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Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective

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Abstract

The Cobb-Douglas production function in the field of economics has a long history. In mathematical economics, it is used to find the functional relationship between the economic inputs and potential outputs. This study applies the Cobb-Douglas production function to predict the cost minimization policies of a running garments industry of Bangladesh. In the study the effects of the variation rate of capital, labor and other inputs with returns to scale in the garments industry of Bangladesh are examined. In multivariable calculus, the method of Lagrange multiplier is a very useful and powerful technique. In this study interpretation of Lagrange multiplier is given to predict the cost minimization policy using Cobb-Douglas production function. An attempt has been taken here to show the production of garments in minimum cost by using statistical analysis.

Keywords: Production function, Cobb-Douglas model, cost minimization, returns to scale

JEL Codes: C1, C6, H7, I3, O1, O4

1. Introduction

In economics, productivity is defined as a relation between the production and the necessary inputs for the production. Cobb-Douglas production function is one of the most widely used production function in economics. It is named after two American scholars, mathematician and economist Charles W. Cobb and economist Paul H. Douglas. In 1928, it is described for the first time to measure the level of physical output in the US manufacturing sector [Cobb & Douglas, 1928]. It not only satisfies the basic economic law but also it is easy in its computation and interpretation of the estimated parameters. It estimates the coefficient of inputs, their marginal productivities, factor shares in total output and degree of returns to scale [Khatun & Afroze, 2016]. It describes the law of productions that is the transformation of factor inputs into outputs at any particular time period, and represents the technology of an industry on the economy as a whole [Gupta, 2016]. It can be applied at the level of individual firms, industries, or entire economies [Cottrell, 2019].

Cobb-Douglas production function helps the industry to make rational decision on the quantity of each factor inputs to employ so as to minimize the production cost for its profit maximization. The industry as a rational economic agent needs information on the marginal productivity of factors to be able to produce at optimum. If the production of an industry becomes at maximum level for each input and also if it maintains the rules of prevention of environment pollution, then the industry is considered as sustainable [Roy at al., 2021].

Labor is measured as the total number of employees in the industry during some period; the capital is measured as the total fixed assets during the same period, the output is taken as the value added [Ahmad & Khan, 2015]. In any industry the production level is described by production function and this industry tries to minimize its cost of production [Onalan & Basegmez, 2018]. The Cobb-Douglas production function is the most ubiquitous form in theoretical and empirical analyses of growth and productivity. We can work on growth, technological change, productivity, and labor even in the 21st century by the use of Cobb-Douglas production function [Felipe & Adams, 2005].

2. Literature Review

David Gordon and Richard Vaughan have explained various types of production functions, such as the Cobb-Douglas, Constant Elasticity of Substitution (CES), and Generalized and Leontief production functions [Gordon & Vaughan, 2011]. Umesh Kumar Gupta has analyzed the Cobb-Douglas production function and cost function in generalized form [Gupta, 2016]. Shaiara Husain and Md. Shahidul Islam work with the Cobb-Douglas production functions for impressive average annual growth of manufacturing sector of Bangladesh. They have collected data of total value of output, total asset, total liabilities, number of permanent workers, etc. of about six major types of industries including garments, textiles, food and food processing, leather and leather products, electronics, and chemicals and pharmaceuticals [Husain & Islam, 2016].

Tahmina Khatun and Sadia Afroze have tried to explore the relationship between real GDP, and labor and capital in case of some Asian countries, such as Bangladesh, India, China, Malaysia and Thailand using the Cobb-Douglas production function to make a comparison among Bangladesh and these selected countries [Khatun & Afroze, 2016]. Thakur Prasad Wagle states that agriculture is the main source of food, income, and employment of Nepal and economic growth of the country depends on both increasing the productivity of existing crops and diversifying the agricultural base for use as industrial inputs. He uses the Cobb-Douglas production function on the agricultural production of Nepal in various spaces and dimensions [Wagle, 2016].

Jeff Biddle uses the Cobb-Douglas model as very innovative as it shows that statistical method can be used to derive empirical relationship between input and output [Biddle, 2012]. Cătălin Angelo Ioan and Gina Ioan have tried to discuss various aspects concerning the Cobb-Douglas production function, such as short- and long-term costs, profit is made both for perfect competition market and maximizes its conditions, the effects of Hicks and Slutsky and the production efficiency problem, the existence of the Cobb-Douglas function, etc. [Ioan & Ioan, 2015]. Aurelia Rybak has tried to present the potential of the production function in relation to such a complicated production process as hard coal mining and to examine the regularities occurring during the extraction process, verify the validity of the current method of resource allocation, disclose the origin of possible problems, and offer efficient corrective solutions. The author also provides the necessary level of the most appropriate combination of labor and capital [Rybak, 2019].

3. Research Methodology of the Study

Word 'Research' is comprised of two syllables, *re-* and *search*. Here *re-* means again, anew or over again, and *search* is a verb to examine closely and carefully, to test and try, or to probe. Together they form a noun describing a careful, systematic, patient study and investigation in some field of knowledge, undertaken to establish facts or principles [Grinnell, 1993]. Research may relate to any subject of inquiry with regard to collection of information, interpretation of facts, and revision of existing theories or laws in the light of new facts or evidence [Adams et al., 2007]. It also emphasizes on creativity that is carried in a systematic way to improve knowledge which consists of human knowledge, culture, and society [OECD, 2002].

'Method' is a word coined of two Greek elements: *meth-* and *odos.* The *meth-* is an element meaning 'after', *odos* means 'way'. A method is, therefore, a following after the way that someone found to be effective in solving a problem, of reaching an objective, in getting a job done [Leedy & Ormrod, 2001]. Greek element *ology* means 'the study of'. Research methodology is the systematic procedure adopted by researchers to solve a research problem that maps out the processes, approaches, techniques, research procedures, and instruments. It may be understood as a science of studying how research is done scientifically [Kothari, 2008].

We have used secondary data to prepare this paper. It can identify the domain, selection, designing and inclusion of various measuring variables in any research. For the collection of secondary data we have used both published and unpublished data sources. The published data are collected from books of famous authors, websites, national and international journals, e-journals, various publications of international organizations, handbooks, theses, magazines, newspapers, various statistical reports, historical documents, information on internet, etc. On the other hand, the unpublished data are collected from diaries, letters, unpublished biographies and autobiographies of scholars, and from various public and private organizations.

In this study we have discussed the Cobb-Douglas production function to obtain minimum cost by showing mathematical calculations in some detail. In Bangladesh, garments sector is the back bone of economy of the country. The country exports garments in many countries of Europe, America and Middle-east. In the study we have tried to provide a suitable suggestion to this sector by the statistical analysis for the sustainable production. The reliability and validity are inevitable issue in any research. In this study we have tried our best to maintain the reliability and validity throughout of the research [Mohajan, 2017b, 2018a, 2020].

4. Objective of the Study

The main objective of this study is to analyze the cost minimization techniques of garments industry of Bangladesh by the help of Cobb-Douglas production function. The other particular objectives are as follows:

- To provide a detail mathematical procedure to show the findings more accurately.
- To discuss returns to scale using Cobb-Douglas production function.
- To increase the production and profit of the industry by minimizing cost in a sustainable way.

5. Preliminary Concepts

In this section we have included some basic concepts of economics and mathematics for those who are novice in this field. We hope all the readers and researchers in this study will capture the full concept efficiently and interestingly.

