Product innovation, diffusion and endogenous growth

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25 June 2021

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June 2021

Abstract: We develop a model of Schumpeterian growth featuring a stochastic diffusion process where the rate of commercial success of product innovations is endogenously determined by advertising intensity. We consider both informative advertising, which young technological leaders use to increase the probability of diffusion, and defensive advertising, which incumbents use to prevent the diffusion of competing products. Economic growth depends positively on the arrival rate of product innovations and the diffusion rate of innovations into the mainstream market. We show that R&D subsidies shift relative investment incentives towards innovation and away from diffusion. This creates an inverted U-shaped relationship between R&D subsidies and both economic growth and welfare as innovations arrive more frequently, but fewer commercialize successfully. We find that lower advertising costs increase diffusion, growth, and welfare when advertising is purely informative. When we include defensive advertising, lower costs lead to socially wasteful increases in resources devoted to advertising without large increases in diffusion, reducing growth and welfare.

Keywords: Endogenous growth; innovation; diffusion; commercialization; advertising; marketing; R&D subsidies

JEL Classification: O31; O33; M30

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"There's two big frustrations of being an inventor. The first is when you can't solve a problem ... The second one [is when] you can't get the world to adopt it." (Nathan Myhrvold, 2020)[1]

1 Introduction

Schumpeterian endogenous growth models routinely assume that successful innovators costlessly and instantaneously capture their entire potential market share. This implies that all consumers immediately recognize the superiority of newly improved products and that incumbent firms are fully displaced as soon as an innovation occurs. Although analytically convenient, this assumption is clearly counterfactual. Firms devote considerable resources to build and maintain market share. As a whole, marketing expenditures are estimated to comprise as much as 8% of GDP, with advertising alone accounting for over 2% (Gourio and Rudanko, 2014; Cavenaile and Roldan-Blanco, 2019). Especially for innovative product categories, advertising is often necessary to establish product awareness and inform consumers of a new product's advantages (Goeree, 2008; Eliaz and Spiegler, 2011). In addition, new product advertising plays a fundamental role in weakening incumbent brand loyalty, reducing perceived switching costs, and overcoming considerable consumer reluctance to change status quo consumption behavior (Shum, 2004; Gourville, 2006).[2]

Still, firms often fail to profitably commercialize innovative technologies and products. Empirical estimates suggest that 40-50% of new product launches fail within their first four years, despite offering technical and functional improvements over competing products in many cases.[3] Moreover, even when commercialization is ultimately successful, it is typically a slow process. Most successful new products experience an initial period of low penetration and slow growth followed eventually by a sharp sales increase or "takeoff" to its market share as a mature product (Agarwal and Bayus, 2002; Golder and Tellis, 2004). Indeed, "the time to sales takeoff can vary considerably across product innovations; some quickly achieve sales takeoff after commercialization, whereas others languish for years with low sales" (Agarwal and Bayus, 2002).

In this paper, we develop a novel theoretical framework to examine the dynamic interaction between product innovation, commercialization, and economic growth. As in standard Schumpeterian quality ladder models, entrepreneurial firms invest in R&D to innovate higher quality products across a fixed (measure one) set of industries and new innovations arrive according to a stochastic Poisson process. However, we introduce an endogenous commercialization process that takes place in two phases. In the first phase, successful innovators instantaneously capture a small share of the market comprised of consumers who immediately recognize the superiority of innovative prod-

[1]Former Chief Technology Officer at Microsoft and co-founder of the firm Intellectual Ventures, speaking on the podcast "People I (Mostly) Admire."

[2]See Bagwell (2007) for an extensive overview of the empirical literature analyzing the effects of advertising new products.

[3]New product failure is defined in the empirical literature as either a total removal from the market or a sufficiently large underperformance relative to a pre-specified sales target. See Asplund and Sandin (1999), Gourville (2006), Chiesa and Frattini (2011) and Castellion and Markham (2013) for a summary of empirical research on new product failure.

ucts. Borrowing marketing terminology, we refer to this subset of consumers as *early adopters*. In the second phase, innovators must invest in costly advertising in order to convince the remaining *mainstream consumers* to recognize their innovative product's quality advantage.

To capture the uncertain nature of sales takeoff, we model this diffusion of innovations into the mainstream market as a stochastic Poisson process whose arrival rate depends upon advertising intensity. We consider two distinct formulations for the relationship between advertising and the probability of diffusion. First, we assume advertising is purely *informative*; only young innovators invest in advertising to expand their market share by communicating their product's advantages to potential consumers. Second, we allow for advertising to be *combative*; incumbent firms also invest in defensive advertising to protect their existing market share against new entrants. In this formulation, the rate of new product diffusion depends upon the advertising contest between young and old firms endogenously battling for consumers through advertising expenditure.[4]

In our framework, firms endogenously cycle through distinct life stages of stochastic length as new innovations arrive and either commercialize successfully or fail. Each new innovator begins life as a young technology leader, serves only early adopters, and invests in advertising to increase its chances of diffusing its product into the mainstream. If a subsequent innovation arrives in the industry before a young firm captures the mainstream market, its product fails. If the young firm instead successfully diffuses into the mainstream prior to the arrival of the next competing innovation, it fully replaces the existing incumbent and begins its tenure as an adult technological leader. Once the next innovation occurs, the now incumbent adult firm transitions to its final stage where it is no longer the technological leader, and early adopters abandon the old product for the newest iteration. However, these old firms still retain a sizable market share until they are fully displaced by the next young firm that diffuses its product successfully. In our combative advertising formulation, old firms also endogenously invest in defensive advertising to protect their market share and prolong their final stage of life.

As in traditional models, economic growth is driven by the incorporation of higher quality products into households' consumption bundles. This implies that the growth rate depends positively on both the rate of innovation and the rate of product diffusion since only innovations that commercialize successfully are adopted by mainstream consumers. We show that this relationship provides novel insights into the role of R&D subsidies in promoting economic growth. Unlike the traditional models in which R&D subsidies *always* promote growth, we find that R&D subsidies can have a non-monotonic effect on growth.[5] This is because young firms' incentives to invest in advertising in order to diffuse their product depend upon the expected length of their reign of market dominance as an adult firm. The more frequently new innovations arrive, the faster technology leaders transition to their old firm stage, and the smaller the incentive to invest in advertising. Thus, although the traditional growth promoting effect of R&D subsidies of stimulating innovation is present in our

[4] This combative advertising formulation reflects empirical evidence that "advertising is often characterized over time by reciprocal cancellation ... new entrants advertise to gain market share and thereby induce increased advertising by incumbents ... in order to limit the sales of new entrants" (Bagwell, 2007).

[5] See for example, Grossman and Helpman (1991), Şener (2008), Chu et al. (2016), and Chu and Cozzi (2018).

model, there is also a competing growth reducing effect as a smaller proportion of innovations commercialize successfully. We show that this fundamental relationship holds in both the informative and combative advertising versions of the model. Using numerical simulations, we find that R&D subsidies exhibit an inverted U-shaped relationship with both economic growth and welfare. In our benchmark case of a 25% product failure rate in the initial equilibrium, an R&D subsidy rate of 15.4% maximizes growth and a subsidy rate of 9.1% maximizes welfare. Furthermore, as the initial product failure rate increases (i.e. less frequent successful diffusion in the baseline equilibrium), the case for R&D subsidies becomes weaker. Indeed, we find that the optimal R&D policy shifts to a tax when the initial failure rate is high but still within an empirically plausible range. Hence, our results suggest that standard endogenous growth models that assume instantaneous innovation diffusion may overstate the case for large R&D subsidies[6]

These findings are consistent with both a significant empirical literature that finds R&D subsidies increase R&D investment and patenting, and recent evidence that suggests R&D subsidies are associated with a lower average market return of new patents and products. For example, Svensson (2013) finds that firms that receive subsidized R&D loans have a significantly lower renewal rate of new patents. Similarly, Czarnitzki et al. (2011) find that R&D subsidies increase the average number of new products introduced by firms, but do not improve general firm performance indicators such as profitability or market share. Indeed, as a possible explanation of their findings, Czarnitzki et al. (2011) suggest that "the reduced cost of R&D funds may shift firms' allocation of funding for innovation activities away from necessary complementary activities such as marketing." Our analysis formalizes this intuition.

Finally, we use the model to examine the economic impact of an exogenous decline in the cost of advertising. Following Grossman and Shapiro (1984) and Dinlersoz and Yorukoglu (2012), we interpret this exercise as a stylized representation of the long run effects of technological advancements in advertising, such as targeted digital advertising. We show that the effect of reduced advertising costs depends critically on whether advertising is informative or combative. When advertising is informative, reduced advertising costs lead to faster product diffusion, which increases economic growth and welfare. However, when advertising is combative, the cost reduction also stimulates defensive advertising by existing incumbents. Since advertising is characterized by reciprocal cancellation in this formulation, the primary effect is a socially wasteful increase in the resources devoted to advertising, without the dynamic benefit of a substantial increase in product diffusion. In this case, economic growth and welfare both fall with the decline in advertising costs. Thus, our analysis suggests that the welfare impact of improved advertising technology hinges on its influence on defensive advertising.

Our paper is related to several strands of endogenous growth literature, including analyses that incorporate non-instantaneous new product diffusion, intangible advertising investment, and the defensive behavior of market incumbents. Both Dinopoulos and Waldo (2005) and Dinopoulos

[6] Examples of Schumpeterian analyses that make the case for optimal R&D subsidies include Segerstrom (2007), Şener (2008), Impullitti (2010), and Minniti et al. (2013).

et al. (2021) allow for innovator market share to evolve over time, but focus on distinct underlying processes. Dinopoulos and Waldo (2005) impose an exogenous diffusion process that mimics a sales takeoff based S-curve. They use the model to explore the relationship between gradual product diffusion and the dynamics of asset prices, but the relationship between R&D and advertising incentives are absent by construction. In several respects, our methodological approach is most similar to Dinopoulos et al. (2021), who allow for endogenous changes to innovator market share over a firm life cycle. However, they focus on employment frictions as young firms expand and use the model to examine the relationship between unemployment and economic growth. Similarly, Chu and Furukawa (2013) and Cozzi and Galli (2014) develop models featuring a two stage innovation process that requires both basic and applied research to bring a product to market. These papers analyze the role of patent policy in shaping incentives through the profit division between distinct basic and applied research firms. In contrast, we focus on the relationship between separate innovation and commercialization stages of product development within a single firm.

To our knowledge, only Grossmann (2008), Cavenaile and Roldan-Blanco (2019), and Cavenaile et al. (2021) have considered the interactions between advertising and innovation in an endogenous growth framework. In these models, advertising acts as a demand shifter that increases consumers' perceived quality of existing incumbent products with an established market position. While advertising impacts innovator profits, and therefore R&D incentives, in these models, it does not impact the dynamics of innovation diffusion into the mainstream market by construction. We contribute to this existing work by focusing on the distinct role of advertising investment in determining the endogenous commercial success and failure of product innovations.[7] In this sense, our work follows the literature that views advertising as fundamental to the process of building customer capital and market share, such as Gourio and Rudanko (2014) and Arkolakis (2010) in the context of international trade. Finally, several papers including Dinopoulos and Syropoulos (2007), Davis and Şener (2012), and Klein (2020) have analyzed the defensive actions, or rent protection activities, of market incumbents in an endogenous growth framework. In all cases, these papers consider defensive actions that increase the effective cost of rival innovation, such as patent infringement litigation. By incorporating defensive advertising, our model advances a distinct form of rent protection activities that targets competitors after they have entered the market.

