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# Patterns of Competitive Interaction

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## Abstract

We explore patterns of price competition in an oligopoly where consumers vary in the set of firms they consider for their purchase and buy from the lowest-priced firm they consider. We study a pattern of consideration, termed “symmetric interactions”, that generalises models used in existing work (duopoly, symmetric firms, and firms with independent reach). Within this class, equilibrium profits are proportional to a firm’s reach, firms with a larger reach set higher average prices, and a reduction in the number of firms (either by exit or by merger) harms consumers. However, increased competition (either by entry or by increased consumer awareness) does not always benefit consumers. We go on to study patterns of consideration with asymmetric interactions. In situations with disjoint reach and with nested reach we find equilibria in which price competition is “duopolistic”: only two firms compete within each price range. We characterize the contrasting equilibrium patterns of price competition for all patterns of consideration in the three-firm case.

**Keywords:** Price competition, consideration sets, price dispersion, entry and merger.

## 1 Introduction

We study oligopoly pricing in a setting where consumers differ in the set of firms they consider for their purchase, and who buy from the firm in their consideration set with the lowest price. Bertrand equilibrium then typically involves firms choosing their prices according to mixed strategies, and a firm chooses from a *range* of prices. The pattern of price competition could take many forms. Firms might all choose from a similar range of prices, or competition might be more segmented with only a small subset of firms competing in a given price range. Who competes with whom at each price is determined

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in equilibrium. How does the equilibrium pattern of price competition depend on the underlying pattern of consumer consideration?

The simplest situation in which this question arises is a duopoly in which each firm has some captive customers, while contested customers are able to buy at the lower of the two firms' prices. A firm then has choice between "fighting", by competing against its rival for the contested consumer segment with a low price, or "retreating" towards its captive base by setting a high price, and in equilibrium these strategies yield the same profit. Even when the firms have different numbers of captive customers they use the same interval range of prices. With more than two firms, though, richer patterns of consumer consideration become possible. Taking the interaction between two firms to be the overlap in the sets of consumers who consider buying from them relative to their reaches, different pairs of firms might have very different strengths of interaction. With more than one rival a firm can compete on several fronts, and richer patterns of pricing also emerge. With a segmented pricing pattern, for instance, a firm might compete against one firm when it charges a low price and another firm when it charges a higher price. Changes in the intensity of competition in one consumer segment might affect the prices offered to other segments, as some firms retreat to their captive base or decide to fight harder for contested customers.

The foundation of our model is the distribution of consideration sets among consumers. There are various reasons why different consumers have different sets of choices open to them. Different consumers might exogenously be better informed about market options than others, for instance because they are "local" (Rosenthal (1980), Varian (1980)). Informative advertising might mean that some consumers become aware of a different set of suppliers than other consumers (Butters (1977), Ireland (1993), McAfee (1994)). Likewise, consumers have different propensities to search (Stahl (1996)), or the search process might be "noisy" and lead different consumers to become aware of different numbers of suppliers (Burdett and Judd (1983)). As in Spiegler (2006), there might be (extreme) horizontal product differentiation such that only a subset of products meet a consumer's needs. Consumers might differ in their ability to make comparisons between offers, and suppliers might make it hard to compare prices, with confused consumers choosing randomly between suppliers or buying from a default seller (Piccione and Spiegler (2012), Chioveanu and Zhou (2013)). Other natural sources of heterogeneity in consumer consideration sets include the

possibility that consumers have different numbers of sellers located near to them, and that different price comparison websites list prices from different sets of sellers.

There is a large empirical literature that documents partial and heterogeneous awareness by consumers of their market options. For instance, see Sovinsky Goeree (2008) for personal computers, Draganska and Klapper (2011) for ground coffee, Giuliatti, Waterson, and Wildenbeest (2014) and Hortacsu, Madanizadeh, and Puller (2017) for electricity, Honka, Hortacsu, and Vitorino (2017) for retail banking, De los Santos, Hortacsu, and Wildenbeest (2012) for online books, and Ho, Hogan, and Scott Morton (2017) for medical insurance. For a recent overview of this body of work, see Honka, Hortacsu, and Wildenbeest (2019). Much of this work takes the supply side as given, and estimates how consumers form their consideration sets and make their choices within them given pricing (and other) strategies of firms. Our paper takes the complementary approach, and focusses on how firms choose their prices for a given pattern of consideration by consumers.

There is also ample evidence about the prevalence of price dispersion in markets with a homogeneous product, both across sellers and across time. (Varian (1980) refers to these respectively as “spatial” and “temporal” forms of price dispersion.) For instance, Kaplan and Menzio (2015) document price dispersion for many homogenous retail products in the bricks and mortar context, and within a three month time period the average normalized standard deviation in the prices paid for given good lies between 19% and 36%, depending on how finely defined are the products. They show that 10% of this variance is due to variation in the average expensiveness of stores, and the remaining variance is approximately equally due to differences in the price of the good sold across similarly expensive stores and to differences in the price of the good over time at the same store.

Our model assumes products are homogeneous, in which case the static Bertrand equilibrium typically involves some of the firms using mixed strategies for their choice of price.<sup>1</sup> This is unlikely to be *literally* true in many markets, although some markets approximate this behaviour over time in the sense that firms engage in temporal price dispersion (as documented by Kaplan and Menzio (2015)), where the identity of lowest price firm changes quite frequently (Lach (2002), Baye, Morgan, and Scholten (2004), Giuliatti, Waterson, and

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<sup>1</sup>Myatt and Ronayne (2019) present a model with homogeneous products that has deterministic pricing in equilibrium. Even with a symmetric pattern of consideration these deterministic prices differ from firm to firm, so there is “stable” price dispersion. Their model has two pricing stages: first each firm announces a “list price”, and then in the second stage firms can adjust this price *downwards* if they wish.

Wildenbeest (2014)). Varian (1980) suggests that temporal price dispersion may be more plausible than spatial dispersion, since in the latter case consumers might be expected to learn over time which were the expensive firms.<sup>2</sup> Over a one-month period, Figure 4 in Kocas and Bohlmann (2008) describes the empirical CDFs for normalized prices used by the fourteen widest-range booksellers on a price comparison site over a 26 day period. This reveals the wide variety of discounting strategies used by firms over this period, with some sellers choosing fairly uniform prices and others offering very dispersed prices.

Our analysis does not take a view on the underlying reason why consumers have different consideration sets. Rather, it takes the distribution of consideration sets in the consumer population as given, and explores the consequences for competition. A considerable literature has explored instances of this general framework, and some patterns of consideration are now well understood: (a) the case with symmetric firms; (b) the case with independent reach, and (c) the “one-or-all” case where consumers are either captive to one firm or can choose between all firms so there are no “partially informed” consumers. Within case (a), which covers the great majority of existing models, Rosenthal (1980) and Varian (1980) considered the situation in which some consumers are randomly captive to particular firms, while others compare the prices of all firms and buy from the cheapest. (Thus these papers also fall under case (c).) Burdett and Judd (1983, section 3.3) analyze a more general symmetric model, in which arbitrary fractions of consumers consider one random firm, two random firms, and so on. In this framework there exists a symmetric equilibrium, and Johnen and Ronayne (2020) show that it is the unique equilibrium if and only if there are some consumers who consider precisely two firms. Bergemann, Brooks, and Morris (2020) extend Burdett and Judd’s model so that firms have information about the number of firms a consumer considers. This information might be public, so all firms see the same information about a consumer, or different firms might have different information, and the authors derive the information structure that maximizes industry profit.<sup>3</sup>

In case (b) with independent reach, the fact that a consumer considers a given firm does not affect the likelihood she considers any other firm. Then the firm that reaches the most consumers also has the largest proportion of captive consumers among the consumers within its reach—i.e., it has the highest captive-to-reach ratio. This model was studied by Ireland (1993) and McAfee (1994), who found an equilibrium in which all firms use

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<sup>2</sup>Salop and Stiglitz (1977) present a model similar to Varian’s which involves spatial price dispersion.

<sup>3</sup>See Armstrong and Vickers (2019) for related analysis in the context of duopoly.

the same minimum price (i.e., have the same minimum price in their price supports), but the maximum price charged is lower for firms with a smaller reach. Thus price supports are nested, so that smaller-reach firms only offer low prices while more ubiquitous firms offer the full range of prices. Since firms use the same minimum price, their profits are proportional to their reach.<sup>4</sup>

Case (c), where consumers either consider just one firm or consider the entire set of firms, was fully solved by Baye, Kovenock, and De Vries (1992). (This framework includes duopoly as a special case, which was studied by Narasimhan (1988).) In the symmetric version of the model (which coincides with the models of Rosenthal and Varian), when there are more than two firms many asymmetric equilibria exist alongside the symmetric equilibrium. When firms have different numbers of captive customers, all but the two smallest firms choose the monopoly price for sure, while the two smallest firms compete using mixed strategies. Intuitively, the two firms with the fewest captive customers have the strongest incentive to fight, leaving firms with more captives with an incentive to retreat to their captive base. This is an extreme instance of the situation where some firms choose only high prices, which will feature in our analysis later.

While these three special cases are natural benchmarks, in practice patterns of consumer consideration will fall outside these cases. For example, in their study of ground coffee Draganska and Klapper (2011) document in their sample that firms are not close to being symmetric, that consumer awareness is far from independent across brands, and that the consideration sets of many consumers consisted neither of a single firm nor of the whole set of firms. Kocas and Bohlmann (2008) show that firms with similar reach often use very different pricing strategies, which is hard to reconcile with these existing models of consumer consideration. Kocas and Bohlmann (2008) also present a triopoly model that falls outside these three cases (a) to (c). Specifically, as well each firm having captive customers and there being some consumers who consider all three firms, there is a segment of consumers who consider only the two largest firms. (This is a special case of our triopoly analysis in section 5.) Within this setup they sometimes find equilibria of a segmented form, whereby one firm only sets low prices, one firm only sets high prices, and the third firm prices across the whole range. This “duopolistic” pricing pattern is strikingly different from

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<sup>4</sup>This equilibrium was subsequently shown by Szech (2011) to be unique. Spiegel (2006) studies the special case of this framework where all firms are equally likely to be considered (which therefore also fits into case (a) with symmetric firms). In her empirical study of the computer market, Sovinsky Goeree (2008) assumes that the reach of the various products is independent.

that seen in the rest of the literature, and it will feature frequently in our own analysis.

The aim of the present paper is to provide a unifying framework that encompasses patterns (a) to (c) as special cases, but which allows us to study richer situations outside these cases as well, and to discover new types of equilibrium interaction. The analysis is organized as follows. In section 2 we present our general framework of consideration sets, and formalize the notion of “interaction” between sets of firms that captures the relevant measure of correlation between the reaches of firms. In section 3 we introduce a pattern of consumer consideration, which we term “symmetric interactions”, which includes (a) to (c) as special cases. With symmetric interactions, the probability that a consumer considers firm  $i$  given that she considers firm  $j$  does not depend on the identity of  $j$ . In this case the unique equilibrium pattern of prices resembles that seen with independent reach, i.e., there is an increasing sequence of prices  $\{p_i\}$ , and the firm with the  $i$ th smallest reach uses prices in the interval  $[p_0, p_i]$ . In particular, all firms use the same minimum price  $p_0$ , profits are proportional to a firm’s reach, and firms with a larger reach stochastically choose higher prices. When the set of firms is reduced, either by exit or by merger, the profits of remaining firms rise and consumers overall are harmed. Enhanced consumer awareness is shown to have mixed effects on consumers overall, depending on which consumer types expand their consideration sets.

Entry by a new firm also has mixed effects on consumers. While entry often pushes down prices, there are natural patterns of interaction where, counter-intuitively, the opposite happens and consumers are harmed by entry. The reason is that more intense competition for contestable consumers induces incumbents to retreat towards their captive base. A related phenomenon was noted by Rosenthal (1980). The insight that entry might raise an incumbent’s price, and perhaps lower consumer surplus, has more often been discussed in the context of markets with differentiated products. For instance, entry by generic pharmaceuticals might cause a branded incumbent to raise its price, as it prefers to focus on those customers who care particularly about its brand. Chen and Riordan (2008) show in a discrete choice framework how entry to a monopoly market can induce the incumbent to raise its price, and they also find situations (including a standard Hotelling setup) where consumer surplus falls with entry despite the increased “variety” enjoyed by consumers.

In the remainder of the paper we study patterns of consideration beyond symmetric interactions. In section 4 we analyse two situations—disjoint reach and nested reach—that

have asymmetric interactions. To illustrate disjoint reach we discuss competition between a chain store and a number of disjoint local rivals. In this case, the chain store uses prices from the whole market range,  $[p_0, 1]$ , and different local firms choose their prices from disjoint intervals within  $[p_0, 1]$  (where local firms with greater interaction with the chain store use lower prices). Thus there is “duopolistic” price competition, and the chain store competes against a single rival at a given price. With nested reach, only the firm with the largest reach has any captive customers, and firms with larger reach chooses prices from a higher range. If the increments between successive firm reaches are non-decreasing we find equilibria with a different form of duopolistic pricing which we term “overlapping duopoly”: there is an increasing sequence of prices  $\{p_i\}$  such that the firm with the  $i$ th smallest reach uses prices in the interval  $[p_{i-1}, p_{i+1}]$ . Hence small firms charge only low prices while large firms charge only high prices, and firms compete only against their immediate neighbours in the nest.<sup>5</sup>

In section 5 we provide a general analysis of the three-firm case. When interactions between pairs of firms are similar, as in section 3, all firms use a common lowest price. In some of these cases, however, we find that the price support of the least competitive firm need not be an interval—the firm might price high or low but not in an intermediate range. By contrast, when one pair of firms has significantly more interaction than other pairs, the equilibrium exhibits the duopolistic pricing seen in section 4—one firm prices low, one high, and one across the full price range. Intuitively, this pair mostly compete with each other, leaving the remaining firm with an incentive to set high prices. While a profitable merger in a three-firm market will always harm consumers (as with symmetric interactions), there are situations where a profitable merger between two firms with a strong interaction can reduce industry profit. The reason is that such a merger opens up a profitable front for the non-merging firms, and induces these firms to fight rather than retreat.

We conclude in section 6 by summarizing the main insights derived from our analysis, and suggesting avenues for further research on this topic.

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<sup>5</sup>A similar pattern of overlapping duopoly pricing is seen in Bulow and Levin (2006). They study a matching model where  $n$  heterogeneous firms each wish to hire a single worker from a pool with  $n$  heterogeneous workers, where the payoff from a match is (in the simplest version of their model) the product of qualities of the firm and worker. In equilibrium, firms offer wages according to mixed strategies, where higher quality firms offer wages in a higher range than lower quality firms.



## 2 A model with consideration sets

There are  $n \geq 2$  firms that costlessly supply a homogeneous product. There is a population of consumers of total measure normalized to 1, each of whom has unit demand and is willing to pay up to 1 for a unit of the product.<sup>6</sup> Consumers differ according to which firms they consider for their purchase, and for each subset  $S \subset N \equiv \{1, \dots, n\}$  of firms the fraction of consumers who consider exactly the subset  $S$  is  $\alpha_S$ . (We slightly abuse notation, and write  $\alpha_1$  rather than  $\alpha_{\{1\}}$  for the fraction who consider only firm 1,  $\alpha_{12} = \alpha_{21}$  for the fraction who consider only firms 1 and 2, and so on.) When there are only few firms the pattern of consideration sets can be illustrated using a Venn diagram, and Figure 1 depicts a market with three firms.<sup>7</sup> Here, a consumer considers a particular subset of firms if she lies inside the “circle” of each of those firms. A consumer is *captive* to firm  $i$  if she considers  $i$  but no other other firm, so there are  $\alpha_i$  such consumers.

It is also useful also to write  $\sigma_S$  for the fraction of consumers who consider all firms in  $S \subset N$  and possibly other firms too. Formally

$$\sigma_S = \sum_{S' | S \subset S'} \alpha_{S'} , \quad (1)$$

so that with the three firms on Figure 1 we have  $\sigma_{12} = \alpha_{12} + \alpha_{123}$ , say. Thus  $\sigma_S \geq \alpha_S$ , with equality for  $S = N$ , and  $\sigma_S$  weakly decreases with  $S$  in the set-theoretic sense.<sup>8</sup> The *reach* of firm  $i$  is the set of consumers who consider the firm, i.e., the firm’s potential customers,

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<sup>6</sup>The equilibrium analysis that follows is not affected if each consumer has a downward-sloping demand function  $x(p)$ , provided revenue  $px(p)$  is an increasing function up to the monopoly price. (Since there is a decreasing relationship between a firm’s profit and the resulting consumer surplus, one can imagine firms as competing to offer the lowest profit to consumers.) However, welfare would then no longer depend just on the total number of consumers served but also on the price they pay, and any welfare analysis (e.g., in our discussion of mergers and entry) requires adjustment with downward-sloping demand. However, in those cases in section 5 where entry raises all prices, it is clear that consumer surplus will fall regardless of the shape of consumer demand.

