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THE ECONOMICS OF STUDENT ATTENDANCE

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1. Introduction

1.1. Introduction to the topic

Education from its very first steps is based on interaction between individuals, direct or indirect. An extremely significant part of this interaction takes place inside the classroom, where individuals with a high level of knowledge and information attempt to transmit this knowledge to those with less. This activity is economic in its very essence. Students allocate time to schooling, in order to build on their human and social capital, but face opportunity costs for doing so. Depending on the technological level and the cultural and social attributes of different populations, the interaction between individuals attempting to transmit knowledge can take and has taken very different forms. In the evolution of institutions in education, we can identify two major breakthroughs that revolutionized the way in which individuals access knowledge. These were the discovery of writing and the discovery of typography. The first allowed knowledge to be recorded and secured in other ways than the oral tradition, while the second resulted in a major drop in the costs of transmitting this knowledge, making it gradually more accessible to much larger parts of the population. Books and written notes very quickly became a central means of transmitting knowledge, and as such became both complements and substitutes for direct learning from a teacher, or what call in this paper attendance. One can easily claim that we are at the beginning of a third technological outburst in the history of education: the information revolution of the Internet.

From as young as 4 or 5 years, individuals attend classes in varying sized groups of peers. In these classes the teachers attempt, with the help of books and other instructional means, to transmit to the individuals knowledge about the world. This imparted knowledge is evaluated and measured in various manners, the most common being by examination.

Numerous expressions of this classroom / teacher framework continue to provide education to individuals for a lifetime. Various rules are applied in these educational institutions as to the level of attendance which individuals must satisfy. Until at least the end of adolescence the rule is

generally, if not ubiquitously, that of Mandatory attendance. In University education there is no consensus about the applied rules, and different professors, institutions and educational systems appear to set their policies independently. Education acquired later in life is mostly associated with free attendance.

The scientific research undertaken is still limited and inconclusive. This is probably due to the multiplicity of the factors at play, which makes it difficult to identify relevance of the various inputs and outputs into education. This lack of science allows intuition, personal benefits and maybe ideology to stand where science should be. This article sheds light on this relatively unexplored scientific area.

1.2. Relevant research

In the last half century concepts and tools have emerged within economic theory that help to identify the conceptual and methodological framework within which the debate should take place. Among the developments that have strongly influenced the field of Educational Economics and more specifically this paper are: the theories of allocation of time and the notion of opportunity costs; interaction between individuals modelled as externalities or in a Game theoretical context; Contract and Information theory and specifically the notions of screening, signalling and moral hazard; Institutional economics and the notions of transaction costs and property rights that turn institutions into a central issue in economic analysis and allow us to study their evolution and efficiency; Behavioural Economics and the concepts of bounded rationality as well as other approaches to “irrational economics”. Moreover recent developments in cognitive science will hopefully provide more information about the learning process and realistic ways to model it. In less than 50 years the economics of education became a central field of research in economics with diverse and manifold research. The main conceptual ancestors of our article, which model educational achievement using time allocation theory, are Kelley (1975) “the student as a utility maximizer”, Becker's (1975) “The university professor as a utility maximizer”, Schmidt (1983) “Who maximizes what a study of student allocation of time”, Levin and Chang (1987) “the economics of student time”, and Bratti and Stafollani (2002) “Student time allocation and educational production functions”. In the existing literature one can find abundant empirical research on student attendance, generally the results of investigations by professors on their own classes. A characteristic example, widely quoted in the literature, is Romers (1993) article “Do students go to class? Should they”. In contrast, very little has been said on the theoretical basis of attendance laws. The literature mentioned above provided valuable insights and the scientific dialogue that was essential for our work. However, although the results there given are interesting

and thought provoking, we consider any normative arguments derived directly from exam results inconclusive or incomplete at best.

1.3. On methodology

The model constructed examines how students make decision regarding their time allocation, and how mandatory attendance rules affect their time allocations and utility levels. In the literature mandatory attendance rules are generally justified on the grounds of irrationality and externalities. These factors are examined in detail. Mandatory attendance rules in place generally specify a minimum level of attendance. We consider both this type of attendance rule and that in which an exact level of attendance is set. We see that under our assumptions the latter often allow institutions to do at least as well as the former.

In our model the theory of the allocation of time is applied as the principal tool of analysis. Our application is very demanding. Although education has good observable inputs, the outputs are often extremely hard to measure as compared to other areas where the theory is used, such as work supply-leisure models or the family literature. The model is an attempt to look primarily at the rationale behind various mandatory attendance rules and the kind of consequences they can have. The study begins in a world of perfect and complete information with no externalities. Individuals allocate time among three activities, attendance, self-study, and leisure, in order to maximise their utility.

Assumptions are then gradually dropped. First the rationality assumption is relaxed, and certain biases are introduced in the decision making procedure of the student. Then the utility obtained by individuals free to choose freely their attendance level (under the assumption that they are myopic) is compared to the utility obtained by the individual under the various mandatory attendance rules fixed by institutions. We consider both the case where institutions are only aware of the correct utility function and the case when they are also aware of the utility function held by the student. This provides us with explicit conditions under which mandatory attendance rules result in welfare gains or losses, and thus when such rules can be justified on the grounds of some form of irrationality.