The remainder of this paper is organized as follows. In Section 2, we develop the informative advertising version of the model. We explore the impact of R&D subsidies on economic growth in Section 3. In Section 4, we examine the welfare properties of the model and analyze optimal R&D policies numerically. Section 5 develops the combative advertising extensions of the model and investigates the impact of lower advertising costs in both versions. Section 6 concludes.

[7] As argued by Agarwal and Bayus (2002), incorporating the "timing and causes of sales takeoff is critically important ... because they have serious short- and long-term resource implications for research and development, product development, marketing, and manufacturing."

2 The Model

2.1 Households and Perceived Quality

The economy is populated by a unit continuum of households indexed by $i \in [0,1]$. Each household is a dynastic family comprised of infinitely lived members that begins with a single member at $t = 0$ and grows at rate $n > 0$. The size of each household at time t equals the population of the economy given by $N(t) = e^{nt}$. Each household i maximizes discounted utility

$$U_i = \int_0^\infty e^{-(\rho-n)t} \, ln(u_i(t))dt, \tag{2.1}$$

where $\rho > n$ is the subjective discount rate. Per capita sub-utility at time t is defined as

$$ln(u_i(t)) = \int_0^1 ln\Big[\sum_k \tilde{q}_i(k,\omega,t)y_i(k,\omega,t)\Big]d\omega, \tag{2.2}$$

where $y_i(k,\omega,t)$ denotes household i's quantity consumed of a product that has experienced k successful innovations in industry $\omega \in [0,1]$ at time t, and $\tilde{q}_i(k,\omega,t)$ denotes household i's *perceived quality* of the associated product. Each household maximizes (2.1) by allocating individual consumption expenditure $c_i(t)$ given prices at time t. Adjusted for perceived quality, products within each industry are perfect substitutes and each household optimally purchases only the product with the lowest perceived quality adjusted price. Products enter utility symmetrically, so households evenly spread consumption expenditure across industries. Establishing notation, household i's demand for the good with the lowest perceived quality adjusted price in a typical industry is

$$y_i(t) = \frac{c_i(t)}{p(t,i)}, \tag{2.3}$$

where $p(t,i)$ is the market price of the good for which perceived quality adjusted price is lowest for consumer i. Maximizing (2.1) subject to the standard intertemporal budget constraint yields

$$\frac{\dot{c}_i(t)}{c_i(t)} = r(t) - \rho, \tag{2.4}$$

where $r(t)$ is the market interest rate.

In each industry ω and time t, there exists a single firm that produces the current state of the art product that all households perceive to be of quality $\tilde{q}(k,\omega,t) = \lambda^{k(\omega,t)}$, where $\lambda > 1$ is the constant step size of the innovation quality ladder. Since all households share this quality perception, this product represents the *definitive quality standard at level* "k." Similar to traditional models, we assume that a mass of challenger firms within each industry may produce products using the previous $k-1$ quality standard, which all consumers perceive to be one step down the λ quality ladder. Challengers invest resources in R&D to innovate new versions of the product that represent possible candidates or *prototypes* for the $k+1$ quality standard. Each $k+1$ prototype eventually either succeeds or fails. A prototype succeeds if it establishes itself as the definitive $k+1$ quality standard by convincing all consumers to perceive it to be of quality $\lambda^{k(\omega,t)+1}$. A prototype fails if all consumers ultimately reject its attempted quality improvement.

There exist two types of households that are differentiated by how they perceive the quality of $k+1$ prototypes. A constant proportion of households, $\phi \in (0,1)$, are *early adopters* who immediately consider the latest prototype to be superior to all other existing options. In the spirit of the traditional consumer classifications in marketing, we assume that early adopters derive value from consuming the newest products.[8] Specifically, these consumers always perceive a new prototype to be a λ size quality improvement over the current quality standard, but view all previously offered prototypes to be of quality level $k-1$ or lower. In contrast, the remaining $1-\phi$ proportion of households are *mainstream consumers* who initially do not consider prototypes to be viable alternatives to the current k level quality standard. Instead, mainstream consumers consider each prototype to be inferior unless they are persuaded, through endogenous advertising efforts detailed in the following section, that the prototype constitutes an actual λ size quality improvement over the current standard.

2.2 Innovation, Diffusion, and Industry Structure

The potential for different perceived quality among consumer types implies that industries and firms endogenously cycle through distinct stages as new prototypes are innovated and either succeed or fail to diffuse into the mainstream market. To see this, consider the evolution of a typical industry up the quality ladder from the k^{th} to the $k^{th}+1$ step.[9] The industry achieves the k^{th} step on the quality ladder when a firm successfully establishes their prototype as the definitive k quality standard. Until the first $k+1$ prototype arrives, this firm enjoys a λ size perceived quality advantage over all other firms that can only produce inferior $k-1$ level products. Standard limit pricing implies that this technology leader captures the industry's entire market share. We refer to such a firm as an adult firm and all such industries served by an adult firm as an A industry.

A mass of challenger firms invest in R&D to innovate new $k+1$ prototypes. As is standard, we model innovation as a stochastic Poisson process that depends on the intensity of R&D investment by challengers within each industry. Specifically, a challenger j that invests $R_j(t)$ in R&D at time t innovates a new $k+1$ prototype with instantaneous probability $I_j(t) = R_j(t)/X_R(t)$, where $X_R(t)$

[8] Within marketing, consumers with this type of novelty seeking preference structure are categorized as "innovative" consumers, which are defined by their "predisposition to buy new and different products and brands rather than remain with previous choices and consumption patterns" (Steenkamp et al., 1999). Empirical evidence shows that "novelty seeking plays an essential role in the early stages of consumer adoption of new products" (Tellis et al., 2009). To avoid confusion with the concept of new product innovation, we choose to label these consumers under the broader related term, early adopters. See also Furukawa et al. (2019, 2020) for recent analyses of the role of novelty seeking preferences in creating demand for innovative products.

[9] Since industries are structurally identical, we omit the ω index to avoid clutter. In the following sections, we show that the model is consistent with a balanced growth equilibrium with a common, constant rate of innovation and diffusion across industries. That is, in equilibrium, we have $I(\omega,t) = I(t)$ for all ω and $\delta(\omega,t) = \delta(t)$ for all prototypes.

captures the difficulty of R&D in the industry. Following the now common approach advanced by Dinopoulos and Segerstrom (1999) and Dinopoulos and Thompson (2000), we specify $X_R(t) = \kappa_R N(t)$, with $\kappa_R > 0$, so that R&D difficulty is proportional to the size of the population and scale effects are eliminated in a simple way.[10] The industry wide innovation rate is obtained by summing across all challengers,

$$I(t) = \sum_j I_{j(t)} = \frac{R(t)}{\kappa_R N(t)}, \qquad \text{where} \qquad R(t) = \sum_j R_j(t). \tag{2.5}$$

As soon as the first $k+1$ prototype is innovated, the measure $\phi \in (0,1)$ of early adopters immediately perceive the prototype to be a λ quality improvement over the k quality standard. Under limit pricing, the innovative, or young, firm's prototype captures this portion of the market and partially displaces the incumbent adult firm. This forces the incumbent into its old firm stage where it is no longer the undisputed technology leader. However, until they are convinced otherwise, the remaining $1-\phi$ mainstream consumers initially perceive the prototype to be of inferior $k-1$ or lower quality, allowing the old firm to continue to serve this portion of the market as before. We label all industries where a young and old firm are both active as a B industry.

In B industries, each young firm either succeeds or fails to establish its prototype as the definitive $k+1$ quality rung by persuading mainstream consumers that its prototype offers λ quality improvement over the k quality standard. If successful, the prototype enjoys a λ perceived quality advantage with all consumers. With limit pricing, the young firm captures the entire industry's market share, fully displaces the previous incumbent's k quality standard product, and takes its place as an adult technology leader. Note that this implies that the industry transitions back to an A type industry at quality level $k+1$, and the process begins anew with the search for $k+2$ prototypes.

To capture the notion of uncertain new product sales takeoff, we model this prototype diffusion into the mainstream market as a stochastic Poisson process that depends upon advertising intensity. Let $\delta(t)$ denote the instantaneous probability of prototype diffusion. A young firm that invests α_y in advertising diffuses with probability,

$$\delta(t) = \frac{\alpha_y(t)}{X_\alpha(t)}, \tag{2.6}$$

where $X_\alpha(t)$ represents the difficulty of diffusion in the industry. In our primary specification, we assume that advertising is purely informative and set $X_\alpha(t) = \kappa_\alpha N(t)$, with $\kappa_\alpha > 0$. That is, we assume that diffusion depends only on young firm advertising as they build product awareness and communicate product advantages to mainstream consumers. In Section 5.1, we consider combative advertising through an alternate specification of $X_\alpha(t) = \kappa_\alpha \alpha_o(t)$, where $\alpha_o(t)$ denotes the endogenous defensive advertising investment by the old firm in the industry. Thus, the alternate specification frames the diffusion process as a marketing contest between young and old firms who

[10] See for example Şener (2001), Impullitti (2010), Chu and Cozzi (2014), and Klein (2021).

battle for the favor of mainstream consumers. Under either specification, until a second $k+1$ prototype is introduced into the industry, the first $k+1$ prototype has an endogenous probability of successfully diffusing per unit time dt equal to $\delta(t)dt$.

However, innovative efforts continue in B industries while prototypes struggle to win over mainstream consumers. Should a second $k+1$ prototype be introduced into the market prior to successful diffusion of the first prototype into the mainstream, early adopters no longer view the now outdated prototype favorably. Recall that early adopters attach a λ quality advantage to the latest prototype only, while viewing all others as quality inferior. Limit pricing implies that the second prototype serves all early adopters and fully displaces the previous young firm from the market. Thus, from the perspective of challengers conducting R&D, the reward for innovating a new prototype in A and B industries is identical; new innovators always serve early adopters and attempt to diffuse their prototype as a young firm. As we will see, this implies that the model is consistent with a balanced growth equilibrium in which A and B industries share a common innovation rate. Finally, note that the introduction of additional prototypes into the market does not impact the quality perception of mainstream consumers, and therefore does not impact the old firm that serves them.

Let $n_A(t)$ and $n_B(t) = 1 - n_A(t)$ denote the proportion of A and B industries in the economy at time t respectively. An A industry cycles to a B industry when an innovation occurs. Over an interval of time dt, the transition rate of A industries to B industries is $n_A(t)I(t)dt$. Similarly, a B industry switches to an A industry when a prototype diffuses successfully. The associated flow into A industries is $(1 - n_A(t))\delta(t)dt$. This implies that the proportion of A industries in the economy evolves endogenously according to,

$$\dot{n}_A = (1 - n_A(t))\delta(t) - n_A(t)I(t). \tag{2.7}$$

We define the diffusion failure rate as the proportion of prototypes that exit the market without successfully diffusing into the mainstream. Over an interval of time dt, each existing prototype has a $\delta(t)dt$ probability of diffusion success and an $I(t)dt$ probability of failure. Therefore, the endogenous rate of diffusion failure for the mass of prototypes is given by,

$$f(t) \equiv \frac{I(t)}{I(t) + \delta(t)}. \tag{2.8}$$

2.3 Labor and Production

Labor is used for three separate tasks: advertising, R&D, and the production of consumption goods. Households supply labor inelastically and labor is freely mobile across industries and tasks. We normalize the wage rate common to all labor to unity and assume that one unit of labor produces one unit of the consumption good in each industry. As discussed in the previous sections, each type of firm (young, adult, old) enjoys a λ size perceived quality advantage over its nearest competitor with a fixed proportion of the population (ϕ, 1, $1-\phi$ respectively). Thus, each firm type optimally captures its respective market share with limit pricing at a common price of $p(t) = \lambda$.

