Should CBA use descriptive or prescriptive discount rates? It should use both!

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SHOULD CBA USE DESCRIPTIVE OR PRESCRIPTIVE DISCOUNT RATES? IT SHOULD USE BOTH!

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Discounting project net flows that exclude financing costs with prescriptive rates fails to reflect costs of capital; discounting them with descriptive rates fails to reflect intertemporal preferences. A hybrid discounting method is proposed by which descriptive rates are used to forecast costs of capital and prescriptive rates are used to discount all-inclusive net welfare flows. An agent-based capital market model audits the performance of alternative discounting approaches. There is no need to reconcile the discounting approaches. They should be viewed as complementary, not as competing. For projects to be economically feasible their rate of return should exceed both the STPR and the SOCR. Following this rule will ensure that proposed public sector projects will be no less effective at converting present consumption into future consumption than what the public can already manage and that the benefits of proposed projects will exceed all their direct and indirect costs.

Keywords: Social discount rate; Prescriptive discounting; Descriptive discounting; Hybrid discounting; Declining discount rates.

JEL classification: D61; H43

1. Introduction

Referring to the familiar descriptive and prescriptive classification of approaches to discounting in cost-benefit analysis (CBA), William Nordhaus (2019) observed that the debate about discounting is "just as unsettled as it was when first raised three decades ago."

The two approaches differ in their response to distortions in the capital market. In an undistorted financial market, the interest rate equals both the marginal rate of substitution (MRS) between present and future consumption of savers, and the marginal rate of transformation (MRT) of producers. If the market is distorted, however, these equalities no longer hold. Those who subscribe to the prescriptive approach to discounting rightly argue that it is not possible to correctly gauge intertemporal welfare without using the correct social time preference rate (SPTR). Those who subscribe to the descriptive approach rightly argue that it is not possible to conduct a proper cost-benefit analysis without knowing the true cost of capital, which can be measured by the social opportunity cost rate (SOCR).

Beyond this fundamental discrepancy, there are further disagreements about the correct discount rate among adherents of both approaches. In the prescriptive camp some, but not all, view the setting of the STPR as primarily an ethical issue. Chichilnisky (1997) thinks that a constant discount rate embodies the "tyranny of the present over the future." The descriptive camp faces the difficulty of identifying the correct cost of capital. According to Baumol (1968) "...we find ourselves forced to hunt for a solution in the dark jungles of the second best." Freeman and Groom (2010) feel that these disagreements "raise the spectre of the near impossibility of reconciling" the prescriptive and the descriptive approaches to discounting.

This paper argues that what is needed is not to reconcile the two approaches, but to recognize that once the STPR and the SOCR diverge, neither, by itself, can compute correct net present values

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(NPV) of projects. This was already recognized by Baumol (1968) "We see now that no optimal rate exists. The rate that satisfies the one requirement cannot possibly meet the conditions of the other."

- The STPR, being a property of the welfare function, defines the correct intertemporal welfare weights, but fails to correctly measure the costs of capital.
- The SOCR, being derived from market interest rates with correction for market distortions and other indirect effects, defines the correct costs of capital, but fails to assign correct intertemporal welfare weights.

This paper proposes a hybrid discounting method: costs of capital are calculated with the help of the SOCR; then added to the project net flows; and the resulting flows are discounted using the STPR. In this way, each discount rate performs that role, and only that role, for which it is suited. The NPV so obtained will then correctly establish whether a project is welfare-enhancing. The intertemporal weights will reflect the STPR, costs of capital will correspond to the SOCR. For a project to be welfare-enhancing its return must exceed the hurdle rates of both the STPR and the SOCR. As both are needed for welfare maximization, they are in fact complimentary, not competing.

These concepts are illustrated by a two-period agent-based capital market model in which the welfare consequences of alternative investments can be measured. The agents are risk-averse consumers who can borrow, lend, and invest in stocks offering stochastic returns. The numerical values used are not intended to reflect real-world magnitudes, but to show how CBA handles some of the most common distortions that call for its use in the first place, such as the displacement of private investment by fiscal borrowing and the taxing of interest income and profits. Two examples illustrate the hybrid discounting method and offer a comparison of the performance of alternative discounting methods.

Section 2 briefly describes the capital market model, which is explained in greater detail in the Appendix. The model itself is available as an Excel file[1]. Section 3 presents the calculation of the STPR and the SOCR in the model. Section 4 employs two project analyses—one with a high return, the other with a low return—to make useful observations. Section 5 demonstrates why both the STPR and the SOCR are necessary hurdle rates, but sufficient only jointly. Section 6 extends the results of the two-period model to the consideration of the distant future. Section 7 presents conclusions and their implications for the discounting debate. Section 8 presents conclusions.

2. Description of the capital market model

We assume that consumers aim to maximize a welfare function of the following form:

$$V(C) = \sum_0^t \frac{U(C_t)}{(1+\rho)^t} \quad (1)$$

where $\rho>0$ is the pure rate of time preference and $U(C_t)$ is a utility function of the constant-intertemporal-elasticity-of-substitution (CIES) type:

$$U(C) = \frac{C^{1-\sigma}-1}{1-\sigma} \quad (2)$$

where consumption $C > 0$, and the elasticity of marginal utility with respect to consumption $\sigma > 0$ but not equal to 1. This is also the measure of the decision maker's constant proportional risk-aversion. When $\sigma = 1$, the utility function takes the form $U(C) = ln(C)$.

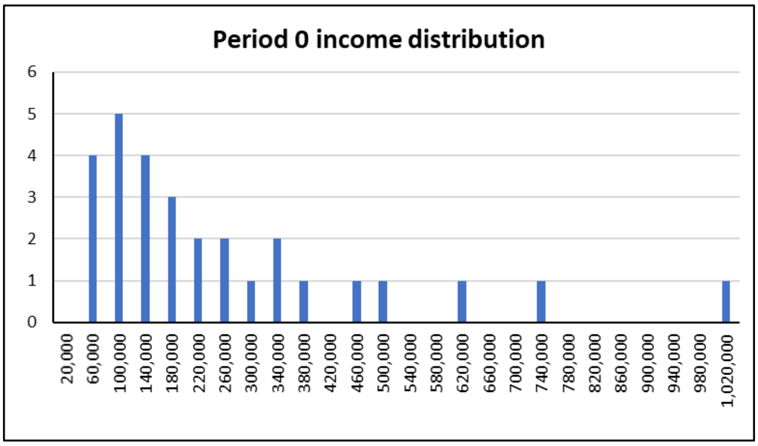
The optimization will only involve two time periods, the present, $t = 0$, and time $t = 1$. The degree of risk aversion chosen for the examples is $\sigma = 1$, following Gandelman and

[1] https://doi.org/10.3886/E132122V2

Hernández-Murillo (2014), which analyzes data from 75 countries and found that "the coefficient of relative risk aversion varies closely around one."[2] We assumed that the pure rate of time preference is $\rho = 2\%$ for all agents.