Subsequently the externalities assumption is relaxed. It is widely accepted that investment in education and human capital has external effects. Various studies estimate the social returns of investment to education, all of which identify some sort of positive externalities, which the most generous estimations evaluating them as up to half of the private returns to education. Interestingly, several of the studies use mandatory attendance laws by way of conducting their natural experiments. A reason that externalities might be underestimated is that many of the external effects of education might concern non-alienated goods such as civic spirit and behaviour

within families. In our model externalities could be seen as some form of social irrationality. The model incorporates this by creating an educational “social welfare function” that corrects for both irrationality and accounts for externalities.

Finally the perfect evaluation methods assumption is dropped. In order to illustrate the problem here we postulate that there are distinct types of individual with different learning capacities. An in depth example demonstrates, using a model with two types, how mandatory rules affect utility in a framework with varying abilities. Then through a contractual setting, using incentive compatible contracts, a solution is suggested as to how division of types might be achieved.

In section 2.6. further interesting avenues of research are discussed. A small discussion on informational benefits of attendance is presented. It also looks at what elements constitute benefits from attendance and how changes in these can affect utility. These are followed by summary conclusions and results.

2. The model

2.1. The simple model

Students have a total amount of time T available to distribute between lecture attendance A, independent study S, and leisure L.

Our utility function is assumed to take the form

$$U_W(A, S) + U_L(L)$$

To avoid boundary solution problem infinite marginal utility rates are assumed at zero, ie.

$$\left. \frac{\partial U_W}{\partial A} \right|_{A=0} = \left. \frac{\partial U_W}{\partial S} \right|_{S=0} = \left. \frac{dU_L}{dL} \right|_{L=0} = \infty$$

2.1.1 Justification of simple model

In the model the utility derived from leisure is assumed to be independent from the level of attendance and independent study, and similarly the utility derived from attendance and self-study is independent from leisure. No such assumption is made between attendance and self-study. A moments thought gives many justifications for this. For example, time spent reviewing lecture notes may make lectures more useful, and attendance may give a better idea of what to study independently. Alternatively self-study can be a substitute for attendance.

It is assumed that students can attend as many lectures as they wish, which often is not the case in reality. The assumption is made to insure an interior solution. However, we can mimic the case in which maximal attendance is A_{\max} by making the marginal utility of attendance zero for $A \geq A_{\max}$. The factors of the model investigated here will behave in almost identical manners under the two formulations.

2.1.2 Solution of simple model and definitions

DEFINITION

Given the model, the optimal levels of attendance and independent study, A^* and S^* , satisfy

$$(A^*, S^*) \in \arg \max_{\substack{(A, S) \in [0, T]^2 \\ A+S \leq T}} \{U_W(A, S) + U_L(T - A - S)\}$$

Giving first order conditions

$$\left. \frac{\partial U_W}{\partial A} \right|_{(A^*, S^*)} = \left. \frac{\partial U_W}{\partial S} \right|_{(A^*, S^*)} = \left. \frac{dU_L}{dL} \right|_{T-A^*-S^*}$$

DEFINITION

For future purposes it will be useful to define $S(A)$, the optimal allocation of time spent on independent study, given a fixed attendance level A

$$S(A) \in \arg \max_{S \in [0, T-A]} \{U_W(A, S(A)) + U_L(T - A - S(A))\}$$

Giving first order conditions

$$\left. \frac{\partial U_W}{\partial S} \right|_{(A, S(A))} = \left. \frac{dU_L}{dL} \right|_{T-A-S(A)}$$

Differentiating one obtains

$$S'(A) = - \frac{\frac{d^2 U_L}{d^2 L}(T - A - S) + \frac{\partial U_W^2}{\partial S \partial A}(A, S(A))}{\frac{d^2 U_L}{d^2 L}(T - A - S) + \frac{\partial U_W^2}{\partial^2 S}(A, S(A))}$$

Thus the sign of $S'(A)$ is unclear, and depends upon the form of U_w

DEFINITION

$U(A)$ is the maximum level of utility the student can achieve given an attendance level A

$$U(A) := U_w(A, S(A)) + U_L(T - A - S(A))$$

Differentiating

$$\begin{aligned} U'(A) &= \left. \frac{\partial U_w}{\partial A} \right|_{(A, S(A))} + S'(A) \left. \frac{\partial U_w}{\partial S} \right|_{(A, S(A))} - (1 + S'(A)) \left. \frac{dU_L}{dL} \right|_{T-A-S(A)} \\ &= \left. \frac{\partial U_w}{\partial A} \right|_{(A, S(A))} - \left. \frac{\partial U_w}{\partial S} \right|_{(A, S(A))} \end{aligned}$$

where the second equality comes from the first order condition for $S(A)$. Note that this is not independent of U_L - U_L affects $S(A)$. It is difficult to make further comments about $U(A)$ without the specific form of U_w and U_L .

2.1.3 Mandatory attendance rules

Two types of attendance rules are considered:

1) Minimum level of attendance

In practice educational institutions generally use mandatory rules of the minimum attendance type. The rationale behind using these and not fixed form attendance rules is primarily motivated by incomplete information on the behalf of the institutions. By fixing what they consider an optimal minimum level of attendance they are able to ensure a minimum student quality and at the same time give students the flexibility of attending more classes if they so desire. The example in section-2.4.1. gives a case where the institution fixes the mandatory attendance at a level optimising the utility of students who learn at a fast rate, and where this level is below that which is optimal for students who learn more slowly. It can thus be seen in this case that if the rule is replaced by a minimum attendance rule of the same level, then the high types do not change their attendance level and the low types alter their behaviour in a utility improving manner. Hence the minimum attendance rule is better.