<sup>7</sup>In a spatial context this Venn diagram has a more literal interpretation: if consumers consider buying from any firm within a specified distance, then the locations of firms determine the centre of the circles on the diagram. In a very different context, Prat (2018) uses a model of consideration sets similar to that presented in this paper.

<sup>8</sup>One can invert (1), and the  $\{\alpha_S\}$  that induce a given feasible  $\{\sigma_S\}$  are given by

$$\alpha_S = \sum_{S' | S \subset S'} (-1)^{|S'/S|} \sigma_{S'} ,$$

where  $|S'/S|$  denotes the number of extra firms in  $S'$  relative to  $S$ . However, not all  $\{\sigma_S\}$  that decrease with  $S$  in the set-theoretic sense can be implemented by some  $\{\alpha_S\}$ , i.e., the above formula sometimes leads to negative  $\alpha_S$ . For example, the situation with  $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_{12} = \sigma_{13} = \sigma_{23} > 0$  and  $\sigma_{123} = 0$  cannot be induced with any pattern of consideration  $\{\alpha_S\}$ .

and the fraction of such consumers is  $\sigma_i$ . Finally, the *captive-to-reach* ratio of firm  $i$  is denoted  $\rho_i \equiv \alpha_i/\sigma_i$ .

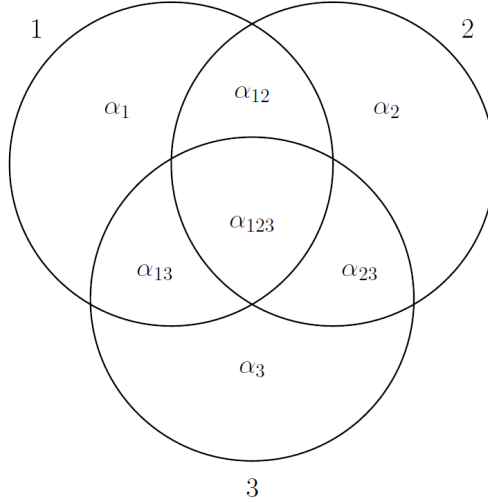


Figure 1: Consideration sets with three firms

Firms compete in a one-shot Bertrand manner, and a consumer buys from the firm she considers that has the lowest price (provided this price is no greater than 1). In particular, a firm offers a uniform price to consumers, and cannot make its price to a consumer contingent on her consideration set. Thus if firm  $i$  sets a price strictly below all its rivals it sells to its entire reach and its demand is  $\sigma_i$ , while if it sets a price strictly above all rivals it sells only to its captive customers and its demand is  $\alpha_i$ . If two or more firms choose the same lowest price, we suppose that the consumer is equally likely to buy from any such firm (although the details of the tie-breaking rule make no difference to the analysis). Since industry profit is a continuous function of the vector of prices chosen, Theorem 5 in Dasgupta and Maskin (1986) shows that an equilibrium exists. Since an individual firm's profit is usually discontinuous in the price vector, the equilibrium will usually involve mixed strategies for some firms.

We make two running assumptions to rule out some extreme and uninteresting configurations. The first requires that there be some competition between firms:

**Assumption 1:** Some consumers consider at least two firms.

(If all consumers were captive, each firm chooses  $p \equiv 1$  for sure.) The second assumption prohibits the possibility that a subset of firms choose the competitive price  $p \equiv 0$  for sure, as such firms play no important role in the analysis:

**Assumption 2:** Every non-empty subset of firms  $S$  contains at least one firm with consumers within its reach who consider no other firm in  $S$ .

For instance, this assumption rules out the situation where two firms reach precisely the same set of consumers. Intuitively, Assumption 2 ensures that no subset  $S$  of firms will set  $p \equiv 0$ , since there is a firm in  $S$  that has some customers with no overlap with other firms in  $S$ , and this firm can profitably raise its price above zero. These two assumptions together imply that there is no equilibrium in pure strategies. As well as these two running assumptions, we sometimes adopt a third assumption:

**Assumption 3:**  $\alpha_{ij} > 0$  for all  $i$  and  $j$ .

This assumption states that each pair of firms has some mutually contested customers, regardless of the prices chosen by other firms. The assumption is satisfied with independent reach, but with more than two firms it rules out the “all-or-one” pattern of consideration.

Suppose that firm  $j \in N$  chooses its price (independently) according to the cumulative distribution function (CDF)  $F_j(p)$ . These strategies then induce a demand function  $q_i(p)$  for firm  $i$ , which is the expected fraction of consumers who buy its product when it chooses price  $p$ . (We present explicit formulas for  $q_i$  shortly.) Equilibrium occurs when for each firm  $i$  there exists a profit level  $\pi_i$  and a CDF  $F_i(p)$  for its price with such that firm  $i$ 's expected profit  $pq_i(p)$  is equal to  $\pi_i$  for every price in the support of  $F_i$  and no higher than  $\pi_i$  for any price outside its support.<sup>9</sup>

It is also useful to introduce the notation  $q_{ij}(p)$ , which is the probability that a consumer considers both firms  $i$  and  $j$  and that every other firm she considers has price at least  $p$ . This represents the fraction of consumers contested by these two firms at price  $p$ , i.e., the increase in its demand if firm  $i$  slightly undercuts firm  $j$  at price  $p$ . (Just as the regular demand function  $q_i$  satisfies  $\alpha_i \leq q_i(p) \leq \sigma_i$ , here we have  $\alpha_{ij} \leq q_{ij}(p) \leq \sigma_{ij}$ .) Using this notation, say that in equilibrium “firms  $i$  and  $j$  compete at price  $p$ ” if  $p$  lies in both firms’ price supports and  $q_{ij}(p) > 0$ . If the two firms have disjoint reach then they do not compete even if they happen to use the same price. However, if Assumption 3 holds then  $q_{ij}(p)$  is always positive, in which case firm  $i$  competes with  $j$  at  $p$  whenever the price lies in both supports.

The following result collects a number of preliminary observations about the structure of

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<sup>9</sup>As usual, the support of firm  $i$ 's price distribution is defined to be the smallest closed set  $\mathcal{P} \subset [0, 1]$  such that the probability the firm chooses a price in  $\mathcal{P}$  equals one.

price competition in equilibrium, some of which are familiar from the existing literature.<sup>10</sup>

**Lemma 1** *In any equilibrium:*

- (i) *firm  $i$  obtains profit  $\pi_i \geq \alpha_i$ , with equality for at least one firm, and the minimum price in its support is no smaller than  $\rho_i$ ;*
- (ii) *the minimum price ever chosen in the market,  $p_0$ , is strictly positive, and firm  $i$  obtains profit  $\pi_i \geq \sigma_i p_0$ ;*
- (iii) *each firm's price distribution is continuous (that is, has no "atoms") in the half-open interval  $[p_0, 1]$ ;*
- (iv) *at least two firms compete at each price in the interval  $[p_0, 1]$ ;*
- (v) *if there are three or more firms, there is at least one price that lies in the support of three or more firms, and*
- (vi)  *$p_0$  lies weakly between the second lowest  $\rho_i$  and the highest  $\rho_i$ . If the firm with the highest  $\rho_i$  has  $p_0$  in its support then  $p_0$  is equal to the highest  $\rho_i$ .*

**Proof.** All proofs are contained in the appendix. ■

Part (iii) of the lemma shows that ties do not occur with positive probability at any price  $p < 1$ . Moreover, there cannot be ties with positive probability even at  $p = 1$  between firms that compete at this price.<sup>11</sup> When firm  $i$  chooses price  $p \leq 1$  it will sell to a consumer when that consumer is within its reach and when none of the other firms the consumer considers offers a lower price. Therefore, when rival firms  $k \neq i$  choose price according to the CDF  $F_k(p)$ , firm  $i$ 's expected demand, i.e., the fraction of consumers who buy from  $i$ , with price  $p$  is

$$q_i(p) \equiv \sum_{S|i \in S} \alpha_S \left( \prod_{k \in S/i} (1 - F_k(p)) \right). \quad (2)$$

Here, the sum is over all consumer segments that consider firm  $i$ , and for each such segment the product is over all rivals for firm  $i$  in that segment.<sup>12</sup> (We use the convention that a product taken over the empty set, i.e., when  $S = \{i\}$  in (2), equals 1.) For a given rival

<sup>10</sup>For instance, see McAfee (1994, page 28).

<sup>11</sup>If  $q_{ij}(1) > 0$  then each of them would obtain a discrete jump in demand if it reduced its price slightly from 1, which cannot occur in equilibrium. So the only way both can have an atom is if  $q_{ij}(1) = 0$ , in which case a firm's demand is not affected if its rival slightly undercuts it at  $p = 1$ .

<sup>12</sup>Further to footnote 6, if each consumer had downward-sloping demand  $x(p)$ , (2) would continue to express the proportion of consumers that buy from  $i$ , though not quantity demanded. Thinking of  $p$  as profit per consumer served, the following analysis would nevertheless apply apart from welfare analysis.

firm  $j$ , by partitioning those sets  $S$  containing  $i$  into sets which also contain  $j$  and those which do not, one can decompose firm  $i$ 's demand in (2) as

$$q_i(p) = q_{i/j}(p) - F_j(p)q_{ij}(p) , \quad (3)$$

where  $q_{i/j}(p)$  is firm  $i$ 's demand in the hypothetical situation where firm  $j$  has exited the market, which is

$$q_{i/j}(p) = \sum_{S|i,j \in S} [\alpha_S + \alpha_{S/j}] \left( \prod_{k \in S/\{i,j\}} (1 - F_k(p)) \right) , \quad (4)$$

while  $q_{ij}(p)$  is  $i$  and  $j$ 's "overlap demand" previously discussed, which is

$$q_{ij}(p) \equiv \sum_{S|i,j \in S} \alpha_S \left( \prod_{k \in S/\{i,j\}} (1 - F_k(p)) \right) . \quad (5)$$

It is often more convenient to express a firm's demand in terms of the  $\sigma_S$  rather than the  $\alpha_S$  parameters, and (2) can be re-written as

$$q_i(p) = \sum_{S|i \in S} (-1)^{|S|-1} \sigma_S \left( \prod_{k \in S/i} F_k(p) \right) . \quad (6)$$

Here,  $(-1)^{|S|-1}$  is plus or minus 1 according to whether  $i$  has an even or odd number of rivals in the given segment. The equivalence of the expressions (2) and (6) can be seen by comparing terms: for each subset  $S$  containing firm  $i$  the term  $\prod_{k \in S/i} F_k$  appears in (2) for each set  $S'$  such that  $S \subset S' \subset N$ , and each time with the same sign  $(-1)^{|S|-1}$ , and so has total weight  $(-1)^{|S|-1} \sigma_S$  in (2), just as in (6).

Finally, expression (6) can be re-written as

$$\frac{q_i(p)}{\sigma_i} = \sum_{S|i \in S} (-1)^{|S|-1} \gamma_S \left( \prod_{k \in S/i} G_k(p) \right) , \quad (7)$$

where we define

$$G_i(p) \equiv \sigma_i F_i(p)$$

and

$$\gamma_S \equiv \frac{\sigma_S}{\prod_{i \in S} \sigma_i} . \quad (8)$$

Thus,  $G_i(p)$  is the probability a consumer considers firm  $i$  and is offered a price below  $p$  from that firm. We say that  $\gamma_S$  is the *interaction* between firms in the subset  $S \subset N$ . These

interaction parameters play a major role in the following analysis, and capture patterns of correlation (or “commonality”) in the reaches of subsets of firms. Clearly,  $\gamma_i \equiv 1$ . The reaches of two firms  $i$  and  $j$  are positively correlated if  $\sigma_{ij} \geq \sigma_i \sigma_j$ , i.e., if  $\gamma_{ij} \geq 1$ , while if two firms have disjoint reach then  $\gamma_{ij} = 0$ . From this perspective, the pattern of consumer consideration is characterized by the reaches,  $\sigma_i$ , together with the way these reaches overlap as captured by the interaction parameters,  $\gamma_S$ .

The patterns of consideration that have been examined in the existing literature, discussed in the introduction, can be interpreted using this notation. With symmetric firms, each of the parameters  $\alpha_S$ ,  $\sigma_S$  and  $\gamma_S$  depends only on the number of firms in  $S$ , not their identity. (Here, the  $\gamma_S$  parameters could be above or below 1.) With independent reach, where a consumer considers firm  $i$  with independent probability  $\sigma_i$ , we have

$$\alpha_S = (\prod_{i \in S} \sigma_i) (\prod_{j \notin S} (1 - \sigma_j)) ; \sigma_S = \prod_{i \in S} \sigma_i ; \gamma_S \equiv 1 ,$$

and (7) factorizes to

$$\frac{q_i(p)}{\sigma_i} = \prod_{k \in N/i} (1 - G_k(p)) . \quad (9)$$

With “all-or-one” consideration,  $\alpha_N > 0$  consumers consider all  $n$  firms and  $\alpha_i$  consumers consider only firm  $i$ , so the reach of firm  $i$  is  $\sigma_i = \alpha_N + \alpha_i$  and  $\sigma_S = \alpha_N$  whenever  $S$  includes more than a single firm. The two smallest firms have the greatest pairwise interaction term  $\gamma_{ij}$ . With duopoly, demand in (7) satisfies

$$\frac{q_i(p)}{\sigma_i} = 1 - \gamma_{12} G_j(p) , \quad (10)$$

while with three firms demand (7) satisfies

$$\frac{q_i(p)}{\sigma_i} = 1 - \gamma_{ij} G_j(p) - \gamma_{ik} G_k(p) + \gamma_{123} G_j(p) G_k(p) . \quad (11)$$

At various points in the remainder of the paper we will discuss the impact of entry, exit and mergers on outcomes. We model entry by a new firm as the introduction of a new “circle” superimposed onto the existing Venn diagram. That is, entry does not affect which consumers consider which incumbent firms, and the reach of an incumbent firm is unaffected by entry, although its number of captive customers will weakly fall.<sup>13</sup> More generally, entry does not alter the interaction parameters  $\gamma_S$  in (8) between sets of

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<sup>13</sup>In particular, there is no danger of “choice overload”, whereby the number of consumers who compare prices falls when there are more firms, as discussed for instance in Spiegler (2011, page 150).

incumbents. Since welfare—consumer surplus plus industry profit—is the total number of consumers reached, it follows that entry (if it is costless) will increase welfare by the number of captive customers of the entrant. For this reason, if entry reduces industry profit it will benefit consumers. Since the entrant’s contribution to welfare (its captive base) is a lower bound on its profit, the external impact of entry on incumbent profit plus consumer surplus is weakly negative. Exit by a firm is just the removal of a “circle”, and leaves the interactions  $\gamma_S$  between the remaining firms unchanged, and weakly reduces welfare. If exit increases industry profit, it will harm consumers.

Mergers also have a natural set-theoretic interpretation in this framework: when two or more firms merge we assume that the merged entity sets the same price to all its customers, and that the reach of the merged entity is the union of the reaches of the separate firms.<sup>14</sup> Thus, a merger has no impact on welfare (if there are no accompanying cost synergies), and harms consumers if and only if it increases industry profit. The fraction of consumers reached by the merged firm, which is  $\sigma_i + \sigma_j - \sigma_{ij}$ , is no greater than the sum of those reached by the separate firms,  $\sigma_i + \sigma_j$ , while the captive base of the merged firm,  $\alpha_i + \alpha_j + \alpha_{ij}$ , is no smaller than the sum of captives of the separate firms,  $\alpha_i + \alpha_j$ . A merger does not alter the interactions  $\gamma_S$  between sets of non-merging firms.

### 3 Symmetric interactions

In this section we suppose that interactions between sets of firms in (8) are *symmetric*, in the sense that  $\gamma_S$  depends only on the number, not the identity, of firms in  $S$ . The familiar cases of symmetric firms, independent reach and duopoly all fall within this more general class of consideration structures. In essence, this class combines the flexibility of the symmetric firm framework, which allows the distribution of the number of firms that are considered to be chosen freely, with the flexibility of the independent reach framework, which allows for asymmetric firms where firm reaches can be chosen freely.<sup>15</sup>

We next describe two natural ways to obtain symmetric interactions that go beyond

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<sup>14</sup>An alternative approach would be for the merged entity to maintain separate brands and to be able to charge distinct prices for each brand.

<sup>15</sup>To illustrate, when there are three firms as on Figure 1, patterns of consumer consideration are described by seven parameters (the number of segments in the Venn diagram). If interactions are constrained to be symmetric the three pairwise interactions  $\gamma_{ij}$  must be equal, and so two degrees of freedom are lost leaving five degrees of freedom remaining (the three reaches plus the pairwise interaction and the three-firm interaction). The cases of symmetric firms and independent reach each have only three degrees of freedom.

those patterns of consideration studied in the literature. The first way can be used to introduce correlation to a scenario with independent reach, while the second can be used to introduce asymmetry into a scenario with symmetric reach.

