Rationality in a general model of choice

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Abstract

In this paper we consider choice correspondences which may be empty-valued. We study conditions under which such choice correspondences are rational, transitively rational, partially rational, partially almost transitive rational, partially almost quasi-transitive rational.

Note: This paper is a revised version of an earlier paper entitled "Choice Problems with a status-quo alternative" that is available as a SSRN Working Paper.

1. Introduction

In economics the conventional method adopted to model a decision making problem is to list the set of alternatives from which the decision maker makes his choice. The act of choice is represented by a function that associates to every menu of options one or more of its chosen alternatives. Such a function is usually referred to as a *choice correspondence*. In such a situation, rationalization of choice is often considered to be a significant issue. While in common parlance rationalization would mean a reasoned justification, in choice theory its meaning is more specific. A rational agent is one whose act of choice results from some kind of optimizing behavior.

Much of choice theory assumes that given a finite set of alternatives, any non-empty finite subset of it could serve as a menu of options for the decision maker. In a sense such a standpoint is at variance with the origins of choice theory. In classical consumer choice theory from where a lot of choice theory arose, it is normally assumed that the consumer chooses from well defined budget sets. The collection of budget sets is a strict subset of the collection of non-empty subsets of the commodity space. The domain of a demand function is assumed to consist of budget sets only. One of the more complete discussions on choices on general domains (which allow for some sets to be excluded from consideration) can be found in Suzumura (1983). However there are equally significant situations, where it makes sense to consider the collection of all non-empty subsets as the domain of decision making. We shall not enter into the debate concerning whether it is or it is not reasonable to allow choices to be made from all subsets of a given set of alternatives. Our approach to the issue will be somewhat different.

There are two ways in which we can describe the domain of a demand function. The first and more conventional method is to say that a demand function is defined on all budget sets. The alternative approach is to say that a demand function is defined on all non-empty subsets of the commodity space but disallows choices from (or assigns the empty set to) subsets which are not a budget set. In a similar vein there are two ways in which we can define a choice correspondence. In the first approach we are given a collection of subsets over which the choice correspondence is defined. In the second approach the choice correspondence which is defined over all non-empty subsets disallows the act of choice from some (if any) non-empty subsets. Whether the set of choices from a given subset of alternatives is empty or not, is a property of the choice correspondence under consideration. The latter approach is what Aizerman and Aleskerov (1995) adopts to define a choice correspondence although the more significant results therein are established under the additional assumption that the decision maker does make non-empty choices from every non-empty subset of alternatives.

In this paper we consider choice correspondences that select (a possibly empty) subset from each nonempty subset of alternatives. Choice theory often considers functions that select exactly one alternative from each non-empty subset of alternatives. Such functions which are special cases of choice correspondences are naturally known as choice functions. The general idea of a choice correspondence is one that models a first stage in a choice procedure to be followed later by a second selection process that is based on some tie-breaking rule. Choice correspondences allow for greater flexibility. A particular type of a choice correspondence that some refer to as a “resolute” choice function allows at most one alternative to be chosen from every pair of alternatives. Given that this paper's concerns are about conditions under which choice correspondences are rational, if we assume that the choice correspondence is a resolute choice function then we will eventually be in a situation where at most one alternative is chosen from every non-empty set of alternatives.

As in much of economic theory where non-market phenomena are analyzed by methods which are initially motivated by models concerned with the market, our paper will concern itself with choice situations that may be far removed from consumer choice theory although we have appealed to the latter in an earlier paragraph. Unlike consumer choice theory, sets comprising two elements will play a central role in this paper. Further unlike consumer choice theory and much else that it motivates, we shall only consider a universal set of alternatives that is finite. Thus our paper is rooted in the tradition of choice theory that formally began with the seminal paper by Arrow (1959).

A type of choice correspondence we consider here is assumed to satisfy a "base domain property". This property is very similar to the one by the same name introduced by Bossert, Sprumont and Suzumura (2006), where it was assumed that the domain of a choice correspondence includes all one and two element subsets of the universal set of alternatives. On the other hand what we mean by "base domain property" is that the choice correspondence succeeds in choosing from all singletons and pairs of alternatives. Subsequently we invoke a weaker version of the base domain property that requires that given any subset of alternatives, choice is possible from each pair comprising a chosen alternative and an alternative in the subset. In the latter case all pairs need not allow non-empty choices. We call this property "weak base domain property".