We first generated 29 agents with the following frequency distribution of Year 0 incomes, which is approximately log-normal:

Figure 1
Income distribution of agents



Next, we doubled their number, by assuming that half the agents would see their income grow by 2% between Year 0 and Year 1, while the other half would see it decline by 2%. This will naturally make half of them be inclined to lend or invest when attempting to smooth their consumption over time, while the other half would be inclined to borrow.

We computed the aggregate indifference curve of this society of 58 consumers, as well as that of the representative agent derived from their average data. See details in the Appendix.

Agents optimize their welfare by choosing amounts to borrow or lend (by selling or buying discount notes) or invest in stocks that have an even chance of either yielding a profit of 40% or a loss of 20%. Their expected yield is 10%. The tax rate on interest income or capital gains is 40%.

A simulation algorithm finds the equilibrium price of the discount notes, thus defining the rate of interest. The supply of stocks is assumed to be infinitely elastic.

The results of the simulation, in the absence of any fiscal borrowing, are described by the following figures. In both figures the vertical axis is dollars borrowed and lent and the horizontal axis is the price of discount notes. The low price of 0.967 corresponds to an annual interest rate of 3.4%, while the high price of 0.971 corresponds to 3.0%. We see in the following Figure 2 that lending declines as the interest rate declines (price of notes increases) while the converse is true for borrowing.

[2] Exhibiting logarithmic utility may be advantageous. Mark Rubinstein (1976) argues that logarithmic utility implies an optimal degree of risk aversion, resulting in the maximization of the expected value of terminal wealth, regardless of initial wealth.

Figure 2
Lending and Borrowing

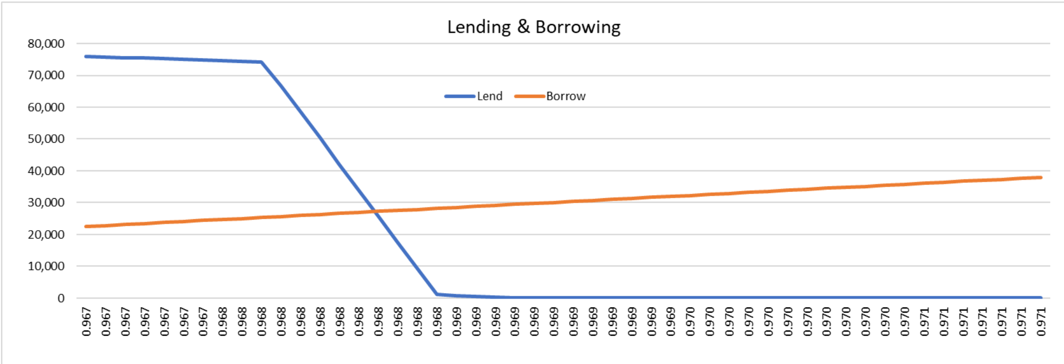
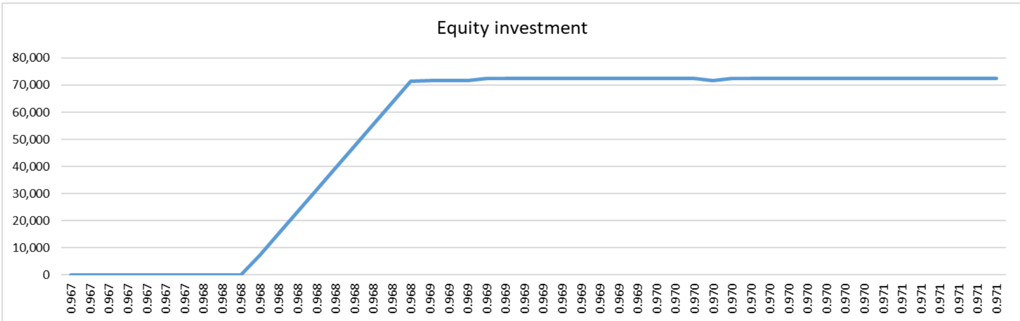


Figure 3 shows a point at which the growth of equity investment begins to appreciably increase. It occurs when interest rates decline to the point at which it becomes worth running the risk of investing in equities.

Figure 3
Equity Investment



Absent fiscal borrowing, the 58 agents borrow and lend $27,200 to each other in the aggregate and invest a total of $45,988 in equities. The equilibrium interest rate at which the amount lent equals the amount borrowed is 3.28%.

3. Calculation of the STPR and the SOCR

There are many approaches to defining the STPR. Some are derived from ethical considerations, such as proposing that there should be no discounting of future values at all, or positing an authoritarian social planner who aims to maximize some welfare function. In this paper we compute the STPR from the preferences of the agents making up the experimental society of our model, as this is what is consistent with the objectives of cost-benefit analysis (CBA), namely, to measure the welfare consequences, measured by revealed preference if possible, of alternative allocations of resources.

Having found the market equilibrium interest rate, all agents have optimized their welfare by either borrowing or lending, and possibly also investing. The Appendix shows that agents who borrow have a MRS that corresponds to the market interest rate of 3.28%. Because they incur no tax liability, there is no market distortion, so this correspondence is a consequence of their consumption path optimization. The MRS of agents who lend, however, always corresponds to 1.97%, which shows that they too optimize to the interest rate that they effectively receive, which in their case is the distorted, after-tax rate.

The STPR reflects the MRS of the entire society. It is computed by aggregating the compensating variation in Year 0 income of all agents (an increase in income) such that their welfare is the same as when given (in the alternative) $1 in Year 1. The result is that this society requires a return higher than 2.62% to consider an investment to be welfare-enhancing, because. This rate is also the weighted average of the rates implicit in the MRSs of the two groups of agents. We conclude that, provided agents can allocate their resources optimally, their rate of time preference equals the effective yield of the safe instruments they use to effect intertemporal transfers. The rate obtained by applying the well-known Ramsey formula to the aggregated consumption values of the agents is also 2.62%.

To measure the SOCR we must compute the welfare impact of fiscal borrowing. For this calculation we assumed that the state would raise $1,000 by issuing discount securities, and then compared the ensuing equilibrium with that of the base-case. The results are shown in Table 1 below, which shows key results of the two scenarios and their differences.

Table 1
Changes attributable to fiscal borrowing

(Dollar amounts)	**Base-case**	**Fiscal borrowing case**	**Attributable changes**
Public borrowing	0	1,000	1,000
Private borrowing	27,200	27,163	-37
Private lending	27,200	28,163	963
Private investments	45,988	45,047	-941
Expected Tax	4,036	3,974	-63

We see that the $1,000 borrowed by the state was sourced from a reduction of private borrowing of $37 and an increase in private lending of $963. The consequence of the latter, however, is a reduction in private investments of $941.

The SOCR must reflect both the direct and indirect costs of fiscal borrowing. In our worked example, we make three adjustments to the market interest rate, which are discussed below. Other possible adjustments can be found in the literature. The values that define the SOCR are shown in Table 2.