5.1 Optimization Techniques

Every industry's first aim is to optimize its costs (minimum), products and profits (maximum) in an efficient and satisfied way. Let us consider a function $f(\mathbf{x})$ of one variable **x**, where $\mathbf{x}=(x_1, x_2, ..., x_n)$. For a function $f(x)$ to be optimum (maximum or minimum) $\frac{df}{dx}=f'(x)=0$. If $\frac{d^2 f}{dx^2}<0$ at $x=x_0$ the function is maximum at a point $x=x_0$, and if $\frac{d^2 f}{dx^2}>0$ at $x=x_0$ the function is minimum at a point $x=x_0$. If $f(x,y)$ be a function of two variables x and y then for optimum, $\frac{\partial f}{\partial x}(i.e. f_x)=0=\frac{\partial f}{\partial y}(\text{i.e., } f_y)$, and $f_{xx}f_{yy}-f_{xy}^2>0$. If $f_{xx}>0$ (and $f_{yy}>0$), then the function has a minimum point, if $f_{xx}<0$ (and $f_{yy}<0$), then the function has a maximum point. For $f_{xx}f_{yy}-f_{xy}^2<0$, there is neither a maximum nor a minimum, but a saddle point. In all cases, the tangent plane at the extremum (maximum or minimum) or a saddle point to the surface $z=f(x,y)$, is parallel to the z-plane [Mohajan, 2018b; Roy et al., 2021] .

5.2 Production function

Production plays a major role in economics. A production function gives the technological relation between quantities of physical inputs and quantities of output of goods [Mishra, 2007]. The growth of economics generally has measured by Gross Domestic Production (GDP) rate in current price. Economic production is effected from various environmental factors, such as capital, labor, and other inputs [Onalan & Basegmez, 2018]. There are various production functions, such as the Cobb-Douglas, Constant Elasticity of Substitution (CES), and Generalized and Leontief production functions [Gordon & Vaughan, 2011]. Production function has been used as an important tool of economic analysis. It is generally believed that in 1894, Philip Henry Wicksteed was the first economist to algebraically formulate the relationship between output and inputs as $Q = f(x_1, x_2, ..., x_n)$, where Q is the quantity of output and $x_1, x_2, ..., x_n$ are the quantities of factor inputs, such as capital, labor, land, raw materials, etc. [Wicksteed, 1894].

But some evidences suggest that Johann von Thünen first formulated it in the 1840s [Humphrey, 1997]. He was perhaps the first economist who implicitly formulated the exponential production function as;

$$P = f(F) = A\prod_{1}^{3}\left(1 - e^{-a_i F_i}\right) \qquad (1)$$

where F_1, F_2, and F_3 are the three inputs, labor, capital and fertilizer, a_i are the parameters and P is the agricultural production [Mishra, 2007; Sickles & Zelenyuk, 2019]. He applied the concept of diminishing returns to a two-input: i) variable proportions, and ii) production function for the first time [Gordon & Vaughan, 2011]. Some studies suggest that production function was made firstly by economist Knut Wicksell in 1906. Later, Cobb-Douglas production function was developed by mathematician Charles W. Cobb and economist Paul H. Douglas in 1928 [Cobb & Douglas, 1928].

5.3 Cobb-Douglas Production Function

In mathematical economics, the Cobb-Douglas production function is a particular functional form of the production function, which is widely used to represent the technological relationship among two or more inputs, such as capital *K*, labor *L*, and other inputs *R*; and the amount of output that can be produced by those inputs. The Cobb-Douglas production function form was developed and tested against statistical evidence by Charles W. Cobb and Paul H. Douglas during 1927-1947. The nature of Cobb-Douglas function is such that it is every time Hicks neutral [Cobb & Douglas, 1928].

The general form of a Cobb-Douglas production function for a set of *n* inputs is,

$$Q = f(x_1, x_2, ..., x_n) = A\prod_{i=1}^{n} x_i^{a_i} \tag{2}$$

where Q is output, $x_1, x_2, ...$ are inputs, and A and a_i are parameters determining the overall efficiency of production and the responsiveness of output to changes in the input quantities [Brown, 2017]. A production function with *n* input factors is called *h* homogeneous, $h > 0$, if [Onalan & Basegmez, 2018],

$$f(kx_1, kx_2, ..., kx_n) = k^h f(x_1, x_2, ... x_n) \tag{3}$$

where *k* is any real number. If $h > 1$, per percent increase in input levels would result greater than per percent increase in the output level (the increasing returns to scale), if $h < 1$, per percent increase in input levels would result less than per percent increase in output (the decreasing returns to scale), and if $h = 1$ represent the constant returns to scale. A simple aggregated Cobb-Douglas production function, with no natural resources is,

$$Q = AK^a (HL)^{1-a} \tag{4}$$

where Q is total output and $0 < a < 1$, and A, K, H, and L are total factor productivity, the stock of physical and human capitals, and the amount of labor employed, respectively [Barros, 2017].

5.4 Returns to Scale

In economics, returns to scale indicates what happens to long-run returns as the scale of production increases, when all input levels including physical capital usage are variables. The concept of returns to scale arises in the context of the production function of an industry [Gelles et al., 1996]. The returns to scale is determined by [Onalan & Basegmez, 2018],

$$\text{Returns to scale } = \frac{\%\partial(\text{quantity of output})}{\%\partial(\text{quantity of input})}. \tag{5}$$

Marginal productivity of capital (MPC) is, $MP_K = \frac{\partial Q}{\partial K}$, marginal productivity of labor (MPL) is, $MP_L = \frac{\partial Q}{\partial L}$, and marginal productivity of other inputs (MPI) is, $MP_R = \frac{\partial Q}{\partial R}$. Therefore, the marginal rate of the technical substitution of labor (*L*) for capital (*K*) is given by,

$$MRTS = \frac{MP_L}{MP_K} = \frac{\partial Q / \partial L}{\partial Q / \partial K}. \tag{6}$$

The marginal rate of the technical substitution of other inputs (*R*) for capital (*K*) is given by,

$$MRTS = \frac{MP_R}{MP_K} = \frac{\partial Q / \partial R}{\partial Q / \partial K}. \tag{6a}$$

There are three types of returns to scale in economics: i) constant returns to scale, ii) increasing returns to scale, and iii) decreasing returns to scale.

5.4.1 Constant Returns to Scale

Constant returns to scale (CRS) explained by a Swedish economist Erik Lindahl (1891-1960) [Lindahl, 1933]. The word scale refers to the long-run situation where all inputs are changed in the same proportion. If we increase all factors (scale) in a given proportion and the output increases in the same proportion, returns to scale are said to be constant. Hence CRS is a constant ratio between inputs and outputs. It occurs when increasing the number of inputs lead to an equivalent increase in the output. A plant with a CRS is equally efficient in producing small batches as it is in producing large batches. Let us consider a homogeneous production function $f(K, L)$ of degree 1, where *K* and *L* are factors of production capital and labor, respectively. Constant returns to scale indicates $f(\alpha K, \alpha L) = \alpha f(K, L)$ where constant $\alpha \geq 0$. CRS exists if an industry increases all resources; labor, capital, and other inputs, by 25% (say), and output also increases by 25%. For example, an industry employs 10,000 workers in factory to produce 10 million units of a product each year. CRS exists if the scale of operation expands to 20,000 workers in that factory and production increases to exactly 20 million units each year [Mohajan, 2018b; Roy et al., 2021].

5.4.2 Increasing Returns to Scale

Increasing returns to scale (IRS) occurs when a firm increases its inputs, and a more than proportionate increase in production results. Mathematically, we can write, an industry has IRS if $f(\alpha K, \alpha L) > \alpha f(K, L)$ where constant $\alpha \geq 0$. For example, in a year an industry employs 10,000 workers, uses 1,000 machines, and produces 10 million products. In the next year, it employs 20,000 workers, uses 2,000 machines (inputs doubled), and produces 25 million products (output more than doubled) [Mohajan, 2018b; Roy et al., 2021].

