Generational Distribution of Fiscal Burdens: A Positive Analysis

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14 May 2021

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Abstract

This study presents a political economy model with overlapping generations to analyze the effects of population aging on fiscal policy formation and the resulting distribution of the fiscal burden across generations. The analysis focuses on the role of endogenous labor supply and shows that increased political weight of the old, arising from population aging, leads to an increase in the ratios of public debt and labor income tax revenue to GDP and an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP. The result fits well with the evidence in OECD countries.

- Keywords: Generational burden, Overlapping generations, Political economy, Population aging, Public debt
- JEL Classification: D70, E24, E62, H60

*The authors gratefully acknowledge financial support from a Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 18K12802, Uchida) and a Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 18K01650, Ono).

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1 Introduction

How is the burden of fiscal policy distributed across generations? How do demographic changes affect the pattern of generational burdens? To answer these questions, several studies explored the political determinants of fiscal policy in overlapping generations models. Examples are works by Renström (1996), Beauchemin (1998), Boldrin and Rustichini (2000), Razin, Sadka, and Swagel (2004), and Razin and Sadka (2007), which are based on tractable models of the economy and voting process. Recently, Forni (2005), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Mateos-Planas (2010), Ono and Uchida (2016), and Bishnu and Wang (2017) study taxation and public expenditure in a framework in which voting yields time consistent policies. All these works assume a balanced government budget in each period, and thus ignore the possibility of a shift of fiscal burdens onto future generations via public debt issuance.

Several researchers address the political economy of public debt, such as Cukierman and Meltzer (1989), Song, Storesletten, and Zilibotti (2012), Müller, Storesletten, and Zilibotti (2016), Röhrs (2016), Arawatari and Ono (2017), Arai, Naito, and Ono (2018), Ono and Uchida (2018), and Andersen (2019). In these studies, labor income tax on the working generation is the only tax instrument; capital income tax on the retired elderly, which is a possible additional instrument, is abstracted away from the analyses. An exception is Arcalean (2018), who considers dynamic fiscal competition over public spending financed by labor and capital taxes and public debt. He focuses on the effects of fiscal cross-border externalities on welfare and growth.[1] In other words, these studies do not fully address the generational conflict over age-specific taxes and the resulting distribution of the fiscal burden across generations. However, as Mateos-Planas (2010) indicates, demographic changes, such as increasing longevity and declining birth rates, affect voters' interests in taxing different factors, and thus drive the change in the mix of capital and labor income taxes over periods.

To address the generational conflict over taxes and public debt, we present an overlapping generations model in which labor supply is elastic and public education expenditure, which benefits both middle and old people, is financed by labor and capital taxes as well as public debt issue. Following Song, Storesletten, and Zilibotti (2012) and the subsequent literature, we assume probabilistic voting (e.g., Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) in which fiscal policy in each period is determined to maximize the weighted sum of utility of the middle and old. Within this setting, we analyze the effects of population aging on the fiscal burden on current and future generations.

[1] Uchida and Ono (2021) also touch on this association, but limit their analysis to the case of inelastic labor supply and productive public expenditure that benefits only the young, so they do not fully address the issue of the fiscal burden across generations. Their analysis rather focuses on the effects of debt ceilings on policy formation and economic growth.

For the analysis, we discuss three representative factors of aging: life expectancy, population growth rate, and the political weight of the old. We consider the political weight of the old as a representative factor of aging because the voter turnout of older people is significantly higher than that of younger ones in developed countries with aging populations (OECD, 2007). We focus on the ratios of public debt, capital income tax revenue, and labor income tax revenue to GDP as measures of the fiscal burden, and show that elastic labor supply and the political weight of the old are the keys to fit the results with empirical evidence in Organisation for Economic Co-operation and Development (OECD) countries.

We start our analysis by focusing on the inelastic labor supply case. The analysis shows that the ratio of government debt to GDP increases as life expectancy increases, but it decreases as the population growth rate decreases and the political weight of the old increases. The former result appears to be consistent with evidence observed in Figure 1, but the latter does not. We also find a negative association between the ratio of capital income tax revenue to GDP and aging factors. This result seems to be consistent with the evidence observed in Figure 1, but the evidence also shows that some countries such as Ireland, Korea, Sweden, and the United States deviate from the association. Therefore, we find a discrepancy between theory and evidence when labor supply is inelastic.

[Figure 1]

To bridge the gap between theory and evidence, we extend the analysis by assuming endogenous labor supply: individuals are endowed with one unit of labor in middle age and supply it elastically in the labor market to balance the marginal costs and benefits of labor supply in terms of utility. This assumption enables us to present the distortionary effect of taxation, which matters for the present investigation. However, the presence of the distortionary effect makes it harder to obtain analytical solutions for the model. We take a numerical approach to overcome this limitation. In particular, we calibrate the parameter that governs the degree of preferences for public goods to match the average ratio of government expenditure to GDP during 1995–2016 for OECD countries.

Our numerical investigation shows that when labor supply is elastic, all three aging factors work to increase the labor income tax rate, which in turn increases the interest rate through the households' choice of labor supply. A higher interest rate leads to higher debt repayment costs, which in turn leads to more public debt issuance. Therefore, population aging results in an increase in the debt-GDP ratio. This result, which is opposite to the result under the assumption of inelastic labor supply, fits well with the evidence observed in OECD countries in Figure 1.

The numerical investigation also shows that under elastic labor supply, the ratio of capital income tax revenue to GDP decreases as the population growth rate declines and life expectancy increases. This negative association between aging and the ratio, which is qualitatively similar to that in the elastic labor supply case, still fails to explain some countries that deviate from the negative association. However, we demonstrate that the increased political weight of the old produces an initial decrease followed by an increase in the ratio. This U-shaped pattern can explain the existence of countries that deviate from the negative association. Our analysis suggests the importance of identifying population aging as a factor when analyzing its impact on capital income taxation. The analysis also suggests that the increased political weight of the old and elastic labor supply are the keys to account for the different patterns of the ratio observed in OECD countries.

The mechanism behind the U-shaped pattern is as follows. The ratio of capital income tax revenue to GDP is the product of two factors: the ratio of capital income to GDP and the capital income tax rate. Population aging, represented by the increased political weight of the old, has the following effects on these two factors. First, as the population ages, the government raises the labor income tax rate and increases public debt issues to pass the fiscal burden onto the young. A higher labor income tax rate reduces labor supply and saving, and a higher level of public debt strengthens the crowding-out effect in the capital market. These two effects in turn slow down capital accumulation and raise the interest rate, and thereby increase the ratio of capital income to GDP. This is the positive effect on the ratio. Second, as the population ages, the government lowers the capital income tax rate to reduce the fiscal burden on the old. However, as the population ages further, the government chooses to raise the capital income tax rate in response to public debt accumulation. Thus, aging produces an initial decrease followed by an increase in the capital income tax rate, yielding the U-shaped effect on the ratio.

The present study is related to recent theoretical contributions on fiscal politics. Razin, Sadka, and Swagel (2004); Razin and Sadka (2007); Bassetto (2008); and Mateos-Planas (2010) focus on the association between population aging and capital income taxation. Mateos-Planas (2010) examines the United States based on a median voter framework and shows that the tax rate initially decreases and then increases as population ages. The present study instead uses a probabilistic voting framework that reflects the preferences of all voters, and shows that the keys to this U-shaped pattern are elastic labor supply and the increased political weight of the old. Song, Storesletten, and Zilibotti (2012) also consider the role of the political weight of the old, but focus on public debt finance and abstract capital income taxation away from their analysis. Thus, our study bridges the gap in the literature by comprehensively evaluating the effects of population aging on fiscal policy formation and the resulting fiscal burden on current and future generations.

The organization of the rest of this paper is as follows. Section 2 presents the model. Section 3 gives the characterization of the political equilibrium and then investigates the policy response to the increased political power of the old. Section 4 provides concluding remarks. The Appendix provides proofs of propositions and supplementary explanations of the analytical and numerical methods.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live at most for two periods: middle and old age.[2] Individuals face uncertain lifetimes in the second period of life. Let $\pi \in [0,1]$ denote life expectancy (i.e., the probability of living in old age). This is idiosyncratic for all individuals and is constant across periods. Each middle individual gives birth to $1+n$ children. The middle population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1} = (1+n)N_t$.

Individuals

Individuals have the following economic behavior over their life cycles. During middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. In old age, they retire and receive and consume returns from savings.

Consider a middle individual in period t. The individual is endowed with one unit of time. He/she supplies it elastically in the labor market and obtains labor income $w_t l_t$, where w_t is wage rate per unit of labor and $l_t \in (0,1)$ is the amount of labor supply. After paying tax $\tau_t w_t l_t$, where τ_t is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings held as an annuity and invested in physical capital, s_t. Therefore, the period-t budget constraint for the middle becomes

$$c_t + s_t \leq (1-\tau_t) w_t l_t. \tag{1}$$

The period $t+1$ budget constraint in old age is

$$d_{t+1} \leq \left(1-\tau_{t+1}^K\right) \frac{R_{t+1}}{\pi} s_t, \tag{2}$$

where d_{t+1} is consumption, τ_{t+1}^K is the period-$t+1$ capital income tax rate, and R_{t+1} is the gross return from savings. If an individual dies at the end of the middle period, then his or her annuitized wealth is transferred to the individuals who live throughout old age via annuity markets. Therefore, the return from saving becomes R_{t+1}/π under the assumption of perfect annuity markets.

The preferences of a middle individual in period t are specified by the following expected utility function:

$$\ln\left(c_t - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\pi \left(\ln d_{t+1} + \theta \ln g_{t+1}\right),$$

[2]In conventional terminology, the first period of life is called young. We refer to it as middle instead of young because in Section 3.2.3, we introduce pre-employment young into the model and extend it to a three-period version.

where g_t is per-capita public goods in period t, $\beta \in (0,1)$ is a discount factor, and $\theta(>0)$ is the degree of preferences for public goods. Following Greenwood, Hercowitz, and Huffman (1988) and Müller, Storesletten, and Zilibotti (2016), we assume that the disutility from labor effort is $(l_t)^{1+1/v}/(1+1/v)$, where $v(>0)$ parameterizes the Frisch elasticity of labor supply. We substitute the budget constraints (1) and (2) into the utility function to form the unconstrained maximization problem:

$$\max_{\{s_t, l_t\}} \ln\left((1-\tau_t)w_t l_t - s - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\pi\left(\ln\left(1-\tau_{t+1}^K\right) R_{t+1} s_t + \theta \ln g_{t+1}\right).$$

By solving this problem, we obtain the following labor supply and savings functions:

$$l_t = [(1-\tau_t)w_t]^v, \tag{3}$$

$$s_t = \frac{\beta\pi}{1+\beta\pi} \cdot \frac{1/v}{1+1/v}[(1-\tau_t)w_t]^{1+v}. \tag{4}$$

The labor supply and savings decrease as the labor income tax rate, τ_t, increases, but they increase as the wage rate, w_t, increases.

Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers and that produce the final output Y_t with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}$. Here, $A(>0)$ is total factor productivity, which is constant across periods, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share in production.

In each period, a firm chooses capital and labor to maximize its profit, $A(K_t)^\alpha (L_t)^{1-\alpha} - \rho_t K_t - w_t L_t$, where R_t is the gross return on physical capital and w_t is the wage rate. The firm's profit maximization leads to

$$K_t : R_t = \alpha A (k_t)^{\alpha-1}(l_t)^{1-\alpha}, \tag{5}$$

$$L_t : w_t = (1-\alpha)A(k_t)^\alpha (l_t)^{-\alpha}, \tag{6}$$

where $k_t \equiv K_t/N_t$ is per-capita capital and $l_t \equiv L_t/N_t$ is per-capita labor. Capital fully depreciates in a single period.

Government budget constraint

Government expenditure is financed by both taxes on capital and labor income and public debt issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is $\tau_t w_t l_t N_t + \tau_t^K (R_{t+1}/\pi) s_{t-1}\pi N_{t-1} + B_{t+1} = R_t B_t + G_t$, where $\tau_t w_t l_t N_t$ is aggregate labor income tax revenue, $\tau_t^K R_t s_{t-1} N_{t-1}$ is aggregate capital income tax revenue, B_{t+1} is newly issued public debt, $R_t B_t$ is debt repayment, and G_t is aggregate public expenditure. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the constraint by N_t, we can obtain a per- capita expression of the government budget constraint:

$$\tau_t w_t l_t + \frac{\tau_t^K R_t s_{t-1}}{1+n} + (1+n)b_{t+1} = R_t b_t + \frac{(1+n)+\pi}{1+n} g_t, \tag{7}$$

where $b_t \equiv B_t/N_t$ is per-capita debt and $g_t \equiv G_t/(N_t + \pi N_{t-1})$ is the per-capita public expenditure.

Capital market-clearing condition

Public debt is traded in the domestic capital market. The market-clearing condition for capital is $K_{t+1} + B_{t+1} = N_t s_t$, which expresses the equality of total savings by middle agents in period t, $N_t s_t$, to the sum of the stocks of aggregate physical capital and aggregate public debt at the beginning of period $t+1$. We can rewrite this condition as

$$(1+n)(k_{t+1} + b_{t+1}) = s_t. \tag{8}$$

Economic Equilibrium

We define an economic equilibrium in the present framework as follows.

- **Definition 1.** Given a sequence of policies, $\{\tau_t, \tau_t^K, g_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t, d_t, s_t, l_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, and prices, $\{w_t, R_t\}_{t=0}^{\infty}$, with the initial conditions $k_0(>0)$ and $b_0(\geq 0)$ such that (i) given $\left(w_t, R_{t+1}, \tau_t, \tau_{t+1}^K, g_t, g_{t+1}\right)$, (c_t, d_{t+1}, s_t) solves the utility maximization problem; (ii) given (w_t, R_t), (K_t, L_t) solves a firm's profit maximization problem; (iii) given $(w_t, R_{t+1}, s_{t-1}, b_t)$, $(\tau_t, \tau_t^K, g_t, b_{t+1})$ satisfies the government budget constraint; (iv) the labor market, $N_t l_t = L_t$, clears; and (v) the capital market, $(1+n)(k_{t+1} + b_{t+1}) = s_t$, clears.

Definition 1 allows us to reduce the economic equilibrium conditions to a system of two difference equations, one representing the government budget constraint and the other representing the capital market-clearing condition, for two state variables, physical capital k and public debt b. To show this, consider the labor supply in (3), the savings in (4), and factor prices in (5) and (6). We write these as functions of physical capital, k_t, and the labor income tax rate, τ_t as follows:[3]

$$l_t = l(\tau_t, k_t) \equiv \left[(1-\tau_t)(1-\alpha)A\left(k_t\right)^{\alpha}\right]^{v/(1+\alpha v)}, \tag{9}$$

$$s_t = s\left(\tau_t, k_t, l(\tau_t, k_t)\right) \equiv \frac{\beta\pi}{1+\beta\pi} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)w\left(k_t, l(\tau_t, k_t)\right)\right]^{1+v}, \tag{10}$$

$$w_t = w\left(k_t, l(\tau_t, k_t)\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha}\left[l(\tau_t, k_t)\right]^{-\alpha}, \tag{11}$$

$$R_t = R\left(k_t, l(\tau_t, k_t)\right) \equiv \alpha A\left(k_t\right)^{\alpha-1}\left[l(\tau_t, k_t)\right]^{1-\alpha}. \tag{12}$$

Using the labor supply function in (9) and the factor prices in (11) and (12), we can reformulate the government budget constraint in Eq. (7) in terms of the state variables, k and b, and the government policy variables, τ, τ^K, and g as follows:

$$\begin{aligned} &TR(\tau_t, k_t) + TR^K\left(\tau_t^K, \tau_t, k_t, b_t\right) + (1+n)b_{t+1} \\ &= R\left(k_t, l(\tau_t, k_t)\right)b_t + \frac{(1+n)+\pi}{1+n}g_t, \end{aligned} \tag{13}$$

where we define $TR(\tau_t, k_t)$ and $TR^K\left(\tau_t^K, \tau_t, k_t, b_t\right)$, representing the tax revenues from labor and capital income, respectively, as follows:

$$\begin{aligned} TR(\tau_t, k_t) &\equiv \tau_t w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t), \\ TR^K\left(\tau_t^K, \tau_t, k_t, b_t\right) &\equiv \tau_t^K R\left(k_t, l(\tau_t, k_t)\right)\left(k_t + b_t\right). \end{aligned}$$

We can also reformulate the capital market-clearing condition in (8) as follows:

$$\begin{aligned} (1+n)(k_{t+1} + b_{t+1}) &= s\left(\tau_t, k_t, l(\tau_t, k_t)\right) \\ &= \frac{\beta\pi}{1+\beta\pi} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)(1-\alpha)A\left(k_t\right)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}}. \end{aligned} \tag{14}$$

Thus, given the initial condition (k_0, b_0) and the sequence of the policy variables, we can characterize $\{\tau_t, \tau_t^K, g_t\}_{t=0}^{\infty}$, the sequence of physical capital and public debt in the economic equilibrium, $\{k_t, b_t\}_{t=0}^{\infty}$, by Eqs. (13) and (14).

In the economic equilibrium, we can express the indirect utility of the middle in period t, V_t^M, and that of the old in period t, V_t^O, as functions of policy variables, physical capital, and public debt. V_t^M becomes:

$$\begin{aligned} V_t^M = &\ln\left[c\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{\left(l(\tau_t, k_t)\right)^{1+1/v}}{1+1/v}\right] \\ &+ \theta\ln g_t + \beta\pi\left[\ln d\left(\tau_{t+1}^K, k_{t+1}, b_{t+1}, l(\tau_{t+1}, k_{t+1})\right) + \theta\ln g_{t+1}\right], \end{aligned} \tag{15}$$

[3] The derivation of (9) - (12) is as follows. First, we substitute (6) into (3) to write the optimal labor supply as a function of τ_t and k_t, as in (9). Second, we reformulate the saving function in (4) using (6) and (9), as in (10). Third, we use firms' profit maximization with respect to L_t in (6) and the labor supply function in (9) to obtain the labor market-clearing wage rate, as in (11). Finally, firms' profit maximization with respect to K_t in (5) and the labor supply function in (9) lead to (12).

where we define $c\left(\tau_t, k_t, l(\tau_t, k_t)\right)$ and $d\left(\tau_{t+1}^K, k_{t+1}, b_{t+1}, l(\tau_{t+1}, k_{t+1})\right)$, representing consumption in middle and old ages, respectively, as follows:

$$c\left(\tau_t, k_t, l(\tau_t, k_t)\right) \equiv (1-\tau_t) w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) - s\left(\tau_t, k_t, l(\tau_t, k_t)\right),$$
$$d\left(\tau_{t+1}^K, k_{t+1}, b_{t+1}, l(\tau_{t+1}, k_{t+1})\right) \equiv \left(1-\tau_{t+1}^K\right) R\left(k_{t+1}, l(\tau_{t+1}, k_{t+1})\right)(1+n)(k_{t+1}+b_{t+1})/\pi.$$

The utility function of the old in period t, V_t^O, is

$$V_t^O = \ln d\left(\tau_t^K, k_t, b_t, l(\tau_t, k_t)\right) + \theta \ln g_t. \tag{16}$$

3 Political Equilibrium

In this section, we consider voting on fiscal policy. We employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As Persson and Tabellini (2000) demonstrate, the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, both the middle and old have an incentive to vote. Thus, the political objective is the weighted sum of the utility of the middle and old, given by $\tilde{\Omega}_t \equiv \pi\omega V_t^O + (1+n)(1-\omega) V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weight placed on the old and middle, respectively. A larger value of ω implies greater political power of the old. We use the gross population growth rate, $(1+n)$ to adjust the weight of the middle and life expectancy (i.e., the probability of living in old age), π to adjust the weight of the old, to reflect their share of the population. To obtain the intuition behind this result, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\pi\omega}{(1+n)(1-\omega)} V_t^O + V_t^M,$$

where the coefficient $\pi\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight of the old.

We substitute V_t^M in (15) and V_t^O in (16) into Ω_t and obtain

$$\begin{aligned}
\Omega_t &= \frac{\pi\omega}{(1+n)(1-\omega)} V^O\left(\tau_t, \tau_t^K, g_t; k_t, b_t\right) + V^M\left(\tau_t, g_t, \tau_{t+1}, \tau_{t+1}^K, g_{t+1}, k_{t+1}; k_t\right) \\
&= \frac{\pi\omega}{(1+n)(1-\omega)} \left[\ln d\left(\tau_t^K, k_t, b_t, l(\tau_t, k_t)\right) + \theta \ln g_t\right] \\
&\quad + \ln\left[c\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{\left(l(\tau_t, k_t)\right)^{1+1/v}}{1+1/v}\right] \\
&\quad + \theta \ln g_t + \beta\pi\left[\ln d\left(\tau_{t+1}^K, k_{t+1}, b_{t+1}, l(\tau_{t+1}, k_{t+1})\right) + \theta \ln g_{t+1}\right].
\end{aligned} \tag{17}$$

The political objective function in (17) suggests that current policy choice affects decisions on future policy via physical capital accumulation. In particular, the period-t

choice of τ_t^K, τ_t, and g_t affect the formation of physical capital in period $t+1$. This in turn influences decision making on period-$t+1$ fiscal policy. To demonstrate this intertemporal effect, we employ the concept of Markov-perfect equilibrium under which fiscal policy today depends on the payoff-relevant state variables.

