$$\rho_p = \frac{cov(P, f(Y))}{cov(P, f(P))} = \frac{\sum(P_j - \overline{P})(f_j(Y) - \overline{f}(Y)}{\sum(P_i - \overline{P})(f_i(P) - \overline{f}(P)} \tag{10}$$

Note that, while G_w and G_p are exogenous parameters (i.e. do not depend on ω), ρ_w and ρ_p - as we will see later - are a function of ω. Therefore, equation (8) can be rewritten as:

$$G_y = \rho_w(\omega) G_w \omega + \rho_p(\omega) G_p (1 - \omega) \tag{11}$$

Equations (8) and (11) tell us that the functional distribution of income is not the only distributional variable that can affect personal distribution and demand. Indeed, provided ρ_w and

[8] In general, for K income sources, the Gini index can be decomposed as $G = \sum_{k=1}^{K} \rho_k G_k S_k$, where s_k is the share of source k in total income.

ρ_p are greater than zero, as it is likely to be in a real economy, every reduction in wage and/or profit inequality can reduce personal inequality and, through equation (5), stimulate aggregate demand.

Equating equation (1) to equation (2) we get the equilibrium rate of capacity utilization:

$$u = \frac{(\gamma - \gamma_u u_n)v}{s(\omega, G_w, G_p) - \gamma_u v} \tag{12}$$

As in all Kaleckian models, Keynesian stability must be assumed in order to have a positive value for u, which in this case means that $s > \gamma_u v$. We can now turn on the impact of functional distribution on aggregate demand.

$$\frac{\partial u}{\partial \omega} = -\frac{(\gamma - \gamma_u u_n)v}{[s(\omega, G_w, G_p]^2} \frac{\partial s}{\partial \omega} \tag{13}$$

The sign of the partial derivative, and the demand regime, clearly depends only on the impact of ω on the saving rate. Accordingly, aggregate demand will be wage-led if the savings rate decreases following an increase in the wage share.

$$\text{If } \frac{\partial s}{\partial \omega} < 0 \Rightarrow \frac{\partial u}{\partial \omega} > 0 \tag{14}$$

Likewise, aggregate demand will be profit-led if the savings rate increases as a result of an increase in the wage share.

$$\text{If } \frac{\partial s}{\partial \omega} > 0 \Rightarrow \frac{\partial u}{\partial \omega} < 0 \tag{15}$$

As shown in Appendix B the sign of $\frac{\partial s}{\partial \omega}$ depends solely on how personal inequality is affected by changes in functional distribution, that is on $\frac{\partial G_y}{\partial \omega}$. Therefore, equations (14) and (15) can be restated as:

$$\text{if } \frac{\partial G_y}{\partial \omega} < 0 \Rightarrow \frac{\partial u}{\partial \omega} > 0 \tag{16}$$

$$\text{if } \frac{\partial G_y}{\partial \omega} > 0 \Rightarrow \frac{\partial u}{\partial \omega} < 0 \tag{17}$$

Therefore, it is sufficient to study the conditions under which an increase of the labour share leads to a higher or lower personal inequality. The reaction of personal distribution to changes in functional distribution is described by the following condition:

$$\frac{\partial G_y}{\partial \omega} = G_w \left[\rho_w + \frac{\partial \rho_w}{\partial \omega}\omega\right] - G_p \left[\rho_p - \frac{\partial \rho_p}{\partial \omega}(1 - \omega)\right] \tag{18}$$

Appendix A discusses the sign of the two partial derivatives and shows that under plausible conditions:

$$\frac{\partial \rho_w}{\partial \omega} \geq 0 \text{ and } \frac{\partial \rho_p}{\partial \omega} \leq 0 \; \forall \; \omega \tag{19}$$

In particular, ρ_w will increase every time an individual a overcome in the income ranking an individual b whose individual share w_i of total labour income is smaller than a, otherwise it will stay constant. Likewise, ρ_p will reduce every time an individual a overcome in the income ranking an individual b whose individual share p_i of total capital income is higher than a, otherwise it will stay constant.

What equation (18) tells us is that the *type* of demand regime - the *sign* of the slope of the demand schedule in the (u, ω) plane - and its *strength* - the *size* of the slope of the demand schedule - depend: on the level of the labour share (ω), on the initial value of parameters ρ_w and ρ_p, on their variation ($\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$) and on inequality among wages (G_w) and profits (G_p). Profits inequality plays the same, but symmetric, role as wage inequality in determining the regime type and its strength, despite being neglected in the literature. In particular, if wage is on average an increasing function of income ranking ($\rho_w > 0$), as it is likely to be in a real economy, the higher wage inequality, the more it is likely that an increase in wage share leads to higher overall personal inequality and, thus, that the demand regime is profit-led. This happens because the higher wage inequality and ρ_w, the more wage income tends to be concentrated in the hands of those at the top of the distribution. Hence, for sufficiently high values of these two parameters, an increase in the labour share would further increase their income, raising personal inequality. In other words, an increase in the wage share would mean increasing the major source of inequality, while the main source of distribution equality - profit income - would shrink. This is true as long as the change in the Gini correlation parameters is not such as to reverse the situation and make wage income more concentrated at the bottom of the distribution and profit income at the top. On the contrary, if wage is on average a decreasing function of income ranking ($\rho_w < 0$), the higher wage inequality, the more it is likely that an increase in wage share leads to a lower overall personal inequality and, thus, that the demand regime is wage-led. This happens because the higher wage inequality and the lower ρ_w, the more wage income tends to be concentrated in the hands of those at the bottom of the distribution. Hence, for sufficiently high values of G_w and sufficiently negative values of ρ_w, an increase in the labour share would increase their income, reducing overall personal inequality, at least until the change in Gini correlation parameters is such that to reverse the situation. The same reasoning applies to profit income. If profit is on average an increasing function of income ranking ($\rho_p > 0$), as it is likely to be, the higher profit inequality, the more it is likely that an increase in wage share leads to a lower overall personal inequality and, thus, that the demand regime is wage-led. Again, this is true as long as the change in the Gini correlation parameters is not such to turn profit income more concentrated at the bottom of the distribution and wage income at the top. On the contrary, if profit is on average a decreasing function of income ranking ($\rho_p < 0$), the higher profit inequality, the more it is likely that an increase in wage share leads to a higher overall personal inequality and, thus, that the demand regime is profit-led. As before, this holds up to a change in Gini correlation parameters such as to reverse capital and labour income concentration in the income distribution.

In the particular case in which, as ω increases, there is no change in income ranking, $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are both equal to zero (Appendix A) and equation (18) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w \rho_w - G_p \rho_p \tag{20}$$

Therefore, as long as the income ranking does not change the first derivative is constant and no regime change can occur, be it initially wage or profit-led.

Equating to zero equation (18) and solving for ω we obtain the critical value for ω, i.e. that value that once passed shifts the economy from a wage-led to a profit-led regime:

$$\omega^* = \frac{\rho_p G_p - \rho_w G_w - \frac{\partial \rho_p}{\partial \omega} G_p}{\frac{\partial \rho_w}{\partial \omega} G_w - \frac{\partial \rho_p}{\partial \omega} G_p} \tag{21}$$

Note that if the income ranking never change, for instance because those who earn a higher wage have a higher profit income too, $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are equal to zero (see Appendix A) and ω^* does not exist.