*Consumers differ in their attentiveness:* Here, in order for a consumer to consider a firm she needs to be sent a message from that firm and to pay attention to this message. Suppose that the pattern of messages is such that a fraction  $\tilde{\sigma}_i$  of consumers are sent a message from firm  $i$ , a fraction  $\tilde{\sigma}_{ij}$  are sent messages from both firm  $i$  and firm  $j$  (and possibly other firms too) and so on, and let  $\tilde{\gamma}_S$  denote the corresponding interaction term for messages akin to (8). A consumer with attentiveness parameter  $a$  pays attention to a message she is sent with probability  $a$ , independently across all messages she is sent. (For instance,  $a$  might measure how much “media”, across which marketing messages are delivered, the consumer chooses to see.) Therefore, the probability that this consumer considers all firms in set  $S$  (and possibly others too) is the probability she is sent a message from all firms in  $S$  and she receives all such messages, which is equal to  $a^{|S|}\tilde{\sigma}_S$ . Integrating over the distribution of  $a$  in the consumer population shows that the induced interaction parameter  $\gamma_S$  is given by

$$\gamma_S = \frac{\mathbb{E}_a[a^{|S|}\tilde{\sigma}_S]}{\prod_{i \in S}(\mathbb{E}_a[a\tilde{\sigma}_i])} = \frac{\mathbb{E}_a[a^{|S|}]}{(\mathbb{E}_a[a])^{|S|}} \times \tilde{\gamma}_S \geq \tilde{\gamma}_S ,$$

where the inequality follows from Jensen’s inequality. Thus, introducing heterogeneous attentiveness by consumers acts to *amplify* interactions between firms. In particular, if the message structure has symmetric interactions, in the sense that  $\tilde{\gamma}_S$  depends only on the number of firms in  $S$ , then so does the induced pattern of consideration when consumers vary in their attentiveness. For instance, if messages are sent *independently*, so that  $\tilde{\gamma}_S \equiv 1$ , then the induced pattern of consideration has symmetric interactions. There is then positive correlation in reaches across firms ( $\gamma_S \geq 1$ ), since if a consumer considers firm  $i$  she is likely to have a high  $a$  and hence is more likely to consider firm  $j$  too.

*Firms differ in their suitability:* Here, in order for a consumer to consider a firm she needs to receive a message from that firm and to find that firm’s product suitable. Suppose that the pattern of messages is as in the previous scenario, with interaction terms  $\tilde{\gamma}_S$ . Suppose that firm  $i$ ’s product will be suitable for a consumer with probability  $b_i > 0$ , which is independent of whether the consumer finds other products suitable and how many messages she receives. (Thus,  $b_i$  could be interpreted as a measure of the firm’s product quality.)



Therefore, the probability that a consumer considers all firms in set  $S$  (and possibly others too) is the probability she is sent a message from all firms in  $S$  and she finds all these firms suitable, which is equal to  $\tilde{\sigma}_S \times \prod_{i \in S} b_i$ . Thus, the induced interaction parameter  $\gamma_S$  is

$$\gamma_S = \frac{\tilde{\sigma}_S \times \prod_{i \in S} b_i}{\prod_{i \in S} (\tilde{\sigma}_i b_i)} = \frac{\tilde{\sigma}_S}{\prod_{i \in S} \tilde{\sigma}_i} = \tilde{\gamma}_S .$$

Thus, introducing these suitability parameters for firms has no impact on the interactions between firms. In particular, if the message structure has symmetric interactions then so does the induced pattern of consideration when suitability varies across firms. For instance, if the pattern of messages is *symmetric* (some consumers receive a message from one random firm, some receive messages from two random firms, and so on), then the induced pattern of consideration has symmetric interactions, although firms might be very asymmetric. (Here,  $\gamma_S = \tilde{\gamma}_S$  could be above or below 1.)

The assumption of symmetric interactions implies that the probability that a consumer considers firm  $i$  given that she considers all firms in  $S$  (where  $i \notin S$ ) depends only on the number of firms in  $S$  and not their identity. Similarly, in markets with symmetric interactions, consumers who consider firm  $i$  and consumers who consider firm  $j$  have the same chance of considering any other subset of firms  $S$  (where  $i, j \notin S$ ), in the sense that  $\Pr\{\text{consider } S \mid \text{consider } i\} = \Pr\{\text{consider } S \mid \text{consider } j\}$ . This latter observation implies that  $q_{i/j}(p)/\sigma_i \equiv q_{j/i}(p)/\sigma_j$ , where  $q_{i/j}$  is defined in (4), as discussed in more detail in the proof of the following useful result.<sup>16</sup>

**Lemma 2** *Suppose interactions are symmetric. Then for any pair of firms  $i$  and  $j$  the demand functions in (7) satisfy*

$$\frac{q_i(p)}{\sigma_i} - \frac{q_j(p)}{\sigma_j} = \frac{q_{ij}(p)}{\sigma_i \sigma_j} [G_i(p) - G_j(p)] , \quad (12)$$

*and captive-to-reach ratios satisfy*

$$\rho_i - \rho_j = \frac{\alpha_{ij}}{\sigma_i \sigma_j} [\sigma_i - \sigma_j] . \quad (13)$$

Recall that  $q_{ij}(p)$  is defined in (5), and if  $q_{ij}(p)$  is positive, which is ensured by Assumption 3, then  $q_i/\sigma_i - q_j/\sigma_j$  has the same sign as  $G_i - G_j$ . Likewise, with Assumption

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<sup>16</sup>Focussing on the underlying  $\alpha_S$  parameters, one can show that the market has symmetric interactions if and only if  $\sigma_i(\alpha_S + \alpha_{S/i}) = \sigma_j(\alpha_S + \alpha_{S/j})$  for all distinct pairs of firms  $i$  and  $j$  and sets of firms  $S$  which contain both  $i$  and  $j$ .

3 expression (13) implies  $\rho_i > \rho_j$  if and only if  $\sigma_i > \sigma_j$ , and so a firm with a larger reach has a higher captive-to-reach ratio.

Expression (12) almost immediately implies our main result concerning symmetric interactions, which is that in equilibrium all firms use the same minimum price,  $p_0$ . For if not, there is a strict subset  $A$  of firms with the market minimum price  $p_0$  in their support, and let firm  $i$  be the firm outside  $A$  with the next lowest minimum price,  $L > p_0$ . Any firm  $j \in A$  weakly prefers price  $p_0$  (when it serves its entire reach) to the price  $L$ , so that  $q_j(L)/\sigma_j \leq p_0/L$ . Firm  $i$ , by contrast, weakly prefers  $L$  to  $p_0$ , and so  $q_i(L)/\sigma_i \geq p_0/L$ . In particular,  $q_i(L)/\sigma_i \geq q_j(L)/\sigma_j$ . But since  $G_j(L) > 0$  and  $G_i(L) = 0$ , this contradicts (12) provided that  $q_{ij}(L) > 0$ . We deduce that in equilibrium all firms must have the same minimum price  $p_0$ , which from part (vi) of Lemma 1 is therefore the highest captive-to-reach ratio among the firms. From (13), this is the captive-to-reach ratio of the firm with the greatest reach. The next result provides further detail about the pricing pattern in equilibrium:

**Proposition 1** *Suppose Assumption 3 holds and interactions are symmetric. Suppose firms are labelled in order of increasing reach, so that  $\sigma_1 \leq \dots \leq \sigma_n$ . Then in the unique equilibrium firm  $i$  has interval price support  $[p_0, p_i]$ , where the minimum price  $p_0$  is equal to the captive-to-reach ratio of the firm with the largest reach (firm  $n$ ) and maximum prices satisfy  $p_1 \leq \dots \leq p_{n-1} = p_n = 1$ . Firms with larger reach set stochastically higher prices, in the sense that  $F_1(p) \geq \dots \geq F_n(p)$  for  $p \in [p_0, 1]$ . Firm  $i$  obtains expected profit  $\pi_i = \sigma_i p_0$ .*

This result shows that with symmetric interactions the equilibrium pricing pattern takes the same form as with independent reach: all firms have the same minimum price and firms with smaller reach have a lower maximum price, so that price supports are nested. Thus introducing correlation in reach, provided that interactions are symmetric, does not affect the equilibrium pattern of prices. The role of Assumption 3 is as in Johnen and Ronayne (2020): if some pairs of firms have no consumers in common who consider just them, the possibility arises that other equilibria exist where some firms price deterministically at  $p = 1$ .

*Exit and mergers:* Markets with symmetric interactions exhibit intuitive properties when the number of firms is reduced, either by exit or by merger. If a firm exits a market with symmetric interactions, then the firms that remain continue to have symmetric interactions

and hence all use the same minimum price. This new minimum price will necessarily be higher than before exit. If two firms merge in a market with symmetric interactions, the new market does not necessarily continue to have symmetric interactions, although if the merger is profitable the minimum price will rise. In either case, the following result shows that consumers overall are harmed.

**Proposition 2** *Suppose Assumption 3 holds and interactions are symmetric. Then:*

- (i) *If a firm exits the minimum price rises and consumer surplus falls;*
- (ii) *If two firms merge profitably the minimum price rises and consumer surplus falls.*

Note that many potential mergers in this framework are unprofitable, and unprofitable mergers can often benefit consumers. For instance, with independent reach the market after a merger continues to have independent reach and so all firms use the same minimum price, but unless the merged entity becomes the largest firm in the market, the merger will be unprofitable and consumers will benefit.<sup>17</sup> Since unprofitable mergers are assumed not to be proposed, only the impact on consumers of a profitable merger is relevant for policy, and Proposition 2 shows such a merger harms consumers.

*Entry and increased consumer awareness:* The previous discussion concerned the impact of reducing competition in the market. Natural ways in which competition might instead be enhanced is if a new firm enters or if consumers consider a wider set of firms. Suppose first that a new firm enters the market. If firms in the post-entry market have symmetric interactions, then Proposition 2 shows that entry will increase consumer surplus. Beyond this case, however, the analysis is less clear cut. When entry occurs only within contested segments, which leads to a market with asymmetric interactions, there is a tendency for it to harm consumers. To illustrate, suppose the incumbents are symmetric firms and the entrant is considered only by consumers who already consider at least two incumbents. (Figure 2 illustrates the case with three incumbents, where the entrant is depicted as the shaded triangle.) This is a natural scenario if only “savvy” consumers consider buying from the entrant, and these are consumers who anyway compared prices in the market. Because incumbents are symmetric, before entry they obtained profit equal to their captive

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<sup>17</sup>If the merged entity does not have the largest reach, the minimum price  $p_0$  is unaffected by the merger as the largest captive-to-reach ratio is unaffected by the merger, but the combined reach of the merged entity strictly falls, and so combined profit of the merged entity falls. Since the profits of non-merging firms are unaffected, total profits fall and so consumer surplus rises.

base. Since the entrant does not reach any of the incumbents' captive customers, entry does not affect an incumbent's captive-to-reach ratio, and so Lemma 1(vi) shows that the minimum price  $p_0$  is unchanged by entry. Thus each incumbent obtains at least as much profit after entry as it did before. Since the entrant also makes a positive profit (at least equal to  $p_0$  times its reach), it follows that industry profit is strictly *higher* after entry. Finally, since the entrant reaches no new consumers total welfare is unchanged, and we deduce that consumer surplus falls with this form of entry.<sup>18</sup>

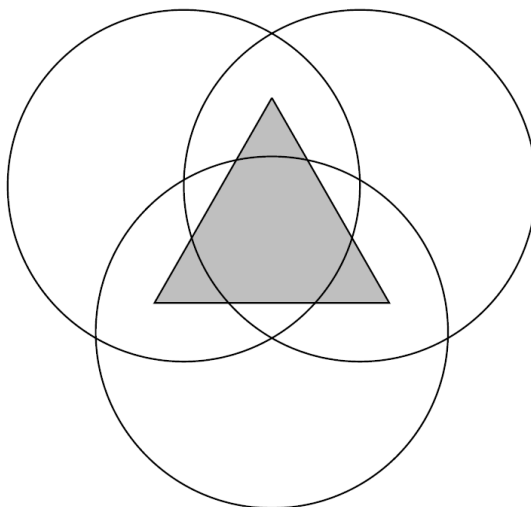


Figure 2: Entry into contested segments

Alternatively, suppose that consumers become more “aware”, in the sense that each consumer considers a weakly wider set of firms. (This might follow a policy intervention to boost consumer awareness of market options, say.) For instance, a consumer who previously considered no firm (and so was excluded from the market) now considers one firm, or a consumer captive to firm  $i$  now considers the firms  $\{i, j\}$ . When does increased awareness lead to a better outcome for consumers?

To make a start on such analysis, consider an “incremental” change in awareness whereby a small fraction  $\epsilon$  of consumers who previously considered the set of firms  $S$  now consider firm  $i \notin S$  as well. (Any increase in awareness can be broken down into a

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<sup>18</sup>This result is related to Rosenthal (1980), who considers the impact of a firm, which hitherto had served only a captive pool of customers, deciding (or being allowed) to serve the contested segment as well. He shows that this causes the rival firms to retreat partially to their own captive bases, and the expected price paid by their captive customers rises as does the expected minimum price paid by the contested customers. (Of course, the price paid by entrant's captive customers falls.) With unit demand, welfare, industry profit and consumer surplus are all unchanged in his scenario, while in ours profit rises and consumer surplus falls with entry.

sequence of these incremental changes.) This incremental change increases the reach of firm  $i$  by  $\epsilon$  and leaves the reach of other firms unchanged. This increases total welfare if and only if it converts an excluded consumer into a captive consumer.

Suppose the market initially had symmetric interactions, where one firm is strictly larger than the rest (i.e.,  $\sigma_n > \sigma_{n-1}$ ). Then for small enough  $\epsilon$  an incremental change will continue to have all firms using the same minimum price, where the identity of the largest firm, firm  $n$ , remains unchanged. Consider first the market-expanding changes that convert an excluded consumer into a captive consumer. Adding  $\epsilon$  new captive customers to firm  $i$ , where  $i < n$ , does not change the minimum price  $p_0 = \rho_n$ , and so increases total profit by  $\epsilon\rho_n$  and increases consumer surplus by  $\epsilon(1 - \rho_n)$ . Adding new captive customers for the largest firm, however, *increases*  $p_0$ . Since firm  $n$ 's gain in profit precisely equals the gain in welfare, and since the other firms' profits rise with  $p_0$ , this incremental change therefore *decreases* consumer surplus. Intuitively, the reduction in competitive intensity due to the largest firm gaining captives outweighs the benefits to consumers of increased market participation.

Consider next incremental changes that do not expand the market, in which case the impact on consumers is opposite to that on industry profit. There are three sorts of these changes: (i) those that convert a consumer captive to the largest firm  $n$  into one who considers  $\{i, n\}$ ; (ii) those where the newly considered firm is the largest firm  $n$ , and (iii) those that do not involve firm  $n$  at all. Change (i) or (ii) causes the minimum price  $p_0$  to fall (since  $n$ 's number of captives falls with (i) or its reach rises with (ii)), and one can check that consumer surplus then rises with a change of these forms. However, change (iii), which is possible only if there are at least three firms, has no impact on  $p_0$ . Since the sum of the reaches increases, industry profit increases and so consumers are again *harmed* by the incremental change. Intuitively, increased competition between smaller firms induces the larger firm to retreat to its captive base. This mechanism is similar to our discussion of entry: increased competition due to greater price comparison has no impact on the minimum price but causes the sum of firm reaches to rise, with the effect that industry profit rises and consumers are harmed.

There is a familiar intuition that an increase in the number of comparison shoppers—consumers who compare prices from several firms rather than just one—induce firms to lower their prices, with positive externalities for all consumers. However, this discussion

has shown that this notion of search externalities needs to be more nuanced when there are more than two firms. Boosting the number of consumers who compare prices of one pair of firms can cause another firm to raise its price, which harms that firm’s captive customers and might harm consumers in aggregate as well.

The class of symmetric interactions, although rich, rules out important patterns of consideration, such as where reach is nested, where some firms have disjoint reach, or where some pairs of firms have reach which is positively correlated and some where reach is negatively correlated. In the remainder of the paper we study in more detail situations outside the case of symmetric interactions, and show how the form of equilibrium can differ markedly. As well as finding situations where only a subset of firms use the minimum price, we will also see instances where a firm has a “gap” in its price support. In contrast to Proposition 2, we will find situations where a profitable merger between firms can increase consumer surplus.

## 4 Asymmetric interactions

In this section we discuss equilibrium pricing patterns when consumer consideration involves asymmetric interactions. We will focus especially on markets where some firms have disjoint reach or where firms have nested reach.