5.4.3 Decreasing Returns to Scale

Decreasing returns to scale (DRS) happens when the firm's output rises proportionately less than its inputs rise. Mathematically, we can write, a firm has DRS if $f(\alpha K, \alpha L) < \alpha f(K, L)$ where constant $\alpha \geq 0$. For example, in year one, an industry employs 20,000 workers, uses 1,000 machines, and produces 20 million products. In the next year, it employs 40,000 workers, uses 2,000 machines (inputs doubled), and produces 15 products million (output less than doubled) [Mohajan, 2018b; Roy et al., 2021].

5.4.4 Elasticity of Substitution

Elasticity of substitution is the elasticity of the ratio of two inputs to a production function with respect to the ratio of their marginal products [Hicks, 1932]. In a competitive market, it measures the percentage change in the two inputs used in response to a percentage change in their prices [Mas-Colell et al., 1995]. The general definition of the elasticity of *X* with respect to *Y* is [Hicks, 1932],

$$E_Y^X = \frac{\%\text{ change in } X}{\%\text{ change in } Y}. \tag{7}$$

The output elasticity of capital *K*, is measured by,

$$E_K = \frac{\%\,\Delta Q}{\%\,\Delta K}. \tag{8}$$

The output elasticity of labor *L*, is measured by,

$$E_L = \frac{\%\,\Delta Q}{\%\,\Delta L}. \tag{9}$$

The output elasticity of other inputs *R*, is measured by,

$$E_R = \frac{\%\,\Delta Q}{\%\,\Delta R}. \tag{10}$$

For infinitesimal changes and differentiable variables (7) becomes,

$$E_Y^X = \frac{dX/dY}{X/Y} = \frac{dX}{dY}.\frac{Y}{X}. \tag{11}$$

For a Cobb-Douglass production function with two inputs *K* and *L*, elasticity of substitution can be written as [Mas-Colell et al., 1995],

$$\sigma = \frac{d\ln(K/L)}{d\ln(MP_L/MP_K)} = \frac{d\ln(K/L)}{d\ln(MRTS)} = 1. \tag{12}$$

The distribution of national income between capital and labor determine the elasticity of substitution. If $\sigma = 1$, any change in *K*/*L* is matched by a proportional change in *w*/*r* and the relative income shares of capital and labor stay constant, where *w* is wage rate and *r* is rental rate of capital [Miller, 2008]. For a production function that has more than two inputs, Hicks elasticity of substitution measure is described as [Onalan & Basegmez, 2018],

$$\sigma_{ij} = -\frac{\partial\ln(X_i/X_j)}{\partial\ln\left(\frac{\partial Q/\partial X_i}{\partial Q/\partial X_j}\right)}. \tag{13}$$

For Cobb-Douglas production function, $\sigma_{ij} = 1$.

5.5 Shadow Price

The shadow price of a commodity is defined as its social opportunity cost, i.e., the net loss (gain) associated with having 1 unit less (more) of it. For example, if $\frac{\partial C}{\partial Q} = \lambda$, then if the firm wants to increase (decrease) 1 unit of its production, it would cause total cost to increase (decrease) by approximately λ units [Mohajan, 2018b].

6. Mathematical Representation of Cobb-Douglass Production Function

We consider that for the fixed price, an industry of Bangladesh wants to produce and deliver quantity *Q* units of a modern dress during a specified time, with the use of *K* quantity of capital, which is represent by the total investment in fixed assets, such as the monetary worth of all machinery, equipment and buildings, *L* quantity of labor, i.e., the total number of person-hours worked in a year, and *R* quantity of other inputs, such as technology, agricultural activities, energy, raw materials, etc. If the industry follows the least cost combination of three factors *K*, *L*, and *R* to produce *Q* quantity of products; to reach its target the industry must minimize its cost function [Moolio & Islam, 2008; Mohajan et al., 2013; Roy et al., 2021];

$$C\left(K, L, R\right) = rK + wL + \rho R, \tag{14}$$

subject to the constraint of production function;

$$Q = f\left(K, L, R\right), \tag{15}$$

where $r = \frac{\partial Q}{\partial K}$ is rate of interest or services of capital per unit of capital *K* that represents the marginal product of capital; $w = \frac{\partial Q}{\partial L}$ is the wage rate per unit of labor *L* that represents the marginal product of labor; and $\rho = \frac{\partial Q}{\partial R}$ is the cost per unit of other inputs *R* that represents the marginal product of other inputs; while *f* is a suitable production function. We assume that second order partial derivatives of the function *f* with respect to the independent variables (factors) *K*, *L*, and *R* exist. Now we apply Lagrange multiplier λ in (14) and (15) with the Lagrangian function *U*, in a four dimensional unconstrained problem as follows [Mohajan, 2017a; Moolio & Islam, 2008; Roy et al., 2021]:

$$U(K, L, R, \lambda) = C(K, L, R) + \lambda\left(Q - f(K, L, R)\right). \tag{16}$$

Here *K*, *L*, *R* are referred to as endogenous (dependent) variables, and *C*, *Q*, and *f* are referred to as exogenous (independent) variables. We assume that the industry minimizes its cost, the optimal quantities K^*, L^*, R^*, λ^* of *K*, *L*, *R*, and λ that necessarily satisfy the first order conditions; which we obtained by partially differentiation of the Lagrangian function (16) with respect to four variables *K*, *L*, *R*, and λ; and setting them equal to zero [Baxley & Moorhouse, 1984],

$$U_\lambda = Q - f(K, L, R) = 0, \tag{17a}$$

$$U_K = C_K - \lambda f_K = 0, \tag{17b}$$

$$U_L = C_L - \lambda f_L = 0, \tag{17c}$$

$$U_R = C_R - \lambda f_R = 0, \tag{17d}$$

where $U_K = \frac{\partial U}{\partial K}$, etc. are partial derivatives.

From (17b) we get,

$$\lambda = \frac{C_K}{f_K}. \tag{18a}$$

From (17c) we get,

$$\lambda = \frac{C_L}{f_L}. \tag{18b}$$

From (17d) we get,

$$\lambda = \frac{C_R}{f_R}. \tag{18c}$$

Combining (18a-c) we get,

$$\lambda = \frac{C_K}{f_K} = \frac{C_L}{f_L} = \frac{C_R}{f_R}. \tag{19}$$

From (14) we observe that cost C is a function of K, L, and R; and also from (15) we observe that quantity of product Q is a function of K, L, and R. Hence using the properties of multivariate calculus we can write for infinitesimal changes of dQ and dC as,

$$dC = C_K dK + C_L dL + C_R dR, \tag{20}$$

$$dQ = f_K dK + f_L dL + f_R dR. \tag{21}$$

Dividing (20) by (21) we get,

$$\frac{dC}{dQ} = \frac{C_K dK + C_L dL + C_R dR}{f_K dK + f_L dL + f_R dR}. \tag{22}$$

If L and R remain constants, K varies then $dL = 0$ and $dR = 0$, hence (22) becomes,

$$\frac{dC}{dQ} = \frac{C_K dK}{f_K dK} = \frac{C_K}{f_K} = \lambda. \tag{23a}$$

If K and R remain constants, L varies then $dK = 0$ and $dR = 0$, hence (22) becomes,

$$\frac{dC}{dQ} = \frac{C_L dL}{f_L dL} = \frac{C_L}{f_L} = \lambda. \tag{23b}$$

If K and L remain constants, R varies then $dK = 0$ and $dL = 0$, hence (22) becomes,

$$\frac{dC}{dQ} = \frac{C_R dR}{f_R dR} = \frac{C_R}{f_R} = \lambda. \tag{23c}$$

Combining (13a-c) we get,

$$\frac{dC}{dQ} = \frac{C_K}{f_K} = \frac{C_L}{f_L} = \frac{C_R}{f_R} = \lambda. \tag{24}$$

Equation (14) can be written as,

$$\frac{dC}{dQ} = \lambda. \tag{25}$$

Hence, the Lagrange multiplier can be interpreted as the marginal cost of production. It indicates that total cost will be increased from the production of an additional unit Q [Mohajan et al., 2013].