In A industries, a single adult firm serves the entire market with corresponding quantity sold equal to $y_a(t) = c(t)N(t)/\lambda$, where $c(t)$ denotes per capita consumption expenditure common to all households. Thus, each adult firm earns flow profits equal to

$$\pi_a(t) = p(t)y_a(t) - y_a(t) = c(t)N(t)\frac{(\lambda - 1)}{\lambda}. \tag{2.9}$$

The situation is identical in B industries after accounting for the proportion of consumers served by young and old firms. Young firms serve the economy's $\phi N(t)$ early adopters, sell $y_y = c(t)\phi N(t)/\lambda$ units, and earn flow profits (gross of advertising expenditure) equal to $\pi_y(t) = c(t)\phi N(t)(1 - \lambda^{-1}) = \phi\pi_a(t)$. Old firms serve the economy's $(1 - \phi)N(t)$ mainstream consumers, sell $y_y = c(t)(1 - \phi)N(t)/\lambda$ units, and earn flow profits equal to $\pi_o(t) = (1 - \phi)\pi_a(t)$. Note that total employment in production is common across A and B industries. For the economy as a whole, labor used in the production of consumption goods is

$$L_c(t) = \frac{c(t)N(t)}{\lambda}. \tag{2.10}$$

Finally, we assume that R&D and advertising are produced under constant returns to scale. Let $\beta_R > 0$ and $\beta_\alpha > 0$ denote the unit labor requirement in R&D and advertising respectively. Note that advertising is conducted only by young firms in B industries and challengers target their R&D efforts at all industries. Thus, total employment in R&D and advertising respectively are given by

$$L_R(t) = \beta_R R(t), \qquad L_\alpha(t) = n_B(t)\beta_\alpha\alpha_y(t). \tag{2.11}$$

2.4 Stock Market Valuations and Optimal Advertising

Let $V_k(t)$ denote the value of firm of type k in a typical industry. The no-arbitrage condition associated with a challenger's R&D investment requires that stock issued by the challenger provides the same expected return as a diversified investment of equal size. Consider a $V_c(t)$ size investment in challenger j. Over a dt unit of time, there is a $I_j(t)dt$ probability of successful innovation and a corresponding realized gain equal to the value of transitioning to a young firm, $V_y(t) - V_c(t)$. Innovation does not occur with probability $(1 - I_j(t)dt)$ and there is an associated capital change of $dV_c(t) = \dot{V}_c(t)dt$. Independent of the innovation outcome, challenger j incurs a R&D cost equal to $(1 - \sigma_R)\beta_R R_j(t)dt$, where $0 \le \sigma_R < 1$ denotes the subsidy rate for R&D investment.

The total equity return of these components must equal the risk-free return of $r(t)V_c(t)dt$. Thus, the no-arbitrage condition for challenger firms is

$$r(t)V_c(t)dt = I_j(t)(V_y(t) - V_c(t))dt - (1 - \sigma_R)\beta_R R_j(t)dt + (1 - I_j(t)dt)\dot{V}_c(t)dt. \tag{2.12}$$

Free-entry into R&D implies that $\dot{V}_c = V_c = 0$. Taking limits as $dt \to 0$ yields the following free-entry condition equating the cost and expected return to R&D, $I_j(t)V_y(t) = (1-\sigma_R)\beta_R R_j(t)$. Given (2.5), the free-entry condition can be rewritten as

$$V_y(t) = (1-\sigma_R)\beta_R \kappa_R N(t), \tag{2.13}$$

which relates the value of a young firm to the cost of innovating a new prototype. Note that (2.13) directly implies that the value of young firms grows with the population, with $\dot{V}_y(t)/V_y(t) = n$.

Next, consider the expected return from holding $V_y(t)$ of stock in a young firm over time interval dt. The young firm earns flow profit $\phi\pi_a(t)dt$ and incurs a cost of advertising $\beta_\alpha \alpha_y(t)dt$. With probability $\delta(t)dt$, the firm successfully diffuses its product and enjoys a capital gain of $V_a(t) - V_y(t)$. With probability $I(t)dt$, a new prototype arrives and displaces the young firm creating a capital loss of $V_y(t)$. With probability $(1 - \delta(t)dt - I(t)dt)$, the firm retains its position as a young firm and there is an associated change in valuation of $\dot{V}_y(t)dt$. Combining terms, the corresponding no-arbitrage condition is

$$\begin{gathered} r(t)V_y(t)dt = \phi\pi_a(t)dt - \beta_\alpha \alpha_y(t)dt + \delta(t)(V_a(t) - V_y(t))dt - \\ I(t)V_y(t)dt + (1 - \delta(t)dt - I(t)dt)\dot{V}_y(t)dt. \end{gathered} \tag{2.14}$$

Each young firm chooses α_y in order to maximize their valuation given by the left hand side of (2.14). Using the definition of $\delta(t)$ from (2.6) and taking limits as $dt \to 0$, the associated first order condition can be written $\delta(t)(V_a - V_y) = \beta_\alpha \alpha_y(t)$, which equates the expected return and cost of advertising expenditures.[11] Substituting this optimal advertising condition into (2.14) yields the following expression for the value of a young firm

$$V_y(t) = \frac{\phi\pi_a(t)}{I(t) + r(t) - n}. \tag{2.15}$$

Much like firms in traditional endogenous growth models, young firms discount flow profits by the rate of replacement $I(t)$ and the effective interest rate $r(t) - n$. After incorporating optimal advertising, the valuation of a young firm does not directly depend on the expected value of diffusing into the mainstream since it is exactly offset by advertising expenditure. Finally, again using (2.6), note that the optimal advertising condition is equivalent to

$$V_a(t) - V_y(t) = \beta_\alpha \kappa_\alpha N(t), \tag{2.16}$$

which provides a useful expression for the value of successful diffusion.

The expected return of holding $V_a(t)$ of stock in an adult firm over interval dt includes the profit flow $\pi_a(t)dt$, minus the $I(t)dt$ probability that a new prototype will be innovated and force the firm into old age with associated capital cost $V_a(t) - V_o(t)$. With probability $(1 - I(t)dt)$, no such innovation occurs and the value of the adult firm changes by $\dot{V}_a(t)dt$. This gives the following

[11] Note from (2.6) that $d\delta(t)/d\alpha_y(t) = \delta(t)/\alpha_y(t)$.

no-arbitrage condition for adult firms

$$r(t)V_a(t)dt = \pi_a(t)dt - I(t)(V_a(t) - V_o(t))dt + (1 - I(t)dt)\dot{V}_a dt. \tag{2.17}$$

Taking limits as $dt \to 0$ and noting that $\dot{V}_a/V_a = n$ from (2.16), yields an expression for value of an adult firm

$$V_a(t) = \frac{\pi_a(t) + I(t)V_o(t)}{I(t) + r(t) - n}. \tag{2.18}$$

Finally, an old firm generates a profit flow $(1-\phi)\pi dt$ and faces a capital loss of $V_o(t)dt$ if the young firm in the industry displaces it by successfully diffusing a new prototype with probability $\delta(t)dt$. If diffusion does not occur, the old firm experiences a change in valuation of $\dot{V}_o(t)dt$. Combining terms yields the no-arbitrage condition for old firms

$$r(t)V_o(t)dt = (1-\phi)\pi_a(t)dt - \delta(t)V_o(t)dt + (1 - \delta(t)dt)\dot{V}_o dt. \tag{2.19}$$

Taking limits once again, we obtain an expression for the value of an old firm

$$V_o(t) = \frac{(1-\phi)\pi_a(t)}{\delta(t) + r(t) - n}. \tag{2.20}$$

2.5 Equilibrium

We now solve for a steady state equilibrium in which $I(t)$, $\delta(t)$, $c(t)$, $f(t)$, and $n_a(t)$ are constant, $\pi_a(t)$, $V_y(t)$, $V_a(t)$, and $V_o(t)$ grow at the rate of population growth n, the labor market clears, the free-entry condition of (2.13) holds, and young firms choose advertising expenditure to maximize their value according to (2.16). Henceforth, we drop the time index for all variables that are constant in equilibrium. We solve for the model's steady state equilibrium by deriving two equilibrium conditions in I and δ.

Imposing $\dot{n}_A = 0$ in (2.7) yields an expression for the equilibrium proportion of A and B type industries in terms of the rate of innovation and diffusion,

$$n_A = \frac{\delta}{\delta + I}, \qquad n_B = 1 - n_A = \frac{I}{\delta + I}. \tag{2.21}$$

Observe from (2.8) that the economy's endogenous failure rate f of new innovations is equal to the proportion of B type industries n_B in equilibrium. This is because industries cycle from their B to A configurations if and only if a prototype diffuses successfully. In traditional models of endogenous growth with instantaneous new product diffusion, all industries exhibit an A type structure and innovations never fail. In our framework, this is equivalent to the limit case of $\delta \to \infty$.

Next, combine the free entry condition (2.13) and the value of a young firm (2.15), noting that $r = \rho$ in equilibrium, to derive the following equilibrium relationship between I and c based on the cost and reward from innovating a new prototype,

$$(1-\sigma_R)\beta_R\kappa_R = \frac{c\phi(\lambda-1)}{\lambda(I+\rho-n)}. \tag{2.22}$$

As in traditional models, a greater level of per capita consumption c implies a greater profit incentive to conduct R&D and a corresponding higher rate of innovation. Similarly, combining the optimal advertising condition (2.16) with the expressions for the value of each firm type (2.15), (2.18), and (2.20), provides an equilibrium relationship between δ, I and c based on the cost and reward from successfully diffusing a prototype into the mainstream,

$$\beta_\alpha\kappa_\alpha = \frac{c(1-\phi)(\lambda-1)]}{\lambda}\Big[\frac{\delta+I+\rho-n}{(I+\rho-n)(\delta+\rho-n)}\Big]. \tag{2.23}$$

Once again, a greater c implies young firms have a greater incentive to advertise since capturing the entire industry's market share as an adult firm becomes more profitable. On the other hand, a greater rate of new prototype innovation I reduces the incentive to invest in advertising since the expected duration of market dominance as an adult firm is shortened.

Combining (2.22) and (2.23) yields our first equilibrium condition in I and δ that captures the relative incentive to invest in R&D over advertising,

$$I = (\rho-n+\delta)\Gamma, \qquad \text{where} \qquad \Gamma \equiv \Big[\frac{\phi\beta_\alpha\kappa_\alpha}{(1-\phi)(1-\sigma_R)\beta_R\kappa_R}-1\Big] \qquad \textbf{[RDAC]} \tag{2.24}$$

We refer to equation (2.24) as the "R&D - advertising curve" (RDAC) Clearly, in order to be consistent with an equilibrium with a positive rate of innovation and diffusion the parameters of the model must produce $\Gamma > 0$. This condition will be met when the benefit of R&D (an initial ϕ market share as a young firm) relative to the cost of R&D (the $(1-\sigma_R)\beta_R\kappa_R$ term) is sufficiently high compared to the benefit of diffusion (capturing an additional $1-\phi$ market share) relative to the cost of diffusion (the $\beta_\alpha\kappa_\alpha$ term). Henceforth, we assume the following,

Assumption 1. $\Gamma > 0$, *as defined in* (2.24).

With this condition in place, the RDAC specifies a linear and upward sloping relationship when graphed in (δ, I) space. To understand why, first recall that the incentives to invest in R&D and advertising both scale proportionally with c. Thus, the c term vanishes in (2.24). Second, although R&D incentives do not directly depend upon the rate of diffusion, advertising incentives are strictly decreasing in the economy's diffusion rate. This is because a higher overall diffusion rate decreases a firm's expected tenure as an old firm through equation (2.20), thereby decreasing the value of adult firm through equation (2.18). For a given level of I, a larger δ implies a greater relative incentive to invest in R&D over advertising. To restore the RDAC, I must change to realign relative incentives. It follows from (2.22) and (2.23) that, although a greater I decreases the reward from both R&D and advertising, the effect is stronger for R&D. Thus, a greater I decreases the relative incentive to invest in R&D and restores equilibrium.