A question that we are concerned with in this paper is the following. Given a choice correspondence is there a reflexive binary relation such that a chosen alternative from a subset of alternatives is at least as good as all other alternatives from the subset? Choice correspondences for which such a binary relation exists are called "partially rational". If in addition the binary relation is complete (i.e. comparable over all pairs of alternatives) then we call such a choice correspondence "rational". If the choice correspondence is at most single-valued i.e. a choice function, then in the latter case the binary relation that rationalizes the choice correspondence is clearly a "tournament" as defined for instance in Moulin (1986) or more recently in Laslier (1997).

In this paper we introduce a status-quo alternative that is selected when the act of choice from a subset of alternatives fails. This status-quo alternative is denoted by the empty set. When presented with a subset of alternatives a decision maker may either choose one or more alternatives from within the subset or he may choose the status-quo alternative. Opting for the status-quo conveys that no alternative from the menu of alternatives under consideration was selected.

Two axioms that play a role in the more general context that we discuss here is the Chernoff axiom and Expansion. The Chernoff axiom says that if a chosen element from a set of alternatives is contained in a subset, then it is chosen from the subset as well. Expansion on the other hand says that if an alternative is chosen from two sets of alternatives then it is also chosen from their union. We are able to show that if a choice correspondence satisfies (weak) base domain property then satisfaction of Chernoff and Expansion is equivalent to it being (partially) rational. Further, if a choice function satisfies weak base domain property then satisfaction of Chernoff, Expansion and another property (Property A) is equivalent to it being partially almost transitive rational. A binary relation is said to be almost transitive if given three alternatives, if the first is at least as good as the second and the second is at least as good as the third, then it is not the case that the third is preferred to the first. Property A says that if an alternative is revealed preferred to a second and the second revealed preferred to a third then given a choice between the first and the third, the first is definitely chosen. In this context we also discuss a property called T-Congruence due Bossert, Sprumont and Suzumura (2006) and show that along with weak base domain property it implies (but is not necessarily implied by) partial almost transitive rationality. We also show that if the choice correspondence satisfies binary domain condition, then the satisfaction of Chernoff, Expansion and Property A is equivalent to transitive rationality. This follows as an immediate corollary of Proposition 2 in our paper.

Another possible relaxation of transitive binary relation that we discuss here is almost quasi-transitivity. A binary relation is said to be almost quasi-transitive if given three alternatives, if the first is preferred to the second and the second is preferred to the third then it should not be that the third is at least as good as the first. In fact almost quasi-transitivity is a generalization of quasi-transitivity. We show here that satisfaction of Chernoff, Expansion and another property (Property B) is equivalent to the choice function being partially almost quasi-transitive rational. Property B says that given three alternatives if in a pair-wise comparison between the first and second only the first is chosen and in a pair-wise comparison between the second and third only the second is chosen then the third is never revealed preferred to the first.

There are several questions that come to mind at this juncture. The first concerns whether there is any significant difference between our framework of choice and the framework of choice where some subsets are exogenously given to be inadmissible. It is true that the results in both frameworks appear to be similar. However, in the framework discussed in this paper whether a given subset of alternatives allows choice to be made from within it or not, is not exogenously given; it is endogenous to the choice correspondence under consideration. In our framework a subset of alternatives may disallow choice in one choice correspondence while allow it for another. This is not the case if the collection of subsets from which choice is permitted is exogenously given and invariant with respect to the choice correspondence. It is also important to bear in mind that an assumption such as weak base domain property is not easily expressible except in the kind of framework discussed in this paper.