Appendix C shows that for wage share levels smaller than the critical value the economy is wage-led. On the contrary, for wage share levels higher than the critical value, the economy is profit-led. In other terms:

$$\text{If } \omega < \omega^* \Rightarrow \frac{\partial u}{\partial \omega} > 0 \Rightarrow \text{ wage-led} \tag{22}$$

$$\text{If } \omega > \omega^* \Rightarrow \frac{\partial u}{\partial \omega} < 0 \Rightarrow \text{ profit-led} \tag{23}$$

Summing up, depending on the distribution of wages and profits, there can be an ω^* such that the regime is wage-led if $\omega < \omega^*$ and the regime is profit-led if $\omega > \omega^*$[9]. Provided the conditions

[9]Note that, as we will see in Section 4, this result would be reversed if the saving function embodied an

for which $0 < \omega^* < 1$ hold and that ρ_w and ρ_p are positive, an increase (decrease) in G_w tends to decrease (increase) ω^*, while an increase (decrease) in G_p tends to increase (decrease) ω^*. In other words, for the same reasons discussed for equation (18), the higher wage inequality and the lower profit inequality, the less is the space for re-distributional policies toward wages with the aim of increasing the level of economic activity, as the boundary beyond which demand turns profit-led become more tighten. The intuition is that an increasing inequality among wages (profits) increases the average propensity to save of those who mainly earn wage (profit) income, weakening the wage-ledness (profit-ledness) of the economy.

Even if the change in the functional distribution of income is not such to make the *sign* of the slope of the demand schedule change, it does not mean that the *size* of the slope does not change at all. In other words even if, as ω increases, a shift from a wage-led to a profit-led demand regime doesn't occur, it does not mean that the *wage-ledness* of the economy remains the same. However, analyzing the sign of the second derivative of equation (12) is not straightforward (Appendix B). What we can say is that, if there is a continuous change in income ranking and ω^* does exist, then there will be a neighborhood of ω^* in which the second derivative is negative. To put it another way, if equation (12) is continuously derivable and has a point of maximum, then its second derivative must be negative in a neighborhood of this point. This implies that, as we get close to such ω^* from the left, the effectiveness of redistributional policies towards wages to stimulate aggregate demand is decreasing in ω. The same is true for a profit share increase in a profit-led regime: as we get close to ω^* from the right, the effectiveness of redistributional policies towards capital to stimulate aggregate demand is decreasing in $1 - \omega$.

2.1 Introducing distributed profits

Let d be the percentage of distributed profits, so that $0 < d < 1$. Personal inequality is now computed not on the total income of the economy but only on income that actually accrue to households, i.e. on total income net of retained profits. Equation (8) becomes:

$$G_y = \rho_w G_w \overline{\omega} + \rho_p G_p (1 - \overline{\omega}) \tag{24}$$

where $\overline{\omega}$ and $1 - \overline{\omega}$ are, respectively, the wage and profit share of total disposable income.

'expenditure cascade' mechanism. If the relationship between personal inequality and the saving rate was negative rather than positive as it is in equation (5), the demand regime would be profit-led for $\omega < \omega^*$ and wage-led for $\omega > \omega^*$.

Rearranging the two shares in terms of the usual wage and profit share, we obtain:

$$G_y = \rho_w G_w \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right) d} + \rho_w G_p \left[1 - \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right) d}\right] \tag{25}$$

Note that if $d = 1$ this condition is nothing but the Gini decomposition of Equation (8), that is, therefore, a special case when all profits are distributed to households.

The equilibrium rate of capacity utilization depends now also on d, since, as stated in equation (5), s is a positive function of G_y, which in turn is a function of d, ω, G_w and G_p (equation 25). Equation (12) becomes:

$$u = \frac{(\gamma - \gamma_u u_n)v}{s(d, \omega, G_w, G_p) - \gamma_u v} \tag{26}$$

We can now focus on the impact of distributed profits on aggregate demand:

$$\frac{\partial u}{\partial d} = \frac{-(\gamma - \gamma_u u_n)v}{s^2} \frac{\partial s}{\partial d} \tag{27}$$

The partial derivative depends only on the response of the saving rate to changes in d.

$$\frac{\partial s}{\partial d} = \frac{\partial s}{\partial G_y} \frac{\partial G_y}{\partial d} \tag{28}$$

Which in turn depends exclusively on $\frac{\partial G_y}{\partial d}$, since $\frac{\partial s}{\partial G_y}$ is always positive, as shown in Appendix B. The main point of interest is, thus, how the percentage of distributed profits affects personal inequality.

$$\frac{\partial G_y}{\partial d} = \frac{(\omega - 1)\omega(\rho_w G_w - \rho_p G_p)}{[d(1-\omega) + \omega]^2} \tag{29}$$

The sign of the partial derivative clearly depends on $\rho_w G_w - \rho_p G_p$. A reasoning similar to that made for equation (18) applies here. If profits are mainly concentrated in the upper (lower) part of the distribution then an increase in d increase (decrease) total inequality (G_y). Indeed, the higher (lower) is $\rho_p G_p$ relative to $\rho_w G_w$ the more profits tends to be concentrated in the upper (lower) tail of the distribution, and an increase in distributed profits would mean further increasing (reducing) their income, raising (lowering) inequality.

$$\text{If } \rho_w G_w < \rho_p G_p \Rightarrow \frac{\partial G_y}{\partial d} > 0 \Rightarrow \frac{\partial u}{\partial d} < 0 \tag{30}$$

$$\text{If } \rho_w G_w > \rho_p G_p \Rightarrow \frac{\partial G_y}{\partial d} < 0 \Rightarrow \frac{\partial u}{\partial d} > 0 \tag{31}$$

3 Some distributional examples

To better understand the properties of the model, I simulate in this section four particular distributions among the infinite possibilities[10]. For the sake of simplicity, the following simulations are based on the standard model without distinguishing distributed and not distributed profits. Firstly, I simulate a 'classical' distribution of income, as it might have been in nineteenth-century capitalism. The population is divided in two groups: 70 % of people earn only labour income and the remaining 30 % draw their income only from capital. Inside those two groups there is perfect equality in wages and profit earned. In statistical terms, there can be 'between' inequality in this distribution, but there is always 'within' equality. This does not imply that G_w and G_p are equal to 0 because the Gini indices are computed on the whole population, not on subgroups. Starting from the bottom-left panel of Figure (2), it can be noted that ω^* exists and is equal to 0.7. For wage share values smaller than 0.7, increasing the wage share to stimulate aggregate demand it is an effective policy, as it results in an increase of capacity utilization. This is no more true for values beyond 0.7, for which, on the contrary, an effective policy would be increasing the profit share. Note that the reaction of aggregate demand to changes in the functional distribution (bottom-left panel) strictly follows the pattern of the response of personal inequality to changes in functional distribution (upper-left panel): the economy is wage-led (profit-led) only as long as an increase in the labour share reduces (increases) personal inequality. In the two right panels it can be noted that to the left of ω^*, ρ_w and ρ_p are equal to -1 and 1, respectively, and constant. The two values reflect the fact that, up to ω^*, wage (profit) income is a decreasing (increasing) function of the income rank, as all wage (profit) earners lie in the bottom (upper) part of the distribution. The situation reverses beyond ω^*, with ρ_w and $_p$ which becomes equal to 1 and -1, respectively. The constancy of the two parameter is instead due to the fact that - as stated in Appendix A - until ω^* and beyond it, as ω increases, there is no change in the income ranking. The only exception is when ω is equal to 0.7, the point in which all the wage earners simultaneously overcome in the income rank all the profit earners.