Proposition 1 provided a sufficient condition for all firms to use the same minimum price. A sufficient condition for the converse, i.e., for only a subset of firms to use the minimum price, can also be derived. If all firms have the minimum price  $p_0$  in their support, then  $q_i(p)/\sigma_i = p_0/p$  for prices just above  $p_0$ , and so  $q'_i(p_0)/\sigma_i = -1/p_0$  for each firm  $i$ . From (7), this entails  $\sum_{j \neq i} \gamma_{ij} G'_j(p_0) = 1/p_0$  for each firm  $i$ . If this system of linear equations in  $G'_1(p_0), \dots, G'_n(p_0)$  has no solution with each  $G'_i(p_0) \geq 0$ , there can be no equilibrium where all firms use the same minimum price  $p_0$ . Farkas’ Lemma implies that there is no such solution if and only if there exists a vector  $(x_1, \dots, x_n)$  that satisfies

$$\sum_{j=1}^n x_j < 0 \text{ and } \sum_{j \neq i} \gamma_{ij} x_j \geq 0 \text{ for all } i. \quad (14)$$

Thus, when the pairwise interactions are such that we can find a vector  $(x_1, \dots, x_n)$  satisfying (14) then any equilibrium has only a subset of firms using the minimum price  $p_0$ . (Note that condition (14) does not depend on the behaviour of  $\gamma_S$  for larger subsets  $S$ .) To illustrate

the argument for  $n = 3$ , suppose interactions are so asymmetric that  $\gamma_{12} > \gamma_{13} + \gamma_{23}$ . Then setting  $(x_1, x_2, x_3) = (\gamma_{23}, \gamma_{13}, -\gamma_{12})$  satisfies (14), and so only two firms will use the minimum price. (In section 5 we will see that it is firms 1 and 2 that price low.)

A similar method can be employed to show that a given subset of firms cannot use the minimum price  $p_0$  in equilibrium. We shortly do this in the case of nested reach to show when only a small number of firms will price low.

*Chain store competition:* An economically natural scenario with disjoint reach is when a “chain store”, firm  $n$ , competes against  $n - 1 \geq 2$  disjoint local rivals (so there are  $n \geq 3$  firms in all), as shown in Figure 3 when  $n = 5$ . In keeping with the assumption throughout the paper that firms set uniform prices, here the chain store follows a “national pricing” strategy (perhaps for reasons of national advertising), and chooses the same price in each local market. With this pattern of consideration,  $\gamma_S = 0$  for every  $S$  that contains two or more local firms. Lemma 1 shows that at least two firms compete at each price in the market range of prices  $[p_0, 1]$ . Since local firms do not compete between themselves, this implies that the chain store chooses its price from the whole range  $[p_0, 1]$  and each price in  $[p_0, 1]$  is also chosen by at least one local firm.

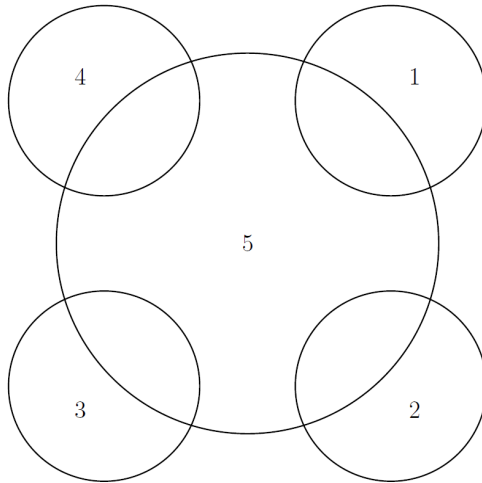


Figure 3: A chain store competing against disjoint local rivals

As in (10), the demand of local firm  $i$  is  $q_i(p) = \sigma_i(1 - \gamma_{in}G_n(p))$ , where  $\gamma_{in}$  is the pairwise interaction term for  $i$  and  $n$ , and  $G_n(p) = \sigma_n F_n(p)$  is the chain store’s adjusted CDF. To avoid knife-edge cases suppose local firms have distinct interaction terms, and label them as  $\gamma_{1n} > \dots > \gamma_{n-1,n}$ . Suppose local firm  $i$  has price  $p_i$  in its support, and

another local firm  $j$  has price  $p_j$  in its support. By revealed preference

$$p_i(1 - \gamma_{in}G_n(p_i)) \geq p_j(1 - \gamma_{in}G_n(p_j)) ; p_j(1 - \gamma_{jn}G_n(p_j)) \geq p_i(1 - \gamma_{jn}G_n(p_i)) ,$$

and subtracting yields

$$(\gamma_{jn} - \gamma_{in})[p_iG_n(p_i) - p_jG_n(p_j)] \geq 0 .$$

Since  $pG_n(p)$  is a strictly increasing function, it follows that local firms with a weaker interaction with the chain store choose weakly higher prices. Moreover, if both firms have both the prices  $p_i$  and  $p_j$  in their supports the argument shows that

$$(\gamma_{jn} - \gamma_{in})[p_iG_n(p_i) - p_jG_n(p_j)] = 0 ,$$

which implies  $p_i = p_j$ . Therefore, there can be no overlap beyond a single price in the price supports of two local firms.

In economic terms, the local firms all compete against the same price distribution from the chain store, and local firms with greater interaction with the chain store have more elastic demand and so offer lower prices. In technical terms, the only way that a single price distribution from the chain store can maintain indifference for each local rival is for different rivals to have disjoint price supports.

Putting this together yields the following result. (Note that we have weak inequalities in  $p_0 < p_1 \leq \dots \leq p_{n-1} = 1$  since it is possible that a number of local firms, but not all, choose the monopoly price  $p = 1$  for sure.)

**Proposition 3** *Suppose a chain store, firm  $n$ , competes with a number of local rivals,  $i = 1, \dots, n - 1$ , where  $\gamma_{1n} > \dots > \gamma_{n-1,n}$ . Then in equilibrium there are threshold prices  $p_0 < p_1 \leq \dots \leq p_{n-1} = 1$  such that the chain store has price support  $[p_0, 1]$  and local firm  $i < n$  has price support  $[p_{i-1}, p_i]$ .*

Thus with this pattern of consideration, the chain store, which interacts with all local firms, uses all prices while firms with disjoint reach use disjoint sets of prices. This is therefore a situation where the pattern of consumer consideration leads very directly to the equilibrium pattern of price competition. In contrast to the pricing pattern with symmetric interactions in Proposition 1, here price competition is “duopolistic”, in the sense that only two firms use a given price range.<sup>19</sup> As will be seen later in section 5,

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<sup>19</sup>With the exception of the threshold prices  $p_1, \dots, p_{n-2}$ , which are in the support of three firms.



this “chain store” pattern of prices can arise in equilibrium even when no pair of firms has disjoint reach.

*Nested reach:* A second situation that involves asymmetric interactions is when consideration sets are nested, in the sense that if firm  $i$  reaches a greater fraction of consumers than firm  $j$ , then all consumers reached by  $j$  also consider firm  $i$ . This is a natural configuration if consumers consider options in the same *ordered* fashion, as may be the case with internet search results (where some consumers just consider the first result, others consider the first two, and so on). With nested reach, only the largest firm has any captive customers,  $\sigma_S = \min_{i \in S} \{\sigma_i\}$ , and pairwise interactions are  $\gamma_{ij} = 1/\max\{\sigma_i, \sigma_j\}$ . For given firm reaches, nested reach has the strongest interactions  $\gamma_S$  among all patterns of consideration.

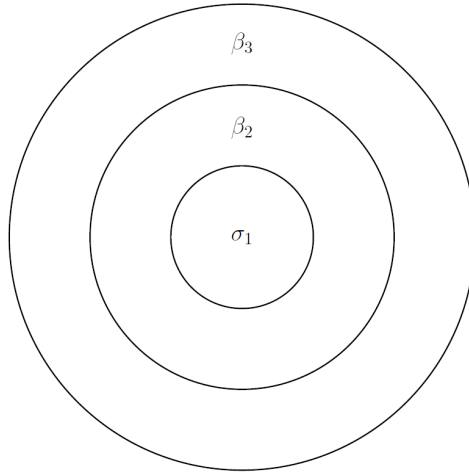


Figure 4: Three firms with nested reach

Suppose there are  $n \geq 3$  firms with nested reach. Let firm  $i$  have reach  $\sigma_i$ , where firms are labelled as  $\sigma_1 < \sigma_2 < \dots < \sigma_n$ , and for  $i \geq 2$  write  $\beta_i = \sigma_i - \sigma_{i-1}$  for the incremental reach of firm  $i$ . (Figure 4 depicts the case with three nested firms.) While it is hard to calculate the precise equilibrium in all nested situations, the following result describes general features of equilibrium pricing and also derives an equilibrium in the case where incremental reach is larger for larger firms.

**Proposition 4** *Suppose  $n \geq 3$  firms, labelled as  $\sigma_1 < \sigma_2 < \dots < \sigma_n$ , have nested reach.*

*(i) If  $L_i$  and  $H_i$  are respectively the minimum and maximum price in firm  $i$ 's support then  $p_0 = L_1 = L_2 \leq L_3 \leq \dots \leq L_n$ ,  $H_1 < H_2 < \dots < H_{n-1} = H_n = 1$ , and  $H_i > L_{i+1}$  for  $i < n$ .*

(ii) If  $\sigma_k > 2\sigma_2$  then no firm  $i \geq k$  uses the minimum price  $p_0$  (i.e.,  $L_k > p_0$ ).

(iii) If

$$\beta_2 \leq \dots \leq \beta_n \quad (15)$$

there is an equilibrium with price thresholds  $p_0 = p_1 < p_2 < \dots < p_{n-1} < p_n = p_{n+1} = 1$  such that the price support of firm  $i$  is  $[p_{i-1}, p_{i+1}]$ . The thresholds are determined recursively by  $p_2 = \frac{\sigma_1 + \beta_2}{\beta_2} p_1$  and for  $1 < i < n$

$$p_{i+1} = p_i + \frac{\beta_i}{\beta_{i+1}} p_{i-1}, \quad (16)$$

where  $p_0 = p_1$  is chosen to make  $p_n = 1$ . The profit of firm 1 is  $\pi_1 = \sigma_1 p_1$  and the profit of firm  $i > 1$  is  $\pi_i = \beta_i p_i$ .

In part (i), the reason that a smaller firm will not choose a price strictly above the maximum price used by a larger firm is that it will then have no demand or profit. The observation that minimum prices weakly increase with size is less immediate, but relates to the intuition that smaller firms face more elastic demand. The implication is that if only a subset of firms use the minimum price  $p_0$ , that subset will consist of the smaller firms. Since  $H_i > L_{i+1}$  the price supports of successive firms overlap. Part (ii) shows that if reaches are spread out then only few firms will price low. In particular, if  $\sigma_3 > 2\sigma_2$  then only the two smallest firms will price low. Together, parts (i) and (ii) shows that if reaches are spread out then the pricing pattern is such that smaller firms only choose low prices while larger firms only choose high prices. Finally, part (iii) shows that if the reaches are spread out enough that (15) holds, then as with the chain store scenario price competition is duopolistic, with only two firms competing at a given price. More precisely, the pricing pattern takes the form of “overlapping duopolies”, where a firm only competes against its two immediate neighbours in the nest.

To illustrate part (iii), consider the case where reach decays with a constant rate of attrition, so that the reach of firm  $i = 1, \dots, n$  is  $\sigma_i = \delta^{n-i}$  for some  $\delta \in (0, 1)$ . In this case  $\beta_i = \delta^{n-i}(1 - \delta)$  which increases with  $i$  as required, and equation (16) becomes  $p_{i+1} = p_i + \delta p_{i-1}$ . When  $n = 2$  the two firms have reaches  $\sigma_1 = \delta$  and  $\sigma_2 = 1$ , and Proposition 1 shows that the minimum price is  $p_0 = 1 - \delta$  and industry profit is  $1 - \delta^2$ . Suppose a third firm enters in nested fashion with reach  $\delta^2$ . Proposition 4(iii) shows that the minimum price falls to  $p_0 = (1 - \delta)/(1 + \delta - \delta^2)$ , while industry profit rises to  $(1 - \delta)(1 + 2\delta)/(1 + \delta - \delta^2)$ . Since welfare is unchanged, it follows consumer surplus falls

with nested entry. This is consistent with the discussion in section 3 that entry within contested segments tends to harm consumers, although in this case the minimum price does fall with entry.

Proposition 4 describes the precise equilibrium only for cases where incremental reach weakly increases. In the next section we analyse the case of triopoly, and obtain results implying for the case of nested reach that (a) when  $\beta_3 > \beta_2$  the equilibrium in part (iii) of Proposition 4 is unique and (b) when  $\beta_3 < \beta_2$  the equilibrium instead has all three firms using the same minimum price.

## 5 The three-firm problem

So far we have encountered two contrasting patterns of pricing: (i) all firms use the same minimum price (the case of symmetric interactions in section 3), and (ii) duopolistic pricing where only two firms use prices in a given price range (cases of disjoint and nested reach in section 4). In this section we study in detail the situation with three firms, and show that the pricing patterns (i) and (ii) are (generically) the *only* possibilities. That is, if pricing is duopolistic for low prices, it continues to be duopolistic throughout the whole market price range.<sup>20</sup>

We first describe a sufficient condition for regime (i) to apply, using a similar argument to that for symmetric interactions in section 3. This argument will also reveal which pair of firms price low when regime (ii) applies. Suppose that an equilibrium has firms 1 and 2 using minimum price  $p_0$ , while 3's minimum price is  $L > p_0$ . Then both firms 1 and 2 have prices  $[p_0, L]$  in their support, and by revealed preference

$$\frac{q_1(L)}{\sigma_1} = \frac{q_2(L)}{\sigma_2} = \frac{p_0}{L} \leq \frac{q_3(L)}{\sigma_3}.$$

From (11), demand functions for distinct firms  $i$ ,  $j$  and  $k$  satisfy

$$\frac{q_i(p)}{\sigma_i} - \frac{q_j(p)}{\sigma_j} = (\gamma_{jk} - \gamma_{ik})G_k(p) + (\gamma_{ij} - \gamma_{ik}G_k(p))(G_i(p) - G_j(p)), \quad (17)$$

where for notational simplicity we have written  $\gamma \equiv \gamma_{123}$ . Note that the term  $(\gamma_{ij} - \gamma_{ik}G_k(p))$  is equal to  $q_{ij}(p)/(\sigma_i\sigma_j) \geq 0$ , and so expression (17) reduces to (12) when interactions are symmetric.

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<sup>20</sup>As discussed after Proposition 5, other possibilities arise on the boundary between these two regimes.

Since 1 and 2 compete alone in the range  $[p_0, L)$ , we must have  $\gamma_{12} > 0$ , and since  $G_3(L) = 0$  expression (17) with  $k = 3$  implies  $G_1(L) = G_2(L)$ . Applying (17) with  $k = 1$ ,  $i = 2$  and  $j = 3$  then implies

$$0 \geq \gamma_{13} + \gamma_{23} - \gamma_{12} - \gamma G_1(L) \geq \gamma_{13} + \gamma_{23} - \gamma_{12} - \gamma \sigma_1 . \quad (18)$$

The same argument with  $i = 1$ ,  $k = 2$  and  $j = 3$  shows that  $0 \geq \gamma_{13} + \gamma_{23} - \gamma_{12} - \gamma \sigma_2$ , and so if only firms 1 and 2 use the minimum price  $p_0$  it is necessary that

$$\gamma \min\{\sigma_1, \sigma_2\} \geq \gamma_{13} + \gamma_{23} - \gamma_{12} . \quad (19)$$

Since  $\gamma_{13} \geq \gamma \sigma_2 \geq \gamma \min\{\sigma_1, \sigma_2\}$  and  $\gamma_{23} \geq \gamma \sigma_1 \geq \gamma \min\{\sigma_1, \sigma_2\}$ , expression (19) implies that  $\gamma_{12} \geq \max\{\gamma_{13}, \gamma_{23}\}$ . Thus, if the equilibrium involves only two firms pricing low those firms must have the greatest interaction among the three pairs. We deduce that if firms are labelled so that firms 1 and 2 have the greatest interaction, then when (19) is violated the equilibrium must have all three firms using the same minimum price.

This discussion is summarized in part (i) of the following result. This result also shows that when only two firms price low, the outcome is duopolistic pricing of the sort seen in section 4. In particular, it cannot be an equilibrium that two firms choose prices over the range  $[p_0, 1]$  while the third firm chooses from an intermediate or upper range of prices.