Table 2
Calculation of the SOCR

(Dollar amounts)	**Cost of Finance**
Interest payment	32.84
Tax impact	62.52
Financial Services Impact	–28.23
Compensating variation	0.11
Total welfare cost	**67.24**

The interest payment amount is due to lenders and is therefore the direct financial cost of borrowing. But it also reflects a welfare cost measured in the consumption numeraire of the analysis. Given that agents have optimized their consumption path, the interest payment received by lenders just compensates them for their willingness to forgo the corresponding Year 0 consumption, so it is the correct welfare cost of the use of funds.

The tax impact is the forgone tax revenue. There are two ways to interpret this amount. On the one hand, forgone tax measures the indirect negative impact of borrowing on the budget, so it is a financial cost. But it also reflects forgone welfare, as the forgone tax on interest and profit represents a combination of forgone consumers' and producers' surplus of borrowers/producers and of savers/investors.

The financial services impact measures the change in the cost of the financial services needed to aggregate investors' funds and make them available to producers, taken to be 3% of the amount invested in equity. Its value is negative in this case because it reflects the value of financial sector services rendered redundant by the fall in the volume of private investments. Being a freeing-up of resources, it is a benefit.

The compensating variation cost measures the welfare change brought about by fiscal borrowing, which altered the market equilibrium. While all agents optimized their consumption path under the new circumstances, their welfare was nonetheless changed. This cost, the calculation of which is explained in the Appendix, reflects the reduction in aggregate welfare due to the state having intervened in the market, beyond what has already been directly accounted for.

Adding the values in Table 1 we get that the total welfare cost of borrowing $1,000 by the state is $67.24, which makes the SOCR equal to 6.72%.

An alternative method of computing the SOCR is shown in the *Opportunity cost* worksheet of the Excel file. It measures the expected value of the forgone pre-tax returns on the amount of displaced equity investment and costs the remaining funding requirement at the market interest rate, and then adds the adjustments for financial services impact and compensating variation. This calculation yields a SOCR of 6.79%, quite close to the first estimate. As the first method considers the portfolio reallocation of agents that results from taking the consequences of fiscal borrowing into account, it might be more accurate, and will be used in our example. It is worth repeating, however, that this paper does not aim to make realistic estimates of the alternative discount rates.

It is interesting to observe that if our model is run assuming no taxes and no equity investment opportunities then the market interest rate is 2.00%, which equals the STPR. If it becomes possible to invest in equity with no taxes, then both the market interest rate and the STPR become 9.70%. This is because even agents that borrow invest in equities, financing their investments with loans. The possibility of equity investment makes borrowers bid up the interest rate but does not introduce a breach between the market interest rate and the STPR. When fiscal borrowing takes place under these circumstances the SOCR declines to 6.80% (below the value of the STPR), because private investments are crowded out, which results in resource savings in the financial services sector. (These results can be obtained by running the model with the appropriate parameters.)

It is also interesting to note that the expected yield of equity investments has an indirect influence on the STPR as well. If we run our model returning to the original assumptions but making the expected yield of equities 10.5% instead of 10%, by increasing the probability of gains from 50% to 50.8%, we find the following in the base-case: (1) investment in equities increases by 72%; (2) lending/borrowing declines by 76%; (3) the market interest rate increases from 3.28% to 3.86%; and (4) the STPR grows from 2.62% to 3.08%. The STPR is again the weighted average of the effective returns of safe instrument utilized by agents. The Ramsey rule confirms this figure, because it is based on the increased consumption in Year 1 that results from the higher yield of equities. The SOCR increases from 6.72% to 7.21%.

Interestingly, a rule of thumb that works in our model in both of the above cases is that the averages of the risk-free interest rate and the expected return of the risky asset are very close to the computed SOCRs (6.64% and 7.17%, respectively).

In our model the SOCR exceeds the STPR, but this might not always be so. No consideration was given to the rate of reinvestment from project generated income, which would tend to lower the SOCR (Marglin, 1963). Broome (1992) points out that market interest rates ignore environmental externalities. Taking them into account could reduce the A small country borrowing abroad would not need to consider the effects of displacing private investment. Its SOCR might just be the rate of interest on its external debt. Finally, it is also unlikely that fiscal borrowing would cause significant displacement of private investments in the current monetary policy environment characterized by the widespread use of quantitative easing.

4. Alternative NPV calculations

In this Section we show the evaluation of two projects: Project A and Project B.

Project A has an investment cost of $1,000 and yields a financial return of 5% (at market prices) and an economic return of 8% (at accounting prices, because of, say, additional external benefits). We will first show three alternative ways of computing this project's financial net present value (FNPV). These correspond to the CBA calculation in the absence of any market distortion or externality. The calculations illustrate a well-known fact that is central to the argument of this paper namely that, besides providing intertemporal valuation, the act of discounting implicitly imputes costs of capital equal to the discount rate used.

Table 3
Alternative FNPV calculations

(Dollar amounts)	Year 0	Year 1
Project net flow	–1,000.00	1.050.00
Borrowing	1,000.00	
Repayment	0.00	–1,000.00
Interest	0.00	–32.84
Net flow to equity	0.00	17.16
FNPV1	16.61	
FNPV2	16.61	

The first line of Table 3 shows the net flow of the project, that is, the difference between revenues and expenditures for investment and operating costs. Notice that the interest cost is explicitly left out, even though it is a project expenditure, precisely because the act of discounting will impute it and we do not want to count it twice. Discounting this net flow at the market interest rate of 3.28% gives us our first FNPV value of $16.61.

The second FNPV is computed from the flow labeled "Net flow to equity." This is the cashflow of the project owner and equals the project net flow less the financing flow. We see that the Year 0 value is zero, as the capital cost is entirely financed, while in Year 1 the interest due is explicitly subtracted from the net flow, along with the repayment of the loan's principal. In other words, the flow to equity is the project net flow after financing.

FNPV2 = FNPV1 = $16.61, which illustrates the fact that discounting the project net flow before financing imputes the entire interest cost implicitly, expressing the result as a present value (FNPV1), using the interest rate to define intertemporal weights. The interest rate both establishes the costs of capital and discounts the future.

There is a third FNPV that can be computed, which derives directly from the definition of present value, namely, it is the sum that will compound to a given future value at the going rate of interest. FNPV3 is also $16.61, because $16.61× 1.0328 = $17.16. This shows the coherence that results from using the same rate for both functions of discounting and illustrates that the discount rate measures the opportunity cost of capital.

Dropping the assumption that there are neither price distortions nor externalities, we analyze the behavior of the SOCR and find that it also displays this coherence. In Table 4 ENPV means economic net present value. The SOCR is an interest rate expressed in the accounting prices of the CBA numeraire, that is, it reflects indirect costs of capital, not just the market interest rate.

Table 4
Alternative ENPV calculations

(Dollar amounts)	**Year 0**	**Year 1**
Project net flow	–1,000.00	1.080.00
Borrowing	1,000.00	
Repayment		–1,000.00
Cost of borrowing		–67.14
Net welfare effect	0.00	12.86
ENPV1	12.05	
ENPV2	12.05	

ENPV1 = \$12.05 is obtained by discounting the project net flow in Table 4 at the SOCR = 6.71%, and ENPV2 = \$12.05 is obtained by discounting the net welfare effect flow at the same rate.