6.1 An Economic Analysis of Cobb-Douglas Production Function

Let us consider the Cobb-Douglas production function f is given by [Humphery, 1997],

$$Q = f\left(K, L, R\right) = AK^{a}L^{b}R^{c}, \tag{26}$$

where A is the efficiency parameter reflecting the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover A also reflects the skill and education level of the workforce. Here a, b, and c are constants; a indicates the output of elasticity of capital measures the percentage change in Q for 1% change in K while L and R are held constants; b indicates the output of elasticity of labor, and c indicates the output of elasticity of other inputs in the production process, are exactly parallel to a. The values of a, b, and c are constants determined by available technologies. Now these three constants a, b, and c must satisfy the following three inequalities [Mohajan et al., 2013; Roy et al., 2021]:

$$0 < a < 1, 0 < b < 1, \text{ and } 0 < c < 1. \tag{27}$$

6.2 Elasticity Coefficients

In the Cobb-Douglass production function; a, b, and c are defined as follows:

Output elasticity coefficient of capital,

$$a = \frac{\partial Q / Q}{\partial K / K}. \tag{28}$$

Output elasticity coefficient of labor,

$$b = \frac{\partial Q / Q}{\partial L / L}. \tag{29}$$

Output elasticity coefficient of other inputs,

$$c = \frac{\partial Q / Q}{\partial R / R}. \tag{30}$$

6.3 Marginal Production and Diminishing Returns

The marginal productivity of factors can be calculated as follows:

Marginal productivity of capital (MPC),

$$MP_K = \left(\frac{\partial Q}{\partial K}\right)_{L,R=\text{ constant}} = aAK^{a-1}L^bR^c = a\left(\frac{Q}{K}\right) > 0. \quad (31)$$

Diminishing returns to capital,

$$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 Q}{\partial K^2} = a(a-1)AK^{a-2}L^bR^c = a(a-1)\left(\frac{Q}{K^2}\right) < 0. \quad (32)$$

The second order derivative is negative, output should increase but at a diminishing rate, where *R* and *L* are constants in this case. An increase in capital raises the marginal product of labor,

$$\frac{\partial MP_K}{\partial L} = abAK^{a-1}L^{b-1}R^c > 0. \quad (33)$$

An increase in capital raises the marginal product of other inputs,

$$\frac{\partial MP_K}{\partial R} = acAK^{a-1}L^bR^{c-1} > 0. \quad (34)$$

Marginal productivity of labor (MPL),

$$MP_L = \left(\frac{\partial Q}{\partial L}\right)_{K,R=\text{ constant}} = bAK^aL^{b-1}R^c = b\left(\frac{Q}{L}\right) > 0 . \quad (35)$$

Diminishing returns to labor,

$$\frac{\partial MP_L}{\partial L} = b(b-1)AK^aL^{b-2}R^c = b(b-1)\left(\frac{Q}{L^2}\right) < 0. \quad (36)$$

The second order derivative is negative, output should increase but at a diminishing rate, where *R* and *K* are constants in this case. An increase in labor raises the necessity of capital,

$$\frac{\partial MP_L}{\partial K} = abAK^{a-1}L^{b-1}R^c > 0. \quad (37)$$

An increase in labor raises the necessity of other inputs,

$$\frac{\partial MP_L}{\partial R} = bcAK^{a-1}L^bR^{c-1} > 0. \quad (38)$$

Marginal productivity of other inputs (MPI),

$$MP_R = \left(\frac{\partial Q}{\partial R}\right)_{L,K=\text{ constant}} = cAK^aL^bR^{c-1} = c\left(\frac{Q}{R}\right) > 0. \quad (39)$$

Diminishing returns to other inputs,

$$\frac{\partial MP_R}{\partial R} = c(c-1)AK^a L^b R^{c-2} = c(c-1)\left(\frac{Q}{R^2}\right) < 0. \tag{40}$$

The second order derivative is negative, output should increase but at a diminishing rate, where *L* and *K* are constants in this case. An increase in other inputs raises the necessity of capital,

$$\frac{\partial MP_R}{\partial K} = acAK^{a-1} L^b R^{c-1} > 0. \tag{41}$$

An increase in other inputs raises the necessity of labor,

$$\frac{\partial MP_R}{\partial L} = bcAK^a L^{b-1} R^{c-1} > 0. \tag{42}$$

6.4 Linearity of Cobb-Douglass Production Function

Relation (26) is a nonlinear equation among production *Q*, capital *K*, labor *L*, and other inputs *R*. To obtain accurate results by displaying graph or arithmetic calculation of nonlinear equation sometimes becomes complicated. To simplify the determination of parameters of the production function for the time of calculation it is reduced to a linear form by operating log in (26) we get,

$$\log Q = \log A + \log K^a + \log L^b + \log R^c$$

$$\log Q = \log A + a \log K + b \log L + c \log R. \tag{43}$$

Now equation (43) represents a linear regression model and in this case drawing of graph for production function or arithmetic calculations will be very easy.

6.5 Returns to Scales of Cobb-Douglass Production Function

We consider an economy in an initial state has initial capital K_0, labor L_0, and other inputs R_0; and the initial production of the industry is Q_0, then (26) becomes [Cottrell, 2019],

$$Q_0 = AK_0^a L_0^b R_0^c. \tag{44}$$

Now suppose we scale the inputs by some common factor α, then let capital $K_1 = \alpha K_0$, labor $L_1 = \alpha L_0$, and other inputs $R_1 = \alpha R_0$, then (26) becomes,

$$\begin{aligned} Q_1 &= AK_1^a L_1^b R_1^c \\ &= A(\alpha K_0)^a (\alpha L_0)^b (\alpha R_0)^c \\ &= \alpha^a \alpha^b \alpha^c AK_0^a L_0^b R_0^c \\ &= \alpha^{a+b+c} Q_0, \quad \text{by (44).} \end{aligned} \tag{45}$$

A strict Cobb-Douglass production function, in which $a+b+c=1$, indicates constant returns to scale, $a+b+c>1$ indicates increasing returns to scale, and $a+b+c<1$ indicates decreasing returns to scale [Besanko & Braeutigam, 2010].