**Proposition 5** *With three firms, label firms so that 1 and 2 have the greatest interaction, i.e.,  $\gamma_{12} \geq \max\{\gamma_{13}, \gamma_{23}\}$ , and that  $\sigma_1 \leq \sigma_2$ .*

(i) *If*

$$\gamma \sigma_1 < \gamma_{13} + \gamma_{23} - \gamma_{12} \quad (20)$$

*then in equilibrium all firms use the same minimum price  $p_0$ , which is the highest captive-to-reach ratio among the firms;*

(ii) *If*

$$\gamma \sigma_1 > \gamma_{13} + \gamma_{23} - \gamma_{12} \quad (21)$$

*then in equilibrium price competition is duopolistic: there are prices  $p_0 < p_1 \leq 1$  such that firm 1 has price support  $[p_0, p_1]$ , firm 2 has support  $[p_0, 1]$  and firm 3 has support  $[p_1, 1]$ . Expressions for the thresholds  $p_0$  and  $p_1$  and for firm profits are given in the proof.*

Part (i) of this result applies when interactions are similar across pairs of firms, as is the case with symmetric interactions, as long as Assumption 3 holds so that  $\gamma \sigma_1 < \gamma_{23}$  in (20).

With nested reach the two smallest firms have the strongest interaction and condition (20) reduces to  $\beta_2 > \beta_3$ . Thus with three nested firms, those cases not covered by Proposition 4 have all firms using the same minimum price in equilibrium. Condition (20) implies that  $\gamma_{12} \leq \gamma_{13} + \gamma_{23}$ , as is consistent with the discussion in section 4 showing that when  $\gamma_{12} > \gamma_{13} + \gamma_{23}$  only two firms will price low.

Part (ii) applies when one pair of firms has significantly stronger interaction than other pairs. The two scenarios covered in section 4 (the chain store case and nested reach with increasing incremental reach), which exhibit duopolistic pricing, both satisfy (21).<sup>21</sup> If firms 1 and 2 are considered by almost the same set of consumers (so their circles on the Venn diagram almost coincide), and if  $\alpha_3 > 0$ , then firms 1 and 2 have the greatest interaction and condition (21) is satisfied, and firm 3 chooses price  $p \approx 1$ . Intuitively, when two firms reach nearly the same set of consumers, they compete fiercely between themselves, leaving the remaining firm to price at or near the monopoly level. Another situation where (21) holds is the asymmetric “one-or-all” specification in Baye *et al.* (1992, Section V), where no consumer considers exactly two firms and  $\alpha_1 \leq \alpha_2 < \alpha_3$ , when the two smallest firms 1 and 2 have the greatest interaction and firm 3 chooses  $p \equiv 1$ .<sup>22</sup>

When part (ii) applies, firm 3 is sure to set a higher price than firm 1. As such, these two firms do not compete in equilibrium, even though in general their reaches overlap.<sup>23</sup> The outcome for consumers and firms in this market is exactly the same as with an alternative pattern of consideration where consumers who previously considered  $\{1, 3\}$  now just consider  $\{1\}$  and consumers who previous considered  $\{1, 2, 3\}$  now consider  $\{1, 2\}$ . This alternative pattern is “coarser”, involving only five segments in the Venn diagram, and is of the chain store form whereby firms 1 and 3 have disjoint reach.

In the boundary case where

$$\gamma\sigma_1 = \gamma_{13} + \gamma_{23} - \gamma_{12} , \tag{22}$$

which is not covered by Proposition 5, there is the possibility that both kinds of equilibrium coexist. For instance, this is so in the symmetric Varian-type market where  $\alpha_{12} = \alpha_{13} =$

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<sup>21</sup> Although in general the “chain store” and the “overlapping duopoly” pricing patterns in section 4 differ, they coincide when there are three firms.

<sup>22</sup> As mentioned in the introduction, Kocas and Bohlmann (2008) study a triopoly model where only the two largest firms have a duopoly segment. The statements of their Propositions 1 and 2 suggest that in all such cases regime (ii) applies (using our terminology). However, this is not the case, and further parameter restrictions are introduced in the body of the proof of their Proposition 1 (see page 140).

<sup>23</sup> Expression (41) in the appendix shows that firm 2, which prices across the whole range, chooses to compete against firm 1, i.e., to set a price below  $p_1$ , with probability  $\sigma_1/\sigma_2$ .

$\alpha_{23} = 0$  and  $\alpha_1 = \alpha_2 = \alpha_3$ , where there is a symmetric equilibrium where all firms price low and also asymmetric equilibria where one of the firms chooses  $p \equiv 1$ . (See Baye *et al.* (1992) for the full range of equilibria in this market.) The chain store case where two local firms are symmetric also satisfies (22) and has multiple equilibria.<sup>24</sup>

*Equilibrium strategies when all firms use the same minimum price:* Proposition 5 provides much information about equilibria in this model—it characterizes equilibrium profit and consumer surplus in the two regimes, and it describes equilibrium strategies when part (ii) applies. However, it does not describe equilibrium pricing strategies for part (i), and the pricing patterns turn out to have interesting features.

Proposition 1 shows that when interactions are precisely symmetric the pricing pattern is such that all three firms are active for low prices, and for higher prices only the two larger firms compete. In the earlier version of this paper (Armstrong and Vickers, 2018, Proposition 2) we calculated an equilibrium whenever part (i) of Proposition 5 applied, which took one of two forms: either (a) the three firms were active in a lower price range and then two were active in a range of higher prices (as with symmetric interactions), or (b) the three firms were active in a lower price range, then only the most interactive pair continued to be active in an intermediate price range, and then another pair of firms were active in a higher range. In particular, in situation (b) one firm (firm 3 using the labelling in Proposition 5) chose low and high prices, but not intermediate prices.

The general analysis was complicated, and here we merely describe an example to show the possibility. Suppose three firms have nested reach, where  $\sigma_1 = \frac{1}{2}$ ,  $\sigma_2 = \frac{4}{5}$  and  $\sigma_3 = 1$ . We show in the appendix that equilibrium with this pattern of consideration has all firms choosing prices in the range  $[\frac{1}{5}, \frac{9}{25}]$ , firms 1 and 2 choosing prices in the range  $[\frac{9}{25}, \frac{16}{25}]$  and firms 2 and 3 choosing prices in the range  $[\frac{16}{25}, 1]$ .

The reason why the largest firm has non-convex price support can be explained as follows. When all firms price low in equilibrium, so that part (i) of Proposition 5 applies, one can calculate that the three CDFs increase in  $p$  for prices just above  $p_0$ , the minimum price. (The condition for the three CDFs to increase in  $p$  above  $p_0$  is that no vector  $(x_1, x_2, x_3)$  can be found to satisfy (14), which corresponds to  $\gamma_{12} < \gamma_{13} + \gamma_{23}$  and which is

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<sup>24</sup>Essentially, only the *sum* of the CDFs of the two local firms are determined in equilibrium in this case. It is then possible that one local firm uses prices in the whole range, while the other uses prices in an interior or upper interval, say. These knife-edge cases have a different pricing pattern from those found generically in Proposition 5.

implied by condition (20).) One can also calculate the smallest price,  $p_1$  say, at which some CDF reaches 1 and above which the two remaining firms compete as duopolists for prices up to 1. (In the nested case, it is the smallest firm’s CDF that first reaches 1, although in the general model more detailed analysis is required to determine which firm first drops out.) However, in some cases—as in this example—firm 3’s candidate CDF (i.e., when we ignore the monotonicity constraint on the CDF) starts to *decrease* in  $p$  before the largest CDF reaches 1, which cannot therefore be a valid CDF. Figure 5 illustrates this. The correct CDF for this firm is then obtained by “ironing” this curve as shown on the figure, so that the largest firm does not choose prices in the interval denoted by the dashed line, which in this example is the interval  $(\frac{9}{25}, \frac{16}{25})$ .<sup>25</sup>

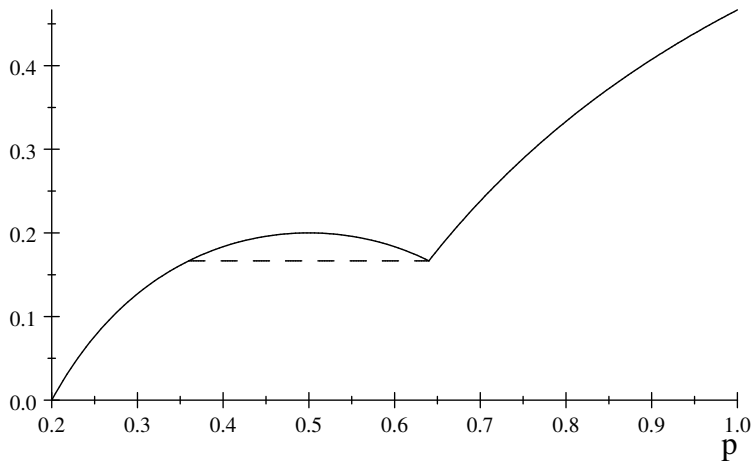


Figure 5: “Ironing”  $F_3(p)$  in a nested market with  $\sigma_1 = 1/2$ ,  $\sigma_2 = 4/5$ ,  $\sigma_3 = 1$

These equilibria with ironing provide insight into the relationship between the two seemingly contrasting parts of Proposition 5. A configuration that is “well inside” the parameter space defined by (20) will have a pattern of prices as with symmetric interactions: all three firms choose low prices, then firm 1 drops out leaving firms 2 and 3 to compete in the range with high prices. As parameters change to approach the boundary (22), the candidate CDF for firm 3 will start to decrease before firm 1’s CDF reaches 1. In this case, the “ironing” procedure is used so that firm 3’s price support has a gap in the middle. As

<sup>25</sup>The CDF on Figure 5 does not reach 1 since this firm has an atom at  $p = 1$  in equilibrium. The ironing procedure shown in Figure 5 is also used in the (otherwise distantly related) analysis in de Clippel, Eliaz, and Rozen (2014). Some of the asymmetric equilibria in Baye *et al.* (1992) also have a gap in one firm’s support: two firms use the whole range  $[p_0, 1]$ , while the third chooses prices in a lower range  $[p_0, p_1]$ , with  $p_1 < 1$ , but also has an atom at  $p = 1$ .

the boundary (22) is reached, the lower price range where all three firms are active shrinks and ultimately vanishes, leaving an equilibrium with duopolistic pricing.

*The impact of a merger:* When a market has symmetric interactions, Proposition 2 shows that a profitable merger harms consumers overall. One can show that the same is always true when the initial market has three firms. Specifically, one can show that a profitable merger between two firms necessarily increases the third firm's profit. For instance, if the non-merging firm used the minimum price  $p_0$  before merger, then the merger between the other two will increase its profit because the minimum price must rise in order for the merger to be profitable.<sup>26</sup> The remaining case to consider is a merger between firms 1 and 2 under the conditions of part (ii) of Proposition 5. More detailed calculations reveal that here too firm 3's profit rises when firms 1 and 2 merge. We deduce that any profitable 3-to-2 merger is detrimental to consumers.

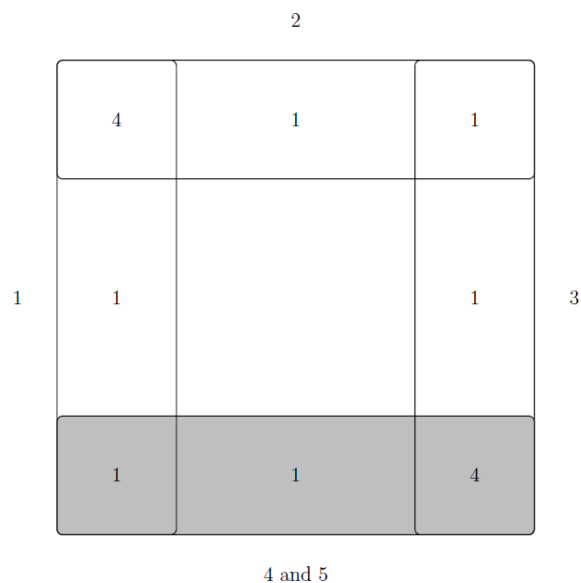


Figure 6: A profitable merger that benefits consumers

However, it is not always true, with more than three firms, that profitable mergers harm consumers. It may be, for example, that a merger between two firms with a strong interaction—which is therefore likely to be profitable—might induce non-merging firms to fight for the newly-profitable consumer segment, with the result that overall industry

<sup>26</sup>Before the merger, the combined profit of the merging firms, say firms  $A$  and  $B$ , was at least  $(\sigma_A + \sigma_B)p_0$ , and since their combined reach falls after the merger, for the merger to be profitable the minimum price must rise.



profits might fall and consumers are made better off. To illustrate this possibility consider the following example, which draws from our analysis of triopoly. Figure 6 depicts the pattern of consumer consideration. There are initially five firms, where firms 4 and 5 reach precisely the same set of consumers (depicted as the shaded set) and hence set price  $p = 0$  in equilibrium.<sup>27</sup> Firms 1, 2 and 3 each have a single captive customer, a single consumer considers each set of firms  $\{2, 3\}$ ,  $\{4, 5\}$  and  $\{1, 4, 5\}$ , while four consumers consider each set of firms  $\{1, 2\}$  and  $\{3, 4, 5\}$ . (No consumers consider more than three firms.)

Since firms 4 and 5 set price zero, firms 1, 2 and 3 compete as triopolists as if the shaded row on Figure 6 was eliminated. Here, firms 1 and 2 have the greatest interaction in this triopoly, and Proposition 5 implies that the equilibrium involves duopolistic pricing with firms 1 and 2 setting low prices and firms 2 and 3 setting high prices. The proof of part (ii) of the Proposition shows that firm 3 obtains its captive profit ( $\pi_3 = 1$ ), while firms 1 and 2 obtain respective profits  $\pi_1 = 5p_0$  and  $\pi_2 = 6p_0$ , where  $p_0 = \frac{2}{7}$  is the minimum price. Since firms 4 and 5 make zero profit industry profit is  $\frac{29}{7}$ . Now suppose firms 4 and 5 merge. (Clearly this is a profitable merger, as before the firms obtained no profit.) Since the four firms are symmetrically placed, each firm now obtains its captive profit ( $\pi_i = 1$ ), in which case industry profit falls to 4 after the merger, and consumers overall are better off.<sup>28</sup> Intuitively, before the merger the market was highly asymmetric, which allowed firms to enjoy high profits, and the merger brings more intense symmetric competition to the market. This example shows that not all profitable mergers in our setting are detrimental to consumers, but such competition-enhancing mergers appear to be relatively rare.

## 6 Conclusions

The aim of this paper has been to explore, in a parsimonious framework with price-setting firms and homogeneous products, how the pattern of consumer consideration determines the pattern of price competition.

In general, patterns of consideration were characterized by firm reaches,  $\sigma_i$ , together

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<sup>27</sup>This violates Assumption 2. However, the same argument works if firms 4 and 5 have reaches that nearly coincide. To simplify the exposition, this example does not normalize the measure of consumers to unity.

<sup>28</sup>Similar analysis shows that imposing a price ceiling on one firm in a market (which is akin to de-merging firms 4 and 5 on the figure) can cause industry profits to increase. Likewise, this example illustrates how entry might increase the profit of an incumbent firm: if the market initially consisted of firms 1 to 4, then entry by firm 5 strictly increases the profits of both firms 1 and 2.

with how those reaches overlap as captured by the interaction parameters,  $\gamma_S$ . We introduced a relatively flexible class of consideration patterns, which we termed symmetric interactions, where the interaction parameters did not depend on the identity of firms in  $S$ . This class includes cases studied in the existing literature—symmetric firms, independent reach, and duopoly—as particular cases. Within this class, in equilibrium all firms use the same minimum price, which implies that profits are proportional to a firm’s reach. Firms choose their prices from an interval, and firms with smaller reach choose lower average prices. Markets within this class have intuitive properties with respect to exit and profitable merger, both of which raise the minimum price and harm consumers.

Outside the class of symmetric interactions, other pricing patterns and more novel comparative statics can emerge. We found equilibria with duopolistic pricing patterns, i.e., where only two firms compete in a given price range. In a market in which a chain store competed against local rivals, the former used the full range of prices while local firms used disjoint price ranges. With nested reach and increasing differences there was an overlapping duopoly price pattern where small firms only use low prices and large firms only use high prices. In the three-firm case we established that if all firms do not use the same minimum price then pricing was necessarily duopolistic. When one pair of firms had significantly greater interaction than other pairs, firms with a strong interaction focus their competitive efforts against each other, leaving a third firm able to set high prices. For some parameter configurations we found equilibria with a gap in one firm’s price support, so that that firm sometimes prices high, and sometimes low, but never in between. We found plausible patterns of consumer consideration in which entry is detrimental to consumers because it softens competition between incumbents, leading them to retreat towards their captive base. Profitable mergers were always detrimental to consumers in the three-firm case, as with symmetric interactions, but not more generally. We also saw how enhanced consumer awareness of market options need not benefit consumers, and increased price comparisons between smaller firms might induce a larger firm to retreat to its captive base which can harm consumers.

