A note on the fertility-income relationship and childcare outside home

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Abstract

This study constructs an overlapping generations model with Stone-Geary preferences and child care outside home. When income is sufficiently large, individuals can afford to have more children due to childcare services outside home. As a result, we demonstrate the demographic transition; thereafter fertility rebound and eventually decreasing fertility.

JEL classifications: J11, J13

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1. Introduction

As has been indicated by many studies and historical data, the shift from positive relationship between income and fertility to negative relationship, i.e., demographic transition has been observed in developed countries. In recent years, some developed countries have experienced increasing total fertility rate (TFR), so-called fertility rebound, and then TFR has decreased in some developed countries.

There are some recent studies on the fertility-income relationship. Nakamura (2018) explain the demographic transition in a simple Solow model with Stone-Geary preferences. Yasuoka and Miyake (2010) and Day (2016) analyze the relationship between fertility and income in an overlapping generations model with human capital and childcare outside home. Yasuoka and Miyake (2010) demonstrate the positive relationship between income and fertility unless the prices of external childcare increases greatly. Day (2016) shows that fertility rebound occurs with increase in the wage rate of skilled worker when the prices of childcare is constant. Yakita (2018) constructs Galor and Weil (1996) model incorporated external childcare services. He demonstrates the fertility rebound occurs as female's wage increases with economic development since female can become to use external childcare services. Focusing on the effects of endogenous longevity and physical child cost on the fertility, Futagami and Konishi (2020) show that demographic transition occurs, and then the fertility increases again.

Many previous studies show demographic transition and the fertility rebound. However, they do not propose the reduction again of the fertility after fertility rebound. This paper shows the non-monotonous fertility-income relationship (without policy intervention), i.e., demographic transition, the fertility rebound and the reduction again of the fertility in a simple overlapping generations model with Stone-Geary preferences and childcare outside home.

2. The model

Consider the competitive equilibrium of an overlapping generations economy. Each individual lives for two periods: childhood and adulthood. In the first period, individuals do not make any decisions. In the second period, individuals raise children and decide consumption and whether or not to use childcare services.

2.2 Individuals

We assume the existence of minimum consumption $\bar{c}$. Hence, People gains utility from consumption $c_t - \bar{c}$ and the number of children n_t. The preference of individual of generation t is expressed by the following Stone-Geary type utility function.

$$u_t = \alpha \frac{(c_t - \bar{c})^{1-1/\sigma}}{1-1/\sigma} + \gamma \frac{n_t^{1-1/\sigma}}{1-1/\sigma}, \tag{1}$$

where $\sigma > 0$, $\alpha \in (0,1)$ and $\gamma \in (0,1)$ represents the elasticity of substitution between consumption and the number of children, the preference of consumption and the preference for children, respectively. Individual decides consumption and the number of children. Thus, her budget constraint become:

$$(1 - l_t)w_t = c_t + P_t x_t, \qquad 0 \leq l_t \leq 1 \tag{2}$$

where l_t is the child-rearing time, x_t is the amount of external childcare outside home purchased, and P_t is the price of childcare services. Let ϕ denote a required time input to rear children, then the total time input necessary to rear n_t is given by $\phi n_t = l_t + x_t$ (See Yakita 2018). Therefore, the cost of rearing children is given by $C_t = l_t w_t + P_t x_t = (w_t - P_t)l_t + \phi P_t n_t$. Following Yakita (2018), we can solve the child cost minimization problem subject to $0 \leq l_t \leq 1$. Hence, we can obtain the following cost function.

$$C(n_t) = \begin{cases} \phi n_t w_t & if \quad w_t \leq P_t \\ \phi P_t w_t & if \quad w_t > P_t, \end{cases} \tag{3}$$

where we assume that $n_t < 1/\phi$. If $w_t > P_t$, then $l_t = 0$ and $x_t = \phi n_t$. If $w_t > P_t$, then $l_t = \phi n_t$ and $x_t = 0$. In other words, if individuals use childcare services outside home, then rearing child time cost is zero. Thus, we can rewrite Eq. (2) as follows:

$$w_t = c_t + C(n_t). \tag{4}$$

From (1) and (2), the utility maximization is formulated as follows:

$$\max_{n_t} \ \alpha \frac{[w_t - C(n_t) - \bar{c}]^{1-1/\sigma}}{1-1/\sigma} + \gamma \frac{n_t^{1-1/\sigma}}{1-1/\sigma}.$$