In the second distribution (Figure 3) wage and profit levels are correlated, as it could be in a population only made of self-employed autonomous workers, where their stock of wealth would be proportional to their wage, assuming a uniform rate of return on capital. All people earn both labour and capital income. Thus, there are no more subgroups here, but the more an individual earns a high wage the higher is also his or her capital income. The parameters ρ_w and ρ_p will

[10] All the following simulations are based on an economy populated by 1000 individuals and the following parameters calibration: $u_n = 0.7$, $\gamma = 0.12$, $\gamma_u = 0.05$, $v = 2$, $a_0 = 0.3$ and $a_1 = 0.2$.

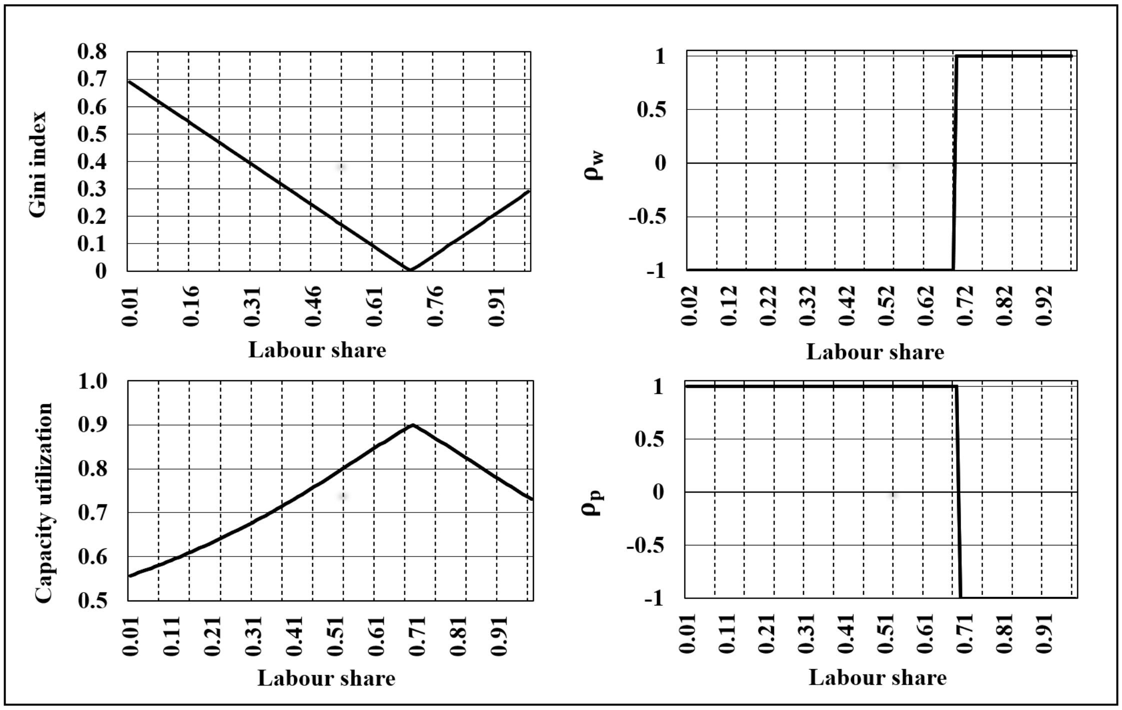

Figure 2: Distribution 1

be both equal to 1 for every ω (right panels), since both the wage and the profit level are an increasing function of the income ranking. In this case the demand regime *type* and its *strength* depend only on which between G_w and G_p is greater. Indeed equation (18) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w - G_p \tag{32}$$

In this example, since it is assumed that $G_w > G_p$, the economy is always profit-led. Besides, ω^* in this case does not exist, since $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are always zero and the denominator of equation (21) is consequently null. These characteristics can be noted looking at the two left panels: an increase in the wage share always generates a raise in the personal inequality (upper-left panel), which in turn leads to a lower capacity utilization (bottom-left panel). A redistribution policy from wage to capital is always effective in stimulating aggregate demand. Summing up, the economy is always wage-led (profit-led) if $G_w < G_p$ ($G_w > G_p$), and a threshold value for ω beyond which there is a regime change does not exist.

The third distribution (Figure 4) is characterized by profit 'quasi-equality'[11] and wage inequality:

[11] Actually, profits are not strictly equal for all, but are generated by a distribution with a very low dispersion which yields negligible individual differences. This modification is made necessary since a profit income strictly

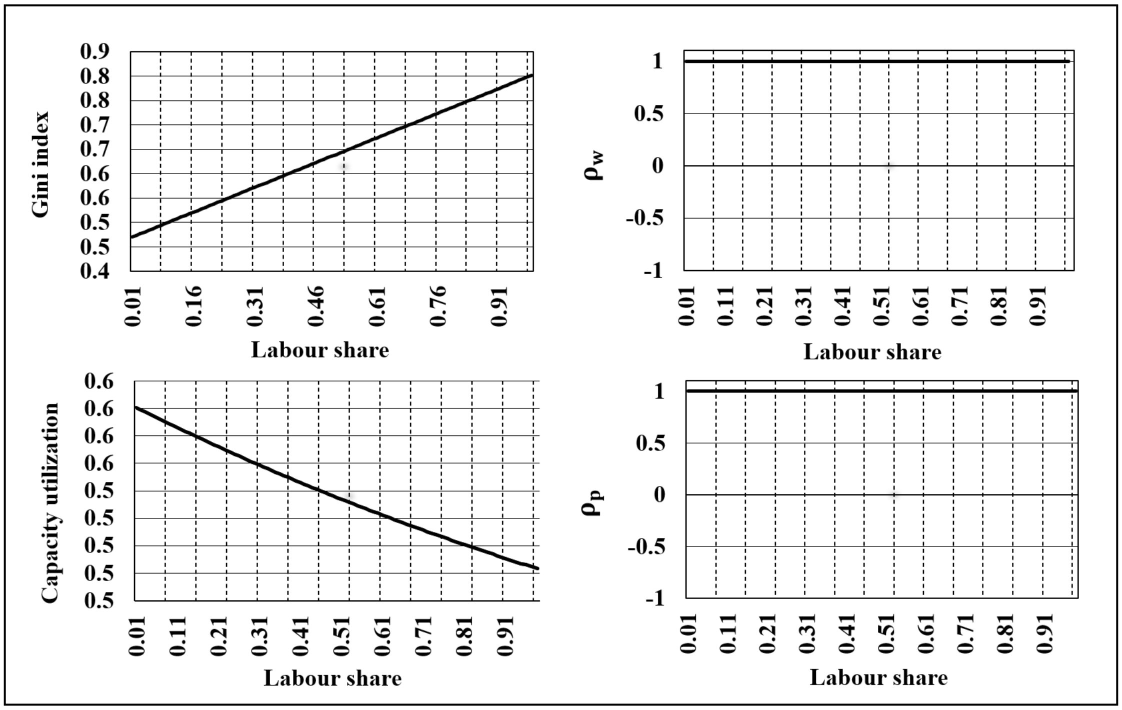

Figure 3: Distribution 2

again, all people have both labour and capital income, all individuals earn the same profits, but they earn different wages. It is a sort of economy of cooperatives where everyone owns the same stock of capital but skills and wages are different. G_p and ρ_p are equal to 0 (bottom-right panel), as profit income is equally distributed and uncorrelated with income ranking. Instead, ρ_w is always equal to 1 (upper-right panel), being wages an increasing function of income ranking. Equation (18) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w \tag{33}$$

Being profits the income source that is equally distributed across the population, an increase of labour share in total income will increase total inequality of an amount exactly equal to the Gini index for wages. As in the previous example, ω^* does not exist, since the denominator of equation (21) is null. These characteristics can be noted looking at the two left panels: an increase in the wage share always generates a raise in the personal inequality - exactly equal to G_y - (upper-left panel), which in turn leads to a lower capacity utilization (bottom-left panel). A redistribution

equal for everyone would have made ρ_p - as well as G_y and u - impossible to compute, as the denominator of Equation (10) would have been null.

policy from wage to capital is effective in stimulating aggregate demand. The opposite occurs with wages equally distributed across the population and profits unequally distributed.