Labor market clearing provides our second equilibrium condition. Market clearing requires that the aggregate supply of labor $N(t)$ equal aggregate demand for labor across production, R&D, and advertising, given by (2.10) and (2.11) respectively. Recall that labor is used for advertising only in B industries, while R&D is conducted across all industries. After eliminating $N(t)$ and using the expressions for the innovation rate (2.5), diffusion rate (2.6), and equilibrium proportion of B industries (2.21), the labor market clearing condition can be written

$$1 = \frac{c}{\lambda} + \beta_R \kappa_R I + \beta_\alpha \kappa_\alpha \frac{\delta I}{I+\delta}. \tag{2.25}$$

Equation (2.25) captures the trade-off inherent to allocating finite labor resources across the three market activities that require labor. Using the relationship between I and c from equilibrium R&D incentives (2.22) to substitute for c, we can express the labor market clearing condition (LMCC) in its final form in terms of I and δ only,

$$1 = \frac{(1-\sigma_R)\beta_R \kappa_R}{\phi(\lambda-1)}(\rho - n + I) + \beta_R \kappa_R I + \beta_\alpha \kappa_\alpha \frac{\delta I}{I+\delta} \qquad \textbf{[LMCC]} \quad (2.26)$$

When graphed in δ, I space, the LMCC is strictly downward sloping. Note that the labor requirement of increasing δ depends on the cost of diffusion in a particular industry $\beta_\alpha \kappa_\alpha$ and the proportion of industries with prototypes attempting to diffuse $n_B = I/(I+\delta)$.

The model's equilibrium is determined by solving the RDAC given by (2.24) and the LMCC given by (2.26) for I and δ. Note that the LMCC provides an upper bound on the rate of innovation the economy's resources can support. Specifically, when $\delta \to 0$, the LMCC implies that $I \to I_{max}$, where

$$I_{max} = \frac{1-\Omega(\rho-n)}{\Omega + \beta_R \kappa_R}, \qquad \text{and} \qquad \Omega = \frac{(1-\sigma_R)\beta_R \kappa_R}{\phi(\lambda-1)}. \tag{2.27}$$

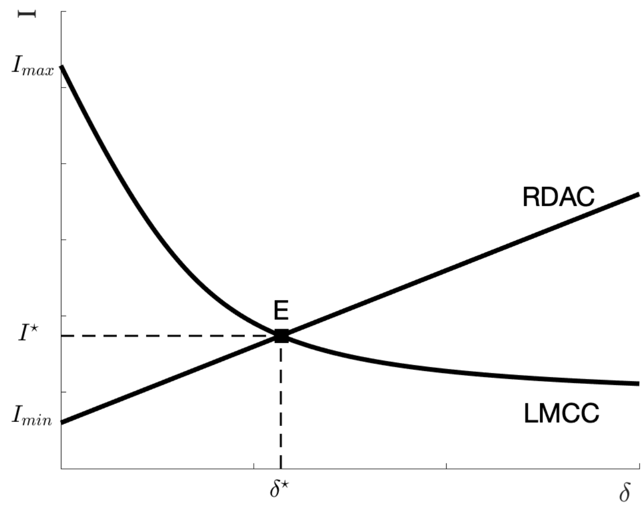
Intuitively, I_{max} represents the rate of innovation in the economy if no resources are devoted to advertising and all firms remain in their young stage until they are displaced by subsequent innovation. Similarly, the RDAC provides a lower bound on the rate of innovation that is consistent with relative R&D and advertising incentives that produce a positive rate of diffusion in equilibrium. That is, when $\delta \to 0$, the RDAC implies that $I \to I_{min}$, where

$$I_{min} = (\rho - n)\Gamma. \tag{2.28}$$

Figure 1 depicts the model's equilibrium by graphing the RDAC and LMCC in (δ, I) space. As illustrated in the figure, the following additional parameter restriction is necessary and sufficient to guarantee a unique steady state equilibrium.

Assumption 2. $I_{max} > I_{min}$, *as defined in* (2.27) *and* (2.28) *respectively.*

Figure 1: Equilibrium



After establishing I and δ, all other endogenous variables can be determined. We follow standard practice in quality ladder models and define economic growth as the growth rate of per capita sub-utility $ln(u(t))$. As shown in the Appendix, we can decompose per capita sub-utility into the following three terms,

$$ln(u(t)) = ln(c/\lambda) + n_B \phi ln(\lambda) + ln(\lambda)(1-f)It. \tag{2.29}$$

The first term captures the standard effect of per capita consumption given limit pricing. This term is common across all industries since all firms charge the same price of $p = \lambda$. On the other hand, in B industries, early adopters and mainstream consumers purchase products of different perceived quality. The second term accounts for the n_B industries in which a ϕ proportion of early adopter consumers purchase products that they perceive to be one λ step up the quality ladder from the current quality standard. The final term captures the dynamic effect of the arrival of new innovations. In our framework however, only innovations that successfully diffuse into the mainstream push the economy up the quality ladder.[12]

Differentiating (2.29) with respect to time yields an expression for the rate of economic growth, g,

$$g = ln(\lambda)(1-f)I = \frac{I\delta}{I+\delta} ln(\lambda). \tag{2.30}$$

Note that innovation and diffusion have a complementary impact on economic growth; the marginal increase in economic growth associated with an increase in innovation (diffusion) depends positively on the rate of diffusion (innovation). As we discuss in the following sections, this relationship will drive the growth impact of policy changes that move the innovation and diffusion rate in opposite

[12]Recall that transition from a B industry to an A industry occurs if and only if a prototype diffuses successfully. Thus, the aggregate rate of successful diffusion is $\delta n_b = \frac{\delta I}{I+\delta}$, which is equal to $(1-f)I$ with f defined as in (2.8).

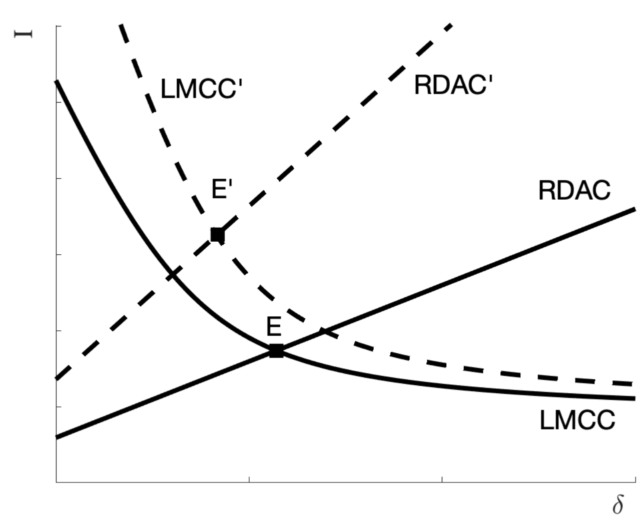
directions.

3 The Impact of R&D Subsidies

In this section, we conduct a comparative statics exercise to examine how the R&D subsidy rate (σ_R) impacts the equilibrium rate of innovation, diffusion, product failure, and economic growth. First, observe that our model includes the traditional innovation enhancing effect of R&D subsidies. By directly reducing the cost of R&D, the subsidy implies a greater rate of innovation at any fixed level of consumption expenditure through the free-entry condition of (2.22). Since the LMCC incorporates R&D incentives, this effect manifests as a rightward shift in the LMCC as depicted in Figure 2. Intuitively, the reduced cost of R&D implies that a lower level of c is required to align incentives at any rate of innovation. The LMCC's rightward shift in (δ, I) space represents the additional labor resources available to both I and δ given this lower required c.

However, the R&D subsidy also shifts *relative* investment incentives towards R&D and away from advertising. This is represented in Figure 2 by a leftward shift in the RDAC curve, and a movement along the LMCC as more of the economy's labor resources are devoted to R&D. While both shifts to the RDAC and LMCC imply a higher equilibrium rate of innovation, note that the change in the equilibrium rate of diffusion is determined by the relative importance of the competing investment incentive effect and the resource allocation effect. In the Appendix, we show that the relative incentive effect always dominates, and the equilibrium rate of diffusion always falls when an R&D subsidy is implemented. This directly implies an increase in the equilibrium product failure rate. However, since the change to I and δ have opposite signs, the overall change to economic growth g is ambiguous in the general case. These findings are summarized in the following proposition,

Figure 2: R&D Subsidy



Proposition 1. *Subsidizing R&D investment decreases the diffusion rate, increases the innovation rate, increases the product failure rate, and has an ambiguous effect on economic growth. That is,* $\frac{\partial \delta}{\partial \sigma_R} < 0$, $\frac{\partial I}{\partial \sigma_R} > 0$, $\frac{\partial f}{\partial \sigma_R} > 0$, *and* $\frac{\partial g}{\partial \sigma_R} >< 0$.

Proof. See Appendix.

To illustrate the potential for a non-monotonic effect of R&D subsidies on the rate of economic growth, we turn to numerical simulations of the model. As depicted in Figure 3, subsidizing R&D always increases the equilibrium innovation rate and reduces the equilibrium rate of diffusion. As a result, a greater proportion of newly innovated prototypes fail to diffuse into the mainstream market. Since only prototypes that diffuse successfully push the economy up the quality ladder, the net effect on the equilibrium rate of growth, $g = ln(\lambda)I(1-f)$, is determined by the competing effects of the increased arrival of new prototypes through innovation against the increased diffusion failure rate of existing prototypes. At low levels of σ_R, the failure rate is sufficiently low so that the innovation effect dominates and economic growth increases with the subsidy. At higher levels of σ_R, the effect of the relatively large failure rate dominates and economic growth decreases with the subsidy. This generates an inverted U-shaped relationship between R&D subsidies and economic growth. In the numerical example presented in Figure 3, growth is maximized at an R&D subsidy rate of $\sigma_R = 0.154$.

Figure 3: Numerical Example: The Impact of R&D Subsidies

(a) I and δ (b) g

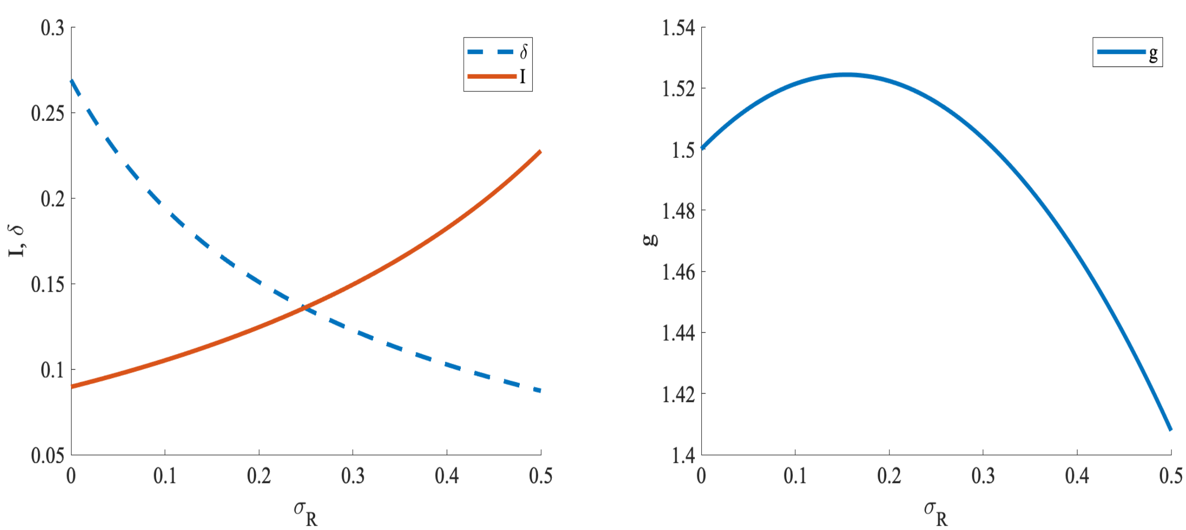


Figure 3 plots equilibrium values of I, δ and g against the R&D subsidy rate σ_R. In this numerical example, the model is calibrated so that $g = 1.5\%$ and $f = 0.25$ when $\sigma_R = 0$. The corresponding growth maximizing subsidy rate is $\sigma_R^* = 0.154$. See Appendix B for a discussion of the calibration and the associated parameter values.