Coherently, ENPV3 = ENPV1 will compound to the net welfare value of Year 1, as shown in Table 5. ENPV3 is first compounded at the real market interest rate of 3.28% and subsequently the indirect effects of financing are added pro-rata to the result, making the effective rate equal to the SOCR.

Table 5
Economic opportunity cost analysis

(Dollar amounts except last column)	**Year 0**	**Year 1**	**Compounding rate**
ENPV3	12.05	12.45	3.28%
Tax impact		0.75	Pro rata adjustments of items in Table 2
Financial Services Impact		–0.34	
Compensating variation		0.00	
Attained total benefit		12.86	6.71%

This coherence of discounting/compounding is exclusively possible when the discount rate both defines the costs of capital and provides the intertemporal weighting. In the no-distortions case of the first example this is natural; there is no choice. When distortions exist, however, there will be two rates: the STPR and the SOCR. The SOCR, which reflects the costs of capital, shares with the undistorted interest rate the property that it uses the same rate both to determine the costs of capital and to define intertemporal weights. Thus, it displays the coherence shown above.

Not so the STPR, or any other set of intertemporal welfare weights that are unrelated to the costs of capital. This has consequences, because in their cases:

- ENPV1 ≠ ENPV2, which means that the imputed costs of capital are incorrect, leading to incorrect NPV results. If STPR < SOCR, the costs of capital will be understated.
- ENPV3 does not exist. ENPV1 compounded to the future at the STPR will not equal the future net benefit and therefore does not comply with the definition of present value.

Table 6 compares alternative NPV calculations for Project A, the high-return project. Four alternative cashflows are discounted at the rates identified in the first column: the net flows before and after financing, at either market or CBA accounting prices (abbreviated Acc).

Table 6
Alternative NPV calculations, high-return project

Discount Rates		**Project net flow before financing (Dollar amounts)**		**Project net flow after financing (Dollar amounts)**	
		Market	**Acc**	**Market**	**Acc**
Market interest rate	3.28%	16.61		16.61	
SOCR	6.71%		12.05		12.05
STPR	2.62%		52.40		**12.53**
Zero	0.00%		80.00		12.86

We see that the same NPV is obtained from both the before and after financing flows only in the first two cases (the cases using the market rate and the social opportunity cost rate), which being descriptive in nature, use the cost of capital to define intertemporal weights.

In the case of prescriptive discount rates, including or excluding financing in the net flow makes for a significant NPV difference, which proves that such rates do not impute the correct capital costs. If a zero discount rate is used, as has often been advocated on normative grounds, it results in the total omission of capital costs from the analysis. (Notice that the computed NPV is the undiscounted aggregation of the project net flow, row 1 of Table 4).

The STPR discounted NPV imputes interest cost at accounting prices of $1,000 × 0.0262 = $26.20, rather than the actually incurred $67.14 ($1,000× 0.0671), thereby understating project costs and overstating project net benefits in the amount of $40.94. Because intertemporal preferences are not a measure the cost of capital, it follows that prescriptive discount rates cannot be used to discount project cash flows that have not accounted for costs of capital explicitly.

By the same token, the SOCR rate has nothing to do with the time preferences of society, as it is derived from the market interest rates and corrections thereof, rather than from the optimal consumption path MRS. Therefore, the SOCR rate should not be used for discounting. It understates/overstates NPV by the extent to which it is greater than/less than the STPR.

The insight that this paper offers is that the twin functions of discounting, (1) establishing costs of capital and (2) providing intertemporal weights, need not be accomplished using the same discount rate. By performing each step using the discount rate that is appropriate for it, the welfare impact of projects can be calculated correctly. This paper advocates a hybrid discounting method that takes the cost of capital defined by the SOCR but does not use the SOCR for discounting. Once the all-inclusive welfare net flow has been computed[3] it is discounted at the STPR, which is not used to calculate costs of capital.

Applying this method to the project example we see that SOCR = 6.71% defines the interest cost of $67.14, which after taking principal repayment into account leaves a net future welfare effect of $12.86 (see Table 4). Then this value is discounted at the STPR = 2.62% to yield a NPV of $12.53, which correctly reflects the welfare impact of this project, because all the welfare costs have been accounted for and the net welfare effect has been discounted at the STPR. The calculations show Project A to be welfare-enhancing, as this value is positive. All other NPV calculations on Table 6 are incorrect, some very seriously so.

The model confirms this result (see the Project A worksheet of the associated Excel file). If $12.86 of net welfare increase is distributed equally among the 58 agents in Year 1 and we compute the compensating variation in their income of Year 0, we obtain amounts of either $0.21 or $0.22 for

[3] As our example only has two time periods, the cost of capital accrues in the single future period. In real life they would accrue at various points of the project flow as a function of the actual debt service schedule, or in whichever period indirect effects are forecast to occur.

each (depending on whether they are borrowers or lenders) which aggregate to $12.53. This shows that hybrid discounting with both the SOCR and the STPR accurately predicts the welfare consequence of undertaking this project. As each rate is used for its intended purpose, the SOCR and STPR are not conflicting but complementary, and only their joint use yields the correct CBA result[4].

By implication, discounting at the STPR is in error. A NPV of $52.40 (see Table 6) requires a Year 1 net welfare of $52.40 × 1.0262 = $53.77, which as we see in Table 4 is just not available. The net welfare flow in Year 1 is $12.86, so $52.40 cannot be its present value.

The same analysis is presented in the Excel file for Project B, the low-return project, which has a financial yield of 5% and an economic yield of 5%. Table 7 shows the economic net flows that correspond to these assumptions.

Table 7

Economic net flows, low-return project

(Dollar amounts)	Year 0	Year 1
Project net flow	–1,000.00	1,050.00
Financing flow	1,000.00	–1,067.14
Net welfare effect	0.00	–17.14

The Year 1 financing cost can be quickly calculated using the SOCR, as follows: $1,000 × (1+0.0671) = $1,067.14, which leaves a net welfare effect in Year 1 of $–17.14.

Table 8 shows the results of the alternative discounting methods for Project B.

Table 8

Alternative NPV calculations, low-return project

Discount Rates		**Project Net flow before financing (Dollar amounts)**		**Project Net flow after financing (Dollar amounts)**	
		Market	**Acc**	**Market**	**Acc**
Financial market rate	3.28%	16.61		16.61	
SOCR	6.71%		-16.06		-16.06
STPR	2.62%		23.17		-16.70
Zero	0.00%		50.00		-17.14

We conclude that Project B not economically feasible because its primary net benefit is insufficient to cover its costs of capital. Its correctly calculated NPV is $–16.70. If we distribute this project's Year 1 welfare loss in the form of a lump sum tax of $0.30 to each member of society, the aggregation of their Year 0 income compensating variation will be –16.70, which is the same as its NPV when discounting the project net flow after financing by the STPR. We see again that conventional discounting of the project's primary net flow at the STPR is wrong.