In the present framework, the payoff-relevant state variables are physical capital, k, and public debt, b. Thus, the expected provision of the public good and the rate of capital income tax for the next period, g_{t+1} and τ_{t+1}^K, respectively, are given by the functions of the period-$t+1$ state variables, $g_{t+1} = G(k_{t+1}, b_{t+1})$ and $\tau_{t+1}^K = T^K(k_{t+1}, b_{t+1})$. We denote by $-\underline{\tau}(<0)$ and $-\underline{\tau}^K(<0)$ the arbitrary lower limits of τ and τ^K, respectively. By using recursive notation with $z'(=k', b', g', \tau^{K\prime})$ denoting the next period $z(=k, b, g, \tau^K)$, we can define a Markov-perfect political equilibrium in the present framework as follows.

- **Definition 2.** A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^K, G, B\rangle$, where $T : \Re_+^2 \to (-\underline{\tau}, 1)$ is the labor income tax rate, $\tau = T(k,b)$, $T^K : \Re_+^2 \to \left(-\underline{\tau}^K, 1\right)$ is the capital income tax rate, $\tau^K = T^K(k,b)$, $G : \Re_+^2 \to \Re$ is the public goods provision rule, $g = G(k,b)$, and $B : \Re_+^2 \to \Re$ is the debt rule, $b' = B(k,b)$, such that given k and b, $\langle T(k,b), T^K(k,b), G(k,b), B(k,b)\rangle$ is a solution to the following problem:

$$\max \Omega = \frac{\pi\omega}{(1+n)(1-\omega)} V^O\left(T(k,b), T^K(k,b), G(k,b); k, b\right)$$
$$+ V^M\left(T(k,b), G(k,b), T(k',b'), T^K(k',b'), G(k',b'), k'; k\right), \tag{18}$$
$$\text{s.t. } (1+n)(k' + B(k,b)) = s(T(k,b), l(T(k,b),k), k), \tag{19}$$
$$TR(\tau,k) + TR^K(\tau^K, \tau, k, b) + (1+n)B(k,b)$$
$$= R(k, l(T(k,b),k))b + \frac{(1+n)+\pi}{1+n} G(k,b), \tag{20}$$
given k and b,

  where (19) comes from the capital market-clearing condition in (14), and (20) comes from the government budget constraint in (13).

3.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following policy functions in the next period:

$$1 - \tau^{K\prime} = \frac{\bar{T}^K}{\alpha} \cdot \frac{k'}{k'+b'}, \tag{21}$$
$$1 - \tau' = \bar{T}, \tag{22}$$
$$g' = \bar{G} \cdot \left[A(k')^\alpha\right]^{\frac{1+v}{1+\alpha v}}, \tag{23}$$

where $\bar{T}^K$, $\bar{T}$, and $\bar{G}$ are positive constant parameters.

Given the conjectures in (21)–(23), we consider the optimization problem described in Definition 2. We solve the problem and obtain the following first-order conditions:

$$\tau : \frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_\tau}{d} + \frac{c_\tau - (l)^{1/v} l_\tau}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right) + \lambda\left(TR_\tau + TR^K_\tau - R_\tau b\right) = 0, \tag{24}$$

$$\tau^K : \frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_{\tau^K}}{d} + \lambda TR^K_{\tau^K} = 0, \tag{25}$$

$$g : \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} - \lambda\frac{(1+n)+\pi}{1+n} = 0, \tag{26}$$

$$b' : \beta\pi\left(\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'}\right) + \lambda(1+n) = 0, \tag{27}$$

where λ is the Lagrangian multiplier associated with the government budget constraint. A variable with a subscript x represents a derivative with respect to x (e.g., $d_\tau = \partial d/\partial\tau$).

We can summarize the first-order conditions in (24)–(27) as follows:

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_\tau}{d} + \frac{c_\tau - (l)^{1/v} l_\tau}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right)}{TR_\tau + TR^K_\tau - R_\tau b} = \frac{\beta\pi}{1+n}\left(\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'}\right), \tag{28}$$

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)} \cdot \frac{d_{\tau^K}}{d}}{TR^K_{\tau^K}} = \frac{\beta\pi}{1+n}\left(\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'}\right), \tag{29}$$

$$\frac{\left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g}}{\frac{(1+n)+\pi}{1+n}} = (-1)\frac{\beta\pi}{1+n}\left(\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'}\right). \tag{30}$$

The expressions in (28)-(30) suggest that the following three effects shape policy: the general equilibrium effect of capital through the interest rate, R'; the disciplining effect through the capital income tax rate, $\tau^{K\prime}$; and the disciplining effect through public goods provision, g'. To understand how these effects work, first consider (28). The numerator on the left-hand side, representing the net marginal benefits of the labor income tax cut, includes the following three effects. First, the term$[\pi\omega/(1+n)(1-\omega)] \cdot (d_\tau/d)$ shows the marginal benefit of the tax cut for the old; lowering the tax rate induces the middle to increase labor supply. This in turn raises return from savings, R, and thus increases the consumption of the old.

Second, the term $\left(c_\tau - (l)^{1/v} l_\tau\right) / \left(c - (l)^{1+1/v} / (1+1/v)\right)$ includes the marginal costs and benefits for the middle. A tax cut causes disposable income to rise, and thus increases the consumption of the middle, as represented by the term c_τ. At the same time, the cut promotes the labor supply of the middle and thus increases their disutility of labor, as the term $(l)^{1/v} l_\tau$ represents.

Third, the term $\beta\pi\left(d'_\tau/d' + \theta g'_\tau/g'\right)$ includes the marginal costs and benefits of the labor income tax cut that the current middle are expected to receive when they become old. The cut in the labor income tax rate increases the disposable income of the middle and thus their savings, which in turn increases consumption in their old age. At the same time, the increase in savings works to reduce the return from savings, R', and so raises the capital income tax rate in the next period, $\tau^{K\prime}$. This in turn reduces consumption in old age. These two opposing effects on old-age consumption is represented by the term d'_{τ}/d'. The term $\theta g'_{\tau}/g'$ implies that the cut in the labor income tax rate increases savings and capital, and thus stimulates public goods provision in the next period. The left-hand side of (28) evaluates the above three effects based on the change in the tax revenue through labor income taxation, as represented by the term $TR_{\tau} + TR^{K}_{\tau} - R_{\tau}b$ in the denominator.

The right-hand side of (28) shows the marginal net costs of public debt issuance. The term $d'_{b'}/d'$ shows the marginal benefits of public debt issuance. The issue of public debt crowds out physical capital accumulation and thus raises the interest rate, R'. This in turn increases consumption in old age. The term $\theta g'_{b'}/g'$ shows that the debt issue crowds out physical capital and thus lowers public goods provision in the next period, g'. Thus, the public debt issuance creates two opposing effects on what the middle expects to receive when they become old. The expression in (28) suggests that the government chooses the labor income tax rate and public debt issuance to balance the above-mentioned costs and benefits.

Next, consider the expression in (29). The left-hand side shows the marginal benefits of a capital income tax cut. The cut increases the consumption of the old and thus makes them better off. We evaluate this effect based on the change in the tax revenue through capital income taxation represented by the term $TR^{K}_{\tau^K}$ in the denominator. The right-hand side is identical to that of (28) and thus represents the net marginal costs of public debt issuance. The expression in (29) therefore suggests that the government chooses the capital income tax rate and public debt issuance to balance these costs and benefits.

Finally, consider the expression in (30). The left-hand side shows the marginal benefit of public goods provision, normalized by the dependency (i.e., beneficiary-contributor) ratio. The right-hand side is equal to that of (28) multiplied by minus one, and so represents the net marginal costs of public debt reduction. The expression in (30) suggests that the government chooses public goods provision and public debt reduction to balance the costs and benefits arising from the choice of these two policy variables.

We can obtain the policy functions that are the solutions to the government optimization problem by solving (28)-(30) and the government budget constraint in (13) for τ, τ^K, g, and b'. To simplify the presentation of the policy functions, we introduce the following notations:

$$\bar{T}^K \equiv \frac{1-\left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)\frac{D_3}{D_1}}{\left[1+\theta+\frac{(1+n)(1-\omega)}{\pi\omega}\left(\theta+\frac{\beta(1+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)\frac{D_2}{D_1}},$$

$$\bar{T} \equiv \frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1},$$

$$\bar{G} \equiv \frac{1+n}{(1+n)+\pi}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^K,$$

$$\bar{B} \equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^K+(1-\alpha)\bar{T}-1\right]+\frac{(1+n)+\pi}{1+n}\bar{G},$$

where D_1, D_2, and D_3 are defined by

$$D_1 \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)-\beta\pi(1+\theta)\alpha(1+v)\left[(-1)\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+1+\frac{v(1-\alpha)}{1+\alpha v}\right],$$

$$D_2 \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},$$

$$D_3 \equiv \left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\frac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.$$

The following proposition describes the optimal policy functions in the present framework.

- **Proposition 1** *There is a Markov-perfect political equilibrium characterized by the following policy functions:*

$$\tau^K = 1-\frac{\bar{T}^K}{\alpha}\cdot\frac{1}{1+b/k},$$

$$\tau = 1-\bar{T},$$

$$g = \bar{G}\cdot\left[A(k)^\alpha\right]^{(1+v)/(1+\alpha v)},$$

$$(1+n)b' = \bar{B}\cdot\left[A(k)^\alpha\right]^{(1+v)/(1+\alpha v)},$$

Proof. See Appendix A.1.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the levels of public goods provision and public debt issues are linear functions of the output. This implies that the government finds it optimal to provide public goods and to issue public debt in proportion to the output. Third, the government borrows in the capital market as long as $\bar{B} > 0$; if this is the case, then the government finds it optimal to shift a part of the burden onto future generations.

Having established the policy functions, we are now ready to demonstrate the accumulation of physical capital. We substitute the policy functions of b' and τ in Proposition 1 into the capital market-clearing condition in (14) (or (19)) and obtain

$$k' = \frac{1}{1+n}\left\{\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}\left[(1-\alpha)\bar{T}\right]^{(1+v)/(1+\alpha v)} - \bar{B}\right\}\left[A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{31}$$

where k' denotes the next-period capital stock. Given the initial condition k_0, Eq. (31) determines the equilibrium sequence $\{k_t\}$. A steady state is defined as an equilibrium sequence with $k = k'$. In other words, per-capita capital is constant in a steady state. Eq. (31) indicates that there is a unique, stable steady-state equilibrium of k.