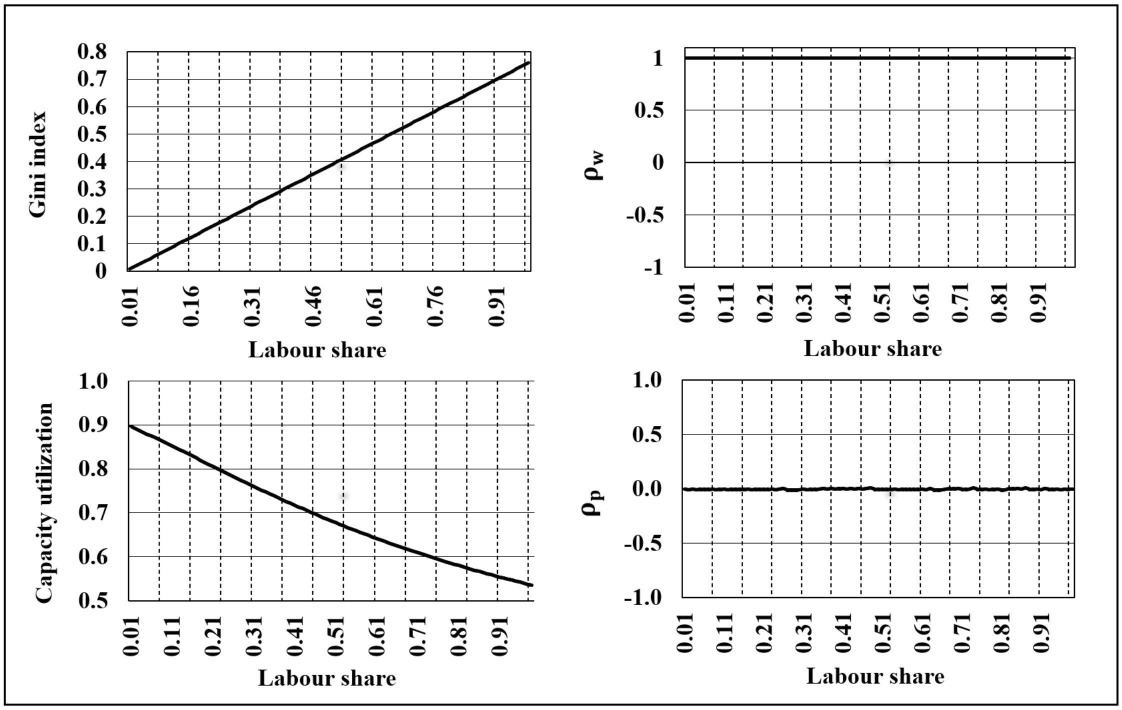

Figure 4: Distribution 3

We have so far investigated very specific income distributions; they are useful to understand model properties, but are far from providing a realistic representation of the economy. The fourth and last distribution example (Figure 5) tries to reproduce some traits of income distribution in a real economy. Individual quotas of wage and profit share are randomly generated by two Pareto distributions. There is, thus, a continuum of individuals, who always differ for income composition and income levels. Differently from previous distributions, as ω changes, there is a continuous change in the income ranking and, as stated in Appendix A, ρ_w and ρ_p are never constant (right panels). These two parameters - despite they change as the labour share increases - are never negative. This reflects the fact that both wage and profit are always *on average* an increasing function of the income rank. From the two left panels, it is evident that ω^* exists and it is equal to 0.67. Therefore, a redistributive policy from from capital towards wage is effective to stimulate aggregate demand up to a wage share level equal to 0.67; further increase in the wage share beyond this value will reduce capacity utilization. Moreover, it is worth noting that the second derivative of the curve in the bottom-left panel is always negative. This implies that, to the left of ω^*, the effectiveness of re-distributional policies towards wage is decreasing in ω and, to the right of ω^*, the effectiveness of re-distributional policies toward capital is decreasing in $1-\omega$. Summing up, redistributing from capital to wage reduces personal inequality (upper-left panel) up to ω equal to 0.67; while beyond this value, the redistribution increases inequality rather than reducing it. This pattern, through the the positive relation between personal inequality and the aggregate saving rate of equation (5), is transmitted to the demand schedule (bottom-left panel). The latter, initially positively sloped in the (ω, u) plane, continuously rotates downward, eventually becoming negatively sloped once outpassed ω^*.

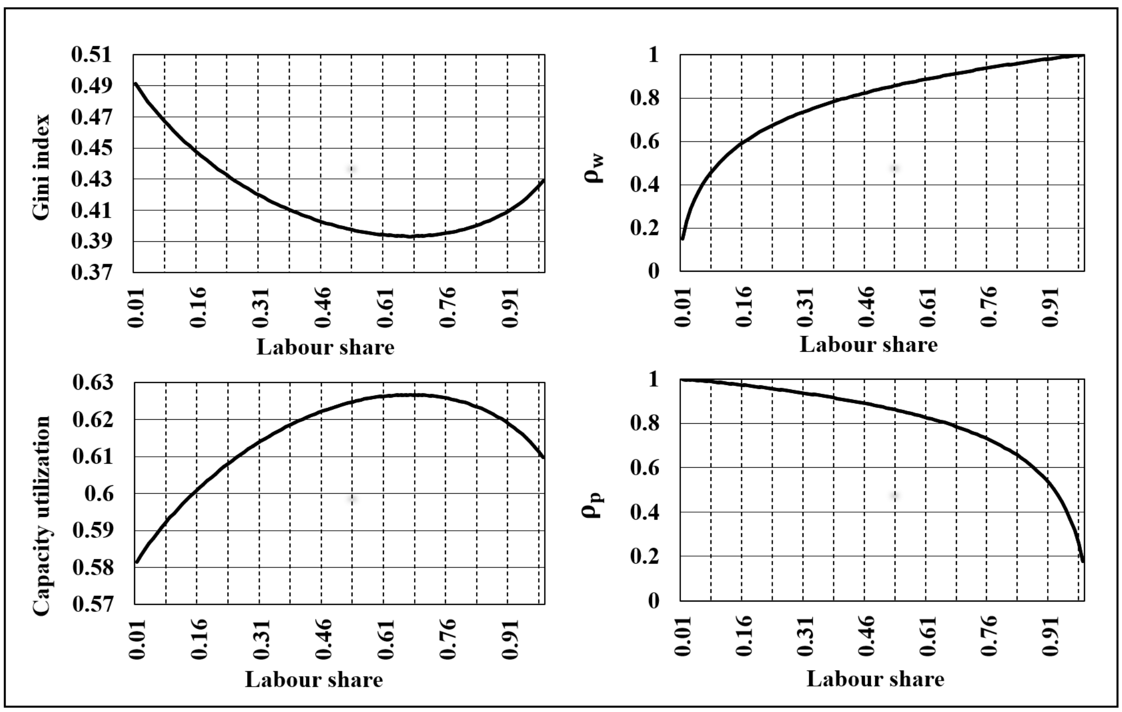

Figure 5: Distribution 4

Functional income distribution is not the only distributional variable that can affect aggregate demand in this model and the last distributional example can be used to analyse also the impact of changes in inequality among wages and profits. As already pointed out when discussing equation (21), provided ρ_w and ρ_p are positive, as it is likely to be in a real economy, a reduction in wage inequality shifts ω^* to the right, increasing the span of ω over which the economy is wage-led (Figure 6). On the contrary, a reduction of profit inequality shifts ω^* to the left, reducing the span of ω over which the economy is wage-led (Figure 7).