Using the SOCR to correctly measure the costs of capital of any project is an indispensable first step of any proper CBA. Only thereafter can correct intertemporal comparisons be made. Serious errors can be avoided by the simple expedient of discounting flows that account for the costs of capital explicitly. The choice of STPR matters but, at least for the values of this example, the resulting differences are not large, provided that the calculation basis is correct.

[4] The NPV so obtained, because it was computed using the STPR, only serves as a hurdle value (acceptable if positive). It is not the amount that can be invested in the market in the expectation of obtaining the future value.

The NPV calculated using the SOCR is also useful by itself, however, to check the optimality of project choice and to optimize project design. If positive, project primary net benefits exceed the welfare costs of capital. Therefore, a positive value is a prerequisite of welfare optimization. It can also be used to compute the opportunity cost of the investment. If the $1,000 invested in Project B were invested instead in the capital market, a future return of $67.14 would be obtained[5]. This would result in all agents receiving $1.15 each, rather than being taxed $0.30, which would be their fate if Project B were undertaken.

Finally, we note that it is impossible to replace hybrid discounting with a single rate that yields the same NPV when applied to a project's primary net flow. Likewise, it is impossible to compute a time-independent factor that would relate the present value of a project's costs of capital to its initial capital outlay.

5. Optimality conditions

The first-order condition for optimal intertemporal resource allocation is that the MRS between present and future consumption equal the rate of transformation between them, that is, the effective rate at which present day savings can be converted into future consumption. This is the interest rate in the undistorted market case, or the after-tax effective interest rate when taxes are present.

The MRS is a locally variable property of the welfare function; it is not a constant but varies with the quantity saved. Examining the aggregate indifference curve of our modeled society (see the *Income distribution* worksheet of the Excel file) we see that for a +/– 20% deviation of Year 0 income the MRS corresponds to interest rates that range between –28% and 58%. Once agents perform their intertemporal optimization, however, their MRS will match the terms under which (expected) consumption between time periods can be traded off.

In our model, the Ramsey discount rate (which is the rate that characterizes the MRS for a pair of present and future consumptions) is 2% for the original income endowment, absent any saving or borrowing, but becomes 2.62%, the weighted average of the after-tax effective interest rates once optimization has taken place. Therefore, the MRS is not a value that can be derived from any social welfare function. Rather, it will equal the rate at which present and future consumption can be traded off.

The Ramsey rate is often used for discounting in CBAs because of the simplicity of its formulation[6]. This well-known expression is often interpreted as defining the discount rate from its remaining variables: the elasticity of marginal utility and the growth of consumption. But this is not the real direction of causation for the typical investor, for whom the interest rate is a given. The growth of consumption is the variable to optimize. Its value is the result of consumption path optimization and not an input to the problem. To illustrate with an empirical example: if we reduce the pure rate of time preference of all agents in our model from 2% to 1.5%, it does not reduce the Ramsey discount rate by a half percentage point, as its defining formula might lead one to believe, but results in a decline from 2.62% to 2.59%, because the change in parameters leads to a different optimization result, after which the Ramsey formula again holds. But the formula did not predict the discount rate[7]. Likewise, an enhanced desire of savers to provide for an uncertain future will not reduce the STPR other than through changes in the quantities saved. So, nothing is gained by tinkering with the Ramsey expression, for once agents optimize their consumption path, their MRS will equal the (distorted) rate at which they can transfer resources between periods regardless of how

[5] Running the model specifying investing $1,000, rather than borrowing it in the market yields the future return of 67.24. The slight difference results from a difference in the value of the compensating variation between the two cases.

[6] $r = \rho + \eta g$, where r is the Ramsey discount rate, ρ is the pure rate of time preference, η is the elasticity of marginal utility and g is the growth of consumption.

[7] A more useful formulation of the Ramsey equation is $g = (r\ \rho) / \eta$ because it reflects that r is given exogenously and g is the optimization result.

their welfare function is defined. This is the consequence of complying with the first-order optimality condition.

Because savers optimize by reference to effective interest rates, the STPR reflects the market distortions in place. It is, nevertheless, the hurdle rate that any investment project must meet to be welfare-enhancing because having optimized their consumption path by reference to it, it measures how savers trade-off present and future income. Therefore, a necessary condition of welfare improvement for any public sector investment is that its return exceed the STPR.

Though necessary, this condition is not sufficient as the STPR carries no information about the welfare cost of providing capital for the project. That is measured by the SOCR. If the project returns less than the SOCR, its net primary benefits are insufficient to cover its costs of capital, which leads to a welfare loss. Thus, another necessary condition for an investment to be welfare-enhancing is that its rate of return (at accounting prices) exceed the SOCR.

Therefore, for a project to be welfare-enhancing its rate of return must exceed *both* the STPR and SOCR hurdle rates. Neither condition is sufficient by itself. Jointly, however, they are.

6. Discounting the distant future

The examples discussed above are set in a one year apart two-period model. In this section we show how hybrid discounting can throw light on the key issue on which disputes about discounting center: the valuation of the distant future.

Advocates of prescriptive discounting complain that discounting by the SOCR gives insufficient weight to benefits accruing to future generations, thereby "tyrannizing them." Pigou (1932) famously characterized this as having "defective telescopic faculty." Hybrid discounting solves this problem, because it does not use the SOCR for discounting but rather the STPR, just as advocates of prescriptive discounting propose. In the following Table 9 a simple project is used to illustrate the application of hybrid discounting to long-lived projects.

Table 9
Net flow of a long-lived project
(Dollar amounts)

Years	Primary net benefits	Financing flow	Net benefits after financing
0	–100.00	100.00	0.00
100	2,008.55	–5,459.82	–3,451.26

Assume that the project's net flows are at accounting prices, with all externalities considered and, for the sake of the example, that STPR = 2.5% for year 100 (Green Book, 2020:122), that SOCR = 4% and that compounding/discounting is continuous. We can then compute the NPVs shown in Table 10.

Table 10
Alternative NPVs of a long-lived project
(Dollar amounts)

Conventional NPV @ STPR = 2.5%	64.87
Conventional NPV @ SOCR = 4%	–63.21
Hybrid NPV @ STPR = 2.5%, SOCR = 4%	–291.51
Hybrid NPV @ STPR = 0%, SOCR = 4%	–3,451.26

If the project's primary net benefits are discounted at the Green Book recommended STPR, the project has a positive NPV of \$64.87 and would receive approval. The NPV discounted at the SOCR is negative, however, which means that this project should be rejected because its benefits are not sufficient to cover its costs of capital. The correct welfare equivalent NPV, computed by the hybrid discounting method, is \$–291.51, which confirms that the correct CBA recommendation is rejection.