3.2 Policy Response to Population Aging

The result established in Proposition 1 indicates that an increase in π (i.e., an increased life expectancy), a decrease in n (i.e., a decreased population growth rate), and an increase in ω (i.e., an increased political weight of the old) affect the policy functions. As mentioned in the introduction the voter turnout of older people is higher than that of younger ones in OECD countries. This implies that the aging of society works in the direction of increasing the political weight of the old. Therefore, we focus on ω as well as π and n to analyze the effects of population aging on the ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP. We analyze the cases of inelastic and elastic labor supply in turn.

3.2.1 Inelastic Labor Supply

We first consider the case of inelastic labor supply, $v = 0$, and obtain the following result.

- **Proposition 2.** *Suppose that labor supply is inelastic:* $v = 0$.
- (i) *If* $\alpha(1+\theta) < 1$ *such that the government borrows in the capital market (i.e.,* $b' > 0$*), then the ratio of government debt to GDP is increasing in the life expectancy and the population growth rate, and decreasing in the political weight of the old:* $\partial(B_{t+1}/Y_t)/\partial\pi > 0$, $\partial(B_{t+1}/Y_t)/\partial n > 0$, *and* $\partial(B_{t+1}/Y_t)/\partial\omega < 0$.
- (ii) *The ratio of capital income tax revenue to GDP is decreasing in the life expectancy and the political weight of the old, and increasing in the population growth rate:* $\partial\left(\tau_t^K R_r s_{t-1} N_{t-1}/Y_t\right)/\partial\pi < 0$, $\partial\left(\tau_t^K R_r s_{t-1} N_{t-1}/Y_t\right)/\partial n > 0$, *and* $\partial\left(\tau_t^K R_r s_{t-1} N_{t-1}/Y_t\right)/\partial\omega <$ 0.
- (iii) *The ratio of labor income tax revenue to GDP is increasing (decreasing) in the life expectancy if* $\omega/(1+n)(1-\omega) > (<) \beta(1-\alpha)$*, decreasing in the population growth rate, and increasing in the political weight of the old:* $\partial(\tau_t w_t N_t/Y_t)/\partial\pi \gtrless 0$ *if*

and only if $\omega/(1+n)(1-\omega) \gtrless \beta(1-\alpha)$, $\partial(\tau_t w_t N_t/Y_t)/\partial n < 0$, *and* $\partial(\tau_t w_t N_t/Y_t)/\partial\omega > 0$.

Proof. See Appendix A.2.

Proposition 2 shows that when labor supply is inelastic, the ratio of labor income tax revenue to GDP increases as the population growth rate declines and the political weight of the old increases. Moreover, when the political weight of the old is large, the ratio increases with the increase in life expectancy. These results are generally consistent with the evidence from Figure 1. The ratio of government debt to GDP is positive as long as $\alpha(1+\theta) < 1$; and under this condition, the ratio increases as life expectancy increases, but it decreases as the population growth rate decreases and the political weight of the old increases. The former result appears to be consistent with the evidence from Figure 1, but the latter does not. Therefore, there is a discrepancy between theory and evidence for the ratio of government debt to GDP as long as labor supply is inelastic.

Proposition 2 also suggests a discrepancy between theory and evidence for the ratio of capital income tax revenue to GDP. The result in Proposition 2 shows that the ratio decreases as the life expectancy and the political weight of the old increase and the population growth rate decreases. This result, implying a negative association between the ratio and aging, seems to be intuitive at first glance because such changes in demographic factors lead to an increase in the political weight of the old, which in turn provides incentives for the government to choose policies favoring the old who bear the capital income tax burden. However, the evidence from Figure 1 shows that the negative association does not hold for some countries. In particular, Ireland, Korea, and the United States show population aging rates below the OECD average, while they show higher ratios of capital income tax revenue to GDP than other countries except for Sweden. In the following analysis, we show that assuming elastic labor supply could solve the discrepancy between theory and the empirical findings.

3.2.2 Elastic Labor Supply

For the analysis, we take a numerical approach owing to the limitations of the analytical approach in the presence of elastic labor supply. Our strategy is to calibrate the model economy such that the steady-state equilibrium matches some key statistics of the average OECD country during 1995–2016. We then use the calibrated economy to run some quantitative experiments.

We fix the share of capital at $\alpha = 1/3$ following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). We introduce a young age into the model; during youth, individuals make no economic decision and depend on their parents for their livelihood. Each period lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt, 2008;

Song, Storesletten, and Zilibotti, 2012; Lancia and Russo, 2016). Our selection of β is 0.99 per quarter, which is also standard in the literature (e.g., Kydland and Prescott, 1982; de la Croix and Doepke, 2002). Since agents in the present model plan over a generation that spans 30 years, we discount the future by $(0.99)^{120}$. Following Lancia and Russo (2016), we set the relative political weight of the old before adjustment for the population ratio, $\omega/(1-\omega)$, to 0.8. In line with Trabandt and Uhlig (2011), we set $v = 3/2$ such that the top of the labor income tax Laffer curve is 60% (see Appendix A.3 for the derivation).

The probability of living in old age, π, is taken from the average life expectancy at birth. The average life expectancy in OECD countries is 78.052 years, so individuals will, on average, live 18.05(=78.05–60) years into old age. In other words, individuals are expected to live 18.052/30 of their 30 years of old age, so $\pi = 0.602$. The net population growth rate, n, is taken from the average annual (gross) population growth rate, 1.00548. The net population growth rate for one period is $(1.00548)^{30} - 1 \simeq 0.178$. The preference weight of public goods provision, θ, is chosen such that the simulated version of the model matches the average ratio of government expenditure to GDP.[4] Table 1 summarizes the parameters.

[Table 1 is here.]

We numerically investigate the effects of aging factors, π, n, and ω on the ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP. The numerical results in Figure 2 show that the ratio of labor income tax revenue to GDP increases as the life expectancy, π, increases, the population growth rate, n, declines, and the political weight of the old, ω, increases. These results are almost qualitatively similar to those in the case of inelastic labor supply presented in Proposition 2, and are consistent with the evidence observed in Figure 1. However, the comparative statics for the ratio of public debt to GDP differs significantly from those obtained under the assumption of inelastic labor supply. The ratio of debt to GDP increases as the population growth rate declines and the political weight of the old increases; the ratio rises with the increase in the life expectancy when its initial level exceeds around 0.2. The effects of declining population growth rates and the increasing political weight of the old are very different from those obtained under inelastic labor supply, but appear to be consistent with the evidence observed in Figure 1.

[Figure 2 is here.]

[4]We define government expenditure as the sum of general government consumption expenditure and general government gross fixed capital formation. Data on the average life expectancy, the average annual population growth rate, and government expenditures, is sourced from OECD.stat. Source: OECD.Stat (https://stats.oecd.org/) (accessed on April 6, 2021).

The comparative statics for the ratio of capital income tax revenue to GDP also differ from those obtained under inelastic labor supply. The ratio of capital income tax revenue to GDP shows a monotone decline against the declining population growth rate and increasing life expectancy. However, it shows a U-shaped pattern against the increasing political weight of the old. The former result is qualitatively similar to that demonstrated in the case of inelastic labor supply, but the latter result differs from that under inelastic labor supply. However, the latter result seems to be consistent with the non-monotonic relationship between aging and the ratio of capital income tax revenue to GDP observed in Figure 1. The numerical results presented in Figure 2 suggest that the elastic labor supply assumption and the political weight of the old are the key to obtaining comparative statics consistent with the evidence. In the following, we examine the implications and mechanisms of the elastic labor supply assumption in detail, focusing on the effects of the political weight of the old, ω.

Ratio of Debt to GDP

To understand the role of the endogenous labor supply assumption, we first consider the ratio of public debt to GDP, B'/Y. From the government budget constraint in (13), the ratio when $v \geq 0$ (including both elastic and inelastic labor supply cases) is

$$\frac{B'}{Y} = (-1)\frac{TR(\tau, k)}{Y} + (-1)\frac{TR^K(\tau, \tau^K, k, b)}{Y} + \frac{G}{Y} + \frac{R\,(k, l(\tau, k))\,B}{Y}. \tag{32}$$

Equation (32) indicates that the ratio depends on the four terms on the right-hand side, TR/Y, TR^K/Y, G/Y, and RB/Y. Figure 3 plots the changes in these four terms as well as B'/Y against the changes in ω to help clarify the mechanism behind the difference between the elastic ($v > 0$) and inelastic ($v = 0$) cases.

[Figure 3 is here.]

As Panel (a) shows, the ratio B'/Y decreases as ω increases when labor supply is inelastic, whereas it increases when labor supply is elastic. The elasticity of labor supply leads to these contrasting results. When the labor supply is elastic, an increase in ω gives the government an incentive to raise the labor income tax rate, which leads to an increase in the interest rate, R, through the household's choice of labor supply. This leads to an increase in the ratio B'/Y through an increase in the debt repayment, RB (Panel (e)). However, when the labor supply is inelastic, this positive effect through the interest rate is absent, so an increase in ω leads to a decrease in the ratio B'/Y.

Ratio of Capital Income Tax Revenue to GDP

Next, we consider the ratio of capital income tax revenue to GDP, $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$. The ratio depends on two factors: the capital income tax rate, τ_t^K, and the ratio of capital income to GDP, $R_t s_{t-1} N_{t-1}/Y_t$. Figure 4 plots changes in $R_t s_{t-1} N_{t-1}/Y_t$, τ_t^K, and $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$ against changes in ω. In each panel, the solid (dashed) curve represents the changes in a concerned variable when labor supply is elastic (inelastic). We first assess the effect through the ratio of capital income to GDP, $R_t s_{t-1} N_{t-1}/Y_t$, and then assess the effect through the capital income tax rate, τ_t^K.

[Figure 4 is here.]

When the labor supply is inelastic such that $v = 0$ holds, the ratio of capital income to GDP reduces to $R_t s_{t-1} N_{t-1}/Y_t = (1-\alpha\theta)/(1+\theta)$, which is independent of the political weight of the old, ω. However, when the labor supply is elastic such that $v > 0$ holds, a change in ω affects the ratio $R_t s_{t-1} N_{t-1}/Y_t$ through the labor supply decision as follows. The government chooses a higher labor income tax rate as the political weight of the old increases. A higher labor income tax rate reduces the supply of labor, resulting in lower labor income and thus lower savings. This leads to a decrease in the ratio of capital income to GDP through the term s_{t-1}. On the other hand, a decrease in savings leads to an increase in the interest rate, R_t, through a decrease in the capital level, which leads to an increase in the ratio of capital income to GDP. In total, the latter effect dominates the former one, implying that the ratio of capital income to GDP increases as ω increases.