The model assumed homogeneous products for two main reasons. First, many relevant markets—such as for electricity or an individual book—do have approximately a homogeneous product, and, as discussed in the introduction, there is evidence of price dispersion broadly consistent with the use of mixed strategies. The second reason was for tractability

and to obtain clean results. Instead of supplying a product which all consumers valued at  $v = 1$ , we might study a model where firm  $i$  supplied a product which a consumer valued idiosyncratically at  $v_i$ .<sup>29</sup> Even in the simplest case where  $v_i$  was identically and independently distributed across firms, though, it may be hard to obtain results about equilibrium prices and profits, or to be sure about whether pricing equilibrium is in pure or mixed strategies. For example, with independent or nested reach it is plausible that smaller firms set lower deterministic prices, but it may be difficult to find natural conditions on the distribution of  $v_i$  which ensure this is so. Nevertheless, it would be worthwhile in future work to try to extend this framework with consideration sets to allow for differentiated products. A useful step in this direction has been made by Chen and Riordan (2007), who study a model with symmetric firms, where consumers either consider a single random firm or a random pair of firms, and in the latter case competition takes a Hotelling form.

The analysis could be extended in a number of other directions. One would be to allow firms to offer multiple “brands”, where a firm is able to charge different prices for its different brands and where consumers might consider some of its brands but not others. In this situation it would be interesting to understand when a multi-brand firm chooses distinct prices for its brands, and, if so, the resulting pattern of pricing within a firm.

Another extension would be to endogenise the pattern of consideration by introducing search by consumers, word-of-mouth communication between consumers, advertising by firms, or the choice of price formats by firms (which affect the ease of price comparisons). For instance, one could extend the scenarios described in section 3 to allow consumers to choose their attentiveness, or firms could invest in product quality as captured by product suitability. Alternatively, Butters (1977) and subsequent papers have studied a market where firms choose both their reach (via costly advertising) and price, under the assumption that reach was *independent*. Using the analysis in this paper, one could study the same issue but with alternative patterns of interaction (i.e., where  $\gamma_S \neq 1$ ), such as when consumers differ in their attentiveness to advertising or when reach is nested rather than independent. Relatedly, if advertising is mediated by platforms and firms pay platforms for reach, those platforms may have an incentive to influence the interactions (e.g., to choose which adverts are shown together) in order to stimulate or stifle competition between firms.

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<sup>29</sup>Section 3 discussed an extreme case of this where  $v_i$  was 0 or 1 depending on whether or not the product was “suitable” for the consumer.

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### Technical Appendix

*Sketch proof of Lemma 1:* We first discuss arguments to do with deletion of dominated prices. In any equilibrium we have  $\pi_i \geq \alpha_i$ , since firm  $i$  can ensure at least this profit by choosing price equal to 1 and serving its captive customers. For this reason, no firm would ever offer a price below  $\rho_i$ , its captive-to-reach ratio, since if it did so it would obtain profit below  $\alpha_i$  even if it supplied its entire reach.

To see that the minimum price  $p_0$  is positive we invoke Assumption 2. There is at least one firm  $i$  that has captive customers, and which will not set price below  $\rho_i > 0$ . From the remaining firms, at least one firm  $j$  has captive customers in the subset of firms excluding  $i$ , and so this firm can set price  $\rho_i$  and be sure to obtain positive profit. Firm  $j$  therefore also has a positive lower bound on its prices. Following the same argument, a firm in the subset of firms excluding both  $i$  and  $j$  can obtain positive profit, and so on until the set of firms is exhausted. In particular, each firm’s minimum price is strictly above zero and hence so is  $p_0$ . If firm  $i$  chooses price  $p_0$  (or just below), it will undercut all rivals and sell to its reach, so achieving profit  $\sigma_i p_0$ . This proves part (ii).

If price  $p < 1$  is in firm  $i$ ’s support then its expected demand  $q_i(\cdot)$  cannot be flat in a neighbourhood of  $p$ , for otherwise the firm could obtain strictly greater profit by raising its price above  $p$ . This implies that this price must be in the support of at least one other firm. More precisely, if price  $p < 1$  is in firm  $i$ ’s support then  $i$  must compete against some other firm  $j$  at this price.

We next turn to arguments concerning the possibility of “atoms” in the price distributions. First observe that two firms cannot both have an atom at price  $p$  if they compete at this price (for otherwise each would have an incentive to undercut the price  $p$  and gain a discrete jump in demand). To see that each firm’s price distribution is continuous in the interval  $[p_0, 1)$ , suppose by contrast that firm  $i$  has an atom at some price  $0 < p < 1$  in its support. We claim that firm’s  $i$  demand  $q_i(\cdot)$  must then be locally flat above  $p$ . As noted above, there cannot be another firm that competes with  $i$  at price  $p$  which also has an atom at  $p$ , and so  $q_i$  does not jump down discretely at  $p$ . In addition, any firm that competes with  $i$  at  $p$  obtains a discrete increase in demand if it slightly undercuts  $p$ , and so such a

firm would never choose a price immediately above  $p$ . Since there is no firm that competes with  $i$  at prices immediately above  $p$ , firm  $i$  loses no demand if it raises its price slightly above  $p$ , which is not compatible with  $p$  maximizing the firm's profit. Therefore, firm  $i$  cannot have an atom at  $p < 1$ , and this completes the proof of part (iii). This implies that each firm's demand  $q_i(p)$  is continuous in the interval  $[p_0, 1)$ .

Similarly, if  $p_0$  is the minimum price ever chosen in the market, then all prices in the interval  $[p_0, 1]$  are sometimes chosen. If  $p$  is in firm  $i$ 's support but no firm is active in an interval  $(p, p')$  above  $p$ , then firm  $i$  has flat demand over the range  $(p, p')$ , and this cannot occur in equilibrium. This completes the proof of part (iv).

Suppose now that there are at least three firms. Let  $P_{ij}$  denote the set of prices in  $[p_0, 1]$  that are in the supports of both firm  $i$  and firm  $j$ , which is a closed set. Part (iv) implies that the collection  $\{P_{ij}\}$  covers the interval  $[p_0, 1]$ , and since each firm participates, at least two of the sets in  $\{P_{ij}\}$  are non-empty. If there were no price in the support of three or more firms then the collection  $\{P_{ij}\}$  would consist of disjoint sets. However, since  $[p_0, 1]$  is connected it cannot be covered by two or more disjoint closed sets, and we deduce that at least two sets in  $\{P_{ij}\}$  must overlap, which proves part (v).

Firms can have an atom at the reservation price  $p = 1$ . However, as noted above, if firm  $i$  has an atom at  $p = 1$  then no potential competitor can also have an atom at 1, which implies that when firm  $i$  chooses  $p = 1$  it sells *only* to its captive customers and so its profit is precisely  $\pi_i = \alpha_i$ . If no firm has an atom at  $p = 1$  then any firm with  $p = 1$  in its support (and there must be at two such firms from part (iv)) has profit equal to  $\alpha_i$ . This completes the proof for part (i).

Let firm  $j$  be a firm that obtains profit equal to  $\alpha_j$ . Then the minimum price ever chosen,  $p_0$ , must be no higher than  $\rho_j$  (for otherwise firm  $j$  could obtain more profit by choosing  $p = p_0$ ), and so  $p_0$  cannot exceed the highest  $\rho_i$ . Since no firm sets a price below its  $\rho_i$ , the minimum price  $p_0$  (which from part (iv) is sometimes chosen by at least two firms) must be weakly above the second lowest  $\rho_i$ . Finally, if the firm with the highest  $\rho_i$  has  $p_0$  in its support, then  $p_0$  cannot be strictly lower than this highest  $\rho_i$ , and so must equal this highest  $\rho_i$ . This completes the proof for part (vi).

*Proof of Lemma 2:* Consider firm  $i$ 's demand in (3), which can be expressed as

$$\frac{q_i(p)}{\sigma_i} = \frac{q_{i/j}(p)}{\sigma_i} - G_j(p) \frac{q_{ij}(p)}{\sigma_i \sigma_j}. \quad (23)$$



Note that the demand expression  $q_{i/j}$  in (4) can be written in terms of the interaction parameters as

$$\frac{q_{i/j}(p)}{\sigma_i} = \sum_{S|i,j \notin S} (-1)^{|S|} \gamma_{S \cup i} (\prod_{k \in S} G_k(p)) . \quad (24)$$

Because the interactions are symmetric,  $\gamma_{S \cup i} = \gamma_{S \cup j}$  for each set  $S$  which excludes  $i$  and  $j$ , and so  $q_{i/j}(p)/\sigma_i = q_{j/i}(p)/\sigma_j$ . Expression (23) therefore implies (12).

Firm  $i$ 's captive-to-reach ratio is  $\alpha_i/\sigma_i = (\alpha_i + \alpha_{ij})/\sigma_i - \alpha_{ij}/\sigma_i$ . Similarly to (24) we have

$$\frac{\alpha_i + \alpha_{ij}}{\sigma_i} = \sum_{S|i,j \notin S} (-1)^{|S|} \gamma_{S \cup i} (\prod_{k \in S} \sigma_k) = \sum_{S|i,j \notin S} (-1)^{|S|} \gamma_{S \cup j} (\prod_{k \in S} \sigma_k) = \frac{\alpha_j + \alpha_{ij}}{\sigma_j} ,$$

and (13) follows.

*Proof of Proposition 1:* The argument in the main text proves that all firms use the same minimum price  $p_0$ . Firm  $i$ 's profit is therefore  $\sigma_i p_0$ , and if price  $p$  is also in its support then  $q_i(p)/\sigma_i = p_0/p$ . Therefore, expression (12) implies that if both firm  $i$  and firm  $j$  have  $p$  in their support then  $G_i(p) = G_j(p)$  in equilibrium.

We claim that this implies that no firm can have a ‘‘gap’’ in its price support, so that firm  $i$ 's support takes the form  $[p_0, p_i]$  for some maximum price  $p_i$ . For if not, there is at least one firm with a gap, and let firm  $i$  be the firm with a gap that starts at the *lowest* price, say  $a$ , and where  $b > a$  is the next price in firm  $i$ 's support. Therefore, firm  $i$  does not use prices in the interval  $(a, b)$  and so  $G_i(a) = G_i(b)$ . If a firm is not active at price  $a$ , it will never use a higher price, and so any firm that is active at  $b$  must also be active at  $a$ . Part (iv) of Lemma 1 implies that at least two firms are active at any price in  $(a, b)$ , and in particular, some firm, say  $j$ , which is active at  $b$  must also be active at prices immediately below  $b$  as well (as well as being active at  $a$ ). Therefore, for this firm  $G_j(b) > G_j(a)$ , which contradicts  $G_i(a) = G_i(b)$  given that both  $i$  and  $j$  are active at both  $a$  and  $b$ .

Since no firm has a gap, the price support of firm  $i$  takes the form  $[p_0, p_i]$ . Since  $G_i(p_i) \equiv \sigma_i$ , if  $p_i > p_j$  then both  $i$  and  $j$  are active at  $p_j$  and so  $\sigma_i > G_i(p_j) = G_j(p_j) = \sigma_j$  and firms with a higher maximum price must have a larger reach, and so with the given labelling of firms we have  $p_1 \leq \dots \leq p_n$ . Since at least two firms must be active at  $p = 1$  we have  $p_{n-1} = p_n = 1$  as claimed. Finally, we show stochastic dominance in price strategies. For  $p \in [p_0, p_1]$  all firms have price  $p$  in their support. Therefore,  $G_1(p) = \dots = G_n(p)$  and so  $F_1(p) \geq \dots \geq F_n(p)$ . For  $p \in [p_{i-1}, p_i]$ , firms  $j < i$  have  $F_j(p) = 1$  while firms  $j \geq i$  have

$p$  in their support. Therefore,  $G_i(p) = \dots = G_n(p)$ , and so again  $F_1(p) \geq \dots \geq F_n(p)$  as claimed.

*Proof of Proposition 2:* First observe that when  $n = 2$ , exit or merger leaves a monopoly and so consumers obtain zero surplus and the result holds trivially. Therefore, in the following suppose  $n \geq 3$ .

(i) Welfare is the total fraction of consumers reached by any firm, which falls by  $\alpha_i$  when firm  $i$  exits. Consumer surplus is welfare minus industry profit. Suppose that before exit the firm with the largest reach is labelled  $n$ , in which case industry profit is  $\rho_n(\sum_i \sigma_i)$ .

Suppose first that the firm that exits is not the largest, so  $i \neq n$ . Then the minimum price increases from  $\alpha_n/\sigma_n$  to  $(\alpha_n + \alpha_{in})/\sigma_n$ , and the change in consumer surplus as a result of exit is

$$-\alpha_i + \frac{\alpha_n}{\sigma_n} \left( \sum_j \sigma_j \right) - \frac{\alpha_n + \alpha_{in}}{\sigma_n} \left( \sum_{j \neq i} \sigma_j \right) = -\frac{\alpha_{in}}{\sigma_n} \left( \sum_{j \neq n} \sigma_j \right) < 0 ,$$

where the equality follows from (13). If instead it is the largest firm, firm  $n$ , which exits, expression (13) implies that all remaining firms have captive-to-reach ratios that are greater than  $\alpha_n/\sigma_n$ , and so again the minimum price rises after exit. Since firm  $n$ 's profit was  $\alpha_n$ , which was its contribution to welfare, the change in consumer surplus is the opposite to the change in the remaining firms' profit, and the latter is positive since the minimum price rises. This completes the proof for part (i).

(ii) Before the merger all firms used the same minimum price,  $p_0$ . Suppose two firms,  $i$  and  $j$ , merge to form firm  $M$ , and the new minimum price in the market is  $\hat{p}_0$ . The reach of firm  $M$  is  $\sigma_M = \sigma_i + \sigma_j - \sigma_{ij}$ , which is less than the sum of the reaches of  $i$  and  $j$ . We will show that  $\hat{p}_0 > p_0$ , which in turn implies that the impact of the merger on the profits of the non-merging firms must be positive (since the profit of a non-merging firm  $k$  is at least  $\sigma_k \hat{p}_0$ , which exceeds its pre-merger profit). Therefore, if the merger is profitable, industry profits rise and consumers are harmed.

Since the post-merger market need not exhibit symmetric interactions, it may be that only a subset of firms have  $\hat{p}_0$  in their price support. First suppose that firm  $M$  uses the minimum price  $\hat{p}_0$  and hence obtains profit  $\sigma_M \hat{p}_0$ . Since  $\sigma_M < \sigma_i + \sigma_j$ , in order for the merger to be profitable it must be that  $\hat{p}_0 > p_0$  as claimed.

Next suppose that firm  $M$  does not use the minimum price  $\hat{p}_0$ , and its minimum price

is  $P > \hat{p}_0$ . The demand function of the merged firm is  $q_M(p) = q_{i/j}(p) + q_{j/i}(p) - q_{ij}(p)$ . Let firm  $k$  be a non-merging firm that does price at  $p_0$ . A similar argument to that used to derive (12) shows that

$$\frac{q_{i/j}(P)}{\sigma_i} - \frac{q_k(P)}{\sigma_k} = \frac{q_{i/j}(P)}{\sigma_i} - \frac{q_{k/j}(P)}{\sigma_k} = q_{ik/j}(P) \left( \frac{F_M(P)}{\sigma_k} - \frac{F_k(P)}{\sigma_i} \right) < 0 .$$

Here, the first equality follows since  $P$  is the minimum price charged by the merged firm, and so firm  $k$ 's demand at  $P$  is the same as if  $j$  was absent, while the second equality follows since the market without  $j$  continues to have symmetric interactions, and so the appropriate adjustment of (12) applies. (The function  $q_{ik/j}$  is the demand from consumers who consider both  $i$  and  $k$  in the market absent  $j$ .) The final inequality follows since  $P$  is the merged firm's minimum price, so  $F_M(P) = 0 < F_k(P)$ . The same argument for  $q_{j/i}$  shows that  $q_{j/i}(P)/\sigma_j < q_k(P)/\sigma_k$ .

Therefore,

$$p_0(\sigma_i + \sigma_j) \leq Pq_M(P) \leq P[q_{i/j}(P) + q_{j/i}(P)] < \frac{P}{\sigma_k} q_k(P)(\sigma_i + \sigma_j) \leq \hat{p}_0(\sigma_i + \sigma_j) ,$$

and hence  $p_0 < \hat{p}_0$  as claimed. Here, the first inequality follows since post-merger profit,  $Pq_M(P)$ , is not below the pre-merger combined profit, and the final inequality follows since firm  $k$  weakly prefers the price  $\hat{p}_0$  to  $P$  in the post-merger market.