For effective implementation, the minimum attendance type of rule often requires less information to be known than the fixed type, and is thus less time consuming and costly.

The student chooses attendance level $A(A_{\min})$ s.t.

$$A(A_{\min}) \in \arg \max_{A \in [A_{\min}, T]} U(A)$$

If $U(A)$ is concave then clearly

$$A(A_{\min}) = \max\{A_{\min}, A^*\}$$

2) Fixed level of attendance

Attendance rules of this form, whereby a level A_{fixed} of attendance is enforced, are rarely used. In this paper it shall be seen however that under certain conditions such rules have significant advantages over those specifying a minimum level of attendance. Also, if $U(A)$ is concave and the institution is interested in increasing an individual's attendance, then the two rules will result in the same outcome. In the rest of the paper this type of attendance rule will be assumed in analysis, and the differences that could arise from instead using minimum attendance rules will be discussed in each case.

2.2. Students with perfect information and no externalities

2.2.1. Assumptions

Perfect information and no externalities are assumed. Thus when evaluating the utilities of time allocations students are assumed to consider all present and discounted future costs and benefits of actions, such as future job satisfaction and remuneration, social interaction in lectures, and the pleasure of learning. Individuals' allocations are assumed to have no effect on the utility of others, so that maximising individual utility is equivalent to maximising society's utility under many social welfare functions.

2.2.2. Effect of mandatory attendance rules

Since the student is assumed to be rational and to act according to the correct utility function, he will choose the optimal allocation of attendance for both him and society. A mandatory attendance rule will decrease his utility by the non-negative amounts $U(A^*) - U(A_{\text{min}})$ in the case of a minimal attendance rule and $U(A^*) - U(A_{\text{fixed}})$ for a set attendance rule. As expected under these conditions mandatory attendance rules can only have negative effects on utility. Thus it shall be assumed for the rest of the paper that there is some difference between the utility function assumed by the student and the correct utility function for society (which is assumed to coincide with the correct utility function for the student unless the student's actions have external effects).

2.3. Imperfect world

2.3.1 Assumptions

It is unrealistic to assume that students have perfect information about their utility functions, especially with regards to future costs and benefits. Students undervaluing time spent studying is a common justification for mandatory attendance rules. Students' actions may also have externalities, whether they are effects on the utilities of other students or other parts of society in general.

The model now addresses the imperfect world - that in which irrationality and externalities may be present. The institutions are assumed to know what is referred to as the correct utility function, denoted as above

$$U_w(A, S) + U_L(L)$$

This utility function corrects for students' irrationality and accounts for the effect of externalities with respect to some social welfare function. It is thus this utility that should be maximised.

The student's utility function is denoted

$$\tilde{U}_w(A, S) + \tilde{U}_L(L)$$

Thus, if for example a student's attendance adds to the utility of others, U will assign a higher utility to attendance than \tilde{U} .

It is important to note that $S(A)$ and $\hat{S}(A)$ are different, where $S(A)$ is as defined above and $\hat{S}(A)$ is the independent study chosen by the student given attendance A under his incorrect utility function.

This has important consequences, as when the institution enforces attendance level A the student will choose $\hat{S}(A)$ of independent study, and not $S(A)$. Thus the realised utility given a mandatory attendance level A, $U^{real}(A)$, is the following

DEFINITION

$$U^{real}(A) = U_w(A, \tilde{S}(A)) + U_L(T - A - \tilde{S}(A))$$

When considering mandatory attendance rules there are two natural cases:

1) $\hat{S}(A)$ unknown to Educational Institution

In this case the institution is ignorant of how the student will allocate its remaining time given any attendance rule the institution imposes. Thus if the institution wishes to instigate a mandatory attendance rule, it has no reason to deviate from setting attendance to A^* , the maximising level when students have the correct utility function. Such an attendance rule, denoting the students optimal attendance under its incorrect utility functions as \tilde{A}^* , will lead to a change in utility from the free case of

$$U^{real}(A^*) - U^{real}(\tilde{A}^*)$$

where \tilde{A}^* is the attendance chosen by student under free decision, ie it optimises the students incorrect utility.

There is no reason for this to be positive in general. Thus, even if the institution knows the correct utility function and implements a mandatory attendance rule based upon it, without knowing the utility function held by the student, or at least how the student will allocate his remaining time given differing attendance rules, the rule could result in a utility loss over free choice.

The effect of setting instead the optimal minimum attendance level is unclear. Depending on the particular functions utility could increase or decrease.

2) $\hat{S}(A)$ known to Educational Institution

In this case the institution knows the effect that changing the mandatory attendance level has on the amount of independent study, and thus this can be taken into account when deciding the optimal attendance rule. The institution would thus set the mandatory level at A_{real} , given by

DEFINITION

$$A^{real} \in \arg \max_{A \in [0, T]} U^{real}(A)$$

Clearly the rule can only have a positive effect on utility. This positive effect will be at least as large, and some times strictly larger than that associated to the optimal rule specifying a minimum attendance level.