Moreover, from equation (8) it is clear that, provided ρ_w and ρ_p are positive, a reduction in

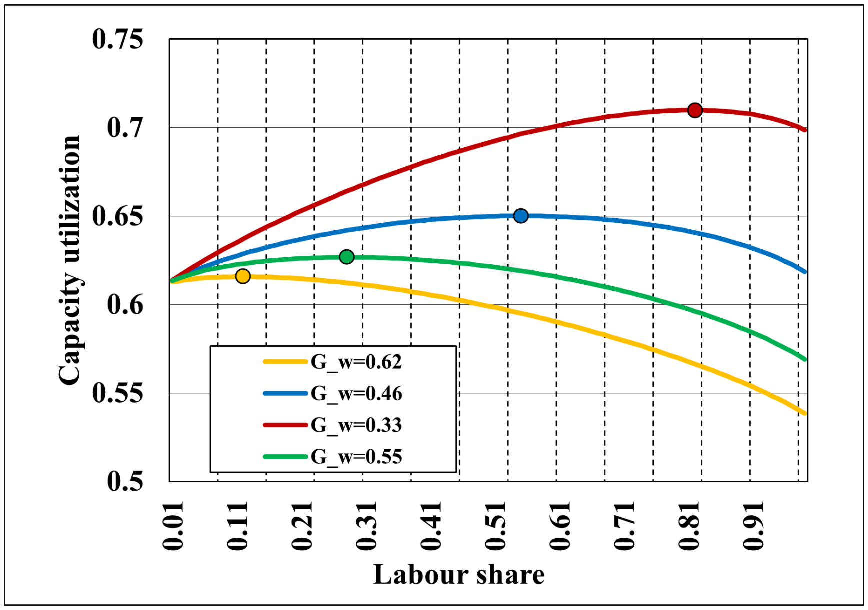

Figure 6: G_w

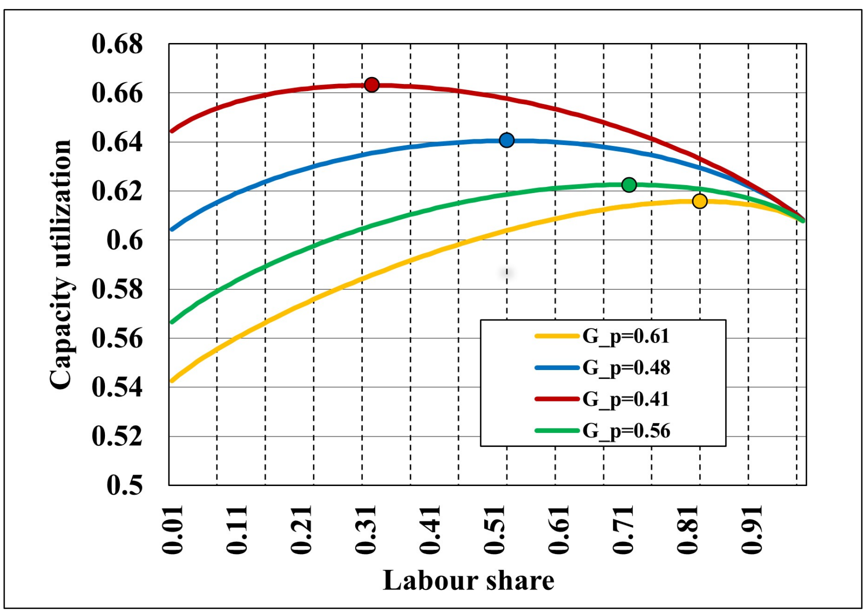

Figure 7: G_p

wage and/or profit inequality always reduce overall inequality, which in turn stimulates aggregate demand. This is evident in Figures (6) and (7), where, following a reduction in G_w or G_p, the new curve always lies above the old one. Thus, as opposed to policies that aim to change the functional distribution, reducing inequality within wages and profits always stimulates aggregate demand, under plausible parameters values.

4 The 'expenditure cascades' case

What we have seen so far was based on a positive relationship between inequality and the aggregate saving rate. As this is the most sensitive assumption of the model, the purpose of this section is to show that the main findings of this paper hold even if the aggregate saving rate and inequality are linked by an inverse relationship. To this aim, let's assume that, in place of equation (5), we have[12]:

$$s = a_0 - a_1 G_y \tag{34}$$

Inserting (34) in (2) we get the new saving function. The rest of the model is the same and it would be redundant to repeat it here. There are only two additional differences. The first regards equation (46) in Appendix B, which now becomes:

$$\frac{\partial s}{\partial G_y} = -a_1 < 0 \tag{35}$$

This condition implies that, differently from the model of sections 2 and 3, a reduction (increase) in inequality increase (reduce) the aggregate saving rate. The second difference, stemming from equation (35), is that, in place of equations (16) and (17), we now have:

$$\text{if } \frac{\partial G_y}{\partial \omega} < 0 \Rightarrow \frac{\partial u}{\partial \omega} < 0 \tag{36}$$

$$\text{if } \frac{\partial G_y}{\partial \omega} > 0 \Rightarrow \frac{\partial u}{\partial \omega} > 0 \tag{37}$$

Thus, the economy is wage-led (profit-led) if an increase in the labour share increases (reduces) inequality. Therefore, as in Section 2, it is sufficient to study the conditions under which an increase of the labour share leads to a higher or lower personal inequality: these conditions are the same stated in equation (18) to (21). In other words, the determinants of ω^* are the same, the only difference is that now the economy is profit-led to the left of ω^* and wage-led to the right. This happens because a reduction in inequality - which as before occurs to the *left* of ω^* - now makes the saving rate increase. Likewise, an increase in inequality - which as before occurs to the

[12] This equation, differently from equation (5), is not microfounded. This is because aggregating individual consumption functions where each individual targets a different consumption level as in Frank et al. (2010) [13], is not so straightforward.

right of ω^* - now makes the saving rate decrease. The only difference with the model of Section 2 concerns the span of ω over which the demand schedule is wage-led or profit-led, which results reverted. However, all the results of Section 2 - the possibility of an endogenous change in the regime *type* and its *strength* and the role played by profit and wage inequality in determining them - are preserved.

The differences with the model of Sections 2 and 3 can be better understood simulating again the distribution 4 of Section 3.