*Proof of Proposition 4:* (i) No firm will ever choose a price strictly above the maximum price used by a larger firm, for it will then be undercut by the larger firm and so have no demand or profit. Therefore,  $H_1 \leq \dots \leq H_n$ . Since at least two firms use the price  $p = 1$ , we have  $H_{n-1} = H_n = 1$ . (Firm  $n$  must have an atom at  $p = 1$  for otherwise firm  $n - 1$  would have zero profit when it chooses price 1. Firm  $n - 1$  cannot also have an atom at  $p = 1$  for otherwise these firms have an incentive to undercut each other.) Since price distributions are continuous, if firm  $i < n - 1$  chooses the price  $H_{i+1}$  it will also have zero demand and profit, and so the inequality is strict for all firms except the largest two.

Turning to the minimum prices  $L_i$ , assume for contradiction that  $L_{i+1} < L_i$  for some  $i < n$ . Define  $A(p) \equiv \prod_{k>i+1} (1 - F_k(p))$  as the probability that all firms larger than  $i + 1$  price above  $p$ . (As usual, if  $i + 1 = n$  then set  $A(p) \equiv 1$ .) As a preliminary point, note that for all  $p$ , the ratio

$$\frac{q_i(p)}{A(p)(1 - F_{i+1}(p))} = [\beta_i + \beta_{i-1}(1 - F_{i-1}(p)) + \beta_{i-2}(1 - F_{i-1}(p))(1 - F_{i-2}(p)) + \dots]$$

is weakly decreasing in  $p$ . In particular, since  $F_{i+1}(L_{i+1}) = 0 < F_{i+1}(L_i)$

$$\frac{q_i(L_{i+1})}{A(L_{i+1})} \geq \frac{q_i(L_i)}{A(L_i)(1 - F_{i+1}(L_i))} > \frac{q_i(L_i)}{A(L_i)}. \quad (25)$$

By revealed preference

$$L_i q_i(L_i) \geq L_{i+1} q_i(L_{i+1}) \quad (26)$$

and

$$\begin{aligned} L_{i+1} q_{i+1}(L_{i+1}) &= L_{i+1} [\beta_{i+1} A(L_{i+1}) + q_i(L_{i+1})] \\ &\geq L_i q_{i+1}(L_i) = L_i \left[ \beta_{i+1} A(L_i) + \frac{q_i(L_i)}{1 - F_{i+1}(L_i)} \right] \\ &> L_i [\beta_{i+1} A(L_i) + q_i(L_i)], \end{aligned} \quad (27)$$

since  $F_{i+1}(L_i) > 0$ . From (26) and (27) we have

$$L_{i+1} A(L_{i+1}) > L_i A(L_i). \quad (28)$$

But multiplying each side of (25) by the corresponding side of (28) implies that

$$L_{i+1} q_i(L_{i+1}) > L_i q_i(L_i)$$

contrary to (26). We deduce that minimum prices  $L_i$  weakly increase with  $i$ .

To show that  $H_i > L_{i+1}$ , note first that if  $H_i < L_{i+1}$  then because  $L_j$  and  $H_j$  increase with  $j$  there is an interval of prices  $(H_i, L_{i+1})$  used by no firm, which is contrary to Lemma 1. If  $H_i = L_{i+1}$ , Lemma 1 shows that at least one firm  $k$  other than  $i$  uses prices just below  $H_i$ . Since  $L_j$  increases with  $j$  we must have  $k < i$ . But since  $i < n$  has no atom at  $p = H_i$ , firm  $k$  will almost surely be undercut by  $i$  when it uses  $p \approx H_i$  in which case it makes zero profit (contrary to Lemma 1). We deduce that  $H_i > L_{i+1}$ .

(ii) If exactly  $k \geq 3$  firms use the minimum price  $p_0$ , part (i) above shows that these will be the firms  $i = 1, \dots, k$ . Following the same argument as at the start of section 4,  $q'_i(p_0)/\sigma_i = -1/p_0$  for each firm  $i \leq k$ . From (7), this entails  $\sum_{j \in \{1, \dots, k\}/i} \gamma_{ij} G'_j(p_0) = 1/p_0$  for each  $i \leq k$ . By Farkas's Lemma, this system of linear equations has no solution with each  $G'_i(p_0) \geq 0$  if and only if there exists a vector  $(x_1, \dots, x_k)$  satisfying

$$\sum_{j=1}^k x_j < 0 \text{ and } \sum_{j \in \{1, \dots, k\}/i} \gamma_{ij} x_j \geq 0 \text{ for each } i \leq k. \quad (29)$$

With nested reach we have  $\gamma_{ij} = 1/\max\{\sigma_i, \sigma_j\}$ . Let  $x_1 = x_2 = 1$ ,  $x_3 = \dots = x_{n-1} = 0$  and  $x_n = -(2 + \varepsilon)$ , which has a negative sum. Then we have  $\sum_{j \in \{1, \dots, k\}/1} \gamma_{1j} x_j =$

$\sum_{j \in \{1, \dots, k\}/2} \gamma_{2j} x_j = \frac{1}{\sigma_2} - \frac{2+\varepsilon}{\sigma_k}$ , for  $2 < i < n$  we have  $\sum_{j \in \{1, \dots, k\}/i} \gamma_{ij} x_j = \frac{2}{\sigma_i} - \frac{2+\varepsilon}{\sigma_k}$ , and finally  $\sum_{j \in \{1, \dots, k\}/k} \gamma_{jk} x_j = \frac{2}{\sigma_k}$ . If  $\sigma_k > 2\sigma_2$  then each of these terms is positive for sufficiently small  $\varepsilon > 0$ , and so (29) holds. Therefore, if  $\sigma_k > 2\sigma_2$  it is not possible that  $k$  firms use the minimum price. Moreover, if  $\tilde{k} > k$  then  $\sigma_{\tilde{k}} > 2\sigma_2$  and so it is also impossible to have a larger number of firms using the minimum price.

(iii) We construct an equilibrium of the stated form. The profit of the largest firm  $n$  is  $\pi_n = \beta_n$ , its number of captive customers, and denote the profit of smaller firms by  $\pi_i$ . In the highest interval  $[p_{n-1}, 1]$  used by the two largest firms, these firms are sure to be undercut by all smaller rivals, and so in this range their CDFs must satisfy

$$\beta_n + \beta_{n-1}(1 - F_{n-1}(p)) = \frac{\beta_n}{p} ; \beta_{n-1}(1 - F_n(p)) = \frac{\pi_{n-1}}{p} .$$

Since  $F_n(p_{n-1}) = 0$  it follows that  $p_{n-1}$  and  $\pi_{n-1}$  are related as  $\pi_{n-1} = \beta_{n-1}p_{n-1}$ . We have  $F_{n-1}(1) = 1$ , while the largest firm has an atom at  $p = 1$  with probability  $1 - F_n(1) = \pi_{n-1}/\beta_{n-1} = p_{n-1}$ .

In the lowest interval  $[p_1, p_2]$  used by the two smallest firms, these firms are sure to undercut all larger rivals, and so in this range their CDFs must satisfy

$$\beta_2 + \sigma_1(1 - F_1(p)) = \frac{\pi_2}{p} ; \sigma_1(1 - F_2(p)) = \frac{\pi_1}{p}$$

and since  $F_1(p_1) = F_2(p_1) = 0$  it follows that

$$\pi_1 = \sigma_1 p_1 ; \pi_2 = (\sigma_1 + \beta_2) p_1 .$$

Since  $F_1(p_2) = 1$  we have  $\pi_2 = \beta_2 p_2$ , which combined with the previous expression for  $\pi_2$  implies that

$$p_2 = \frac{\sigma_1 + \beta_2}{\beta_2} p_1 . \quad (30)$$

If there are just three firms, these are the two price intervals in the equilibrium. With more than three firms there are intermediate intervals, and in the interval  $[p_i, p_{i+1}]$ , where  $1 < i < n - 1$ , firms  $i$  and  $i + 1$  are active and will be undercut by smaller rivals and undercut their larger rivals. Therefore, in this range their CDFs must satisfy

$$\beta_{i+1} + \beta_i(1 - F_i(p)) = \frac{\pi_{i+1}}{p} ; \beta_i(1 - F_{i+1}(p)) = \frac{\pi_i}{p} . \quad (31)$$

Since  $F_{i+1}(p_i) = 0$  it follows that  $\pi_i = \beta_i p_i$ .

An intermediate firm  $i$ , where  $2 \leq i \leq n - 1$ , is active in both the intervals  $[p_{i-1}, p_i]$  and  $[p_i, p_{i+1}]$ , and its CDF  $F_i$  needs to be continuous across the threshold price  $p_i$ . At the price  $p_i$  we therefore require that

$$\frac{\pi_{i-1}}{\beta_{i-1}p_i} = 1 - F_i(p_i) = \frac{1}{\beta_i} \left( \frac{\pi_{i+1}}{p_i} - \beta_{i+1} \right), \quad (32)$$

where in the case of  $i = 2$  we have written  $\beta_1 = \sigma_1$ . If we write  $p_n = 1$  then we have  $\pi_i = \beta_i p_i$  for all firms  $1 \leq i \leq n$ , and so for  $2 \leq i \leq n - 1$  expression (32) entails expression (16). This is a second-order difference equation in  $p_i$  where  $p_1$  is free,  $p_2$  is given in (30), and the terminal condition  $p_n = 1$  serves to pin down  $p_1$ . It is clear from (30) and (16) that the sequence  $p_1, p_2, p_3, \dots$  is an increasing sequence of price thresholds. This completes the description of the candidate equilibrium.

We next show that firms have no incentive to deviate from these strategies. By construction, firm  $i$  is indifferent between choosing any price in the interval  $[p_{i-1}, p_{i+1}]$ , assuming its rivals follow the stated strategies. We need to check that a firm's profit is no higher if it chooses a price outside this interval. Consider first an upward price deviation, which is only relevant if  $i < n - 1$ . If  $i < n - 2$  and firm  $i$  chooses a price above  $p_{i+2}$  it has no demand since firm  $i + 1$  is sure to set a lower price and all firm  $i$ 's potential customers also consider firm  $(i + 1)$ 's price. Suppose then that  $i < n - 1$  and firm  $i$  chooses a price  $p \in [p_{i+1}, p_{i+2}]$ , in which case it has demand  $\beta_i$  if its price is below the prices of both rivals  $i + 1$  and  $i + 2$ . Therefore, from (31) its profit with such a price is

$$p\beta_i[1 - F_{i+1}(p)][1 - F_{i+2}(p)] = \frac{\beta_i\pi_{i+1}}{\beta_{i+1}^2} \left( \frac{\pi_{i+2}}{p} - \beta_{i+2} \right) = p_{i+1} \frac{\beta_i\beta_{i+2}}{\beta_{i+1}} \left( \frac{p_{i+2}}{p} - 1 \right).$$

This profit decreases from  $\pi_i = \beta_i p_i$  at  $p = p_{i+1}$  to zero at  $p = p_{i+2}$ . We deduce that firm  $i$  cannot increase its profit by choosing a price above  $p_{i+1}$ .

Next consider a downward price deviation, so that firm  $i$  chooses a price below  $p_{i-1}$  (which is only relevant when  $i > 2$ ). Suppose that this firm chooses a price in the interval  $[p_j, p_{j+1}]$ , where  $j \leq i - 2$ . The firm will undercut all firms larger than firm  $j + 1$ , and so obtain demand at least  $\beta_{j+2} + \dots + \beta_i$ . It will also serve the segment  $\beta_{j+1}$  if it undercuts firm  $j + 1$  and it will additionally serve the segment  $\beta_j$  if it undercuts both firms  $j$  and  $j + 1$ . Putting this together implies that the firm's profit with price  $p \in [p_j, p_{j+1}]$  is

$$p \left\{ \beta_{j+2} + \dots + \beta_i + (1 - F_{j+1}(p))(\beta_{j+1} + \beta_j(1 - F_j(p))) \right\}. \quad (33)$$

Given the CDFs in (31), this profit is a convex function of  $p$  and so must be maximized in this range either at  $p_j$  or at  $p_{j+1}$ . Therefore, we can restrict our attention to deviations by firm  $i > 2$  to the threshold prices  $\{p_1, p_2, \dots, p_{i-2}\}$ . If it chooses price  $p_j$  where  $2 \leq j \leq i-2$ , expression (33) implies its profit is  $p_j \{\beta_{j+1} + \dots + \beta_i + \beta_j(1 - F_j(p_j))\}$ . Expression (32) implies that  $\beta_j(1 - F_j(p_j))$  is equal to  $\beta_{j+1}(\frac{p_{j+1}}{p_j} - 1)$ , in which case the above deviation profit with price  $p_j$  is

$$p_j \left( \beta_{j+1} + \dots + \beta_i + \beta_{j+1} \left( \frac{p_{j+1}}{p_j} - 1 \right) \right) = \beta_{j+1} p_{j+1} + (\beta_{j+2} + \dots + \beta_i) p_j. \quad (34)$$

One can check that expression (34) holds also for  $j = 1$ . We need to show that (34) is no higher than firm  $i$ 's equilibrium profit, which is  $\pi_i = \beta_i p_i$ . We do this in two steps: (i) we show that (34) is increasing in  $j$  given  $i$ , so that  $j = i - 2$  is the most tempting of these deviations for firm  $i$ , and (ii) we show (34) is below  $\beta_i p_i$  when  $j = i - 2$ .

To show (i), suppose that  $i \geq 4$ , which is the only relevant case, and suppose that  $1 \leq j \leq i - 3$ . Then firm  $i$ 's deviation profit with price  $p_{j+1}$  from (34) is

$$\begin{aligned} & \beta_{j+2} p_{j+2} + (\beta_{j+3} + \dots + \beta_i) p_{j+1} = \beta_{j+1} p_j + \beta_{j+2} p_{j+1} + (\beta_{j+3} + \dots + \beta_i) p_{j+1} \\ & \geq \beta_{j+1} p_j + \beta_{j+2} p_{j+1} + (\beta_{j+3} + \dots + \beta_i) p_{j+1} - (\beta_{j+2} - \beta_{j+1})(p_{j+1} - p_j) \\ & = \beta_{j+1} p_{j+1} + \beta_{j+2} p_j + (\beta_{j+3} + \dots + \beta_i) p_{j+1} \geq \beta_{j+1} p_{j+1} + (\beta_{j+2} + \dots + \beta_i) p_j \end{aligned}$$

where the final expression is the firm's deviation profit with price  $p_j$ , which proves claim (i). (Here, the first equality follows from (16), the first inequality follows from (15) and the fact that  $\{p_j\}$  is an increasing sequence, while the final inequality follows from  $\{p_j\}$  being an increasing sequence.)

To show claim (ii), suppose that  $i \geq 3$ , which is the only relevant case, and observe that

$$\begin{aligned} \beta_i p_i &= \beta_{i-1} p_{i-2} + \beta_i p_{i-1} \\ &\geq \beta_{i-1} p_{i-2} + \beta_i p_{i-1} - (\beta_i - \beta_{i-1})(p_{i-1} - p_{i-2}) \\ &= \beta_{i-1} p_{i-1} + \beta_i p_{i-2} \end{aligned}$$

where the final expression is (34) when  $j = i - 2$ . (Here, the first equality follows from (16) and the inequality follows from  $\{\beta_i\}$  being an increasing sequence.) This completes the proof that the stated strategies constitute an equilibrium.

*Proof of Proposition 5:* Part (i) was demonstrated in the main text, so suppose now that (21) is satisfied. Lemma 1 shows that there is at least one price in all three price supports, and let  $L$  and  $H$  denote respectively the lowest and highest price among the prices in all three supports. (The set of prices in all three supports is closed.) If condition (21) holds we will show that  $L = H$  so there is only one price in all three supports.

Suppose by contradiction that we have  $H > L$ . Either all three firms have the same minimum price  $p_0$  (i.e.,  $L = p_0$ ) or only two firms do, and in the latter case the proof for part (i) shows that it must be firms 1 and 2 that price low. In either case firms 1 and 2 use  $p_0$ , and we have  $G_1(L) = G_2(L) = \delta \geq 0$ . Let  $g = \max\{G_1(H), G_2(H)\}$ . Since we cannot have only one firm active in the open interval  $(L, H)$ , one or both of 1 and 2 must choose prices in  $(L, H)$ , and so  $\delta < g$ .