Next, consider the effect of ω through the capital income tax rate, $\tau_t^K = 1-\bar{T}^K/\left[\alpha(k_t+b_t)/k_t\right]$. The expression shows that a change in ω affects the tax rate through $\bar{T}^K$ and $\alpha(k_t+b_t)/k_t$. The term $\bar{T}^K$ increases and thus, τ_t^K decreases as ω increases, irrespective of the status of labor supply, v. The negative effect on the capital income tax rate reflects the preferences of the old who want to reduce their fiscal burden of capital income taxation. The term $\alpha\left(k_t+b_t\right)/k_t$, which is equal to $R_t s_{t-1} N_{t-1}/Y_t$, is independent of ω when $v = 0$, while it is increasing in ω when $v > 0$, as we argued in the last paragraph. The effects through the two terms suggest that when $v = 0$, the capital income tax rate decreases as ω increases. However, when $v > 0$, the negative effect through the term $\bar{T}^K$ outweighs the positive effect through the capital income to GDP ratio, $\alpha\left(k_t+b_t\right)/k_t$, for low values of ω; the opposite result holds for high values of ω. Thus, an increase in the political power of the old produces an initial decrease followed by an increase in the capital income tax rate.

Up to now, the results have the following implications for the ratio of capital income tax revenue to GDP. When the labor supply is inelastic, $v = 0$, the effect through the ratio of capital income to GDP does not appear; the effect through the capital income tax rate remains and works to lower the ratio of capital income tax revenue to GDP as ω increases. However, when the labor supply is elastic, $v > 0$, the positive effect through the ratio of capital income to GDP may outweigh the negative effect through the capital income tax rate for high values of ω. Which effect dominates depends on the initial value of ω. Therefore, the political weight of old and the elastic labor supply play important roles in determining the ratio of capital income tax revenue to GDP.

4 Conclusion

This study analyzed the distribution of the fiscal burden across generations in a political economy model of fiscal policy. The model includes (i) two tax instruments: capital and labor income taxes, accompanied by debt finance; and (ii) household decisions on labor supply. The first element enables us to investigate the impact of population aging on the distribution of the fiscal burden across generations; the second element allows us to present the effects of aging on policy variables via households' labor decisions.

Given these features, we showed that aging, which implies increased political weight of the old, leads to (i) an increase in the ratios of debt to GDP and labor income tax revenue to GDP; and (ii) an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP. These model predictions fit well with the evidence observed in OECD countries. In particular, the latter result suggests that the political weight of the old is a key factor in the different patterns of the ratio observed among OECD countries sharing similar demographic characteristics.

The results of this study provide important and useful information to predict the generational burden of fiscal policy in aging societies. Population aging first produces a shift of the tax burden from older to younger generations. As the population ages further, the tax burden on both younger and older generations increases. This suggests a U-shaped pattern of the fiscal burden on the old. Mateos-Planas (2010) predicts this pattern, but limits his analysis to the balanced government budget case. The present study instead allowed for government deficits, showing that the ratio of public debt to GDP increases as population ages. The result suggests that when debt finance is allowed, a shift of the fiscal burden from older to younger generations could stronger in the early stage of population aging. However, further aging leads to an increased fiscal burden on both younger and older generations. The increased fiscal burden is an inevitable consequence of population aging in the long run.

A Mathematical Appendix

A.1 Proof of Proposition 1

Based on the specification of the utility and production functions, we can reformulate the first-order conditions in (28)-(30) as follows:

$$\frac{(-1)\pi\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}+\frac{\beta\pi\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}$$
$$\times\frac{\alpha\left[\left(1-\tau\right)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{(1+n)k'}=0, \tag{A.1}$$

$$(-1)\left[\frac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\frac{1}{1+\alpha v}\frac{1}{1-\tau}$$
$$+\frac{\beta\pi\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}\cdot\frac{\left(1-\tau\right)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}}{(1+n)k'}$$
$$\times\left\{(-1)\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+\frac{1}{1-\tau}\frac{v}{1+\alpha v}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}=0, \tag{A.2}$$

$$\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}-\beta\pi\left(1+\theta\right)\frac{\alpha(1+v)}{1+\alpha v}\frac{\frac{(1+n)+\pi}{1+n}}{(1+n)k'}=0. \tag{A.3}$$

Equations (A.1), (A.2), and (A.3) correspond to (29), (28), and (30), respectively. We present the derivation of (A.1)–(A.3) in Appendix B.

The procedure to find the optimal policy functions is as follows. First, substitute the first-order condition with respect to τ^K in (A.1) into the first-order condition with respect to g in (A.3) to write g as a function of τ^K and $\tau : g = g(\tau^K, \tau)$. Second, substitute $g = g(\tau^K, \tau)$ into the capital market-clearing condition in (14) to write k' as a function of τ^K and $\tau : k' = k'\left(\tau^K, \tau\right)$. Third, substitute $k' = k'\left(\tau^K, \tau\right)$ into the first-order condition with respect to τ^K in (A.1) and τ in (A.2) to obtain the two optimal relations between τ^K and τ, and solve them for τ^K and τ. Fourth, substitute the solutions for τ^K and τ into $g = g(\tau^K, \tau)$ to obtain the optimal policy function of g. Finally, substitute the optimal policy functions of τ^K, τ, and g into the government budget constraint in (13) to obtain the optimal policy function of b'.

Recall the first-order condition with respect to g in (A.3), which we rewrite as

$$\frac{(1+n)+\pi}{1+n}g=\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{1+\alpha v}{\beta\pi\left(1+\theta\right)\alpha(1+v)}(1+n)k'.$$

We substitute the first-order condition with respect to τ^K in (A.1) into the above expression to obtain $g = g(\tau^K, \tau)$, or

$$\frac{(1+n)+\pi}{1+n}g=\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\alpha\left[\left(1-\tau\right)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{\frac{\pi\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}}. \tag{A.4}$$

Next, we substitute (A.4) into the capital market-clearing condition in (14) to obtain

$$
\begin{aligned}
(1+n)k' = {} & \frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& - \alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{b}{k} \\
& - \left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{\frac{\pi\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}} \\
& + \frac{\tau}{1-\tau}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{1/(1+\alpha v)}\cdot\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{v/(1+\alpha v)} \\
& + \tau^K\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right).
\end{aligned}
$$

Rearranging the terms, we have

$$
\begin{aligned}
(1+n)k' = {} & \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& \times\left\{\left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right. \\
& \left. -\left[1+\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}. \qquad \text{(A.5)}
\end{aligned}
$$

Eq. (A.5) shows that we can express $(1+n)k'$ as a function of τ^K and τ.

Third, we substitute (A.5) into the first-order condition with respect to τ^K in (A.1) and obtain

$$
\begin{aligned}
& \frac{\pi\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^K} \\
& = \frac{\beta\pi\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}\times\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{X} \\
& = \frac{\beta\pi\left(1+\theta\right)\alpha(1+v)}{1+\alpha v} \\
& \times\frac{\alpha\left(1+\frac{b}{k}\right)}{\left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1},
\end{aligned}
$$

where X is defined by

$$
\begin{aligned}
X \equiv {} & \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& \times\left\{\left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}
\end{aligned}
$$

Rearranging the terms, we have

$$
\begin{aligned}
& \left(\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)+1 \\
& = \left[1+\theta+\frac{(1+n)(1-\omega)}{\pi\omega}\left(\theta+\frac{\beta\pi\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}\right)\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right). \qquad \text{(A.6)}
\end{aligned}
$$

This equation describes the optimal relationship between τ^K and τ.

Third, we substitute (A.5) in the first-order condition with respect to τ into (A.2) to obtain

$$\begin{aligned}
&\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left\{\left(\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right.\\
&\left.-\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}\\
&=\beta\pi\left(1+\theta\right)\alpha(1+v)\left(1-\tau\right)\left(1-\alpha\right)\\
&\times\left\{(-1)\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+\tfrac{v}{1+\alpha v}\tfrac{1}{1-\tau}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}\\
&=\beta\pi\left(1+\theta\right)\alpha(1+v)\left(1-\alpha\right)\\
&\times\left\{\left[(-1)\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right](1-\tau)+\tfrac{v}{1+\alpha v}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\tfrac{v}{1+\alpha v}\right\}.
\end{aligned}$$

Rearranging the terms, we have

$$\begin{aligned}
&\underbrace{\left\{\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}-1\right)-\beta\pi\left(1+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right]\right\}}_{D_1}\\
&\times(1-\tau)(1-\alpha)\\
&=\underbrace{\left\{\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\tfrac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_2}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)\\
&-\underbrace{\left\{\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_3},
\end{aligned}\tag{A.7}$$

where D_1, D_2, and D_3 are defined by

$$\begin{aligned}
D_1&\equiv\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}-1\right)-\beta\pi\left(1+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right],\\
D_2&\equiv\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\tfrac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},\\
D_3&\equiv\left[\tfrac{\pi\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta\pi(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.
\end{aligned}$$

Eqs. (A.6) and (A.7) characterize the atonal τ and τ^K. Substituting (A.7) into (A.6) yields

$$1-\tau^K=\underbrace{\frac{1-\left(\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_3}{D_1}}{\left[1+\theta+\tfrac{(1+n)(1-\omega)}{\pi\omega}\left(\theta+\tfrac{\beta\pi(1+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\tfrac{\beta\pi}{1+\beta\pi}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_2}{D_1}}}_{=\bar{T}^K}\cdot\frac{1}{\alpha}\cdot\frac{1}{1+b/k},\tag{A.8}$$

thus verifying the conjecture of τ^K in (21). In addition, we substitute (A.8) into (A.7) to obtain

$$1-\tau=\frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1}\equiv\bar{T},\tag{A.9}$$

thus verifying the conjecture of τ in (22).

Fourth, we substitute (A.8) and (A.9) into (A.4) to derive the policy function of g :

$$g = \underbrace{\frac{1+n}{(1+n)+\pi}\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^{K}}_{=\bar{G}}\left[A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}. \tag{A.10}$$

Finally, substituting τ^K, τ, and g into the government budget constraint in (13) leads to the following policy function of b':

$$(1+n)b' = \bar{B}\left[A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{A.11}$$

where $\bar{B}$ is defined by

$$\bar{B} \equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^{K}+(1-\alpha)\bar{T}-1\right]+\frac{(1+n)+\pi}{1+n}\bar{G}.$$

■

A.2 Proof of Proposition 2.