4 Welfare

We consider a social planner who chooses c, I, and δ to maximize social welfare, subject to the aggregate resource constraint (2.25). As is standard, we assume that the social planner cannot impact firm pricing decisions. Using our expression for per capita sub-utility (2.29), welfare discounted to time zero can be written as

$$(\rho - n)U = \frac{ln(\lambda)I\delta}{(\rho - n)(I + \delta)} + ln(c) - ln(\lambda)\Big[1 - \phi n_B\Big]. \tag{4.1}$$

The Lagrangian associated with the social planner's optimization problem is

$$\mathcal{L}(c, I, \delta, \mu) = (\rho - n)U + \mu\Big[1 - \frac{c}{\lambda} - \beta_R\kappa_R I - \beta_\alpha\kappa_\alpha\delta n_B\Big], \tag{4.2}$$

where μ is the Lagrange multiplier and $(\rho - n)U$ is given by (4.1). The welfare maximizing solution for I is obtained from $\partial\mathcal{L}/\partial I = 0$, which equates the social cost and return to R&D at the margin:

$$\beta_R\kappa_R + \beta_\alpha\kappa_\alpha\delta\frac{\partial n_B}{\partial I} = \frac{c}{\lambda}\frac{ln(\lambda)\delta}{(\rho - n)}\frac{\partial n_B}{\partial I} + \phi ln(\lambda)\frac{c}{\lambda}\frac{\partial n_B}{\partial I}. \tag{4.3}$$

The analogous expression for the market cost and return to R&D is given by (2.22).

Comparing these expressions, we see several reasons for the market and socially optimal levels of R&D to differ. To begin with, we capture two standard effects that are well established in the Schumpeterian growth literature. The first is the *monopoly distortion effect*. The social planner considers the utility benefit of a marginal innovation measured by $ln(\lambda)$, whereas firms consider the monopoly mark-up rate of $\lambda - 1$. Since $\lambda - 1 > ln(\lambda)$, this effect implies the market overinvests in R&D. The second is the *intertemporal spillover effect*. The social planner discounts the benefits of innovation by $\rho - n$, while the effective market discount rate of $I + \rho - n$ incorporates the expected capital loss due to replacement. With $I > 0$, this effect implies that the market underinvests in R&D.

Furthermore, our modeling of endogenous diffusion generates three novel effects.

1. *The diffusion resource burden of marginal innovation.* A greater innovation rate raises the proportion of industries, n_B, in which young firms invest resources in advertising to diffuse their newly innovated prototypes. At a given diffusion rate δ, this reduces the resources available for R&D and production. The corresponding social cost of R&D is captured by the $\beta_\alpha\kappa_\alpha\delta(\partial n_B/\partial I) > 0$ term on the left hand side of (4.3). Since private firms do not consider this additional resource burden on the aggregate economy, this effect implies that the market overinvests in R&D.

2. *The dynamic effect of stochastic diffusion.* The social planner scales down the dynamic impact of a marginal innovation since prototype diffusion is uncertain (prototypes fail to diffuse at rate f). The first term on the right hand side of (4.3) captures this scaling factor of $\delta(\partial n_B/\partial I) = (1 - f)^2 \in (0, 1)$. Thus, the social planner recognizes that only prototypes that

diffuse successfully push the economy up the quality ladder and generate economic growth.[13] Although private R&D incentives also incorporate the effect of stochastic diffusion, firms consider only the internal cost and benefits of their efforts to use advertising to gain market share in the value of an innovation. Private firms scale the entire dynamic benefit of innovation by $\phi \in (0,1)$ to reflect their initial market share of early adopters, but market R&D incentives do not directly incorporate the economy's failure rate. Holding all else constant, this effect implies that the market underinvests in R&D if $\phi < (1-f)^2$.

3. *The the early adopter market effect.* The social planner considers the one-time utility gain of early adopters from marginal innovation. A higher innovation rate raises the aggregate mass of the early-adopter market ϕn_B and generates a social gain that is independent of the dynamic effect of prototype diffusion. This utility gain is captured by $(\phi cln(\lambda)/\lambda)(\partial n_B/\partial I) > 0$, the last term on the right hand side of (4.3). Since private R&D firms do not consider this social gain associated with early adopters, this effect implies that the market underinvests in R&D.

We now turn to the welfare maximizing solution for δ. This is obtained from $\partial \mathcal{L}/\partial \delta = 0$, which equates the social cost and benefit of prototype diffusion at the margin.

$$\beta_\alpha \kappa_\alpha = \frac{cln(\lambda)}{(\rho - n)\lambda} - \frac{\phi cln(\lambda)}{\lambda I}. \tag{4.4}$$

The analogous expression for the market based cost and return to diffusion is given by (2.23). In addition to the monopoly distortion effect, we identify three novel differences between the market-determined and socially optimal rates of diffusion.

1. *Dynamic effect of life-cycle replacement.* The social planner recognizes that prototype diffusion always pushes the economy up the quality ladder and generates economic growth. Thus, the social planner applies a $(\rho - n)$ discount factor to the welfare benefit of diffusion. In contrast, young firms discount the benefit of diffusion based on the different rate of replacement they face over the firm life-cycle. That is, firms enjoy the adult status they gain from diffusion only until an innovation occurs and they transition to their old stage. Once old, they are displaced from the market entirely at the rate of diffusion. The corresponding market discount rate is captured by the second term in brackets in (2.23). For all $\delta > 0$ and $I > 0$, this effect implies that the market underinvests in diffusion.

2. *The effect of early adopters on the benefit of diffusion.* The social planner recognizes that early adopters do not immediately benefit from the diffusion of new prototypes since they already purchase the prototype prior to diffusion. Instead, early adopters benefit only after the next innovation arrives, which enables them to consume a prototype at the next quality ladder step. The social planner accounts for this effect by subtracting $\phi cln(\lambda)/(\lambda I)$ from

[13] If diffusion success were instantaneous as assumed in traditional treatments, then $\delta \to \infty$, $\partial n_B/\partial I \to 1$, and this externality vanishes.

the overall benefit of diffusion in (4.4).[14] Young firms engaged in diffusion do not take this effect into consideration. For any positive and finite I, this effect implies that the market overinvests in diffusion.

3. *Mainstream market effect.* Since young firms already serve the early adopter market, they only consider the benefit of the additional $1 - \phi$ market share gained by capturing the mainstream market. In contrast, the social planner considers the dynamic full impact of diffusion through the aggregate growth rate. This effect implies that the market underinvests in diffusion.

4.1 Optimal R&D Subsidies

In this section, we conduct numerical simulations to examine the welfare maximizing R&D policy. As the preceding discussion makes clear, the model's market equilibrium features two potential sources of dynamic inefficiency: the traditional allocative inefficiency from over or underinvestment in R&D and a novel source of inefficiency from over or under investment in diffusion promoting advertising. In general, using an R&D subsidy as a single policy instrument will be insufficient to correct both potential distortions and achieve the socially optimal allocation. Nonetheless, we will show that R&D policy may still improve welfare through its impact on the relative size of these two distortions.

In all simulations, we choose the following benchmark parameters: $\lambda = 1.25$, $\rho = 0.07$, $n = 0.01$, $\kappa_R = \kappa_\alpha = 1$, and $\phi = 0.20$. We then calibrate the unit labor requirements of R&D (β_R) and advertising (β_α) to deliver a target rate of economic growth and diffusion failure. To highlight the importance of the initial diffusion failure rate in determining the optimal R&D policy, we separately examine three distinct target values of $f_0 = [0.01, 0.25, 0.45]$. In each case, we target a common initial growth rate of $g = 1.5\%$. See Appendix B for parameter details. Numerical results are displayed in Table 1 and Figure 4.[15]

Our central result is that the welfare maximizing R&D policy $\sigma^{\star}_{R,U}$ can be a subsidy or a tax depending on the initial diffusion failure rate f_0. Specifically, we find that it is optimal to subsidize R&D at low levels of f_0 and tax R&D at high levels of f_0. To clarify the intuition, we first note that the market equilibrium exhibits underinvestment in both R&D and diffusion across each value of f_0.[16] However, since the growth rate is held constant at 1.5%, a higher initial failure rate necessarily implies a higher rate of innovation and a lower rate of diffusion in equilibrium. This difference in the relative resource allocation to R&D and advertising determines the relative size of the equilibrium distortion from underinvestment in innovation and diffusion.

[14] If the next innovation occurred instantaneously with $I \to \infty$, early adopters would immediately benefit from diffusion, and this effect vanishes.

[15] We find these numerical results to be generally robust to a range of alternate parameter values and target rates. Computational files are available upon request.

[16] This corresponds to the typical finding in endogenous growth models that the market equilibrium exhibits underinvestment in growth promoting activity. In traditional models, this equates to R&D. In our model, both R&D and diffusion contribute positively to growth. One can show this underinvestment directly using the five effects identified in the previous section that determine the social desirability of a marginal innovation and the analogous three effects of marginal diffusion.

Table 1: Optimal R&D Subsidies

	$f_0 = 0.01$		$f_0 = 0.25$		$f_0 = 0.45$	
	$\sigma_R = 0$	$\sigma^\star_{R,U} = 0.238$	$\sigma_R = 0$	$\sigma^\star_{R,U} = 0.091$	$\sigma_R = 0$	$\sigma^\star_{R,U}$ = -0.078
$g(\%)$	1.500	1.651	1.500	1.520	1.500	1.499
I	0.068	0.104	0.090	0.104	0.122	0.101
δ	6.636	0.259	0.269	0.199	0.149	0.174
c	1.104	1.076	1.092	1.086	1.087	1.090
f	0.010	0.286	0.250	0.342	0.450	0.387
U	2.097	2.306	2.106	2.127	2.167	2.177
$\sigma^\star_{R,g}$	0.317	–	0.154	–	-0.038	–
$\sigma^\star_{R,U}$	0.238	–	0.091	–	-0.078	–

Table 1 presents equilibrium values of the model's endogenous variables for three distinct initial values of the failure rate, $f_0 = [0.01, 0.25, 0.45]$, and $g = 1.5\%$. In each case, we present results for the initial equilibrium with $\sigma_R = 0$ and with R&D subsidized at the welfare maximizing level. The final two rows of Table 1 display the associated R&D subsidy rates that maximize economic growth ($\sigma^\star_{R,g}$) and welfare ($\sigma^\star_{R,U}$) respectively.

Second, recall from Proposition 1 that subsidizing R&D always increases the equilibrium rate of innovation and decreases the rate of diffusion. This implies that subsidizing R&D will reduce the distortion associated with underinvestment in R&D but *magnify* the distortion associated with underinvestment in advertising.