7. Cost Minimization Analysis by Cobb-Douglass Production Function

A Cobb-Douglass production function is optimized subject to a budget constraint [Mohajan, 2018b]. Now using (14), (15), and (26) in (16) we get [Mohajan et al., 2013; Moolio & Islam, 2008; Roy et al., 2021],

$$U(K, L, R, \lambda) = rK + wL + \rho R + \lambda\left(Q - AK^a L^b R^c\right). \tag{46}$$

Taking the partial differentiations in (46), for minimization we get;

$$U_\lambda = Q - AK^a L^b R^c = 0, \tag{47a}$$

$$U_K = r - a\lambda AK^{a-1} L^b R^c = 0, \tag{47b}$$

$$U_L = w - b\lambda AK^a L^{b-1} R^c = 0, \tag{47c}$$

$$U_R = \rho - c\lambda AK^a L^b R^{c-1} = 0. \tag{47d}$$

From (47a) we get,

$$K^a L^b R^c = \frac{Q}{A}. \tag{48a}$$

From (47b) we get,

$$\lambda = \frac{r}{aAK^{a-1} L^b R^c} = \frac{rK}{aAK^a L^b R^c}. \tag{48b}$$

From (47c) we get,

$$\lambda = \frac{w}{bAK^{a}L^{b-1}R^{c}} = \frac{wL}{bAK^{a}L^{b}R^{c}}. \tag{48c}$$

From (47d) we get,

$$\lambda = \frac{\rho}{cAK^{a}L^{b}R^{c-1}} = \frac{\rho R}{cAK^{a}L^{b}R^{c}}. \tag{48d}$$

Combining (48b-d) we get,

$$\lambda = \frac{rK}{aAK^{a}L^{b}R^{c}} = \frac{wL}{bAK^{a}L^{b}R^{c}} = \frac{\rho R}{cAK^{a}L^{b}R^{c}}. \tag{49}$$

$$\Rightarrow \frac{rK}{a} = \frac{wL}{b} = \frac{\rho R}{c}. \tag{50}$$

From (47a) we get,

$$K^{a} = \frac{Q}{AL^{b}R^{c}}$$

$$\Rightarrow K = \frac{Q^{1/a}}{A^{1/a}L^{b/a}R^{c/a}}. \tag{51a}$$

Similarly from (47a) we get,

$$L = \frac{Q^{1/b}}{A^{1/b}K^{a/b}R^{c/b}}. \tag{51b}$$

$$R = \frac{Q^{1/c}}{A^{1/c}K^{a/c}L^{b/c}}. \tag{51c}$$

From (47a) we get,

$$Q = AK^{a}L^{b}R^{c}. \tag{52a}$$

From (47b) we get,

$$r = a\lambda AK^{a-1}L^{b}R^{c}. \tag{52b}$$

From (47c) we get,

$$w = b\lambda AK^{a}L^{b-1}R^{c}. \tag{52c}$$

From (47d) we get,

$$\rho = c\lambda AK^{a}L^{b}R^{c-1}. \tag{52d}$$

Dividing (52b) by (52c) we get,

$$\frac{r}{w}=\frac{a\lambda AK^{a-1}L^{b}R^{c}}{b\lambda AK^{a}L^{b-1}R^{c}}=\frac{aL}{bK}$$

$$\Rightarrow w=\frac{rbK}{aL}. \tag{53a}$$

From (51a) we get,

$$R^{c}=\frac{Q}{AK^{a}L^{b}}.$$

(53b)

$$\Rightarrow R=\left(\frac{Q}{AK^{a}L^{b}}\right)^{1/c}=\frac{Q^{1/c}}{A^{1/c}K^{a/c}L^{b/c}}. \tag{53c}$$

From (51c) we get,

$$R^{c}=\frac{w}{b\lambda AK^{a}L^{b-1}}. \tag{53d}$$

From (51d) we get,

$$\rho=c\lambda AK^{a}L^{b}R^{c-1}. \tag{54a}$$

$$\Rightarrow \rho=c\lambda AK^{a}L^{b}\frac{R^{c}}{R}. \tag{54b}$$

Using (53c) and (53d) in (54b) we get,

$$\Rightarrow \rho=c\lambda AK^{a}L^{b}.\frac{w}{b\lambda AK^{a}L^{b-1}}.\frac{A^{1/c}K^{a/c}L^{b/c}}{Q^{1/c}}. \tag{54c}$$

Using (53a) in (54c) we get,

$$\Rightarrow \rho=c\lambda AK^{a}L^{b}.\frac{1}{b\lambda AK^{a}L^{b-1}}.\frac{rbK}{aL}.\frac{A^{1/c}K^{a/c}L^{b/c}}{Q^{1/c}}$$

$$\Rightarrow \rho=\frac{crK^{\frac{a+c}{c}}L^{b/c}A^{1/c}}{aQ^{1/c}}$$

$$\Rightarrow K^{\left(\frac{a+c}{c}\right)}=\frac{a\,\rho\,Q^{1/c}}{c\,rA^{1/c}L^{b/c}}$$

$$\Rightarrow K = \left[\frac{a \, \rho \, Q^{1/c}}{c \, r \, A^{1/c} L^{b/c}} \right]^{\left(\frac{c}{a+c}\right)}$$

$$\Rightarrow K = \frac{a^{\left(\frac{c}{a+c}\right)} \rho^{\left(\frac{c}{a+c}\right)} Q^{\left(\frac{1}{a+c}\right)}}{c^{\left(\frac{c}{a+c}\right)} r^{\left(\frac{c}{a+c}\right)} A^{\left(\frac{1}{a+c}\right)} L^{\left(\frac{b}{a+c}\right)}}. \tag{55a}$$

Similarly we get,

$$L = \frac{b^{\left(\frac{c}{b+c}\right)} \rho^{\left(\frac{c}{b+c}\right)} Q^{\left(\frac{1}{b+c}\right)}}{c^{\left(\frac{c}{b+c}\right)} w^{\left(\frac{c}{b+c}\right)} A^{\left(\frac{1}{b+c}\right)} K^{\left(\frac{a}{b+c}\right)}}. \tag{55b}$$

Now using the values of K and L from equation (54a) and (55b) respectively in (53a), we get,

$$K = \frac{awL}{br} = \frac{awb^{\left(\frac{c}{b+c}\right)} \rho^{\left(\frac{c}{b+c}\right)} Q^{\left(\frac{1}{b+c}\right)}}{brc^{\left(\frac{c}{b+c}\right)} w^{\left(\frac{c}{b+c}\right)} A^{\left(\frac{1}{b+c}\right)} K^{\left(\frac{a}{b+c}\right)}}$$

$$\Rightarrow K^{\left(\frac{a+b+c}{b+c}\right)} = \frac{awb^{\left(\frac{c}{b+c}\right)} \rho^{\left(\frac{c}{b+c}\right)} Q^{\left(\frac{1}{b+c}\right)}}{brc^{\left(\frac{c}{b+c}\right)} w^{\left(\frac{c}{b+c}\right)} A^{\left(\frac{1}{b+c}\right)}}$$

$$\Rightarrow K = \left[\frac{awb^{\left(\frac{c}{b+c}\right)} \rho^{\left(\frac{c}{b+c}\right)} Q^{\left(\frac{1}{b+c}\right)}}{brc^{\left(\frac{c}{b+c}\right)} w^{\left(\frac{c}{b+c}\right)} A^{\left(\frac{1}{b+c}\right)}} \right]^{\left(\frac{b+c}{a+b+c}\right)}$$

$$= \frac{a^{\left(\frac{b+c}{a+b+c}\right)} w^{\left(\frac{b}{a+b+c}\right)} \rho^{\left(\frac{c}{a+b+c}\right)} Q^{\left(\frac{1}{a+b+c}\right)}}{b^{\left(\frac{b}{a+b+c}\right)} c^{\left(\frac{c}{a+b+c}\right)} r^{\left(\frac{b+c}{a+b+c}\right)} A^{\left(\frac{1}{a+b+c}\right)}}$$

$$K = K^{*} = \frac{a^{\left(\frac{b+c}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} r^{\left(\frac{b+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}. \tag{56a}$$

where $\Omega = a + b + c$.