Utility is still less than it would be if the individuals had correct utility functions. Correcting the students utility functions would result in a utility gain of

$$U(A^*) - U^{real}(A^{real})$$

Hence if the institution is able to alter the students behaviour, at a utility cost of less than this amount and in such a way that the student devotes time $S(A^*)$ to self-study when attendance is at A^* , it should do so. Possible ways to alter $S(A^*)$ include giving some incentive to self-study, such as setting homework and giving financial or academic rewards based upon its quality, or devoting some lecture time to educating the students, directly or indirectly, about the errors of their utility functions. Obviously the latter would also effect the value of A^* , which would also need to be taken into account.

2.4. Two types of students

In reality, educational institutions have to set attendance rules for classes comprising of a range of students each of which possesses their own utility function.

In the model to avoid complications it is assumed here that there are just two types of student, type 1 and type 2, in proportions β and $1-\beta$ respectively. The institution knows this as well as both the

“correct” and the student held utility function for each type. However, the institution is unable or unwilling to tell which type any particular individual is. The institution will attempt to implement a mandatory attendance rule which maximises its social welfare function. The assumption remains that each student’s utility is independent of the actions of the others, and in addition it is assumed that the institution is utilitarian. Thus the institution wishes to maximise the following weighted sum of utilities, where notation has been extended from the one type of individual case in the natural manner

$$\beta U_1^{real}(A) + (1 - \beta)U_2^{real}(A)$$

If the institution is only able to impose one level of mandatory attendance then it will choose the A maximising this. In this framework there are simple cases where imposing instead a minimum level of attendance rule would be better – for instance when trying to improve the attendance of a student very averse to studying without reducing the attendance of somebody who correctly finds it very useful (as seen in example in section 2.4.1.). However there are still cases where set level attendance rules lead to better outcomes.

Utility can be improved if the institution is able to offer two set levels of attendance. The best such pair of levels (\bar{A}_1, \bar{A}_2) solves the following contracting problem.¹

DEFINITION

$$(\bar{A}^1, \bar{A}^2) \in \arg \max_{(A^1, A^2)} \left\{ \beta U_1^{real}(A^1) + (1 - \beta)U_2^{real}(A^2) \left| \begin{array}{l} (A^1, A^2) \in [0, T]^2 \\ \tilde{U}^1(A^1) \geq \tilde{U}^1(A^2) \\ \tilde{U}^2(A^2) \geq \tilde{U}^2(A^1) \end{array} \right. \right\}$$

If the incentive compatibility constraints hold at (A_1^{real}, A_2^{real}) , ie.

$$\tilde{U}^1(A_1^{real}) \geq \tilde{U}^1(A_2^{real})$$

$$\tilde{U}^2(A_2^{real}) \geq \tilde{U}^2(A_1^{real})$$

Then $(\bar{A}_1, \bar{A}_2) = (A_1^{real}, A_2^{real})$, and so no utility loss arises from having a class of mixed types.

If not then at least one type of student will attend a level of classes different from that which is optimal for that type. The overall loss of utility from being unable to enforce attendance levels selectively is thus

$$\beta(U_1^{real}(A_1^{real}) - U_1^{real}(\bar{A}^1)) + (1 - \beta)(U_2^{real}(A_2^{real}) - U_2^{real}(\bar{A}^2))$$

To reduce this utility loss educational institutions can potentially offer contracts giving additional incentives / penalties to the two attendance levels in such a way that the students pick the contracts designed for them. For example, if both types would choose A_2^{real} when offered (A_1^{real}, A_2^{real}) , adding a reward to attending amount A_1^{real} might correct type 1s behaviour without affecting type

¹ Offering two minimum attendance rules is not helpful, as it is the same as offering just the rule with minimum attendance the lower of the two

2s. When deciding whether to implement such schemes their cost would have to be compared to the utility gain. Rewards associated to following a particular attendance level could for instance take the form of financial remuneration or additional examination marks.

2.4.1. Example

In order to illustrate our model a simple example is constructed with two types to see how utility levels of individuals are affected by changing the various parameters considered. The utility function $U_w(A, S) + U_L(L)$ from the general model refers to the rational individual's utility function and $\tilde{U}_w(A, S) + \tilde{U}_L(L)$ refers to the irrational individual's utility function. We now ascribe a particular functional to the above utility function. It takes the form:

$$\sqrt{A} + \sqrt{\{f(A) / \theta_i\}S} + (K / \theta_i)\sqrt{L}$$

s.t.