The second question concerns the relevance of our general model of possibly empty choice sets in decision making problems. How does such a framework relate to real world decision making situations? In recent times behavioral economists have obtained evidence "from psychology, as well as casual observation and introspection" that "real-life behavior often depends on observable information, other than the set of feasible alternatives, which is irrelevant in the rational assessment of the alternatives but nonetheless affects behavior"(Salant and Rubinstein (2008)). Such additional information is referred to as a *frame* and to the dependence on the frame as a *framing effect*. Both Salant and Rubinstein (2008) and an earlier paper by Bernheim and Rangel (2007) consider a choice function which given a frame, associates to each feasible set exactly one alternative from the feasible set. A choice correspondence in the traditional sense results when one associates with each feasible set all those alternatives from the feasible set that are chosen with respect to <u>some</u> frame. What if it were the case that the frame based choice function (known in the literature as an extended choice function) instead of being single-valued was at most single-valued? Why is it imperative that for every frame the decision maker is able to make a choice from every set of feasible alternatives? Since the framing effect acts on the mind of the decision maker, it could at times compel the decision maker to opt for the status-quo rather than choose a feasible alternative. Received theory is remarkably silent on such a predicament that the decision maker may face. It is not as though the rationality of an extended choice function is not discussed. Salant and Rubinstein (2008) use the word "salient condition" to describe single-valued extended choice functions which are rational for each frame. What gets ignored in their research is that the extended choice function may more often than not be at most single valued rather than single-valued. The choice correspondences we study here may thus make a small contribution towards generalizing the concept of an extended choice function in order to make it appear more meaningful. Alternatively, this paper could be considered to be a possible extension of the received theory of choice functions on finite sets as summarized in Moulin (1984).

2. The Model

Let X be a non-empty finite set of alternatives and let P(X) denote the set of all non-empty subsets of X. Let 2^X denote the power set of X.
A *choice correspondence* is a function C: P(X) $\rightarrow 2^X$ such that (i) for all A$\in$P(X): C(A) $\subset$ A; (ii) for all x$\in$X: C({x}) = {x}.

A *choice function* is a choice correspondence which is at most single valued.
If C is a choice function then there exists a function c: P(X) $\rightarrow$ X$\cup${ϕ} such that (i) for all A$\in$P(X): (i) if C(A) $\neq \phi$, then C(A) = {c(A)}; (ii) if C(A) = ϕ, then c(A) = ϕ.

Given a choice correspondence C (on X) let *dom*(C) denote the set {A$\in$P(X)/ C(A) $\neq \phi$}, i.e. the set of all non-empty subsets of X for which C is non-empty valued.
Clearly for all x$\in$X: {x}$\in$*dom*(C).
Let R_C denote the direct revealed preference relation $\bigcup_{A\in dom(X)} C(A)\times A$, i.e. for all x,y$\in$X: x$R_C$y if and only if there exists A$\in$P(X) such that x$\in$C(A) and y$\in$A; let R_C^* = $\bigcup_{A\in dom(X)} C(A)\times(A\setminus C(A))$, i.e. for all x,y$\in$X: x R_C^* y if and only if there exists A$\in$P(X) such that x$\in$C(A) and y$\in$A\C(A); and R^C = $\bigcup_{\{x,y\}\in dom(X)} C(\{x,y\})\times\{x,y\}$, i.e. for all x,y$\in$X: x$R^C$y if and only if x$\in$C({x,y}).

A choice correspondence C is said to be **partially rational** if there exists a binary relation R on X satisfying the following:
(1) R is *reflexive*: For all x$\in$X, xRx;
(2) For all A$\in$*dom*(C) and x$\in$A: [x$\in$C(A)] if and only if [xRy for all y$\in$A].
In this case R is said to be a *partial rationalization* of C.

A partial rationalization R of C is said to be a *rationalization* of C if R is *complete*, i.e. for all $x,y \in X$ with $x \neq y$: either xRy or yRx.
If a choice correspondence C has a rationalization then we say that it is **rational**.

The original version of the following property is due to Bossert, Sprumont and Suzumura (2006).

Base Domain Property: A choice correspondence C is said to satisfy the Base Domain Property (BD) if for all $x,y \in X$: $\{x,y\} \in$ *dom*(C).

In other words, the base domain property requires that the decision maker is able to choose from every two element set (and thus does not opt for the status-quo).

A weaker version of the above property is the following:

Weak Base Domain Property: A choice correspondence C is said to satisfy the Weak Base Domain property (WBD) if for all $A \in P(X)$, $x \in C(A)$ and $y \in A$ with $x \neq y$: $\{x,y\} \in$ *dom*(C).