This example illustrates a key finding of this paper: discounting projects' primary net flows at prescriptive discount rates fails to properly account for the costs of capital and hence yields incorrect results. Trying to remedy the perceived excessive discounting of the future by lowering the conventionally used discount rate is incorrect because it comes at the price of invalidating the result. Indeed, those conventionally discounting with the STPR can be faulted for having a "defective telescopic faculty" of their own by failing to correctly account for the costs of capital. It is the hybrid NPV discounted at the super egalitarian STPR of 0%—which, setting the telescope to full magnification, values the welfare of future generations on a par with that of present ones—that illustrates the travesty that would be perpetrated on future generations by undertaking this project. They would be forced to pay \$3,451.26 in incremental taxes above the value of the benefits generated by the project[8]. Accepting this project would truly be an act of tyranny of the present over the future, born from the failure to properly account for the welfare costs of capital. The telescopic faculty metaphor supports the contention of prescriptive advocates only if the telescope trains exclusively on benefits. Once costs of capital come into its view as well, the picture changes.

Nothing prevents the CBA analyst from looking at the very distant future. The value of an infinite series of benefits can be easily ascertained. Having \$3 per annum forever has a present value of \$120, discounted at the STPR of the last example (3 / 0.025 = 120). If the investment required to generate that benefit stream is \$100 (implying an IRR of 3%) the project would clear the hurdle rate of the STPR but would not clear the hurdle rate of the SOCR (3% < 4%), again using the rate of the last example. Therefore, the project would not be welfare enhancing according to the conclusions of the previous section.

This is confirmed by computing the hybrid NPV of the project. Annual cost of capital would be \$4 (100 × 0.04 = 4) leaving a post financing net flow of \$–1, the present value of which, discounted at the STPR, is \$–40 (–1 / 0.025 = –40). Another way to look at this is to note that investing \$100 at the SOCR rate would generate an infinite stream of benefits of \$4, instead of one of just \$3. The present value of this is \$160 (4 / 0.025 = 160). The present value of the additional \$1/year foregone by investing in the project, discounted at the STPR, is \$40 (1 / 0.25 = 40), congruently the difference between the \$120 and the \$160 that we computed. This conclusion is not due to benefits in the distant future being excessively discounted, as all NPV calculations were carried out at the STPR. It shows, rather, the consequence of optimizing correctly, which prevents shortchanging future generations by \$1 per year, forever.

In the discounting debate these questions arise when discussing the economic feasibility of environmental projects. It is often argued that distant benefits are undervalued by discounting. Hybrid discounting undervalues nothing, as we have seen, but as it also accounts for costs of capital, the dilemma remains: when is it worth investing in the present for the sake of benefits in the distant future? Using a different metaphor: when would it make sense to invest in a sprinkler system that would prevent serious fire damage in the distant future? Whenever the preventive measure is so effective that compounding its cost at the SOCR would result in a future value that is less than the cost of the damage it prevents. Notice that the comparison to be made is between two alternative future amounts and not a comparison between a present cost and a future benefit. Therefore, intertemporal weighting plays no role in finding the answer to the question. What needs to be ascertained is if the proposed project is more efficient at transforming present day resources into

[8] To illustrate: savers today buy 100-year government bonds and bequeath them to their heirs. When the notes come due, future taxpayers redeem them and the heirs receive payment. From an inter-generational point of view the transfer is between the savers and their heirs, but because current day government made the wrong decision, a welfare loss is induced in the future, because the welfare cost of the taxes raised to pay for the interest due exceeds the welfare benefits of the project.

future benefits than the alternative that defines the opportunity cost of the resources it would use. The welfare of future generations would only be enhanced by investing in a project if it is more effective in converting present day resources into future benefits than the capital market. As Becker, Murphy and Topel (2011) stated: “Future generations would not thank us for investing in a low-return project.”

Some may feel that the opportunity cost of capital is a hypothetical concept, somehow less real and tangible than an incurred cost. This is not so, however, for in CBA all costs are opportunity costs, given the requirement of establishing the difference in aggregate welfare levels between the with-project and the without-project states of the world. Furthermore, at least when public projects are financed by issuing debt, the opportunity cost is the incurred cost: they are two faces of the same coin.

Investment projects incur costs of capital because the difference between the flows of benefits and costs is initially negative and this difference must be financed by someone who provides resources that have alternative uses, hence an opportunity cost. This gives rise to a stock of debt that keeps growing as long as accumulated project costs exceed accumulated project benefits. The welfare cost of providing this funding is the lenders’ opportunity cost of funds. If lenders have optimized their consumption path by reference to the capital market, their opportunity cost will be the interest/dividend income they would have earned by investing elsewhere the amount lent to the project. Therefore, the cost of capital of the project is the opportunity cost of the lenders that cover its direct and indirect funding requirements, which is the same as the social opportunity cost of devoting funds to the project.

This line of reasoning shows why the costs of capital must be computed at the SOCR using compound interest rules, as is done in hybrid discounting. It also shows that a project that is unable to cover the welfare cost of the capital it employs from its benefits is welfare destroying. This can be detected by its failure to clear both the STPR and the SORC hurdle rates, or by its yielding a negative hybrid NPV.

Costs of capital and opportunity cost of capital diverge, however, when projects are funded by direct taxation rather than debt. In such cases the hybrid NPV of a project might be positive despite its NPV at the SOCR being negative. What can be said about such projects is that while they are not welfare destroying outright, investing in them is not an optimal use of funds.

7. Conclusions

This paper extends the classic consumption path optimization model on which the discourse on discounting is based. Instead of having just one decision maker and just one savings instrument this paper uses a general equilibrium model of multiple decision makers and two savings instruments: debt and equity, the latter with stochastic returns. Besides providing experimental data for the paper and corroborating its results, the model used in this paper yields important observations for both social discount rates.

The model indicates that the SOCR estimation derived from direct and indirect financing costs of fiscal borrowing corresponds to that calculated from the forgone expected yield of displaced private sector investments. Therefore, the opportunity cost of displacing private sector investments has equivalent tangible budgetary consequences.

STPRs based on welfare maximization will always equal the weighted average of distorted interest rates because of the optimizing behavior of savers. Interestingly, the expected yield of stochastic investment opportunities only plays an indirect role in the determination of the STPR through its effect on the market interest rate. The Ramsey formula cannot compute the discount rate because the latter is not an exclusive function of its variables. A better formulation of the Ramsey discount rate expression uses consumption growth as the dependent variable, and all other parameters, including the discount rate, as independent variables, bearing in mind that the Ramsey relationship holds only after savers have optimized their consumption path by reference to the effective interest rate available to them.

The choice of discounting approach is the fundamental question in the discounting debate and is the focus of this paper. Yet, perhaps because of the impossibility of reconciling the two approaches noted by Baumol (1968), this question has not been directly addressed in the recent literature, which focused instead on the perceived problem of excessive discounting of the distant future and which, therefore, mostly follows the prescriptive approach. Ben Groom et al (2005:459) observes that it is "because we are uncertain about the long-run market rate of return that the social rate of time preference is frequently used for CBA."