Suppose that $v=0$ holds. The policy functions of b_{t+1}, τ_t^K, and τ_t presented in Proposition 1 then reduce to

$$\begin{aligned}
b_{t+1} &= \frac{1}{1+n}\cdot\frac{\beta\pi\left(1-\alpha(1+\theta)\right)\frac{(1+n)(1-\omega)}{\pi\omega}}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta\pi)\right)}A(k)^{\alpha},\\
\tau_t^K &= 1-\frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}(1+\alpha\beta\pi)\right)}\cdot\frac{1}{\alpha\left(1+b_t/k_t\right)},\\
\tau_t &= 1-\frac{\frac{1+\beta\pi}{1-\alpha}\cdot\frac{(1+n)(1-\omega)}{\pi\omega}}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}(1+\alpha\beta\pi)\right)}.
\end{aligned}$$

The ratio of B_{t+1} to Y_t becomes

$$\frac{B_{t+1}}{Y_t}=\frac{(1+n)b_{t+1}N_t}{A(k)^{\alpha}N_t}=\frac{\beta\pi\left(1-\alpha(1+\theta)\right)}{(1+\theta)\left(\frac{\pi\omega}{(1+n)(1-\omega)}+(1+\alpha\beta\pi)\right)}.$$

The expression above indicates that the ratio of B_{t+1}/Y_t is positive if $\alpha(1+\theta)<1$, and that the ratio is increasing in π and decreasing in ω and n if $\alpha(1+\theta)<1$.

The ratio of $\tau_t^K R_r s_{t-1}N_{t-1}$ to Y_t becomes

$$\begin{aligned}
\frac{\tau_t^K R_r s_{t-1}N_{t-1}}{Y_t} &= \frac{\tau_t^K \alpha A(k)^{\alpha-1}(k_t+b_t)(1+n)N_{t-1}}{A(k)^{\alpha}N_t}\\
&= \alpha\left(1+\frac{b_t}{k_t}\right)-\frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}(1+\alpha\beta\pi)\right)}.
\end{aligned}$$

In period 0, the ratio becomes

$$\frac{\tau_0^K R_0 s_{-1} N_{-1}}{Y_0} = \alpha\left(1+\frac{b_0}{k_{t0}}\right) - \frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}(1+\alpha\beta\pi)\right)}.$$

Given the initial conditions of k_0 and b_0, the equation indicates that the ratio $\tau_0^K R_0 s_{-1} N_{-1}/Y_0$ is decreasing in π and ω and increasing in n. In period $t \geq 1$, we have

$$1+\frac{b_t}{k_t} = \frac{1}{\alpha(1+\theta)}.$$

Thus, the ratio becomes

$$\frac{\tau_t^K R_r s_{t-1} N_{t-1}}{Y_t} = \frac{1}{(1+\theta)} - \frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}(1+\alpha\beta\pi)\right)},$$

showing that $\tau_t^K R_r s_{t-1} N_{t-1}/Y_t$ is decreasing in π and ω and increasing in n.

The ratio of $\tau_t w_t N_t$ to Y_t becomes

$$\frac{\tau_t w_t N_t}{Y_t} = \frac{\tau_t(1-\alpha)A(k)^\alpha N_t}{A(k)^\alpha N_t} = (1-\alpha) - \frac{1+\beta\pi}{(1+\theta)\left(\frac{\pi\omega}{(1+n)(1-\omega)}+(1+\alpha\beta\pi)\right)}.$$

The equation indicates that the ratio of $\tau_t w_t N_t/Y_t$ is decreasing in n and increasing in ω. To see the effect of a higher π on the ratio, we take the first derivative of the ratio with respect to π and obtain

$$\partial\left(\frac{\tau_t w_t N_t}{Y_t}\right)/\partial\pi = \frac{(-1)}{(1+\theta)\left(\frac{\pi\omega}{(1+n)(1-\omega)}+(1+\alpha\beta\pi)\right)^2}\left[-\frac{\pi\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\right].$$

Thus, $\partial(\tau_t w_t N_t/Y_t)/\partial\pi \gtrless 0$ if $\frac{\pi\omega}{(1+n)(1-\omega)} \gtrless \beta(1-\alpha)$ holds.

■

A.3 Calibration of v

The labor income tax revenue is

$$\begin{aligned}\tau_t w_t l_t N_t &= \tau_t(1-\alpha)A(k_t)^\alpha (l_t)^{-\alpha} l_t N_t \\ &= [(1-\alpha)A(k_t)^\alpha]^{(1-\alpha)v/(1+\alpha v)} N_t \tau_t (1-\tau_t)^{(1-\alpha)v/(1+\alpha v)},\end{aligned}$$

where we obtain the equality in the second line by substituting the labor supply function in Eq. (9) into the expression in the first line. The revenue-maximizing tax rate, denoted by $\tau_{\max}$, satisfies the following first-order condition:

$$(1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)} - \tau_{\max}\frac{(1-\alpha)v}{1+\alpha v}(1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)-1} = 0,$$

which leads to

$$\tau_{\max} = \frac{1 + \alpha v}{1 + v}.$$

Following Trabandt and Uhlig (2011), we set v such that the top of the labor income tax Laffer curve is at 60%. Setting $\tau_{\max} = 0.6$ and $\alpha = 1/3$, we obtain $v = 3/2$.

■

B Online Appendix

B.1 Reformulation of V_t^M in (15) and V_t^O in (16)

The utility function for the middle in period t, V_t^M, is

$$V_t^M = \ln \left(c_t - \frac{(l_t)^{1+1/v}}{1+1/v} \right) + \theta \ln g_t + \beta\pi \left(\ln d_{t+1} + \theta \ln g_{t+1} \right).$$

We rewrite the term $c_t - (l_t)^{1+1/v} / (1+1/v)$ as follows:

$$\begin{aligned} c_t - \frac{(l_t)^{1+1/v}}{1+1/v} &= (1-\tau_t) w l_t - s_t - \frac{(l_t)^{1+1/v}}{1+1/v} \\ &= (1-\tau_t) w \left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) - s\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{(l(\tau_t, k_t))^{1+1/v}}{1+1/v}, \end{aligned} \tag{B.1}$$

where the first line comes from the budget constraint in middle age in (1), and the second line comes from the labor market-clearing wage rate in (11), the labor supply function in (9), and the saving function in (10). Rearranging the terms, we can reduce the expression in (B.1) to

$$c_t - \frac{(l_t)^{1+1/v}}{1+1/v} = \frac{1}{1+\beta\pi} \cdot \frac{1/v}{1+1/v} \left[(1-\tau_t)(1-\alpha) A (k_t)^\alpha \right]^{(1+v)/(1+\alpha v)}. \tag{B.2}$$

We rewrite the term d_{t+1} as follows:

$$\begin{aligned} d_{t+1} &= \left(1 - \tau_{t+1}^K\right) R_{t+1} s_t \\ &= \left(1 - \tau_{t+1}^K\right) \frac{R\left(k_{t+1}, l(\tau_{t+1}, k_{t+1})\right)}{\pi} s\left(\tau_t, k_t, l(\tau_t, k_t)\right), \end{aligned} \tag{B.3}$$

where the equality in the second line comes from (10) and (12).

With (9), (10), (11), and (12), we can reformulate the equation in (B.3) further as follows:

$$\begin{aligned} d_{t+1} = \left(1 - \tau_{t+1}^K\right) \cdot \frac{\alpha}{\pi} \left[(1-\tau_{t+1})(1-\alpha) \right]^{(1-\alpha)v/(1+\alpha v)} \left[A (k_{t+1})^\alpha \right]^{(1+v)/(1+\alpha v)} \frac{1}{k_{t+1}} \\ \times \frac{\beta\pi}{1+\beta\pi} \cdot \frac{1/v}{1+1/v} \left[(1-\tau_t)(1-\alpha) A (k_t)^\alpha \right]^{\frac{1+v}{1+\alpha v}}. \end{aligned} \tag{B.4}$$

Thus, with (B.2) and (B.4), we can reformulate the expression in (15) as

$$\begin{aligned} V_t^M &= V^M \left(\tau_t, g_t, \tau_{t+1}, \tau_{t+1}^K, g_{t+1}, k_{t+1}; k_t \right) \\ &\simeq \underbrace{(1+\beta\pi) \frac{1+v}{1+\alpha v} \ln(1-\tau_t)}_{(\#1)} + \theta \ln g_t + \beta\pi \ln \left(1 - \tau_{t+1}^K\right) \\ &\quad + \underbrace{\beta\pi \frac{(1-\alpha) v}{1+\alpha v} \ln(1-\tau_{t+1})}_{(\#2)} + \underbrace{(-1)\beta\pi \frac{1-\alpha}{1+\alpha v} \ln k_{t+1}}_{(\#3)} + \beta\pi\theta \ln g_{t+1}, \end{aligned} \tag{B.5}$$

where we omit the irrelevant terms from the expression in (B.5). Term (#1) includes the effects of the period-t labor income tax rate on $c_t - (l_t)^{1+1/v} / (1+1/v)$ and s_t; term (#2) includes the effect of the period-$t+1$ labor income tax rate on the interest rate R_{t+1} through the labor supply l_{t+1}; and term (#3) includes the effect of physical capital on the interest rate R_{t+1}.

Using (9) and (12), we reformulate the expression in (16) as follows:

$$V_t^O = V^O\left(\tau_t, \tau_t^K, g_t; k_t, b_t\right) \simeq \ln\left(1-\tau_t^K\right) + \frac{(1-\alpha)v}{1+\alpha v}\ln(1-\tau_t) + \theta\ln g_t, \tag{B.6}$$

where we omit the irrelevant terms from the expression.