Three axioms that are well known in the choice theory literature are the following.

Chernoff Axiom: A choice correspondence C is said to satisfy Chernoff Axiom (CA) if for all $A \in P(X)$ and $B \in$ *dom*(C): $[B \subset A]$ implies $[C(A) \cap B \subset C(B)]$.

Expansion (E): A choice correspondence C is said to satisfy Expansion (E) if for all $A,B \in P(X)$ with $A \cup B \in$ *dom*(C): $C(A) \cap C(B) \subset C(A \cup B)$.

3. Partial Rational Choice and Weak Base Domain Property

Proposition 1: Let C be a choice correspondence satisfying WBD. Then C is partially rational if and only if C satisfies CA and E.

Proof: Let C be a choice correspondence satisfying WBD.
(a) Suppose C satisfies CA and E.
Let $A \in$ *dom*(C).
Let $x \in C(A)$. Then xR_Cy for all $y \in A$. Thus $C(A) \subset \{x \in A: xR_Cy \text{ for all } y \in A\}$.
On the other hand if $x \in A$ and xR_Cy for all $y \in A$, then for all $y \in A$ there exists a set $A_y \in$ *dom*(C) such that $y \in A_y$ and $x \in C(A_y)$.
By WBD, $\{x,y\} \in$ *dom*(C) for all $y \in A$.
By CA applied to A_y and $\{x,y\}$ we get that $x \in C(\{x,y\})$ for all $y \in A$.
By E and since $A = \bigcup_{y \in A} \{x, y\}$ we get $x \in \bigcap_{y \in A} C(\{x, y\}) \subset C(A)$.
Thus $\{x \in A: xR_Cy \text{ for all } y \in A\} \subset C(A)$.
Combining the two inclusions we get that $C(A) = \{x \in A: xR_Cy \text{ for all } y \in A\}$.
Clearly R_C is reflexive.

Thus C is partially rational with R_C being a partial rationalization of C.
(b) In the other direction, suppose C is partially rational.
Thus there exists a binary relation R on X satisfying the following:
(1) R is *reflexive*: For all $x\in X$, xRx;
(2) For all $A\in dom(C)$ and $x\in A$: $[x\in C(A)]$ if and only if [xRy for all $y\in A$].
Let $A\in P(X)$ and $B\in dom(C)$, $B \subset A$ and $x\in C(A)\cap B$.
Thus $A\in dom(C)$ and xRy for all $y\in A$.
Hence xRy for all $y\in B$.
Since C is partially rational with R being a partial rationalization, in view of $B\in dom(C)$ we get $x\in C(B)$.
Thus C satisfies CA.
Now let $A,B\in P(X)$ with $A\cup B\in dom(C)$ and $x\in C(A)\cap C(B)$. Thus $A,B\in dom(C)$.
If $y\in A$ then since $x\in C(A)$ we get xRy. On the other hand if $y\in B$ then since $x\in C(B)$ we get xRy. Thus $x\in C(A\cup B)$ and $C(A)\cap C(B) \subset C(A\cup B)$.
Thus C satisfies E. Q.E.D.

While proving the above proposition we established the following result:
Let C be a choice correspondence satisfying WBD. If C satisfies CA and E, then R_C is a partial rationalization of C.

An immediate corollary of Proposition 1 is the following:

Corollary of Proposition 1: Let C be a choice correspondence satisfying BD. Then C is rational if and only if C satisfies CA and E.

A binary relation R on X is said to be *transitive* if for all $x,y,z\in X$: [xRy & yRz] implies [xRz].

A choice correspondence C is said to be (partially) *transitively rational* if there exists a (partial) rationalization R of C that is transitive.

We now provide an example of a choice correspondence that satisfies BD, CA and E but is not transitively rational.

Example 1: Let $X = \{x,y,z,u\}$; $C(\{a\}) = \{a\}$ for all $a\in X$, $C(A\cup\{u\}) = \{u\}$ for all non-empty subsets A of $\{x,y,z\}$, $C(\{x,y\}) = \{x\}$, $C(\{y,z\}) = \{y\}$, $C(z,x\}) = \{z\}$, $C(\{x,y,z\}) = \phi$. It is easy to see that C satisfies BD, CA and E.
Suppose R is any rationalization of C. Then R is not transitive since we have xRyRz but *not* xRz.