In our low failure case with $f_0 = 0.01$, we find a large optimal subsidy of $\sigma^\star_{R,U} = 23.8\%$. This is because the social benefit of reducing the substantial underinvestment in R&D dominates the social cost of exacerbating the market's relatively less severe underinvestment in advertising. Although the reallocation generated by the subsidy increases the failure rate sharply from 0.01 to 0.28, the increase in the rate of innovation is sufficient to raise economic growth from 1.5% to 1.65%. Despite a small decline in the level of consumption per capita as fewer resources are devoted to manufacturing, the dynamic welfare gain from higher growth and the static welfare gains due to the increased share of early adopter markets (recall that $f = n_b$) dominate the static losses from lower consumption. In the intermediate case with $f_0 = 0.25$, it remains optimal to subsidize R&D, but at a substantially lower rate with $\sigma^\star_{R,U} = 9.1\%$. The direction of change to all endogenous variables mirrors the $f_0 = 0.01$ case, just with smaller magnitudes. In contrast, when the failure rate is relatively high with $f_0 = 0.45$, the size of equilibrium underinvestment in diffusion is sufficiently large relative to underinvestment in R&D that it becomes optimal to tax R&D, with $\sigma^\star_{R,U} = -7.8\%$. In this case, the increase in diffusion following the R&D tax almost completely offsets the negative growth impact of reducing innovation. The social cost associated with the minor decreases in economic growth and the share of early adopter markets is dominated by the static benefit of greater consumption as more resources are available for manufacturing.

This simulation exercise clearly demonstrates that the optimal R&D policy is highly sensitive to the initial failure rate, even though the growth rate is held constant at 1.5%. In this way, the model frames the typical finding of large optimal R&D subsidies in Schumpeterian models as conditional on the implicit assumption of instantaneous diffusion and failure rate of zero. In fact, even in the low initial failure rate case with $f_0 = 0.01$, we emphasize that the relationship between R&D subsidies and the endogenous rate of diffusion plays an important role in determining the optimal subsidy level. This is because the equilibrium failure rate always increases with the R&D subsidy. The negative growth and welfare effects of increasing the failure rate eventually dominate the positive effects of stimulating innovation, even in cases where it is initially growth and welfare enhancing to subsidize R&D. As illustrated in Figure 4, this dynamic implies that the relationship between the R&D subsidy level and welfare and that between the R&D subsidy and growth are both characterized by similar inverted U-shaped curves for each initial equilibrium failure rate.

Figure 4: Optimal R&D Subsidies

(a) g (b) Welfare

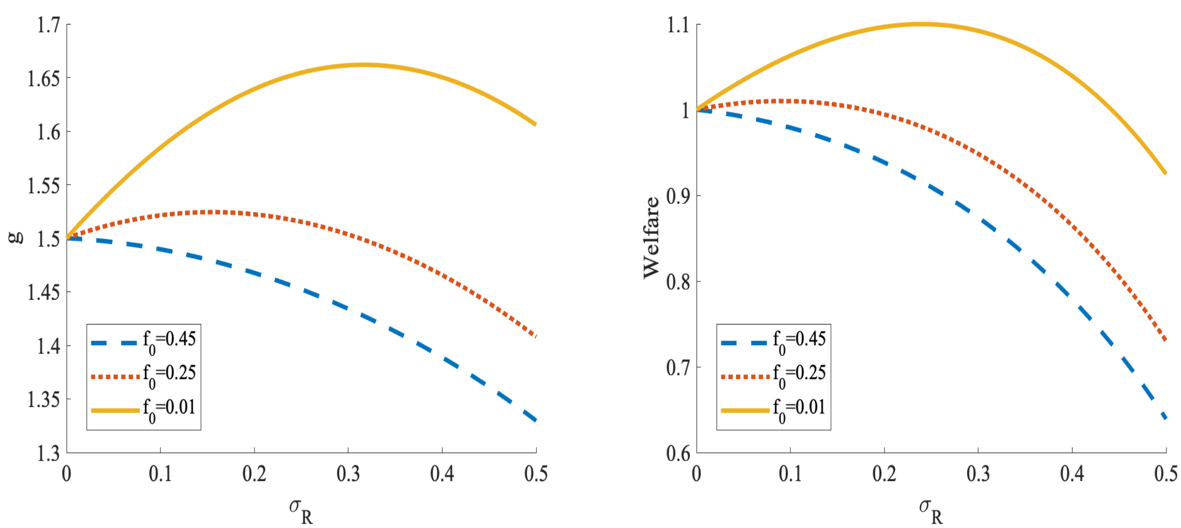


Figure 4 plots equilibrium values of growth g and welfare against the R&D subsidy rate σ_R for three distinct initial values of the failure rate f_0. In each case, we calibrate the model so that $g = 1.5\%$ and $f_0 = [0.01, 0.25, 0.45]$ when $\sigma_R = 0$. Panel (b) plots U as specified in equation (4.1) after normalizing its value to one when $\sigma_R = 0$. See Appendix B for a discussion of the calibration and the associated parameter vales.

5 Extensions

5.1 Combative Advertising

In this section, we consider an extension to the baseline model to incorporate defensive advertising by old firms. As with advertising by young firms, we assume that the effectiveness of an old firm's investment in advertising, α_o, decreases in the size of its costumer base. We now define the instantaneous probability of prototype diffusion as,

$$\delta(t) = \frac{\alpha_y(t)/\kappa_\alpha N(t)}{\alpha_o(t)/N(t)} = \frac{\alpha_y(t)}{\kappa_\alpha \alpha_o(t)}. \tag{5.1}$$

Under this specification, the rate of diffusion is determined by an advertising contest between young and old firms battling for market share.

We assume a constant unit labor requirement for defensive advertising of $\beta_o > 0$. Each old firm chooses $\alpha_o(t)$ to maximize its value. We include the cost of advertising $\beta_o\alpha_o(t)$ in (2.19) and maximize with respect to α_o. This yields the an optimality condition of $\beta_o\alpha_o(t) = \delta V_o(t)$, which equates the total cost of advertising to the expected capital loss. This condition is directly analogous to the optimality condition for young firm advertising of $\beta_y\alpha_y(t) = \delta(t)(V_a(t) - V_y(t))$, which is unchanged aside from a change in the notation for the labor requirement of young firm advertising to β_y to avoid ambiguity. Incorporating optimal advertising expenditure into (2.19) provides an expression for the value of an old firm

$$V_o(t) = \frac{(1-\phi)\pi_a(t)}{2\delta(t) + r(t) - n}. \tag{5.2}$$

As before, we solve for the model's steady state equilibrium by deriving two equilibrium conditions in I and δ. Labor market clearing provides our first equilibrium condition. Total employment in advertising is now given by $L_\alpha(t) = n_B(t)(\beta_y\alpha_y(t) + \beta_o\alpha_o(t))$. Note that, since the combative advertising formulation accommodates reciprocal cancelation, advertising has the potential to be directly socially wasteful. That is, proportional increases in young and old advertising draw labor resources away from production and R&D without increasing the diffusion rate. As shown in the Appendix, the labor market clearing condition can be written as

$$\frac{1}{\beta_R\kappa_R} = I + (1-\sigma_R)(\rho - n + I)\left[\frac{1}{\phi(\lambda-1)} + \frac{\delta I}{I+\delta}\left[\frac{(1-\phi)[\beta_y\kappa_\alpha\delta + \beta_o]}{\phi\beta_o(2\delta + \rho - n)}\right]\right] \quad \textbf{[LMCC]} \tag{5.3}$$

We establish in the Appendix that, as long as young firm advertising is sufficiently costly relative to old firm advertising, (5.3) is strictly downward sloping in (δ, I) space. That is, an increase to the diffusion rate always requires more total labor resources devoted to advertising, and the labor market clearing condition under combative advertising continues to reflect a trade-off between the resources allocated towards innovation and diffusion.

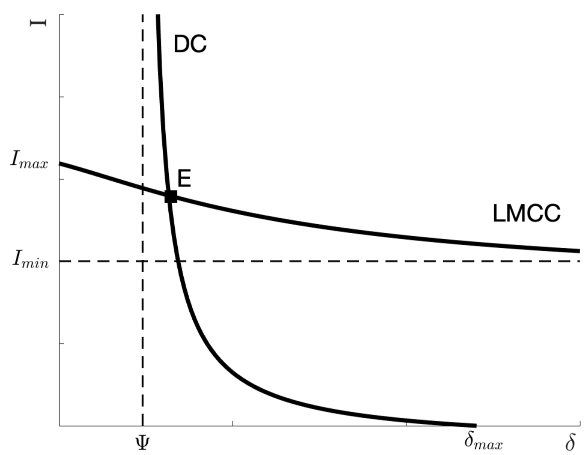
Our second equilibrium condition is derived from the equilibrium determination of the diffusion rate based on the optimized advertising expenditures of old and young firms. Combining these two advertising optimality conditions yields the following equilibrium diffusion curve (DC) that captures relative advertising incentives,

$$I = \frac{\Psi(\rho - n) + \delta(2\Psi - (\rho - n))}{\delta - \Psi} \quad \textbf{[DC]} \tag{5.4}$$

where $\Psi \equiv \beta_o/\beta_y\kappa_\alpha$ captures the effective relative cost of old and young firm advertising. When graphed in (δ, I) space, this diffusion curve is strictly downward sloping. This is because a greater innovation rate does not directly impact old firms' incentives to invest in defensive advertising, but it reduces young firms' advertising incentives since the reward from successful diffusion, a firm's tenure as an adult firm, is reduced. Note that the equilibrium determination of δ under combative advertising is now entirely independent of the equilibrium consumption level, c. Although greater equilibrium consumption increases the reward from successful diffusion, thereby incentivizing young firms to increase their advertising expenditure, it generates a proportional increase in the value of serving the market as an old firm. The corresponding increase in defensive advertising exactly offsets the effect of increased advertising by young firms. The net effect is an increase in total advertising volume, without changing the diffusion rate.

In the following proposition, we establish the existence and uniqueness of the model's balanced growth equilibrium. The equilibrium is illustrated in Figure 5. Furthermore, we show that the inclusion of combative advertising does not change the model's predictions for the effect of subsidies to R&D. Subsidizing R&D impacts relative advertising incentives only indirectly through the general equilibrium effect of the associated increase in the innovation rate. The increased innovation rate decreases young firm advertising incentives, resulting in a decrease in the equilibrium diffusion rate. As in the informative advertising specification, subsidizing R&D has an ambiguous effect on economic growth when advertising is combative.[17] Further details are provided in the Appendix.

Figure 5: Combative Advertising Equilibrium



Proposition 2. *The model with combative advertising has a unique equilibrium with $I > 0$ and $\delta > 0$ under the following parameter restriction,*

$$\frac{2\beta_o}{\beta_y \kappa_\alpha} < \rho - n < \frac{\phi(\lambda - 1)}{(1 - \sigma_R)\beta_R \kappa_R}.$$

[17]Numerical simulations confirm that the quantitative effects of R&D subsidies are also very similar in the informative and combative advertising frameworks.

Furthermore, subsidizing R&D investment decreases the diffusion rate, increases the innovation rate, increases the product failure rate, and has an ambiguous effect on economic growth. That is, $\frac{\partial \delta}{\partial \sigma_R} < 0$, $\frac{\partial I}{\partial \sigma_R} > 0$, $\frac{\partial f}{\partial \sigma_R} > 0$, *and* $\frac{\partial g}{\partial \sigma_R} >< 0$.

Proof. See Appendix.