■

B.2 Derivation of (A.1)

We reformulate the terms d_{τ^K}/d, $TR^K_{\tau^K}$, $d'_{b'}/d'$, and $g'_{b'}/g'$ in (29) as follows. First, consider the terms d_{τ^K}/d and $TR^K_{\tau^K}$. Given $d = \left(1-\tau^K\right)R\left(k, l(\tau,k)\right)(1+n)(k+b)$ and $TR^K = \tau^K R\left(k, l(\tau,k)\right)(k+b)$, we have

$$d_{\tau^K} = -R\left(k, l(\tau,k)\right)(1+n)(k+b) \Rightarrow \frac{d_{\tau^K}}{d} = \frac{-1}{1-\tau^K}, \tag{B.7}$$

$$TR^K_{\tau^K} = R\left(k, l(\tau,k)\right)(k+b). \tag{B.8}$$

Next, consider the term $d'_{b'}/d'$. Note that we can rewrite d' as

$$\begin{aligned} d' &= \left(1-\tau^{K\prime}\right)\frac{R\left(k', l(\tau',k')\right)}{\pi}s\left(\tau,k,l(\tau,k)\right) \\ &= \frac{\bar{T}^K}{\alpha}\frac{(1+n)k'}{s\left(\tau,k,l(\tau,k)\right)}\frac{1}{\pi}\alpha A\left(k'\right)^{\alpha-1}\left[\left(1-\tau'\right)(1-\alpha)A\left(k'\right)^{\alpha}\right]^{(1-\alpha)v/(1+\alpha v)}s\left(\tau,k,l(\tau,k)\right) \\ &= \frac{\bar{T}^K}{\alpha}(1+n)\frac{\alpha A}{\pi}\left[\left(1-\tau'\right)(1-\alpha)A\right]^{(1-\alpha)v/(1+\alpha v)}\left(k'\right)^{\alpha(1+v)/(1+\alpha v)}, \end{aligned} \tag{B.9}$$

where the equality in the second line comes from (10) and (12). The capital market clearing condition, $(1+n)(k'+b') = s$, implies $\partial k'/\partial b' = -1$. Thus, we have

$$\frac{d'_{b'}}{d'} = \frac{\partial d'}{\partial k'}\frac{\partial k'}{\partial b'}\frac{1}{d'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}\frac{1}{k'}. \tag{B.10}$$

Finally, consider the term $g'_{b'}/g'$. Based on the conjecture of the policy function in (23), we have

$$\frac{g'_{b'}}{g'} = \frac{\partial g'}{\partial k'}\frac{\partial k'}{\partial b'}\frac{1}{g'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}\frac{1}{k'}. \tag{B.11}$$

With (B.10) and (B.11), we obtain

$$\frac{d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}(1+\theta)\frac{1}{k'}. \tag{B.12}$$

By using (B.7), (B.8), (B.10), and (B.12), we can reformulate (29) as

$$\frac{\frac{\pi\omega}{(1+n)(1-\omega)}\frac{-1}{1-\tau^K}}{R\left(k,l(\tau,k)\right)(1+n)(k+b)}+\frac{\beta\pi}{1+n}\frac{(1+v)\alpha}{1+\alpha v}\left(1+\theta\right)\frac{1}{k'}=0,$$

or as in (A.1).

■

B.3 Derivation of (A.2) and (A.3)

Equation (A.3) is immediate from substituting (B.12) in (30). The derivation of (A.2), which is equivalent to (28), is as follows.

We reformulate the terms in (28) as follows. First, consider the term $\frac{\pi\omega}{(1+n)(1-\omega)}\cdot\frac{d_\tau}{d}$, which expresses the first derivative of $\frac{\pi\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^K\right)R(k,l(\tau,k))s$ with respect to τ. From (9) and (12), R is given by

$$R=\alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{1}{k}.$$

We substitute this into the term $\frac{\pi\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^K\right)R(k,l(\tau,k))s$ and obtain

$$\begin{aligned}&\frac{\pi\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^K\right)R(k,l(\tau,k))s\\&=\frac{\pi\omega}{(1+n)(1-\omega)}\ln(1-\tau^K)\alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{1}{k}(1+n)(k+b).\end{aligned}$$

Differentiation with respect to τ leads to

$$\frac{\pi\omega}{(1+n)(1-\omega)}\frac{d_\tau}{d}=(-1)\frac{\pi\omega}{(1+n)(1-\omega)}\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}.\tag{B.13}$$

Next, consider the term $\left[c_\tau-(l)^{1/v}l_\tau\right]/\left[c-(l)^{1+1/v}/(1+1/v)\right]$, which expresses the first derivative of $\ln\left\{c(\tau,k,l(\tau,k))-\frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ. Using (B.2), we have

$$\ln\left\{c(\tau,k,l(\tau,k))-\frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}=\ln\frac{1}{1+\beta\pi}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}.$$

Differentiating $\ln\left\{c(\tau,k,l(\tau,k))-\frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ leads to

$$\frac{c_\tau-(l)^{1/v}l_\tau}{c-(l)^{1+1/v}/(1+1/v)}=\frac{1+v}{1+\alpha v}\frac{-1}{1-\tau}.\tag{B.14}$$

Third, consider the term $\beta\pi\left(d'_\tau/d'+\theta g'_\tau/g'\right)$, which expresses the first derivative of $\beta\pi\ln d'+\beta\pi\theta\ln g'$ with respect to τ. Using (9) and (12) and the conjecture of the policy function in (23), we can write

$$\begin{aligned}\beta\pi\ln d'+\beta\pi\theta\ln g'&=\beta\pi\ln\frac{\bar{T}^K}{\alpha}\frac{(1+n)k'}{s}\frac{R'}{\pi}s+\beta\pi\theta\ln\left(k'\right)^{\alpha(1+v)/(1+\alpha v)}\\&\simeq\beta\pi\frac{\alpha(1+v)}{1+\alpha v}(1+\theta)\ln k'.\end{aligned}$$

Thus, we have

$$\beta\pi\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)=\beta\pi\,(1+\theta)\,\frac{\alpha(1+v)}{1+\alpha v}\frac{k'_\tau}{k'}. \tag{B.15}$$

Fourth, consider the term $TR_\tau+TR^K_\tau-R_\tau b$, which expresses the first derivative of $TR+TR^K-Rb$ with respect to τ. Using (9), (11), and (12), we can reformulate the term $TR+TR^K-Rb$ as follows:

$$\begin{aligned}TR+TR^K-Rb&=\tau w\,(k,l(\tau,k))\,l(\tau,k)+\tau^K R\,(k,l(\tau,k))\,(k+b)-R\,(k,l(\tau,k))\,b\\&=\left[(1-\alpha)\,A\,(k)^\alpha\right]^{(1+v)/(1+\alpha v)}(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[\tau+\tau^K\frac{\alpha}{1-\alpha}\left(1+\frac{b}{k}\right)-\frac{\alpha}{1-\alpha}\frac{b}{k}\right].\end{aligned} \tag{B.16}$$

Differentiating $TR+TR^K-Rb$ in (B.16) with respect to τ leads to

$$\begin{aligned}TR_\tau+TR^K_\tau-R_\tau b=&\left\{(-1)\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}\left[\tau+\tau^K\frac{\alpha}{1-\alpha}\left(1+\frac{b}{k}\right)-\frac{\alpha}{1-\alpha}\frac{b}{k}\right]+1\right\}\\&\times(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\,(k)^\alpha\right]^{(1+v)/(1+\alpha v)}.\end{aligned} \tag{B.17}$$

Using (B.12) and (B.13)–(B.17) derived so far, we can rewrite (28) as

$$\begin{aligned}&(-1)\frac{\pi\omega}{(1+n)(1-\omega)}\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}+\frac{1+v}{1+\alpha v}(-1)\frac{1}{1-\tau}+\beta\pi\,(1+\theta)\,\frac{\alpha\,(1+v)}{1+\alpha v}\frac{k'_\tau}{k'}\\&=\left\{(-1)\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}\left[\tau+\tau^K\frac{\alpha}{1-\alpha}\left(1+\frac{b}{k}\right)-\frac{\alpha}{1-\alpha}\frac{b}{k}\right]+1\right\}\\&\times(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\,(k)^\alpha\right]^{(1+v)/(1+\alpha v)}(-1)\frac{\beta\pi}{1+n}\frac{\alpha\,(1+v)}{1+\alpha v}\,(1+\theta)\,\frac{1}{k'}.\end{aligned} \tag{B.18}$$

The remaining task is to compute k'_τ. Recall the capital market clearing condition in (14). Differentiating k' with respect to τ yields

$$k'_\tau=(-1)\frac{1+v}{1+\alpha v}\frac{1}{1+n}\frac{\beta\pi}{1+\beta\pi}\frac{1/v}{1+1/v}(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\,(k)^\alpha\right]^{(1+v)/(1+\alpha v)}. \tag{B.19}$$

Substituting (B.19) into (B.18) and rearranging the terms, we obtain (A.2).

■

References

[1] Andersen, T.M., 2019. Intergenerational conflict and public sector size and structure: A rationale for debt limits? *European Journal of Political Economy* 57, 70–88.

[2] Arai, R., Naito, K., and Ono, T., 2018. Intergenerational policies, public debt, and economic growth: A politico-economic analysis. *Journal of Public Economics* 166, 39–52.

[3] Arawatari, R., and Ono, T., 2017. Inequality and public debt: A positive analysis. *Review of International Economics* 25, 1155–1173.

[4] Arcalean, C., 2018. Dynamic fiscal competition: A political economy theory. *Journal of Public Economics* 164, 211–224.

[5] Bassetto, M., 2008. Political economy of taxation in an overlapping-generations economy. *Review of Economic Dynamics* 11, 18–43.

[6] Beauchemin, K., 1998. Intergenerational politics, fiscal policy and productivity. *Review of Economic Dynamics* 1, 835–858.

[7] Bishnu, M., and Wang, M., 2017. The political intergenerational welfare state. *Journal of Economic Dynamics and Control* 77, 93–110.

[8] Boldrin, M., and Rustichini, A., 2000. Political equilibria with social security. *Review of Economic Dynamics* 3, 41–78.

[9] Cukierman, A., and Meltzer, A.H., 1989. A political theory of government debt and deficits in a neo-Ricardian framework. *American Economic Review* 79, 713–732.

[10] de la Croix, D., and Doepke, M., 2002. Inequality and growth: Why differential fertility matters. *American Economic Review* 93, 1091–1113.

[11] Forni, L., 2005. Social security as Markov equilibrium in OLG models. *Review of Economic Dynamics* 8, 178–194.

[12] Gonzalez-Eiras, M., and Niepelt, D., 2008. The future of social security. *Journal of Monetary Economics* 55, 197–218.

[13] Gonzalez-Eiras, M., and Niepelt, D., 2012. Ageing, government budgets, retirement, and growth. *European Economic Review* 56, 97–115.

[14] Greenwood, J., Hercowitz, Z., Huffman, G.W., 1988. "Investment, capacity utilization, and the real business cycle. *American Economic Review* 78, 402–417.

[15] Kydland, F.E., and Prescott, E.C., 1982. Time to build and aggregate fluctuations. *Econometrica* 50, 1345–1370.

[16] Lancia, F., and Russo, A., 2016. Public education and pensions in democracy: A political economy theory. *Journal of the European Economic Association* 14, 1038–1073.

[17] Lindbeck, A.J., and Weibull, W., 1987. Balanced-budget redistribution as the outcome of political competition. *Public Choice* 52, 273–297.