4. Partial Almost Transitive and Almost Quasi-Transitive Rationality

Given a binary relation R on X let P(R) denote the *asymmetric* part of R (i.e. for all $x,y\in X$:xP(R)y if and only if xRy but *not*[yRx]) and I(R) its *symmetric* part (i.e. for all $x,y\in X$:xI(R)y if and only if xRy *and* [yRx]).

A binary relation R on X is said to be *almost transitive* if there does not exist three distinct alternatives $x,y,z \in X$ such that xRy, yRz and zP(R)x.

A choice correspondence C is said to be partially *almost transitive rational* if there exists a partial rationalization R of C that is almost transitive.

Property A: A choice correspondence C is said to satisfy Property A if for all $x, y, z \in X$: $[xR_Cy, yR_Cz$ and $\{x,z\} \in dom(C)]$ implies $[x \in C(\{x,z\})]$.

Proposition 2: Let C be a choice correspondence that satisfies WBD. C is partially almost transitive rational if and only if C satisfies CA, E and Property A.

Proof: Suppose C is a choice correspondence that satisfies WBD, CA, E and Property A. By the first three properties we get that C is partially rational with R_C being a partial rationalization of C.
Let $x,y,z \in X$ with xR_Cy and yR_Cz. Towards a contradiction suppose that $zP(R_C)x$. Thus zR_Cx.
Hence there exists $A \in dom(C)$ such that $z \in C(A)$ and $x \in A$.
By WBD, $\{x,z\} \in dom(C)$.
By Property A we get $x \in C(\{x,z\})$ contradicting $zP(R_C)x$.
Thus R_C is almost transitive rational.
Now suppose C is partially almost transitive rational with R being the necessary partial almost transitive rationalization.
Thus for all $A \in dom(C)$: $C(A) = \{x \in A$: xRy for all $y \in A\}$.
By Proposition 1, C satisfies CA and E. Let us show that C satisfies Property A.
Let $x, y, z \in X$ with xR_Cy, yR_Cz and $\{x,z\} \in dom(C)$. Towards a contradiction suppose that $x \notin C(\{x,z\})$. Thus $C(\{x,z\}) = \{z\}$.
Hence zP(R)x.
xRy, yRz and zP(R)x contradicts the almost transitivity of R.
Thus C satisfies Property A. Q.E.D.

The following corollary of Proposition 2 is easily established.

Corollary of Proposition 2: Let C be a choice correspondence that satisfies BD. C is transitive rational if and only if C satisfies CA, E and Property A.

In Bossert, Sprumont and Suzumura (2006) we can find a stronger version of the Weak Congruence Axiom due to Richter (1966). This stronger version is referred to as T-Congruence.

A choice correspondence C is said to satisfy **T-Congruence** if for all $x,y,z \in X$ and $A \in dom(C)$: $[xR_Cy, yR_Cz, x \in A$ and $z \in C(A)]$ implies $[x \in C(A)]$.
The special case where y = z corresponds to the definition of Weak Congruence.

Bossert, Sprumont and Suzumura (2006) show that provided a choice correspondence satisfies BD it is transitive rational if and only if it satisfies T-Congruence. On the other hand, if we merely assume that a choice correspondence satisfies WBD then we cannot obtain such a strong result. What we can show is the following.

Proposition 3: Let C be a choice function that satisfies WBD. If C satisfies T-Congruence then it is almost transitively rational. The converse is however not true.