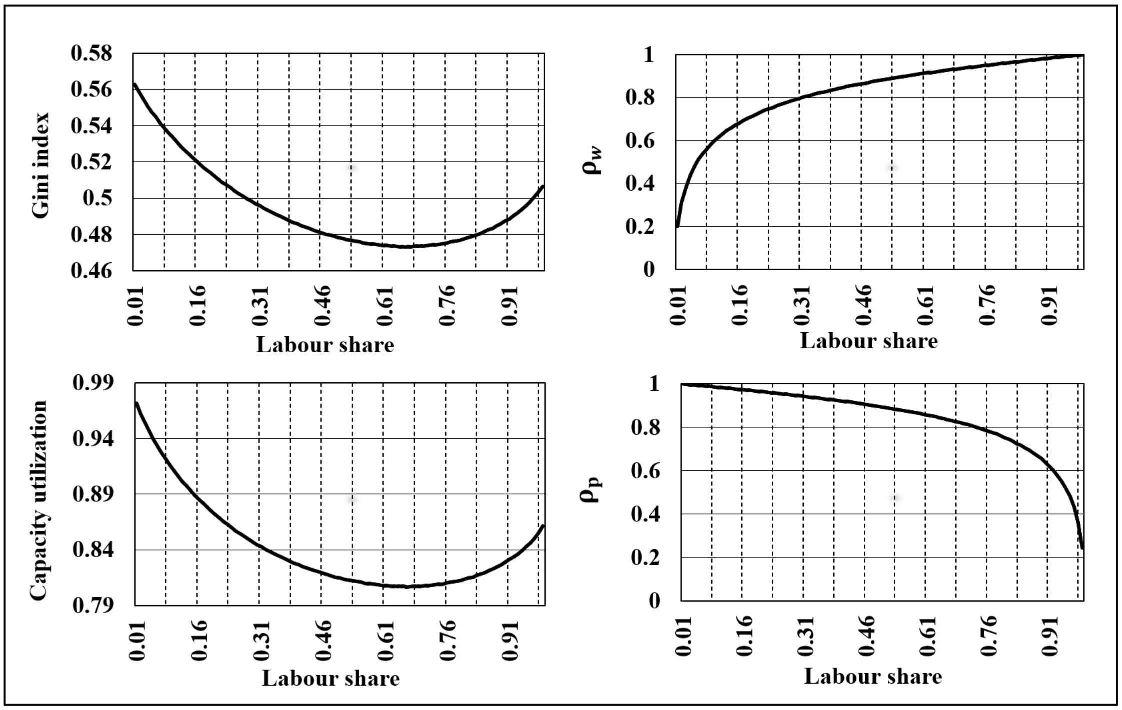

Figure 8: Distribution

As can be noted, the only difference with Figure 5 concerns the demand schedule in the (u, ω) plane, which now results reversed.

5 Conclusion and considerations

This paper shows that, if saving is a function of personal rather than functional income distribution, that is if individuals make their consumption decisions based on their position in the income rank and not on their income type: (i) wage and profit inequality play the same, but symmetric, role in determining the demand regime *type* and its *strength* - the degree of wage or profit-ledness of the demand regime; (ii) as the labour share increases, depending on the distribution of wages and profits, there may be an endogenous regime change, i.e. there can be a threshold value of the wage share where the economy shifts from a wage-led to a profit-led demand regime. (iii) Even without passing such threshold, as the labour share varies, depending on the distribution of wages and profits, the degree of *wage* or *profit-ledness* of the demand regime - its *strength* - may not be constant and will be negative in a neighborhood of the the threshold value. Thus, the effectiveness of re-distributional policies - the regime *strength* - is decreasing as we get close to such threshold. (iv) The percentage of distributed profits amplifies the impact of profits distribution on inequality (and aggregate demand). These results hold even if the saving function was of the 'expenditure cascade' type rather than a la Carvalho and Rezai (2016), that is if the relationship between inequality and the saving rate was negative rather than positive. The only difference concerns the span of the wage share over which the demand schedule is wage-led or profit-led, which results reverted. There are other points that affect our results that are worth discussing. The first regards the choice to keep the investment function in its simplest form, where investment demand does not depend on any distributional variable. Obviously, if the investment function was a function of functional distribution, different results become possible. The same is true for the *distribution schedule*, here simply assumed flat. The last point concerns the assumption that individual shares of total wealth are constant. As the flows of savings do not sum up to the individual stock of wealth over time, this confines the model to a short to medium-run perspective. In a long-run framework, as highlighted by Ederer and Rehm (2019) [7] and Palley (2017) [17], individual shares of total wealth must be endogenized or, put it differently, wealth distribution must be taken into account. Further research along this line is required to extend the model to a long-run perspective.

Appendices

A

Let's define:

$$\rho_w = \frac{cov(W, f(Y))}{cov(W, f(W))} = \frac{\sum(W_j - \overline{W})(f_j(Y) - \overline{f}(Y))}{\sum(W_i - \overline{W})(f_i(W) - \overline{f}(W))} = \frac{A}{B} \tag{38}$$

The first partial derivative of ρ_w with respect to ω is:

$$\frac{\partial \rho_w}{\partial \omega} = \frac{\frac{\partial A}{\partial \omega} B - \frac{\partial B}{\partial \omega} A}{B^2} \tag{39}$$

Where B is always greater than zero, $\frac{\partial B}{\partial \omega}$ is always positive and constant, A can be either positive or negative, and $\frac{\partial A}{\partial \omega}$ will be smaller than zero if A is negative and greater than zero if A is positive. The sign of equation (39) depends on the sign of the numerator which will always be greater or equal to zero as can be noted in figure (9) and (10). In those two figures random values[13] of w_i and p_i are generated from a Pareto distribution with different combinations of the parameter α[14]. For each of them, the responses of ρ_w and ρ_p to changes in ω are plotted.

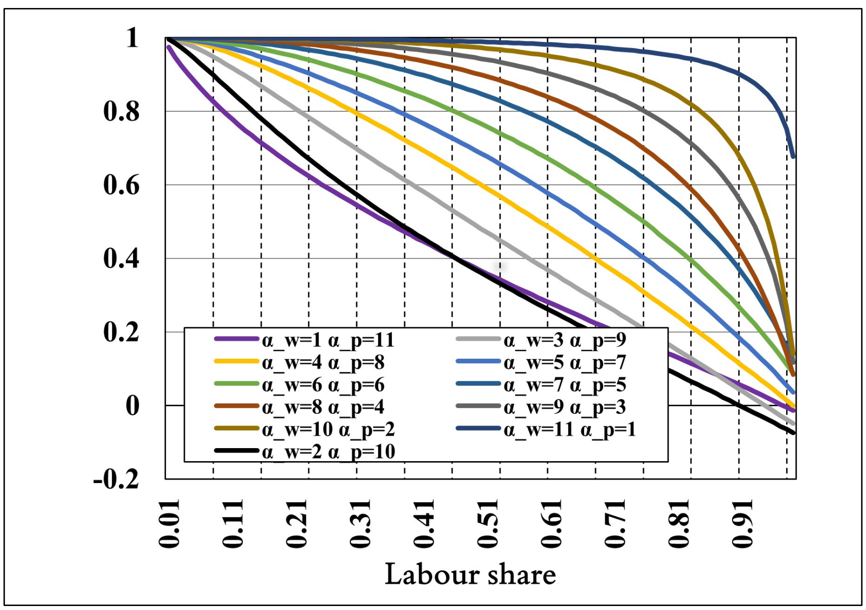

Figure 9: ρ_w

[13] 1000 values are generated, which corresponds to an economy populated by 1000 individuals.

[14] When income follows a Pareto distribution the Gini index has a closed-form solution. In this case: $G_w = \frac{1}{2\alpha_w - 1}$ and $G_p = \frac{1}{2\alpha_p - 1}$.