$$A + S + L = 16$$

$$S \in \arg \max \sqrt{\{f(A) / \theta_i\}S} + (K / \theta_i)\sqrt{L}$$

and

$$\theta_i, i = l, h$$

and

$$f(A) = 1/2 + \sqrt{A/8}$$

The utility from attendance (\sqrt{A}) is a standard utility form i.e. strictly increasing in the time allocated to lecture attendance with a decreasing marginal utility. We believe this functional form closely resembles reality. It is assumed, as the general utility function indicates, that attendance level affects the choice of self-study. This relationship is described by the $f(A)$ function given by $f(A) = 1/2 + \sqrt{A/8}$. This is chosen under the assumption that self-study is a complement to attendance. Formulations are also considered where self-study could be a substitute or independent of attendance but choose this particular formulation for exposition purposes, as is the case where attendance seems to be beneficial to self-study, and even under such a formulation we would like to explore validity of mandatory rules where they have a greater chance of being beneficial. It is assumed in the particular functional form that self-study does not affect attendance in order to keep the model simple. Lastly utility from leisure $K\sqrt{L}$ is a standard strictly increasing, but at a decreasing rate, formulation. Moreover the coefficient k of leisure is assumed to be the importance the student attaches to leisure. When introducing irrationality, in the sense defined before, the value of k is increased, whereby the student overvalues leisure. The θ_i in the function are to distinguish between the two types. As can be clearly seen a higher value of θ_i implies less utility from self-study and leisure and hence a lower type. It is assumed that the high type learns faster through

lectures, but due to the decreasing marginal utility given through the strictly concave utility function will have to attend less classes than the low type in order to maximize his utility.

Though one could also consider the case where a high type is able to benefit more of lectures because of better understanding, and therefore be better of choosing a higher level of attendance, in this paper the focus is put on the undergraduate level, where lectures are geared towards providing the student with a minimum required level of skills and knowledge. On a postgraduate level, where lectures tend to be more specialised, the latter case might be more appealing, but at this level one anyway should be facing a lower level of irrationality as students already have experience with university education and therefore should automatically reduce the error in their decision making process.

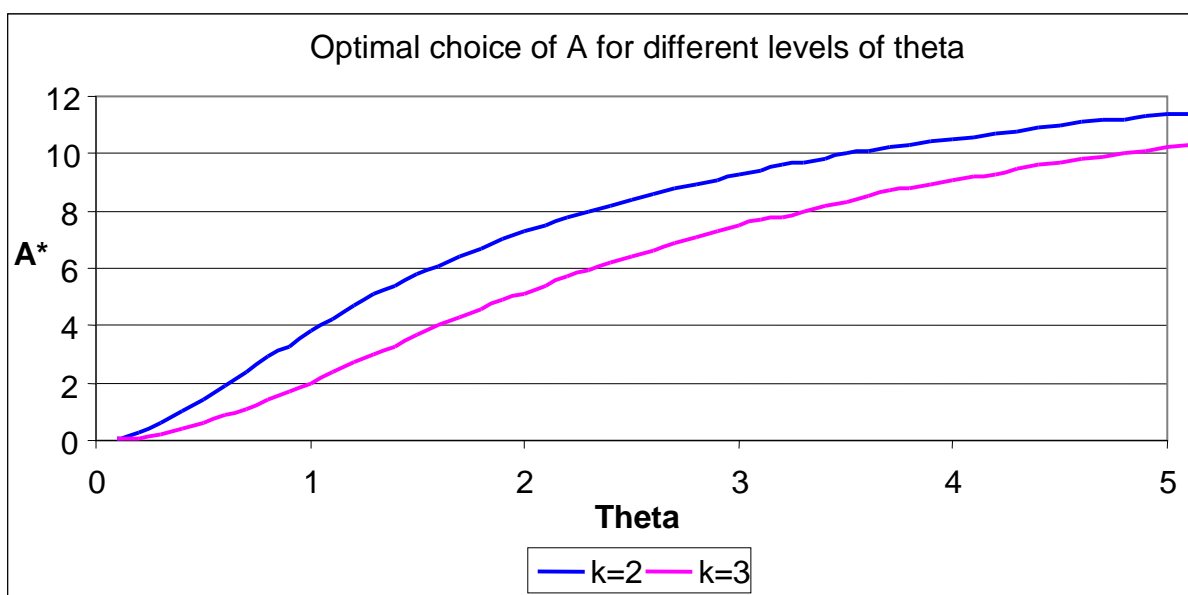


Figure 1: attendance level depending on student type θ

Now certain values are assigned to the various parameters, namely θ_h , θ_l and k . A rational individual is assumed to be described by a parameter $k=2$. The high type is assumed to have $\theta_h=1$ while the low type is endowed with $\theta_l=1.5$. The proportion of θ_l -individuals is assumed to be 0.55. In order to introduce irrationality the value of k is increased for both types to the value 3. Through this formulation irrationality is introduced in the sense of overvaluing leisure.