Proof: Let C satisfy WBD and T-Congruence. Let $A \in dom(C)$.
If $x \in C(A)$ then xR_Cy for all $y \in A$. Thus $C(A) \subset \{x \in A$: xR_Cy for all $y \in A\}$.
Now suppose $x \in A$ and xR_Cy for all $y \in A$.
Since for all $y \in A$ we have yR_Cy, it follows by T-Congruence that $x \in C(A)$.
Thus $\{x \in A$: xR_Cy for all $y \in A\} \subset C(A)$.
Combining the two inclusions we get $C(A) = \{x \in A$: xR_Cy for all $y \in A\}$.
Clearly R_C is reflexive. Now let us show that R_C is almost transitive.
Let $x,y,z \in X$ with xR_Cy and yR_Cz. Towards a contradiction suppose that $zP(R_C)x$.
Then there exists $A \in dom(C)$ such that $z \in C(A)$ and $x \in A \backslash C(A)$.
However [xR_Cy and yR_Cz, $A \in dom(C)$, $z \in C(A)$ and $x \in A$] implies by T-Congruence that [$x \in C(A)$], contradicting [$x \in A \backslash C(A)$].
Thus *not* $zP(R_C)x$ and hence R_C is almost transitive.
Thus C is partially almost transitive rational.
To show that the converse is not true let $X = \{x,y,z\}$. Let $C(\{x,y\}) = \{x,y\}$, $C(\{y,z\}) = \{y,z\}$, $C(\{x,z\}) = \phi$ and $C(X) = \{y\}$. Clearly C is partially almost transitive rational with R^C being the necessary partial almost transitive rationalization. However, C does not satisfy T-congruence since xR_Cy, yR_Cy, $y \in C(X)$ and $x \in X \backslash C(X)$. Q.E.D.

A binary relation R on X is said to be *quasi-transitive* if given $x,y,z \in X$: [$xP(R)y$ and $yP(R)z$] implies [zRx].
A binary relation R on X is said to be *almost quasi-transitive* if there does not exist three distinct alternatives $x,y,z \in X$ such that $xP(R)y$, $yP(R)z$ and zRx.

It is easy to see that a binary relation that is **almost transitive** is also **almost quasi-transitive**, though the converse need not be true.

A choice correspondence C is said to be partially *almost quasi-transitive rational* if there exists a partial rationalization R of C that is almost quasi-transitive.

Property B: A choice correspondence C is said to satisfy Property B if for all $x, y, z \in X$: [$\{x\} = C(\{x,y\})$ & $\{y\} = C(\{y,z\})$] implies [*not* zR_Cx].

Proposition 4: Let C be a choice correspondence that satisfies WBD. C is partially almost quasi-transitive rational if and only if C satisfies CA, E and Property B.

Proof: Suppose C is a choice correspondence that satisfies WBD, CA, E and Property B. By the first three properties we get that C is partially rational with R_C being a partial rationalization of C.
Let $x,y,z\in X$ with $xP(R_C)y$, $yP(R_C)z$.
Thus there exists $A,B\in dom(C)$ such that $x\in C(A)$, $y\in C(B)\cap(A\backslash C(A))$ and $z\in B\backslash C(B)$.
By WBD, $\{x,y\}, \{y,z\}\in dom(C)$.
By CA and $xP(R_C)y$ we get $\{x\} = C(\{x,y\})$.
By CA and $yP(R_C)z$ we get $\{y\} = C(\{y,z\})$.
Thus by Property B we get *not* zR_Cx.
Thus R_C is almost quasi-transitive.
Now suppose C is partially almost quasi-transitive rational with R being the necessary partial almost quasi-transitive rationalization.
Thus for all $A\in dom(C)$: $C(A) = \{x\in A: xRy \text{ for all } y\in A\}$.
By Proposition 1, C satisfies CA and E. Let us show that C satisfies Property B.
Let $x, y, z\in X$ with $\{x\}= C(\{x,y\})$ & $\{y\} = C(\{y,z\})$. Towards a contradiction suppose that zR_Cx.
Thus $xP(R)y$ and $yP(R)z$ and zRx, contradicting the almost quasi-transitivity of R.
Hence C satisfies Property B. Q.E.D.

A choice correspondence C is said to be *quasi-transitive rational* if there exists a rationalization R of C that is quasi-transitive.

The following corollary of Proposition 4 is easy to establish.

Corollary of Proposition 4: Let C be a choice correspondence that satisfies BD. C is quasi-transitive rational if and only if C satisfies CA, E and Property B.

6. Conclusion

In this paper we have discussed choice correspondences that may be empty valued. This framework throws up possibilities that are absent in the classical framework of choice theory. This is only a modest beginning that has been made in a more general setting than what choice theory has been mainly concerned with. Much remains to be done if we want to understand fully the scope and nature of the kind of choice correspondences that we discuss here.

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