Similarly we get,

$$L = L^{*} = \frac{b^{\left(\frac{a+c}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} w^{\left(\frac{a+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}, \text{ and} \tag{56b}$$

$$R = R^* = \frac{c^{\left(\frac{a+b}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{v}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{a+b}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}. \tag{56c}$$

Using (56a, b) in (47b) we get the optimal value of Lagrange multiplier λ^* as;

$$\lambda = \frac{rK}{aAK^a L^b R^c}$$

$$= \frac{r\dfrac{a^{\left(\frac{b+c}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} r^{\left(\frac{b+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}}{aA\left[\dfrac{a^{\left(\frac{b+c}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} r^{\left(\frac{b+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}\right]^a \left[\dfrac{b^{\left(\frac{a+c}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} w^{\left(\frac{a+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}\right]^b \left[\dfrac{c^{\left(\frac{a+b}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{a+b}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}\right]^c}.$$

After some mathematical manipulation we yield,

$$\lambda = \lambda^* = \frac{r^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1-\Omega}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}. \tag{56d}$$

Now using the values of K^*, L^* and R^* in (27) we can write,

$$C = r\frac{a^{\left(\frac{b+c}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} r^{\left(\frac{b+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}} + w\frac{b^{\left(\frac{a+c}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} w^{\left(\frac{a+c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}} + \rho\frac{c^{\left(\frac{a+b}{\Omega}\right)} r^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{a+b}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}$$

$$= \frac{ar^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)} + br^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)} + cr^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}$$

$$C^* = \frac{\Omega r^{\left(\frac{a}{\Omega}\right)} w^{\left(\frac{b}{\Omega}\right)} \rho^{\left(\frac{c}{\Omega}\right)} Q^{\left(\frac{1}{\Omega}\right)}}{a^{\left(\frac{a}{\Omega}\right)} b^{\left(\frac{b}{\Omega}\right)} c^{\left(\frac{c}{\Omega}\right)} A^{\left(\frac{1}{\Omega}\right)}}. \tag{57}$$

Equation (57) is the optimal cost in terms of *r*, *w*, *A*, *a*, *b*, *c*, *Q*, *ρ*, and $\Omega = a + b + c$. Now putting the known values of in right side of (57) we can easily calculate the value of minimum cost *C*.

8. Analysis of Lagrange Multiplier

From (14) and (47b-d) we get [Mohajan, 2017a; Roy et al., 2021],

$$\frac{\partial C}{\partial Q} = C_K \frac{\partial K}{\partial Q} + C_L \frac{\partial L}{\partial Q} + C_R \frac{\partial R}{\partial Q}$$

$$= r\frac{\partial K}{\partial Q} + w\frac{\partial L}{\partial Q} + \rho\frac{\partial R}{\partial Q}. \quad (58)$$

From (47b-d) we get,

$$r = a\lambda AK^{a-1}L^b R^c, \quad (59a)$$

$$w = b\lambda AK^a L^{b-1} R^c, \quad (59b)$$

$$\rho = c\lambda AK^a L^b R^{c-1}. \quad (59c)$$

Using (59a-c) in (58) we get,

$$\frac{\partial C}{\partial Q} = \lambda \left[aAK^{a-1}L^b R^c \frac{\partial K}{\partial Q} + \beta AK^a L^{b-1} R^c \frac{\partial L}{\partial Q} + \lambda AK^a L^b R^{c-1} \frac{\partial R}{\partial Q} \right]. \quad (60)$$

Differentiating (47a) with respect to *Q* we get,

$$1 = aAK^{a-1}L^b R^c \frac{\partial K}{\partial Q} + bAK^a L^{b-1} R^c \frac{\partial L}{\partial Q} + cAK^a L^b R^{c-1} \frac{\partial R}{\partial Q}. \quad (61)$$

From (60) and (61) we get,

$$\frac{\partial C}{\partial Q} = \lambda,$$

$$\Rightarrow \frac{\partial C^*}{\partial Q} = \lambda^*. \quad (62)$$

We have observed that (62) verifies (25). So that, Lagrange multiplier λ^* indicates that if the industry wants to increase (decrease) one unit of its production, it would cause total cost to increase (decrease) by approximately λ^* units, i.e., the Lagrange multiplier is a shadow price [Mohajan, 2017a, 2018b].

9. Special Cases of Returns Scale

In this section we will discuss returns scale by choosing various elasticity of coefficient. We will try to obtain a suitable technique of production to minimize production cost of a garments industry. In the study we will analyze three returns scales and try to provide a suitable tool for the sustainability of the industry. In this section all the data are provided on the basis of garments industry of Bangladesh. We hope, the garments sector of Bangladesh will be benefited from our works.

9.1 Case I: Constant Returns Scale

We consider the constant returns scale such that, $a = b = c = \dfrac{1}{3}$; so, $\Omega = 3a = 1$, then from (57) we get the minimum cost as;

$$C^* = \frac{\frac{1}{3} r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q + \frac{1}{3} r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q + \frac{1}{3} r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q}{\left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} \left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} \left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} A}$$

$$= \frac{r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q}{\frac{1}{3} A}$$

$$= \frac{3r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q}{A}. \tag{63}$$

For $a = b = c = \frac{1}{3}$, and $\Omega = 3a = 1$, then from (56d) we get the Lagrange multiplier as;

$$\lambda^* = \frac{r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q^{\left(\frac{2}{3}\right)}}{\left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} \left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} \left(\frac{1}{3}\right)^{\left(\frac{1}{3}\right)} A}$$

$$\lambda^* = \frac{3r^{\left(\frac{1}{3}\right)} w^{\left(\frac{1}{3}\right)} \rho^{\left(\frac{1}{3}\right)} Q^{\left(\frac{2}{3}\right)}}{A}. \tag{64}$$

9.2 Case II: Increasing Returns Scale

We consider the increasing returns scale such that, $a = b = \frac{1}{2}, c = \frac{3}{4}$; so, $\Omega = \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \frac{7}{4}$, then from (57) we get the minimum cost as;

$$C^* = \frac{\frac{7}{4} r^{\left(\frac{2}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{3}{7}\right)} Q^{\left(\frac{4}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{2}{7}\right)} \left(\frac{1}{2}\right)^{\left(\frac{2}{7}\right)} \left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)} A^{\left(\frac{4}{7}\right)}}$$

$$C^* = \frac{\frac{7}{4} r^{\left(\frac{2}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{3}{7}\right)} Q^{\left(\frac{4}{7}\right)}}{\left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)} A^{\left(\frac{4}{7}\right)}}. \tag{65}$$

We consider the increasing returns scale such that, $a = b = \frac{1}{2}, c = \frac{3}{4}$; so,

$\Omega = \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \frac{7}{4}$, then from (56d) we get the Lagrange multiplier as;

$$\lambda^* = \frac{r^{\left(\frac{2}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{3}{7}\right)} Q^{\left(\frac{-3}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{2}{7}\right)} \left(\frac{1}{2}\right)^{\left(\frac{2}{7}\right)} \left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)} A^{\left(\frac{4}{7}\right)}}$$

$$\lambda^* = \frac{r^{\left(\frac{2}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{3}{7}\right)}}{\left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)} Q^{\left(\frac{3}{7}\right)} A^{\left(\frac{4}{7}\right)}} . \tag{66}$$

9.3 Case II: Decreasing Returns Scale

We consider the decreasing returns scale such that, $a = \frac{1}{2}, b = \frac{1}{4}, c = \frac{1}{8}$; so,

$\Omega = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$, then from (57) we get the minimum cost as;

$$C^* = \frac{\frac{7}{8} r^{\left(\frac{4}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{1}{7}\right)} Q^{\left(\frac{8}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{4}{7}\right)} \left(\frac{1}{4}\right)^{\left(\frac{2}{7}\right)} \left(\frac{1}{8}\right)^{\left(\frac{8}{7}\right)} A^{\left(\frac{8}{7}\right)}} . \tag{67}$$