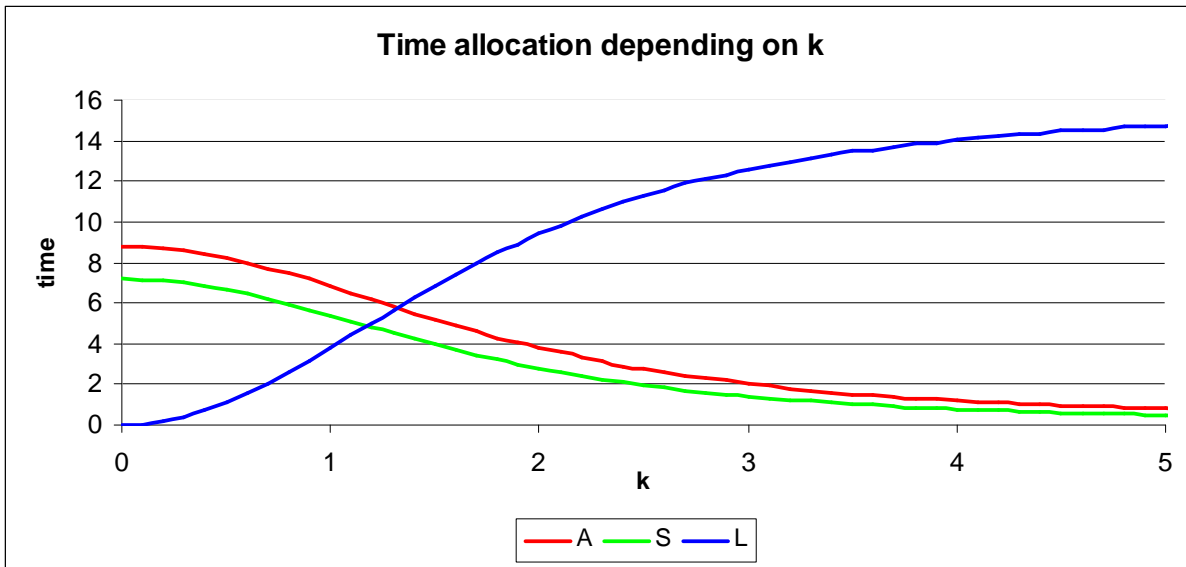


Figure 2: Time allocation depending on k

Now optimal time allocation and associated utility for both types is computed; firstly $U(A^*)$ when assumed to be rational and secondly $U(A_{fd})$ under irrationality but when they are free to decide between leisure, lecture attendance and self-study. The results are illustrated in the table below where the utility level has been normalized to 1 under the case of rationality.

	$U(A^*)$	$U(A_{fd})$
high type	1.0000	0.9874
Low type	1.0000	0.9875

Table 1: Utility levels of optimal and irrational free choice attendance (normalized)

Given that the utility level under irrationality is lower than the first best case the more important question is how this level compares to the utility obtained under various mandatory attendance rules. These shall be explored now.

The university is assumed to have perfect knowledge of the utility functions in both cases, rationality and irrationality, as well as the distribution of types among the students. However they cannot distinguish between the types and hence are forced to implement a universal mandatory rule not contingent on types. Hereby they have to consider, that given their set attendance level \bar{A} the student still solves for his time allocation between self-study and leisure with his incorrect utility function. Hence we factor this into account by assuming the university solves the following functions to fix mandatory attendance:

$$U^{real}(A) = U_w(A, \tilde{S}(A)) + U_L(T - A - \tilde{S}(A))$$

where

$$A^{real} \in \arg \max_{A \in [0, T]} U^{real}(A)$$

Four different attendance rules are considered:

1) The university solves the above problem in order to optimise the utility level of the low type by setting the mandatory level at what is denoted by \bar{A}_1 . Given this level of attendance the high and the

low type solve their irrational utility functions for time allocated to self-study and leisure as illustrated in table 2.

	time allocation of high type			time allocation of low type		
	A	S	L	A	S	L
First best solution	3.8	3.19	9.0	5.8	3.20	7.0
Free decision (irrationality)	2.0	1.40	12.6	3.7	1.65	10.7
\bar{A}_l	5.8	1.33	8.9	5.8	2.58	7.6
\bar{A}_h	3.8	1.42	10.8	3.8	2.80	9.4
$\bar{A}_{average}$	4.9	1.38	9.7	4.9	1.95	9.1
\bar{A}_{max}	8.0	1.14	6.9	8.0	2.18	5.8

Table 2: Utility levels of mandatory attendance levels

As can be seen in table 3 (where the utility level has been normalized to 1 under the case of rationality) the utility levels under time allocation as illustrated in table 3 endow the low type with an increased utility compared to the utility obtained under free decision, whereas the high type is worse off by being forced to visit more lectures than he would under free decision. Hence this form of mandatory attendance benefits some students at the expense of others. The validity of this trade-off is normative question which is not addressed here.

	$U(A^*)$	$U(A_{fd})$	$U(\bar{A}_l)$	$U(\bar{A}_h)$	$U(\bar{A}_{average})$	$U(\bar{A}_{max})$
High type	1.0000	0.9874	0.9798	0.9909	0.9873	0.9464
Δ high to $U(A^*)$	0.0000	-0.0126	-0.0202	-0.0091	-0.0127	-0.0536
Δ high to $U(A_{fd})$	0.0126	0.0000	-0.0076	0.0036	-0.0001	-0.0410
low type	1.0000	0.9875	0.9971	0.9858	0.9873	0.9851
Δ low to $U(A^*)$	0.0000	-0.0125	-0.0029	-0.0142	-0.0127	-0.0149
Δ low to $U(A_{fd})$	0.0125	0.0000	0.0096	-0.0017	-0.0002	-0.0024
Total utility	2.0000	1.9748	1.9769	1.9767	1.9746	1.9315
Δ to $U(A^*)$	0.0000	-0.0252	-0.0231	-0.0233	-0.0254	-0.0685
Δ to $U(A_{fd})$	0.0252	0.0000	0.0020	0.0019	-0.0002	-0.0434

Table 3: Time allocation of mandatory attendance levels

2) The next rule considered is the symmetrical case where the university solves the above problem, but now in order to optimize the utility level of the high type by setting the mandatory level at what is denoted by \bar{A}_h . Given this level of attendance the high and the low type solve their irrational utility functions for time allocated to self-study and leisure.