This paper answers the choice of discounting-approach question by arguing that there is no need to choose between them, nor is there a need to reconcile them. Rather, they should be viewed as complementary, not competing. Both are necessary and are only jointly sufficient to achieve optimality in intertemporal resource allocation. The hybrid discounting method proposed meets the objectives of both approaches. It can accommodate any STPR, including the zero discount rate desired by some adherents of the prescriptive approach, while simultaneously taking full account of the appropriate opportunity cost of capital. It will, therefore, always calculate the correct NPV of any investment project.

Full optimality in resource allocation requires that project rates of return exceed both the STPR and the SOCR, and when the latter is greater than the former, as it is generally assumed to be the case, it is the latter that becomes the effective hurdle rate of return that projects must clear. In such circumstances many of the discount methods/rates discussed in the more recent literature and in CBA manuals will only have a secondary role. Using such rates by themselves in conventional discounting leads to erroneous results.

Significant strands of recent research have concentrated on analyzing the impact of uncertainty on discounting. It has been argued that either out of precaution, given the uncertainty of future incomes, or because of the uncertainty of interest rates (Weitzman, 1998), certainty equivalent discount rates should be declining functions of time[9]. Two statements may be in order in this regard. First, to the extent that these proposed rates constitute STPRs, they will only have a secondary role when SOCR > STPR, regardless of their merits. Second, such rates assume risk aversion in decision making, whereas it is standard CBA practice to assume risk neutrality in public sector decisions following Arrow and Lindt (1970).

The long-standing absence of consensus regarding discounting and the proliferation of proposed discount rates and methods paint a disconcerting picture that we hope this paper will help to clarify. By showing that the twin functions of discounting—establishing costs of capital and providing intertemporal weights—need not be accomplished using the same discount rate, it is possible to resolve the discounting-approach question and to formulate a simple optimality rule: for projects to be economically feasible their rate of return should exceed both the STPR and the SOCR. Following this rule will ensure that proposed public sector projects will be no less effective at converting present consumption into future consumption than what the public can already manage[10], and that the benefits of proposed projects will exceed all their direct and indirect costs.

[9] Weitzman's proposal has been shown to be based on a fallacy: see Szekeres (2020) which looks at the evidence for declining discount rates (DDR) in general, including cases of uncertainty of interest rates and future income that the present paper does not address.
[10] or what an authoritarian social planner requires.

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APPENDIX

1. Introduction

This appendix provides details of the agent-based model used to generate the numerical examples in this paper. It is best read in conjunction with the accompanying Excel workbook so as to view the results discussed, which, because of their availability there, are not presented here.

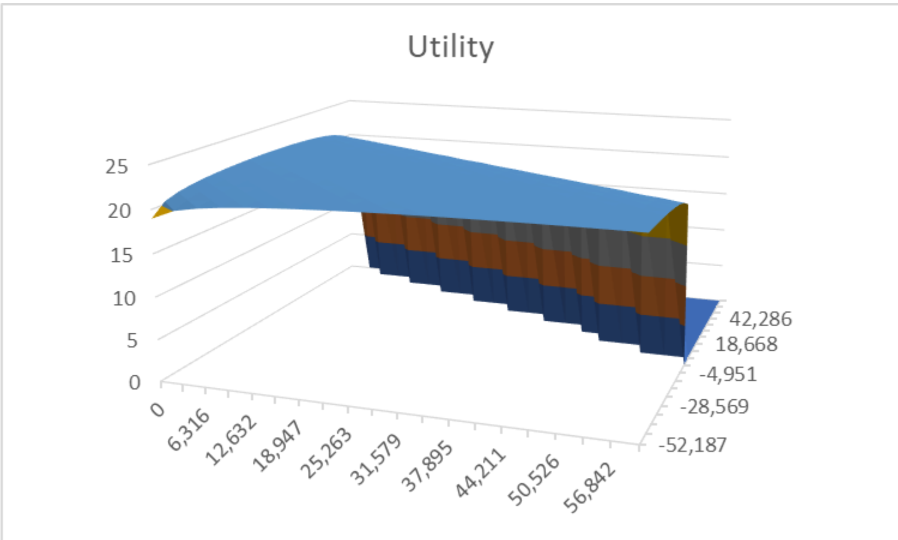
An overview of the model is that 58 agents are endowed with pairs of incomes in Years 0 and Year 1, and each finds his own optimal intertemporal resource reallocation action maximizing the welfare function described in the main text. To find the optimal course of action, the interest rate on lending/borrowing is determined endogenously as the market-clearing rate.

This requires two levels of optimization. First, for any given interest rate, the optimal welfare maximizing action must be found for each agent. If at the first tested interest rate there is a disequilibrium between the demand for loanable funds and their supply, then the interest rate must be appropriately changed, which is done iteratively until the imbalance between lending and borrowing falls below \$0.5.

The algorithms doing the optimizations are written in the MS Visual Basic language in which Excel macros are written and can be viewed by clicking on the *Visual Basic* icon in the *Developer* tab of Excel. Most of the macros are in Module 1, but there are also some in Sheets 2, 5 and 7.

The optimization macros are straightforward, without recourse to sophisticated search methods, and the code is documented, so is easy to follow. The individual agent's optimization routine evaluates the utility function at all feasible combinations of lending/borrowing and investing, arrayed in a grid of 50 rows and columns, and finds the best combination.

Figure 1
Utility of borrowing/lending and investing combinations



In Figure 1 the vertical axis shows utilities, invested amounts are shown in the breadth of the plot, and lending (negative if borrowing) in the depth.

Once the best combination of assets is found, the immediate neighborhood of the optimal value is subdivided into a 50 × 50 grid to again find the optimum. The process is repeated until the desired precision is achieved.

A simplified version of this search that has a coarser grid and operates without macros is available on the *Explore* worksheet for illustrative purposes. The utilities are computed by formulas that can be traced without any programming knowledge. (The case is simplified: it includes no taxes, and the utility is logarithmic only). By manually changing the possible ranges of risky and safe asset amounts, the reader can find the optimal allocation to any desired degree of precision. The cell containing the utility of the best combination is highlighted in orange.

The market-clearing interest rate is found by an algorithm that uses interval halving, which will stop when the market-clearing interest rate is found.

There are several additional macros, some that help audit the optimization results, and some that perform operations that are necessary to derive needed outputs. These will be described in the following sections.

2. Income distribution

The Year 0 income levels considered are shown in Column A of the *Income distribution* worksheet starting at row 4. Thirty income levels are used, growing from $20,000 in increments of $40,000 to reach $1,220,000. These are converted to natural logs in Column B. Column C contains the normal cumulative probability distribution of these log income values, with a mean of 12 and a variance of 1[11]. In Column D we have the cumulative number of agents up to the income level of the corresponding row, and finally Colum E has the number of agents for each income level. However, the number of agents so generated is only 29, with the lowest income being $60,000 and the highest $1,020,000. These are the values plotted in Figure 1 of the main text and at the top of this worksheet.