[18] Mateos-Planas, X., 2010. Demographics and the politics of capital taxation in a life-cycle economy. *American Economic Review* 100, 337–363.

[19] Müller, A., Storesletten, K., and Zilibotti, F., 2016. The political color of fiscal responsibility. *Journal of the European Economic Association* 14, 252–302.

[20] Ono, T., and Uchida, Y., 2016. Pensions, education, and growth: A positive analysis. *Journal of Macroeconomics* 48, 127–143.

[21] Ono, T., and Uchida, Y., 2018. Human capital, public debt, and economic growth: A political economy analysis. *Journal of Macroeconomics* 57, 1–14.

[22] Organisation for Economic Co-operation and Development (OECD), 2007. Society at a glance 2006: OECD social indicators. Paris: OECD publishing.

[23] Persson, T., and Tabellini, G., 2000. Political Economics: Explaining Economic Policy. MIT Press, Cambridge, Massachusetts.

[24] Razin, A., and Sadka, E., 2007. Aging population: The complex effect of fiscal leakages on the politico-economic equilibrium. *European Journal of Political Economy* 23, 564–575.

[25] Razin, A., Sadka, E., and Swagel, P., 2004. Capital income taxation under majority voting with aging population. *Review of World Economics* 140, 476–495.

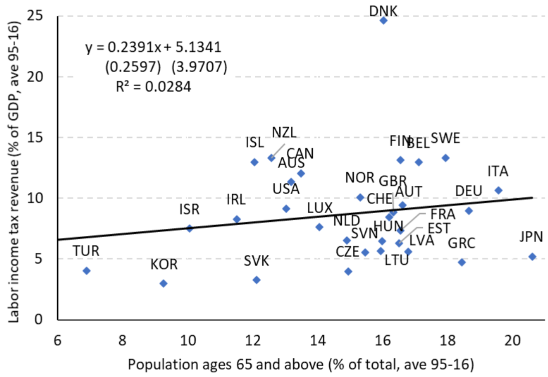
[26] Renström, T.I., 1996. Endogenous taxation: An overlapping generations approach. *Economic Journal* 106, 471–482.

[27] Röhrs, S., 2016. Public debt in a political economy. *Macroeconomic Dynamics* 20, 1282–1312.

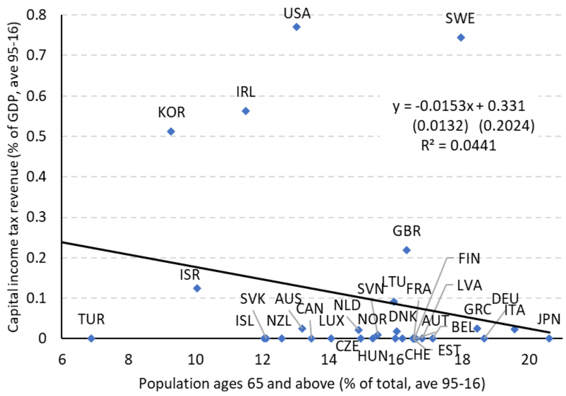
[28] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

[29] Trabandt, M., and Uhlig, H., 2011. The Laffer curve revisited. *Journal of Monetary Economics* 58, 305–327.

[30] Uchida, Y., and Ono, T., 2021. Political economy of taxation, debt ceilings, and growth. *European Journal of Political Economy.* https://doi.org/10.1016/j.ejpoleco.2020.101996



(a)



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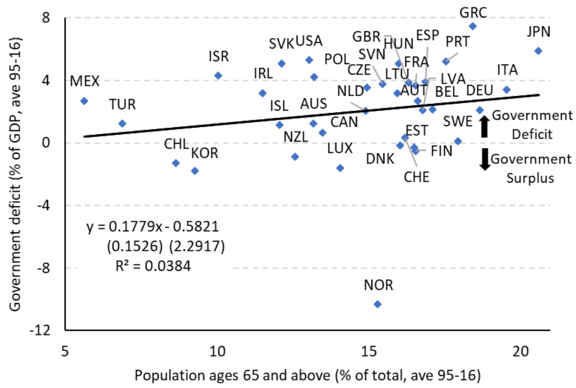

(c)

Figure 1: Each panel plots the data for OECD countries during 1995–2016. The horizontal axis represents the average share of the population aged 65 years and over. The vertical axis represents the average ratio of labor income tax revenue to GDP (Panel (a)), the average ratio of capital income tax revenue to GDP (Panel (b)), and average ratio of deficit to GDP (Panel (c)). In Panel (c), the budget deficit is an approximate variable for the public debt. Each panel presents the OLS equation estimated results. The numbers in parentheses represent the standard errors. Source: OECD.Stat (https://stats.oecd.org/) (accessed on September 25, 2019).

α: Capital share of output	1/3
β: Discount factor	$(0.99)^{120}$
$\omega/(1-\omega)$: Relative political weight of the old	0.8
v: Frisch elasticity of labor supply	3/2
π: Probability of living in old age	0.602
n: Population growth rate	0.178
θ: Preferences for public goods	0.667

Table 1: Calibration

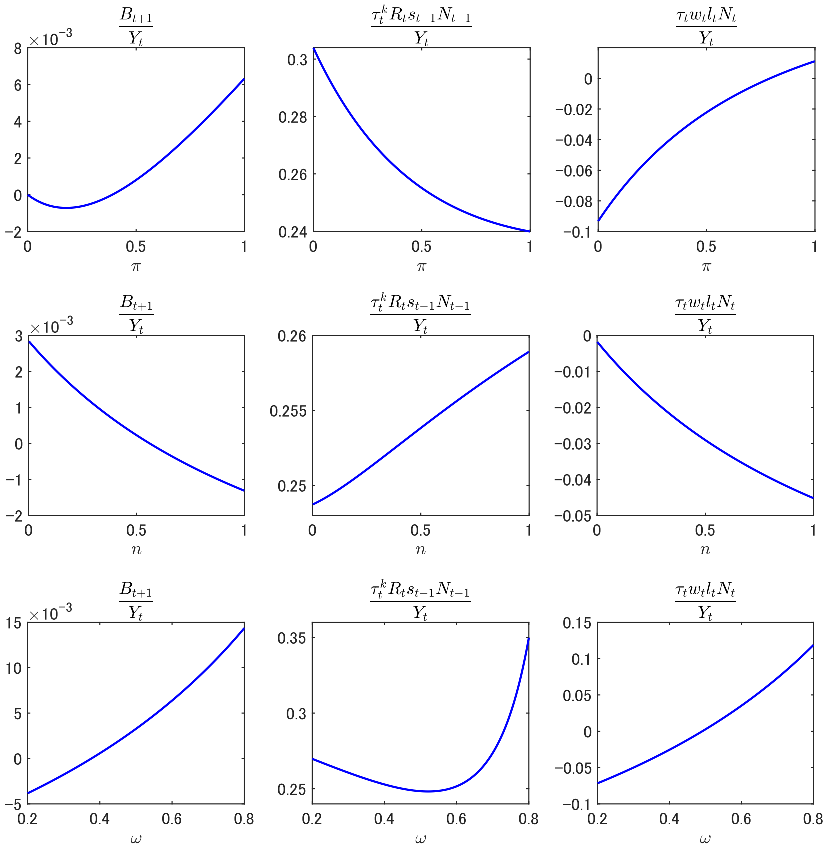

Figure 2: Predicted ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP against changes in π, n, and ω.

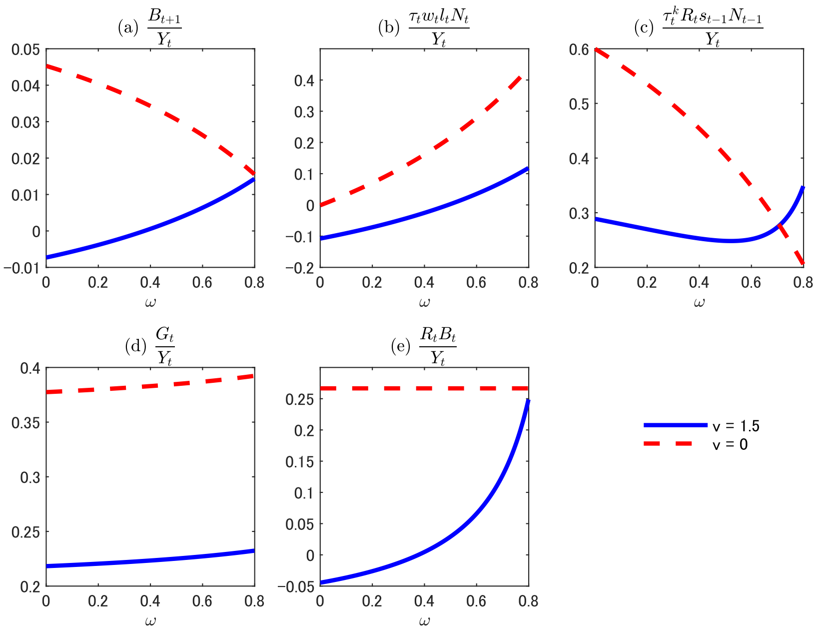

Figure 3: Numerical illustration of the effects of ω on B_{t+1}/Y_t, $\tau_t w_t l_t N_t/Y_t$, $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$, G_t/Y_t, and $R_t B_t/Y_t$. The dotted and solid curves plot the results when $v = 0$ and 1.5, respectively.

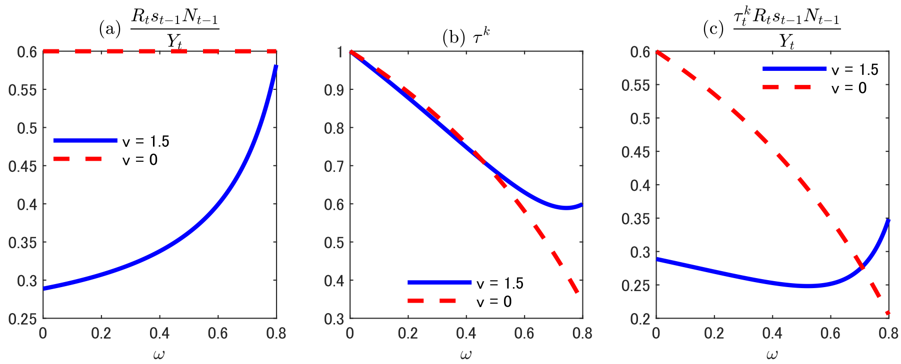

Figure 4: Numerical illustration of the effects of ω on $R_t s_{t-1} N_{t-1}/Y_t$, τ_t^K, and $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$. The dotted and solid curves plot the results when $v = 0$ and 1.5, respectively.