As can be seen in table 2 the utility levels under the time allocation in table 3 endow the high type with an increased utility compared to the utility obtained under free decision, whereas the low type is worse off by being forced to visit less lectures than he would under free decision. Here the high types benefit at the expense of the low type.

3) Now the university implements a mandatory attendance level $\bar{A}_{average}$ which is an arithmetic average of attendance levels \bar{A}_l and \bar{A}_h set in case one and two based on the distribution of types among the students. As table two illustrates both types have a small loss of utility compared to the

utility obtained under free choice and irrationality. This rule is an example of a situation where mandatory attendance rules might be undesirable under the framework considered.

4) At last the case where students have to attend all lectures offered is considered, which in our case is fixed by \bar{A}_{\max} . Unsurprisingly both types experience a decrease in utility, whereby the high type even suffers a considerable reduction of utility. Our simple model seems to suggest that complete mandatory attendance has non-desirable consequences for both types.

	$U(A^*)$	$U(A_{fd})$	$U(\bar{A}_l)$	$U(\bar{A}_h)$	$U(\bar{A}_{average})$	$U(\bar{A}_{max})$
high type	1.0000	0.9691	0.9669	0.9779	0.9750	0.9339
Δ high to $U(A^*)$	0.0000	-0.0309	-0.0331	-0.0221	-0.0250	-0.0661
Δ high to $U(A_{fd})$	0.0309	0.0000	-0.0022	0.0088	0.0059	-0.0352
Low type	1.0000	0.9603	0.9853	0.9735	0.9713	0.9739
Δ low to $U(A^*)$	0.0000	-0.0397	-0.0147	-0.0265	-0.0287	-0.0261
Δ low to $U(A_{fd})$	0.0397	0.0000	0.0251	0.0132	0.0110	0.0136
Total utility	2.0000	1.9294	1.9522	1.9514	1.9463	1.9078
Δ to $U(A^*)$	0.0000	-0.0706	-0.0478	-0.0486	-0.0537	-0.0922
Δ to $U(A_{fd})$	0.0706	0.0000	0.0228	0.0220	0.0169	-0.0216

Table 4: Utility levels of mandatory attendance levels

Next the example is extended to a case of stronger irrationality on part of the students. A value of $k=4$ is assigned. As can be seen in table 4 under the rules \bar{A}_h and $\bar{A}_{average}$ both types are better off with a mandatory attendance rule than under free decision making.

This example shows that pareto-improvement through attendance rules is a possibility and hence institutions have to factor degrees of irrationality into their decision making process.

When increasing or decreasing the variance of distribution of types it becomes difficult to obtain mandatory attendance rules which are a pareto-improvement. Clearly under no mandatory attendance scheme the student can be better off than under his optimal time allocation under rationality.

The simple model illustrates that there are a lot of complexities that need to be taken into account while devising mandatory attendance rules. It serves to illustrate that institutions need to follow a more decentralized approach taking into account the specific circumstances (namely degree of irrationality, distribution and variance of types) whether or not to implement mandatory attendance rules, and if they do, which ones.

2.5. Using mandatory attendance rules to improve screening

Economically, education has at least two values to society: it increases the human capital of individuals, and it increases efficiency by providing a signal to employers about individuals' suitability for jobs. The signal is given largely by exam results, and thus improvements in the accuracy of exam results are beneficial to society.

It is assumed here that there are two types of individual, a low type and a high type. Examination results range from 0 to 100, and there is a cut off point α above which students are labelled high type, and beneath which they are labelled low type. There is some positive probability of a low type being classed as a high type

$$P(R_L > \alpha) > 0$$

Screening will be improved if there is a way of reducing the results of the low types without decreasing those of the high types, or improving the results of the high types without increasing the results of the low types. Two ways in which this could be done are

2.5.1. Enforcing non-optimal mandatory attendance levels

Setting a non-optimal level of attendance for the low type, such that the incentive compatibility constraints still hold, and under which the low types achieve lower examination results, would decrease the probability of incorrect labelling. Similarly, setting a non-optimal attendance level for the high types which increase their examination results but maintains the incentive compatibility constraints will do the same. Such actions, as well as improving screening, have a negative affect the utility and productivity of one type of individual, so the net effect must be considered. This net effect will depend upon the relative importance placed on screening and productivity.

2.5.2. Relating examination results to attendance

This case is more interesting. Assume that optimal solutions for the two types satisfy the incentive compatibility constraints. Then any slackness in the incentive compatibility constraints can be used to increase screening without distorting attendance levels, and thus giving a strict improvement in screening at essentially no cost.

Assume that lectures are aimed towards low types, so that the optimal attendance is higher for low types. An examination result penalty (eg. Subtract 10 from the final mark) is associated to selecting the higher level of mandatory attendance, such that the incentive compatibility constraints still hold under the associated changes of utility. Then we have increased screening whilst maintaining the same productivity levels². Similarly, if lectures are aimed towards high types, so that at optimal attendances high types attend more, we can associate an examination mark bonus to the higher level of attendance to obtain a similar gain in screening.

² This is not strictly true. More effective screening is likely to alter the amount of time students devote to independent study. However this would occur regardless of how examinations are improved, so in a sense the above just provides a free way of improving examinations.