The number of agents is then doubled because half see their Year 1 income grow and half see it decline. The rates are given in Rows 1 and 2 of Column Q. There is a macro that can be launched by clicking on the button labeled *Populate* that will fill Columns P-R starting with Row 5 as appropriate, creating the required Income 0 and income growth combinations. The other parameters of the agents are computed by formulas. The pure rate of time preference and coefficient of risk aversion of each agent will be the ones assigned to the first agent, but these can be changed if desired. Income 1 is computed from the growth rate of income assigned to each agent.

This worksheet contains a second plot, that of an indifference curve. It is possible to request the indifference curve of any one of the agents, or of the entire market, or of the representative agent, by entering a number in Cell J21. The indifference curve is constructed by placing in Column G the Income 0 range of +/– 20% around the endowment value of the chosen agent in 1% steps (or the aggregation of incomes for the entire market option, or their average for the representative agent). Column H contains the similarly defined Income 1 for the endowment case (row 58). For all other rows of this column a macro, launched by clicking on the *Generate Indiff* button, finds an Income 1 value such that the total utility of it and the corresponding Income 0 value leaves the agent's utility unchanged from the endowment base-case.

Column I contains the slope defined by adjacent combinations of Income 0 and Income 1 values, which defines the MRS, and Column J contains the corresponding interest rate. Be mindful of the fact that these interest rates are only approximately correct because they correspond to arcs defined by 1% steps in Income 0.

When the representative agent option is exercised the same output is displaced to the adjacent light blue shaded area, so that this value will not be overwritten by subsequent requests. Looking at the values for the entire market and those of the representative agent, we see that the interest rates corresponding to the MRS values are identical, which means that the representative agent is indeed

[11] There is no particular reason to have adopted these values other than to obtain the skewed density function shown in Figure 1 of the main text.

representative. From the latter's values the discount rate defined by the Ramsey equation is shown in Cell O35.

3. Simulation

The *Simulation* worksheet is where simulations are launched. The first two rows specify the amount of fiscal borrowing (negative if the state buys bonds) and the parameters of the risky asset (percent of gain and loss of the invested amount and the probability of gain). The tax rate is specified, as well as the time elapsed in years between the first and the second modeled periods. This is set to 1 and it should not be changed, because higher values have not been tested yet.

Simulations are launched by clicking on the *Simulate* button. Simulations take a while, during which progress report messages appear. Usually around 40 rounds are needed to find a market-clearing interest rate. The results of the simulation appear in Columns A-C below Row 5. Aggregate values are given, as well as the capital market actions of all agents. Negative amounts of the safe asset indicate borrowings.

The equilibrium safe security price can be verified by clicking on the *Verify equilibrium* button. This launches a new equilibrium search by a different search method. It uses a hill-climbing algorithm that starts with a low price, goes in steps until it overshoots the solution, and then turns around in smaller steps. After each sign-change of the difference between saved and borrowed amounts is shown and the user is given the option of quitting or continuing.

There is a third market equilibrium algorithm as well that operates on the *Plot* worksheet. This subdivides a price search range specified by the user into 50 segments and computes, for each price, the supply and demand of loanable funds as well as the amount invested in equities and plots the results (this is the source of Figures 1 and 2 in the main text). By running this over ever-narrowing ranges, the market-clearing price can be found to any desired degree of precision. Click on *Run* to start. Runs take a while. *It is advisable to use this option to determine if a market-clearing interest rate exists when choosing a new set of model parameter values*. For some combination of values there might be no lending or borrowing and, in such cases, the primary market equilibrium finding algorithm will fail to converge, as there is nothing to converge to.

There is also the option of verifying the equilibrium of any single agent. Specify the agent number in Cell J5 on the *Simulation* worksheet and click on *Start*. This will show the 50 × 50 grid mentioned in section 1 of this Appendix, identify the optimal cell as well as the optimal row and column. All the values required to compute the utility value of the optimal combination will be presented below the grid to permit an audit of the result. To continue to zoom in on the answer with greater precision click on *Continue*. This can be done until the desired precision has been reached.

Click on *Clear* to clear the verification area.

4. Opportunity cost

The Opportunity cost worksheet is where the model's key outputs can be seen. There are three blocks of data: results of a reference case run, those of a public borrowing/lending case, and the difference between the two. In a fourth area the direct and indirect costs of fiscal borrowing/lending are calculated. To make the correct comparison, the two cases must be run independently.

First, the reference simulation without public borrowing is run, as described in the previous section. Next, on the Opportunity cost worksheet click on the *Copy* button found in the reference case area. This transcribes the results from the just finished simulation from the Simulation worksheet, including the yields of the safe and risky asset, the amount borrowed, lent or invested both globally and individually by each agent. Lending is rationed pro-rata to the extent to which lending exceeds borrowing, to ensure that they match exactly.

The routine also computes agents' expected tax payments, attained utility and MRS. The latter are analytically computed from the agents' welfare function. This procedure also aggregates Year 0 and Year 1 consumption across all agents, from which a cell formula computes the discount rate that corresponds to the Ramsey rule.

Clicking on the *Indiff rate* button computes the STPR by finding the compensating variation in Year 0 income that leaves each agent's utility unchanged after receiving $1 in Year 1. The aggregated compensating variations and amount of added Year 1 income define the social MRS, from which the computed STPR is derived.

After performing a second simulation with fiscal borrowing ($1,000 in our example), the copying procedure just described is performed for the borrowing/lending case by clicking on the homologous buttons in the area that corresponds to this case.

Once both cases have been copied, the differences in their results are automatically displayed in the third area that is labeled as CHANGES. Changes in expected tax payments and attained utility are displayed for each agent. Clicking on the *Compensating Variation* button found in the CHANGES area quantifies the welfare gain or loss suffered by each agent because of the changes induced by fiscal borrowing, which induced them to change their behavior. The aggregation of the compensations needed (negative in some cases, as some agents benefit) gives the welfare cost of fiscal borrowing that is not compensated by the interest paid.

The final panel shows how the SOCR is derived from the detected changes, as described in the main text. Cell Q8 contains the percent of equity investments that the cost of financial services represents. In our example we assumed this to be 3%.

An alternative SOCR calculation method is shown below that. In this the first line is the forgone pre-tax return of the displaced equity investments. As the displaced volume is less than fiscal borrowing, the balance is costed at the market interest rate, which is shown on the second line. The subsequent adjustments are the same as in the first SOCR estimation.

5. Project appraisal

Worksheets *Project A* and *Project B* contain the analyses pertaining to the projects. These are the sources of Tables 3-7 in the main text, where they have been explained.

Each project worksheet has a button labeled *Com Var NPV*. Clicking on this, computes for each agent the equivalent compensating variation in Year 0 that leaves his utility on a par with receiving, in Year 1, his share (1/58) of the net future welfare effect of the project in question (Cell C5). The aggregation of these compensating variations is the welfare equivalent NPV of the project. It is the same as discounting the net welfare effect by the STPR.

The individual compensating variation amounts are shown, as well as the interest rate corresponding to each agent's MRS.