5.2 Reduced Advertising Costs

In this section, we examine the economic impact of an exogenous decline in the cost of advertising. Following Grossman and Shapiro (1984) and Dinlersoz and Yorukoglu (2012), we interpret this cost reduction as a stylized representation of recent improvements to advertising technology. In particular, we have in mind the role of targeted digital advertising in enhancing the reach and effectiveness of firms' efforts to disseminate information to their potential customer base. For simplicity, we assume that the cost of advertising decreases by a fixed proportion, $0 < \gamma < 1$.[18]

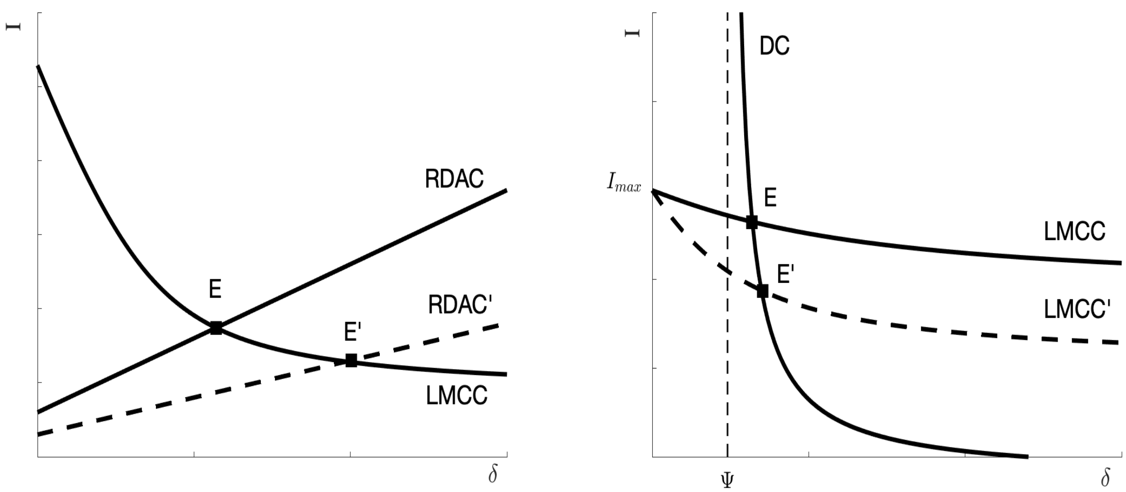
The advertising cost reduction changes the condition for young firms' optimal advertising expenditure to $(1-\gamma)\beta_y \alpha_y(t) = \delta(t)(V_a(t) - V_y(t))$. In the informative advertising version of the model, this impacts the RDAC, equation (2.24), by reducing the cost of advertising relative to R&D. In other words, the cost reduction shifts relative investment incentives away from R&D towards advertising, and young firms endogenously devote more resources to their attempts to diffuse products into the mainstream. This is represented in Panel (a) of Figure 6 by a rightward shift in the RDAC. Although there is no direct change to the LMCC, equation (2.26), the cost reduction induces movement along the curve as fewer labor resources are devoted to production and R&D. The end result is an increase in the rate of diffusion, a decrease in the innovation rate, and a decrease in the rate of product failure. Since the change to I and δ have opposite signs, and the overall change to economic growth g is ambiguous in general.

In the combative advertising version of the model, the cost decrease also changes the optimality condition for defensive advertising expenditure to $(1-\gamma)\beta_o \alpha_o(t) = \delta V_o(t)$. Since the cost of all advertising decreases by the same proportion, there is no direct change to the relative advertising incentives of young and old firms captured by the DC, equation (5.4). However, defensive advertising intensity determines the difficulty of diffusion in the combative advertising framework. Since the cost reduction stimulates defensive advertising, there is an increase in the labor resources required to maintain a constant rate of diffusion δ. This effect generates a leftward shift in the LMCC, equation (5.3), as depicted in Panel (b) of Figure 6. Although relative advertising incentives are not directly affected by the cost reduction, the movement along the downward sloping DC curve represents the increased relative incentive for young firm advertising as a lower I implies a longer expected reign as an adult firm if they diffuse successfully. Thus, a decline in advertising costs has the same *qualitative* impact on the model's endogenous variables in both the informative and combative advertising specifications. This result is summarized in the following proposition,

[18] As argued by Mandel (2019), digital advertising has created "a supply-side change in the advertising market ... increasing supply of digital ads at a relatively low price compared to the price of 'equally effective' print ads."

Figure 6: Reduced Advertising Costs

(a) Informative Advertising (b) Combative Advertising



Proposition 3. *In both the informative and combative advertising models, a decline in advertising costs increases the diffusion rate, decreases the innovation rate, decreases the product failure rate, and has an ambiguous effect on economic growth. That is,* $\frac{\partial \delta}{\partial \gamma} > 0$, $\frac{\partial I}{\partial \gamma} < 0$, $\frac{\partial f}{\partial \gamma} < 0$, *and* $\frac{\partial g}{\partial \gamma} >< 0$.

Proof. See Appendix.

Despite this qualitative equivalence, the growth and welfare effects of declining advertising costs can be markedly different when advertising is informative or combative. To illustrate, we turn to numerical simulations and examine the effects of an exogenous 15% decline in advertising costs from an initial equilibrium with $f = 0.25$ and $g = 1.5\%$. Results are reported in Table 2. See Appendix B for a discussion of the calibration and the associated parameter values.

In the informative advertising case, the decline in advertising costs shifts investment incentives towards advertising and generates a huge increase in the diffusion rate from 0.269 to 1.005. This is equivalent to a decrease in the expected duration before sales takeoff from $1/\delta = 3.7$ years to about 1 year. Although the innovation rate decreases modestly, the large increase in the diffusion rate creates a substantial increase in economic growth from 1.5% to 1.78% and a decrease in the failure rate from $f = 0.25$ to $f = 0.079$. Furthermore, since $n_b = f$, this lower failure rate implies fewer industries use labor resources for advertising. Consequently, the large increase in the diffusion rate is achieved with only an additional 1.9% of the labor force used in advertising.[19] Although this

[19] Although we do not target the proportion of the labor force used in advertising and R&D, we note that the respective 10% and 2.6% generated by the model in the initial equilibrium is roughly in line with available estimates. For example, Gourio and Rudanko (2014) report that 11% of US workers are employed in broadly defined sales-related activities. Estimates for the proportion of the labor share employed in R&D range from 1% to 5% (Segerstrom, 2007; Şener, 2008).

Table 2: Reduced Advertising Costs

	Informative		Combative	
	$\gamma = 0$	$\gamma = 0.15$	$\gamma = 0$	$\gamma = 0.15$
$g(\%)$	1.500	1.779	1.500	1.490
I	0.090	0.087	0.090	0.087
δ	0.269	1.005	0.269	0.287
c	1.092	1.070	1.082	1.063
f	0.250	0.079	0.250	0.233
U	2.106	2.414	1.940	1.612
L_{α_y}	0.100	0.119	0.089	0.103
L_{α_o}	–	–	0.019	0.021
L_R	0.026	0.025	0.026	0.025

Table 2 presents results from a 15% reduction in advertising costs in both the informative and combative advertising versions of the model. In both cases, the model is calibrated to an initial equilibrium with $f_0 = 0.25$ and $g = 1.5\%$. The final three rows report the proportion of labor resources allocated to young firm advertising, old firm advertising (if applicable), and R&D.

still implies a static welfare loss from a reduced proportion of labor in manufacturing and lower consumption, this is dominated by the dynamic welfare gain of increased economic growth.

In the combative advertising case however, the same decline in advertising costs leads to lower economic growth and a decrease in welfare. Since combative advertising is characterized by reciprocal cancelation of young and old firm advertising, the 1.4% increase in labor devoted to diffusion promoting young firm advertising is partially offset by the 0.2% increase in defensive old firm advertising. The net result of the full 1.6% increase in labor in advertising is but a small decrease in expected duration before sales takeoff from 3.7 to 3.5 years. This is insufficient to compensate for the decreased innovation rate, and economic growth falls. This dynamic welfare loss is coupled with a static welfare loss from lower consumption.

6 Conclusion

In this paper, we analyze the dynamic interaction between product innovation, diffusion, and economic growth. We contribute to the Schumpeterian growth literature by introducing a stochastic diffusion process in which the rate of commercial success of product innovations is determined by advertising intensity. Through this mechanism, firms endogenously cycle through distinct life stages of stochastic length as new innovations arrive and either commercialize successfully or fail. Unlike traditional quality ladder models, our framework is consistent with empirical evidence that (1) firms devote substantial resources to advertising innovative products, (2) a large proportion of product launches fail despite these advertising efforts, and (3) even when ultimately successful, most new products experience an initial period of low market penetration before an eventual sales takeoff to their mature market share.

In this framework, economic growth depends positively on both the rate of innovation and the rate of product diffusion since only product innovations that commercialize successfully enter the mainstream market. In contrast to traditional models where R&D subsidies always promote growth, we find that R&D subsidies can have a non-monotonic effect on growth by shifting incentives towards R&D but away from the complementary advertising investment needed for innovation diffusion. In particular, we show that this effect gives rise to an inverted U-shaped relationship between R&D subsidies and both economic growth and welfare. Our simulation exercises illustrate that the optimal R&D policy is highly sensitive to the initial failure rate of newly innovated products, even though the growth rate is held constant. While we find that a large subsidy is optimal when the initial failure rate is very low, the optimal policy shifts to a modest tax when the initial failure rate is high, but still within the empirically plausible range. In general, the scope for using R&D subsidies to improve welfare diminishes with the failure rate because the social benefit of stimulating innovation is lower when a smaller proportion of new innovations succeed. We argue that these results suggest that standard endogenous growth models that assume costless and instantaneous innovation diffusion may overstate the case for large subsidies to R&D.

In addition, we use the model to investigate the economic impact of an exogenous decline in the cost of advertising. We interpret this cost reduction as a stylized representation of recent improvements in the effectiveness and reach of advertising through targeted digital media. When advertising is purely informative, we show that lower advertising costs generate a large increase in the diffusion rate, which boosts economic growth and improves welfare. However, when advertising is combative, much of the increase in diffusion promoting advertising by young technological leaders is offset by the increase in defensive advertising by incumbent firms. The net result is a decrease in economic growth and welfare as there is a substantial increase in resources devoted to advertising, without the expected large increase in diffusion. Our analysis highlights the importance of understanding the responsiveness of different types of advertising to changes in advertising technology.