We consider the decreasing returns scale such that, $a = \frac{1}{2}, b = \frac{1}{4}, c = \frac{1}{8}$ so,

$\Omega = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$, then from (56d) we get the Lagrange multiplier as;

$$\lambda^{*} = \frac{r^{\left(\frac{4}{7}\right)} w^{\left(\frac{2}{7}\right)} \rho^{\left(\frac{1}{7}\right)} Q^{\left(\frac{1}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{4}{7}\right)} \left(\frac{1}{4}\right)^{\left(\frac{2}{7}\right)} \left(\frac{1}{8}\right)^{\left(\frac{1}{7}\right)} A^{\left(\frac{8}{7}\right)}}. \tag{68}$$

10. Statistical Analysis of Returns Scale

In this section we present statistical analyses on the results to show which returns scale will give more satisfactory result for the benefit of the industry, as well as, that of Bangladesh. The country is situated in South Asia. It is a developing and densely populated country. In this country, the availability of labor for the industry is satisfactory and the industry can manage labors with lower wages. In all three cases, we have used the values of parameters from three industries of Bangladesh. The industry authorities provide their crude data in the condition that we cannot use their industries' name in the article. So that we have used the disguise name of three industries as *X*, *Y*, and *Z*. We have collected data of year 2020. Due to COVID-19 pandemic, the garments industries of the country remain closed for four months.

10.1 Case I: Constant Returns Scale

In this section we have used the data provided by the industry *X*. This industry tries to follow constant returns scale. Here the industry provides us its data of the year 2020. In this industry, $Q = 10{,}000{,}000$ units, $A = \frac{1}{2} = 0.5$, $r = \$0.02$, $w = \$5$, and $\rho = \$2$, then from (63) we get the minimum cost as;

$$C^{*} = 2 \times 3 \times (0.02)^{\frac{1}{3}} (5)^{\frac{1}{3}} (2)^{\frac{1}{3}} \times 10{,}000{,}000 = \$35{,}088{,}213. \tag{69}$$

Now using same data, $Q = 10{,}000{,}000$ units, $A = \frac{1}{2} = 0.5$, $r = \$0.02$, $w = \$5$, $\rho = \$2$ in (64) we get the Lagrange multiplier of this industry as;

$$\lambda^{*} = 6 \times (0.02)^{\frac{1}{3}} (5)^{\frac{1}{3}} (2)^{\frac{1}{3}} \times (10{,}000{,}000)^{\frac{2}{3}} = \$162{,}865. \tag{70}$$

10.2 Case II: Increasing Returns Scale

In this section we have used the data provided by the industry *Y*. This industry tries to follow increasing returns scale. Here this industry also provides us its data of the year 2020.

In this industry, $Q = 20{,}000{,}000$ units, $A = 0.75$, $r = \$0.05$, $w = \$7$, $\rho = \$1.5$, then then from (65) we get the minimum cost as;

$$C^* = \frac{\frac{7}{4}(0.05)^{\left(\frac{2}{7}\right)}(7)^{\left(\frac{2}{7}\right)}(1.5)^{\left(\frac{3}{7}\right)}(20{,}000{,}000)^{\left(\frac{4}{7}\right)}}{\left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)}(0.75)^{\left(\frac{4}{7}\right)}} = \$25{,}688. \tag{71}$$

Again using same data, $Q = 20{,}000{,}000$ units, $A = 0.75$, $r = \$0.05$, $w = \$7$, and $\rho = \$1.5$, in (66) we get the Lagrange multiplier of this industry as;

$$\lambda^* = \frac{(0.05)^{\left(\frac{2}{7}\right)}(7)^{\left(\frac{2}{7}\right)}(1.5)^{\left(\frac{3}{7}\right)}}{\left(\frac{3}{4}\right)^{\left(\frac{3}{7}\right)}(20{,}000{,}000)^{\left(\frac{3}{7}\right)}(0.75)^{\left(\frac{4}{7}\right)}} = \$8.73228\times10^{-4}. \tag{72}$$

10.3 Case II: Decreasing Returns Scale

In this section we have used the data provided by the industry *Z*. This industry tries to follow decreasing returns scale due to run short of capital, labor, and other inputs, and also for dislocation. We have collected data of this industry of the year 2020. In this industry, $Q = 1{,}000{,}000$ units, $A = 0.55$, $r = \$0.07$, $w = \$6$, $\rho = \$3$, then then from (67) we get the minimum cost as;

$$C^* = \frac{\frac{7}{8}(0.07)^{\left(\frac{4}{7}\right)}(6)^{\left(\frac{2}{7}\right)}(3)^{\left(\frac{1}{7}\right)}(1{,}000{,}000)^{\left(\frac{8}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{4}{7}\right)}\left(\frac{1}{4}\right)^{\left(\frac{2}{7}\right)}\left(\frac{1}{8}\right)^{\left(\frac{8}{7}\right)}(0.55)^{\left(\frac{8}{7}\right)}} = \$63{,}948{,}913. \tag{73}$$

Again using same data, $Q = 1{,}000{,}000$ units, $A = 0.55$, $r = \$0.07$, $w = \$6$, $\rho = \$3$, then from (68) we get the Lagrange multiplier as;

$$\lambda^* = \frac{r^{\left(\frac{4}{7}\right)}w^{\left(\frac{2}{7}\right)}\rho^{\left(\frac{1}{7}\right)}Q^{\left(\frac{1}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{4}{7}\right)}\left(\frac{1}{4}\right)^{\left(\frac{2}{7}\right)}\left(\frac{1}{8}\right)^{\left(\frac{1}{7}\right)}A^{\left(\frac{8}{7}\right)}}$$

$$\lambda^* = \frac{(0.07)^{\left(\frac{4}{7}\right)}(6)^{\left(\frac{2}{7}\right)}(3)^{\left(\frac{1}{7}\right)}(1{,}000{,}000)^{\left(\frac{1}{7}\right)}}{\left(\frac{1}{2}\right)^{\left(\frac{4}{7}\right)}\left(\frac{1}{4}\right)^{\left(\frac{2}{7}\right)}\left(\frac{1}{8}\right)^{\left(\frac{1}{7}\right)}(0.55)^{\left(\frac{8}{7}\right)}} = \$27. \tag{74}$$

11. Results and Discussion

In the light of statistical analysis in section 10 we have found that minimum cost in constant returns scale in industry X is \$35,088,213, in increasing returns scale in industry Y is \$25,688, in decreasing returns scale in industry Z is \$63,948,913. In the statistical analysis we have obtained the minimum cost in increasing returns scale. Therefore, garments sector of Bangladesh will be very satisfactory in increasing returns scale for sustainable production. Our suggestion to this sector is that all the industry must run to increasing return to scale production to obtain maximum profit and sustainable business.

On the other hand, the Lagrange multiplier, i.e., shadow price in constant returns scale in industry X is \$62,865, in increasing returns scale in industry Y is

$\$8.73228 \times 10^{-4}$, and in decreasing returns scale in industry Z is \$27. There is a very few change of shadow price in increasing returns scale, in decreasing returns scale it is very small, and in constant returns scale it is very high. Hence, there is no risk in increasing returns scale, but other two have high risks. Our suggestion to the garments sector of Bangladesh is that it should run to the increasing returns scale production for the betterment of the industry and for the welfare of economy of Bangladesh.