From first-order condition for maximization, we have optimal the number of children.

$$n_t = \begin{cases} \dfrac{\gamma^\sigma (w_t - \bar{c})}{\gamma^\sigma \phi w_t + (\alpha \phi w_t)^\sigma} & if \quad w_t \leq P_t \\[2ex] \dfrac{\gamma^\sigma (w_t - \bar{c})}{\gamma^\sigma \phi P_t + (\alpha \phi P_t)^\sigma} & if \quad w_t > P_t, \end{cases} \tag{5}$$

2.3 Goods sector

To simplify analysis and to focus on the relationship between fertility and income, we assume that production function is linear in labor.

$$Y_t = AL_t^Y , \tag{6}$$

where A and L_t^Y denote productivity in goods sector and the labor in goods sector in period t, respectively. Let $L_t = L_t^Y + L_t^X$ denote aggregate labor in period t, where L_t^X is labor in childcare sector. Therefore, per capita output becomes

$$y_t = A\lambda_t^Y , \tag{7}$$

where $\lambda_t^Y = L_t^Y/L_t$ is the populations share of labor employed in goods sector. Hence, the wage (and income) is always equal to technology in the equilibrium, i.e., $w_t = w = A$ for all t.

2.4 Childcare sector

Aggregate child care services represent $X_t = \mu L_t^X$, where $\mu > 1$ means the productivity in childcare sector. In addition, we assume the total goods cost to maintain the level of childcare services as with Yakita (2018). The childcare sector profit is represented as follows:

$$\pi_t = P_t X_t - w_t L_t^X - BX_t , \tag{8}$$

where BX_t is the total goods cost in childcare sector. The zero-profit condition in childcare sector is given by

$$P_t = \frac{w_t}{\mu} + B . \tag{9}$$

From Eq. (9), we can obtain $P_t \gtreqless w_t$ as $\frac{\mu B}{\mu - 1} \gtreqless w_t$. Hence, if individual's income is sufficiently low relative to price of childcare services, then childcare outside home will not be used and the childcare services will not be produced.[1]

3. Fertility-Income Relationship

Suppose that $\bar{w} \equiv \mu\beta/(\mu - 1)$. From Eq. (9), we can rewrite Eq. (5) as follows:

$$n_t = \begin{cases} \dfrac{\gamma^\sigma (w_t - \bar{c})}{\gamma^\sigma \phi w_t + (\alpha\phi w_t)^\sigma} & if \quad w_t \le \bar{w} \\[2ex] \dfrac{\gamma^\sigma (w_t - \bar{c})}{\gamma^\sigma \phi (w_t/\mu + B) + [\alpha\phi(w_t/\mu + B)]^\sigma} & if \quad w_t > \bar{w} , \end{cases} \tag{10}$$

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[1] The supply of childcare services is represented as $X_t^S = \mu L_t^X = \mu(1 - \lambda_t)L_t$ and the demand of childcare services is represented as $X_t^D = \phi n_t L_t$. In equilibrium, $\mu(1 - \lambda_t)L_t = \phi n_t L_t$ and hence, $\lambda_t = 1 - \phi n_t/\mu$.

The effect of income on the number of children becomes

$$\frac{\partial n_t}{\partial w_t}=\begin{cases}\dfrac{(\gamma\phi)^{\sigma}w_t^{\sigma-1}[\sigma\alpha^{\sigma}\bar{c}+\gamma^{\sigma}\bar{c}(\phi w_t)^{1-\sigma}-(\sigma-1)\alpha^{\sigma}w_t]}{[\gamma^{\sigma}\phi w+(\alpha^{\sigma}\phi w_t)^{\sigma}]^2}, & if \quad w_t\leq\bar{w}\\ \dfrac{\theta^1(w_t/\mu+B)^{\sigma-1}\{\theta^2+\theta^3(w_t/\mu+B)^{1-\sigma}-(\sigma-1)\alpha^{\sigma}w_t\}}{\{\gamma^{\sigma}\phi(w_t/\mu+B)+[\alpha\phi(w_t/\mu+B)]^{\sigma}\}^2}, & if \quad w_t>\bar{w}\end{cases} \tag{11}$$

where $\theta^1\equiv(\gamma\phi)^{\sigma}/\mu$ and $\theta^2\equiv\alpha^{\sigma}(\sigma\bar{c}+\mu B)$ and $\theta^3\equiv\gamma^{\sigma}\phi^{1-\sigma}(\bar{c}+\mu B)$. When $\sigma\leq1$, the effect is always positive, i.e., $\partial n_t/\partial w_t>0$. In other words, the fertility increases monotonically with income. On the other hand, when $\sigma>1$, the fertility dynamics depends crucially on the level of income. To focus on non-monotonous fertility dynamics, we assume that $\sigma>1$ in what follows.