Firms 1 and 2 obtain respective profits  $p_0\sigma_1$  and  $p_0\sigma_2$ , and let  $\pi_3 \geq p\sigma_3$  denote firm 3's profit. Expression (11) shows that a price  $p$  in firm 3's support satisfies

$$\frac{\pi_3}{\sigma_3 p} = 1 - \gamma_{13}G_1(p) - \gamma_{23}G_2(p) + \gamma G_1(p)G_2(p) ,$$

and setting  $p = L, H$  in the above and subtracting implies that

$$\begin{aligned} \frac{\pi_3}{\sigma_3} \left( \frac{1}{L} - \frac{1}{H} \right) &= \gamma_{13}G_1(H) + \gamma_{23}G_2(H) - \gamma G_1(H)G_2(H) \\ &\quad - \gamma_{13}G_1(L) - \gamma_{23}G_2(L) + \gamma G_1(L)G_2(L) \\ &\leq \gamma_{13}g + \gamma_{23}g - \gamma g^2 - \gamma_{13}\delta - \gamma_{23}\delta + \gamma\delta^2 \\ &= (g - \delta)(\gamma_{13} + \gamma_{23} - \gamma(g + \delta)) . \end{aligned} \tag{35}$$

Here, the inequality follows since  $\gamma_{13} \geq \gamma G_2(H)$  and  $\gamma_{23} \geq \gamma G_1(H)$ , and so the initial expression is weakly increased if we replace  $G_1(H)$  and  $G_2(H)$  by  $g = \max\{G_1(H), G_2(H)\}$ . Likewise, and using that fact that  $G_3(L) = 0$ , for firm  $j = 1, 2$  we have

$$\begin{aligned} p_0 \left( \frac{1}{L} - \frac{1}{H} \right) &= \gamma_{12}G_i(H) + \gamma_{j3}G_3(H) - \gamma G_3(H)G_i(H) - \gamma_{12}G_i(L) \\ &\geq \gamma_{12}g + \gamma_{j3}G_3(H) - \gamma g G_3(H) - \gamma_{12}\delta \\ &\geq (g - \delta)\gamma_{12} . \end{aligned}$$

Since  $\pi_3 \geq \sigma_3 p_0$  and  $g - \delta > 0$ , it follows that

$$\gamma(g + \delta) \leq \gamma_{13} + \gamma_{23} - \gamma_{12} . \tag{36}$$



If  $\gamma = 0$  this inequality contradicts (21), so we deduce that it is not possible to have  $H > L$  when (21) holds and  $\gamma = 0$ . Therefore, suppose henceforth that  $\gamma > 0$ . Then since  $g > 0$  the inequality (36) contradicts the first inequality in (18), which holds whenever  $L > p_0$ . We deduce that if  $H > L$  then all three firms use the same minimum price  $p_0$ .

We show next that if all three firms use the same minimum price, then (21) cannot hold. Suppose that all three firms use  $p_0$  and that  $H$  is the highest price in all three price supports. Then  $\frac{q_1(H)}{\sigma_1} = \frac{q_2(H)}{\sigma_2} = \frac{q_3(H)}{\sigma_3}$  and (17) implies

$$(\gamma_{12} - \gamma_{23})G_2(H) + (\gamma_{13} - \gamma G_2(H))(G_3(H) - G_1(H)) = 0, \quad (37)$$

$$(\gamma_{12} - \gamma_{13})G_1(H) + (\gamma_{23} - \gamma G_1(H))(G_3(H) - G_2(H)) = 0. \quad (38)$$

Condition (21) implies that  $\gamma_{12} > \max\{\gamma_{13}, \gamma_{23}\}$ , and so these expressions imply that  $G_3(H) < \min\{G_1(H), G_2(H)\}$  and also that the terms  $(\gamma_{13} - \gamma G_2(H))$  and  $(\gamma_{23} - \gamma G_1(H))$  are strictly positive. At least one of the  $G_i(H)$  must equal  $\sigma_i$ .

Suppose  $G_1(H) = \sigma_1$ . If  $\sigma_1 \geq G_2(H)$  then (38) implies that  $G_3(H)$  has the sign of

$$(\gamma_{23} - \gamma\sigma_1)G_2(H) - (\gamma_{12} - \gamma_{13})\sigma_1 \leq (\gamma_{23} - \gamma\sigma_1)\sigma_1 - (\gamma_{12} - \gamma_{13})\sigma_1 < 0,$$

where the final strict inequality follows from (21). Since  $G_3$  cannot be negative, this is not possible. Likewise, if  $\sigma_1 \leq G_2(H)$  then (37) implies that  $G_3(H)$  has the sign

$$(\gamma_{13} - \gamma G_2(H))\sigma_1 - (\gamma_{12} - \gamma_{23})G_2(H) \leq (\gamma_{13} - \gamma\sigma_1)\sigma_1 - (\gamma_{12} - \gamma_{23})\sigma_1 < 0.$$

Therefore,  $G_1(H) < \sigma_1$ . If instead  $G_2(H) = \sigma_2$  then we must have  $G_1(H) \leq \sigma_2$ . Then (37) again implies that  $G_3(H) < 0$ . We conclude that  $G_3(H) = \sigma_3$ .

If  $H = 1$  then both firms 1 and 2 have an atom at  $p = 1$ , which requires  $\alpha_{12} = 0$ . However, if  $\alpha_{12} = 0$  then  $\gamma_{12} = \gamma\sigma_3$  and so (21) implies that  $\gamma(\sigma_1 + \sigma_3) > \gamma_{13} + \gamma_{23} \geq \gamma(\sigma_1 + \sigma_2)$ , and so  $\sigma_3 \geq \sigma_2$ , which contradicts the previous condition  $\sigma_3 = G_3(H) < \min\{G_1(H), G_2(H)\}$ . Therefore, we deduce that  $H < 1$ .

Since only firms 1 and 2 are active above price  $H$ , we must have  $\alpha_{12} > 0$  and so  $\gamma_{12} > \gamma\sigma_3$ . Expression (17) with  $k = 3$  then implies that  $(G_1(p) - G_2(p))$  is constant for  $p \in [H, 1]$ , so that  $G_1(1) - G_1(H) = G_2(1) - G_2(H) \equiv \eta > 0$ . For  $k \neq 3$  we have  $\sigma_k p_0 = H q_k(H) = q_k(1)$  and hence

$$p_0 \left( \frac{1}{H} - 1 \right) = \frac{q_k(H) - q_k(1)}{\sigma_k} = (\gamma_{12} - \gamma\sigma_3)\eta, \quad (39)$$

whereas  $\sigma_3 p_0 = H q_3(H) \leq q_3(1)$  and so

$$\begin{aligned} p_0 \left( \frac{1}{H} - 1 \right) &\leq \frac{q_3(H) - q_3(1)}{\sigma_3} \\ &= (\gamma_{13} + \gamma_{23})\eta - \gamma[G_1(1)G_2(1) - G_1(H)G_2(H)] \\ &= [(\gamma_{13} + \gamma_{23}) - \gamma(G_1(H) + G_2(H) + \eta)]\eta . \end{aligned}$$

With (39) this implies

$$\begin{aligned} \gamma_{12} - \gamma_{13} - \gamma_{23} &\leq \gamma[\sigma_3 - (G_1(H) + G_2(H) + \eta)] \\ &< -\gamma(G_k(H) + \eta) = -\gamma G_k(1) \end{aligned}$$

for  $k = 1, 2$ . (Here, the strict inequality follows since  $\sigma_3 = G_3(H) < \min\{G_1(H), G_2(H)\}$ .) As 1 and 2 cannot both have atoms at  $p = 1$ ,  $G_k(1) = \sigma_k \geq \sigma_1$  for some  $k$ , and (21) is contradicted.

In sum, we have shown that when (21) holds, there is only one price in the support of all three firms, say  $p_1$ , which strictly exceeds  $p_0$ , and only firms 1 and 2 are active in the range  $[p_0, p_1)$ . If  $p_1 = 1$  then the proof is complete. If  $p_1 < 1$  then only two firms are active in this range, one of which is firm 3. The remaining issue is which of firms 1 and 2 is the other firm active above  $p_1$ . Expression (17) implies that  $\sigma_1 F_1(p) = \sigma_2 F_2(p)$  in the range  $[p_0, p_1]$ . If  $\sigma_1 = \sigma_2$  then  $F_1 = F_2$ , and so one of these firms cannot drop out before the other and we must have  $p_1 = 1$ . If  $\sigma_2 > \sigma_1$  then in the range  $[p_0, p_1]$  we have  $F_2 > F_1$  and so it is firm 1 that drops out first.

The final step in the proof is to determine the profits of the three firms, as well as the price thresholds  $p_0$  and  $p_1$ . Since firms 1 and 2 have  $p_0$  as their minimum price in this equilibrium, their profits are  $\pi_1 = \sigma_1 p_0$  and  $\pi_2 = \sigma_2 p_0$ . In the range  $[p_0, p_1]$  their adjusted CDFs are given by

$$\gamma_{12} G_j(p) = 1 - \frac{p_0}{p} ,$$

and firm 1 drops out at price  $p_1$ , so that the ratio  $p_0/p_1$  satisfies

$$\gamma_{12} \sigma_1 = 1 - \frac{p_0}{p_1} \tag{40}$$

and so

$$G_2(p_1) = \sigma_1 . \tag{41}$$

Either firm 2 or 3 (or both) obtains exactly its captive profit.<sup>30</sup> Suppose first that firm 3 obtains its captive profit, so that  $\pi_3 = \alpha_3$ . For prices in the upper range  $[p_1, 1]$  only firms 2 and 3 compete and are sure to be undercut by firm 1, so from (11) firm 2's CDF satisfies

$$1 - \gamma_{13}\sigma_1 - \gamma_{23}G_2 + \gamma\sigma_1G_2 = \frac{\rho_3}{p} ,$$

where recall that  $\rho_3$  is firm 3's captive-to-reach ratio. In order for  $G_2(\cdot)$  to be continuous at the threshold price  $p_1$ , (41) implies that

$$1 - \gamma_{13}\sigma_1 - \gamma_{23}\sigma_1 + \gamma\sigma_1^2 = \frac{\rho_3}{p_1} ,$$

which determines  $p_1$ . Expression (40) in turn implies that

$$p_0 = p_1(1 - \gamma_{12}\sigma_1) = \frac{\rho_3(1 - \gamma_{12}\sigma_1)}{1 - \gamma_{13}\sigma_1 - \gamma_{23}\sigma_1 + \gamma\sigma_1^2} . \quad (42)$$

It is convenient to write  $P$  for the right-hand side above, so that

$$P = \frac{\rho_3(1 - \gamma_{12}\sigma_1)}{1 - \gamma_{13}\sigma_1 - \gamma_{23}\sigma_1 + \gamma\sigma_1^2} = \frac{\alpha_3(\alpha_2 + \alpha_{23})}{\alpha_3\sigma_2 + \alpha_{23}(\sigma_2 - \sigma_1)} , \quad (43)$$

where the second equality follows by routine manipulation. Note from the first expression for  $P$  above that the condition  $P < \rho_3$  corresponds to (21). Expression (42) implies

$$p_1 = \frac{\alpha_3\sigma_2}{\alpha_3\sigma_2 + \alpha_{23}(\sigma_2 - \sigma_1)} . \quad (44)$$

Alternatively, suppose firm 2 obtains its captive profit, so that  $\pi_2 = \alpha_2$ . Since the firm has  $p_0$  as its lowest price it follows that

$$p_0 = \rho_2 . \quad (45)$$

Expression (40) then implies that

$$p_1 = \frac{\alpha_2}{\alpha_2 + \alpha_{23}} . \quad (46)$$

For prices in the upper range  $[p_1, 1]$  firm 2's CDF now satisfies

$$1 - \gamma_{13}\sigma_1 - \gamma_{23}G_2 + \gamma\sigma_1G_2 = \frac{\pi_3}{\sigma_3 p} ,$$

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<sup>30</sup>If one of these firms has no atom at  $p = 1$  then the other obtains its captive profit when it chooses  $p = 1$ . If both have an atom at  $p = 1$  then for neither to have an incentive to undercut the other we must have  $\alpha_{23} = 0$ , in which case both firms obtain their captive profit at  $p = 1$ .

where  $\pi_3$  is firm 3's profit. For  $G_2$  to be continuous at  $p_1 = \alpha_2/(\alpha_2 + \alpha_{23})$ , (41) implies

$$1 - \gamma_{13}\sigma_1 - \gamma_{23}\sigma_1 + \gamma\sigma_1^2 = \frac{\alpha_2 + \alpha_{23}}{\alpha_2} \cdot \frac{\pi_3}{\sigma_3},$$

which determines  $\pi_3$ . This can be expressed as

$$\pi_3 = \frac{\alpha_3\rho_2}{P} \quad (47)$$

where  $P$  is given in (43).

Finally, we determine when it is that firm 2 or firm 3 obtains its captive profit. When firm 3 obtains its captive profit, firm 2's minimum price is  $P$  in (43), which must be no lower than  $\rho_2$  if firm 2 is willing to offer this price. Therefore, if  $P < \rho_2$  the equilibrium must instead have firm 2 obtaining its captive profit, in which case the threshold prices and firm 3's profit are given respectively by (45), (46) and (47). Conversely, when firm 2 obtains its captive profit, firm 3's profit is given in (47). This profit cannot be below its captive profit  $\alpha_3$ , which therefore requires  $P \leq \rho_2$ . Therefore, if  $P > \rho_2$  the equilibrium must involve firm 3 obtaining its captive profit, so  $\pi_3 = \alpha_3$ , and the threshold prices  $p_0$  and  $p_1$  are given respectively by (43) and (44). Finally, in the knife-edge case where  $P = \rho_2$  the two equilibria coincide, and firms 2 and 3 each obtain their captive profit. This completes the proof.

*Details for the nested example in section 5:* Recall that the example for Figure 5 has nested reach with  $\sigma_1 = \frac{1}{2}$ ,  $\sigma_2 = \frac{4}{5}$  and  $\sigma_3 = 1$ . Then part (i) of Proposition 5 applies, and all firms use the same minimum price  $p_0 = \frac{1}{5}$  and have profits  $\sigma_i p_0$ . In this example we have  $\gamma_{13} = \gamma_{23} = 1$  and  $\gamma_{12} = \gamma = \frac{5}{4}$ , and so expression (11) implies that for any price in the support of all three firms we have

$$1 - \frac{5}{4}G_2 - G_3 + \frac{5}{4}G_2G_3 = 1 - \frac{5}{4}G_1 - G_3 + \frac{5}{4}G_1G_3 = 1 - G_1 - G_2 + \frac{5}{4}G_1G_2 = \frac{1}{5p}. \quad (48)$$

These simultaneous equations can be solved to give

$$G_1(p) = G_2(p) = \frac{4}{5} - \frac{2}{5}\sqrt{\frac{1-p}{p}}; \quad G_3(p) = 1 - \frac{2}{5}\sqrt{\frac{1}{p(1-p)}}. \quad (49)$$

Here, each  $G_i$  is zero at  $p = p_0$  and  $G_1$  and  $G_2$  increase with  $p$  for prices above  $p_0$ .

A candidate solution is that all three firms choose prices in the range  $[p_0, p_1]$ , then firm 1 drops out leaving firms 2 and 3 active in the range  $[p_1, 1]$ . Here  $F_1$  reaches 1, i.e.,  $G_1$

reaches  $\sigma_1 = \frac{1}{2}$ , at  $p_1 = \frac{16}{25}$ . For prices above  $p_1$  firms 2 and 3 compete alone, with firm 1 sure to undercut them, in which case the required adjusted CDFs in (48) are given by

$$G_2(p) = \frac{4}{3} - \frac{8}{15p} ; G_3(p) = 1 - \frac{8}{15p} .$$

The problem with this candidate solution, however, is that  $G_3 (= F_3)$  in (49) increases with  $p$  only for prices below  $\frac{1}{2}$ , and thereafter it decreases with  $p$  as depicted as the solid curve on Figure 5. The correct solution is then obtained by “ironing” this CDF as shown as the dashed line on the figure so that  $F_3$  is flattened to be no greater than the level  $F_3(p_1) = \frac{1}{6}$  for prices below  $p_1$ . The smaller root of  $G_3 = \frac{1}{6}$  in (49) is  $\hat{p} = \frac{9}{25}$ .

In this example, all three firms are active in the price range  $[\frac{1}{5}, \frac{9}{25}]$ , only firms 1 and 2 are active in the interior range  $[\frac{9}{25}, \frac{16}{25}]$ , and only firms 2 and 3 are active in the range  $[\frac{16}{25}, 1]$ . In the interior range  $[\frac{9}{25}, \frac{16}{25}]$ , the adjusted CDFs  $G_1$  and  $G_2$  need modifying from (49) to reflect that they will be undercut by firm 3 with the constant probability  $F_3(p_1) = \frac{1}{6}$  in this range (in which case they have no demand), so that

$$G_1(p) = G_2(p) = \frac{4}{5} - \frac{24}{125p} .$$

(Again,  $G_1$  reaches  $\sigma_1 = \frac{1}{2}$  at  $p_1 = \frac{15}{25}$ .) With these CDFs, one can check that firm 3 does not gain by choosing a price in the interior range  $[\frac{9}{25}, \frac{16}{25}]$ , and that firm 1 has no incentive to choose a price above  $p_1 = \frac{16}{25}$ , so that this is indeed an equilibrium.