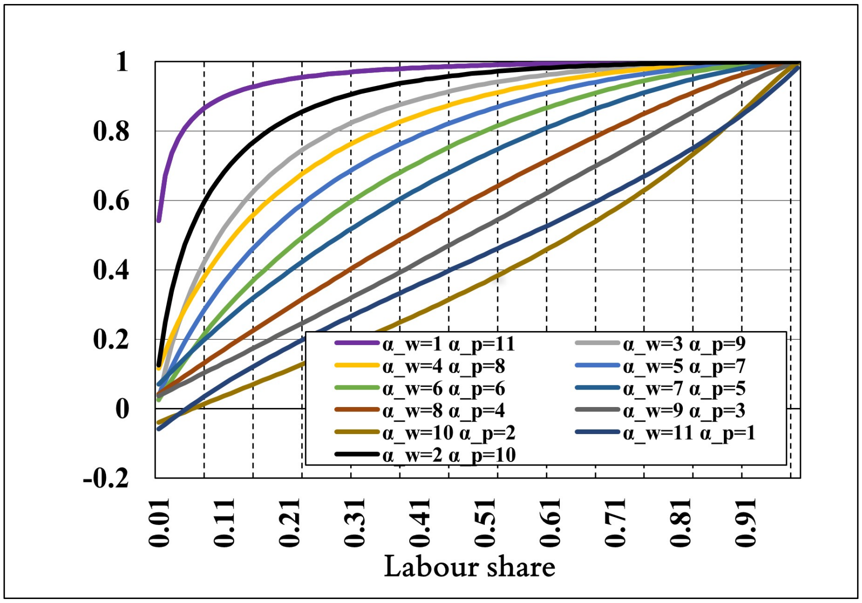

Figure 10: ρ_p

In particular, as ω increases, if at least one individual will overtake in the income ranking someone else with a lower individual share w_i of total wage mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} > \frac{A}{B} \Rightarrow \frac{\partial \rho_w}{\partial \omega} > 0 \tag{40}$$

If instead, the increase in labour remuneration is not sufficient to make at least one individual to be better-off someone else with a lower individual share w_i of total wage mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} = \frac{A}{B} \Rightarrow \frac{\partial \rho_w}{\partial \omega} = 0 \tag{41}$$

An analogous reasoning applies to ρ_p. By expressing ρ_p in the same form as equation (38), as ω increases, if at least one individual will overtake in the income ranking someone else with a higher individual share p_i of total profit mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} < \frac{A}{B} \Rightarrow \frac{\partial \rho_p}{\partial \omega} < 0 \tag{42}$$

While, if the increase in labour remuneration is not sufficient to make at least one individual to be better-off someone else with a lower individual share w_i of total wage mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} = \frac{A}{B} \Rightarrow \frac{\partial \rho_p}{\partial \omega} = 0 \tag{43}$$

This behaviour of parameters ρ_w and ρ_p can be better understood in the next example, where, given the small number of individuals, the change in income ranking is not continuous.

A.1 Example with a three persons economy

Suppose the economy is populated by three individuals, this make it easier to understand what happens to ρ_w and ρ_p when ω varies. Each of the three earns the following shares w_i of the total wage mass ωY:

- $w_1 = 0.3$
- $w_2 = 0.7$
- $w_3 = 0$

And the following shares p_i of the total profit mass $(1 - \omega)Y$:

- $p_1 = 0.1$
- $p_2 = 0.3$
- $p_3 = 0.6$

Thus, individuals sorted by w_i in ascending order are $i = (3, 1, 2)$, while individuals sorted by p_i in ascending order are $i = (1, 2, 3)$. In figure (11) the level of individual incomes (in euros or dollars), ρ_w and ρ_p are plotted. As we have stated before in equation (43), as ω increases, ρ_w remains constant unless there is a change in the income ranking. This happens at $\omega = 0.29$ where individual 2 overtakes individual 3 in the income ranking, and at $\omega = 0.62$ where individual 1 overtakes individual 3. In those two points $\frac{\partial \rho_w}{\partial \omega} > 0$ and $\frac{\partial \rho_p}{\partial \omega} < 0$ as stated in equation (42).

In brief, as ω increases, ρ_w will increase every time an individual a overcomes in the income ranking an individual b whose individual share w_i of total labour income is smaller than a, otherwise it will stay constant. Likewise, ρ_p will reduce every time an individual a overcomes in the income ranking an individual b whose individual share p_i of total capital income is higher than a, otherwise it will stay constant.

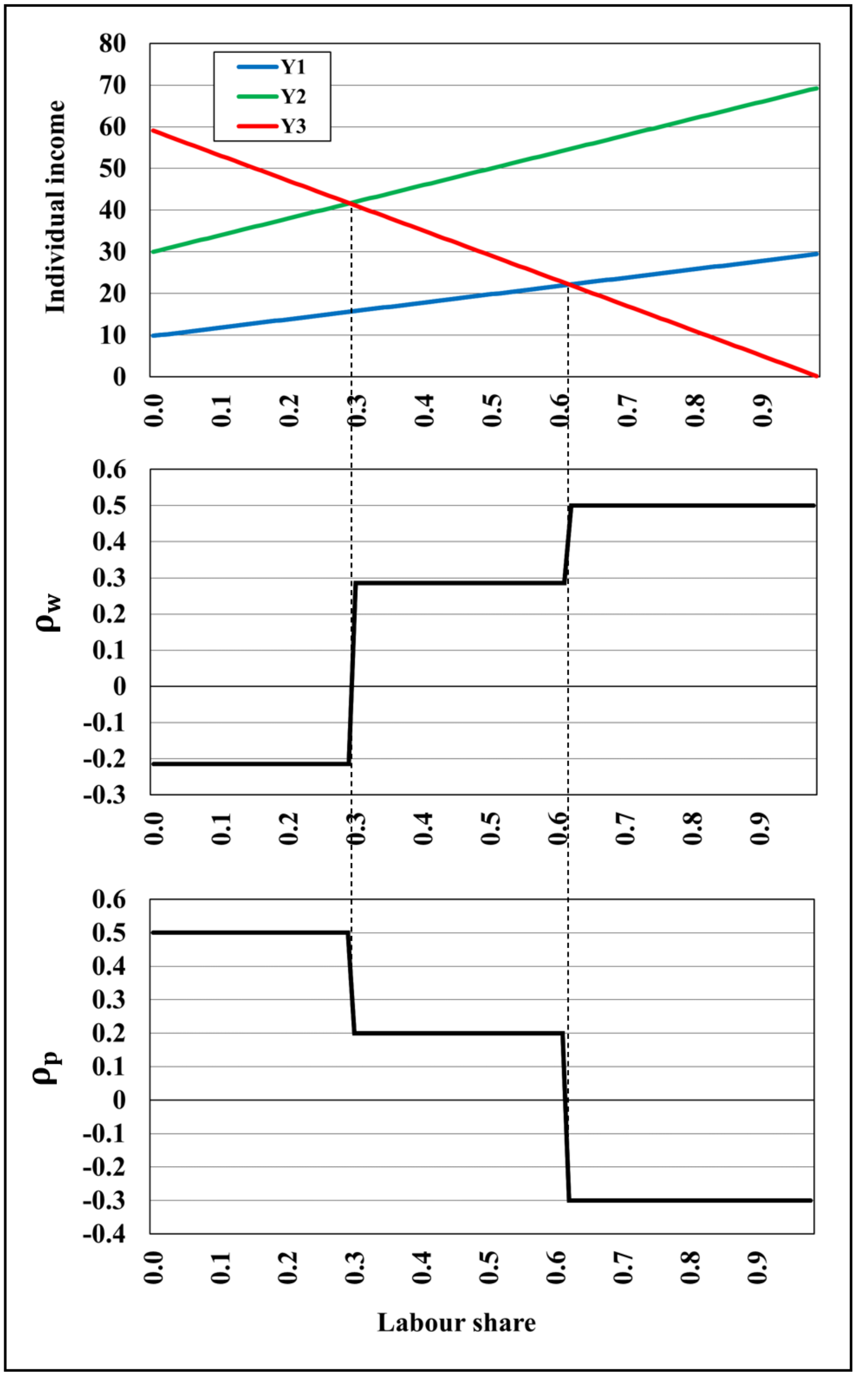

Figure 11: ρ_w and ρ_p in a three persons economy

B

The partial derivative of equation (12) with respect to ω is:

$$\frac{\partial u}{\partial \omega} = \frac{-(\gamma - \gamma_u u_n)v}{s^2}\frac{\partial s}{\partial \omega} \tag{44}$$

Where:

$$\frac{\partial s}{\partial \omega} = \frac{\partial s}{\partial G_y}\frac{\partial G_y}{\partial \omega} \tag{45}$$

The first term is always positive, indeed:

$$\frac{\partial s}{\partial G_y} = a_1 4^{G_y/(1-G_y)}\left[\frac{2(1+G_y) - (1-G_y)\ln 4}{(1+G_y)^3}\right] > 0 \tag{46}$$

Therefore, the sign of $\frac{\partial u}{\partial \omega}$ depends only on $\frac{\partial G_y}{\partial \omega}$ as described in equations (13) to (17).

The second partial derivative of u with respect of ω is:

$$\frac{\partial^2 u}{\partial \omega^2} = \frac{-(\gamma - \gamma_u u_n)v\left[\frac{\partial^2 s}{\partial \omega^2}s^2 - 2s(\frac{\partial s}{\partial \omega})^2\right]}{s^4} \tag{47}$$

Where, from equation (45):

$$\frac{\partial^2 s}{\partial \omega^2} = \frac{\partial^2 s}{\partial \omega \partial G_y}\frac{\partial G}{\partial \omega} + \frac{\partial^2 G_y}{\partial \omega^2}\frac{\partial s}{\partial G_y} \tag{48}$$

Therefore equation (47) can be rewritten as:

$$\frac{\partial^2 u}{\partial \omega^2} = \frac{-(\gamma - \gamma_u u_n)v\left[\frac{\partial^2 s}{\partial \omega \partial G_y}\frac{\partial G_y}{\partial \omega}s^2 + \frac{\partial^2 G_y}{\partial \omega^2}\frac{\partial s}{\partial G_y}s^2 - 2s\left(\frac{\partial s}{\partial G_y}\frac{\partial G_y}{\partial \omega}\right)^2\right]}{s^4} \tag{49}$$

Where:

$$\frac{\partial^2 G_y}{\partial \omega^2} = G_w\left[\frac{\partial^2 \rho_w}{\partial \omega^2}\omega + 2\frac{\partial \rho_w}{\partial \omega}\right] + G_p\left[\frac{\partial^2 \rho_p}{\partial \omega^2}(1-\omega) - 2\frac{\partial \rho_p}{\partial \omega}\right] \tag{50}$$

In the special case in which, as ω increases, the income ranking does not change, $\frac{\partial^2 G_y}{\partial \omega^2} = 0$ as $\frac{\partial G_y}{\partial \omega} = 0$. Equation (50) reduces to:

$$\frac{\partial^2 u}{\partial \omega^2} = \frac{-(\gamma - \gamma_u u_n)v\left[\frac{\partial^2 s}{\partial \omega \partial G_y}\frac{\partial G_y}{\partial \omega}s^2 - 2s\left(\frac{\partial s}{\partial G_y}\frac{\partial G_y}{\partial \omega}\right)^2\right]}{s^4} \tag{51}$$

C

Substituting $\omega = \omega^* - \varepsilon$ in equation (18), where ω^* is defined as in equation (21) and $\varepsilon > 0$, we obtain:[15]

$$\frac{\partial G_y}{\partial \omega} = G_p \frac{\partial \rho_p}{\partial \omega}\varepsilon - G_w \frac{\partial \rho_w}{\partial \omega}\varepsilon \leq 0 \tag{52}$$

Which along with equation (16) proves equation (22).

Likewise, substituting $\omega = \omega^* + \varepsilon$ in equation (18), we obtain:

$$\frac{\partial G_y}{\partial \omega} = G_w \frac{\partial \rho_w}{\partial \omega}\varepsilon - G_p \frac{\partial \rho_p}{\partial \omega}\varepsilon \geq 0 \tag{53}$$

Which along with equation (17) proves equation (23).

[15] $\frac{\partial \rho_p}{\partial \omega} \leq 0$ and $\frac{\partial \rho_w}{\partial \omega} \geq 0$ from equation (19)

D

The aggregate saving function can be derived in a similar way as Carvalho and Rezai (2016) [3]. Assuming that income distribution follows a Pareto type I distribution, the mean μ and the median z of a Pareto distribution are defined as:

$$\mu = \frac{x\alpha}{\alpha - 1} \text{ and } z = 2^{\frac{1}{\alpha}} \tag{54}$$

Where x is the Pareto index.

Individual saving decisions are described by Equation (4) here repeated for convenience:

$$S_i = a_0 Y_i + a_1 (Y_i - Y_m) \tag{4}$$

The aggregate saving is equal to the sum of all individual savings:

$$S = \int \left[a_0 Y_i + a_1 (Y_i - Y_m)\right] f(Y) dY = a_0 \mu + a_1 (\mu - z) = \left[a_0 + a_1 \left(1 - \frac{z}{\mu}\right)\right] \mu \tag{55}$$

With a constant coefficients production function it is true that $\mu L = \frac{uK}{v}$ or normalizing L to one ($L \equiv 1$):

$$\mu = \frac{uK}{v} \tag{56}$$

From equation (54) $x = \mu \frac{\alpha - 1}{\alpha}$ or, substituting equation (56), $x = \frac{\alpha-1}{\alpha} \frac{uK}{v}$. Equation (56) can therefore be rewritten as:

$$\mu = \frac{\alpha - 1}{\alpha} \frac{uK}{v} \frac{\alpha}{\alpha - 1} \tag{57}$$

Substituting equation (57) in equation (55):

$$S = \left[a_0 + a_1 \left(1 - \frac{2^{\frac{1}{\alpha}} (\alpha - 1)}{\alpha}\right)\right] \frac{uK}{v} \tag{58}$$

Dividing by $Y = \frac{uK}{v}$ we obtain the saving function in terms of the saving rate:

$$s = \left[a_0 + a_1 \left(1 - \frac{2^{\frac{1}{\alpha}} (\alpha - 1)}{\alpha}\right)\right] \tag{59}$$

Bearing in mind that the Gini index of a Pareto distribution is defined as $G_y = \frac{1}{2\alpha - 1}$, equation (59) can be rearranged as:

$$s = a_0 + a_1 \left(1 - 4^{\frac{G_y}{1+G_y}} \frac{1 - G_y}{1 + G_y}\right) \tag{5}$$

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