When $\sigma>1$, the sign of $\partial n_t/\partial w_t$ depends on the level of income as shown in Fig. 1. When $w_t\leq\bar{w}$, as income increases, the fertility turns from increasing to decreasing after a certain threshold of income $\hat{w}$ as illustrated in Fig. 1 (a). When income is sufficiently low, the marginal utility from consumption around $\bar{c}$ is very high, and hence individuals choose higher consumption instead of the number of children. In other words, the substitutability between consumption and the number of children is low. Hence, the income effect is larger than the substitution effect when income is sufficiently low, i.e., $w_t\leq\hat{w}$. As income increases, individual's consumption is larger than $\bar{c}$. When income is sufficiently large, i.e., $w_t>\hat{w}$, the substitution effect is larger than the income effect. As a result, the positive relationship between income and fertility is changed to the negative.

[Insert Fig.1 about here]

Let us turn to $w_t>\bar{w}$. Since this study is interested in non-monotonous fertility-income relationship observed developed countries, we assume that $\hat{w}<\bar{w}$. This assumption implies that individuals with low income, such as those who consume around $\bar{c}$, do not use childcare services. In other words, we assume that the price of childcare services is sufficiently large. Hence, we impose following assumption in what follows.

Assumption

$$\hat{w}<\bar{w}\equiv\frac{\mu B}{\mu-1}$$

When $w_t>\bar{w}$, individuals will purchase the childcare services outside home. When income is sufficiently high, rearing-children time cost is also higher, and therefore individuals have fewer children. However, individuals can afford to have more children by substituting rearing children time cost for childcare services outside home. Hence, the fertility increases with income when $\bar{w} < w_t \leq \tilde{w}$ as illustrated in Fig. 1 (b). When income becomes higher relative to the price of childcare services, i.e., $w_t > \tilde{w}$, the income effect is smaller, and hence the substitution effect is larger than the income effect. Thus, the fertility decreases with income. As a result, we obtain the following proposition.

Proposition The fertility-income relationship is non-monotonous. The fertility increases with income for $w_t < \hat{w}$, and it decreases for $\hat{w} < w_t \leq \bar{w}$. It increases again for $\bar{w} < w_t \leq \tilde{w}$ and it decreases eventually for $w_t > \tilde{w}$.

$$\frac{\partial n_t}{\partial w_t} \begin{cases} > 0 & if \quad w_t < \hat{w} \\ < 0 & if \quad \hat{w} < w_t \leq \bar{w}, \\ > 0 & if \quad \bar{w} < w_t \leq \tilde{w}\ , \\ < 0 & if \quad w_t > \tilde{w} \end{cases} \tag{12}$$

As shown in Fig. 2, as income increases, the demographic transition occurs, and the fertility increases again due to using childcare services outside home; it eventually decreases due to substitution effect. As a result, we show non-monotonous fertility-income relationship observed in developed countries

[Insert Fig.2 about here]

4. Conclusion

This paper presents non-monotonous fertility-income relationship observed in developed countries. Constructing an overlapping-generations model with Stone-Geary type utility function and childcare outside home, we demonstrate the demographic transition; thereafter fertility rebound and eventually decreasing fertility. In particular, childcare services outside home plays crucial role in fertility dynamics. When income is sufficiently large, individuals use the external child care services. Since individuals can afford to have more children using childcare outside home, the fertility rebound occurs. However, the fertility decreases since the substitution effect is larger than income effect.