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Appendix A

A.1 Growth Rate Derivation

At any point in time, an n_A proportion of industries are served by an adult firm whose product enjoys a perceived quality of $k(\omega, t)$ with all consumers. In the remaining $(1 - n_A)$ proportion of B industries, old firms serve $1 - \phi$ proportion of mainstream consumers that purchase a product of perceived quality at the $k(\omega, t)$ quality standard, while young firms serve ϕ proportion of early adopters that purchase a prototype they perceive to be one step up the λ quality ladder. All products have a common price of $p = \lambda$. This gives the following expression for instantaneous per capita sub-utility,

$$ln(u(t)) = \int_{n_A} ln\Big[\frac{c(t)\lambda^{k(\omega,t)}}{\lambda}\Big]d\omega + \int_{(1-n_A)(1-\phi)} ln\Big[\frac{c(t)\lambda^{k(\omega,t)}}{\lambda}\Big]d\omega + \int_{(1-n_A)\phi} ln\Big[\frac{c(t)\lambda^{k(\omega,t)+1}}{\lambda}\Big]d\omega \quad (A.1)$$

$$= ln(c(t)) + \int_0^1 ln\Big[\lambda^{k(\omega,t)-1}\Big]dw + \int_{(1-n_A)\phi} ln\lambda d\omega$$

$$= ln(c(t)) - ln(\lambda) + (1-n_A)\phi ln(\lambda) + \int_0^1 ln(\lambda^{k(\omega,t)})d\omega$$

$$= ln(c/\lambda) + (1-n_A)\phi ln(\lambda) + ln(\lambda)(1-f)It \quad (A.2)$$

where the last line follows since $I(1 - f)$ is the expected aggregate rate of progress up the quality ladder. Differentiating $ln(u(t))$ gives the rate of utility growth

$$g \equiv \frac{\dot{u}}{u} = ln(\lambda)I(1-f) = ln(\lambda)\frac{I\delta}{I+\delta} \quad (A.3)$$

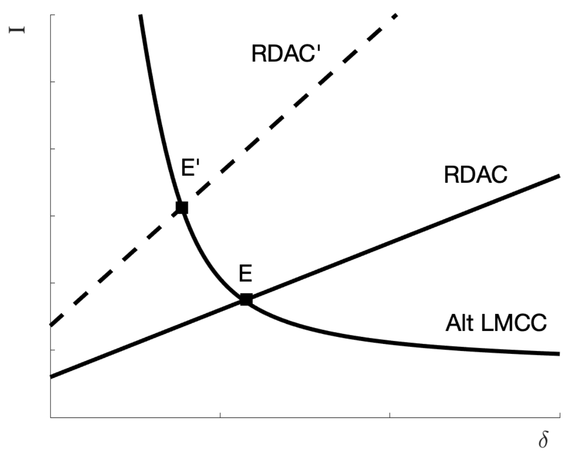
A.2 Proof of Proposition 1

The LMCC used in the main text eliminated c by substituting the R&D condition (2.22) into the labor market clearing condition (2.25). Instead, consider an alternate LMCC derived by using the advertising condition (2.23) instead of (2.22) to eliminate c. This results in,

$$1 = \frac{\beta_\alpha\kappa_\alpha}{(1-\phi)(\lambda-1)}\frac{(\rho-n+I)(\rho-n+\delta)}{(\rho-n+I+\delta)} + \beta_R\kappa_R I + \beta_\alpha\kappa_\alpha\frac{\delta I}{I+\delta}. \quad (A.4)$$

The alternate LMCC remains strictly downward sloping in (δ, I) space, but no longer directly depends upon σ_R. As illustrated in Figure A.1, subsidizing R&D always increases the rate of innovation and decreases the rate of diffusion.

Figure A.1: R&D Subsidy - Alternate LMCC



A.3 Derivation of the Labor Market Clearing Condition With Combative Advertising

As in the primary model, total employment in production is cN/λ and employment in R&D is $\beta_R R$. With combative advertising, total employment in advertising is $L_\alpha = n_B(\beta_y \alpha_y + \beta_o \alpha_o)$. The labor market clearing condition requires that this total labor demand equals the total supply of labor N. Using the equilibrium expression of n_B given by (2.21) and the definition of δ from (2.6), we have

$$L_\alpha = \frac{I}{I+\delta}\Big[\alpha_o(\beta_y \kappa_\alpha \delta + \beta_o)\Big]. \tag{A.5}$$

Substituting the optimality condition for old firm advertising to eliminate α_o and the expression for the value of an old firm from (5.2) gives

$$L_\alpha = \frac{I\delta}{I+\delta}\Big[\frac{(1-\phi)(\lambda - 1 + \sigma_p)cN}{\lambda(\rho - n + 2\delta)}\Big(\frac{\beta_y \kappa_\alpha}{\beta_o}\delta + 1\Big)\Big]. \tag{A.6}$$

Finally, we use the equilibrium relationship between I and c from the R&D condition (2.22) to eliminate c from both (A.6) and production employment. Rearranging terms gives the expression for the LMC in the main text (5.3).

A.4 Proof of Proposition 2

First, we prove the existence and uniqueness of an equilibrium with $I > 0$ and $\delta > 0$ by establishing a single crossing of the LMCC displayed in (5.3) and the DC displayed in (5.4). As noted in the main text the DC is strictly downward sloping in (δ, I) space. Plugging $I = 0$ into the DC gives its horizontal intercept, δ_{max}, where

$$\delta_{max} = \frac{\Psi(\rho - n)}{\rho - n - 2\Psi} > \Psi > 0, \tag{A.7}$$

since the parameter restriction in Proposition 2 implies that $\rho - n > 2\Psi$. Next, note that the DC curve implies an asymptote of $I \to \infty$ as $\delta \to \Psi$ from the right. Thus, the DC is well defined with $0 < I < \infty$ for $\delta \in (\Psi, \frac{\Psi(\rho-n)}{\rho-n-2\Psi})$.

Let $h(\delta, I)$ denote the right hand side of the LMCC as written in (5.3). Clearly, $\frac{\partial h(\delta,I)}{\partial I} > 0$. This implies that, if $\frac{\partial h(\delta,I)}{\partial \delta} > 0$, then the LMCC is strictly downward sloping in (δ, I) space. Differentiating, we have

$$\frac{\partial h(\delta, I)}{\partial \delta} = \frac{(1-\sigma_R)(\rho-n+I)I}{(I+\delta)^2(2\delta+\rho-n)^2}\Big[I(2\delta+\rho-n)(\beta_y\kappa_\alpha\delta+\beta_o)+\delta(I+\delta)(\beta_y\kappa_\alpha(\rho-n)-2\beta_o)\Big] \tag{A.8}$$

Since the parameter restriction in Proposition 2 ensures that $\rho - n > 2\beta_o/\beta_y\kappa_\alpha$, we have that $\frac{\partial h(\delta,I)}{\partial \delta} > 0$, and the LMCC is strictly downward sloping. As mentioned in the main text, this parameter restriction implies that young firm advertising is sufficiently costly relative to old firm advertising. To establish the single crossing, we show that the rate of innovation implicitly defined by the LMCC is positive and finite over the domain $\delta \in (0, \infty)$. When $\delta \to 0$, the LMCC gives

$$\frac{1}{\beta_R\kappa_R} = I_{max} + \frac{(1-\sigma_R)(\rho-n+I_{max})}{\phi(\lambda-1)}, \tag{A.9}$$

and when $\delta \to \infty$, we have that

$$\frac{1}{\beta_R\kappa_R} = I_{min} + (1-\sigma_R)(\rho-n+I_{min})\Big[\frac{1}{\phi(\lambda-1)} + \frac{(1-\phi)\beta_y\kappa_\alpha}{\phi\beta_o}\Big]. \tag{A.10}$$

The parameter restriction in Proposition 2 ensures that $\rho - n < \frac{\phi(\lambda-1)}{(1-\sigma_R)\beta_R\kappa_R}$, which implies that $0 < I_{min} < I_{max} < \infty$. Therefore, single crossing obtains as illustrated in Figure 5,

To obtain the rest of Proposition 2, observe that σ_R enters only the LMCC. An increase to σ_R shifts the LMCC right in 5, generating movement along the downward sloping DC as young firms respond to the higher innovation rate by decreasing advertising expenditure. As a result, the innovation rate increases and the diffusion rate decreases, just as in the informative advertising case.

A.5 Proof of Proposition 3

As mentioned in the main text, an advertising cost reduction enters the equilibrium condition for optimal advertising investment for young firms $(1-\gamma)\beta_y\alpha_y(t) = \delta(t)(V_a(t) - V_y(t))$ and old firms $(1-\gamma)\beta_o\alpha_o(t) = \delta V_o(t)$. Following the solution method used in the informative advertising model of Section 2, the corresponding expression for the RDAC is,

$$I = (\rho - n + \delta)\Big[\frac{\phi(1-\gamma)\beta_\alpha\kappa_\alpha}{(1-\phi)(1-\sigma_R)\beta_R\kappa_R} - 1\Big] \tag{A.11}$$

The results of Proposition 3 follow immediately from the rightward shift in equation (A.11) as depicted in Figure 6.

In the combative advertising model, total employment in advertising remains $L_\alpha(t) = n_B(t)(\beta_y \alpha_y(t) + \beta_o \alpha_o(t))$. The same procedure discussed in Section A.3 delivers the following labor market clearing condition,

$$\frac{1}{\beta_R \kappa_R} = I + (1-\sigma_R)(\rho - n + I)\left[\frac{1}{\phi(\lambda-1)} + \frac{\delta I}{I+\delta}\left[\frac{(1-\phi)[\beta_y \kappa_\alpha \delta + \beta_o]}{\phi(1-\gamma)\beta_o(2\delta+\rho-n)}\right]\right]. \tag{A.12}$$

Equation (A.12) is nearly identical to its counterpart in the main text (5.3) and remains downward sloping in (δ, I) space under the same parameter restriction. The only difference is the $(1-\gamma)$ term in the denominator of the final term on the right hand side. This captures the increased labor resources needed to maintain a constant diffusion rate δ when the cost of advertising declines. Intuitively, the subsidy encourages advertising expenditure by old firms, which increases the difficulty of diffusion in the combative advertising framework. The results of Proposition 3 for the combative advertising model follow immediately from the leftward shift in equation (A.12) as depicted in Figure 6.

Appendix B - Numerical Simulations

B.1 Parameter Values

All numerical simulations use the following pre-set parameter values: $\rho = 0.07$, $n = 0.01$, $\kappa_R = \kappa_\alpha = 1$, $\lambda = 1.25$ and $\phi = 0.20$. As is standard, we set ρ to 0.07 to reflect a 7% long run real return of the U.S. stock market, and $n = 0.01$ to approximate the average growth rate of the U.S. labor force. Since κ_R and κ_α impact the cost of R&D and advertising respectively only through their values relative to β_R and β_α, we normalize $\kappa_R = \kappa_\alpha = 1$. The size of each innovation's quality improvement is set to a standard value of $\lambda = 1.25$ to reflect intermediate empirical estimates of firm gross markup over marginal cost of about 25%. Finally, recall that ϕ determines the proportion of households that comprise our two broad consumer categories, early adopters (ϕ) and mainstream consumers ($1 - \phi$). These two categories represent a simplified version of the classic partition of consumers used in the innovation diffusion marketing literature into five groups: innovators, early adopters, early majority, late majority, and laggards. We choose $\phi = 0.20$ to align with traditional estimates of the combined size of the innovator and early adopter marketing categories of 12.5% to 23% of the market and the corresponding size of the remaining three categories of 77% to 87.5% (Mahajan et al., 1990).

In the informative advertising model, we consider three distinct initial values of the proportion of prototypes that fail to diffuse into the mainstream market, $f_0 = [0.01, 0.25, 0.45]$. As noted in the introduction, the rate new product failure is typically estimated in the range of 40-50%. In this empirical literature, the definition of product failure often includes products that exhibit a sufficiently large underperformance relative to sales targets, in addition to products that are removed from the market entirely. In our model however, a new prototype fails only when it is fully displaced from the market. To account for this difference, we analyze a case of $f_0 = 0.45$ to represent a relatively high rate of failure along side a more conservative value of $f_0 = 0.25$. We also analyze a case of a very low rate of failure $f_0 = 0.01$ to approximate traditional models of endogenous growth in which new products diffuse instantaneously and never fail. The calibrated values of β_R and β_α corresponding to an initial equilibrium with $g = 1.5\%$ and $f_0 = [0.01, 0.25, 0.45]$ are presented in Table B.1.

Table B.1: Calibrated Parameters

	$f_0 = 0.0.1$	$f_0 = 0.25$	$f_0 = 0.45$
β_R	0.3451	0.2921	0.2385
β_α	1.3944	1.4864	1.5110

In Section 5.2, we also examine a numerical simulation of the combative advertising model using parameters calibrated to the intermediate case of $f_0 = 0.25$ and $g = 1.5\%$. Since the diffusion rate is determined by the relative intensity of young and old firm advertising, we normalize $\beta_o = \kappa_\alpha = 1$

and treat $\Psi = \beta_o/\beta_y\kappa_\alpha$ and β_R as free parameters. The corresponding calibrated parameter values are $\Psi = 0.0585$ and $\beta_R = 0.2891$.