2.6. Further investigations

Apart from studying mandatory attendance rules, the model could potentially provide important information about time allocation decisions of individuals given our utility functions in cases that have further scientific interest. External shocks would change the relative pay-off of leisure and the rational levels of K , and the changes in the form of $F(A)$ function would virtually change the nature of the relation between attendance and self study as well as the relative importance of each one of them separately.

First we consider the case where an external event changes the relative pay-off of leisure. This could result from external shocks that make leisure for a rational individual relatively more important. As an example consider a case such as the importance of acquiring skills outside the strictly academic environment. For instance learning foreign languages contributes to the formation of social capital. In this case the student's value of k would increase, and this would not be considered irrationality. Mandatory attendance would deteriorate the student's utility by constraining the student's available time for the widely interpretable variable we defined as leisure. Other external shocks that could shift or change the curvature of our utility function through changes in $F(A)$, the variable for interdependency between self-study and lecture attendance, could occur through the development of other educational resources. Recently online resources, such as video lectures of MIT, have provided the student with alternative methods to substitute lecture attendance. In this case $F(A)$ might have to be a decreasing function of A in order to take into account that a student might be able to receive the same information whilst saving transportation time and being able to schedule the learning session at his own time convenience. Here again students would face a loss in utility through a mandatory attendance level.

Further we could also consider cases of groups of individuals that have peculiar learning functions, as ADHD types. In this case we know that the performance of individuals might strongly differ from the usual types considered in our model. A strong dispersion of types makes universal attendance rules less desirable. Here the idea of incentive compatible contracts could provide an exciting new field of research.

A further question that arises is: which is the way to maturity and rationality? Could rationality be taught from a very early stage and at what welfare loss would this happen? The interested scientist could implement a version model with many periods representing student's educational course, in which students become gradually more mature and rational. This model could answer if it is worthwhile pursuing an educational course that aims at cultivating maturity and rationality in individuals as a way to capture the later welfare losses that occur from paternalistic behaviour.

In addition to the model one could treat the behaviour of individual in different periods of time in what is defined as an academic year, as in the end of sessions i.e. exams, the beginning of the course, or in the middle of it. The exams for example can provide very powerful incentives for class attendance and intensive self study during that period of time. As attendance is constrained the individuals will concentrate on self study, before exam periods. The fact that in many cases before exams students self study a lot can be attributed to the pressure that the exams provide and its possible relation to learning or possible changes in the relevant material from the side of the professors. On the other hand in the beginning of the course individuals might attend more in order to collect the information that will allow them to form their preferences and take decisions on their attendance in the rest of the class and limit self study as they are not sure about potential interest and the relevant literature that they will have to consult. In the middle of the session one might observe relatively reduced attendance if the student decides his potential interest is low but has to take the course as part of his studies.

The existing empirical research in student attendance and performance in exams identifies a strong relationship between attendance and exam result. However we would like to present another possible reason that reinforces the positive relationship between attendance and performance. Attendance could be more informative about the nature and the form of the examination system than self study, ranging in degrees from negligible to significant. In this case the students attending are more informed, we might observe cases where two types of similar ability levels have exerted exactly the same learning effort but the informed one does relatively better. This case could be expressed in our model through the incorrect utility functions. In further empirical experiments we should attempt to control the possible information parameters.

Finally the model might prove useful research not strictly related to student time allocation and mandatory attendance law. It might find applicability in various situations in every day life where the theorist decides that individual time could be represented in three distinct periods in order to study how an individual pursues his goals under a constraint maximisation time allocation model.

3. Conclusion

The paper investigates the economics of student attendance under the existing system of mandatory rules. A model is developed from which several interesting conclusions emerge. Firstly the unsurprising result that in a perfect world any mandatory attendance rules are unjustified and result in utility losses is shown. Then externalities, irrationalities and asymmetric information is introduced into the model. It is assumed that the educational institution knows the “correct” utility function and fixes mandatory attendance rules based on this. Two types of mandatory attendance rules are considered: those that set a minimum level of attendance, and those that specify a fixed

level of attendance. The former are those generally in use, but it is argued that in many cases the latter lead to more optimal solutions.

In the case of one student it is shown that instigating the mandatory attendance rule that is optimal with respect to the information known to the institution can actually lead to a loss in utility. Then the model is generalised to include two types of student. In this case, if the institution offers only one level of mandatory attendance, whether as a fixed level or the minimum level, the optimal mandatory attendance rule can again lead to a lower utility than that observed under optional attendance, regardless of whether or not the institution knows the students behaviour. Thus institutions must exercise caution in implementing any form of mandatory rules. This is especially true when they are unaware of individuals' decision making processes (the case which most closely relates to reality).

The case of two students is then treated as a contracting problem, where two levels of attendance can be offered. A condition is thus given for when teaching the two types of student in the same class leads to a utility loss.

An example with selected utility function is developed to show many of the results discussed in the model, and this is followed by a brief discussion of a controversial idea using mandatory attendance rules to aid screening.

Finally several possible extensions of the model identify further directions for research and investigation.

The main conclusion to be drawn is that educational institutions need to be flexible and ready to react to both exogenous and endogenous changes. Real life evidence that mandatory attendance levels often stay the same for long periods of times and over significant changes in the internal and external environment suggest that institutions need to reconsider the way they decide upon mandatory attendance rules.

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