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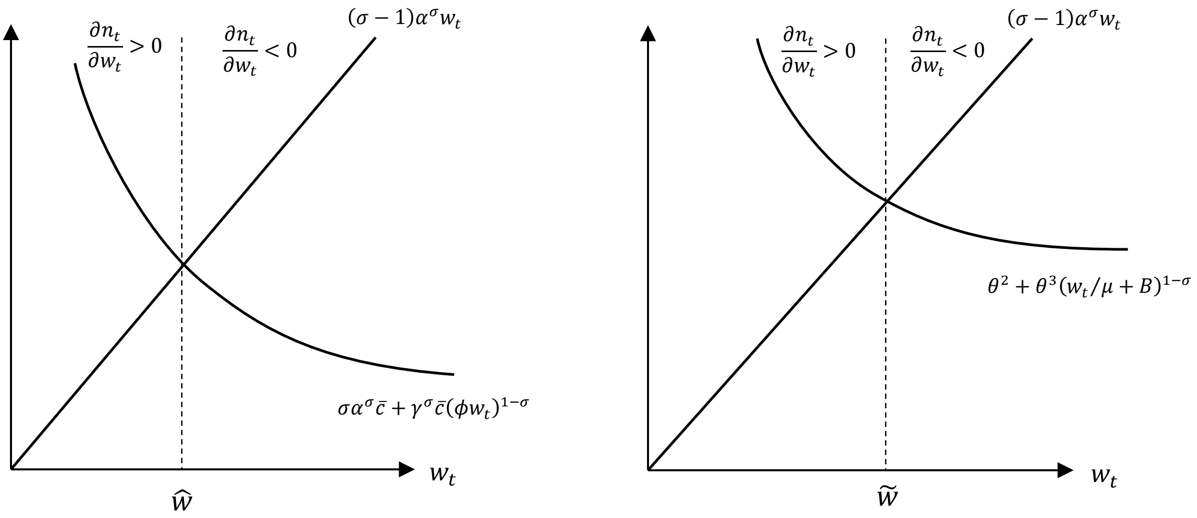

(b) Threshold $\hat{w}$

(b) Threshold $\tilde{w}$

Fig. 1 Threshold of income level

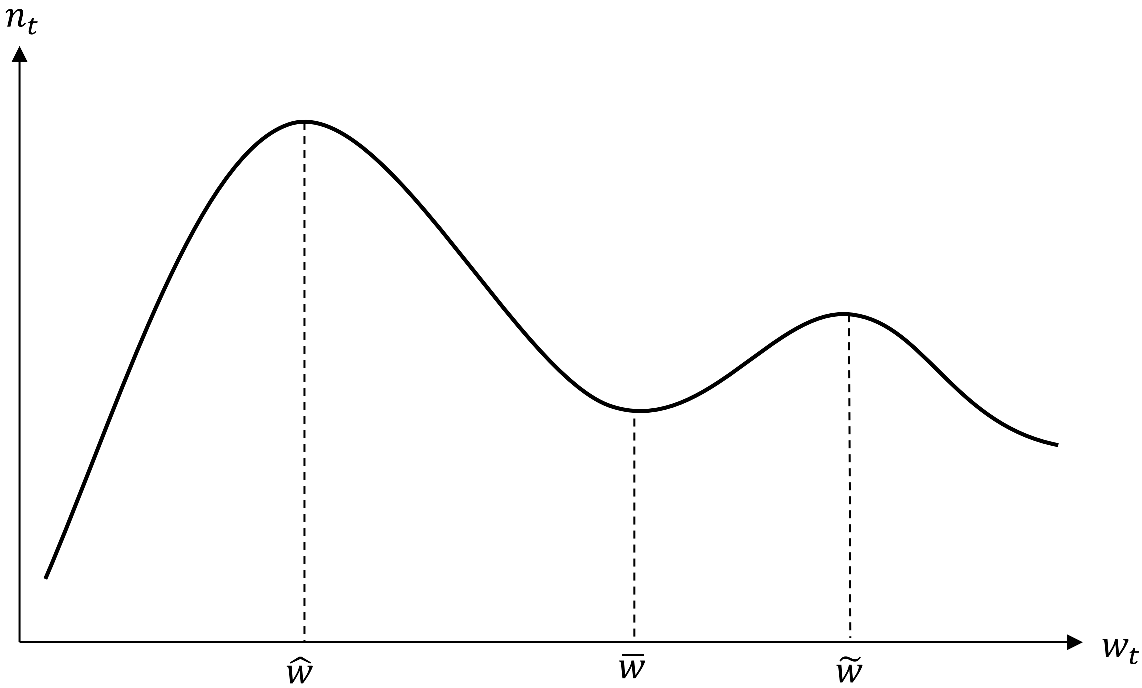

Fig. 2 Relationship between fertility and income