12. Conclusion and Recommendation

It is clear that Cobb-Douglas production function plays an important role in economics. In this study we have discussed the production function and cost function of Cobb-Douglas model. The capital, labor, and raw materials are main elements to increase production of an industry. We observe that in an industry if production is increased; use of various inputs also increase. Consequently generates employment both in government and private sectors. In the study we have discussed the Cobb-Douglass production function with mathematical calculations and statistical analysis for the social welfare, and for national and global economic development. In the study we have tried to provide a reasonable interpretation of the Lagrange multiplier. We observe that the value of the Lagrange multiplier is positive, and our study it indicates shadow price. In the statistical analysis we confirm that the garments sector of Bangladesh has better future if it moves to increasing returns to scale production.

References

Arrow, K. J., Dasgupta, P., Goulder, L. H., Mumford, K. J., & Oleson, K. (2010). Sustainability and the Measurement of Wealth. *NIESR Discussion Paper No. 369*. National Institute of Economic and Social Research, Dean Trench Street, London.

Mohajan, H. K. (2021). Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299, doi: https://doi.org/10.26458/21212

Adams, J., Khan, H. T. A., Raeside, R., & White, D. (2007). *Research Methods for Graduate Business and Social Science Students*. Sage Publications Ltd., London.

Ahmad, A., & Khan, M. (2015). Estimating the Cobb-Douglas Production Function. *International Journal of Research in Business Studies and Management*, 2(5), 32-33.

Barros, A. R. (2017). Relative Declining in the 19th Century, Roots of Brazilian Relative Economic Backwardsness. ScienceDirect, Academic Press, Elsevier Inc.

Baxley, J. V., & Moorhouse, J. C. (1984). Lagrange Multiplier Problems in Economics. *The American Mathematical Monthly*, 91(7), 404-412.

Besanko, D. A., & Braeutigam, R. R. (2010). *Microeconomics* (4th Ed.). John Wiley & Sons.

Biddle, J. (2012). The Introduction of the Cobb-Douglas Regression. *Journal of Economic Perspective*, 26(2), 223-236.

Brown, M. (2017). Cobb-Douglas Functions. *The New Palgrave Dictionary of Economics*. Palgrave Macmillan, UK.

Cobb, C. W., & Douglass, P. H. (1928). A Theory of Production. *American Economics Review*, 18(1), 139-165.

Cottrell, A. (2019). *The Cobb-Douglas Production Function*. Economics, 207, Lal Bahadur Shastri Institute of Management, Economics 111001.

Felipe, J., & Adams, F. G. (2005). The Estimation of the Cobb-Douglas Function: A

Mohajan, H. K. (2021). Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299, doi: https://doi.org/10.26458/21212

Retrospective View. *Eastern Economic Journal*, 31(3), 427-444.

Gelles, G. M., & Mitchell, D. W. (1996). Returns to Scale and Economies of Scale: Further Observations. *Journal of Economic Education*, 27(3), 259-261.

Grinnell, R. Jr. (Eds**.**) (1993). *Social Work Research and Evaluation* (4th Ed.). Itasca, I L, F. E. Peacock.

Gordon, D., & Vaughan, R. (2011). The Historical Role of the Production Function in Economics and Business. *American Journal of Business Education*, 4(4), 25-30.

Gupta, U. K. (2016). An Analysis for the Cobb-Douglas Production Function in General Form. *International Journal of Applied Research*, 2(4), 96-99.

Hicks, J. R. (1932). *The Theory of Wages*. Macmillan & Co., London.

Humphery, T. M. (1997). Algebraic Production Functions and Their Uses before Cobb-Douglas. *Federal Reserve Bank of Richmond Economic Quarterly*, 83(1), 51-83.

Husain, S., & Islam, S. M. (2016). A Test for the Cobb-Douglas Production Function in Manufacturing Sector: The Case of Bangladesh. *International Journal of Business and Economics Research*, 5(5), 149-154.

Ioan, C. A., & Ioan, G. (2015). The Complete Theory of Cobb-Douglas Production Function. *Acta Universitatis Danubius. Œconomica*, 11(1), 74-114.

Khatun, T., & Afroze, S. (2016). Relationship between Real GDP and Labour & Capital by Applying the Cobb-Douglas Production Function: A Comparative Analysis among Selected

Mohajan, H. K. (2021). Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299, doi: https://doi.org/10.26458/21212

Asian Countries. *Journal of Business Studies*, XXXVII(1), 113-119.

Kothari, C. R. (2008). *Research Methodology: Methods and Techniques* (2nd Ed.). New Delhi: New Age International (P) Ltd.

Leedy, P. D. & Ormrod, J. (2001). *Practical Research: Planning and Design* (7th Ed.). Upper Saddle River, NJ: Merrill Prentice Hall. Thousand Oaks: Sage Publications.

Lindahl, E. (1933). The Concept of Income. In G. Bagge (Ed.), *Economic Essays in Honor of Gustav Cassel*, London: George Allen & Unwin.

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic Theory*. New York, NY: Oxford University Press.

Miller, E. (2008). An Assessment of CES and Cobb-Douglas Production Functions. Working Paper 2008-05, Congressional Budget Office.

Mishra, S. K. (2007). *A Brief History of Production Functions*. MPRA Paper No. 5254.

Mohajan, H. K. (2017a). Optimization Models in Mathematical Economics. *Journal of Scientific Achievements*, 2(5), 30-42.

Mohajan, H. K. (2017b). Two Criteria for Good Measurements in Research: Validity and Reliability. *Annals of Spiru Haret University Economic Series*, 17(3), 58-82.

Mohajan, H. K. (2018a). Qualitative Research Methodology in Social Sciences and Related Subjects. *Journal of Economic Development, Environment and People*, 7(1), 23-48.

Mohajan, H. K. (2021). Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299, doi: https://doi.org/10.26458/21212

Mohajan, H. K. (2018b). *Aspects of Mathematical Economics, Social Choice and Game Theory*. PhD Dissertation, Jamal Nazrul Islam Research Centre for Mathematical and Physical Sciences (JNIRCMPS), University of Chittagong, Chittagong, Bangladesh.

Mohajan, H. K. (2020). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People*, 9(4), 52-79.

Mohajan, H. K., Islam, J. N., & Moolio, P. (2013). *Optimization and Social Welfare in Economics*. Lambert Academic Publishing, Germany.

Moolio, P., & Islam, J. N. (2008). Cost Minimization of a Competitive Firm. *Indus Journal of Management and Social Sciences*, 2(2), 184-160.

OECD (2002). *Frascati Manual: Proposed Standard Practice for Surveys on Research and Experimental Development* (6th Ed.). OECD Publication Service, Paris Cedex, France.

Onalan, O., & Basegmez, H. (2018). Estimation of Economic Growth Using Grey Cobb-Douglas Production Function: An Application for US Economy. *Journal of Business, Economics and Finance*, 7(2), 78-190.

Roy, L., Molla, R., & Mohajan, H. K. (2021). Cost Minimization is Essential for the Sustainability of an Industry: A Mathematical Economic Model Approach. *Annals of Spiru Haret University Economic Series*, 21(1).

Rybak, A. (2019). Application of the Cobb-Douglas Production Function to Study the Results of the Production Process and Planning under Turbulent Environment Conditions. *Mineral Resources Management*, 35(3), 99-118.

Mohajan, H. K. (2021). Estimation of Cost Minimization of Garments Sector by Cobb-Douglas Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299, doi: https://doi.org/10.26458/21212

Sickles, R. C. & Zelenyuk, V. (2019). *Measurement of Productivity and Efficiency: Theory and Practice*. Cambridge University Press, Cambridge.

Wagle, T. P. (2016). Spatial Analysis of Cob-Douglas Production Function in Agriculture Sector of Nepal: An Empirical Analysis. *Journal of Advanced Academic Research*, 3(II), 101-114.

Wicksteed, P. H. (1894). *An Essay on the Co-ordination of the Laws of Distribution*. Macmillan & Co., London.